Great idea: Uncertainty modeled by probability

$P(H) + P(T) = 1$

if fair coin, $P(H) = P(T) = \frac{1}{2}$

otherwise $P(H) = p \in [0,1]$, $P(T) = 1-p$

Law of Large numbers

flip N times, outcomes $x_1 \in \{H, T\}, x_2, \cdots, x_N$

$$\mathbb{1}[x_i = H] = \begin{cases} 1 & \text{if } x_i = H \\ 0 & \text{if } x_i = T \end{cases}$$

$$\lim_{N \to \infty} \left(\frac{\sum_{i=1}^{N} \mathbb{1}[x_i = H]}{N} \right) = p$$ The frequency interpretation of probability

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fair: $\left(\frac{1}{6} \cdots\cdots \frac{1}{6}\right)$ probability vector (face 1 — face 6)

loaded: $(P_1 \text{ ---- } P_6)$ $\quad \sum_{j=1}^{6} P_j = 1$, $P_j \in [0, 1]$, $j = 1 \cdots 6$

outcome (which face of die) $x \in \{1, 2, \ldots, 6\}$

$x_1, \ldots, x_N$

$$\lim_{N \to \infty} \left(\frac{\sum_{i=1}^{N} \mathbb{1}[x_i = j]}{N} \right) = P_j, \text{ for } j = 1 \cdots 6$$

indicator function $\mathbb{1}[z] = \begin{cases} 1 & \text{if } z \text{ is True} \\ 0 & \text{if } z \text{ is False} \end{cases}$ where z is a boolean function

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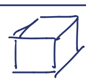
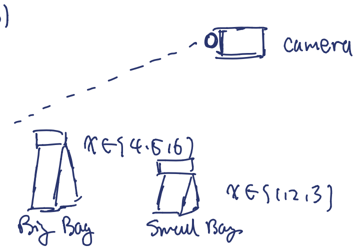
$P(x \in \{2, 4, 6\}) = P(x=2) + P(x=4) + P(x=6)$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$



$\longrightarrow x \in \{1, 2, \cdots, 6\}$

1. Take X cookies
2. if X even
   do AI HW
   else
   do chemistry HW



The camera observes if x is big or small

Q: if Big bag, what's the chance "I submit AI HW"? $\frac{2}{3}$

small bag $\frac{1}{3}$

The subjective degree of belief interpretation of probability

Outcome: $X \in \{1, \ldots, 6\}$

Event: $\underbrace{X \in S}_{\text{boolean}}$ where $S \subseteq \{1, \ldots, 6\}$

$A := x \in \{2, 4, 6\}$ $\quad x$ is Even

$B := x \in \{4, 5, 6\}$ $\quad x$ is Big

$P(A) = \sum_{j \in \{2,4,6\}} P_j = P_2 + P_4 + P_6 = \frac{1}{2}$

$P(B) = P_4 + P_5 + P_6 = \frac{1}{2}$

Joint Probability $P(A, B) \rightarrow$ Both events A and B are true

$$P(A,B) = P(x \in \{2,4,6\} \text{ and } x \in \{4,5,6\})$$
$$= P(x \in [\{2,4,6\} \cap \{4,5,6\}])$$
$$= P(x \in \{4,6\})$$
$$= p_4 + p_6 = \frac{1}{3}$$

Negation (complement)

$$\bar{A} = \neg A = A^c = x \in \{1 \cdots 6\} \setminus A = x \in \{1,3,5\}$$
$$P(\bar{A}) = 1 - P(A)$$

Marginalization:

$$P(A,B) + P(\bar{A},B) = P(B)$$

Conditional Probability

$P(A|B)$ prob of A given B (B observed)

$$:= \frac{P(A,B)}{P(B)} = \frac{P(A,B)}{P(A,B) + P(\bar{A},B)}$$

i.e. probabilitic reasoning in AI
P(prediction | observation)

$P(\text{"1"} | \text{"Big"}) = 0$
$P(\text{"4"} | \text{Big}) = 1$

$P(\text{"1"} | \overline{\text{Big}}) = \frac{1}{3}$

Ex: "Jerry has a big bag" $\rightarrow$ B is true

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{\frac{2}{6}}{\frac{1}{2}} = \frac{2}{3}$$

will do AI HW

Ex. Given that ① 1 in 100 people has a flu $P(F) = \frac{1}{100}$ $P(\bar{F}) = \frac{99}{100}$

② If you have the flu, 9 out of 10 times you have headache

$P(H|F) = \frac{9}{10}$ $P(\bar{H}|F) = \frac{1}{10}$

③ 1 in 10 people have headache $P(H) = \frac{1}{10}$

$$P(H|F) = \frac{P(H,F)}{P(F)}$$

You have a headache, what's the chance you have flu?

$$P(F|H) = \frac{P(F,H)}{P(H)} = \frac{P(H|F) \cdot P(F)}{P(H)} = \frac{\frac{9}{10} \cdot \frac{1}{100}}{\frac{1}{10}} = \frac{9}{100}$$

$$= \frac{P(F,H)}{P(F)}$$

This formula is easy to use
Bayes Rule