Section 16.8 Stokes' Theorem

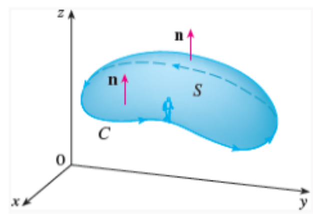
Recall that Green's Theorem gives us a way to calculate a line integral over a simple closed curve C in the xy-plane. For Green's Theorem, we must be in 2D. Stokes' Theorem gives us a way to handle line integrals over closed curves in 3D.

Stokes' Theorem

Let S be an oriented surface that is bounded by a simple, closed curve C with positive (counterclockwise) orientation. Let $\mathbf{F}$ be a vector field that contains S whose components have continuous partial derivatives. Then,

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

If C is positively-oriented, i.e. counterclockwise, when viewed from above, then the surface is oriented upward.



Use Stokes' Theorem to find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle z^2,\ y^2,\ xy \rangle$ and C is the part of the plane $3x + 2y + z = 6$ in the first octant. Orient C to be counterclockwise when viewed from above.

Use Stokes' Theorem to find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle z^2,\ 2x,\ y^2 \rangle$ and C is the curve of intersection of the plane $2y + z = 6$ and the cylinder $x^2 + y^2 = 4$. Orient C to be counterclockwise when viewed from above.

(If time) Use Stokes' Theorem to set up an integral that will find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle xy,\ yz,\ z^2 \rangle$ and C is the curve of intersection of the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. C is oriented counterclockwise when viewed from above.

Section 16.9 The Divergence Theorem

The Divergence Theorem gives us a way to evaluate a surface integral over a **closed surface**.

Divergence Theorem

Let E be a simple solid region whose boundary surface S has positive (outward) orientation. (Note that S is a closed surface since E is a solid region.) Let $\mathbf{F}$ be a vector field that contains E whose component functions have continuous partial derivatives. Then,

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div } \mathbf{F} \, dV$$

Let S be the surface of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane. Use the Divergence Theorem to evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle x^3,\ 2xz^2,\ 3y^2z \rangle$.

Use the Divergence Theorem to evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \left\langle x + z\sin z, 2y + \arccos x, 3z + \tan^2 y \right\rangle$ and S is the sphere $x^2 + y^2 + z^2 = 4$.

Find the flux of the vector field $\mathbf{F} = \langle z\cos y,\ x\sin z,\ xz\rangle$ across S where S is the solid tetrahendron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x + y + z = 2$.

Remember that Green's Theorem, Stokes' Theorem, and the Divergence Theorem all require a CLOSED curve or surface.

- Green's Theorem – Used to evaluate line integrals over CLOSED curves in the xy-plane.
- Stokes' Theorem – Used to evaluate line integrals over CLOSED curves in three dimensions. (These curves are usually viewed as boundaries of surfaces.)
- Divergence Theorem – Used to evaluate surface integrals over CLOSED surfaces.