Problem set 4 Yancen Dong

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Question 1:

$P(S)=q \quad P(R)=1-q$

(a) $E[u(\text{bring})] = 6(1-q) + 9q = 6+3q$

$E[u(\text{not bring})] = 0 + 10q = 10q$

$6+3q > 10q$
$6 > 7q$
$\frac{6}{7} > q$

only when $E(u)$ of bringing umbrella is greater that of not bringing, I will bring the umbrella.

$\therefore$ only when $q < \frac{6}{7}$, $E(u(\text{bring}) > E(u(\text{not bring})]$.

(b) $P(s|S)=0.7 \quad P(r|S)=0.3$
$P(r|R)=0.7 \quad P(s|R)=0.3$

$$P(S|ss) = \frac{P(s|S)\cdot P(s|S)\cdot P(S)}{P(s|S)^2\cdot P(S) + P(s|R)^2\cdot P(R)}$$

$$= \frac{0.7^2 q}{0.7^2 q + 0.3^2(1-q)} = \frac{0.49q}{0.49q + 0.09(1-q)}$$

(C) if $P(S)=0.6 \quad P(R)=1-0.6=0.4$

$$\therefore P(S|ss) = \frac{0.49\times0.6}{0.49\times0.6+0.09\times0.4} = \frac{0.294}{0.294+0.036} = 89.09\%$$

So, will not bring umbrella because the $P(S|ss)$ is 89.09%.

(d) Because aware of bias

$\therefore$ Sophisticated, true state is S

$P(\tilde{s}|S) = 0.7+0.3\times0.2 = 0.76$

$P(\tilde{s}|R) = 0.3+0.7\times0.2 = 0.44$

$$P(S|\tilde{s}\tilde{s}) = \frac{P(\tilde{s}|S)\cdot P(\tilde{s}|S)\cdot P(S)}{P(\tilde{s}|S)^2\cdot P(S) + P(\tilde{s}|R)^2\cdot P(R)}$$

$$= \frac{0.76^2\times0.6}{0.76^2\times0.6+0.44^2\times0.4} = \frac{0.34656}{0.34656+0.07744} = 81.736\%$$

$\because 81.7\% < 89.09\%$, you will bring the umbrella because you're less certain about the prob of sunny day.

Question 2

(a) $P(5\text{ cold days}) = 0.1^5 = 0.00001$

(b) $P(\text{observed}|F) = 0.2^3\times0.7^2 = 0.00392$

$P(O|W) = 0.7^3\times0.3^2 = 0.03087$

$$P(F|O) = \frac{P(O|F)\cdot P(F)}{P(O|F)\cdot P(F)+P(O|W)\cdot P(W)} = \frac{0.00392\times0.9}{0.00392\times0.9+0.03087\times0.1}$$

$$= \frac{0.003528}{0.003528+0.003087} = 53\%$$

(C) $P(O|F) = \frac{2}{10}\times\frac{7}{9}\times\frac{1}{8}\times\frac{0}{7}\times\frac{6}{6} = 0$

$P(O|W) = \frac{7}{10}\times\frac{3}{9}\times\frac{6}{8}\times\frac{5}{7}\times\frac{2}{6} = \frac{1260}{30240} = \frac{1}{24}$

$$P(F|O) = \frac{P(O|F)\cdot P(F)}{P(O|F)\cdot P(F)+P(O|W)\cdot P(W)} = \frac{0\times0.9}{0\times0.9+\frac{1}{24}\times0.1} = \frac{0}{0.9+\frac{1}{240}} = 0$$

$\therefore$ the probability of actually in fall is 0

Question 3

(a) $$P(H|\text{says }H) = \frac{P(\text{says}H|H)\cdot P(H)}{P(\text{say}H|H)\cdot P(H) + P(\text{says}H|L)\cdot P(L)} = \frac{1\times0.2}{1\times0.2+0.8\times0.5} = 33.3\%$$

(b) $$P(H|\text{says}H) = \frac{P(\text{says}H|H)\cdot P(H)}{P(\text{say}H|H)\cdot P(H)+P(\text{say}H|L)\cdot P(L)} = \frac{0.95\times0.2}{0.95\times0.2+0.8\times0.05} = 82.6\%$$

(C) From (a), since Jane is aware of teacher's bias, so the $P(H|\text{say}L)$ is high, so
① she tend to not believe in teacher's speaking and not study if payoff is low.
From (b), since Jane does not think teacher is biased so her belief of $P(H/\text{say}L)$ is 0.05. Thus, she tend to believe in teacher that completing education is good.

This proves that the policy targeting high graduation rate intends to tell students that the teachers and school are not biased, which means the sayings from school's teachers is believable. Thus, the $P(H|\text{say}H)$ will be high and $P(H|\text{say}L)$ will be low; therefore, the posterior of students of $P(H|\text{says}H)$ is higher, indicating they will tend to return school and the graduation rate will be high.

② I think correct beliefs about bias do not always lead to improved outcomes.
For example, when teachers are biased in the outcome of returning to school, they always say returning to high school's payoff is high no matter it is truly high or low.
△ If students know the teachers are biased, they will have lower probability belief of $P(H|\text{says}H)$. Thus, they tend to not return to study, which is not good for the student's education and the school's graduation rate.
But, if students do not know the teachers are biased, they tend to have higher $P(H|\text{say}H)$. Thus, they tend to back to school, which contributes to better education for students and higher graduation rate for schools.

So, correct beliefs about the bias of an source do not always lead to improved outcomes.