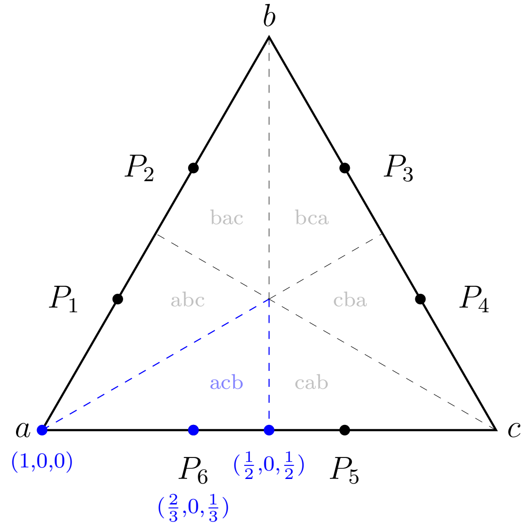
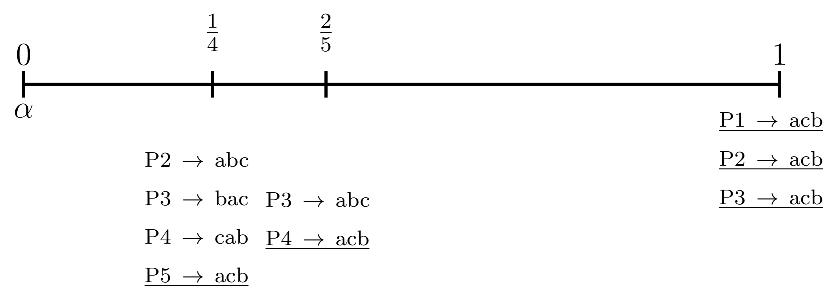
Weeks of 6/23 + 6/30
Kit Levy



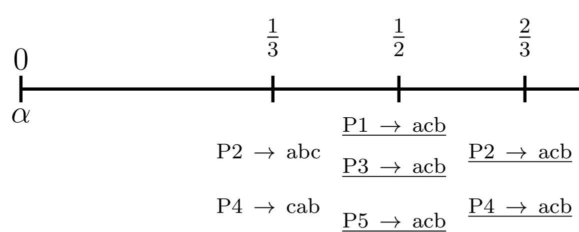
Threshold α values for each change in preferences:

	P1 → **acb**	P2 → abc	**P2** → **acb**	P3 → bac	P3 → abc	P3 → cba	P3 → cab	**P3** → **acb**	P4 → cab	**P4** → **acb**	**P5** → **acb**
$(1,0,0)$	1	$\frac{1}{4}$	1	$\frac{1}{4}$	$\frac{2}{5}$	-	-	1	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{1}{4}$
$(\frac{2}{3},0,\frac{1}{3})$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	-	-	-	-	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$
$(\frac{1}{2},0,\frac{1}{2})$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{4}{7}$	-	-	$\frac{2}{5}$	$\frac{4}{7}$	1	$\frac{2}{5}$	1	1

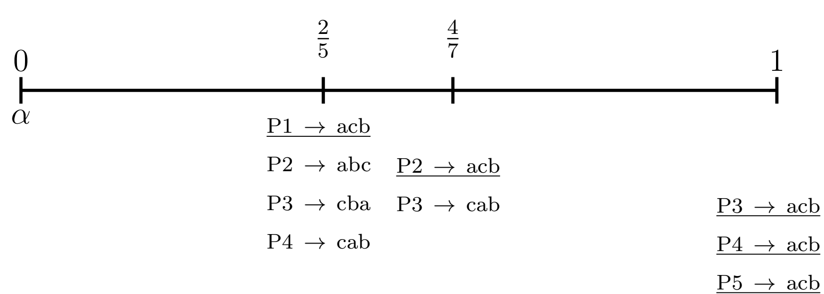
$(1,0,0)$:



$(\frac{2}{3},0,\frac{1}{3})$:



$(\frac{1}{2},0,\frac{1}{2})$:



Voters who rank $a > c > b$ as a function of α:

$w = (1, 0, 0)$: $P_{acb} = P6 + \mathbb{1}\{\alpha = 1\}(P1 + P2 + P3) + \mathbb{1}\{\alpha \geq \frac{2}{5}\}P4 + \mathbb{1}\{\alpha \geq \frac{1}{4}\}P5$

$w = (\frac{2}{3}, 0, \frac{1}{3})$: $P_{acb} = P6 + \mathbb{1}\{\alpha \geq \frac{1}{2}\}(P1 + P3 + P5) + \mathbb{1}\{\alpha \geq \frac{2}{3}\}(P2 + P4)$

$w = (\frac{1}{2}, 0, \frac{1}{2})$: $P_{acb} = P6 + \mathbb{1}\{\alpha \geq \frac{2}{5}\}P1 + \mathbb{1}\{\alpha \geq \frac{4}{7}\}P2 + \mathbb{1}\{\alpha = 1\}(P3 + P4 + P5)$

Choosing w to achieve $a > c > b$ based on known α (ignoring effects of IA):

α	**Best w (highest P_{acb})**
$\alpha < \frac{1}{4}$	None
$\frac{1}{4} \leq \alpha < \frac{2}{5}$	$(1, 0, 0)$
$\frac{2}{5} \leq \alpha < \frac{1}{2}$	$(1, 0, 0)$ if $P4 + P5 > P1$ $(\frac{1}{2}, 0, \frac{1}{2})$ if $P1 > P4 + P5$
$\frac{1}{2} \leq \alpha < \frac{4}{7}$	$(1, 0, 0)$ if $P4 > P1 + P3$ $(\frac{2}{3}, 0, \frac{1}{3})$ if $P1 + P3 > P4$
$\frac{4}{7} \leq \alpha < \frac{2}{3}$	$(1, 0, 0)$ if $P4 > P1 + P3$ and $P4 + P5 > P1 + P2$ $(\frac{2}{3}, 0, \frac{1}{3})$ if $P1 + P3 > P4$ and $P3 + P5 > P2$ $(\frac{1}{2}, 0, \frac{1}{2})$ if $P1 + P2 > P4 + P5$ and $P2 > P3 + P5$
$\frac{2}{3} \leq \alpha < 1$	$(\frac{2}{3}, 0, \frac{1}{3})$
$\alpha = 1$	Any

Movement into and out of IA according to α and w:

Voter	w	α for which voter is in IA	"Time" spent in IA
P1	Any	None	0
P2	$(1, 0, 0)$ $(\frac{2}{3}, 0, \frac{1}{3})$ $(\frac{1}{2}, 0, \frac{1}{2})$	$\frac{1}{4} \leq \alpha < 1$ $\frac{1}{3} \leq \alpha < \frac{2}{3}$ $\frac{2}{5} \leq \alpha < \frac{4}{7}$	$\frac{3}{4}$ $\frac{1}{3}$ $\frac{6}{35}$
P3	$(1, 0, 0)$ $(\frac{2}{3}, 0, \frac{1}{3})$ $(\frac{1}{2}, 0, \frac{1}{2})$	$\frac{1}{4} \leq \alpha < 1$ None $\frac{2}{5} \leq \alpha < 1$	$\frac{3}{4}$ 0 $\frac{3}{5}$
P4	$(1, 0, 0)$ $(\frac{2}{3}, 0, \frac{1}{3})$ $(\frac{1}{2}, 0, \frac{1}{2})$	$\frac{1}{4} \leq \alpha < \frac{2}{5}$ $\frac{1}{3} \leq \alpha < \frac{2}{3}$ $\frac{2}{5} \leq \alpha < 1$	$\frac{3}{20}$ $\frac{1}{3}$ $\frac{3}{5}$
P5	Any	None	0

Plurality votes as a function of α:

$w = (1, 0, 0)$:

$\text{plurality}_a(\alpha) = P6 + P1 + \mathbb{1}\{\alpha \geq \frac{1}{4}\}(P2 + P5) + \mathbb{1}\{\alpha \geq \frac{2}{5}\}(P3 + P4)$

$\text{plurality}_b(\alpha) = \mathbb{1}\{\alpha < \frac{1}{4}\}P2 + \mathbb{1}\{\alpha < \frac{2}{5}\}P3$
$\text{plurality}_c(\alpha) = \mathbb{1}\{\alpha < \frac{2}{5}\}P4 + \mathbb{1}\{\alpha < \frac{1}{4}\}P5$

$w = (\frac{2}{3}, 0, \frac{1}{3})$**:**

$\text{plurality}_a(\alpha) = P1 + P6 + \mathbb{1}\{\alpha \geq \frac{1}{3}\}P2 + \mathbb{1}\{\alpha \geq \frac{1}{2}\}(P3 + P5) + \mathbb{1}\{\alpha \geq \frac{2}{3}\}P4$
$\text{plurality}_b(\alpha) = \mathbb{1}\{\alpha < \frac{1}{3}\}P2 + \mathbb{1}\{\alpha < \frac{1}{2}\}P3$
$\text{plurality}_c(\alpha) = \mathbb{1}\{\alpha < \frac{2}{3}\}P4 + \mathbb{1}\{\alpha < \frac{1}{2}\}P5$

$w = (\frac{1}{2}, 0, \frac{1}{2})$**:**

$\text{plurality}_a(\alpha) = P1 + P6 + \mathbb{1}\{\alpha \geq \frac{2}{5}\}P2 + \mathbb{1}\{\alpha = 1\}(P3 + P4 + P5)$
$\text{plurality}_b(\alpha) = \mathbb{1}\{\alpha < \frac{2}{5}\}(P2 + P3)$
$\text{plurality}_c(\alpha) = \mathbb{1}\{\frac{2}{5} \leq \alpha < 1\}P3 + \mathbb{1}\{\alpha < 1\}(P4 + P5)$

Borda votes as a function of α:
$w = (1, 0, 0)$**:**

$\text{borda}_a(\alpha) = 2P1 + P2 + P5 + 2P6 + \mathbb{1}\{\alpha \geq \frac{1}{4}\}(P2 + P3 + P4 + P5) + \mathbb{1}\{\alpha \geq \frac{2}{5}\}(P3 + P4)$
$\text{borda}_b(\alpha) = \mathbb{1}\{\alpha < 1\}(P1 + P2 + P3) + \mathbb{1}\{\alpha < \frac{1}{4}\}(P2 + P4) + \mathbb{1}\{\alpha < \frac{2}{5}\}P3$
$\text{borda}_c(\alpha) = P4 + P5 + \mathbb{1}\{\alpha = 1\}(P1 + P2 + P3) + \mathbb{1}\{\alpha < \frac{1}{4}\}(P3 + P5) + \mathbb{1}\{\alpha < \frac{2}{5}\}P4$

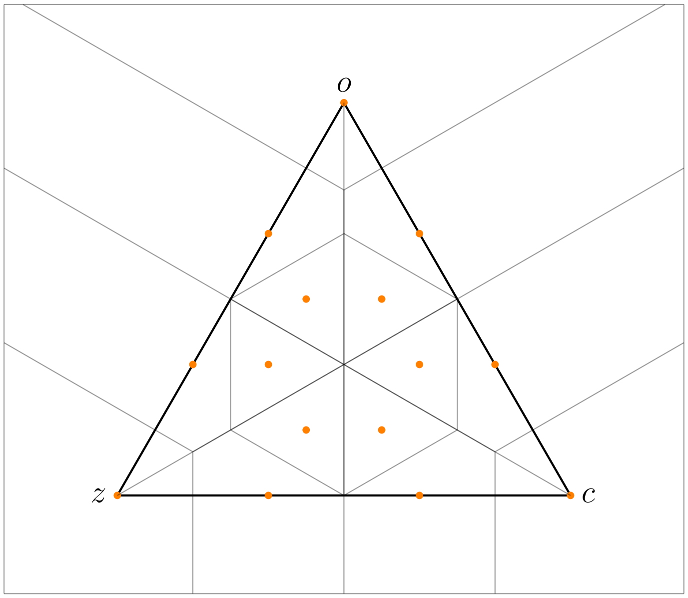
$w = (\frac{2}{3}, 0, \frac{1}{3})$**:**

$\text{borda}_a(\alpha) = 2P1 + P2 + P5 + 2P6 + \mathbb{1}\{\alpha \geq \frac{1}{3}\}(P2 + P4) + \mathbb{1}\{\alpha \geq \frac{1}{2}\}(2P3 + P5) + \mathbb{1}\{\alpha \geq \frac{2}{3}\}P4$
$\text{borda}_b(\alpha) = \mathbb{1}\{\alpha < \frac{1}{2}\}(P1 + 2P3) + \mathbb{1}\{\alpha < \frac{1}{3}\}(P2 + P4) + \mathbb{1}\{\alpha < \frac{2}{3}\}P2$
$\text{borda}_c(\alpha) = P3 + P4 + P5 + \mathbb{1}\{\alpha \geq \frac{1}{2}\}P1 + \mathbb{1}\{\alpha \geq \frac{2}{3}\}P2 + \mathbb{1}\{\alpha < \frac{2}{3}\}P4 + \mathbb{1}\{\alpha < \frac{1}{2}\}P5$

$w = (\frac{1}{2}, 0, \frac{1}{2})$**:**

$\text{borda}_a(\alpha) = 2P1 + P2 + P5 + 2P6 + \mathbb{1}\{\alpha \geq \frac{2}{5}\}(P2 + P4) + \mathbb{1}\{\alpha \geq \frac{4}{7}\}P3 + \mathbb{1}\{\alpha = 1\}(P3 + P4 + P5)$
$\text{borda}_b(\alpha) = \mathbb{1}\{\alpha < \frac{2}{5}\}(P1 + P2 + P3 + P4) + \mathbb{1}\{\alpha < \frac{4}{7}\}(P2 + P3)$
$\text{borda}_c(\alpha) = P3 + P4 + P5 + \mathbb{1}\{\alpha \geq \frac{2}{5}\}P1 + \mathbb{1}\{\alpha \geq \frac{4}{7}\}P2 + \mathbb{1}\{\frac{2}{5} \leq \alpha < 1\}P3 + \mathbb{1}\{\alpha < 1\}(P4 + P5)$

Forced full preferences problem (IRV):



Conditions for z to win under partial prefs, c win under forced full prefs (assuming o eliminated in round 1):

$z_{\text{first choice}} + oz_{\to ozc} + ozc_{\to ozc} > c_{\text{first choice}} + oc_{\to ocz} + ocz_{\to ocz}$
$z_{\text{first choice}} + o_{\to ozc} + oz_{\to ozc} + ozc_{\to ozc} < c_{\text{first choice}} + o_{\to ocz} + oc_{\to ocz} + ocz_{\to ocz}$
Key condition: $o_{\to ocz} - o_{\to ozc} > z_{partial} - c_{partial}$

Example 1:

Initial vote distribution:
$zo_{\to zoc} = \frac{1}{2} - \epsilon$, $co_{\to coz} = \frac{1}{4} + 2\epsilon$, $o_{\to ocz} = \frac{1}{4} - \epsilon$
Round 2 (forced full ballots):
$z_{\text{total}} = z_{\to zoc} = \frac{1}{2} - \epsilon$, $c_{\text{total}} = co_{\to coz} + o_{\to ocz} = \frac{1}{2} + \epsilon$
Round 2 (partial ballots):
$z_{\text{total}} = z_{\to zoc} = \frac{1}{2} - \epsilon$, $c_{\text{total}} = co_{\to coz} = \frac{1}{4} + 2\epsilon$
Exhausted votes $= \frac{1}{4} - \epsilon$

Example 2:

Initial vote distribution:
$zo_{\to zoc} = \frac{1}{3} + 2\epsilon$, $co_{\to coz} = \frac{1}{3}$, $o_{\to ocz} = \frac{1}{6} + \epsilon$, $ozc_{\to ozc} = \frac{1}{6} - 3\epsilon$
Round 2 (forced full ballots):
$z_{\text{total}} = z_{\to zoc} + ozc_{\to ozc} = \frac{1}{2} - \epsilon$, $c_{\text{total}} = co_{\to coz} + o_{\to ocz} = \frac{1}{2} + \epsilon$
Round 2 (partial ballots):
$z_{\text{total}} = z_{\to zoc} + ozc_{\to ozc} = \frac{1}{2} - \epsilon$, $c_{\text{total}} = co_{\to coz} = \frac{1}{3}$
Exhausted votes $= \frac{1}{6} + \epsilon$

Example 3:

Initial vote distribution:
$zo_{\to zoc} = \frac{2}{5}, co_{\to coz} = \frac{2}{5} - \epsilon, o_{\to ocz} = \frac{1}{10} + 2\epsilon, o_{\to ozc} = \frac{1}{10} + \epsilon$