A few more review problems on 15.6 and 15.7:

Find the volume of the tetrahedon formed by the plane $2x + 3y + z = 6$ and the coordinate planes by....

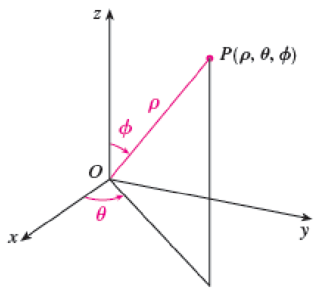
- using the order $dz\ dx\ dy$

- using the order $dx\ dz\ dy$

Evaluate $\iiint_E xy\ dV$ where E is the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 12 - 3x^2 - 3y^2$.

Section 15.8 Triple Integrals in Spherical Coordinates

In the spherical coordinate system, a point P is represented by an ordered triple (ρ, θ, ϕ) where ρ is the distance from the origin to P, θ is the angle made with the positive x-axis in the xy-plane, and ϕ is the angle between the positive z axis and the line segment OP. By convention, we choose $\rho \geq 0$ and $0 \leq \phi \leq \pi$.



The relationship between rectangular and spherical coordinates are

$$x = \rho \sin\phi \cos\theta$$

$$y = \rho \sin\phi \sin\theta$$

$$z = \rho \cos\phi$$

along with the fact that

$$\rho^2 = x^2 + y^2 + z^2$$

Examples:

- Convert $\left(4, \frac{\pi}{4}, \frac{\pi}{6}\right)$ from spherical to rectangular coordinates.

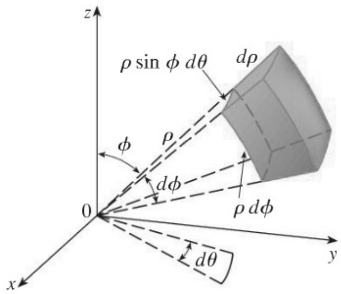
- Convert $\left(-1, \sqrt{3}, 2\right)$ from rectangular to spherical coordinates.

Write each Cartesian equation in spherical coordinates.

$x^2 + y^2 + z^2 = 25$ (a sphere)

$z = \sqrt{x^2 + y^2}$ (a cone)

When dealing with a spherical region, instead of chopping up the solid into boxes, we chop it up into spherical wedges.



While in Cartesian coordinates, $dV = dx\ dy\ dz$, in spherical coordinates $dV = \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta$. So we must now multiply by this factor $\rho^2 \sin\phi$ when switching to spherical coordinates.

(Note: I will show why this is true in the next section when we discuss more general change of coordinates.)

$$\iiint_E f(x,y,z)\, dV = \iiint_E f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho\cos\phi)\, \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta$$

Evaluate $\iiint_E \sqrt{x^2+y^2+z^2}\, dV$ where E is the region between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=9$.

Evaluate $\iiint_E xz\, dV$ where E is the region bounded above by the sphere $x^2+y^2+z^2=4$ and below by the cone $z=\sqrt{x^2+y^2}$.

Evaluate $\displaystyle\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} z\,dz\,dy\,dx$ by converting to spherical coordinates.

Find the volume of the solid that lies within the sphere $x^2+y^2+z^2=4$, above the xy plane, and below the cone $z=\sqrt{3x^2+3y^2}$ in the first octant (i.e., where $x\geq 0$ and $y\geq 0$).