Karlsruhe Institute of Technology

Master-Thesis

High Quality Hypergraph Partitioning via Max-Flow-Min-Cut Computations

Tobias Heuer

Advisors: Prof. Dr. Peter Sanders
M. Sc. Sebastian Schlag

Institute of Theoretical Informatics, Algorithmics
Department of Informatics
Karlsruhe Institute of Technology

Date of submission: 05.01.2018

Contents

1 Experimental Results

1.1 Flow Algorithms and Networks

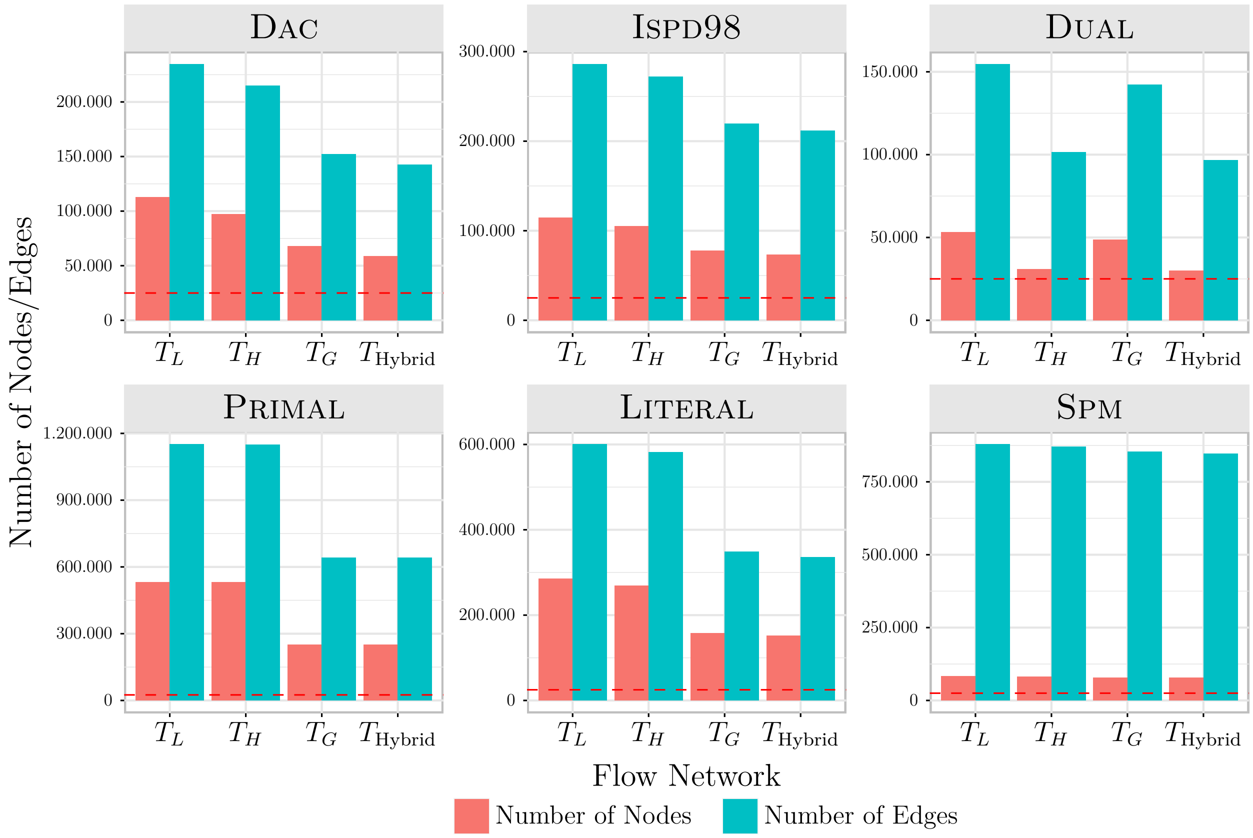

Figure 1: Comparison of the number of nodes and edges induced by flow problems of size $|V'| = 25000$ on our flow networks for different benchmark types. The red dashed lines indicates 25000 nodes.

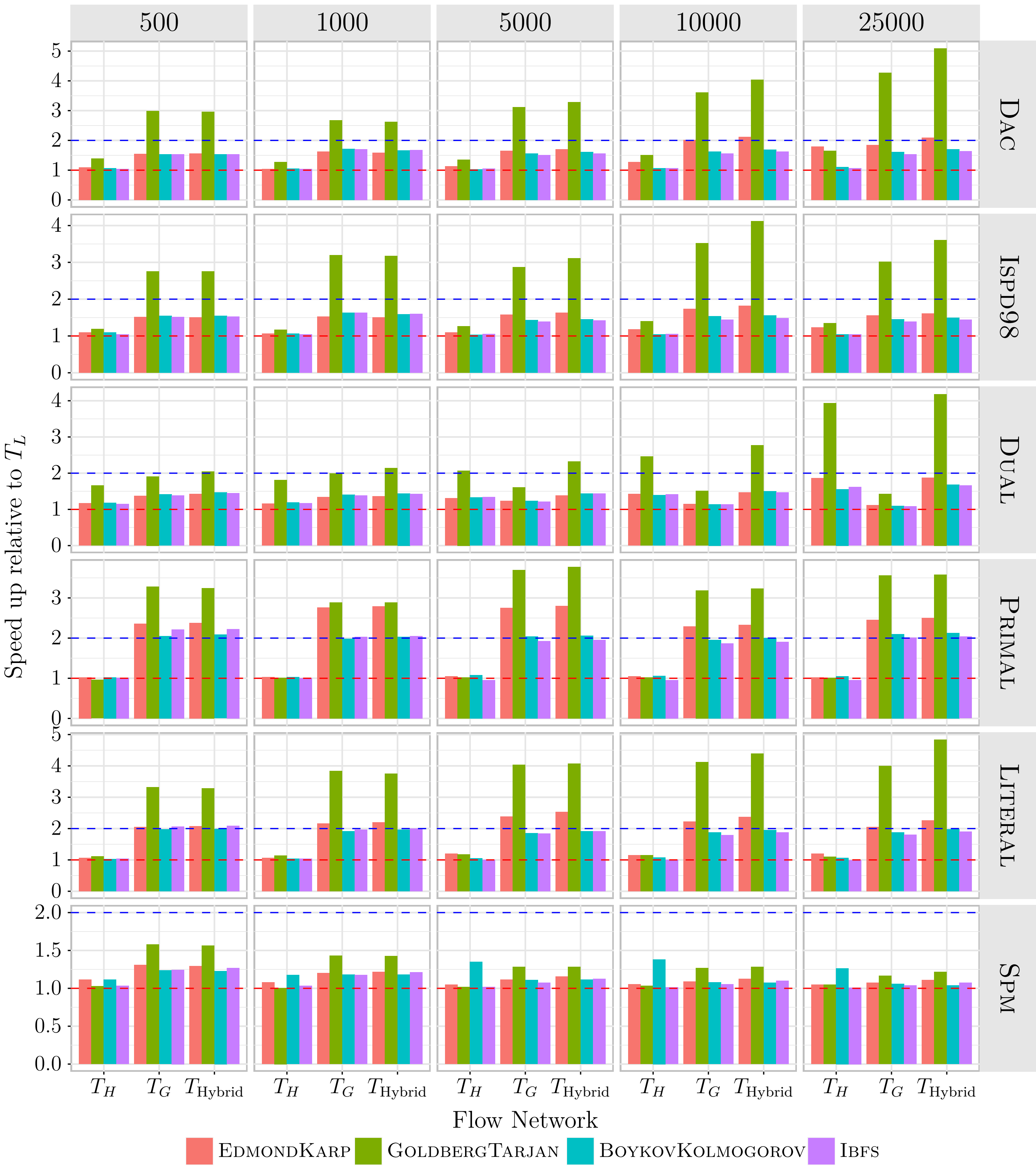

Figure 2: Speed-ups of the flow algorithms on different flow networks relative to execution on T_L for different problem sizes and types. The red resp. blue dashed line indicate a speed-up of 1 resp. 2.

$\lvert V' \rvert$	Ibfs $t[ms]$	BoykovKolmogorov $t[\%]$	GoldbergTarjan $t[\%]$	EdmondKarp $t[\%]$
500	0.81	1.79	24.56	**-7.36**
1000	**1.91**	12.92	26.88	13.90
5000	**13.52**	38.11	63.68	108.43
10000	**28.40**	53.52	92.56	182.79
25000	**64.18**	50.19	95.25	157.03

Table 1: Average running times of our maximum flow algorithms on flow network T_{Hybrid}. Note, all values in the table are in percentage relative to the running time of the Ibfs algorithm. In each line the fastest variant is marked bold.

1.2 Configuring direct k-way Flow-based Refinement

	M2 - BoykovKolmogorov							
Config.	(+F,-M,-FM)		(+F,+M,-FM)		(+F,+M,+FM)		Constant128	
α'	Avg[%]	$t[s]$	Avg[%]	$t[s]$	Avg[%]	$t[s]$	Avg[%]	$t[s]$
1	−6.10	13.51	−5.62	14.22	0.23	15.19	0.53	55.75
2	−3.20	16.89	−2.08	18.23	0.74	17.97	1.09	87.93
4	−1.82	22.23	−0.20	24.29	1.21	22.50	1.61	144.42
8	−0.85	31.49	0.98	34.43	1.71	30.58	2.16	257.41
16	−0.20	48.66	1.75	53.23	2.21	45.04	2.69	498.29
Ref.	(-F,-M,+FM)		6373.88	13.73				
	M1 - GoldbergTarjan							
Config.	(+F,-M,-FM)		(+F,+M,-FM)		(+F,+M,+FM)		Constant128	
α'	Avg[%]	$t[s]$	Avg[%]	$t[s]$	Avg[%]	$t[s]$	Avg[%]	$t[s]$
1	−15.48	12.94	−15.26	13.29	0.14	14.99	0.32	67.38
2	−10.50	16.07	−10.12	16.93	0.36	16.93	0.62	139.21
4	−5.98	21.22	−5.08	23.01	0.67	20.76	1.03	274.60
8	−3.22	30.73	−1.64	33.72	1.25	28.65	1.67	558.81
16	−1.52	50.89	0.51	56.39	1.87	46.17	2.44	1220.92
Ref.	(-F,-M,+FM)		6373.88	13.73				

Table 2: Table contains results for different configurations of our flow-based refinement framework for increasing α'. The quality in column *Avg.* is relative to our baseline configuration (-F,-M,+FM).

Config.	(+F,-M,-FM)		(+F,+M,-FM)		(+F,+M,+FM)	
α'	M1 - Avg [%]	M2 - Avg [%]	M1 - Avg [%]	M2 - Avg [%]	M1 - Avg [%]	M2 - Avg [%]
1	-15.48	-6.10	-15.26	-5.62	0.14	0.23
2	-10.50	-3.20	-10.12	-2.08	0.36	0.74
4	-5.98	-1.82	-5.08	-0.20	0.67	1.21
8	-3.22	-0.85	-1.64	0.98	1.25	1.71
16	-1.52	-0.20	0.51	1.75	1.87	2.21
Ref.	(-F,-M,+FM)		6373.88			

Table 3: Comparison on quality of our framework with different source and sink set modeling approaches. M1 represents the approach of Sanders and Schulz and M2 is our new variant.

	M2 - BoykovKolmogorov		
Config.	(+F,-M,-FM)	(+F,+M,-FM)	(+F,+M,+FM)
α'	Avg[%]	Avg[%]	Avg[%]
1	-6.08	-5.57	0.23
2	-3.22	-2.10	0.72
4	-1.90	-0.26	1.18
8	-0.91	0.92	1.67
16	-0.29	1.65	2.15
Ref.	(-F,-M,+FM)	6376.03	
	M1 - GoldbergTarjan		
Config.	(+F,-M,-FM)	(+F,+M,-FM)	(+F,+M,+FM)
α'	Avg[%]	Avg[%]	Avg[%]
1	-15.45	-15.19	0.17
2	-10.51	-10.08	0.38
4	-5.97	-5.10	0.68
8	-3.22	-1.66	1.26
16	-1.67	0.46	1.84
Ref.	(-F,-M,+FM)	6376.03	

Table 4: Table contains results of the effectiveness test for different configurations of our flow-based refinement framework for increasing α'. The quality in column *Avg.* is relative to our baseline configuration (-F,-M,+FM).

1.3 Speed-Up Heuristics

	M2 - BoykovKolmogorov			
Variant	Avg [%]	Min [%]	$t_{\text{flow}}[s]$	$t[s]$
KaHyPar-CA	7077.20	6820.17	-	29.26
KaHyPar-MF	−2.48	−2.13	51.76	81.02
KaHyPar-MF$_{(R1)}$	−2.41	−2.05	41.21	70.47
KaHyPar-MF$_{(R1,R2)}$	−2.40	−2.04	35.56	64.82
KaHyPar-MF$_{(R1,R2,R3)}$	−2.41	−2.05	26.64	55.90
	M1 - GoldbergTarjan			
Variant	Avg [%]	Min [%]	$t_{\text{flow}}[s]$	$t[s]$
KaHyPar-CA	7077.20	6820.17	-	29.26
KaHyPar-MF	−2.13	−1.80	52.28	81.54
KaHyPar-MF$_{(R1)}$	−2.05	−1.74	41.48	70.74
KaHyPar-MF$_{(R1,R2)}$	−2.05	−1.73	35.27	64.54
KaHyPar-MF$_{(R1,R2,R3)}$	−2.04	−1.75	27.62	56.88

Table 5: Results of our flow-based refinement framework with different speedup heuristics.

1.4 Comparison with other Hypergraph Partitioners

Partitioner	M2 - BoykovKolmogorov Running Time $t[s]$						
	All	Dac	Ispd98	Primal	Literal	Dual	Spm
KaHyPar-MF	63.33	505.75	21.10	71.81	140.53	98.06	32.64
KaHyPar-CA	30.85	368.97	12.35	32.91	64.65	68.27	13.63
hMetis-R	78.47	446.36	29.03	66.25	142.12	200.36	40.64
hMetis-K	57.36	240.92	23.18	44.23	94.89	125.55	35.08
PaToH-Q	5.84	28.34	1.89	6.90	9.24	10.57	3.35
PaToH-D	1.21	6.45	0.35	1.12	1.58	2.87	0.75
Partitioner	M1 - GoldbergTarjan Running Time $t[s]$						
	All	Dac	Ispd98	Primal	Literal	Dual	Spm
KaHyPar-MF	62.24	637.58	22.29	71.63	140.84	106.24	29.61
KaHyPar-CA	31.05	368.97	12.35	32.91	64.65	68.27	13.91
hMetis-R	79.23	446.36	29.03	66.25	142.12	200.36	41.79
hMetis-K	57.86	240.92	23.18	44.23	94.89	125.55	35.95
PaToH-Q	5.89	28.34	1.89	6.90	9.24	10.57	3.42
PaToH-D	1.22	6.45	0.35	1.12	1.58	2.87	0.77

Table 6: Comparing the average running time of KaHyPar-MF with KaHyPar-CA and other hypergraph partitioners.

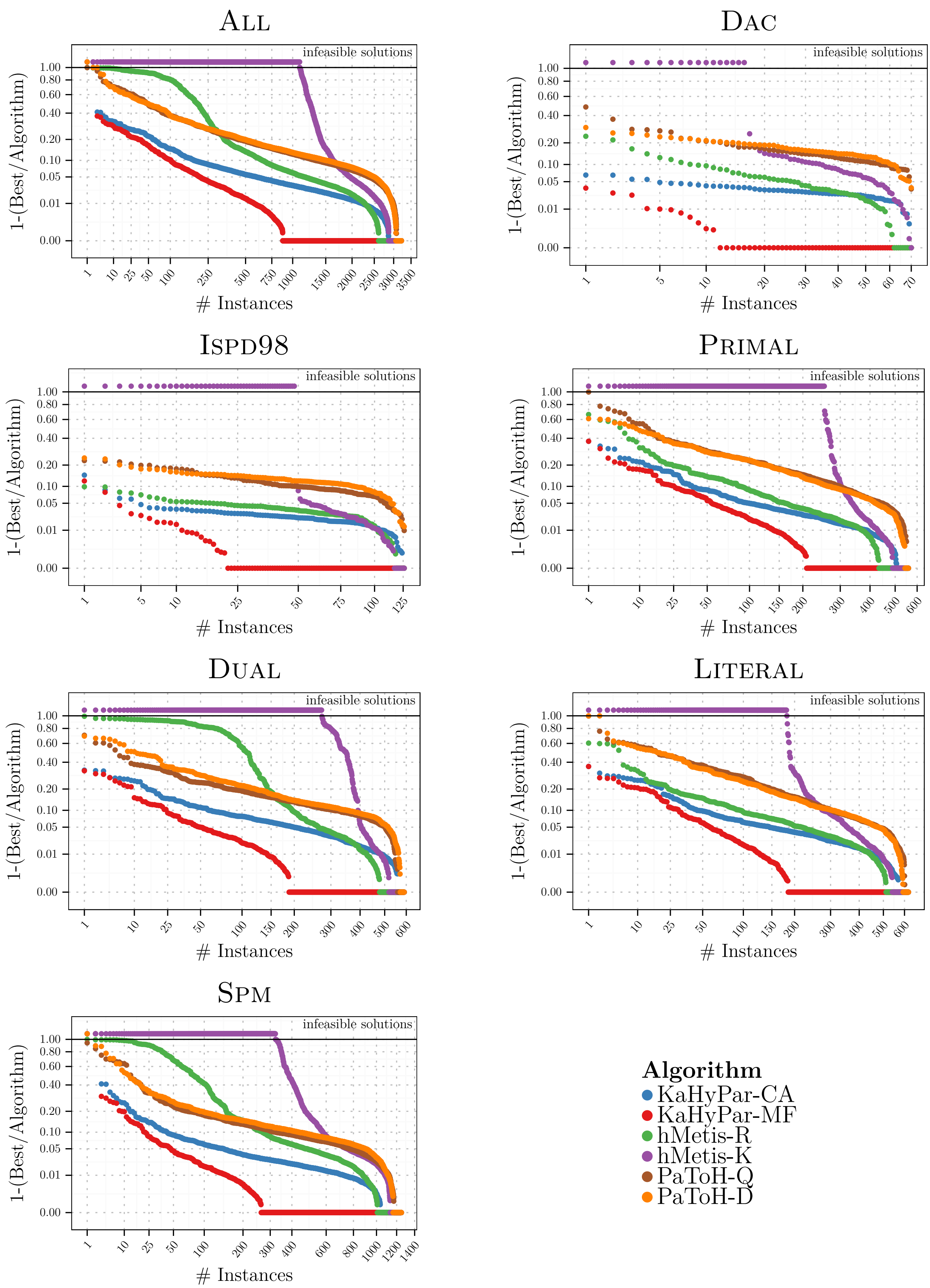

Figure 3: Min-Cut performance plots comparing KaHyPar-MF with KaHyPar-CA and other partitioners. Plots are explained in Section ??.

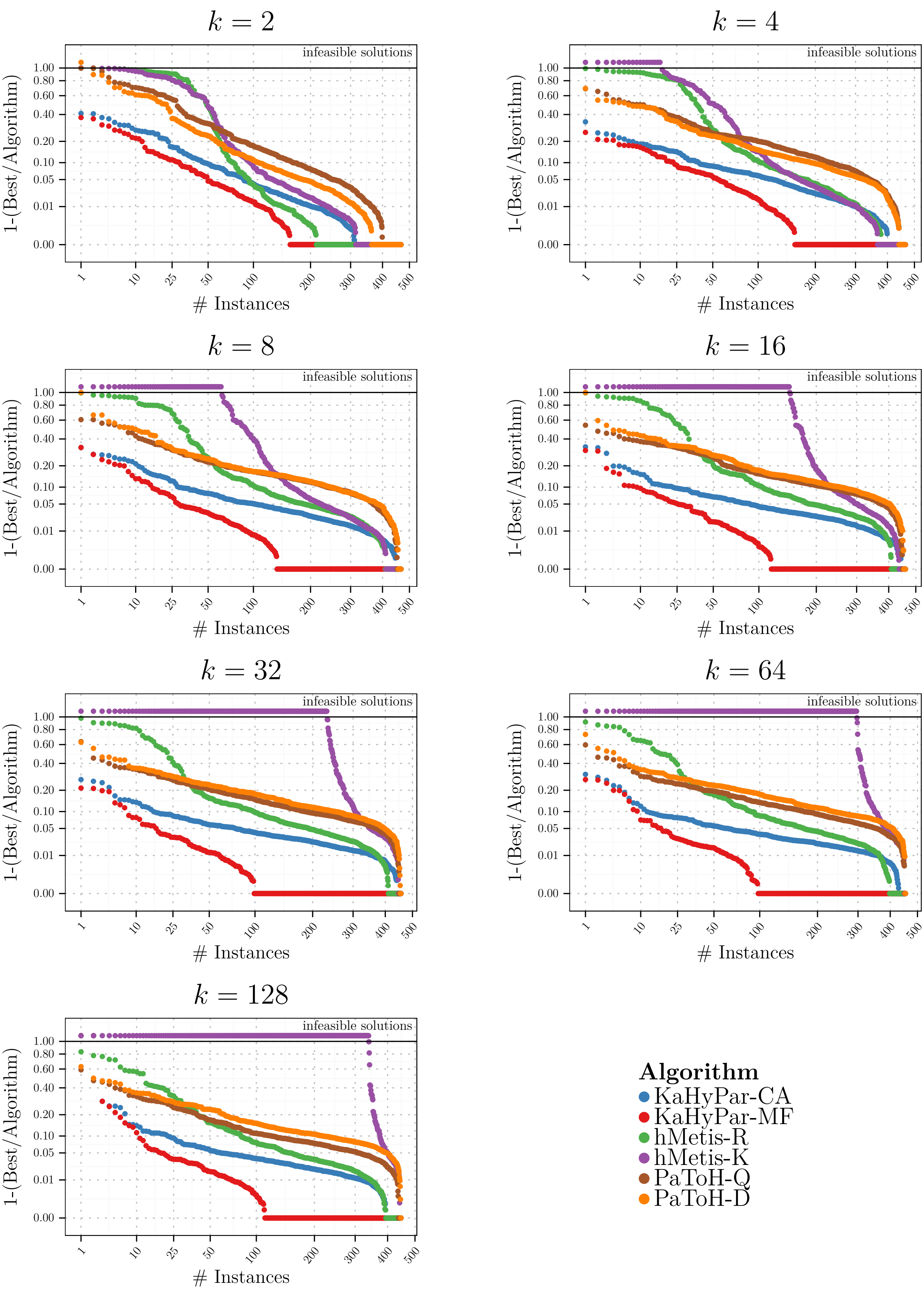

Figure 4: Min-Cut performance plots comparing KaHyPar-MF with KaHyPar-CA and other partitioners for different values of k.

Partitioner	M2 - BoykovKolmogorov Average $\lambda - 1$						
	All	Dac	Ispd98	Primal	Literal	Dual	Spm
KaHyPar-MF	7727.97	17 480.70	5644.61	15 863.61	15 769.49	3038.31	6027.03
KaHyPar-CA	2.49	3.11	2.20	2.08	2.74	3.51	2.08
hMetis-R	15.79	3.64	1.62	2.07	2.80	43.44	19.77
hMetis-K	15.25	8.46	1.38	4.05	9.29	28.65	19.54
PaToH-Q	9.51	13.57	7.92	12.14	13.35	8.80	6.79
PaToH-D	16.77	23.75	15.08	18.27	21.54	18.31	12.92
Partitioner	M1 - GoldbergTarjan Average $\lambda - 1$						
	All	Dac	Ispd98	Primal	Literal	Dual	Spm
KaHyPar-MF	7819.11	17 590.10	5671.37	15 923.74	15 844.61	3061.94	6165.74
KaHyPar-CA	2.03	2.47	1.72	1.69	2.25	2.71	1.75
hMetis-R	15.21	2.99	1.14	1.69	2.31	42.33	19.22
hMetis-K	14.71	7.78	0.90	3.66	8.77	27.66	19.09
PaToH-Q	8.98	12.86	7.41	11.72	12.81	7.96	6.37
PaToH-D	16.21	22.98	14.54	17.83	20.97	17.40	12.50

Table 7: Comparison of average $(\lambda - 1)$ metric of KaHyPar-MF with KaHyPar-CA and other partitioners on different benchmark types. The results are in percentage relative to KaHyPar-MF.

Partitioner	M2 - BoykovKolmogorov Running Time $t[s]$						
	$k = 2$	$k = 4$	$k = 8$	$k = 16$	$k = 32$	$k = 64$	$k = 128$
KaHyPar-MF	23.56	39.84	56.44	68.69	86.73	108.35	125.74
KaHyPar-CA	12.68	17.02	23.70	30.78	41.38	56.95	76.05
hMetis-R	27.87	51.03	73.94	90.09	107.94	127.26	147.72
hMetis-K	25.47	31.92	42.06	52.87	73.30	108.15	151.63
PaToH-Q	1.93	3.58	5.39	6.95	8.32	9.97	11.34
PaToH-D	0.43	0.76	1.11	1.40	1.69	2.00	2.27
Partitioner	M1 - GoldbergTarjan Running Time $t[s]$						
	$k = 2$	$k = 4$	$k = 8$	$k = 16$	$k = 32$	$k = 64$	$k = 128$
KaHyPar-MF	22.13	38.51	55.04	67.83	85.75	108.97	128.04
KaHyPar-CA	12.68	17.16	23.88	31.01	41.69	57.35	76.61
hMetis-R	27.87	51.59	74.74	91.09	109.13	128.66	149.34
hMetis-K	25.47	32.27	42.50	53.41	74.00	109.12	152.92
PaToH-Q	1.93	3.61	5.44	7.01	8.40	10.06	11.44
PaToH-D	0.43	0.77	1.12	1.42	1.71	2.02	2.29

Table 8: Comparing the average running time of KaHyPar-MF with KaHyPar-CA and other partitioners for different values of k.

Partitioner	M2 - BoykovKolmogorov Average $\lambda - 1$						
	$k = 2$	$k = 4$	$k = 8$	$k = 16$	$k = 32$	$k = 64$	$k = 128$
KaHyPar-MF	1057.94	3105.80	5988.27	9292.89	14 582.34	21 735.78	31 477.02
KaHyPar-CA	2.32	2.62	2.86	2.73	2.55	2.28	2.08
hMetis-R	27.19	18.98	16.97	15.81	12.86	10.70	8.49
hMetis-K	27.59	17.84	15.75	15.58	11.78	10.25	8.48
PaToH-Q	11.74	9.14	9.14	10.01	9.38	9.05	8.10
PaToH-D	15.29	16.60	19.17	19.91	16.15	15.78	14.52
Partitioner	M1 - GoldbergTarjan Average $\lambda - 1$						
	$k = 2$	$k = 4$	$k = 8$	$k = 16$	$k = 32$	$k = 64$	$k = 128$
KaHyPar-MF	1064.06	3147.96	6062.80	9406.00	14 756.03	21 978.89	31 820.94
KaHyPar-CA	1.73	2.06	2.36	2.28	2.11	1.90	1.73
hMetis-R	26.46	18.26	16.34	15.25	12.33	10.23	8.08
hMetis-K	26.86	17.19	15.18	15.06	11.29	9.83	8.10
PaToH-Q	11.10	8.50	8.57	9.49	8.87	8.60	7.70
PaToH-D	14.62	15.94	18.55	19.34	15.62	15.31	14.09

Table 9: Comparison of average $(\lambda - 1)$ metric of KaHyPar-MF with KaHyPar-CA and other partitioners for different values of k. The results are in percentage relative to KaHyPar-MF.