1. Soln: Let T denote the number of rolls required to produce the first head.

Then it's obviously that $T \sim Geo(p)$.

So $P(\text{"A tosses the first head"})$

$$
\begin{aligned}
&= P(T \text{ is odd}) \\
&= P(T=1) + P(T=3) + P(T=5) + \dots \\
&= p + q^2 p + q^4 p + \dots \\
&= \frac{p}{1-q^2} \\
&= \frac{p}{(1+q)(1-q)} \\
&= \frac{p}{(1+q)p} \\
&= \frac{1}{1+q} \quad \left(> \frac{1}{2} \quad \text{if } q<1.\right)
\end{aligned}
$$

A has more chance to toss the first head!!! $\square$

2 Soln: a). $X \sim Negb(b, \frac{b}{w+b})$, and its pmf is

$$P(X=k) = \binom{k-1}{b-1}\left(\frac{b}{w+b}\right)^b\left(\frac{w}{b+w}\right)^{k-b} \text{ for } k \geq b. \quad \square$$

b) When drawing without replacement, X could only take value from b to $b+w$.

For $k = b, b+1, \dots, b+w$,

$P(X=k) = P(b-1 \text{ blacks in first } k-1 \text{ draws}) \times$

$P(k\text{th draw is black} \mid b-1 \text{ blacks in first } k-1 \text{ draws})$

pmf of HyperGeo(k-1, b, b+w) at b-1

$$= \frac{\binom{b}{b-1}\cdot\binom{w}{k-b}}{\binom{b+w}{k-1}} \cdot \frac{b-(b-1)}{b+w-k+1}$$

$$= \frac{b \cdot \frac{w!}{(k-b)!(w-k+b)!}}{\frac{(b+w)!}{(k-1)!(b+w-k+1)!}} \cdot \frac{1}{b+w-k+1}$$

$$= \frac{b\,w!\,(k-1)!}{(k-b)!\,(b+w)!} \qquad \square$$

3. Sol: Two extreme cases:

If the first n trials are all successes (or failures), then $V_n = n$.

If the first $2(n-1)$ trials are $(n-1)$ successes and $(n-1)$ failures, then $V_n = 2(n-1)+1 = 2n-1$.

So V_n takes values in $\{n, n+1, \dots, 2n-1\}$.

For $k = n, n+1, \dots, 2n-1$,

$\{V_n = k\} = \{V_n = k$, the k-th trial is success$\} \cup \{V_n = k$, the k-th trial is failure$\}$.

and these 2 events are mutually exclusive.

$P($"$V_n = k$, the k-th trial is success"$)$

$= P($"exactly $n-1$ successes in first $k-1$ trials, the kth trial is success"$)$

$= P($"exactly $n-1$ successes in first $k-1$ trials"$) \cdot P($"the kth trial is success"$)$

$$= \binom{k-1}{n-1} p^{n-1} q^{k-n} \cdot p$$

$$= \binom{k-1}{n-1} p^{n} q^{k-n}$$

It's exactly the pmf of $NegB(n,p)$ at k.

By symmetry, we could get

$$P(\text{"}V_n = k, \text{ the } k\text{-th trial is failure"}) = \binom{k-1}{n-1} q^n p^{k-n}$$

Therefore, for $k = n, n+1, \dots, 2n-1$,

$$P(V_n = k) = \binom{k-1}{n-1} p^n q^{k-n} + \binom{k-1}{n-1} q^n p^{k-n}$$

$$= \binom{k-1}{n-1} \left(p^n q^{k-n} + q^n p^{k-n} \right) \qquad \square$$