

Ex.



\$1



\$0

Event  $E$  = we originally picked the \$1 bag

$$P(E) = P(\bar{E}) = \frac{1}{2}$$

$R$  = ball is red

$$P(R|E) = \frac{1}{2}$$

$$P(\bar{R}|E) = \frac{1}{2}$$

$$P(R|\bar{E}) = 0$$

$$P(\bar{R}|\bar{E}) = 1$$

$$\begin{aligned} P(R) &\stackrel{\text{marginalize}}{=} P(R, E) + P(R, \bar{E}) \\ &\stackrel{\text{conditional}}{=} P(E)P(R|E) + P(\bar{E})P(R|\bar{E}) \\ &= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot 0 = \frac{1}{4} \end{aligned}$$

Switch if  $P(E|\bar{R}) < \frac{1}{2}$ ,  
hold otherwise why?

$$P(E|\bar{R}) = \frac{P(E, \bar{R})}{P(\bar{R})} \stackrel{\text{Bayes}}{=} \frac{P(\bar{R}|E)P(E)}{P(\bar{R})} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{1 - P(R)} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{3}{4}} = \frac{1}{3}$$

If we do not switch, the EXPECTED money we get

$$P(E|\bar{R}) \cdot 1 + P(\bar{E}|\bar{R}) \cdot 0 = \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 0 = \$\frac{1}{3}$$

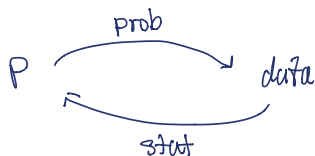
If we switch, the Expected money we get is  $\$ \frac{2}{3}$

In general, assume  $P(E|\bar{R}) = x$ , the <sup>not switch</sup> expected amount =  $x \cdot 1 + (1-x) \cdot 0 = x$

The switch expected amount =  $(1-x) \cdot 1 + x \cdot 0 = 1-x$

Therefore, decision rule, switch if  $x < \frac{1}{2}$

↑↑  
Probability:  
Statistics:  
↓↓



coin head prob  $p \in (0, 1)$ , flip  $n$  times,  $x$  heads observed.

outcomes  $o_1, o_2, \dots, o_n \in \{H, T\}$

$$P(o_1, o_2, \dots, o_n) \stackrel{\text{independence}}{=} P(o_1) \cdot P(o_2) \cdot \dots \cdot P(o_n)$$

$$\underline{\underline{p^x (1-p)^{n-x} \text{ likelihood}}}$$

likelihood function  $L(p) = p^x (1-p)^{n-x}$

One way to estimate  $p$ : Maximum Likelihood Estimate (MLE)

$$\hat{p} = \arg \max_{p \in (0,1)} L(p)$$

↳ arguments  $p$  that achieve  $L(p)$  max