

age	income	student	credit rating	buys computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

• จำแนก class

$$\text{Info}(D) = \sum_{i=1}^m p_i \log_2(p_i)$$

$$= I(9,5) = \frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right)$$

$$= 0.41 + 0.53$$

$$= 0.94$$

• จำแนก Feature

$$\text{Info}_{\text{age}}(D) = \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j)$$

$$= \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2)$$

$$= \frac{5}{14} \left[-\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) \right] + \frac{4}{14} \left[-\frac{4}{4} \log_2\left(\frac{4}{4}\right) - \frac{0}{4} \log_2\left(\frac{0}{4}\right) \right] + \frac{5}{14} \left[-\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) \right]$$

$$= \frac{5}{14} (0.529 + 0.442) + \frac{5}{14} (0.442 + 0.529)$$

$$= 0.347 + 0.347$$

$$= 0.694 \approx 0.694$$

$$\text{Info}_{\text{income}}(D) = \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j)$$

$$= \frac{4}{14} I(2,2) + \frac{6}{14} I(4,2) + \frac{4}{14} I(3,1)$$

$$= \frac{4}{14} \left[-\frac{2}{4} \log_2\left(\frac{2}{4}\right) - \frac{2}{4} \log_2\left(\frac{2}{4}\right) \right] + \frac{6}{14} \left[-\frac{2}{6} \log_2\left(\frac{2}{6}\right) - \frac{4}{6} \log_2\left(\frac{4}{6}\right) \right] + \frac{4}{14} \left[-\frac{3}{4} \log_2\left(\frac{3}{4}\right) - \frac{1}{4} \log_2\left(\frac{1}{4}\right) \right]$$

$$= \frac{4}{14} (0.5 + 0.5) + \frac{6}{14} (0.39 + 0.528) + \frac{4}{14} (0.31 + 0.5)$$

$$= 0.286 + 0.394 + 0.222$$

$$= 0.912$$

$$\text{Info}_{\text{student}}(D) = \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j)$$

$$= \frac{7}{14} I(3,4) + \frac{7}{14} I(6,1)$$

$$= \frac{7}{14} \left[-\frac{3}{7} \log_2\left(\frac{3}{7}\right) - \frac{4}{7} \log_2\left(\frac{4}{7}\right) \right] + \frac{7}{14} \left[-\frac{6}{7} \log_2\left(\frac{6}{7}\right) - \frac{1}{7} \log_2\left(\frac{1}{7}\right) \right]$$

$$= \frac{7}{14} (0.584 + 0.461) + \frac{7}{14} (0.191 + 0.401)$$

$$= 0.493 + 0.396$$

$$= 0.789$$

$$\text{Info}_{\text{credit}}(D) = \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j)$$

$$= \frac{8}{14} I(6,2) + \frac{6}{14} I(3,3)$$

$$= \frac{8}{14} \left[-\frac{6}{8} \log_2\left(\frac{6}{8}\right) - \frac{2}{8} \log_2\left(\frac{2}{8}\right) \right] + \frac{6}{14} \left[-\frac{3}{6} \log_2\left(\frac{3}{6}\right) - \frac{3}{6} \log_2\left(\frac{3}{6}\right) \right]$$

$$= \frac{8}{14} (0.311 + 0.5) + \frac{6}{14} (0.5 + 0.5)$$

$$= 0.464 + 0.429$$

$$= 0.893$$

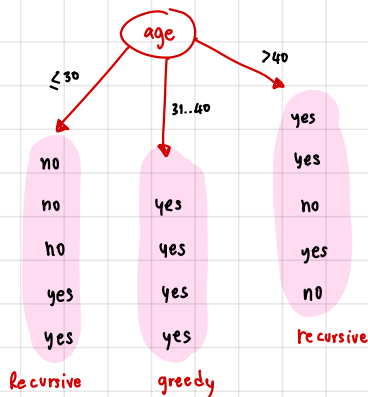
$$* * \text{Gain}(A) = \text{Info}(D) - \text{Info}_A(D)$$

$$\text{Gain}(\text{age}) = 0.94 - 0.694 = 0.241 \rightarrow \text{เลือกเป็น root node}$$

$$\text{Gain}(\text{income}) = 0.94 - 0.912 = 0.028$$

$$\text{Gain}(\text{student}) = 0.94 - 0.789 = 0.151$$

$$\text{Gain}(\text{credit}) = 0.94 - 0.893 = 0.047$$



F1 age ≤ 30

age	income	student	credit_rating	buys_computer
≤30	high	no	fair	no
≤30	high	no	excellent	no
≤30	medium	no	fair	no
≤30	medium	no	excellent	no
≤30	low	yes	fair	yes
≤30	low	yes	excellent	yes
≤30	medium	yes	excellent	yes
≤30	medium	yes	fair	yes
≤30	high	yes	excellent	yes
≤30	high	yes	fair	yes

• คำนวณ class

$$\begin{aligned}
 \text{Info}(D) &= \sum_{i=1}^m p_i \log_2(p_i) \\
 &= I(2,3) = -\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) \\
 &= 0.5288 + 0.4422 \\
 &= 0.971
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}(D)_{\text{income}} &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\
 &= \frac{2}{5} I(0,2) + \frac{2}{5} I(1,1) + \frac{1}{5} I(1,0) \\
 &= \frac{2}{5} \left[-\frac{0}{2} \log_2\left(\frac{0}{2}\right) - \frac{2}{2} \log_2\left(\frac{2}{2}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] \\
 &\quad + \frac{1}{5} \left[-1 \log_2\left(\frac{1}{5}\right) - 0 \log_2(0) \right] \\
 &= \frac{2}{5} (0.5 + 0.5) \\
 &= 0.4
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}(D)_{\text{credit}} &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\
 &= \frac{3}{5} I(1,2) + \frac{2}{5} I(1,1) \\
 &= \frac{3}{5} \left[-\frac{1}{3} \log_2\left(\frac{1}{3}\right) - \frac{2}{3} \log_2\left(\frac{2}{3}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] \\
 &= \frac{3}{5} (0.5288 + 0.3999) + \frac{2}{5} (0.5 + 0.5) \\
 &= 0.4 + 0.551 \\
 &= 0.951
 \end{aligned}$$

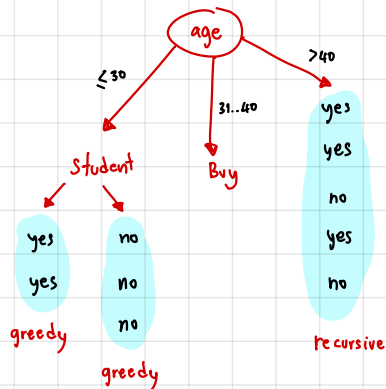
$$\begin{aligned}
 \text{Info}(D)_{\text{student}} &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\
 &= \frac{3}{5} I(0,3) + \frac{2}{5} I(1,0) \\
 &= \frac{3}{5} \left[-\frac{0}{3} \log_2\left(\frac{0}{3}\right) - \frac{3}{3} \log_2\left(\frac{3}{3}\right) \right] + \frac{2}{5} \left[-\frac{2}{2} \log_2\left(\frac{1}{2}\right) - \frac{0}{2} \log_2\left(\frac{0}{2}\right) \right] \\
 &= 0
 \end{aligned}$$

** Gain(A) = Info(D) - Info_A(D)

Gain(income) = 0.971 - 0.4 = 0.571

Gain(student) = 0.971 - 0 = 0.971 → **เลือก**

Gain(credit) = 0.971 - 0.951 = 0.020



F₂ age > 40

age	income	student	credit rating	buys computer
<30	high	no	fair	no
<30	high	no	excellent	no
31..40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
<30	low	yes	excellent	yes
<30	medium	no	fair	no
<30	medium	yes	fair	yes
>40	medium	yes	fair	yes
<30	medium	yes	excellent	yes
31..40	medium	yes	excellent	yes
<30	high	yes	fair	yes
>40	medium	no	excellent	no

• จำแนก class

$$\begin{aligned}
 \text{Info}(D) &= \sum_{i=1}^m p_i \log_2(p_i) \\
 &= I(3,2) = -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) \\
 &= 0.4422 + 0.5288 \\
 &= 0.971
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}_{\text{credit}}(D) &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\
 &= \frac{3}{5} I(3,0) + \frac{2}{5} I(0,2) \\
 &= \frac{3}{5} \left[-\frac{3}{3} \log_2\left(\frac{3}{3}\right) - \frac{0}{3} \log_2\left(\frac{0}{3}\right) \right] + \frac{2}{5} \left[-\frac{0}{2} \log_2\left(\frac{0}{2}\right) - \frac{2}{2} \log_2\left(\frac{2}{2}\right) \right] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}_{\text{income}}(D) &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\
 &= \frac{3}{5} I(2,1) + \frac{2}{5} I(1,1) \\
 &= \frac{3}{5} \left[-\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] \\
 &= 0.551 + 0.4 \\
 &= 0.951
 \end{aligned}$$

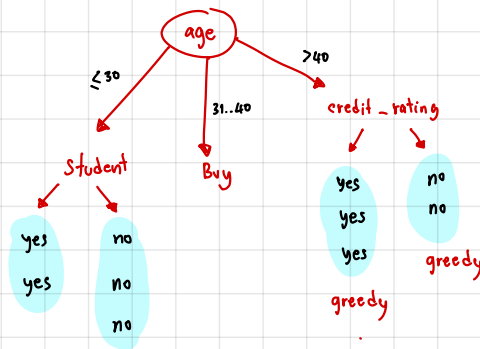
$$\begin{aligned}
 \text{Info}_{\text{student}}(D) &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\
 &= \frac{3}{5} I(1,1) + \frac{2}{5} I(2,1) \\
 &= \frac{3}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] + \frac{2}{5} \left[-\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) \right] \\
 &= 0.4 + 0.551 \\
 &= 0.951
 \end{aligned}$$

** Gain(A) = Info(D) - Info_A(D)

Gain(income) = 0.971 - 0.951 = 0.2

Gain(student) = 0.971 - 0.951 = 0.2

Gain(credit) = 0.971 - 0 = 0.9710 → ดีกว่า



Decision Tree

