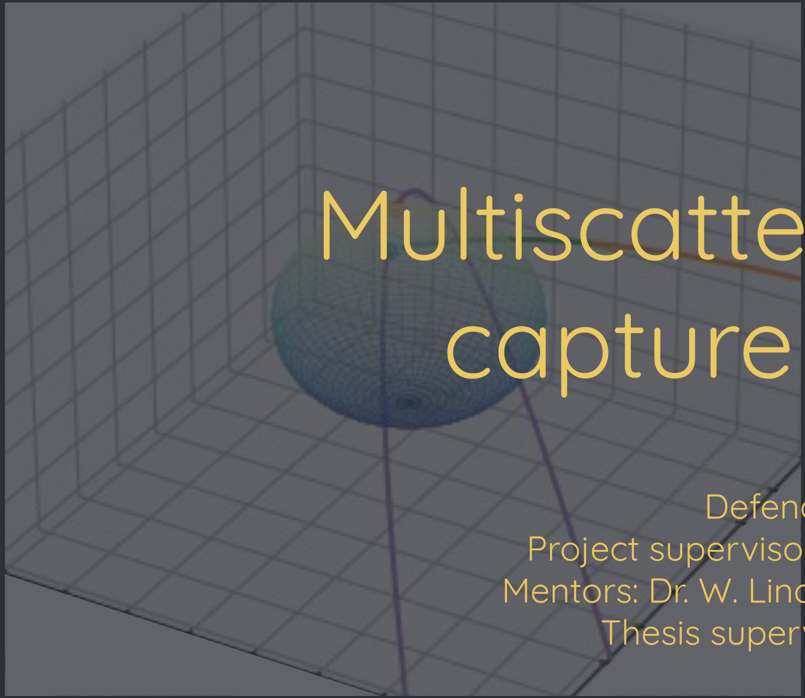
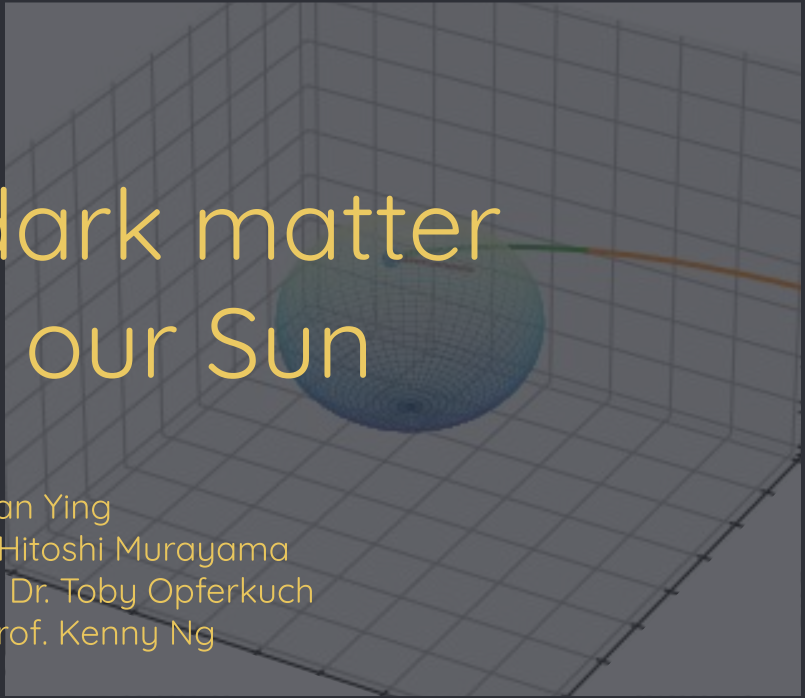
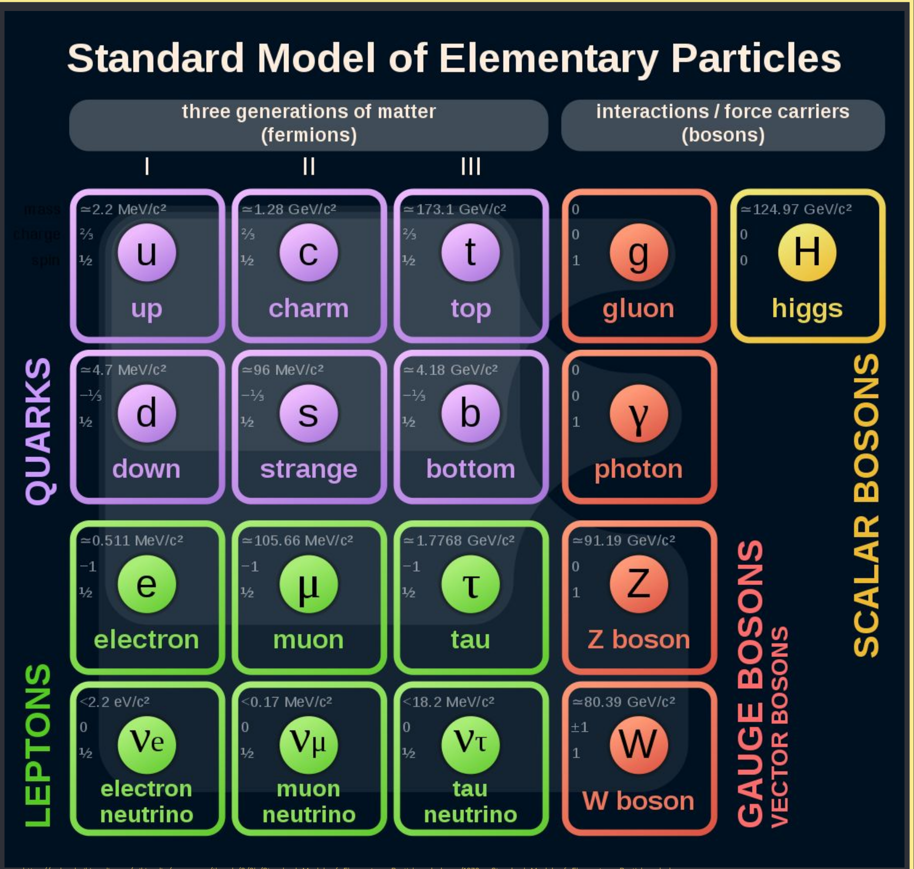



Multiscatter dark matter capture of our Sun

Defender: Chan Ying
Project supervisor: Prof. Hitoshi Murayama
Mentors: Dr. W. Linda Xu & Dr. Toby Opferkuch
Thesis supervisor: Prof. Kenny Ng

Motivation to searching for Dark Matter (DM)

Beyond



https://upload.wikimedia.org/wikipedia/commons/thumb/2/2b/Standard_Model_of_Elementary_Particles_dark.svg/1070px-Standard_Model_of_Elementary_Particles_dark.svg.png

? ψ_{DM}

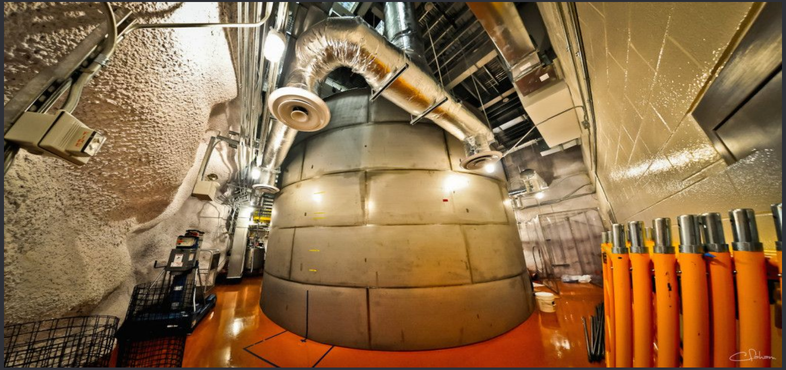
1. DM particle nature?

2. DM interactions?

Direct detection

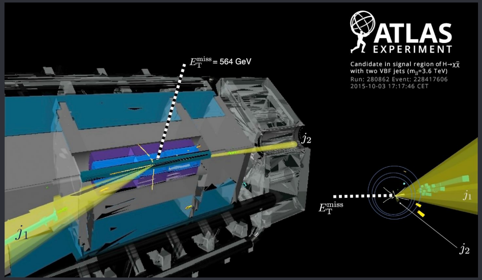
etc.

1. LUX-ZEPLIN (LZ)



https://lz.lbl.gov/wp-content/uploads/sites/6/2014/07/LUX_watertank-1024x587.jpg

2. Collider Production



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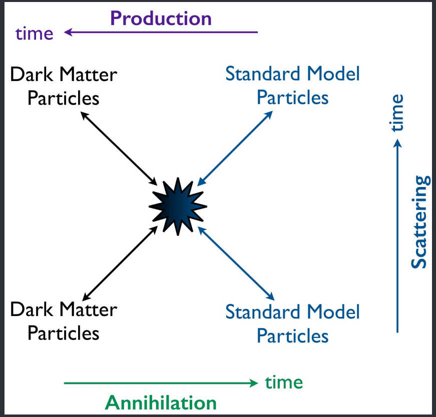


https://particleastro.brown.edu/files/2019/10/Screenshot-from-2019-10-22-17-11-47.png

Theoretical prediction

Lagrangian

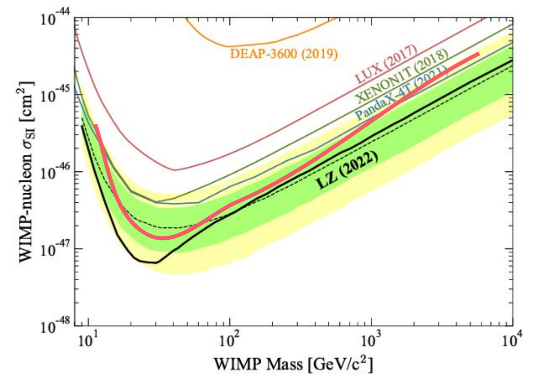
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}D\psi + h.c.$$
$$+i\psi_i y_{ij}\psi_j\phi + h.c.$$
$$+|D_\mu\phi|^2 - V(\phi) + ?$$

J. Aalbers, 2022 Fig 5

Predicted

<u>Monte Carlo simulation</u>

⇒ Covers a range of models



Direct detection

etc.

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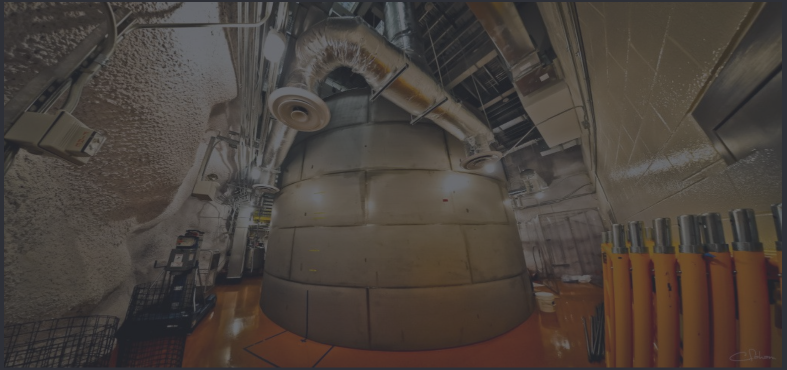

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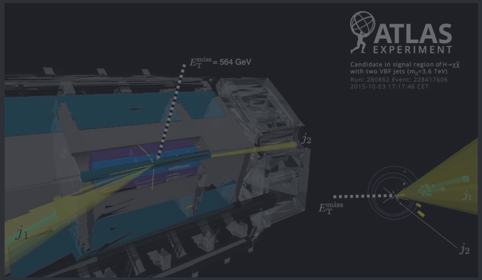

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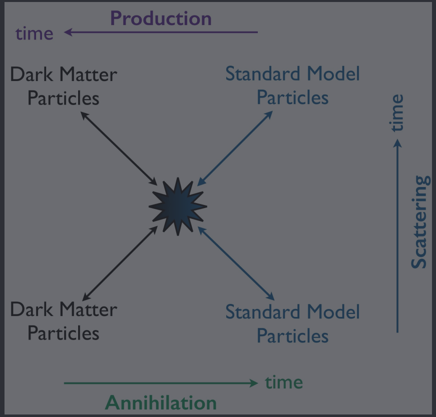

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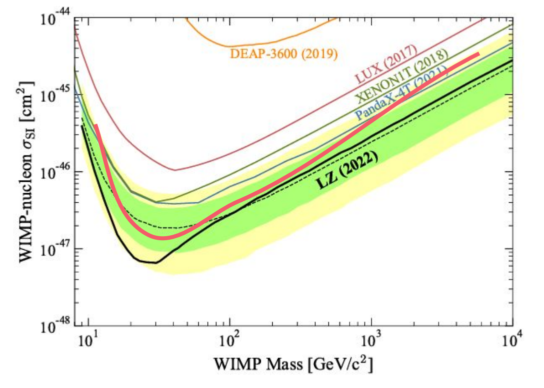
J. Aalbers, 2022 Fig 5

Predicted

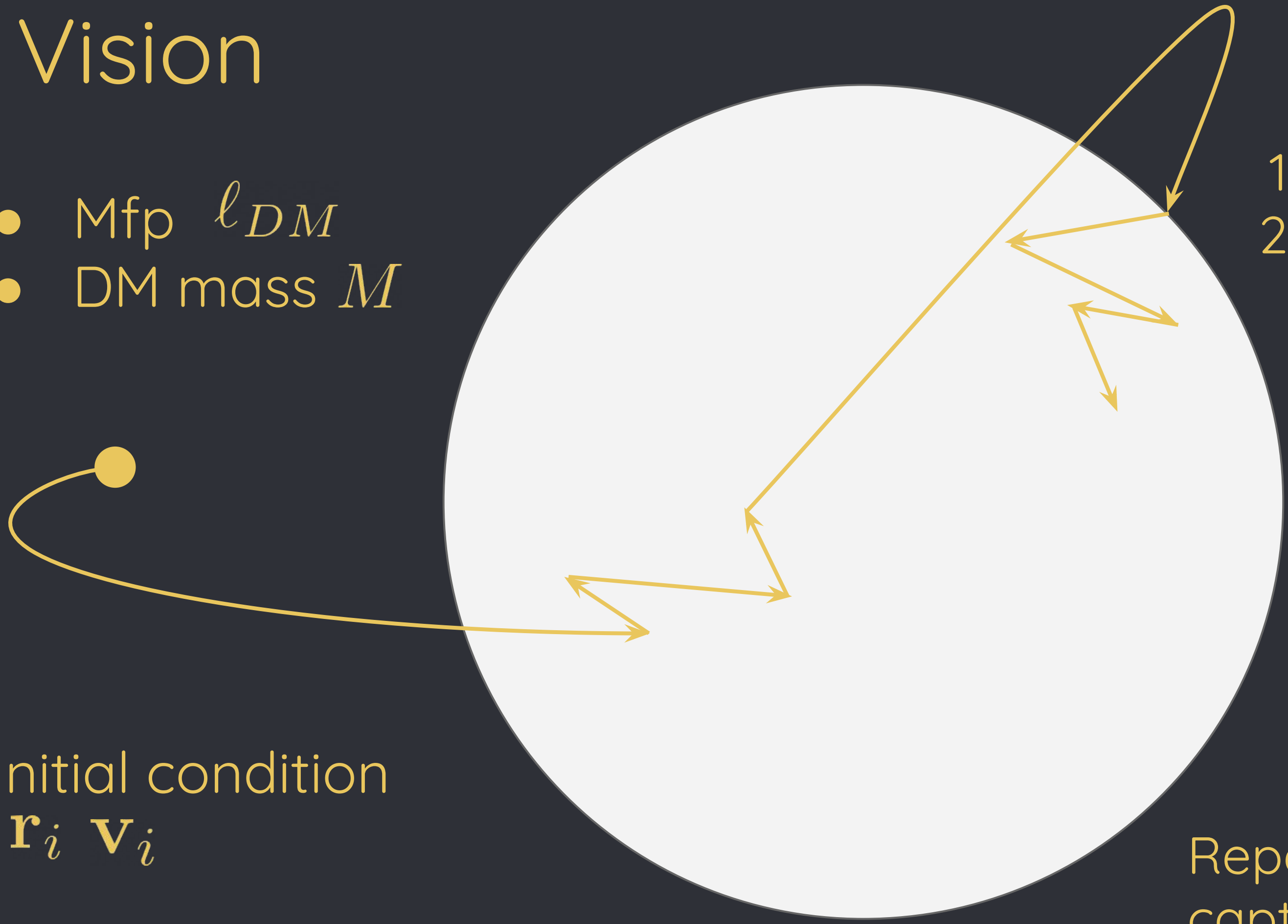


<u>Monte Carlo simulation</u>

⇒ Covers a range of models

Vision

- Mfp ℓ_{DM}
- DM mass M



1. Orbit around Sun
2. Enter Sun
3. Scattering after entrance
4. Leaves if energy < escape energy

Initial condition
$\mathbf{r}_i$ $\mathbf{v}_i$

Repeat 1, 2, 3 until DM is captured

- <u>Simulate successful DM capture event</u>

Content

1. Motivation and recent efforts to DM detection

2. Theoretical overview of DM multiscattering

3. Monte Carlo simulation of DM capture

Content

1. Motivation and recent efforts to DM detection
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Classical scattering

1. Mean free path: ℓ_{DM}

2. Kinematic parameters: $\vec{p}\,,\, E$

Mean free path $\ell_{DM} \approx \frac{\text{length of path}}{\text{number of collisions}}$

stellar mass

$$M = N m_{avg}$$

75% H, 25% He

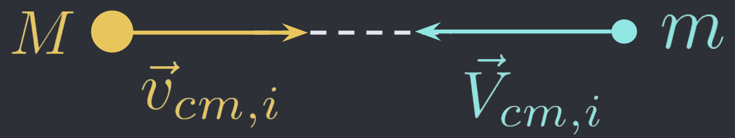
$$n_\odot = \frac{N}{V} = \frac{M}{m_{avg}} \frac{1}{V} = \frac{\rho_\odot}{m_{avg}}$$

$$\ell_{DM} \approx \frac{v\Delta t}{(\sigma_c \times \Delta t) n_\odot}$$

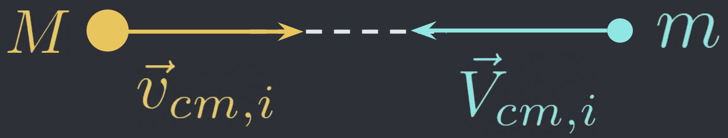
$$\boxed{\ell_{DM} \approx \frac{1}{\sigma_c n_\odot}}$$

Stellar rest frame

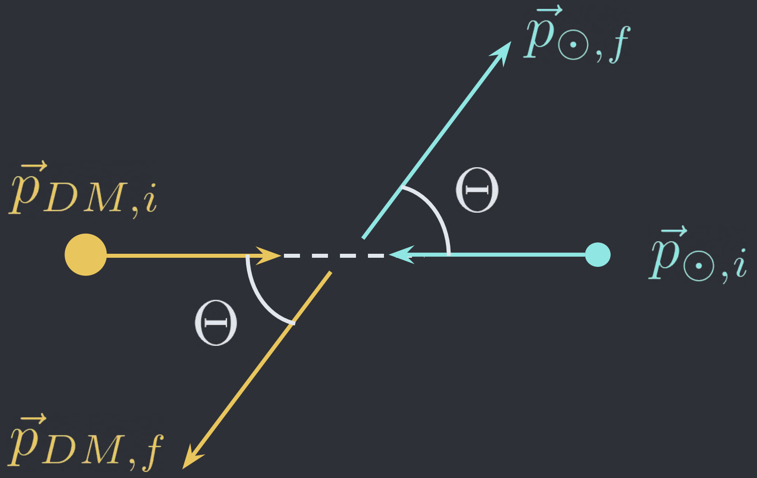
Center-of-mass frame



Center-of-mass frame



Center-of-mass frame $\dot{\mathbf{R}} = 0$



1. Elastic scattering

$$\frac{|\vec{p}_{DM,i}|^2}{2M} + \frac{|\vec{p}_{\odot,i}|^2}{2m} = \frac{|\vec{p}_{DM,f}|^2}{2M} + \frac{|\vec{p}_{\odot,f}|^2}{2m}$$

$$\Rightarrow M(v_{cm,i}^2 - v_{cm,f}^2) = -m(V_{cm,i}^2 - V_{cm,f}^2)$$

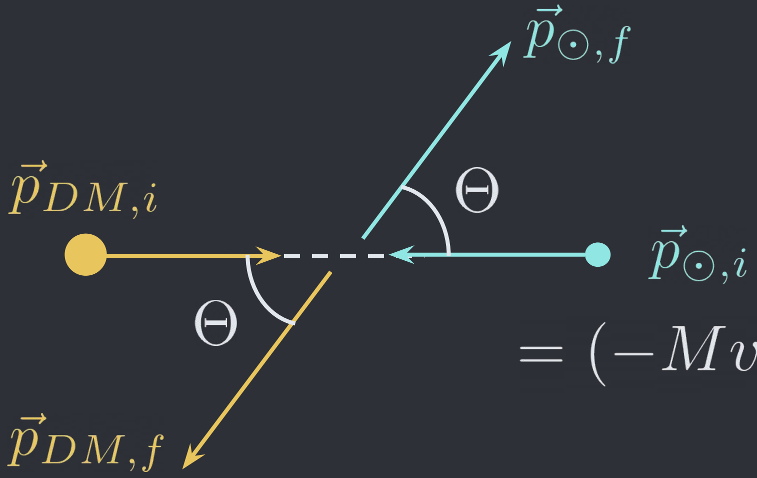
$$\vec{p}_{DM,i} = M v_{cm,i} \left(0 \quad , 1 \quad , 0\right)$$

$$\vec{p}_{DM,f} = M v_{cm,f} \left(0 \quad , -\cos\Theta \quad , -\sin\Theta\right)$$

$$\vec{p}_{\odot,i} = m V_{cm,i} \left(0 \quad , -1 \quad , 0\right)$$

$$\vec{p}_{\odot,f} = m V_{cm,f} \left(0 \quad , \cos\Theta \quad , \sin\Theta\right)$$

Center-of-mass frame $\dot{\mathbf{R}} = 0$



2. Momentum conservation

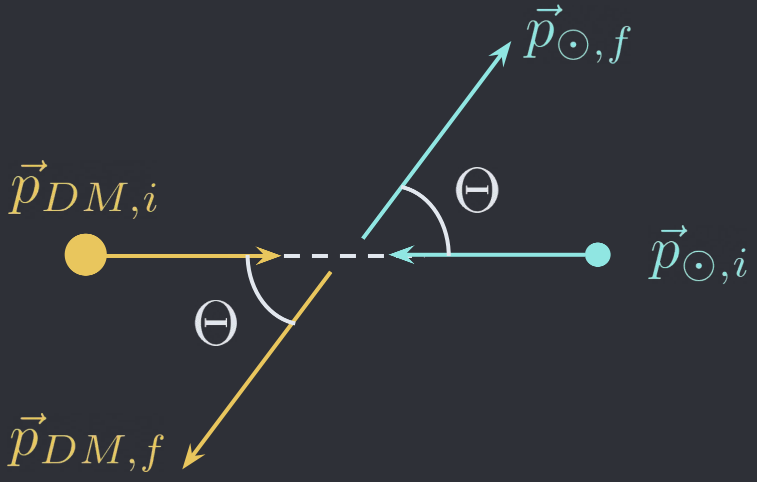
$$(Mv_{cm,i} - mV_{cm,i})\,(0 \quad ,1 \quad ,0)$$
$$= (-Mv_{cm,f} + mV_{cm,f})\,(0 \quad ,\cos\Theta \quad ,\sin\Theta)$$

Initial momentum = 0,

$$\Rightarrow Mv_{cm,f} = mV_{cm,f} \quad \&$$

$$\vec{p}_{DM,i} = Mv_{cm,i}\,(0 \quad ,1 \quad ,0)$$
$$\vec{p}_{DM,f} = Mv_{cm,f}\,(0 \quad ,-\cos\Theta \quad ,-\sin\Theta) \Rightarrow |v_{cm,i}| = |v_{cm,f}|$$
$$\vec{p}_{\odot,i} = mV_{cm,i}\,(0 \quad ,-1 \quad ,0)$$
$$\vec{p}_{\odot,f} = mV_{cm,f}\,(0 \quad ,\cos\Theta \quad ,\sin\Theta)$$

Center-of-mass frame $\dot{\mathbf{R}} = 0$



1. Elastic scattering

$$M(v_{cm,i}^2 - v_{cm,f}^2) = -m(V_{cm,i}^2 - V_{cm,f}^2)$$

2. Momentum conservation

$$|v_{cm,i}| = |v_{cm,f}|$$

$$Mv_{cm,f} = mV_{cm,f}$$

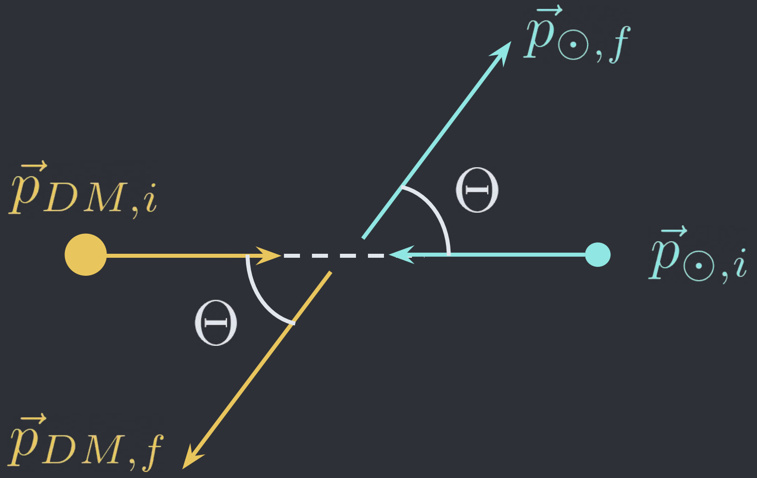
$$\vec{p}_{DM,i} = Mv_{cm,i}\,(0 \quad ,1 \quad ,0)$$

$$\vec{p}_{DM,f} = Mv_{cm,f}\,(0 \quad ,-\cos\Theta \quad ,-\sin\Theta)$$

$$\vec{p}_{\odot,i} = mV_{cm,i}\,(0 \quad ,-1 \quad ,0)$$

$$\vec{p}_{\odot,f} = mV_{cm,f}\,(0 \quad ,\cos\Theta \quad ,\sin\Theta)$$

Center-of-mass frame $\dot{\mathbf{R}} = 0$



1. Elastic scattering
2. Momentum conservation

$$|v_{cm,i}| = |v_{cm,f}|$$

$$M v_{cm,f} = m V_{cm,f}$$

$$E_{cm} = \frac{1}{2} M v_{cm,i}^2 + \frac{1}{2} m V_{cm,i}^2$$

$$\vec{p}_{DM,i} = M v_{cm,i} \left(0 \quad ,1 \quad ,0\right)$$

$$\vec{p}_{DM,f} = M v_{cm,f} \left(0 \quad ,-\cos\Theta \quad ,-\sin\Theta\right)$$

$$\vec{p}_{\odot,i} = m V_{cm,i} \left(0 \quad ,-1 \quad ,0\right)$$

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Stellar rest frame

?

Center-of-mass frame

$$|v_{cm,i}| = |v_{cm,f}|$$

$$Mv_{cm,f} = mV_{cm,f}$$

$$E_{cm} = \frac{1}{2}Mv_{cm,i}^2 + \frac{1}{2}mV_{cm,i}^2$$

Stellar rest frame *

$$E_* = E_{cm} + \frac{1}{2}(M+m)|\dot{\mathbf{R}}|^2$$

$$\dot{\mathbf{R}} = \frac{M\mathbf{v}_i + m\mathbf{V}_i}{M+m}$$

$$\frac{1}{2}Mv_i^2 + \frac{1}{2}mV_i^2 = \frac{1}{2}\left(M + \frac{M^2}{m}\right)v_{cm,i}^2 + \frac{1}{2}(M+m)|\dot{\mathbf{R}}|^2$$

$$v_{cm,i} = \frac{\mu}{M}\sqrt{v_i^2 + V_i^2}$$ w/ $|v_{cm,i}| = |v_{cm,f}|$

Stellar rest frame *

$$\vec{v}_{cm,f} = \frac{\mu}{M}\sqrt{v_i^2 + V_i^2}$$

$$\vec{v}_{cm,f} = \mathbf{v}_f - \dot{\mathbf{R}}$$

$$\dot{\mathbf{R}} = \frac{M\mathbf{v}_i + m\mathbf{V}_i}{M+m}$$

$$\mathbf{v}_f = \frac{\mu}{M}\sqrt{v_i^2 + V_i^2}\,(0 \quad , \cos\Theta \quad , \sin\Theta) + \dot{\mathbf{R}}$$

$$|\mathbf{v}_f|^2 \approx \frac{\mu^2}{M^2}(v_i^2 + V_i^2) + |\dot{\mathbf{R}}|^2$$

Averaging $\Theta \in (0, 2\pi]$

Classical scattering

1. Mean free path: ℓ_{DM}
2. Kinematic parameters: $\vec{p}\,,\,E$

$\Rightarrow\ \Delta E$ for multiple scatterings

Stellar rest frame

$$*\quad |\mathbf{v}_f|^2 \approx |\dot{\mathbf{R}}|^2 + \frac{\mu^2}{M^2}(v_i^2 + V_i^2)$$

$$\Delta E = \frac{1}{2}M\left[\left(\frac{M\mathbf{v}_i + m\mathbf{V}_i}{M+m}\right)^2 + \frac{\mu^2}{M^2}\left(v_i^2 + V_i^2\right) - v_i^2\right]$$

$$\approx -\frac{mM^2}{(m+M)^2}v_i^2 + \frac{m^2 M}{(m+M)^2}V_i^2$$

$\times 2$

Premature Black Hole Death of Population III Stars by Dark Matter

Sebastian A. R. Ellis

Département de Physique Théorique, Université de Genève,
24 quai Ernest Ansermet, 1211 Genève 4, Switzerland
and
Université Paris-Saclay, CNRS, CEA, Institut de Physique Théorique, 91191, Gif-sur-Yvette, France

$$\boxed{\Delta E = \frac{1}{2}m_{\rm DM}(|\vec{v}'|^2 - |\vec{v}|^2) \simeq -\frac{2m_{\rm DM}^2 m_i|\vec{v}|^2}{(m_{\rm DM}+m_i)^2} + \frac{2m_{\rm DM}m_i^2|\vec{v}_i|^2}{(m_{\rm DM}+m_i)^2}\,,}$$

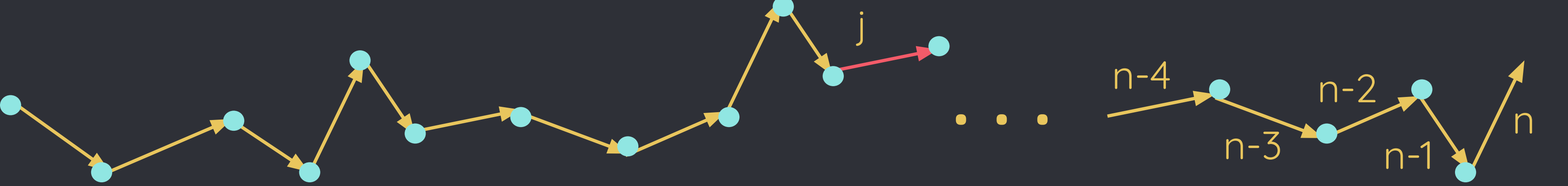
Consider a series of continuous scattering, $\Delta E_{tot} = \sum_{i}^{n} (\Delta E)_i$

For $i = 1$,

$$\Delta E_1 \approx \frac{\mu^2}{mM}\left(-Mv_i^2 + mV_i^2\right) \qquad \Delta E_1 = \frac{1}{2}M\left(v_1^2 - v_i^2\right)$$

Defining a new variable, $\beta_+ \equiv \frac{4Mm}{(M+m)^2}$

$$v_1^2 = \left(1 - \frac{\beta_+}{2}\right)v_i^2 + \frac{2m^2}{(m+M)^2}V_i^2$$



Repeating the same procedures, we propagate $v_i \to v_1 \to v_2 \to \cdots \to v_j$

For $i = j$,

$$\Delta E_j \approx \frac{\mu^2}{mM}\left(-Mv_{j-1}^2 + mV_i^2\right) \qquad \Delta E_j = \frac{1}{2}M\left(v_{j+1}^2 - v_j^2\right)$$

$$v_j^2 = \left(1 - \frac{\beta_+}{2}\right)^j v_i^2 + \overset{0}{\frac{2m^2}{(m+M)^2}}\left(2 - \frac{\beta_+}{2}\right)^{j-1} V_i^2$$

$$\Rightarrow \Delta E_{tot} = \frac{1}{2}M\left(v_n^2 - v_i^2\right) \approx \frac{1}{2}Mv_i^2\left[\left(1 - \frac{\beta_+}{2}\right)^n - 1\right]$$

$$\Delta E_{tot} \approx \frac{1}{2} M v_i^2 \left[\left(1 - \frac{\beta_+}{2}\right)^n - 1 \right]$$

$$\Delta E_{\rm tot} = \frac{1}{2} m_{\rm DM} v_0^2 \left[\left(1 - \frac{\beta_+}{2}\right)^{n_{\rm coll}} - 1 \right] .$$

Premature Black Hole Death of Population III Stars by Dark Matter

Sebastian A. R. Ellis

Multiscatter stellar capture of dark matter

Joseph Bramante,[1,2] Antonio Delgado,[1] and Adam Martin[1]

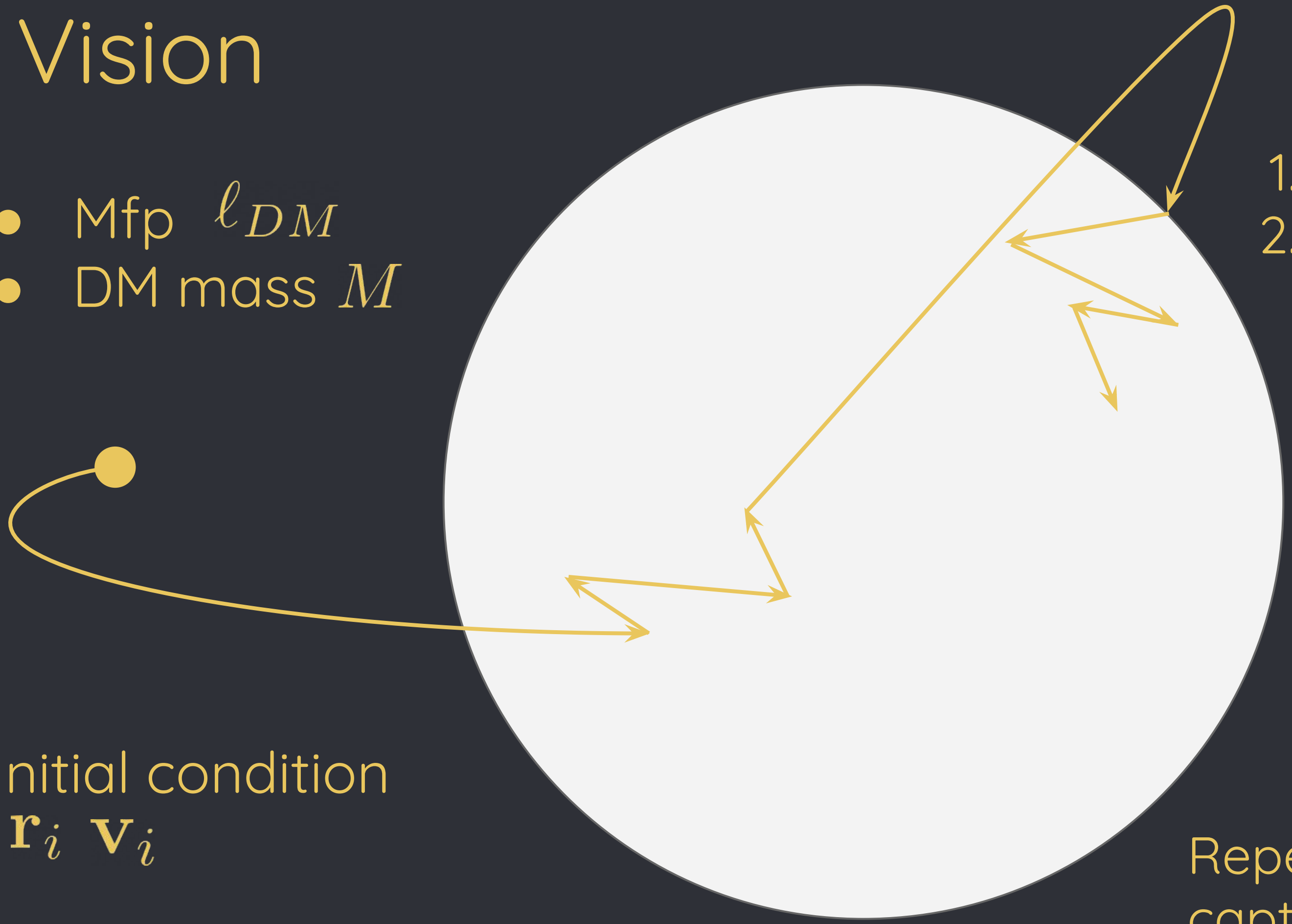
where we remind the reader that m is the mass of the stellar constituent with which the DM scatters. A kinematic analysis shows that, in the star's rest frame, the fraction of DM energy lost in a single scatter is evenly distributed over the interval $0 < \Delta E/E_0 < \beta_+$. For single scatter capture, the required fraction of DM kinetic energy loss is u^2/w^2, which is the ratio of DM's kinetic

Content

1. Motivation and recent efforts to DM detection
2. Theoretical overview of DM multiscattering
3. Monte Carlo simulation of DM capture

Vision

- Mfp ℓ_{DM}
- DM mass M



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Initial condition
$\mathbf{r}_i \ \mathbf{v}_i$

Repeat 1, 2, 3 until DM is captured

- <u>Simulate successful DM capture event</u>

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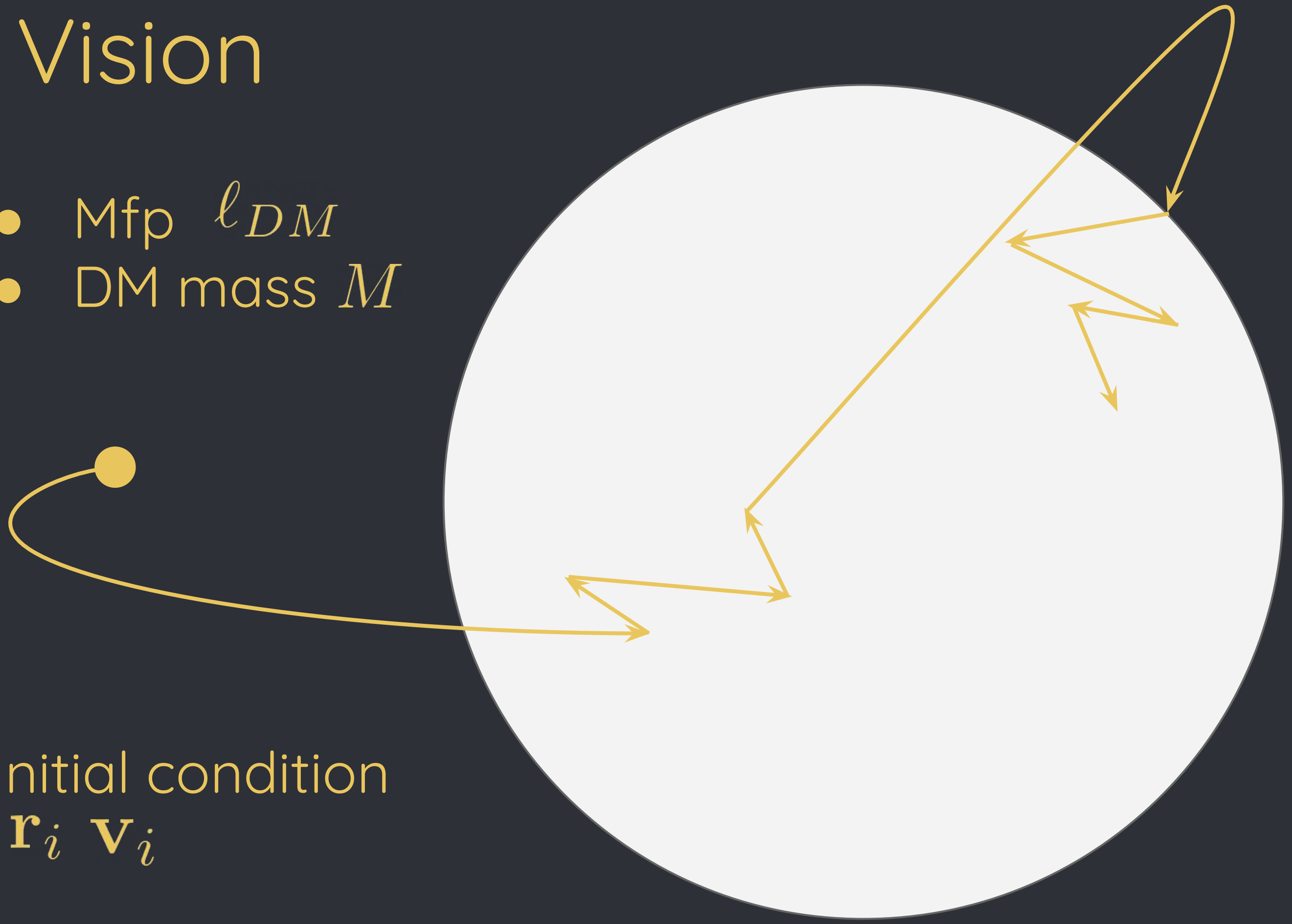

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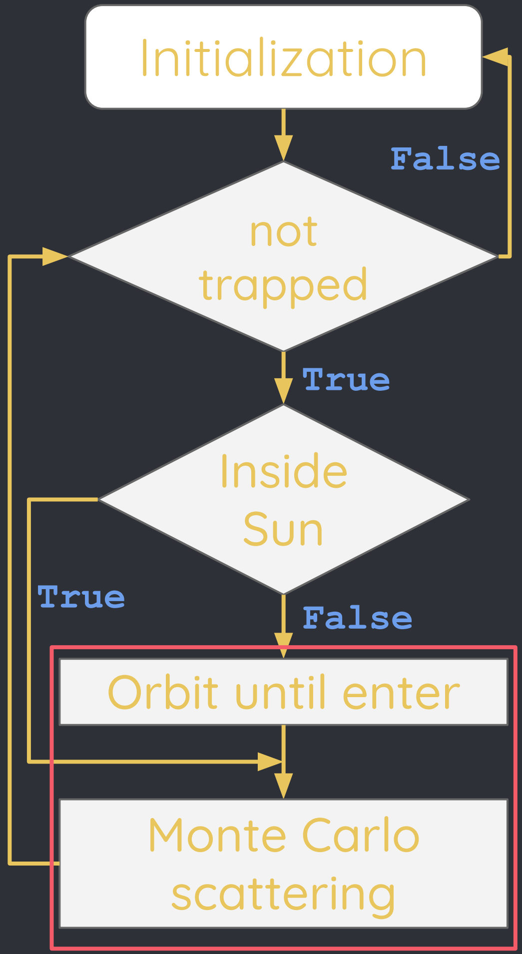
- <u>Simulate successful DM capture event</u>



Tasks to simulate DM capture

1. Orbiting: `swifter` (Fortran + Python)
2. Scattering algorithm (Python)

Tasks to simulate DM capture

1. Orbiting: `swifter` (Fortran + Python)
    - Inputs and outputs by `swifter`
2. Scattering algorithm (Python)
    - Fortran: reduces computation time
    - Sympletic fourth order T+U integrator

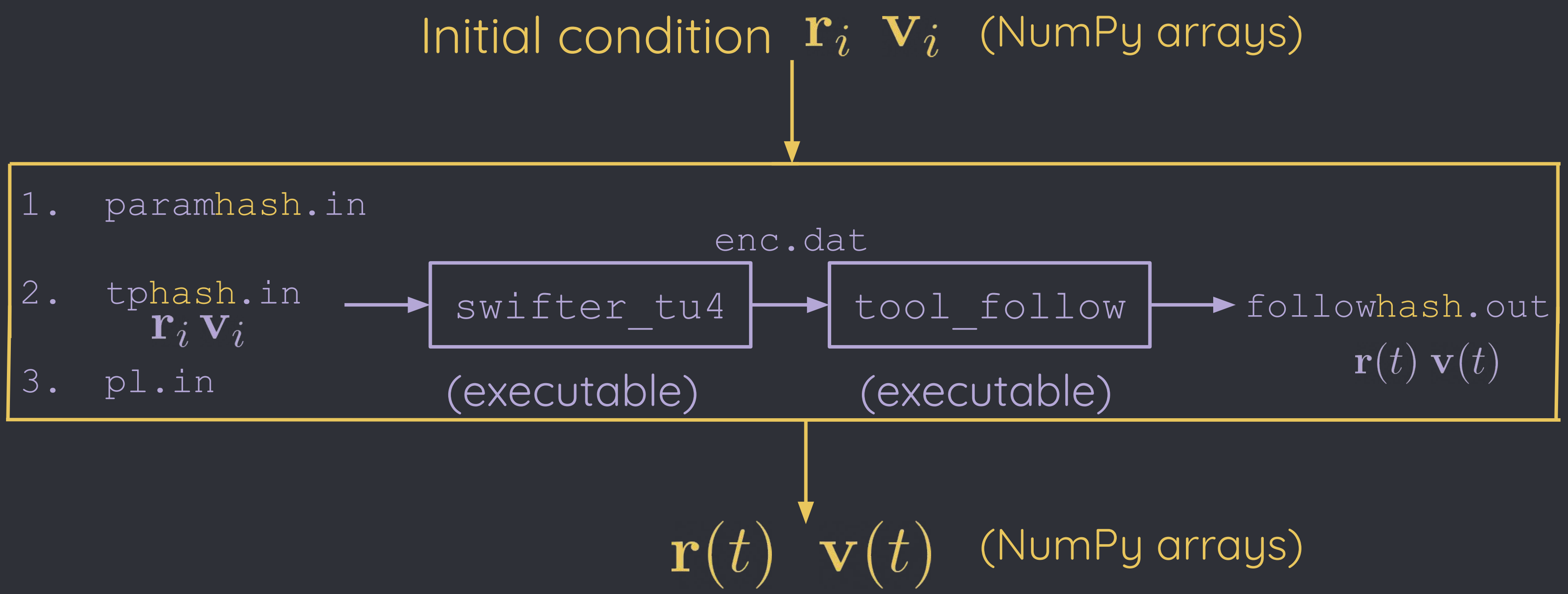
- Automation inside Python using `subprocess`

1. `param.in`
2. `tp.in` $\mathbf{r}_i$ $\mathbf{v}_i$
3. `pl.in`

- Recompilation for `tool_follow` solving display numerical data display problem

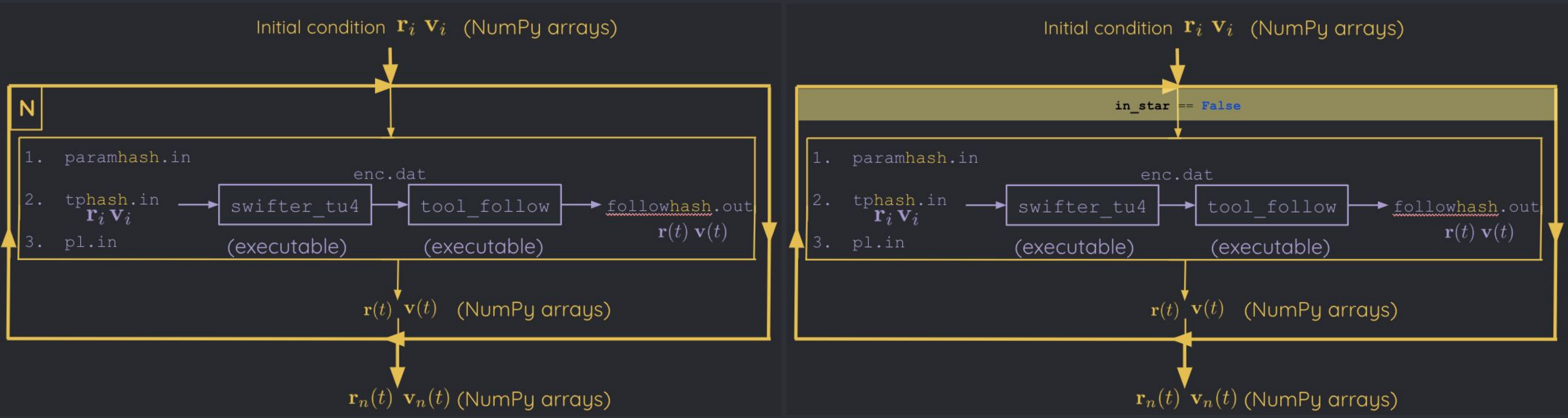
star2 > recom > swifter > example > follow1386961.out

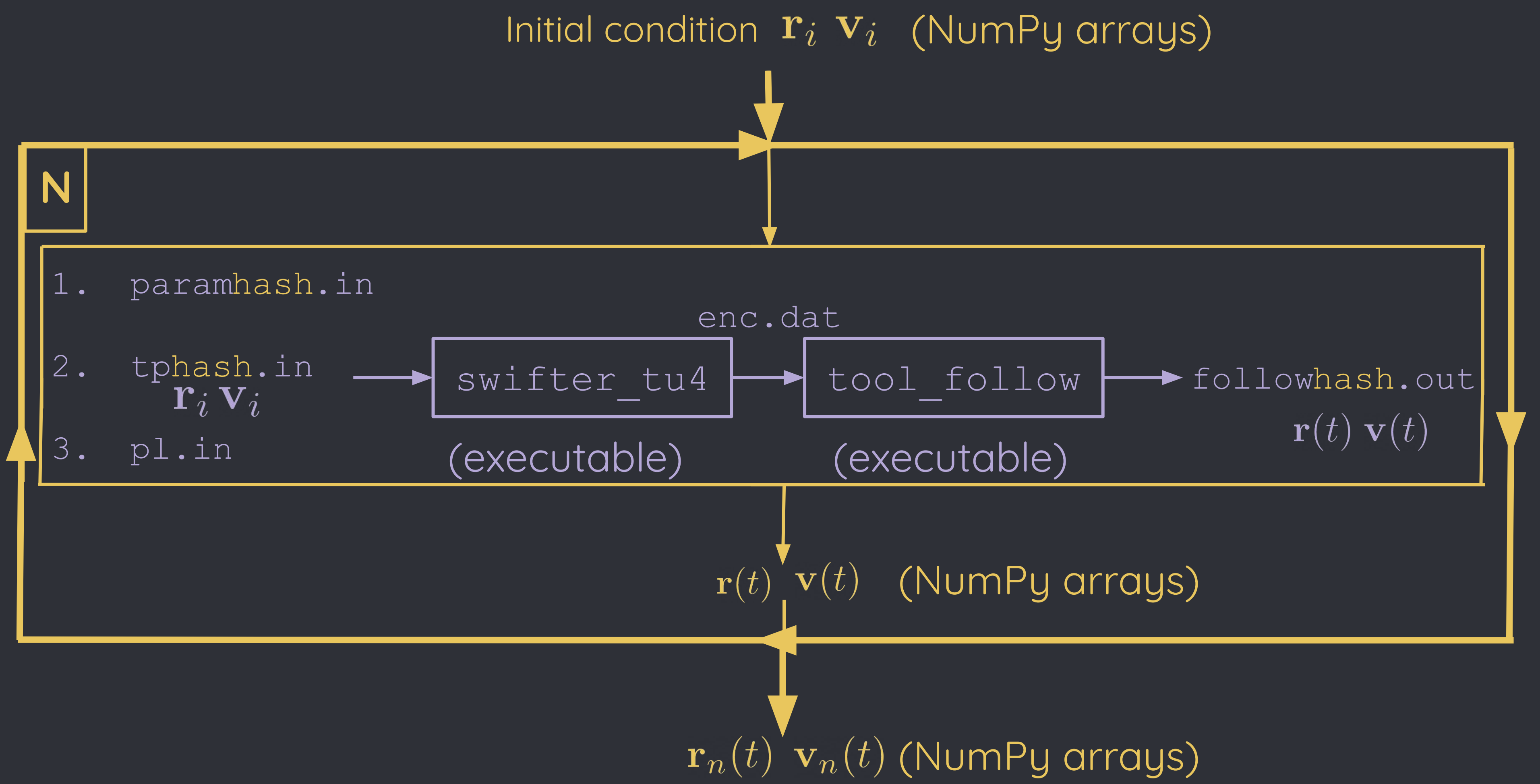
0.0000000E+00	6	0.0000000E+00	0.1000000E+01	0.0000000E+00	0.1721421E-01	0.0000000E+00	0.0000000E+00
0.1826250E+02	6	0.3092288E+00	0.9510601E+00	0.0000000E+00	0.1637180E-01	-0.5315278E-02	0.0000000E+00
0.3652500E+02	6	0.5881984E+00	0.8090490E+00	0.0000000E+00	0.1392802E-01	-0.1010842E-01	0.0000000E+00
0.5478750E+02	6	0.8096442E+00	0.5879089E+00	0.0000000E+00	0.1012450E-01	-0.1390972E-01	0.0000000E+00
0.7305000E+02	6	0.9519809E+00	0.3093183E+00	0.0000000E+00	0.5336218E-02	-0.1634866E-01	0.0000000E+00
0.9131250E+02	6	0.1001407E+01	0.5323903E-03	0.0000000E+00	0.3333827E-04	-0.1719000E-01	0.0000000E+00
0.1095750E+03	6	0.9532212E+00	-0.3083121E+00	0.0000000E+00	-0.5265945E-02	-0.1635575E-01	0.0000000E+00
0.1278375E+03	6	0.8122377E+00	-0.5871486E+00	0.0000000E+00	-0.1004639E-01	-0.1393125E-01	0.0000000E+00
0.1461000E+03	6	0.5922714E+00	-0.8088968E+00	0.0000000E+00	-0.1384542E-01	-0.1015529E-01	0.0000000E+00
0.1643625E+03	6	0.3147641E+00	-0.9520647E+00	0.0000000E+00	-0.1629695E-01	-0.5395968E-02	0.0000000E+00
0.1826250E+03	6	0.6694207E-02	-0.1002797E+01	0.0000000E+00	-0.1716543E-01	-0.1147488E-03	0.0000000E+00
0.2008875E+03	6	-0.3020255E+00	-0.9561881E+00	0.0000000E+00	-0.1636754E-01	0.5177564E-02	0.0000000E+00
0.2191500E+03	6	-0.5814201E+00	-0.8167448E+00	0.0000000E+00	-0.1397984E-01	0.9969122E-02	0.0000000E+00
0.2374125E+03	6	-0.8043296E+00	-0.5979613E+00	0.0000000E+00	-0.1023169E-01	0.1379540E-01	0.0000000E+00
0.2556750E+03	6	-0.9490287E+00	-0.3210406E+00	0.0000000E+00	-0.5484248E-02	0.1628353E-01	0.0000000E+00
0.2739375E+03	6	-0.1001343E+01	-0.1287438E-01	0.0000000E+00	-0.1967962E-03	0.1718859E-01	0.0000000E+00
0.2922000E+03	6	-0.9560572E+00	0.2965401E+00	0.0000000E+00	0.5116682E-02	0.1641838E-01	0.0000000E+00
0.3104625E+03	6	-0.8174676E+00	0.5770077E+00	0.0000000E+00	0.9937064E-02	0.1404391E-01	0.0000000E+00
0.3287250E+03	6	-0.5990080E+00	0.8010926E+00	0.0000000E+00	0.1379113E-01	0.1029407E-01	0.0000000E+00
0.3469875E+03	6	-0.3219693E+00	0.9468291E+00	0.0000000E+00	0.1629898E-01	0.5534242E-02	0.0000000E+00
0.3652500E+03	6	-0.1342532E-01	0.9999100E+00	0.0000000E+00	0.1721266E-01	0.2307837E-03	0.0000000E+00



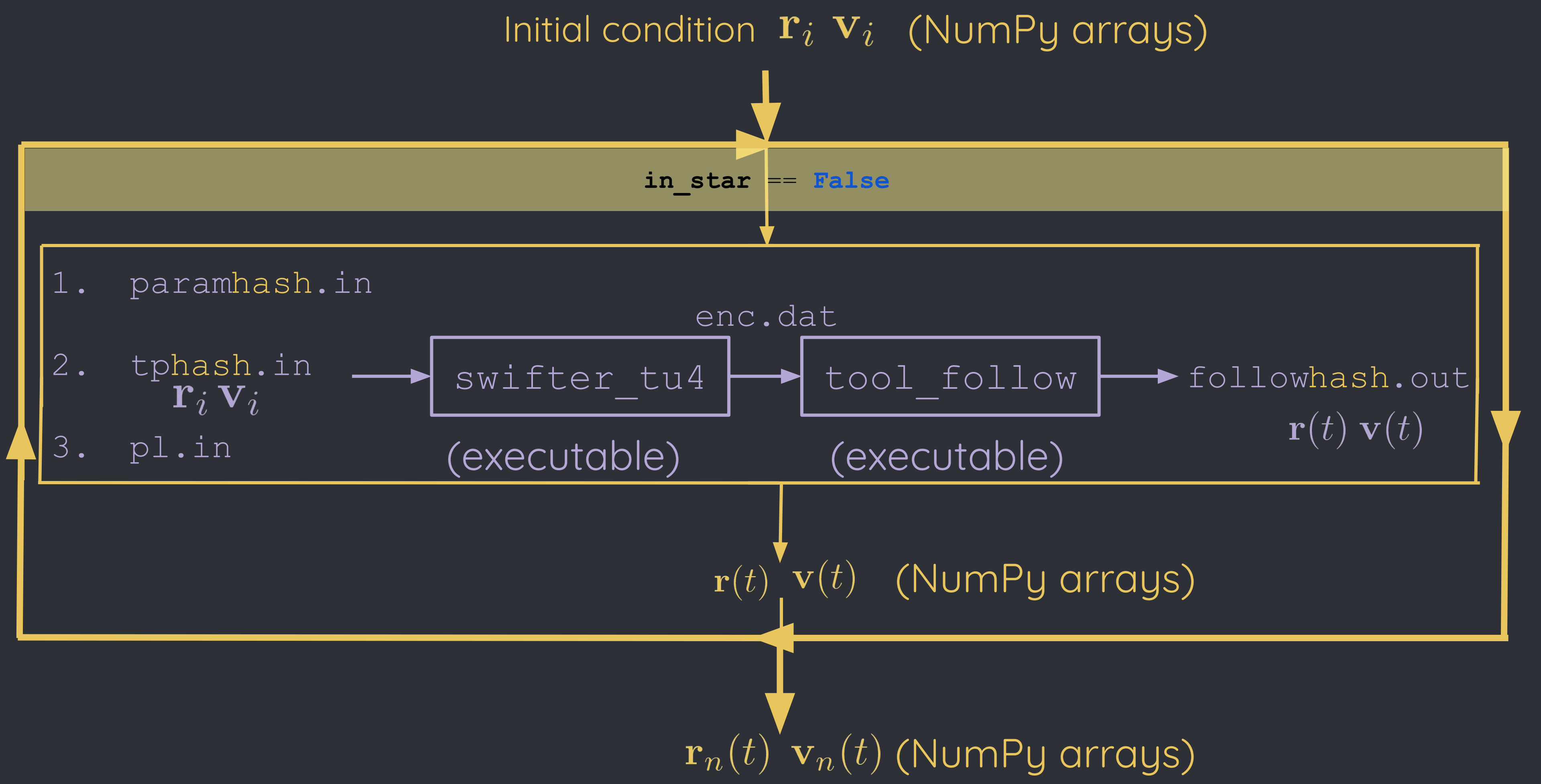
Initial condition $\mathbf{r}_i$ $\mathbf{v}_i$ (NumPy arrays)
1. paramhash.in
2. tphash.in
$\mathbf{r}_i$ $\mathbf{v}_i$
3. pl.in
swifter_tu4
(executable)
enc.dat
tool_follow
(executable)
followhash.out
$\mathbf{r}(t)$ $\mathbf{v}(t)$
$\mathbf{r}(t)$ $\mathbf{v}(t)$ (NumPy arrays)

Rewrite two functions to wrap the `swifter subprocess` as loops



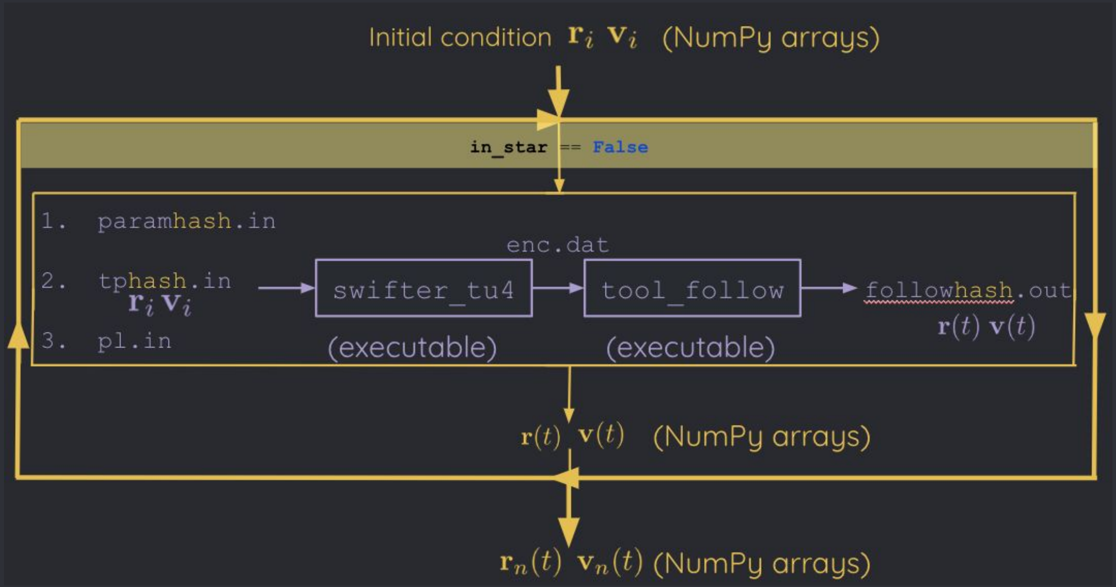


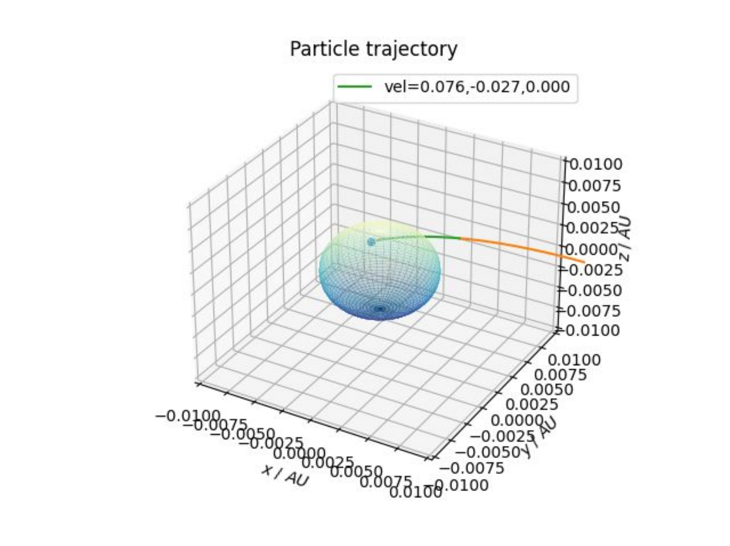
Initial condition $\mathbf{r}_i$ $\mathbf{v}_i$ (NumPy arrays)
N
1. paramhash.in
2. tphash.in
$\mathbf{r}_i\,\mathbf{v}_i$
3. pl.in
enc.dat
swifter_tu4
(executable)
tool_follow
(executable)
followhash.out
$\mathbf{r}(t)$ $\mathbf{v}(t)$
$\mathbf{r}(t)$ $\mathbf{v}(t)$ (NumPy arrays)
$\mathbf{r}_n(t)$ $\mathbf{v}_n(t)$ (NumPy arrays)



Initial condition $\mathbf{r}_i$ $\mathbf{v}_i$ (NumPy arrays)
in_star == False
1. paramhash.in
2. tphash.in
$\mathbf{r}_i \, \mathbf{v}_i$
3. pl.in
enc.dat
swifter_tu4
(executable)
tool_follow
(executable)
followhash.out
$\mathbf{r}(t)$ $\mathbf{v}(t)$
$\mathbf{r}(t)$ $\mathbf{v}(t)$ (NumPy arrays)
$\mathbf{r}_n(t)$ $\mathbf{v}_n(t)$ (NumPy arrays)

`while` loop `swifter` until entering the Sun





Tasks to simulate DM capture

1. Orbiting: `swifter` (Fortran + Python)
   - Inputs and outputs by `swifter`
2. Scattering algorithm (Python)
   - Restoring Maxwell-Boltzmann Statistics
   - Scattering Energy
   - Randomization of scattering processes

Initial condition $\mathbf{r}_i$ $\mathbf{v}_i$

Maxwell-Boltzmann distribution

$$f(\vec{v}) = \sqrt{\left(\frac{m}{2\pi kT}\right)^3} 4\pi v^2 e^{-\frac{mv^2}{2kT}}$$

- Box-muller transform

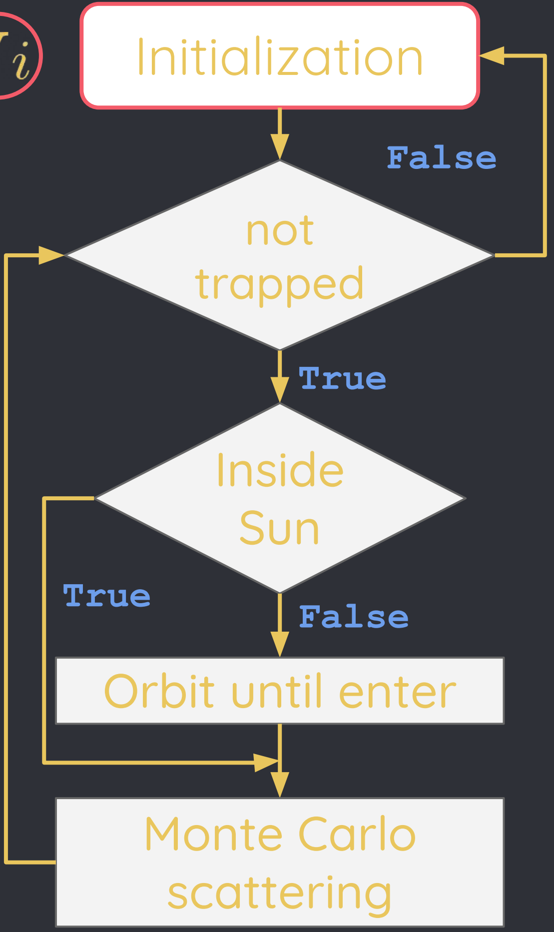
Independent uniform random variables x_1, x_2

$$y = \sqrt{-2\ln x_1} \cos(2x_2)$$

$$v_i = \sqrt{\frac{kT}{m}} \times y$$

For each dimension (x, y, z)

With the knowledge of using natural units $G = k = 1$



Tasks to simulate DM capture

1. Orbiting: `swifter` (Fortran + Python)
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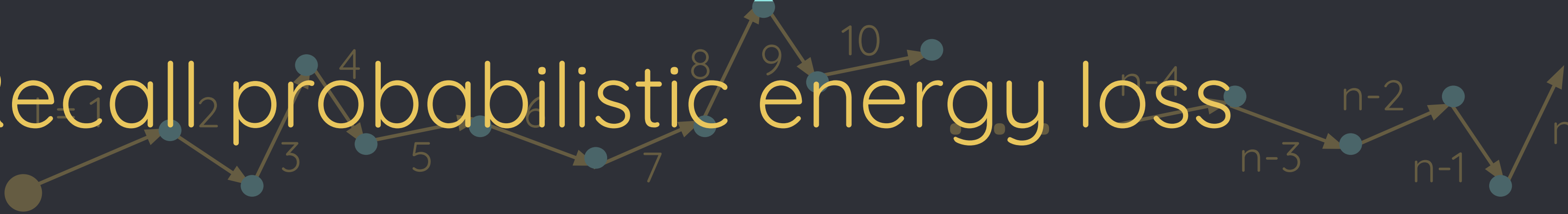
Recall probabilistic energy loss



$$\Delta E_{tot} \approx \frac{1}{2} M v_i^2 \left[\left(1 - \frac{\beta_+}{2} \right)^n - 1 \right]$$

$$\parallel$$

$$\Delta E_{\text{tot}} = \frac{1}{2} m_{\text{DM}} v_0^2 \left[\left(1 - \frac{\beta_+}{2} \right)^{n_{\text{coll}}} - 1 \right] .$$

Premature Black Hole Death of Population III Stars by Dark Matter

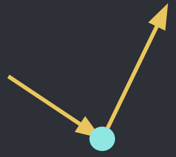
Sebastian A. R. Ellis

Multiscatter stellar capture of dark matter

Joseph Bramante,[1,2] Antonio Delgado,[1] and Adam Martin[1]

where we remind the reader that m is the mass of the stellar constituent with which the DM scatters. A kinematic analysis shows that, in the star's rest frame, the fraction of DM energy lost in a single scatter is evenly distributed over the interval $0 < \Delta E/E_0 < \beta_+$. For single scatter capture, the required fraction of DM kinetic energy loss is u^2/w^2, which is the ratio of DM's kinetic

Following only 1 scattering

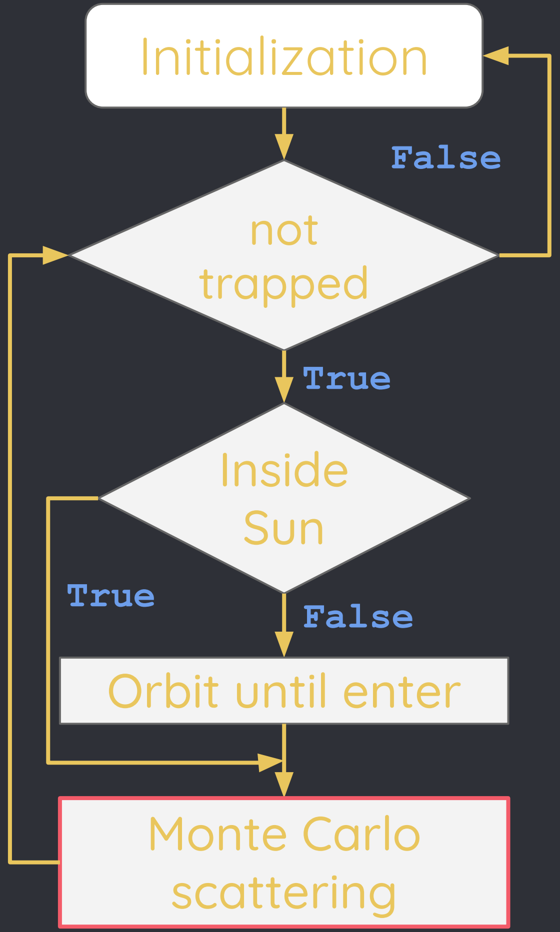


$$\frac{|\Delta E|}{E_i} \in [0, \beta_+]$$

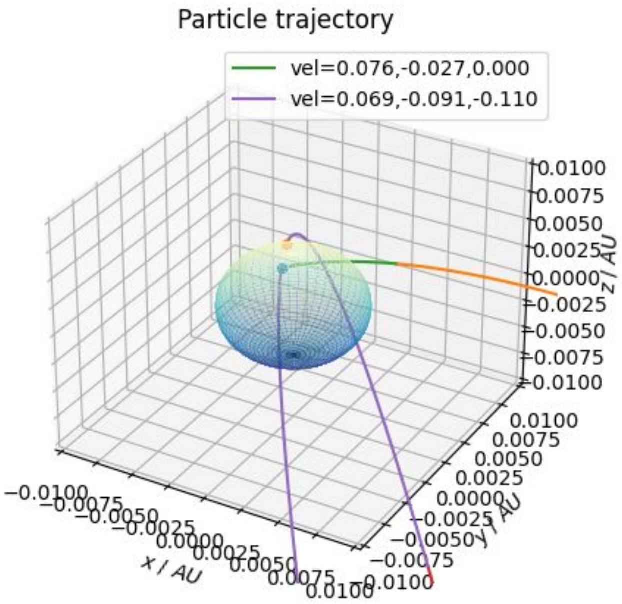
- Uniform probabilistic fractional energy change
- Make use of v_{j-1} from previous run, energy after scattering will be sampled
- $E_j(v_j) \rightarrow v_j(E_j)$ the magnitude of new velocity is obtained
- project the $v_j(E_j)$ and ℓ_{DM} to a random unit vector

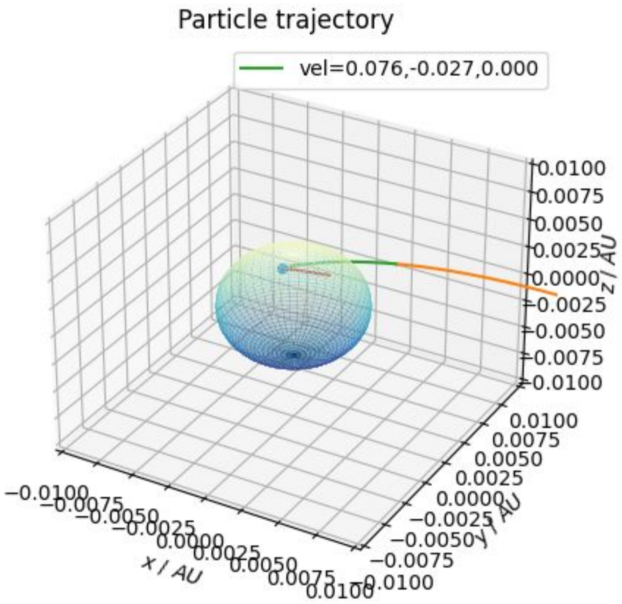
$$\mathbf{r} = |\mathbf{r}| \left(\cos u \sin v \quad, \sin u \sin v \quad, \cos v\right)$$

- where we generate 2 independent uniform random variable u, v

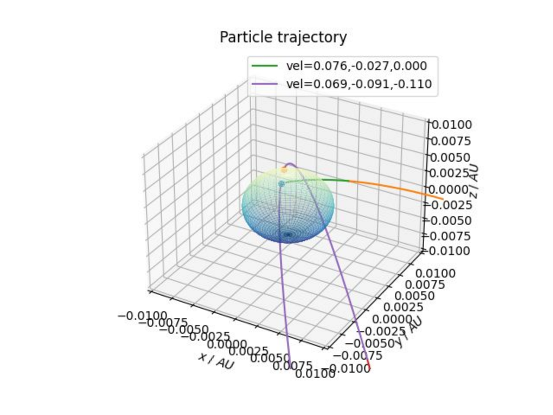


Successfully simulated DM capture events!

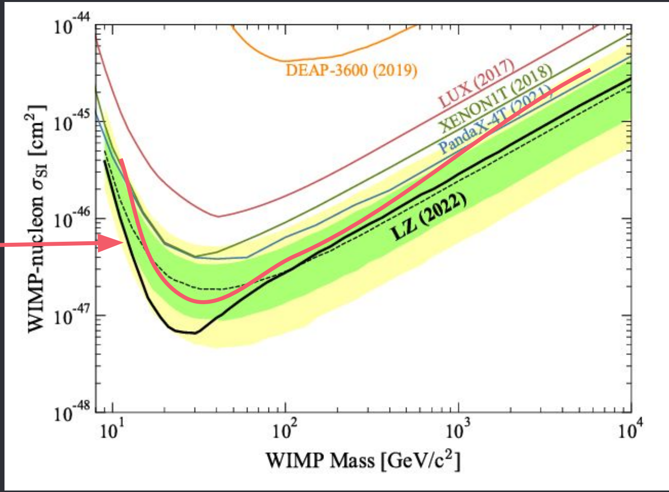


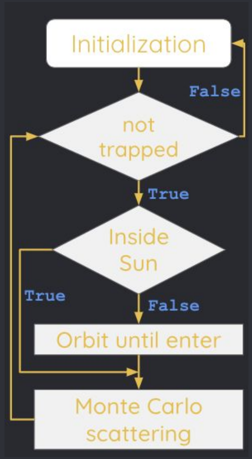


Future work

- 

$\times N$



- 

apply to other astronomical bodies and make radial profile for $\rho(\mathbf{r})$

Acknowledgements

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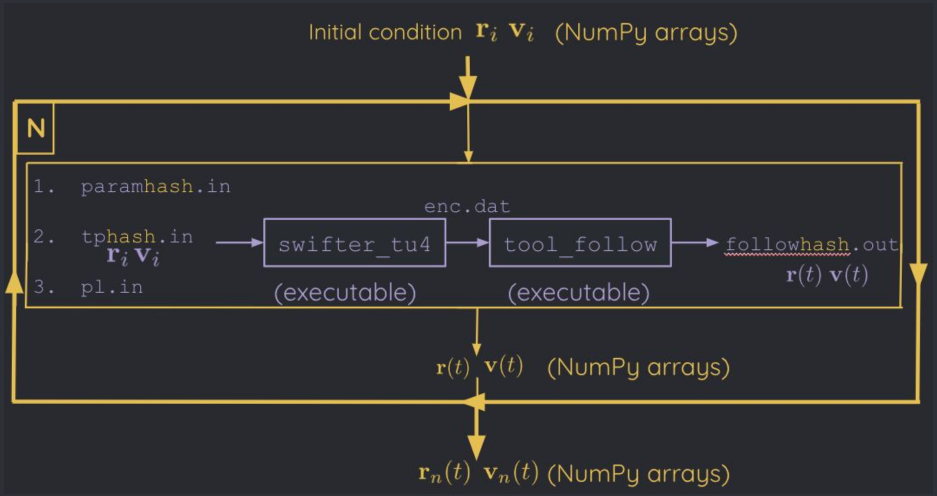
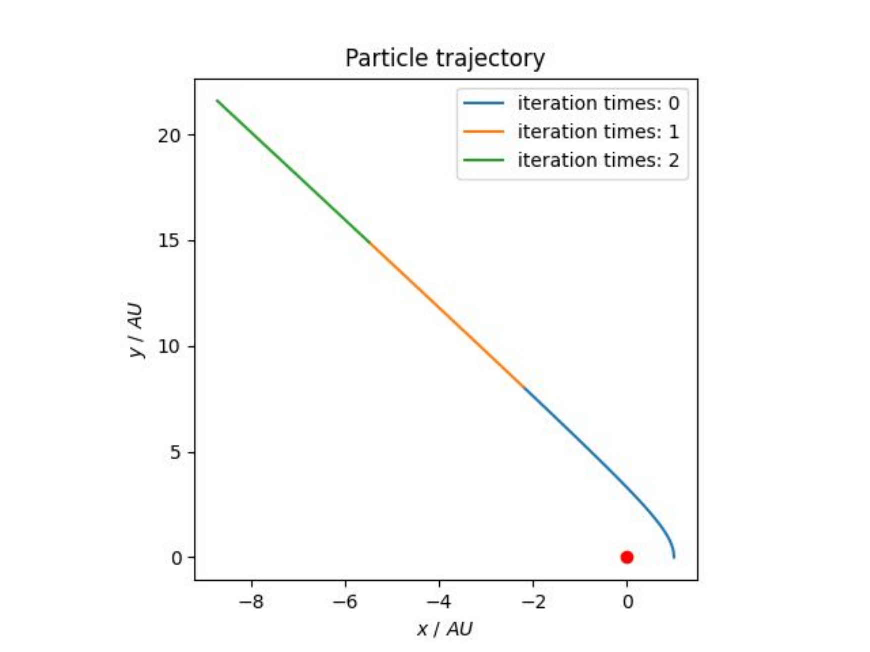
Q&A

Appendix

Sanity checking

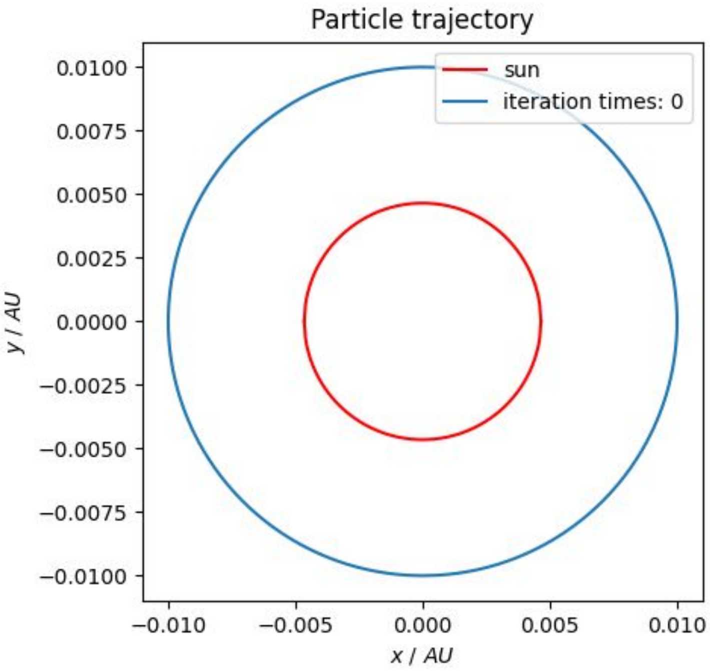
Sanity checking: continuous I/O

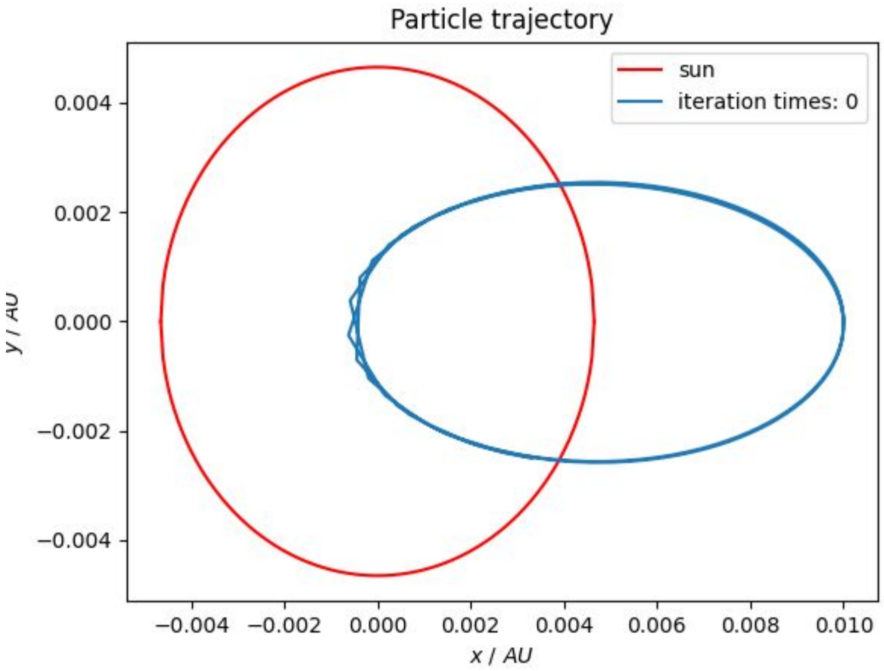




N=3

Sanity checking: `swifter`

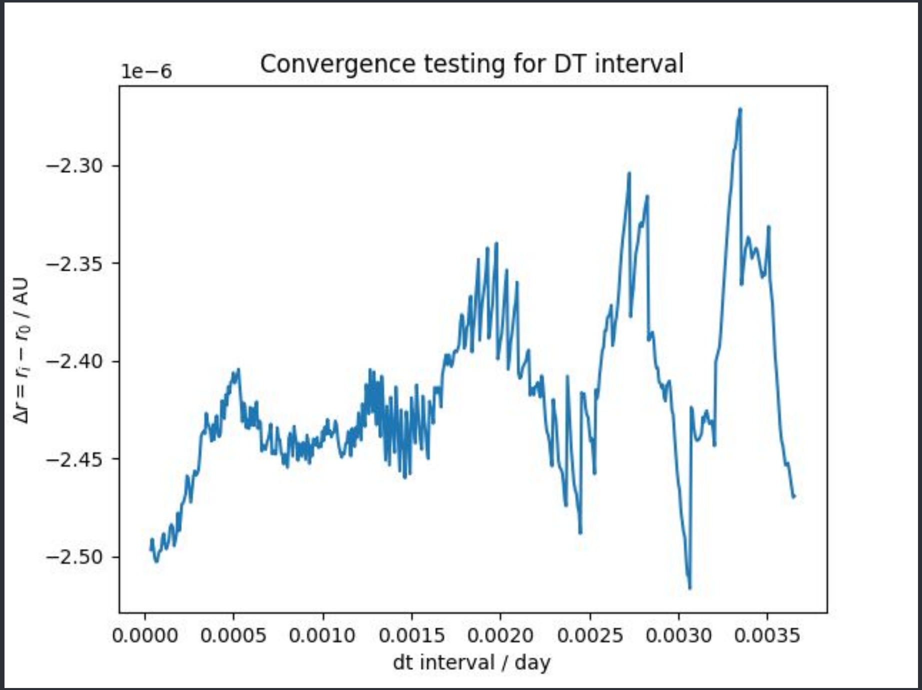




Initial conditions of Earth, slight perturbation ⇒ confirmed symplectic

Convergence testing

Convergence testing



- Fluctuations decreases from large dt to small dt