1. Sol: Think of 324 passengers as 324 Bernoulli trials, and each passenger show up as the "Success" event.

Define the the number of those 324 passengers show up as X, then

$$X \sim Bin(324, 0.9).$$

So P("the flight is overbooked")

$= P(X > 300)$.

Since $np = 324 \cdot 0.9 = 291.6 > 5$

$npq = 324 \cdot 0.9 \cdot 0.1 = 29.2 > 5$,

we could use normal approximation $N(291.6, 29.2)$ to calculate this Binomial tail prob.

Continuity correction

$$\text{Hence } P(X > 300) = P(X \geq 301) \approx P\left(\frac{X - 291.6}{\sqrt{29.2}} > \frac{301 - 0.5 - 291.6}{\sqrt{29.2}}\right)$$

(where $Z \sim N(0,1)$, and $\Phi(x)$ is the standard Normal c.d.f.)

$$= P(Z > 1.65)$$

$$= 1 - \Phi(1.65)$$

$$\approx 0.0495.$$

$\square$

2. Sol: Let $X = int(T)$.

Since T takes values in $(0, +\infty)$, X takes values in $\{0, 1, 2, \cdots\}$, and for any $k = 0, 1, 2, \cdots$,

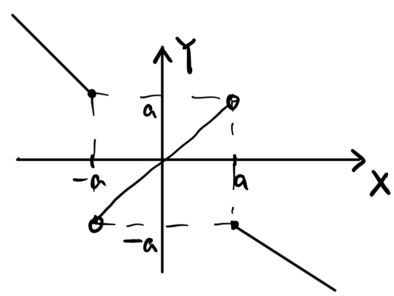
$$
\begin{aligned}
P(X=k) &= P(k \le T < k+1) \\
&= P(T<k+1) - P(T<k) \\
&= [1-e^{-\lambda(k+1)}] - [1-e^{-\lambda k}] \\
&= e^{-\lambda k} - e^{-\lambda(k+1)} \\
&= e^{-\lambda k}(1-e^{-\lambda}) \\
&= (e^{-\lambda})^k \cdot (1-e^{-\lambda}) \\
&\overset{\text{def}}{=} q^k p
\end{aligned}
$$

where $p = 1-e^{-\lambda}$, $q = 1-p = e^{-\lambda}$.

So $X \sim \text{Geo}(1-e^{-\lambda})$ on $\{0,1,2,\dots\}$. $\square$

3. Sol:

The graph of Y v.s. X.



We want to find the distribution of Y by calculating its c.d.f. $F_Y(y)$ in the following 3 cases, respectively.

① If $y \le -a$:

$$
\begin{aligned}
F_Y(y) = P(Y \le y) &= P(-X \le y) \\
&= P(X \ge -y) \\
&= 1 - P(X \le -y) \\
&= 1 - \Phi(-y) \\
&= 1 - [1-\Phi(y)] = \Phi(y)
\end{aligned}
$$

where $\Phi(\cdot)$ is the c.d.f. for $N(0,1)$.

② If $-a < y \le a$,

$$F_Y(y) = P(Y \le y) = P(Y \le -a) + P(-a < Y \le y)$$
$$= \Phi(-a) + P(-a < X \le y)$$

(from case ①, as $-a \le -a$).

$$= \Phi(-a) + \Phi(y) - \Phi(-a)$$
$$= \Phi(y).$$

③ If $y > a$,

$$F_Y(y) = P(Y \le y) = P(Y \le a) + P(a < Y \le y)$$
$$= \Phi(a) + P(a < -X \le y)$$

(from case ②, as $a \le a$)

$$= \Phi(a) + P(-y \le X < -a)$$
$$= \Phi(a) + \Phi(-a) - \Phi(-y)$$
$$= \Phi(a) + [1 - \Phi(a)] - [1 - \Phi(y)]$$
$$= \Phi(a) + 1 - \Phi(a) - 1 + \Phi(y)$$
$$= \Phi(y)$$

Hence $F_Y(y) = \Phi(y)$ for $y \in (-\infty, +\infty)$.

Therefore $Y \sim N(0,1)$ too! $\square$