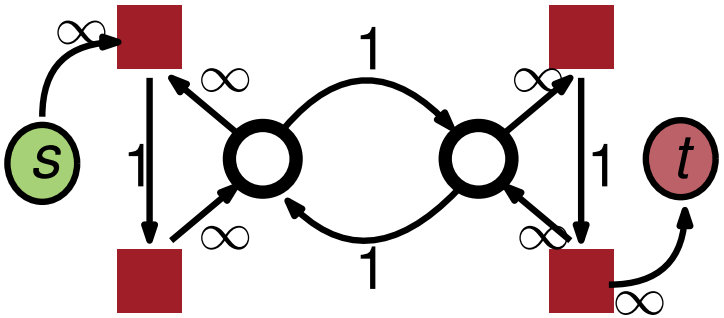
Karlsruhe Institute of Technology

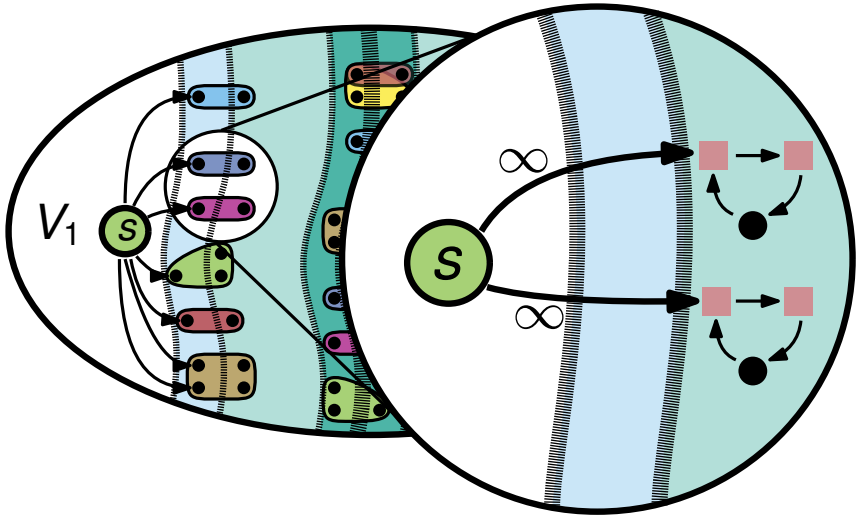
High Quality Hypergraph Partitioning via Max-Flow-Min-Cut Computations

Master Thesis · February 16, 2018

Tobias Heuer

INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP



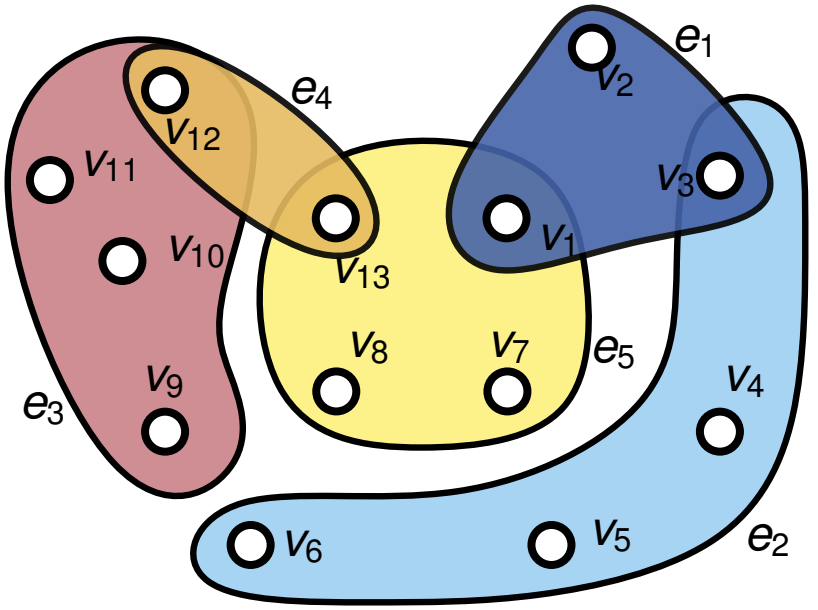


Hypergraphs
[from SEA'17]

- Generalization of graphs
  $\Rightarrow$ hyperedges connect ≥ 2 nodes

- Graphs $\Rightarrow$ dyadic (**2-ary**) relationships
- Hypergraphs $\Rightarrow$ (**d-ary**) relationships

- Hypergraph $H = (V, E, c, \omega)$
  - Vertex set $V = \{1, ..., n\}$
  - Edge set $E \subseteq \mathcal{P}(V) \setminus \emptyset$
  - Node weights $c : V \to \mathbb{R}_{\geq 1}$
  - Edge weights $\omega : E \to \mathbb{R}_{\geq 1}$



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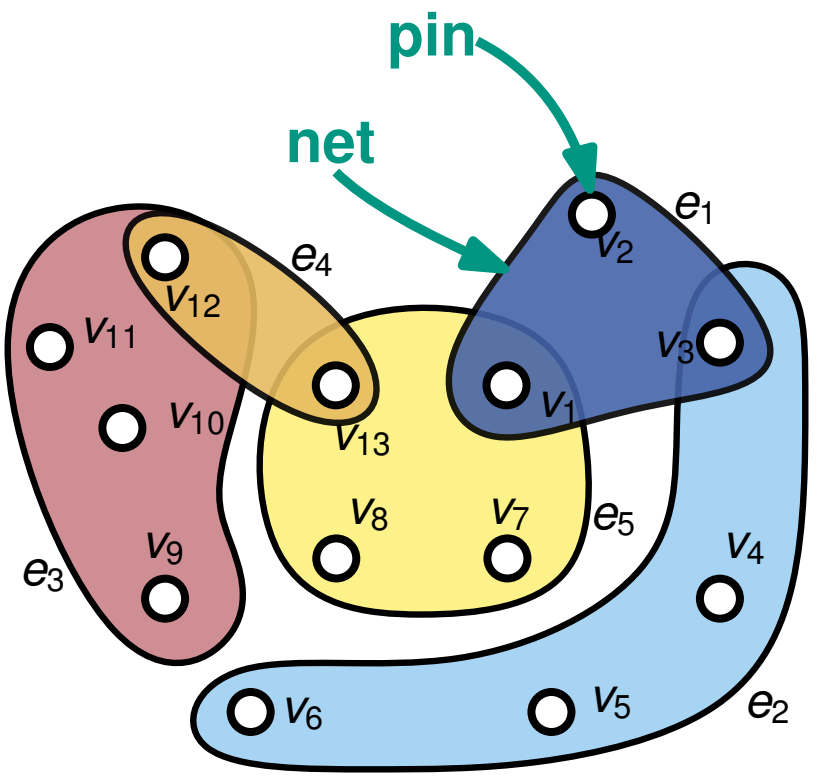

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Hypergraph Partitioning Problem

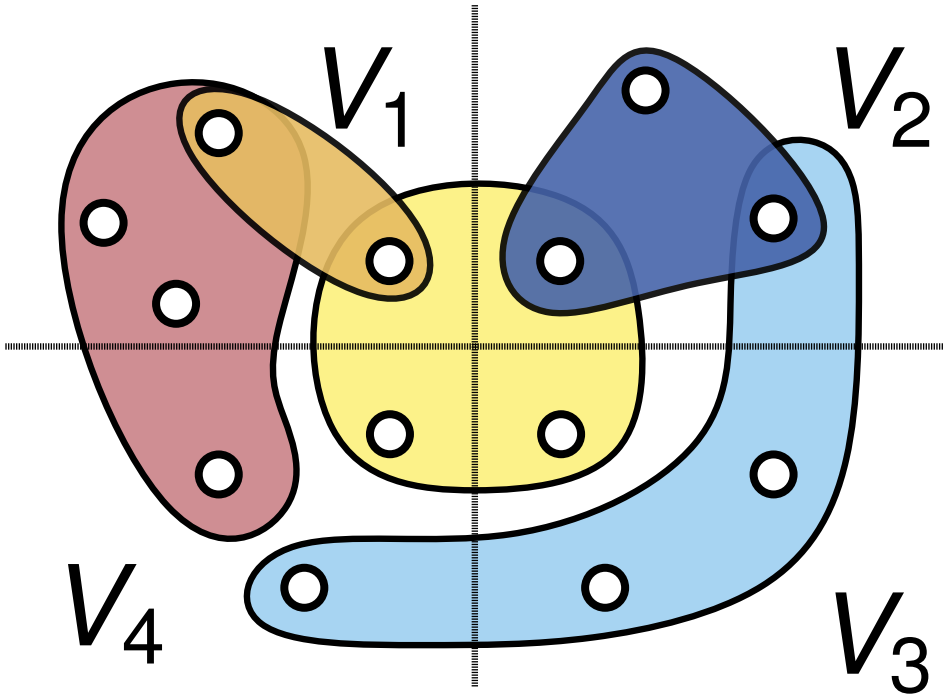
[from SEA'17]

Partition hypergraph $H = (V, E, c, \omega)$ into $\mathbf{k}$ non-empty disjoint blocks $\Pi = \{V_1, \ldots, V_k\}$ such that:

- blocks V_i are **roughly equal-sized**:

imbalance parameter

$$c(V_i) \leq (1 + \varepsilon) \left\lceil \frac{c(V)}{k} \right\rceil$$



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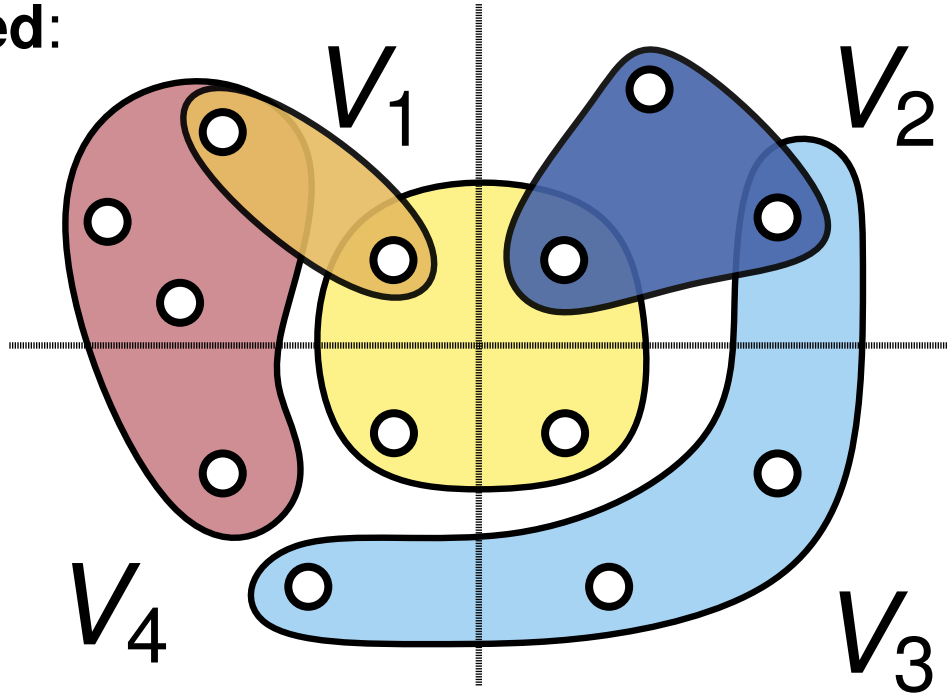

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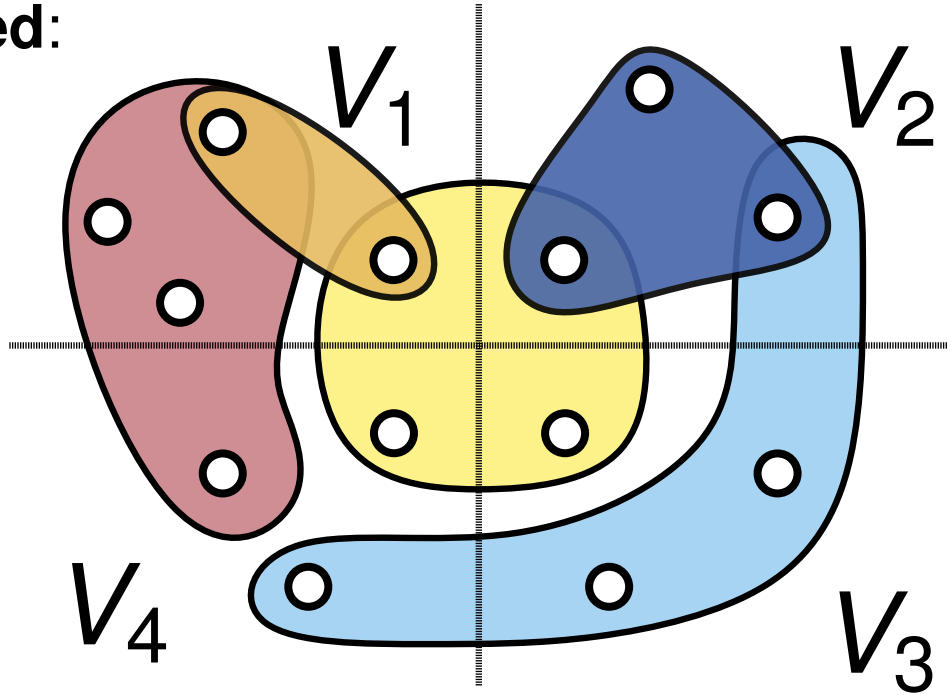
$$c(V_i) \leq (1 + \varepsilon) \left\lceil \frac{c(V)}{k} \right\rceil$$

- **connectivity** objective is **minimized**:

$$\sum_{e \in \text{cut}} (\lambda(e) - 1)\, \omega(e)$$

connectivity:
 # blocks connected by net e



Applications

[from SEA'17]

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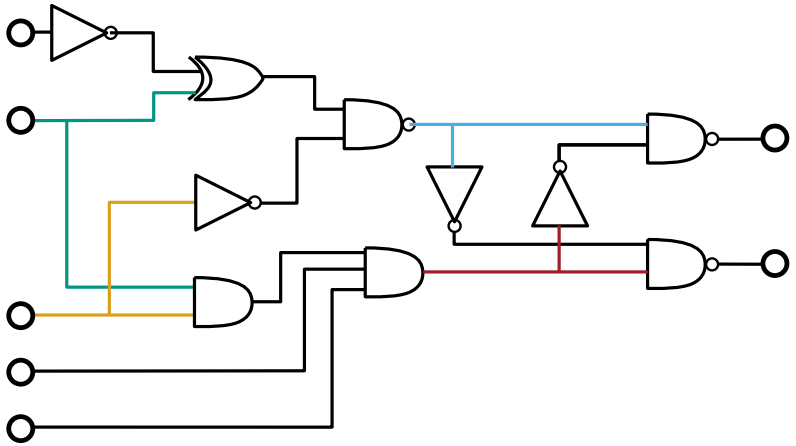
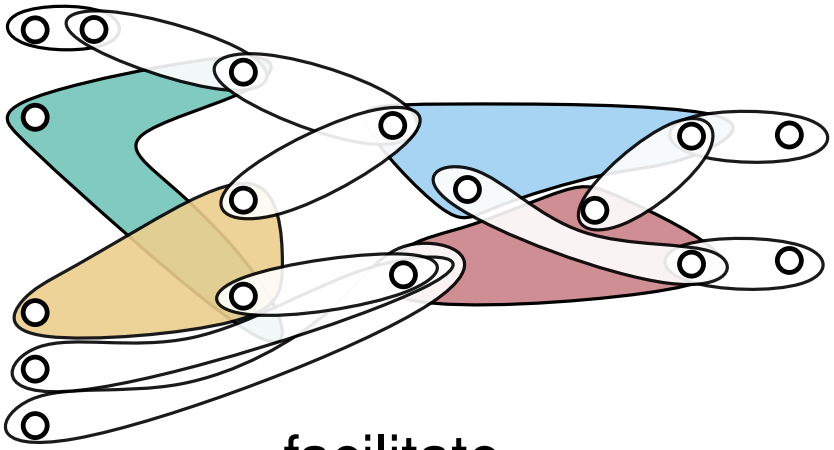
VLSI Design





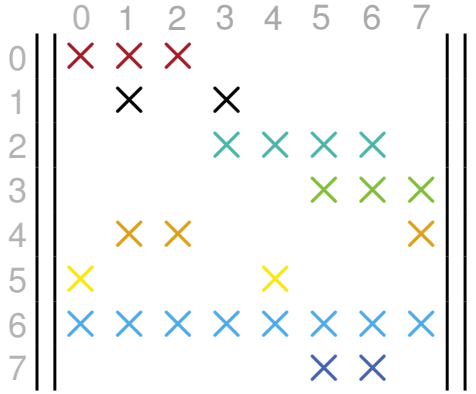
facilitate
floorplanning & placement

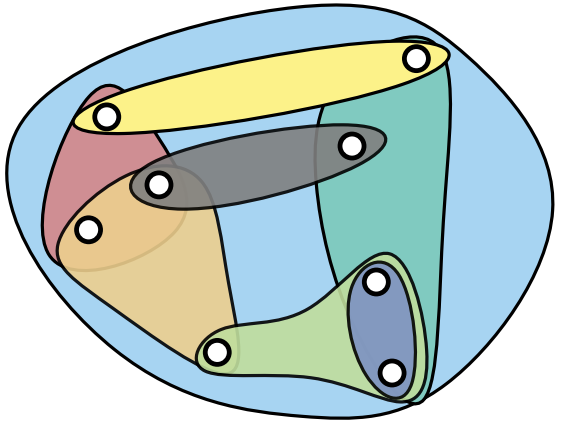
Application Domain

Hypergraph Model

Goal

Scientific Computing





minimize
communication

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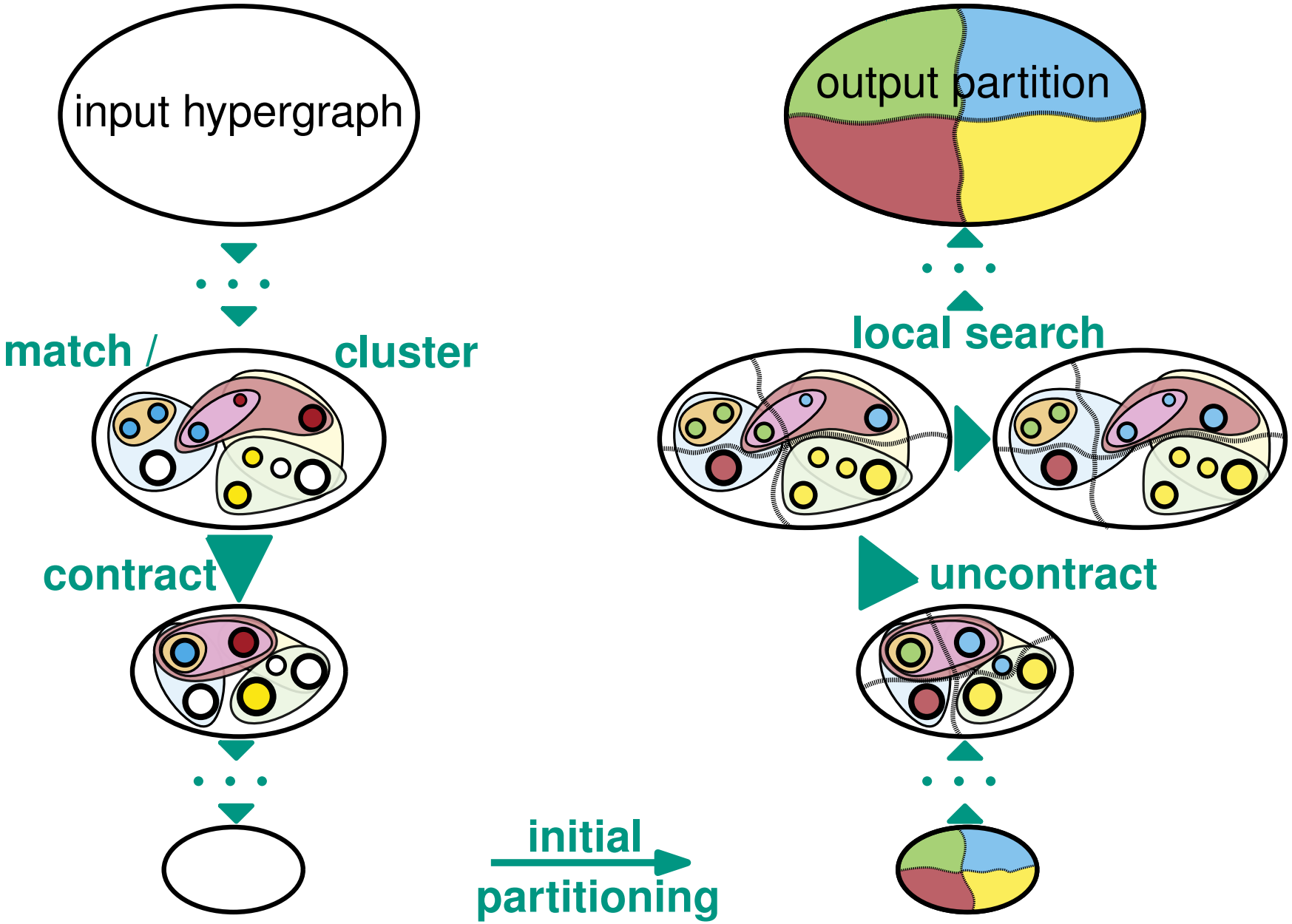
The Multilevel Framework

[from SEA'17]



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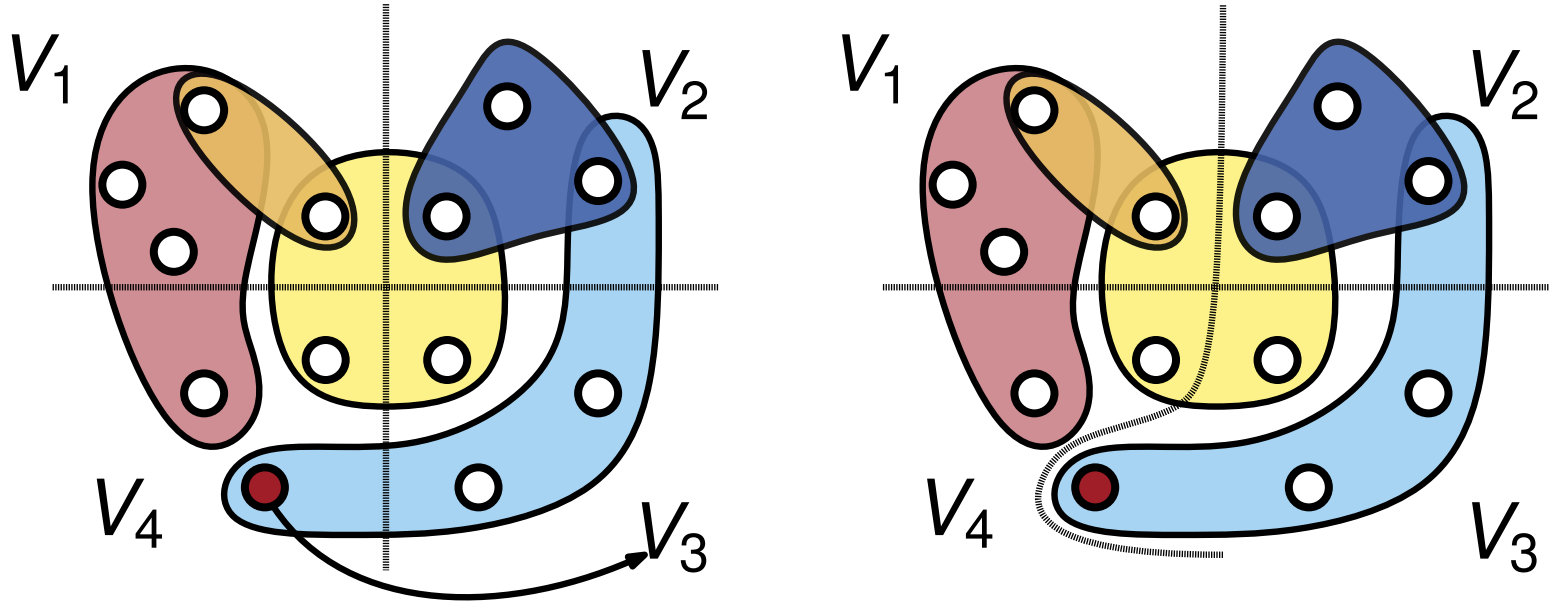
FM Algorithm

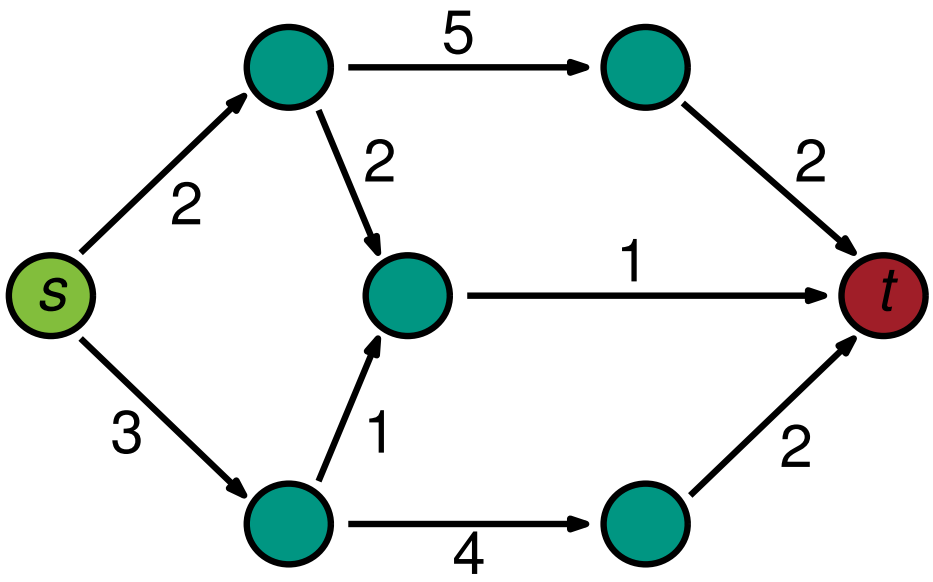
[Fiduccia, Mattheyses 88]



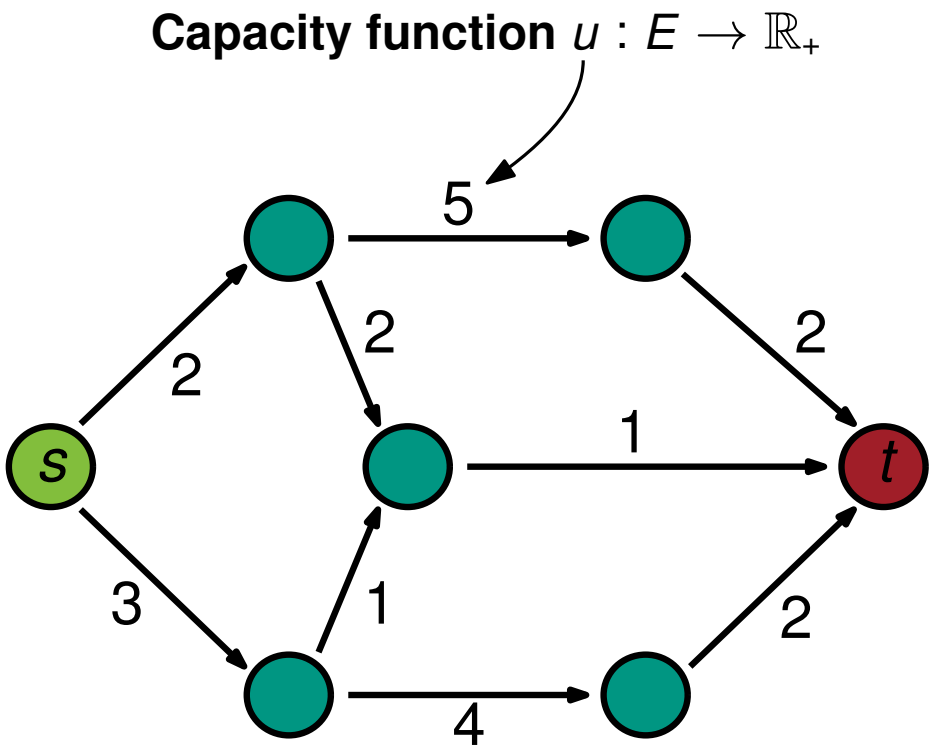
Moving ● from V_4 to V_3 reduces cut by 1

gain

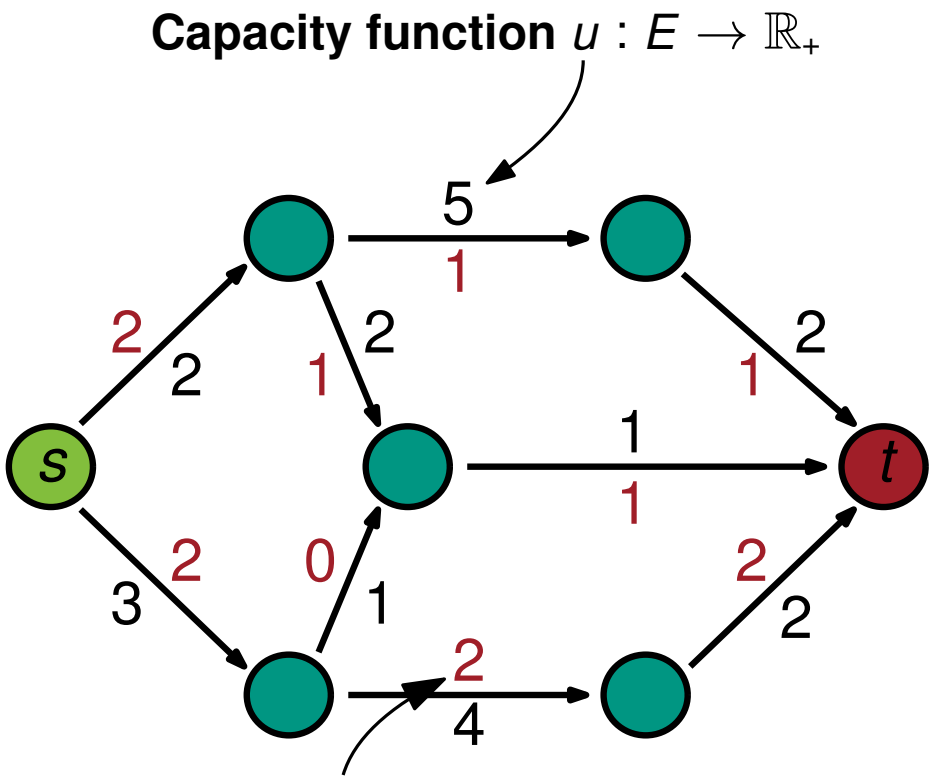
Flows



Flows

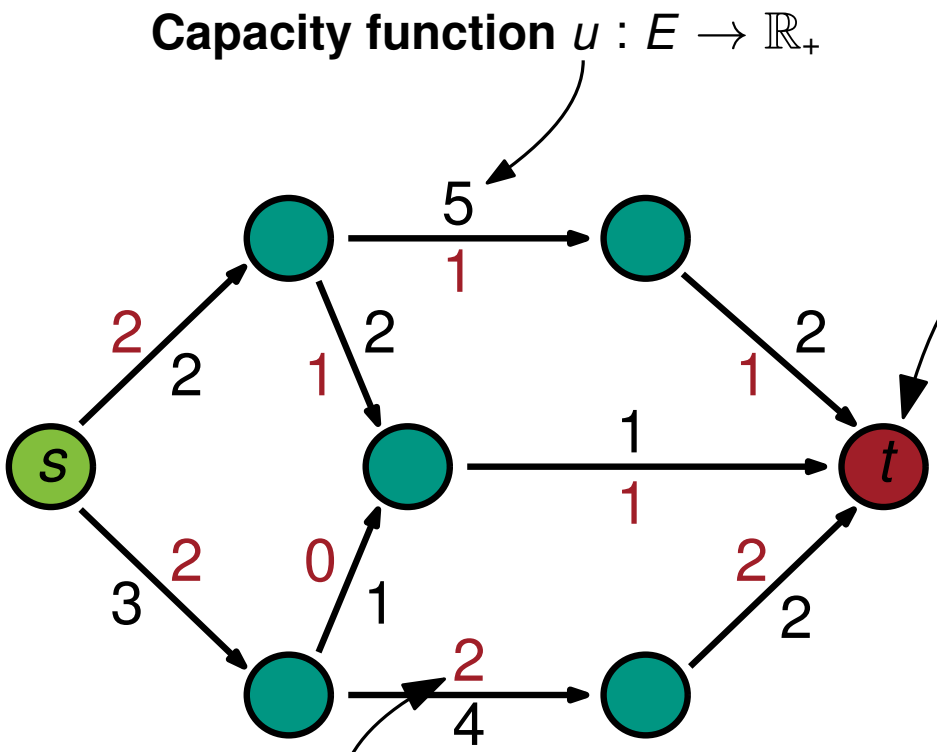


Flows



- $\forall (v, w) \in E : f(v, w) \leq u(v, w)$
- $\forall v \in V \setminus \{s, t\} : \sum_{(w,v) \in E} f(w, v) = \sum_{(v,w) \in E} f(v, w)$

Flows



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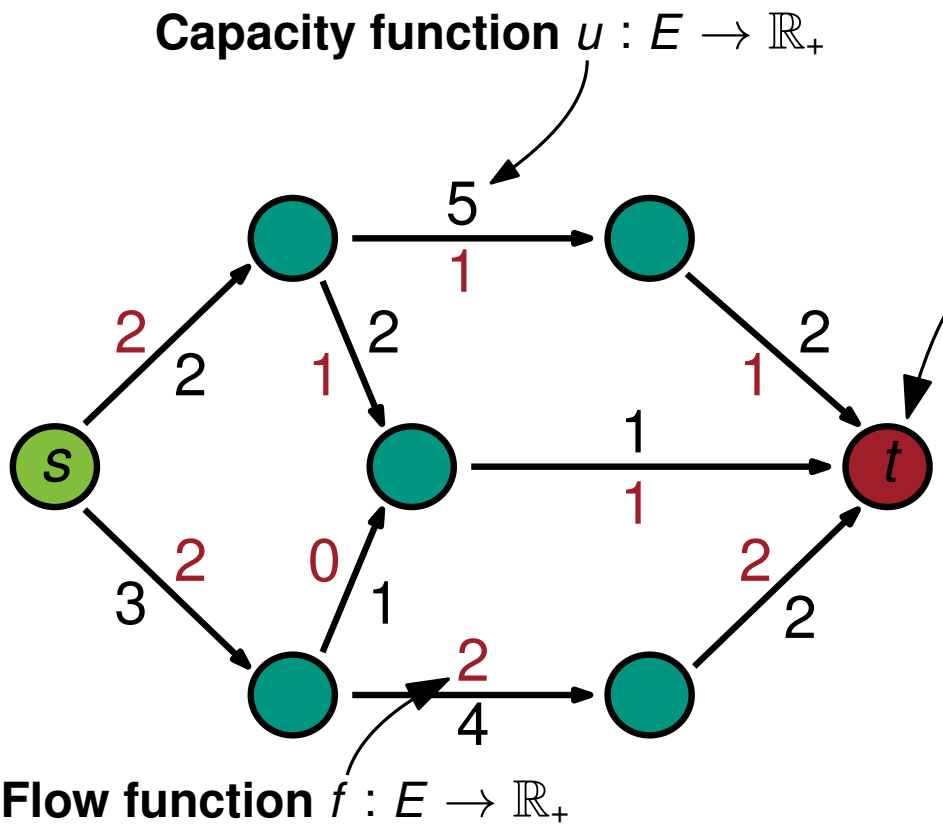
Flows



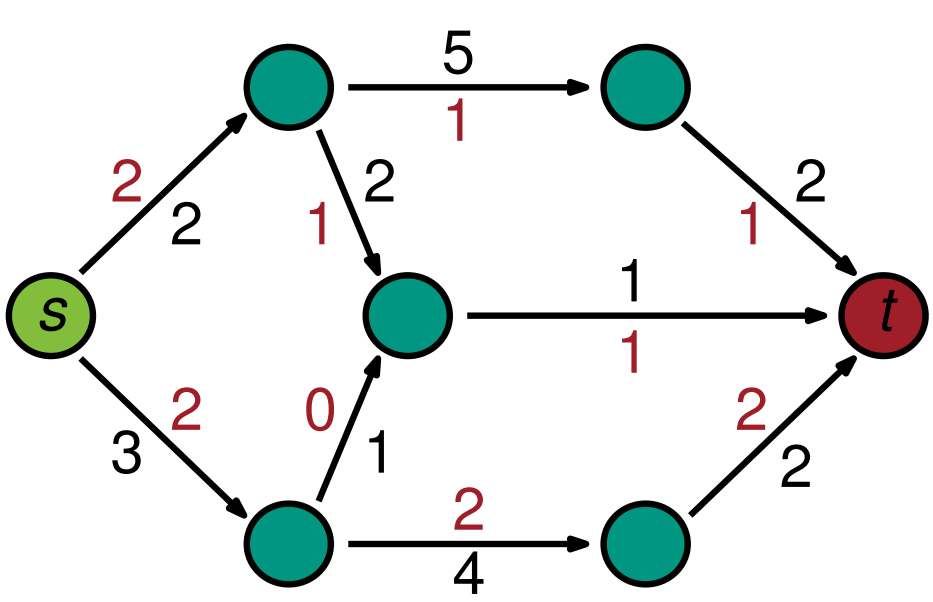
Value of flow $|f| = \sum_{(v,t)\in E} f(v,t)$

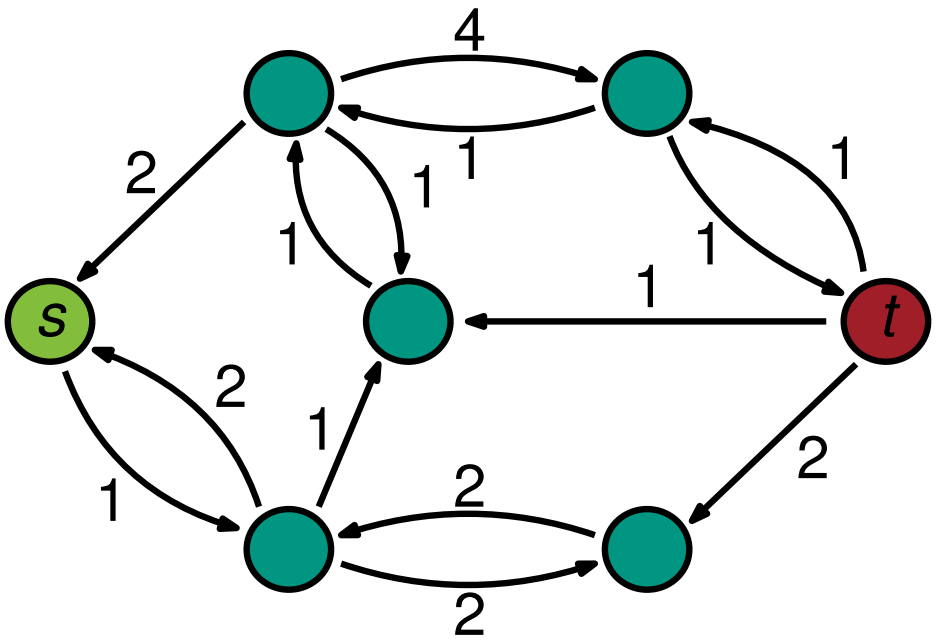
Maximum Flow Problem

Find a maximum flow f from s to t such that $\forall f' : |f'| \leq |f|$

- $\forall (v,w) \in E : f(v,w) \leq u(v,w)$
- $\forall v \in V \setminus \{s,t\} : \sum_{(w,v)\in E} f(w,v) = \sum_{(v,w)\in E} f(v,w)$

Flows





Residual Graph G_f

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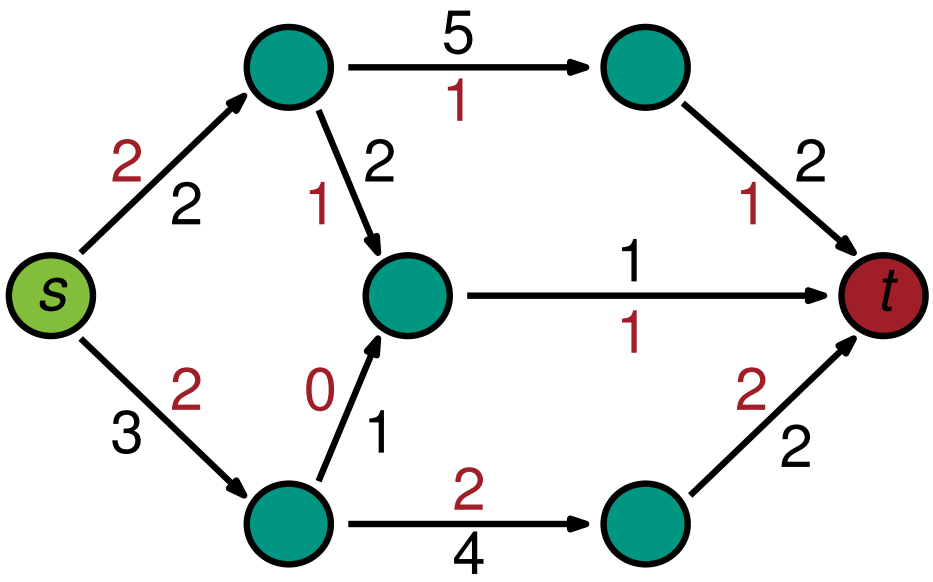
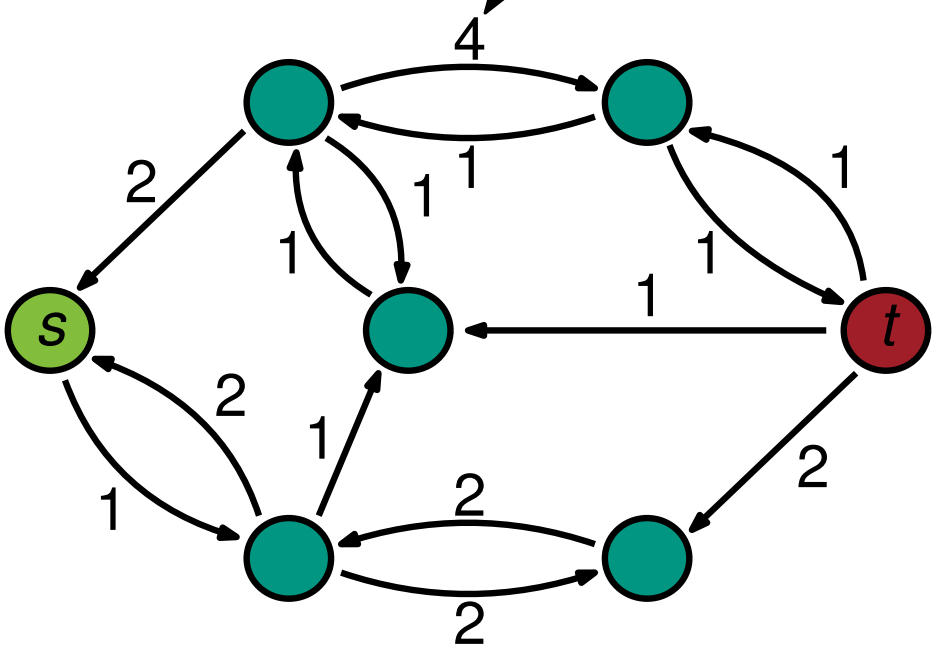
Flows



Residual capacity $r_f : V \times V \to \mathbb{R}_+$



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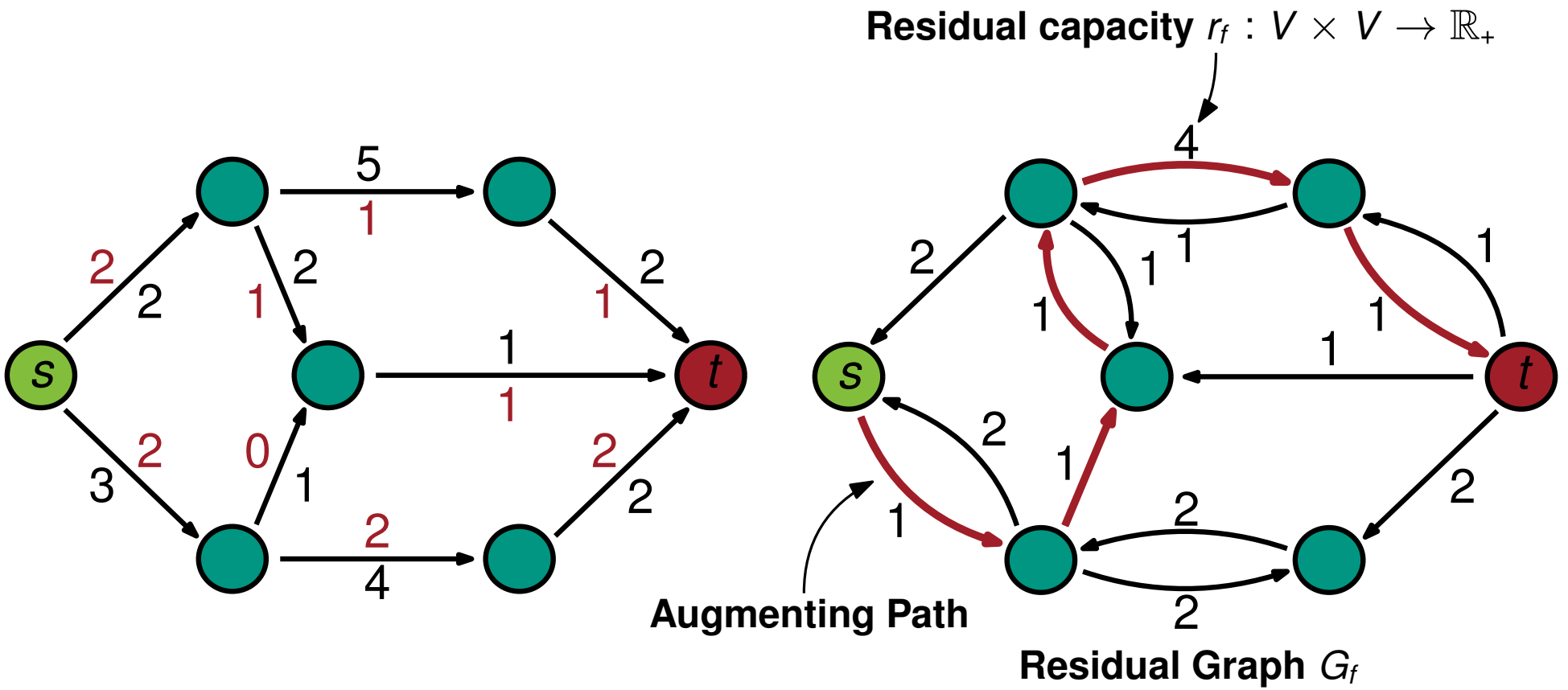

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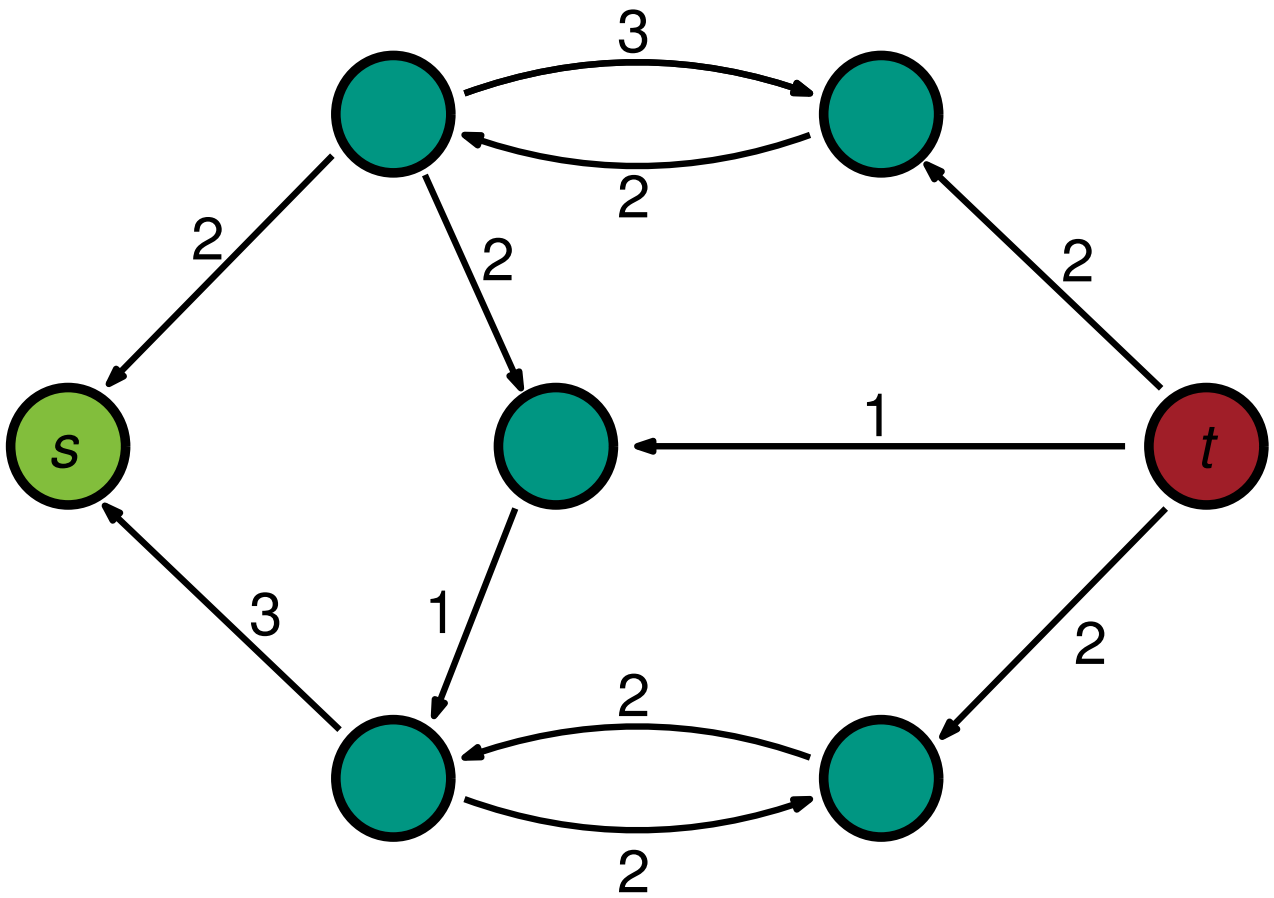
Minimum (s, t)-Bipartition

All nodes *reachable* from s are part of V_1 and $V_2 = V \setminus V_1$



Residual Graph G_f of a maximum flow f

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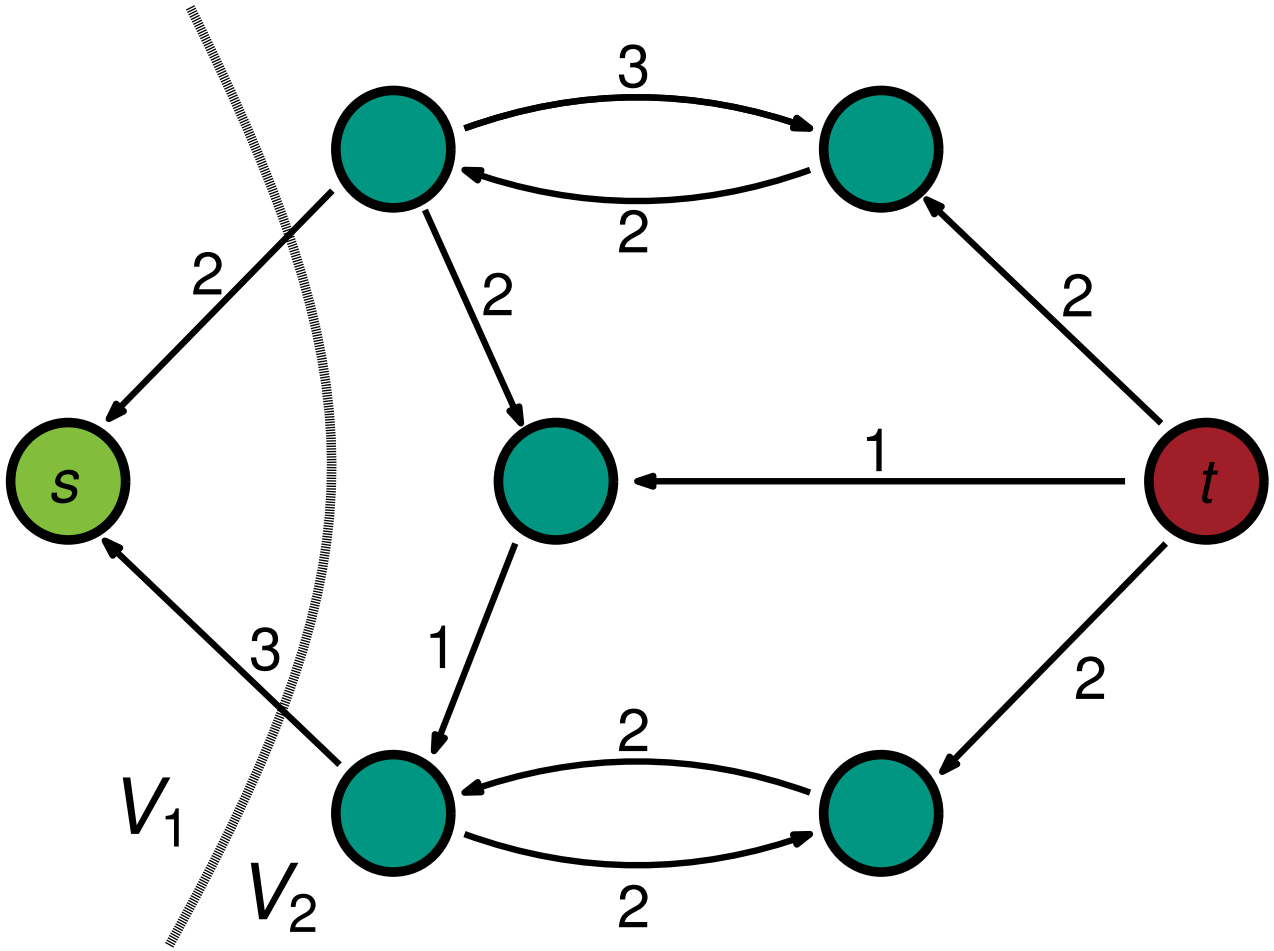
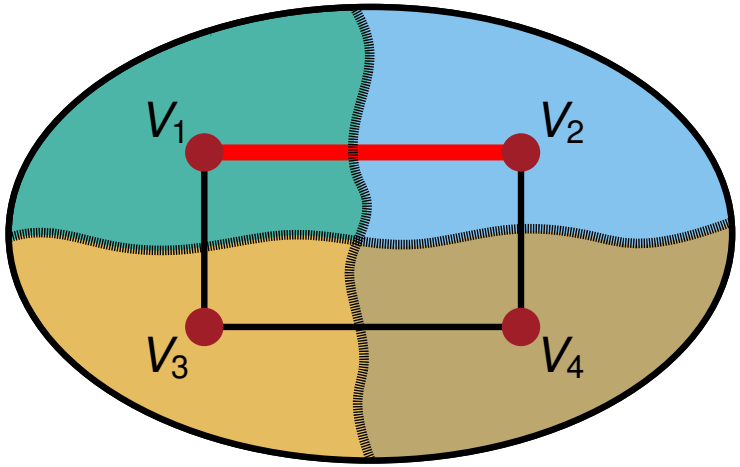
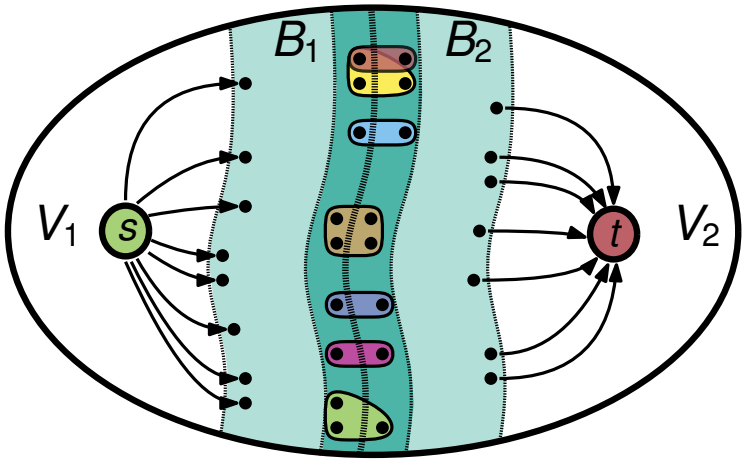


Residual Graph G_f of a maximum flow f

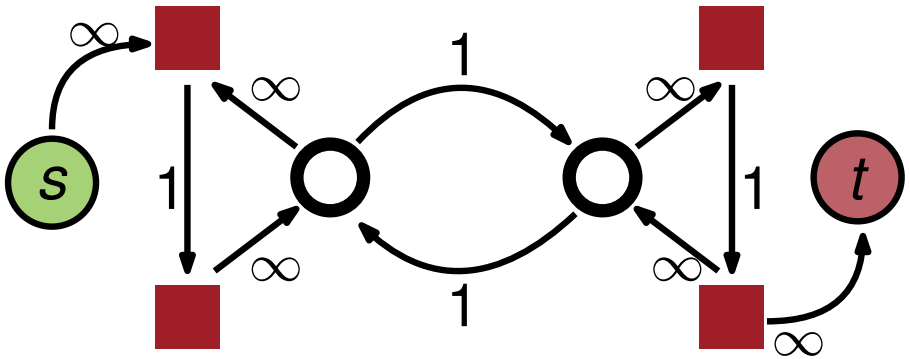
Our Flow-Based Refinement Framework



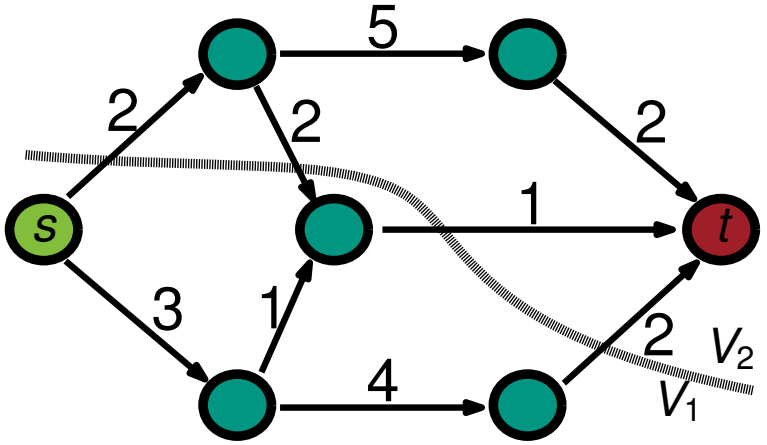
Select two adjacent blocks for refinement



Build Flow Problem



Solve Flow Problem



Find feasible minimum cut

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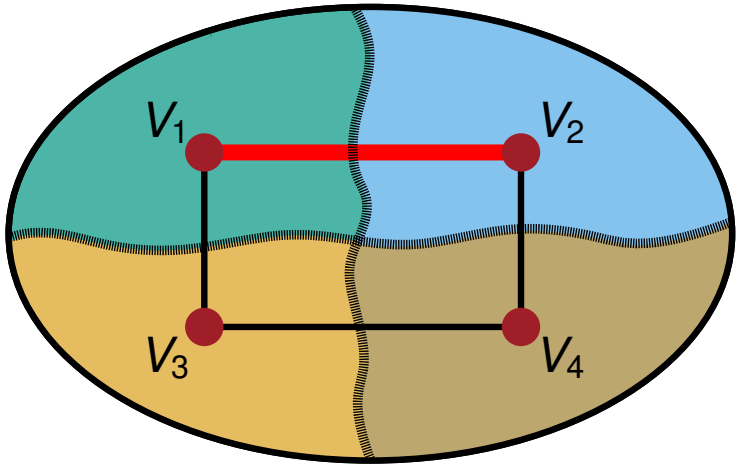
Our Flow-Based Refinement Framework



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Build Flow Problem

Solve Flow Problem

Find feasible minimum cut

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Our Flow-Based Refinement Framework

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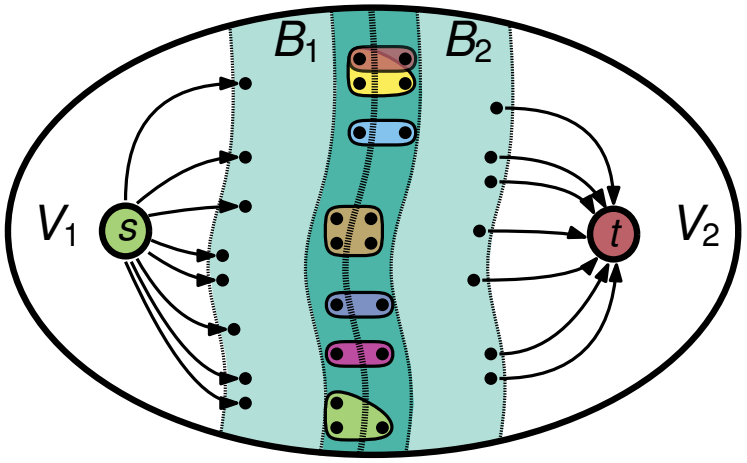

Build Flow Problem

Solve Flow Problem

Find feasible minimum cut

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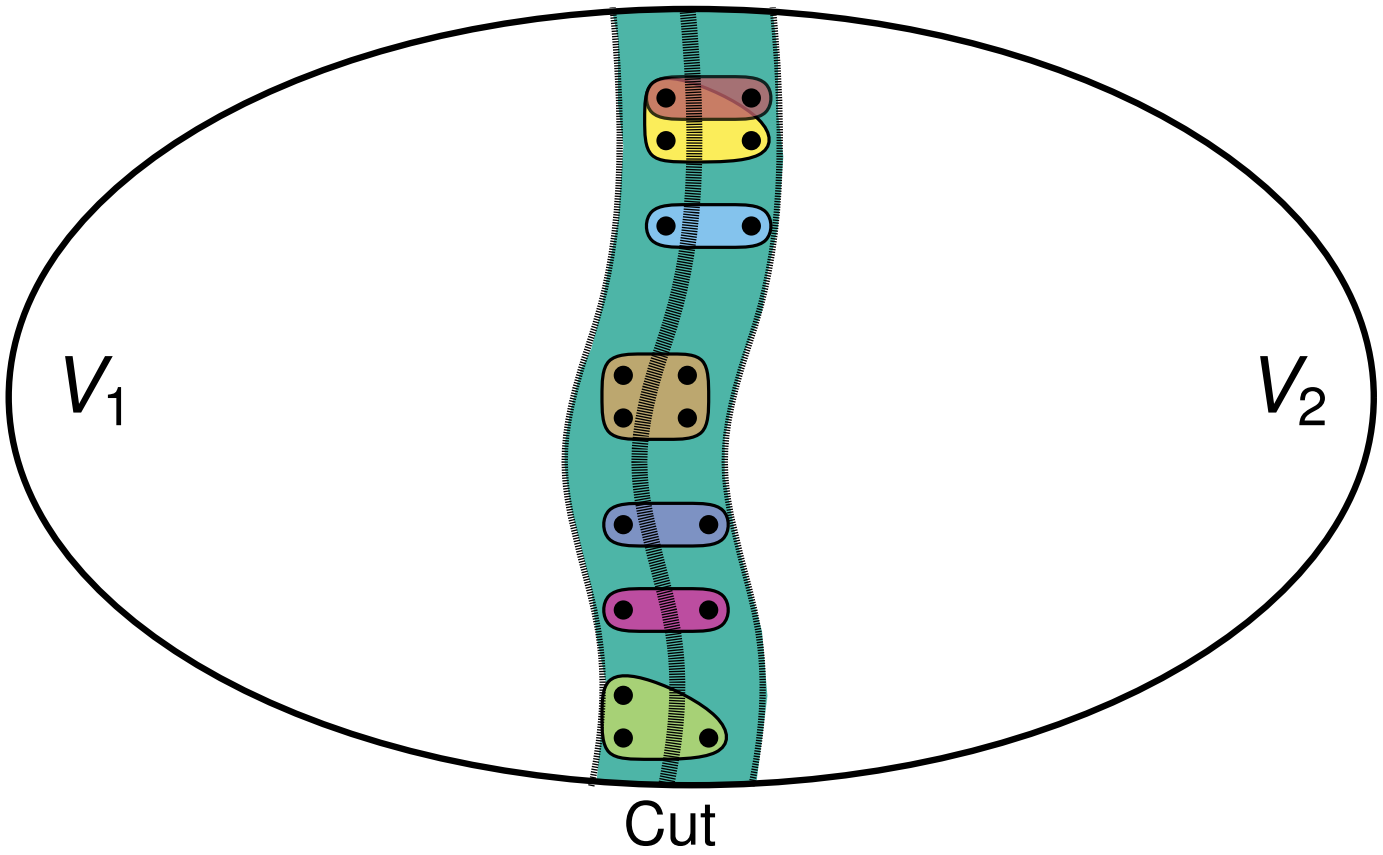
Adaptive Flow Iterations

KaFFPa [Sanders, Schulz 11]



Adaptive Flow Iterations

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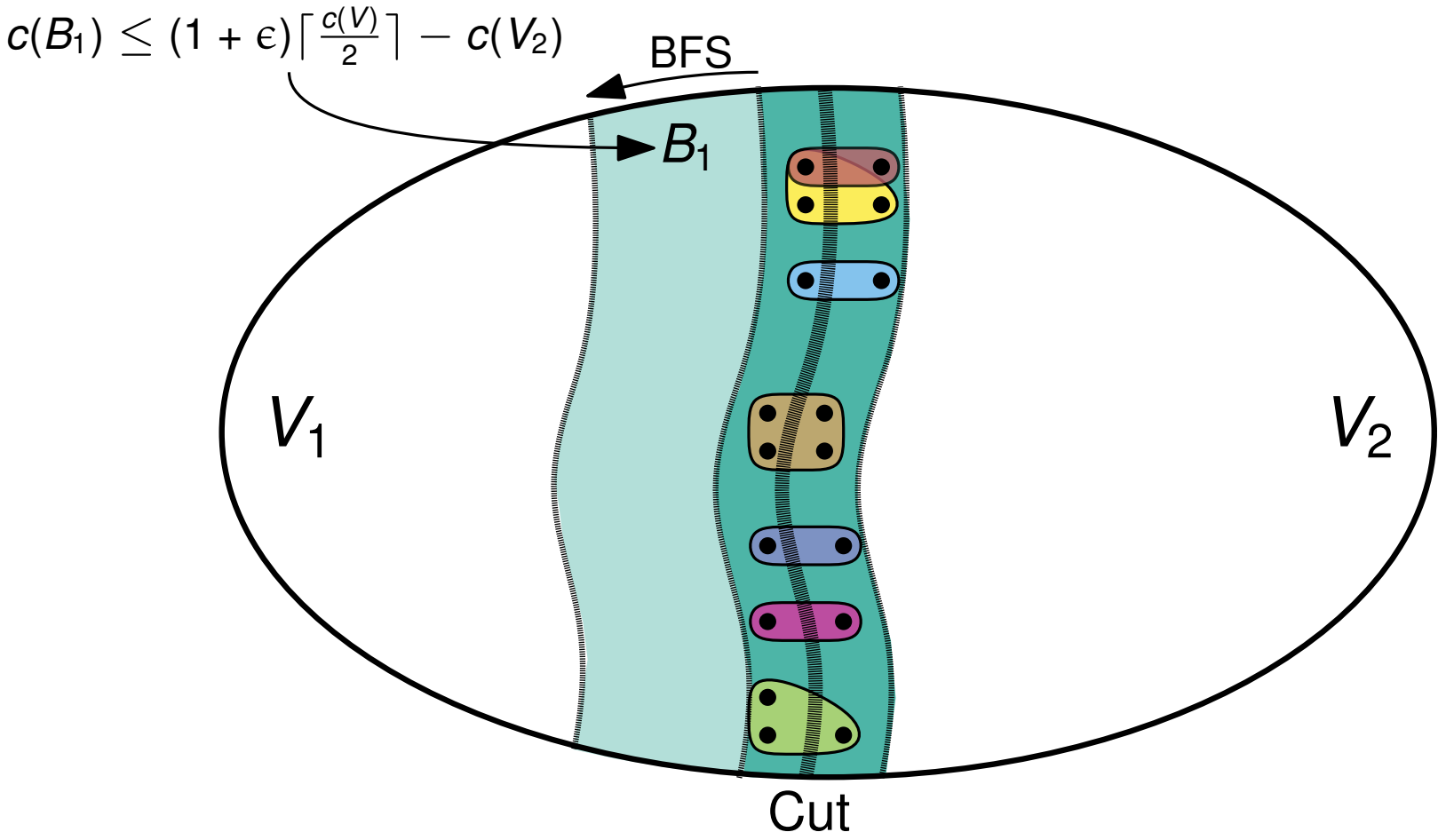

Adaptive Flow Iterations

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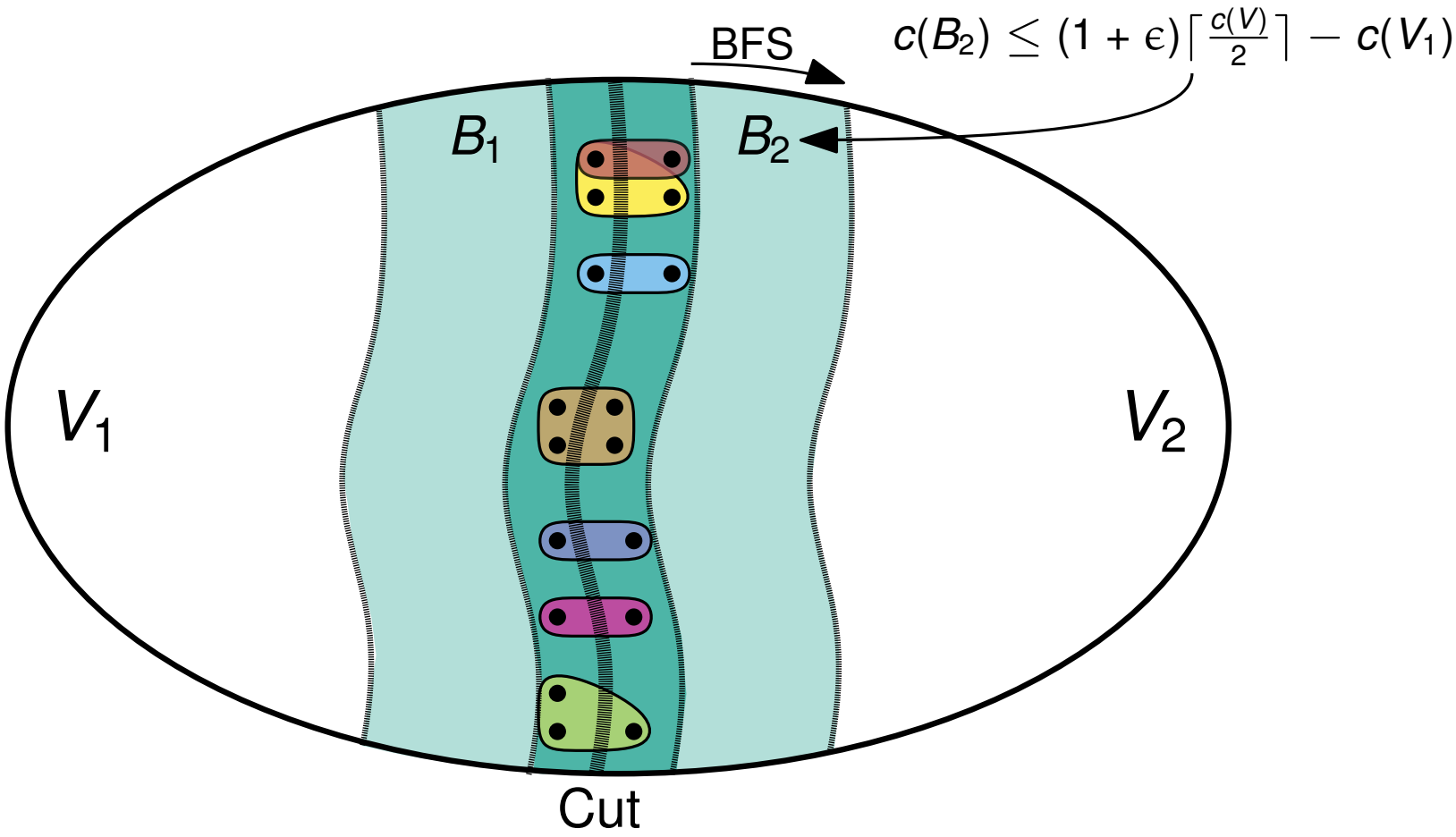

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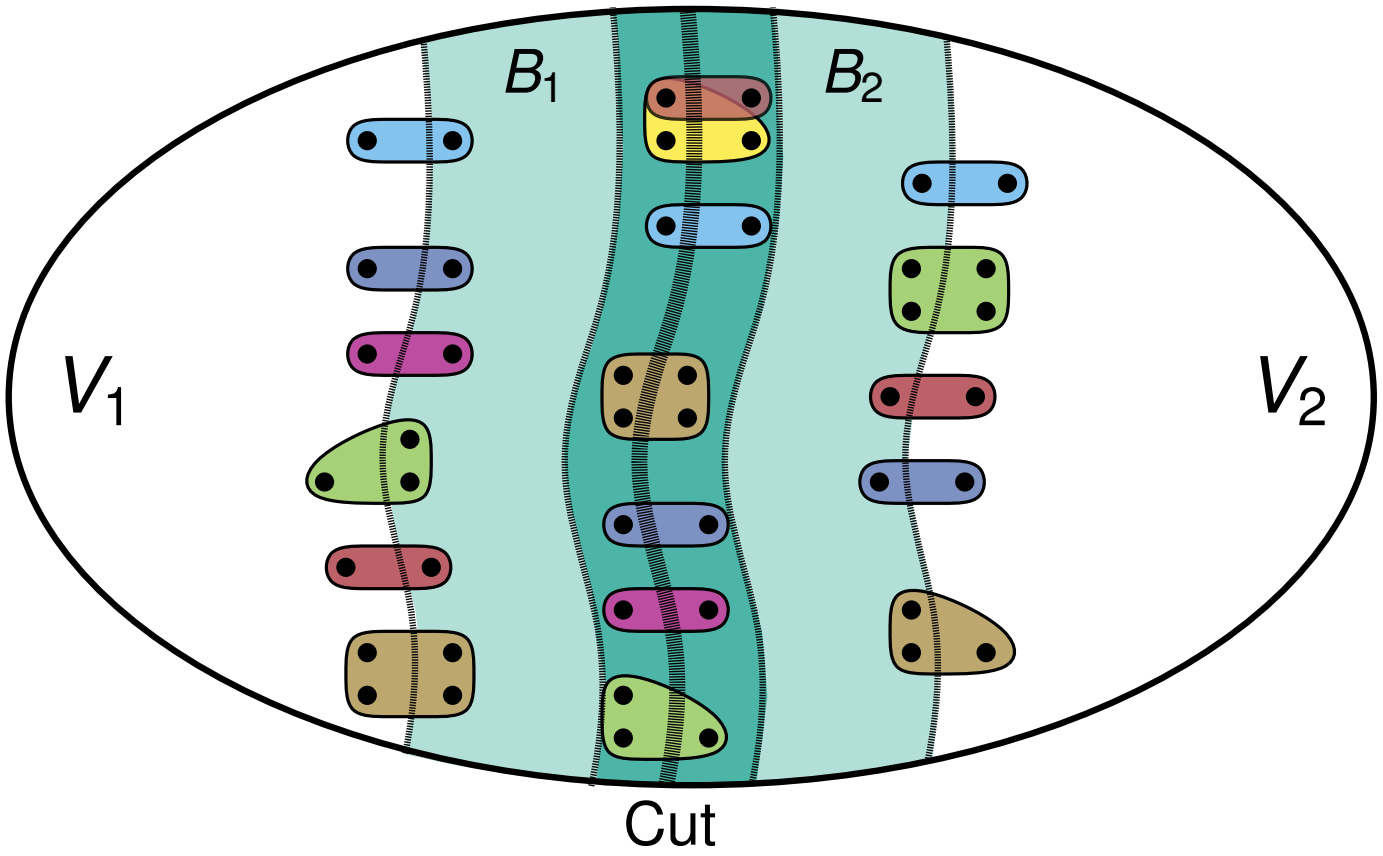

Adaptive Flow Iterations

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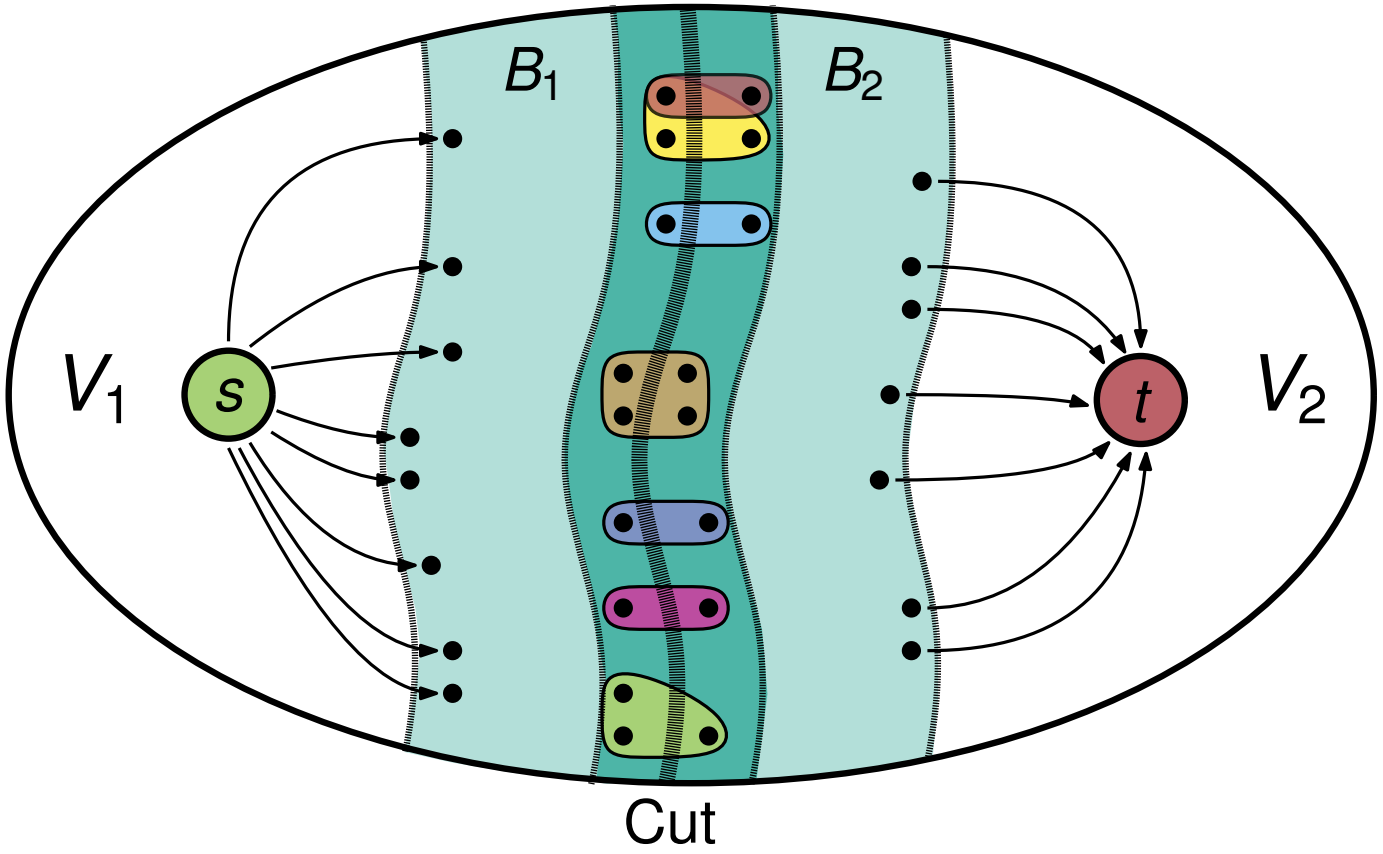

Adaptive Flow Iterations

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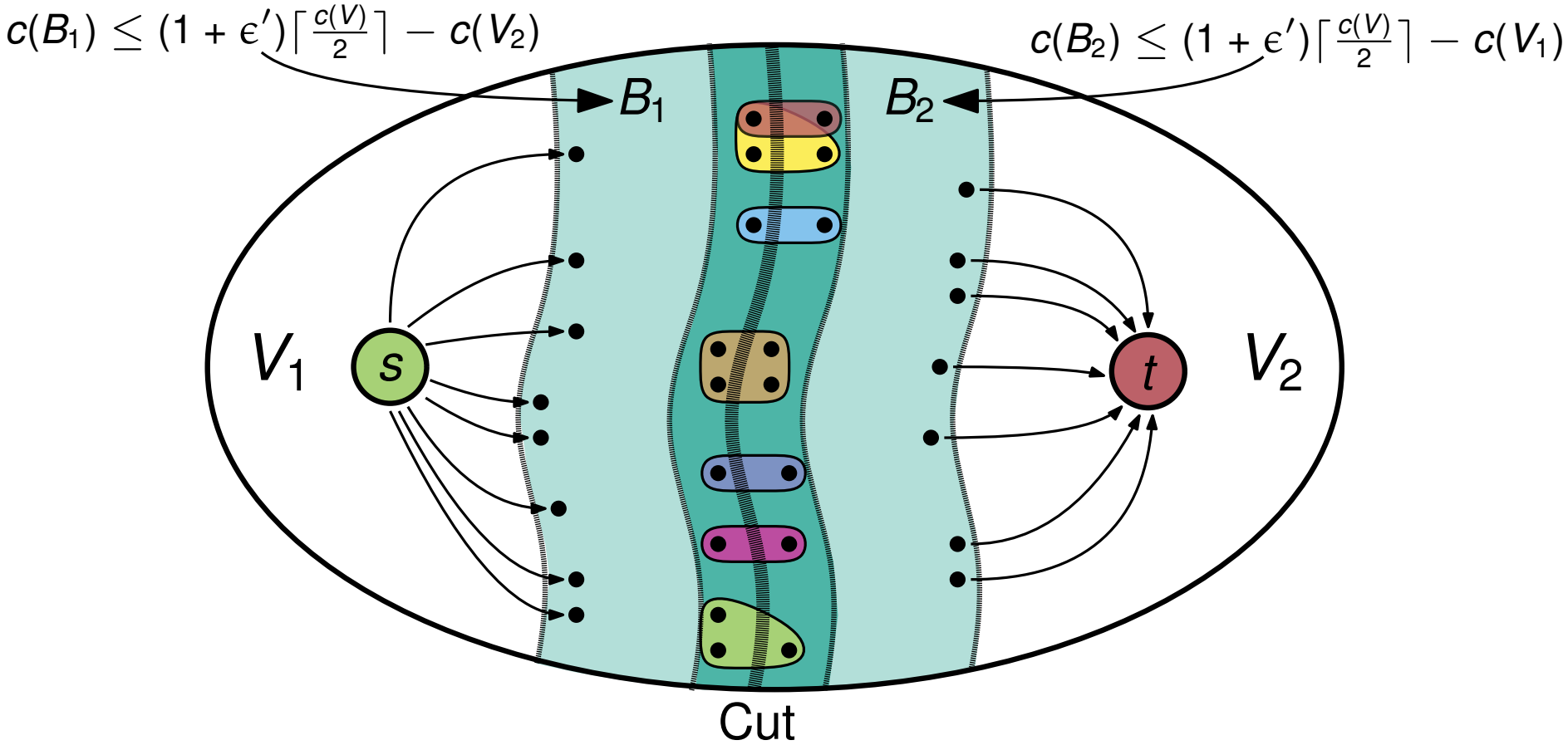
Use $\epsilon' = \alpha\epsilon$ instead of ϵ



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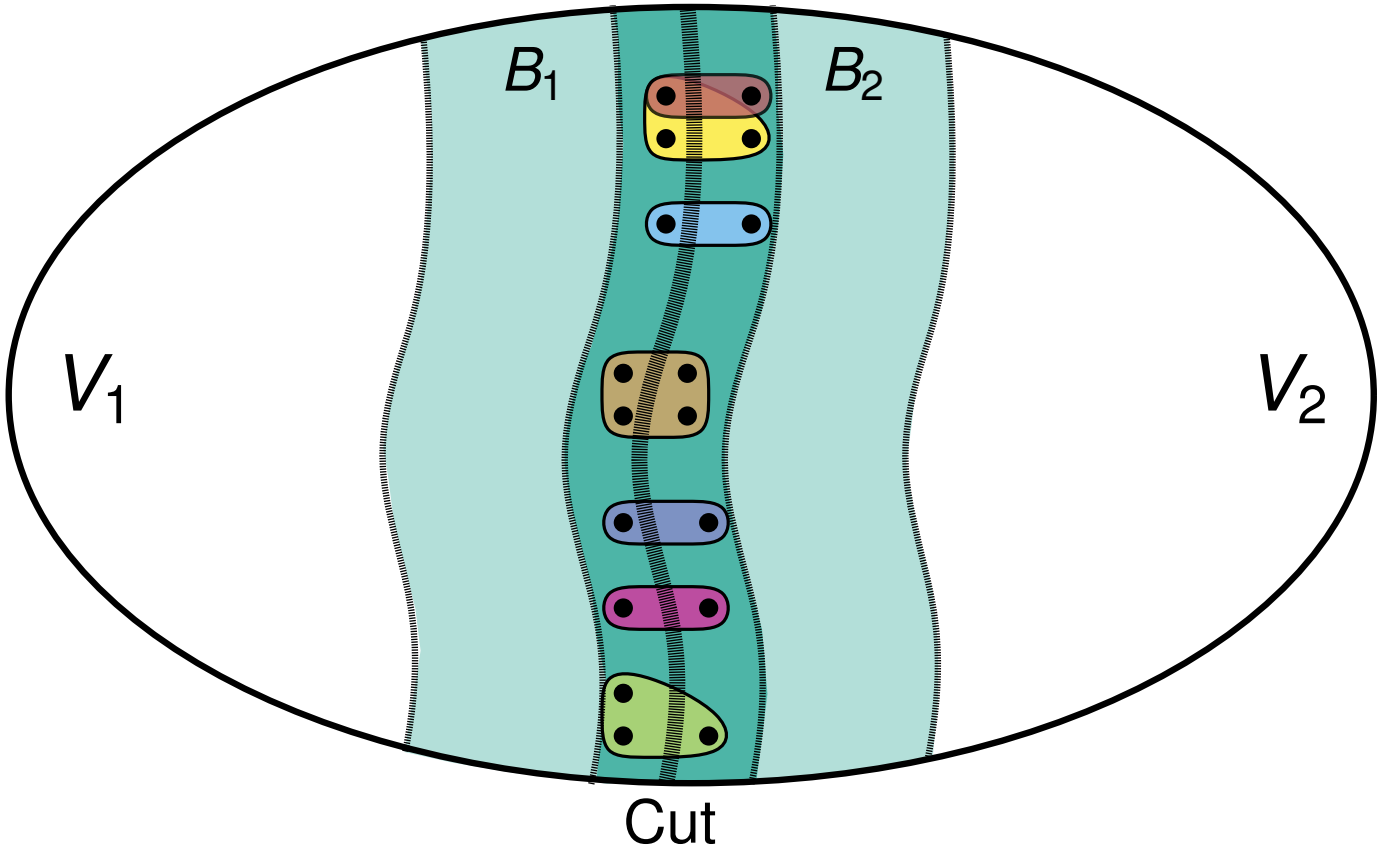
Adaptive Flow Iterations

KaFFPa [Sanders, Schulz 11]

Use $\epsilon' = \alpha\epsilon$ instead of ϵ

$\alpha = 1 \Rightarrow$ **Improvement Found** $\Rightarrow \alpha = \max(2\alpha, \alpha') = 2$

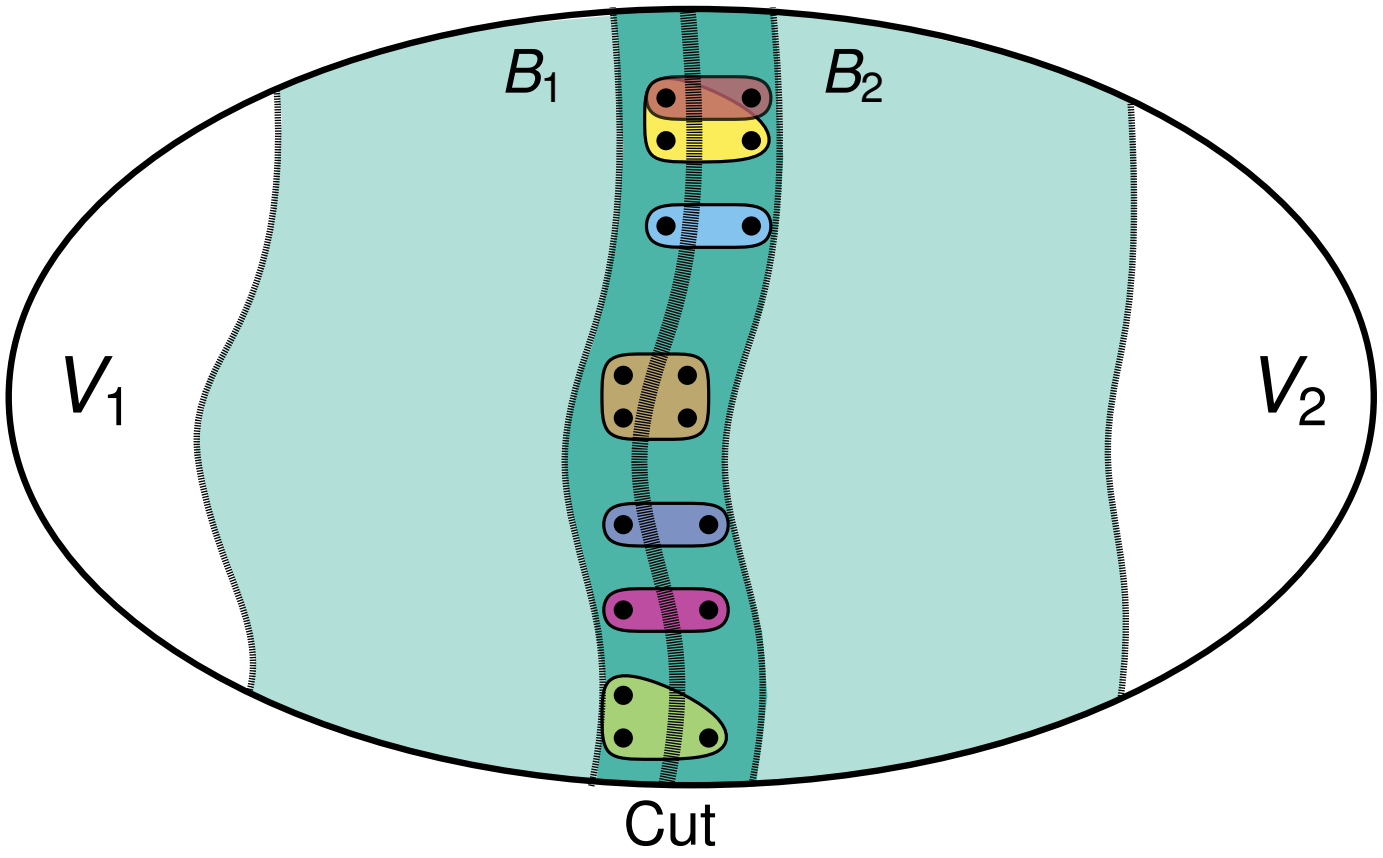


Adaptive Flow Iterations

KaFFPa [Sanders, Schulz 11]

Use $\epsilon' = \alpha\epsilon$ instead of ϵ

$\alpha = 2 \Rightarrow$ **No Improvement** $\Rightarrow \alpha = \min(\frac{\alpha}{2}, 1) = 1$



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Adaptive Flow Iterations

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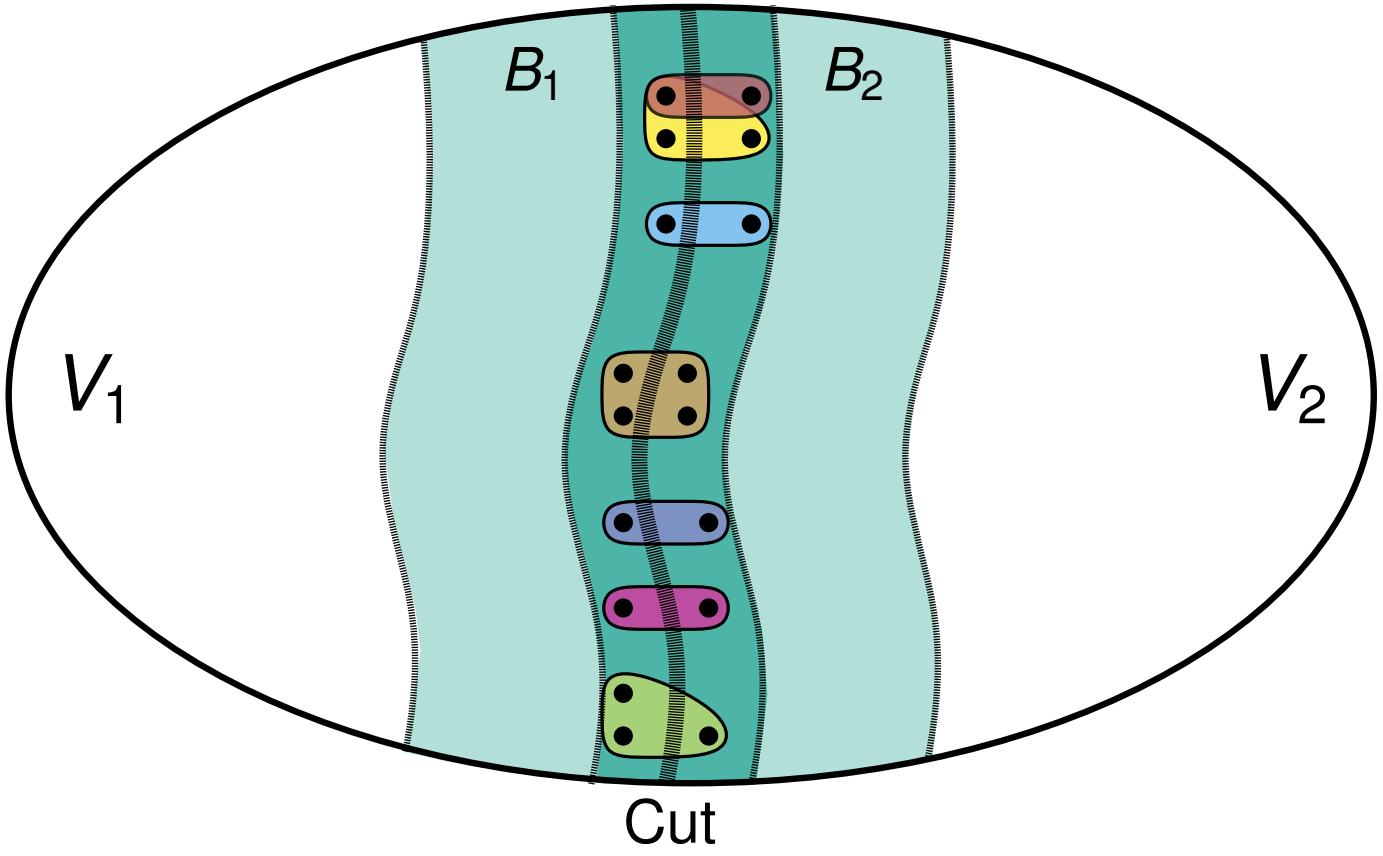
Use $\epsilon' = \alpha\epsilon$ instead of ϵ

$\alpha = 1 \Rightarrow$ **No Improvement** $\Rightarrow$ **Terminate**



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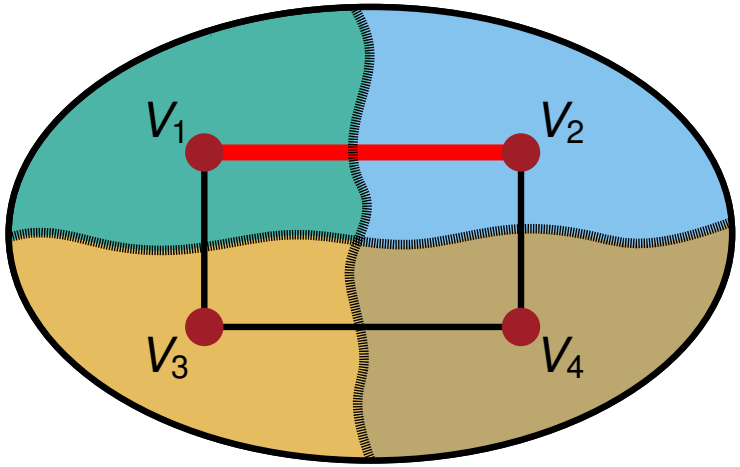
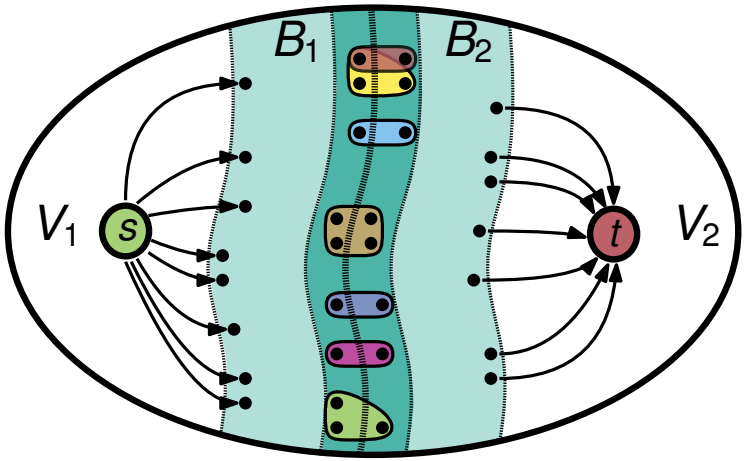
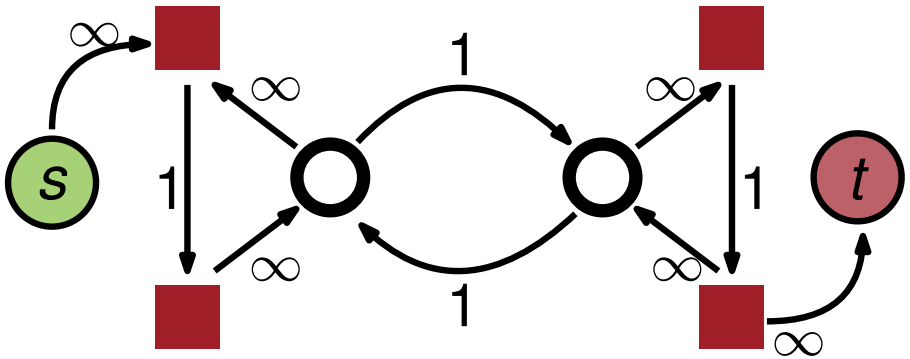


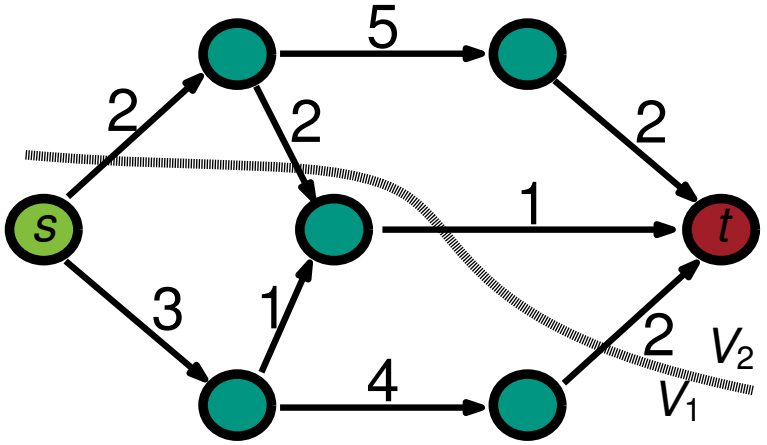
Select two adjacent blocks for refinement



Build Flow Problem



Solve Flow Problem



Find feasible minimum cut

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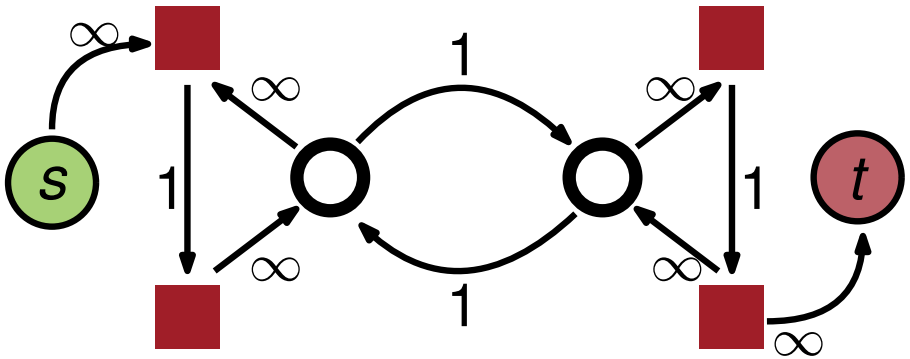
Our Flow-Based Refinement Framework

Select two adjacent blocks for refinement

Build Flow Problem



Solve Flow Problem

Find feasible minimum cut

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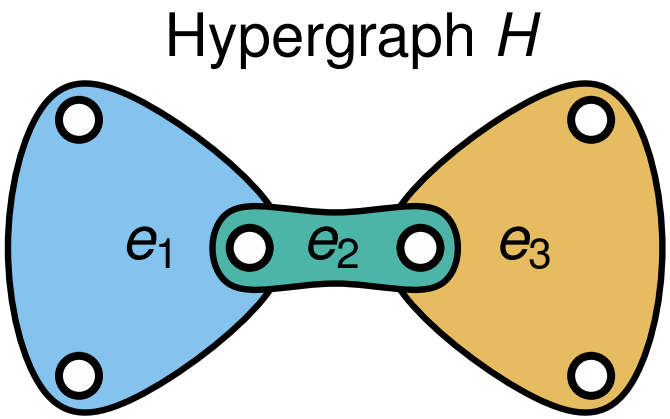
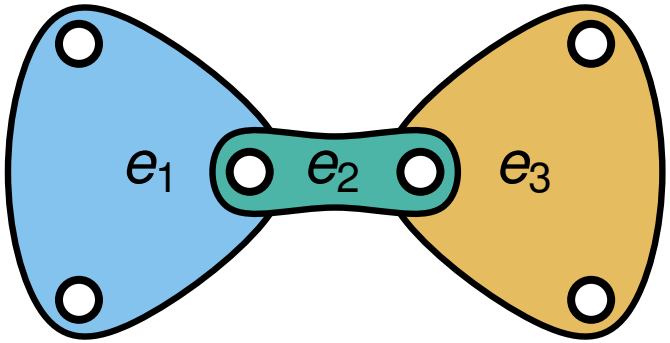
Hypergraph Flow Network

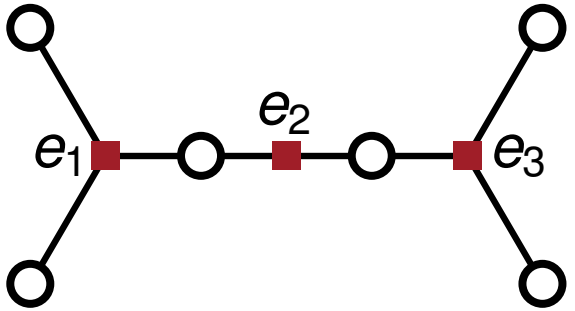


Hypergraph Flow Network

Hypergraph H

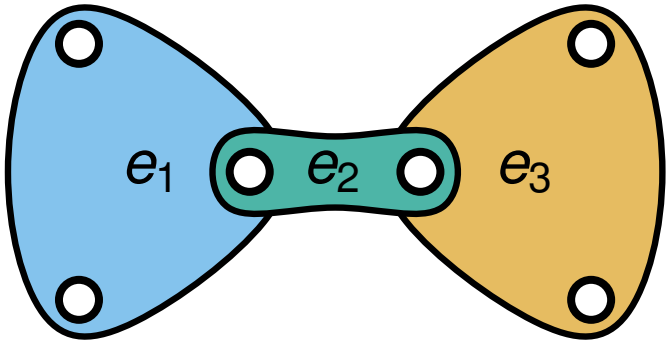


Bipartite Graph $G_*(H)$

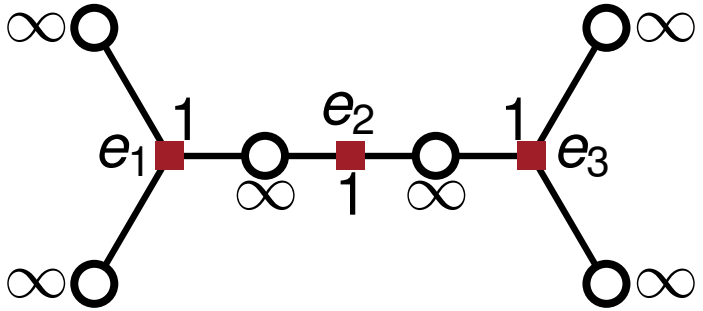


Hypergraph Flow Network

Hypergraph H



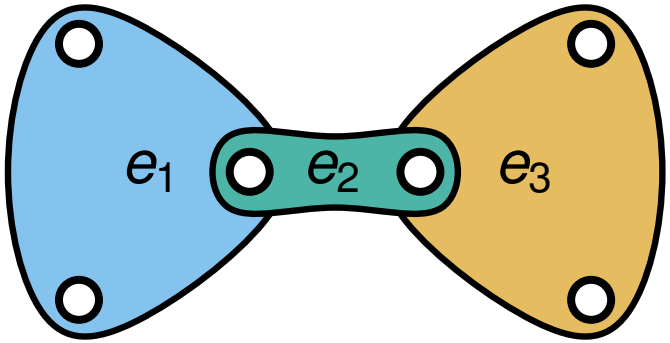
Bipartite Graph $G_*(H)$



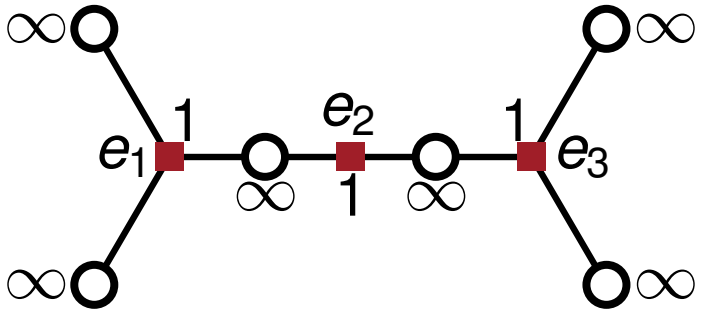
Vertex Separator Problem [Hu, Moerder 85]

Hypergraph Flow Network

Hypergraph H

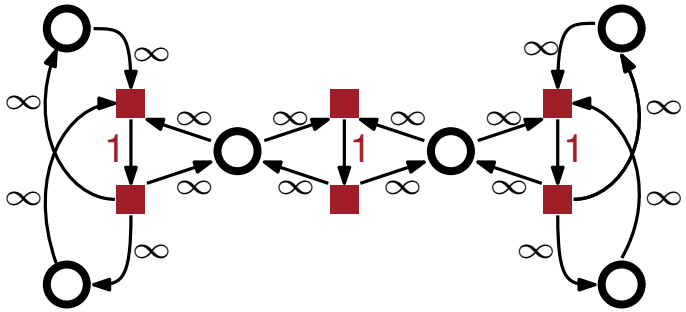


Bipartite Graph $G_*(H)$



Vertex Separator Problem [Hu, Moerder 85]

Lawler Network [Lawler 73]



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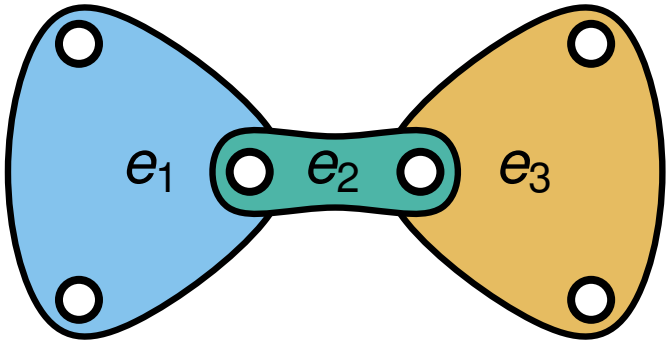
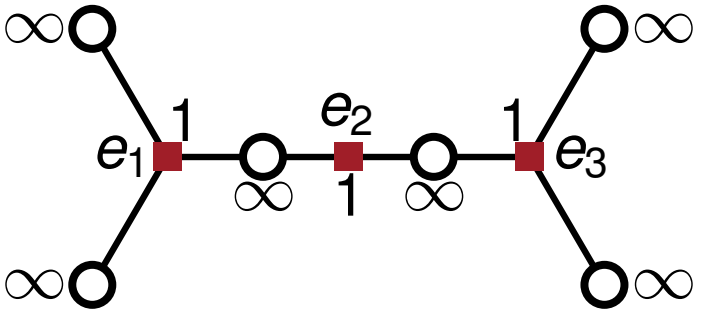
Hypergraph Flow Network

Hypergraph H

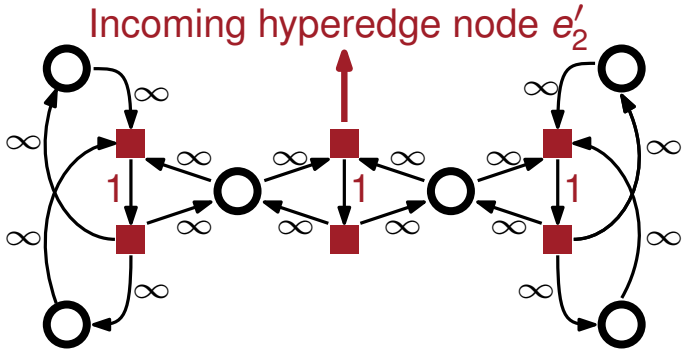


Bipartite Graph $G_*(H)$



Vertex Separator Problem [Hu, Moerder 85]

Lawler Network [Lawler 73]



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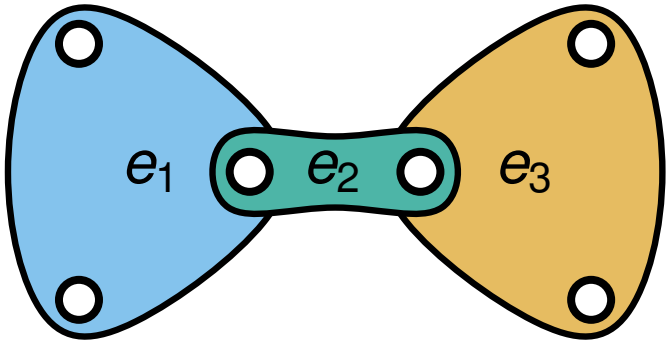
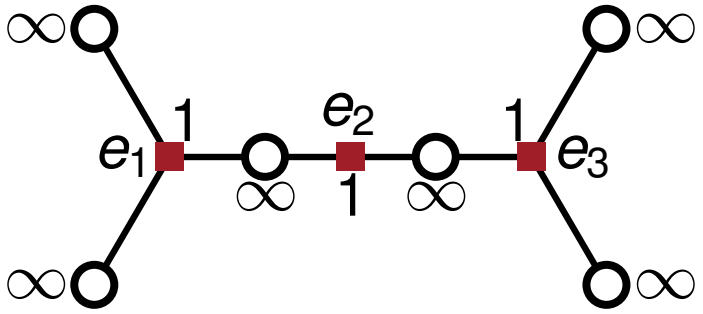
Hypergraph Flow Network

Hypergraph H



Bipartite Graph $G_*(H)$



Vertex Separator Problem [Hu, Moerder 85]

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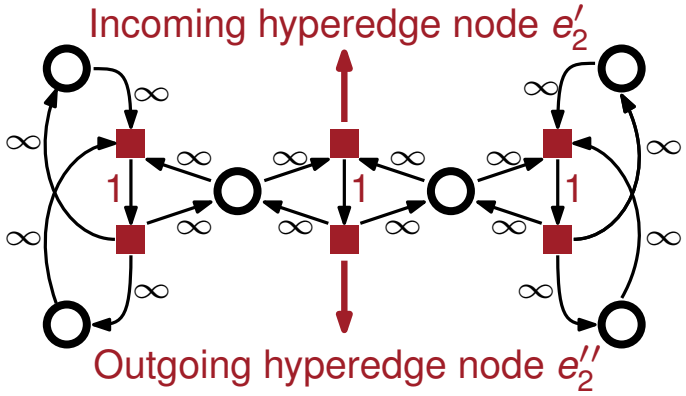

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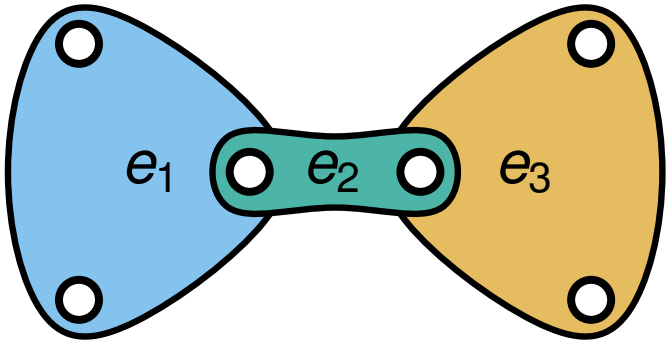
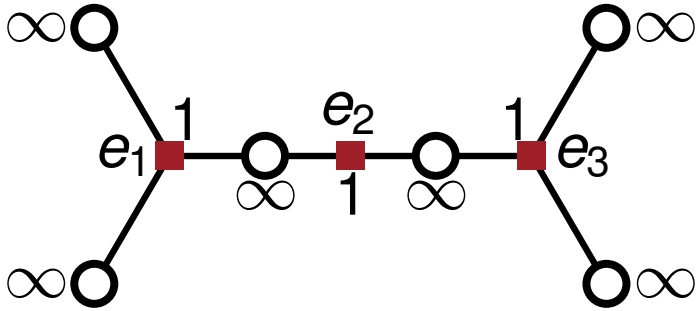
Hypergraph Flow Network - Low Degree Vertices

Hypergraph H



Bipartite Graph $G_*(H)$



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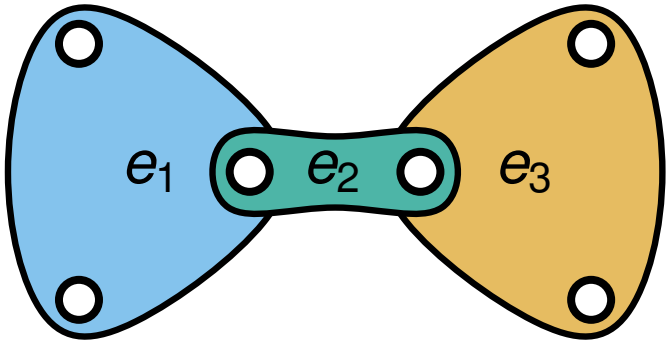
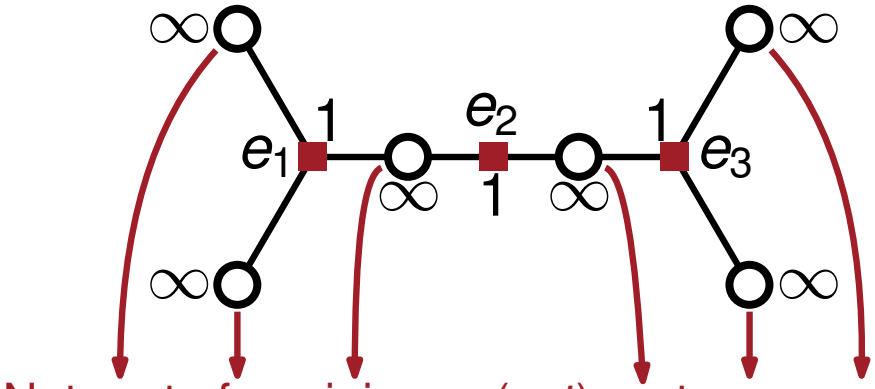
Hypergraph Flow Network - Low Degree Vertices

Hypergraph H



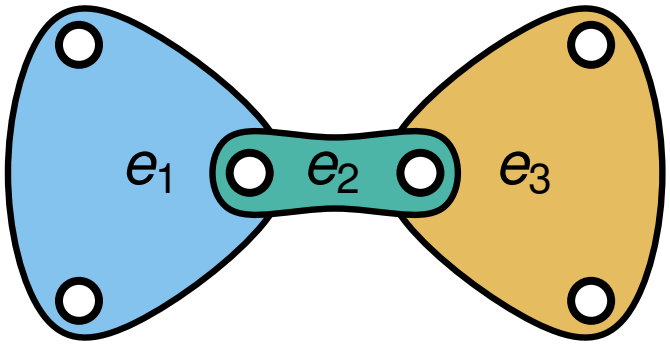
Bipartite Graph $G_*(H)$

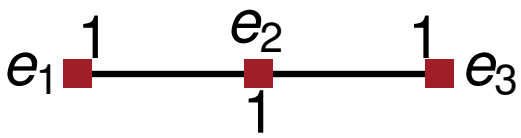


Not part of a minimum (s, t)-vertex separator

Hypergraph Flow Network - Low Degree Vertices

Hypergraph H

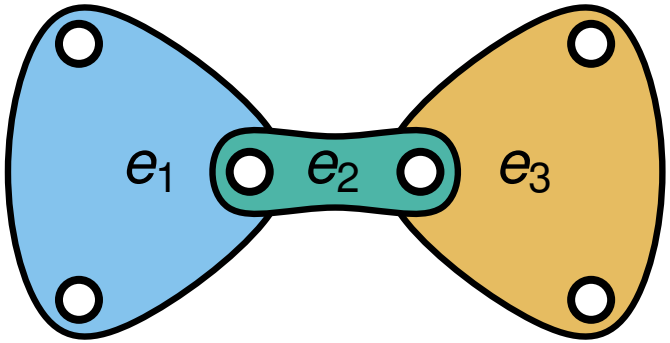


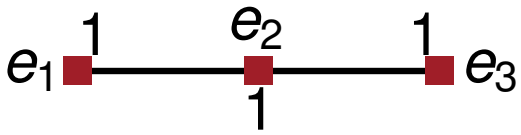


Remove all vertices by adding a clique

Hypergraph Flow Network - Low Degree Vertices

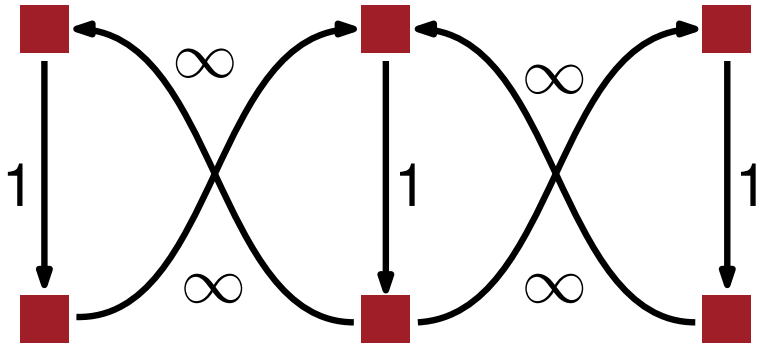
Hypergraph H





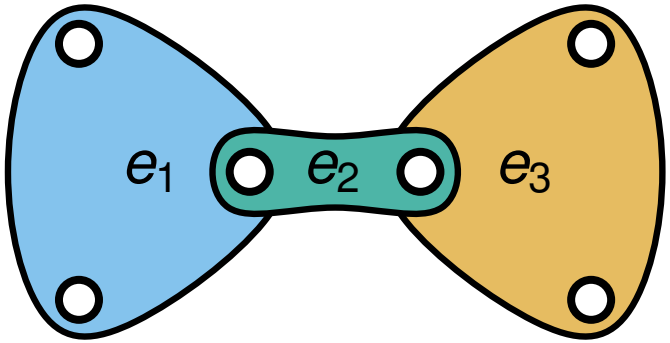
Remove all vertices by adding a clique

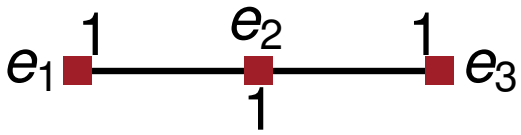
Our Network



Hypergraph Flow Network - Low Degree Vertices

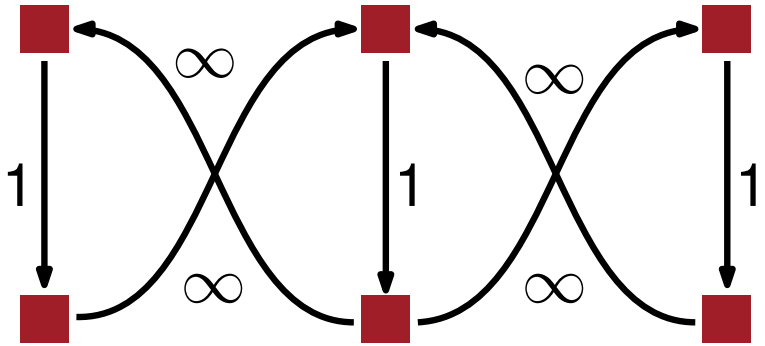
Hypergraph H





Remove all vertices by adding a clique

Our Network



A hypernode v induces . . .

- . . . $2d(v)$ edges in the Lawler Network
- . . . $d(v)(d(v)-1)$ edges in our network

If $d(v) \leq 3$, then $d(v)(d(v)-1) \leq 2d(v)$

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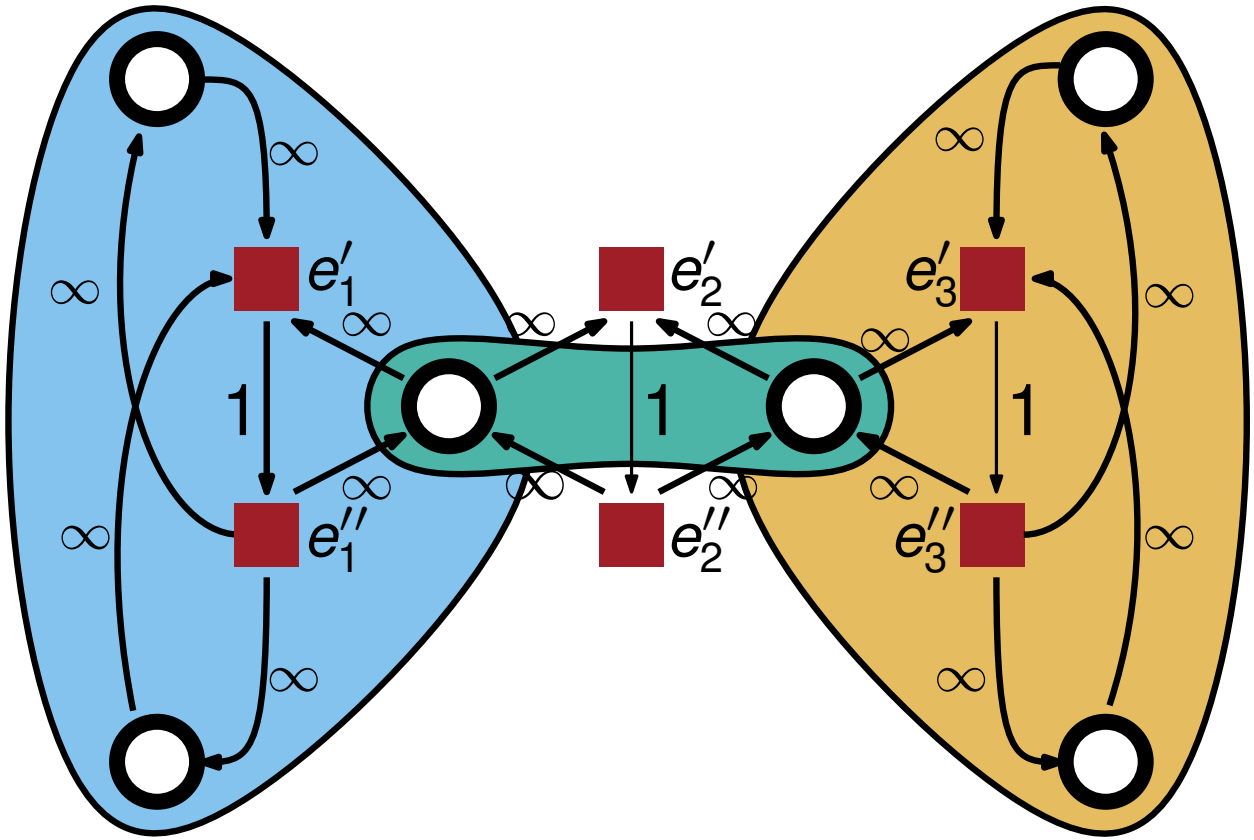
Hypergraph Flow Network - Summary



Lawler Network [Lawler 73]

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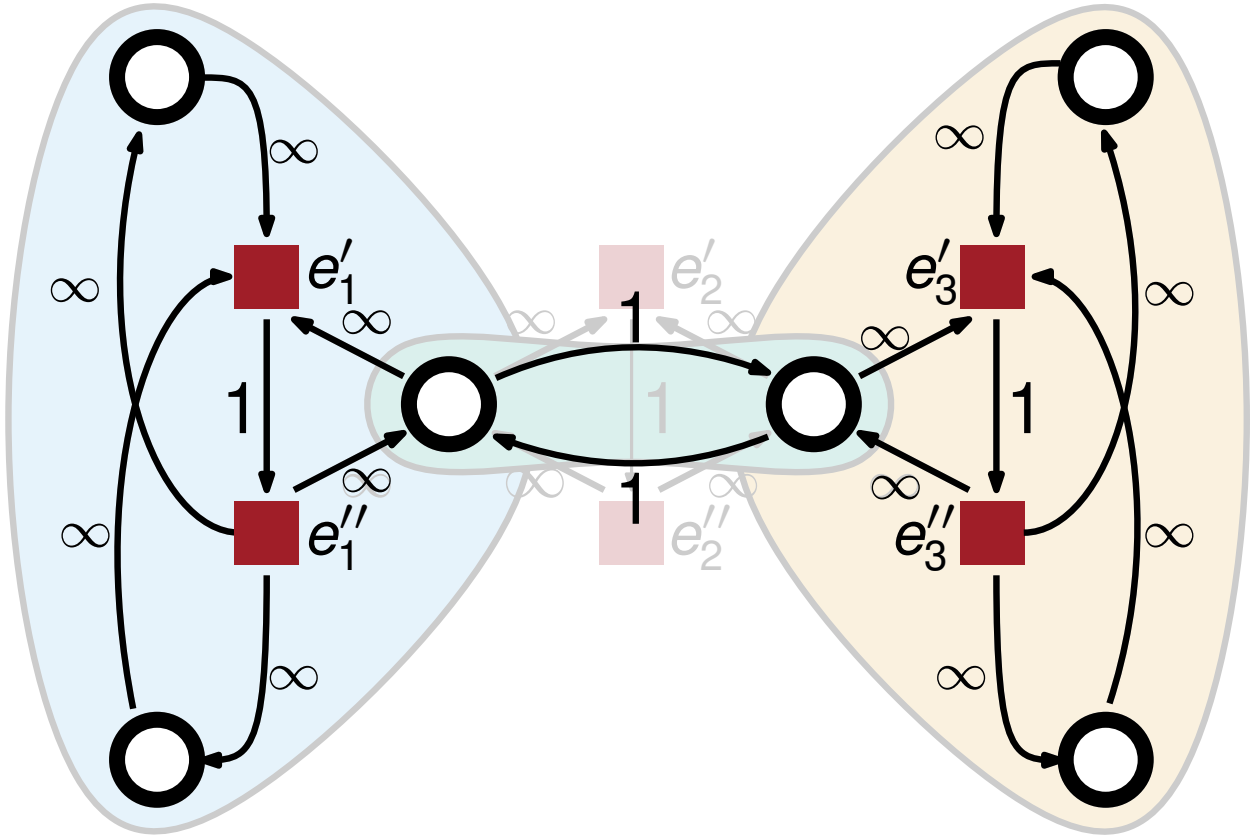
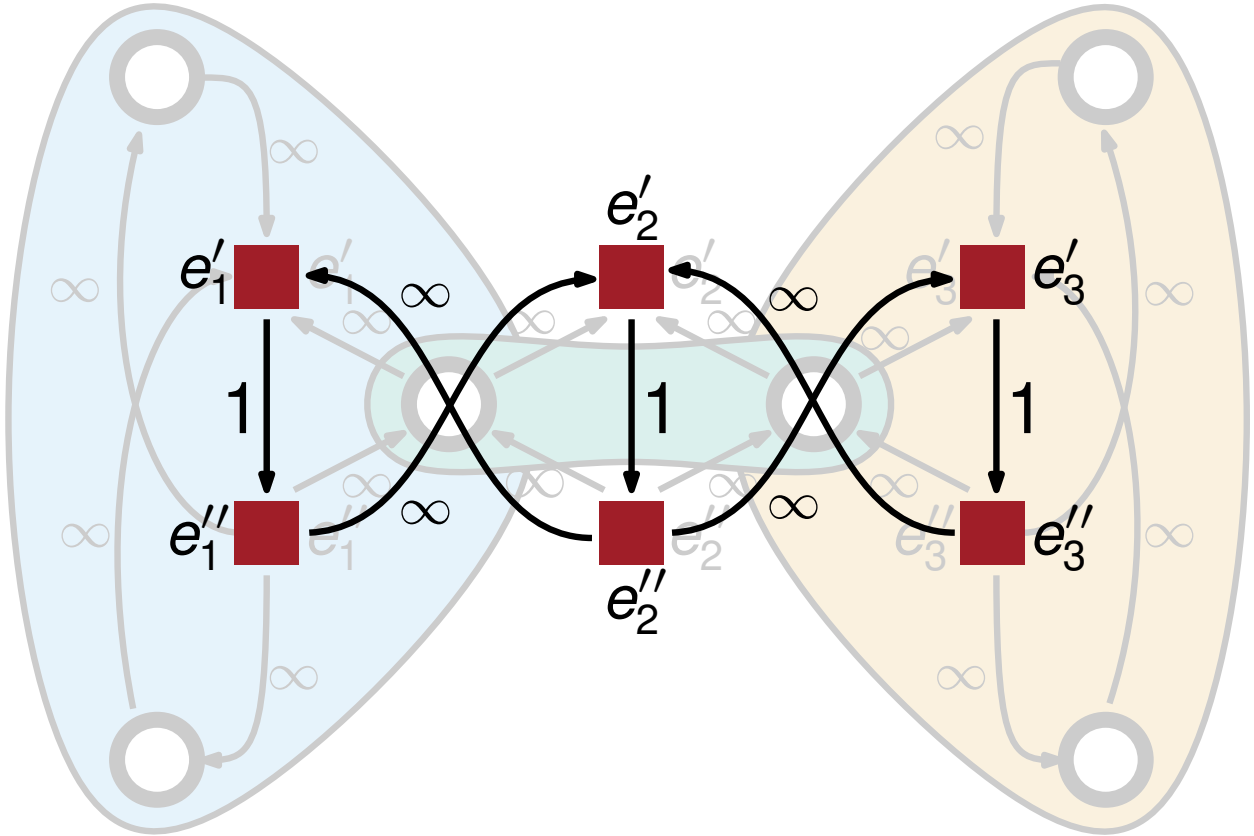
Hypergraph Flow Network - Summary



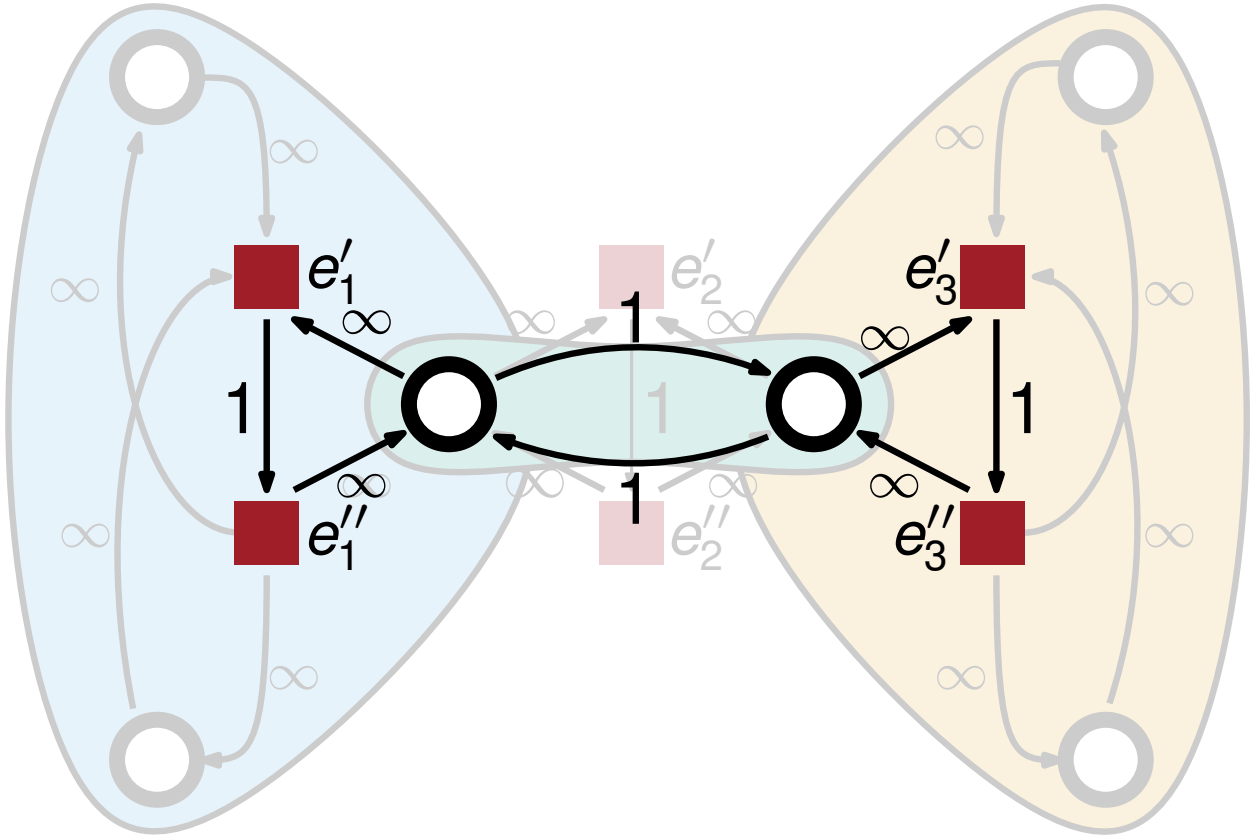
Wong Network [Liu, Wong 98]

Hypergraph Flow Network - Summary



Our Network

Hypergraph Flow Network - Summary



Hybrid Network

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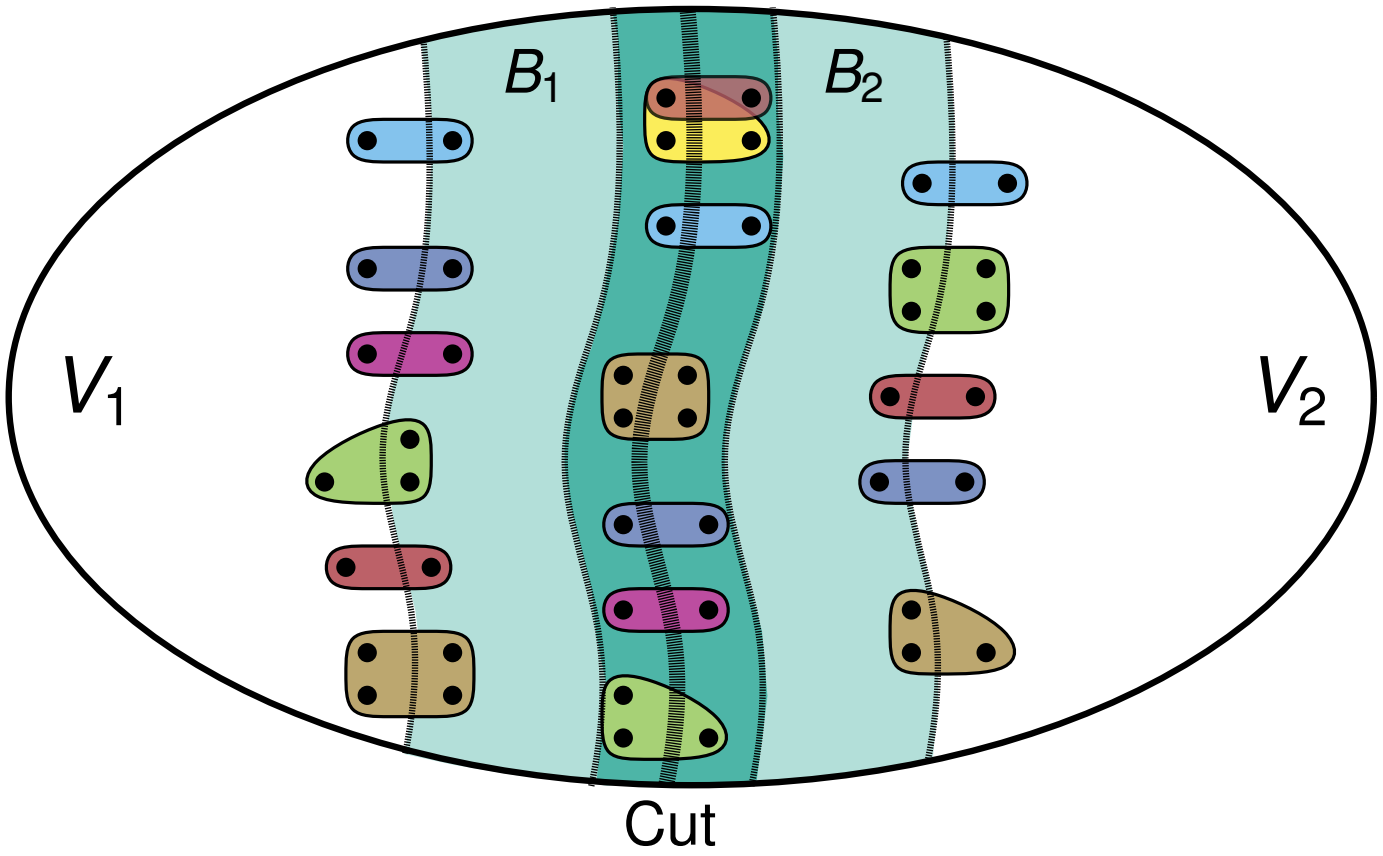
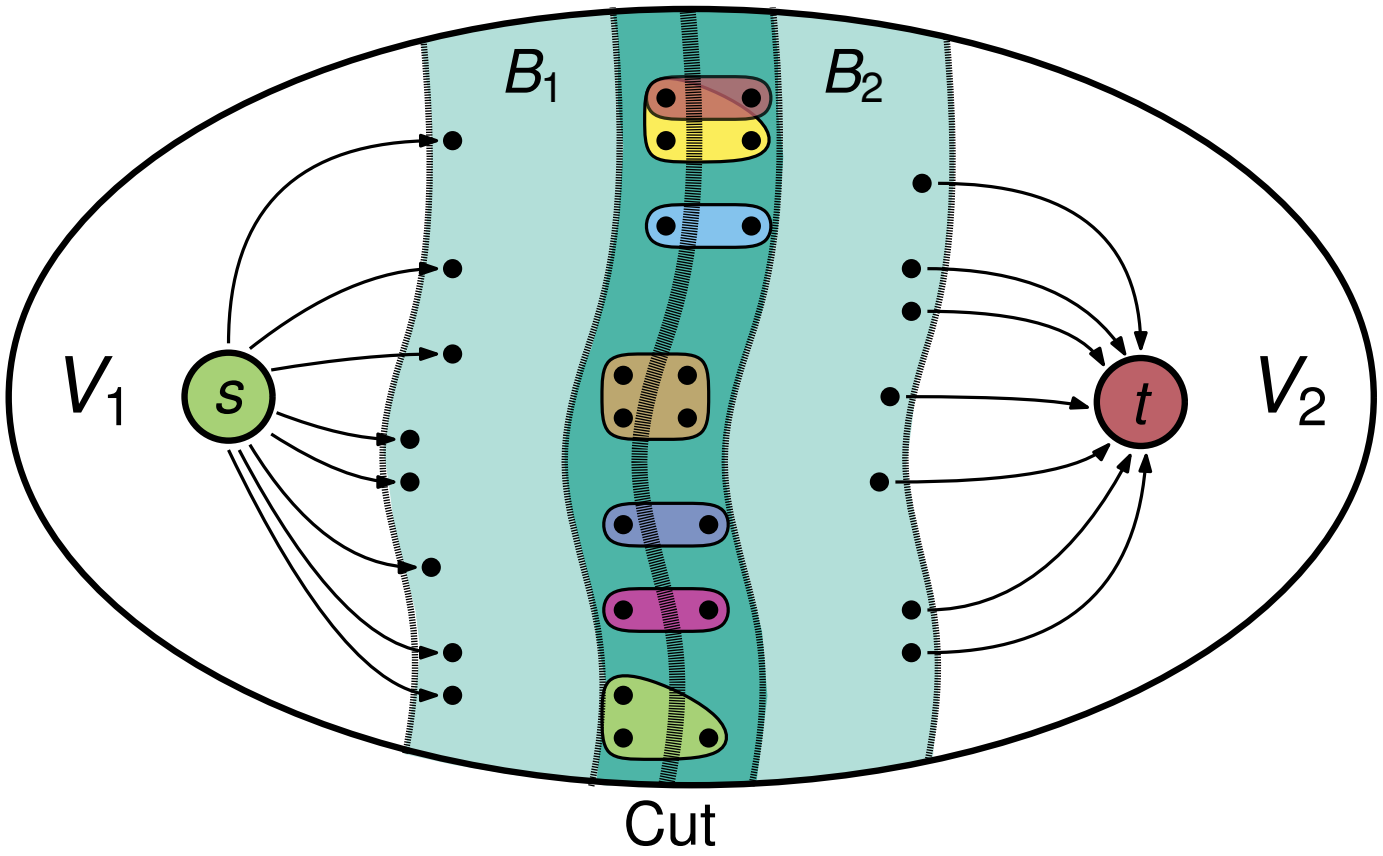
Optimized Flow Problem Modeling Approach

Modeling Approach in *KaFFPa*



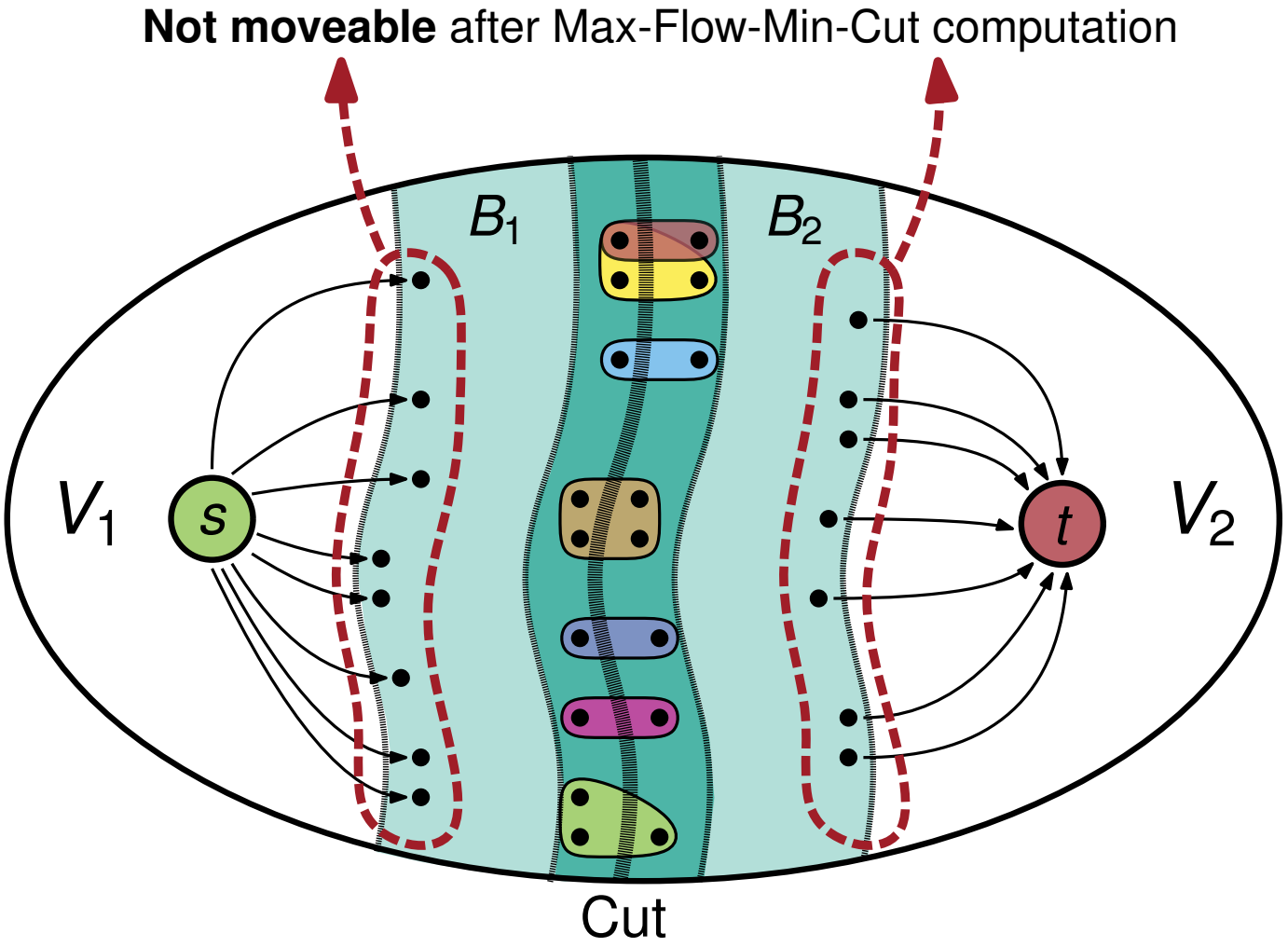
Optimized Flow Problem Modeling Approach

Modeling Approach in *KaFFPa*



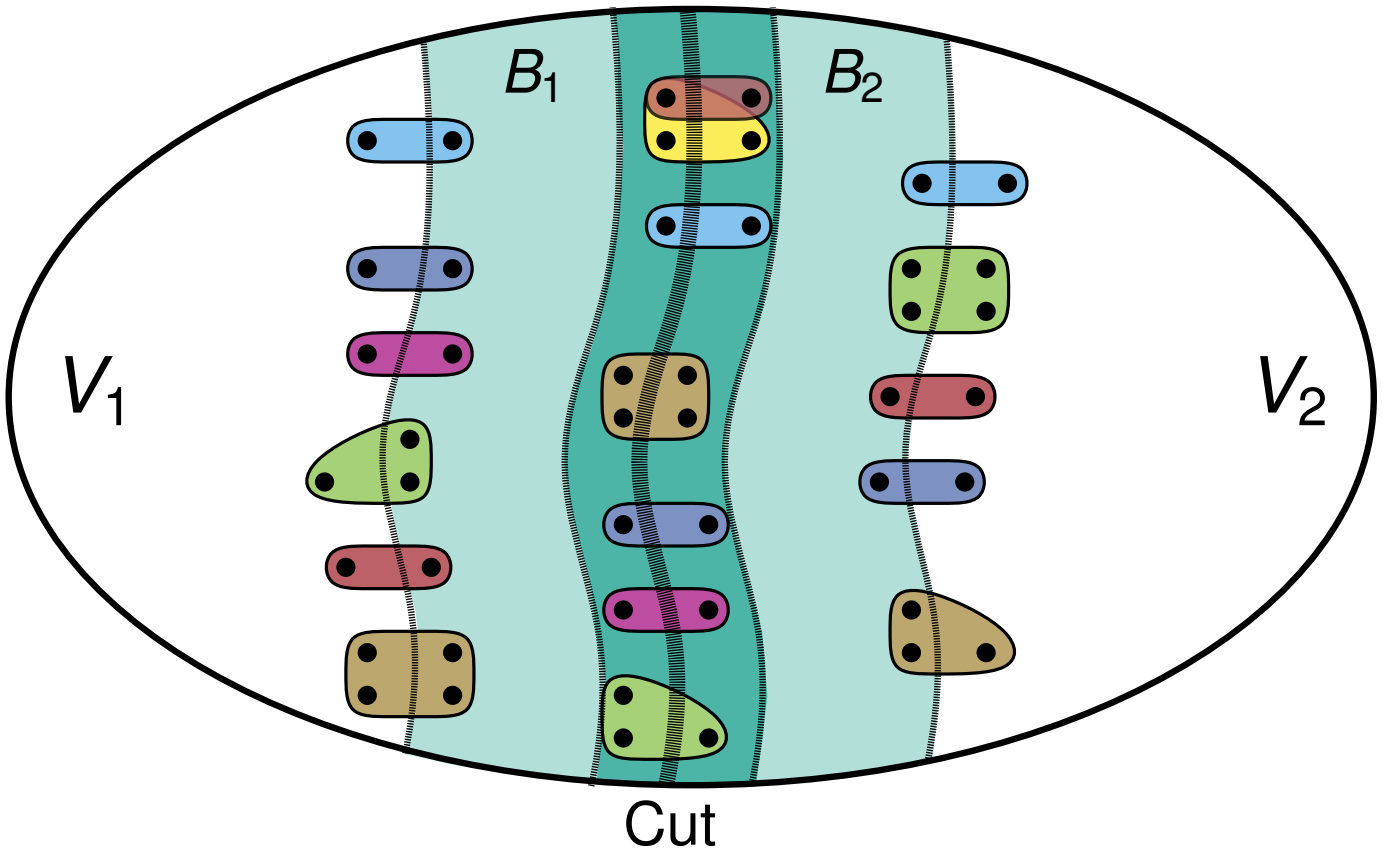
Optimized Flow Problem Modeling Approach

Modeling Approach in *KaFFPa*



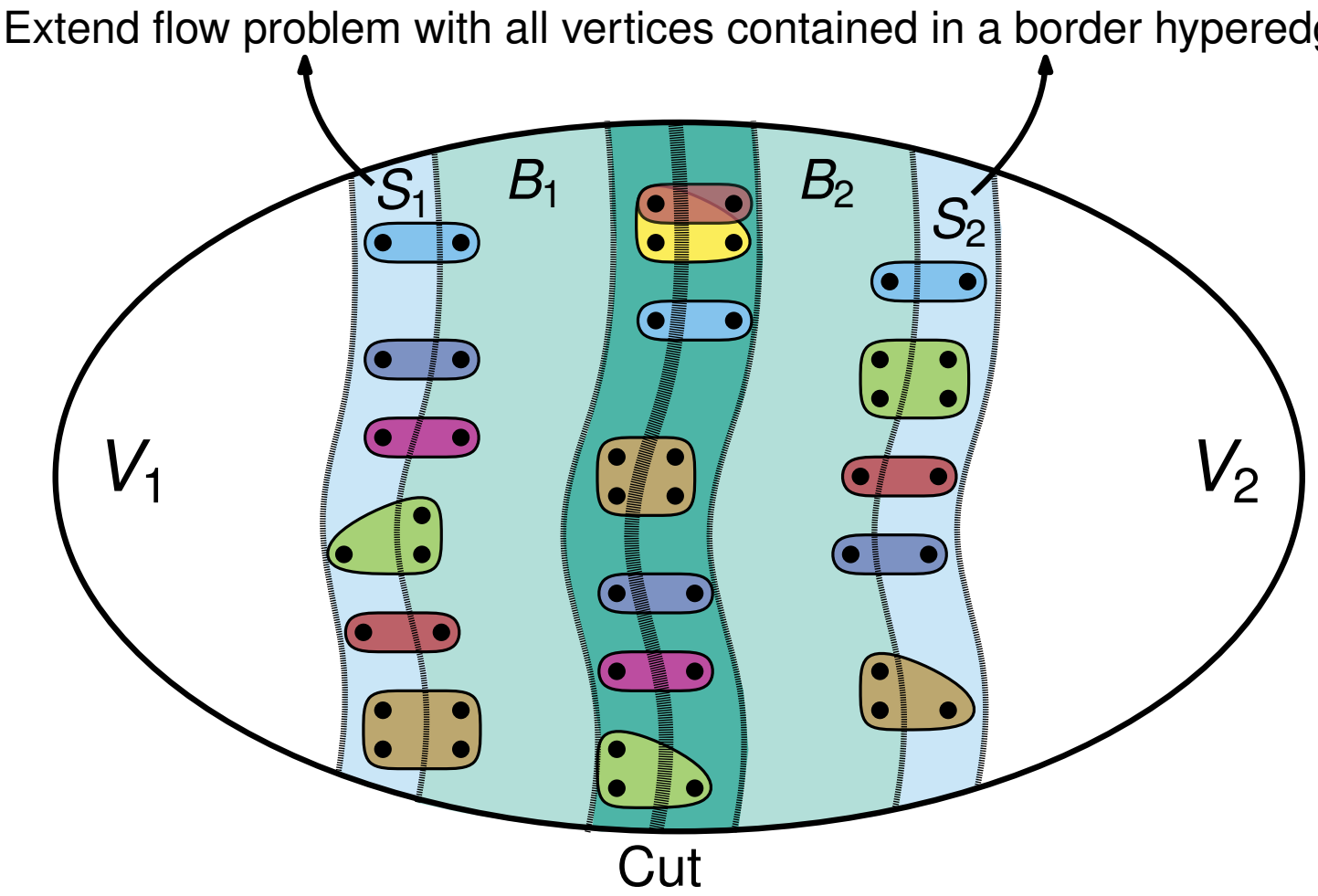
Optimized Flow Problem Modeling Approach

Modeling Approach in *KaHyPar*



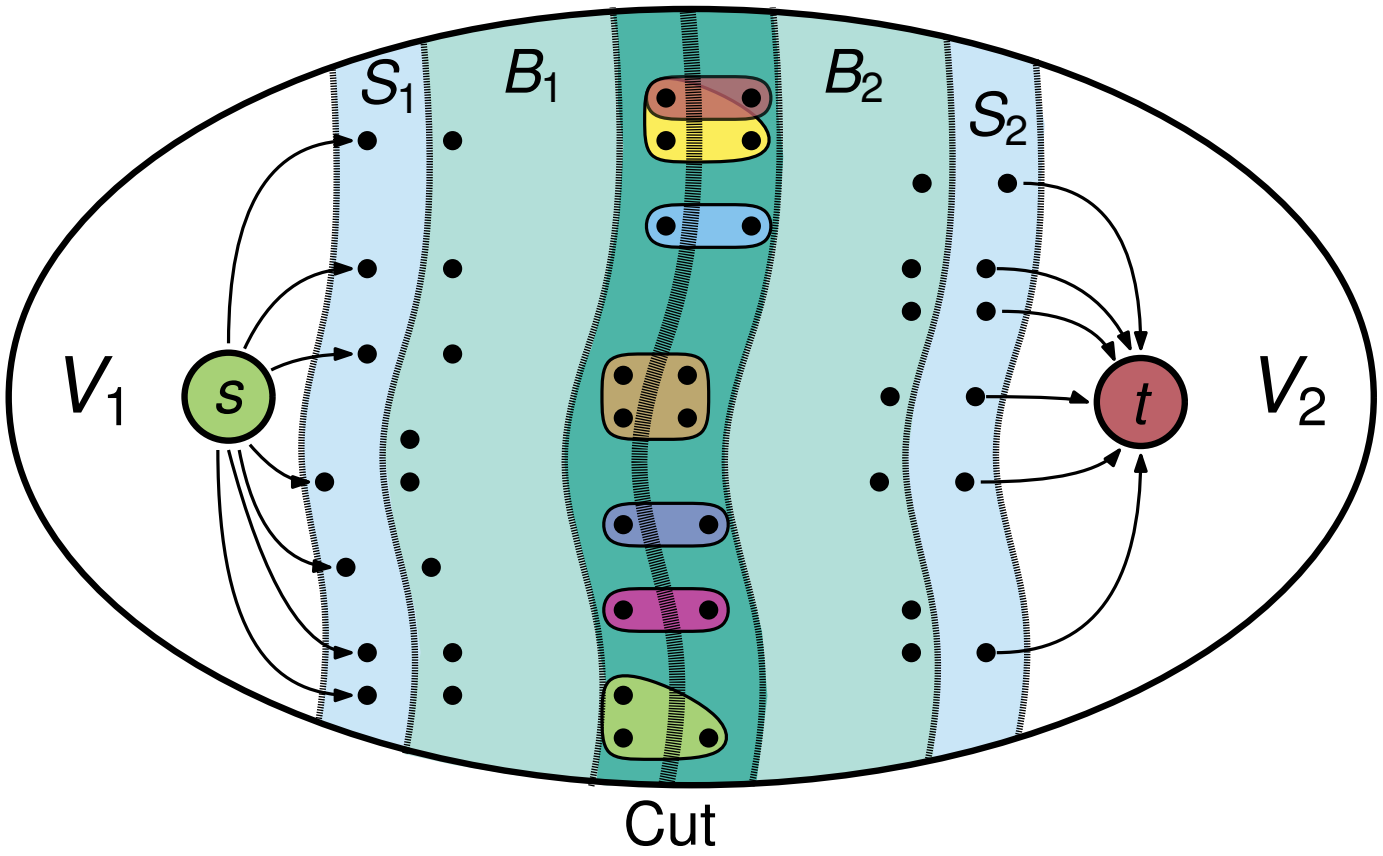
Optimized Flow Problem Modeling Approach

Modeling Approach in *KaHyPar*



Optimized Flow Problem Modeling Approach

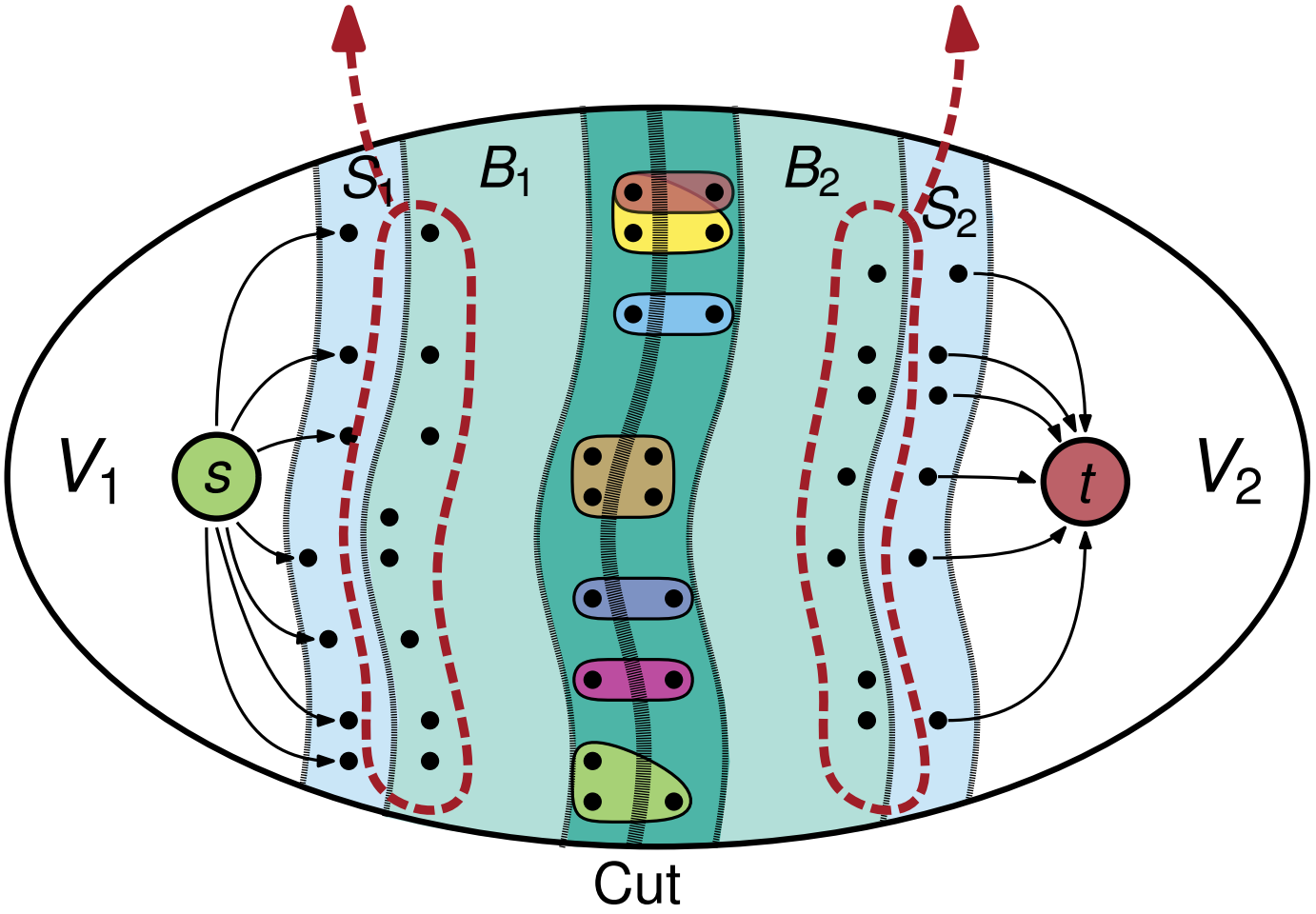
Modeling Approach in *KaHyPar*



Optimized Flow Problem Modeling Approach

Modeling Approach in *KaHyPar*

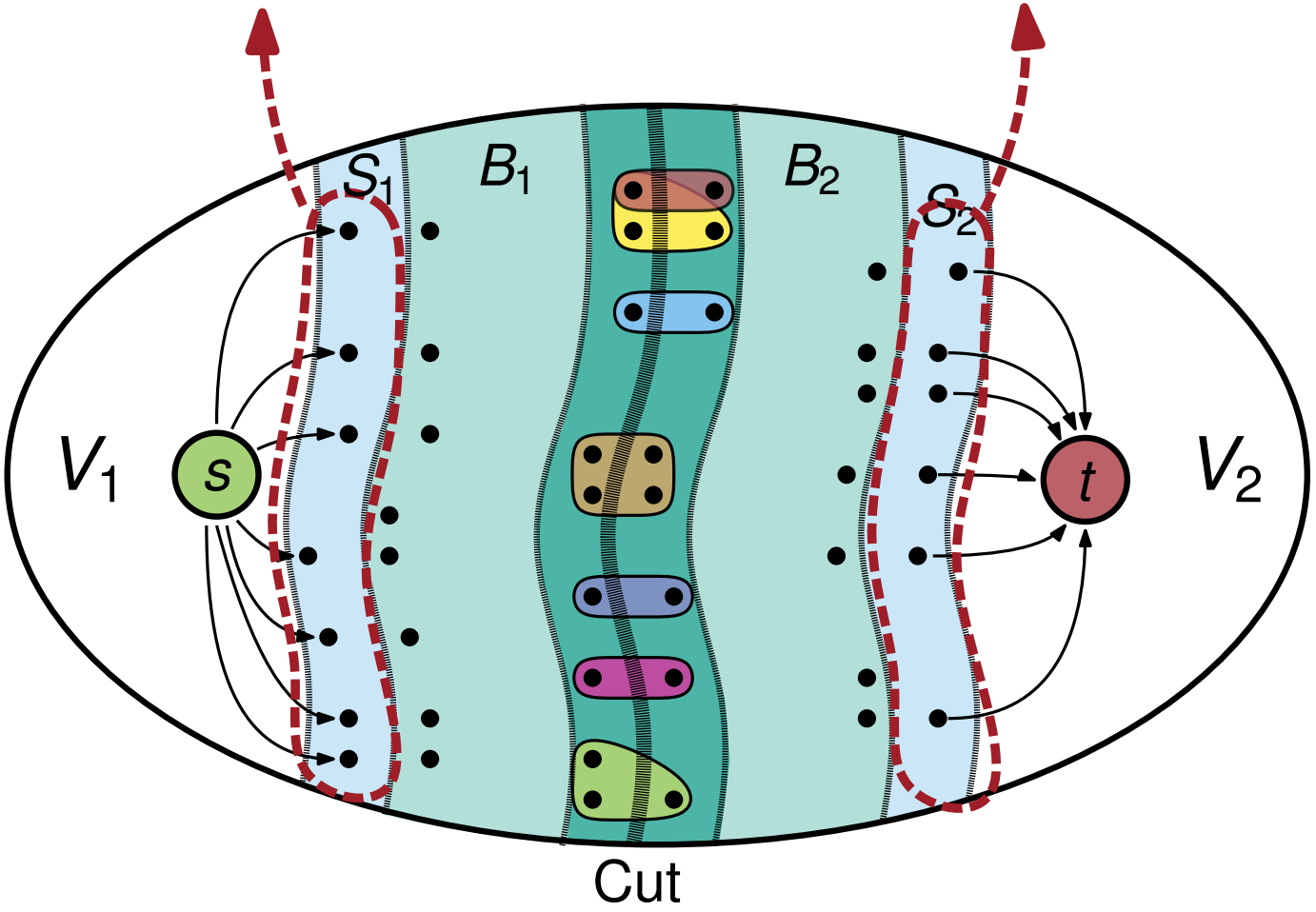
Moveable after Max-Flow-Min-Cut computation, but . . .



Optimized Flow Problem Modeling Approach

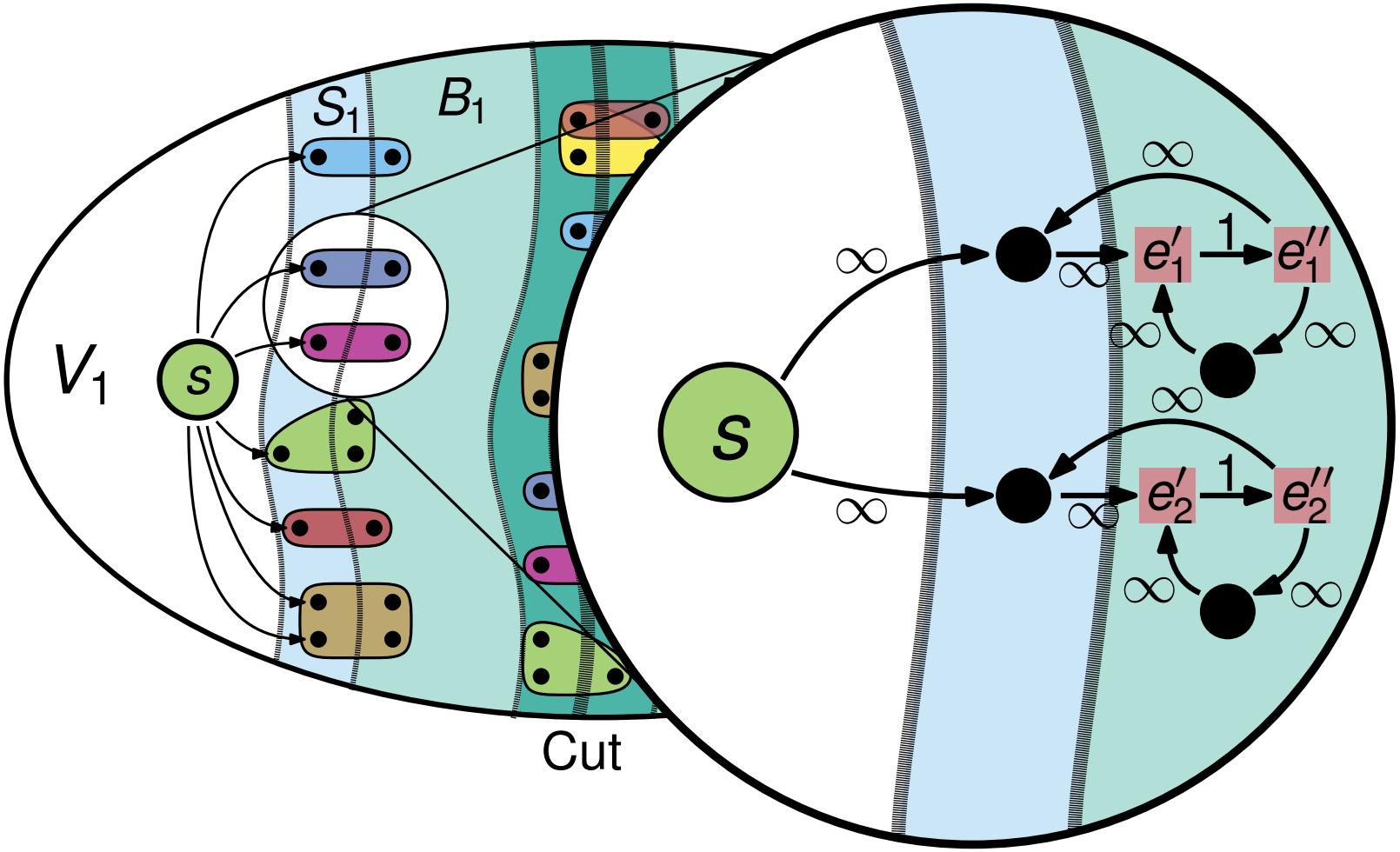
Modeling Approach in *KaHyPar*

. . . flow problem has significantly more nodes and edges.



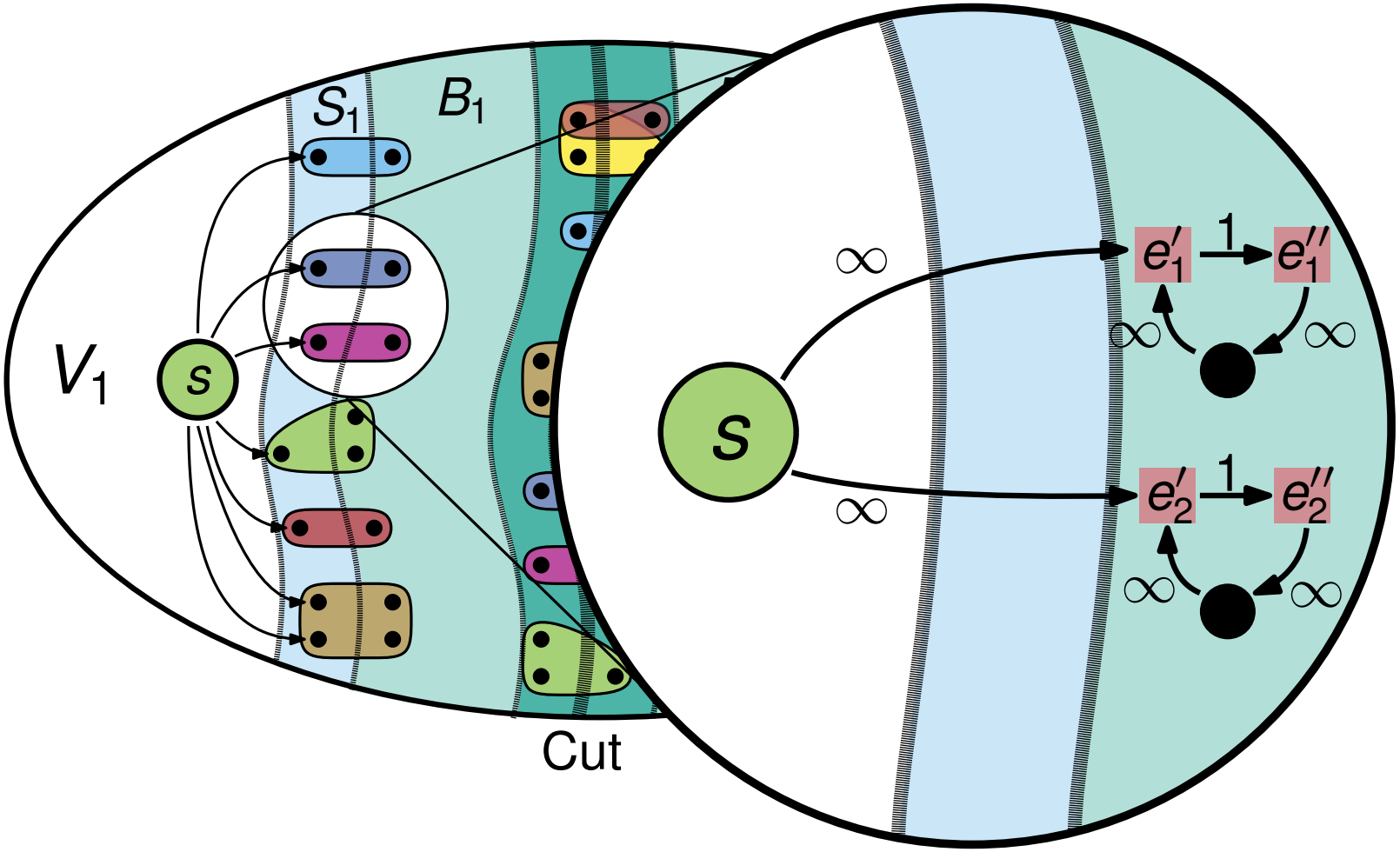
Optimized Flow Problem Modeling Approach

Modeling Approach in *KaHyPar*



Optimized Flow Problem Modeling Approach

Modeling Approach in *KaHyPar*

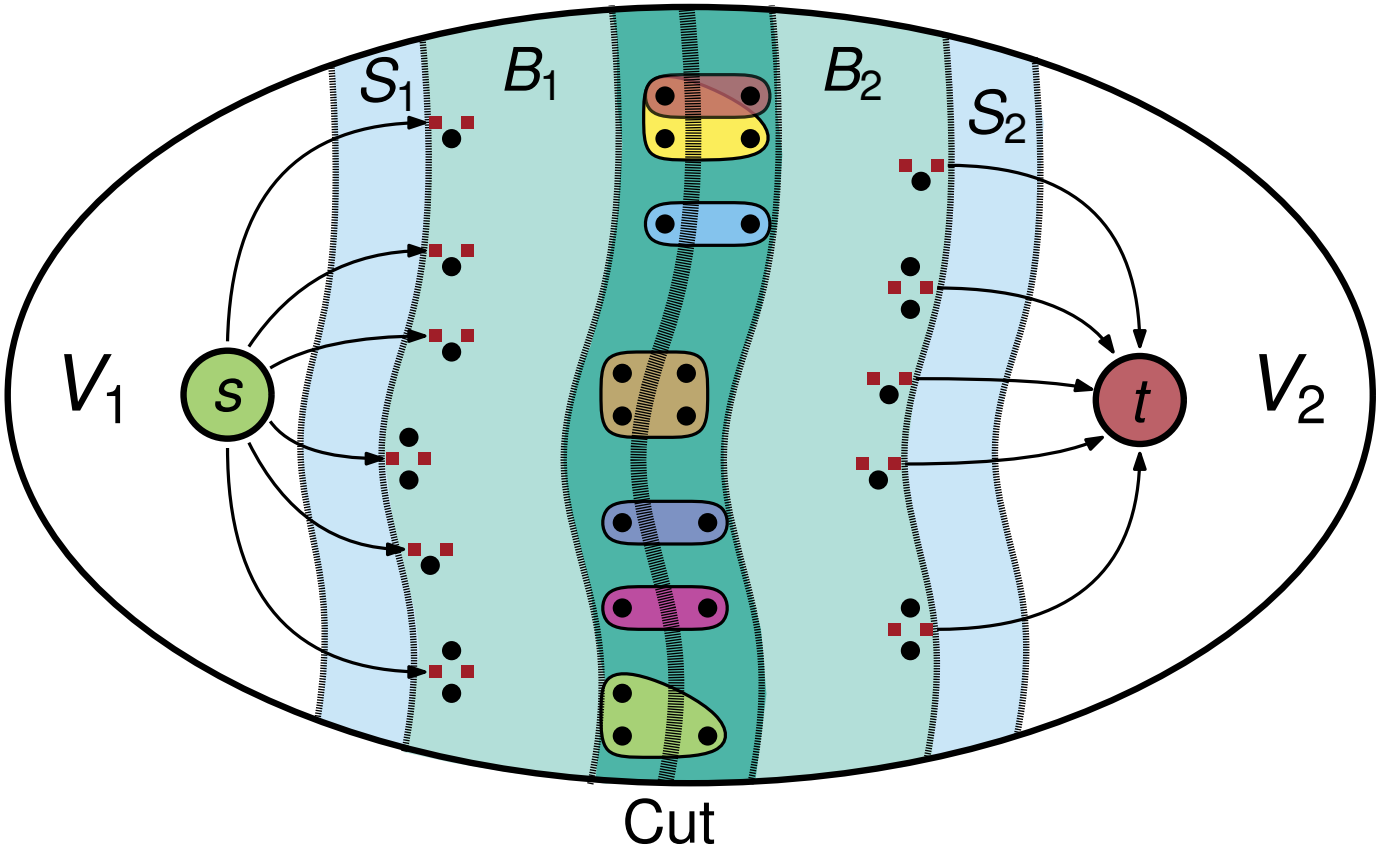


Optimized Flow Problem Modeling Approach

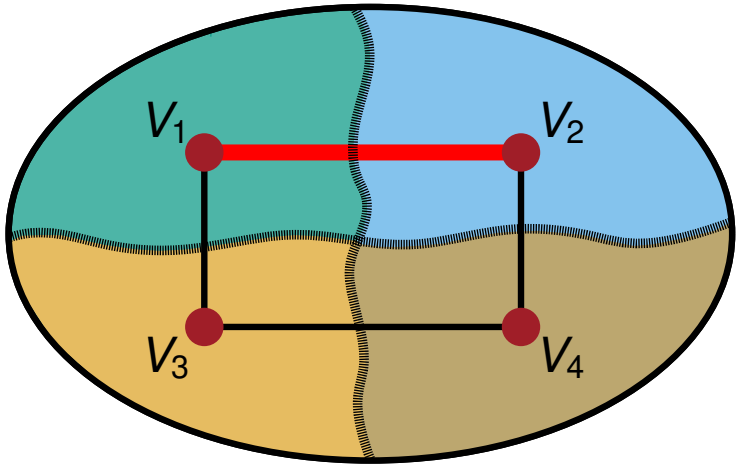
Modeling Approach in *KaHyPar*

$$S = \{e' \mid e \in I(S_1)\}$$

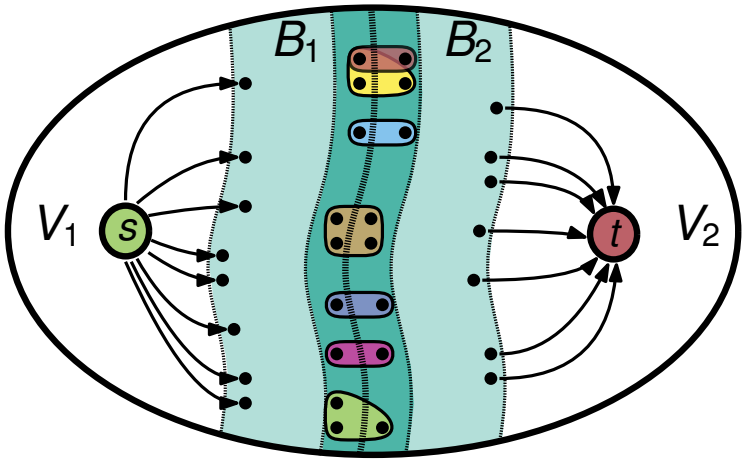
$$T = \{e'' \mid e \in I(S_2)\}$$



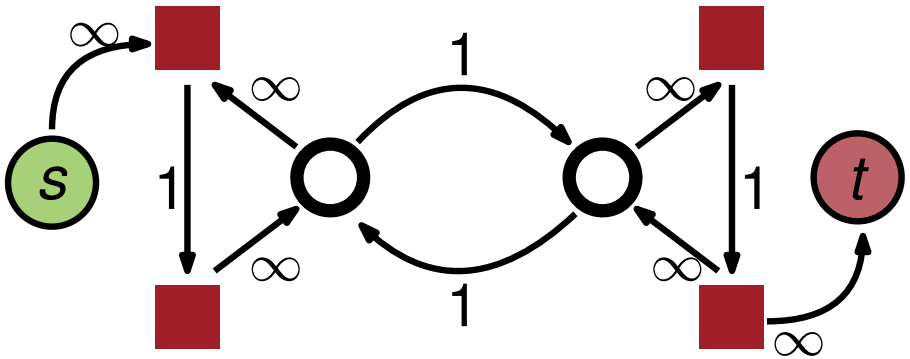
Our Flow-Based Refinement Framework



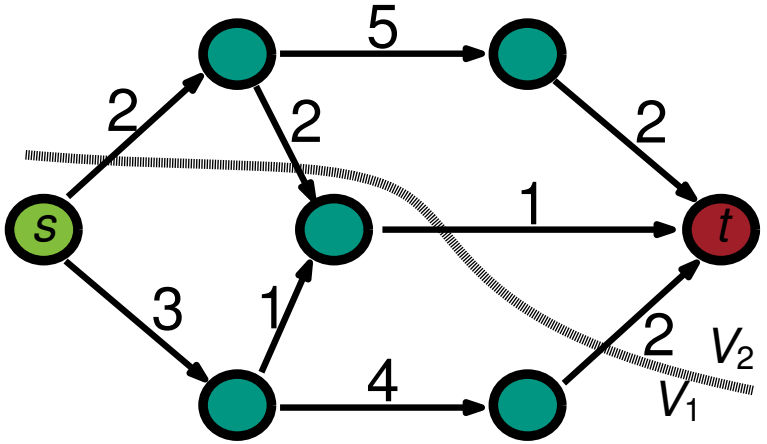
Select two adjacent blocks for refinement



Build Flow Problem



Solve Flow Problem



Find feasible minimum cut

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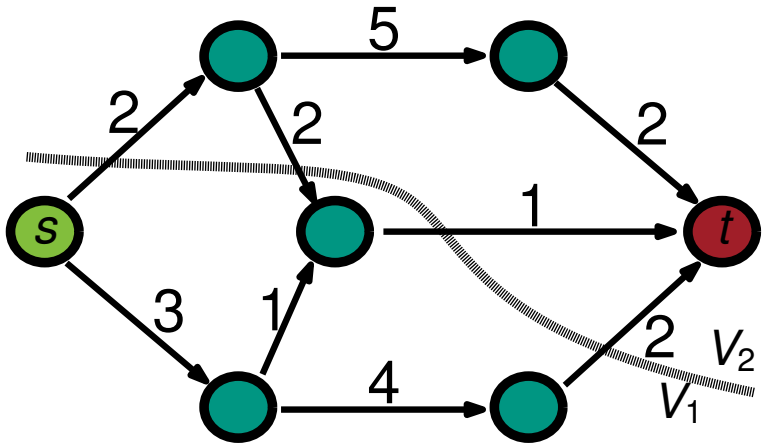
Our Flow-Based Refinement Framework

Select two adjacent blocks for refinement

Build Flow Problem

Solve Flow Problem



Find feasible minimum cut

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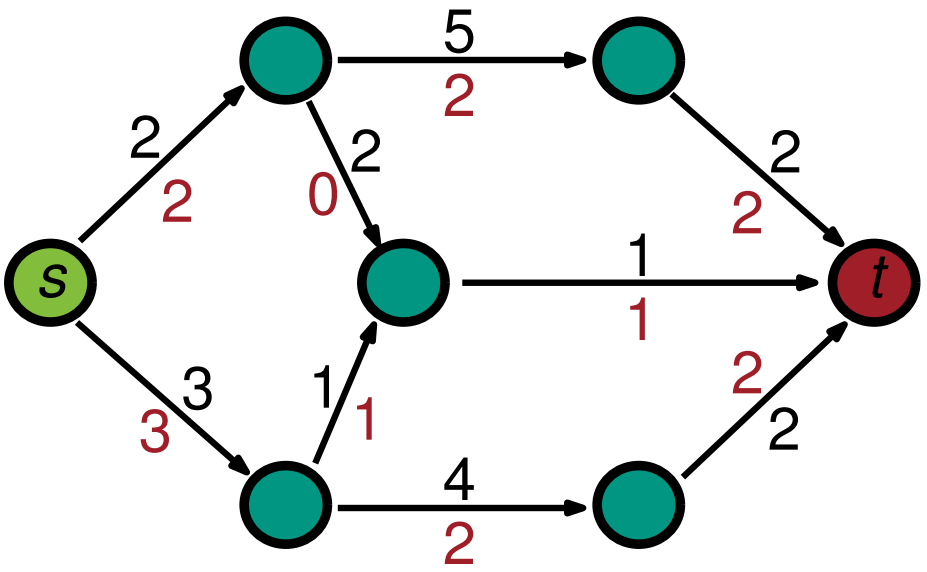
Most Balanced Minimum Cut

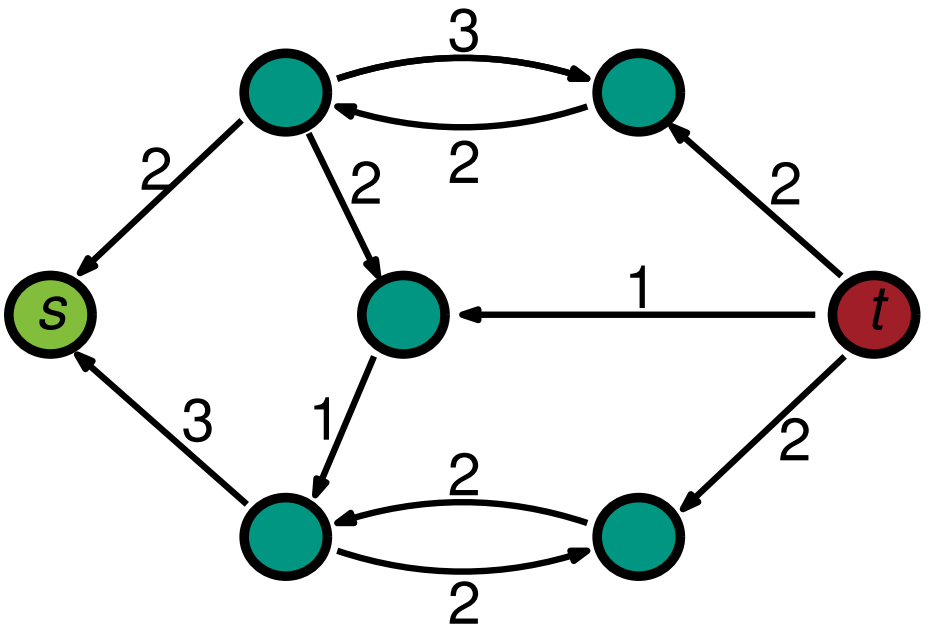
KaFFPa [Sanders, Schulz 11]

One maximum flow f has enough information to enumerate all minimum (s, t)-cuts [Picard, Queyranne 80]

Flow Graph



Residual Graph



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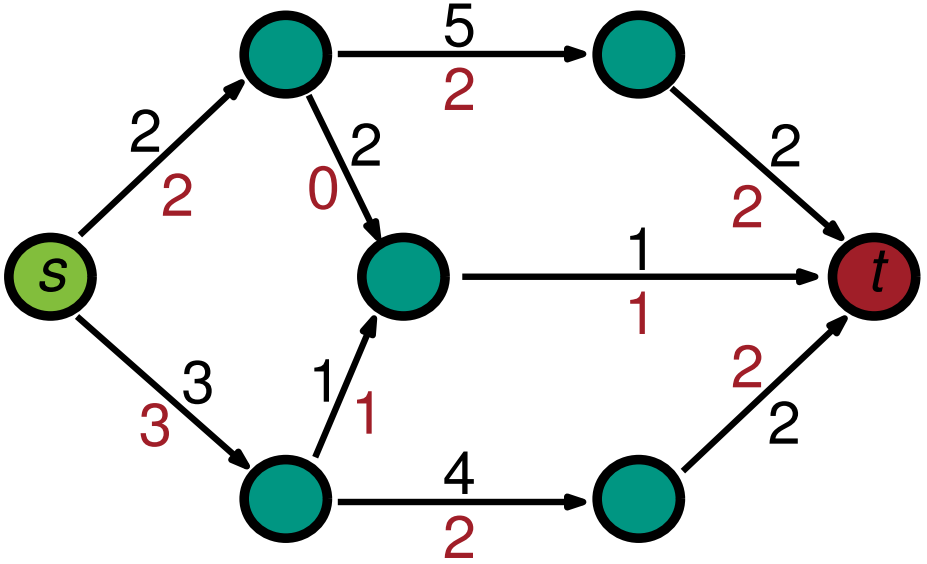
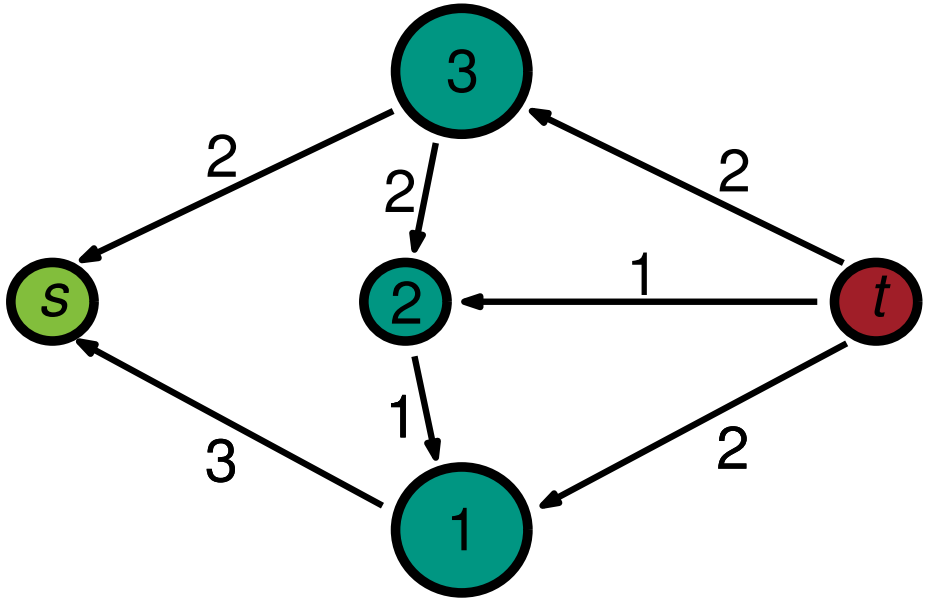


Picard-Queryanne DAC



Contract all *strongly connected components* in the residual graph

Most Balanced Minimum Cut

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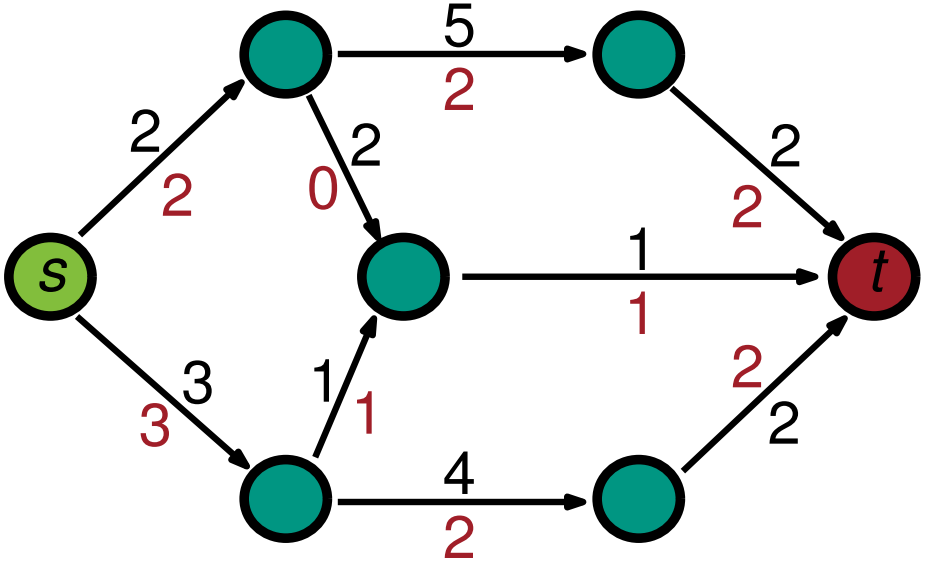
Flow Graph



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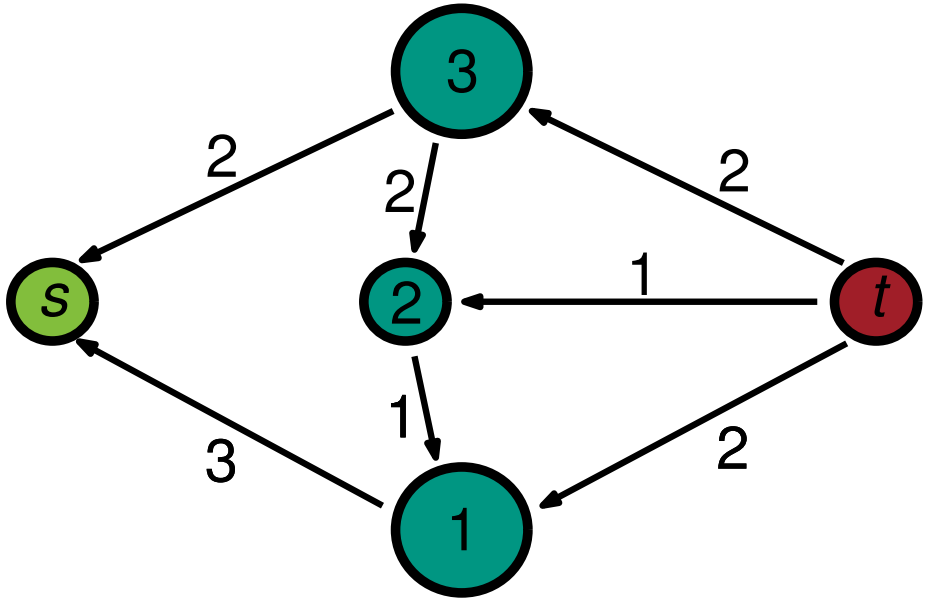

$\langle t, \quad 3, \quad 2, \quad 1, \quad s \rangle$

Find *topological order*

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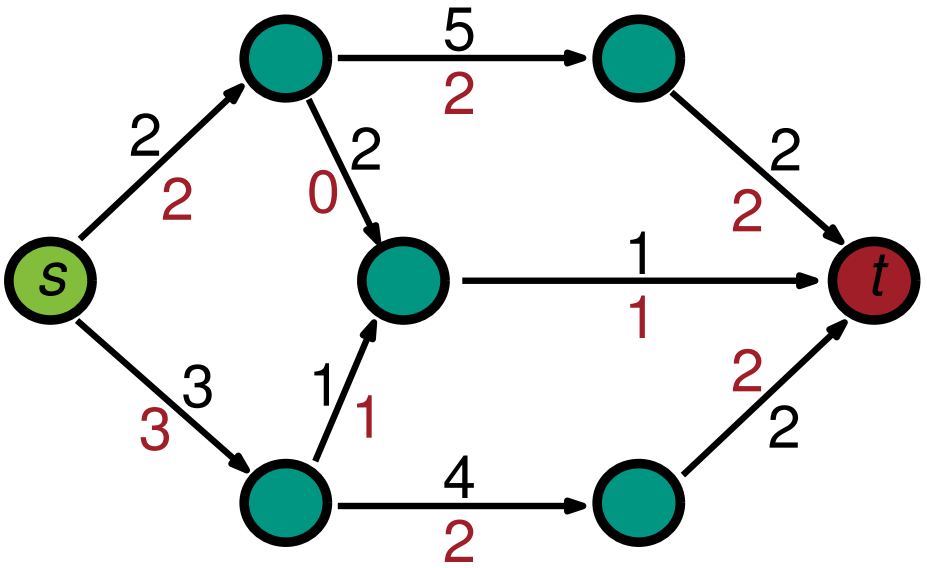
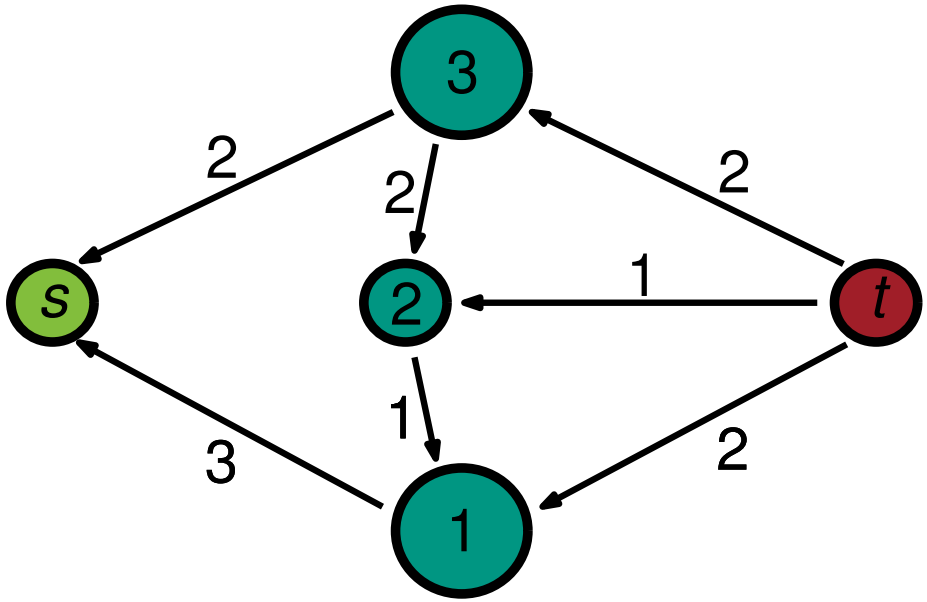


Picard-Queryanne DAC



$\langle t, \quad 3, \quad 2, \quad 1, \quad s \rangle$

Sweep through **reverse** topological order

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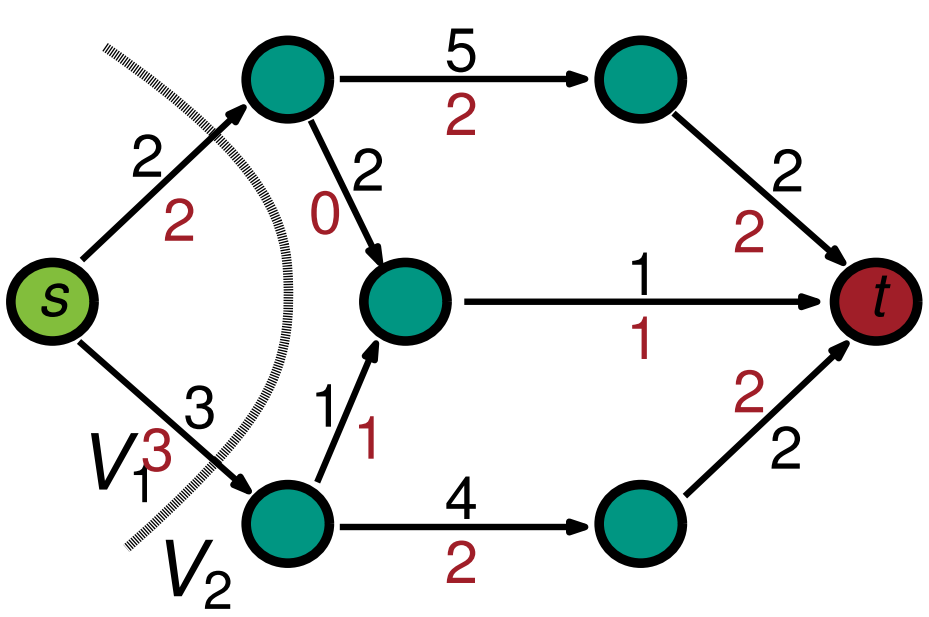
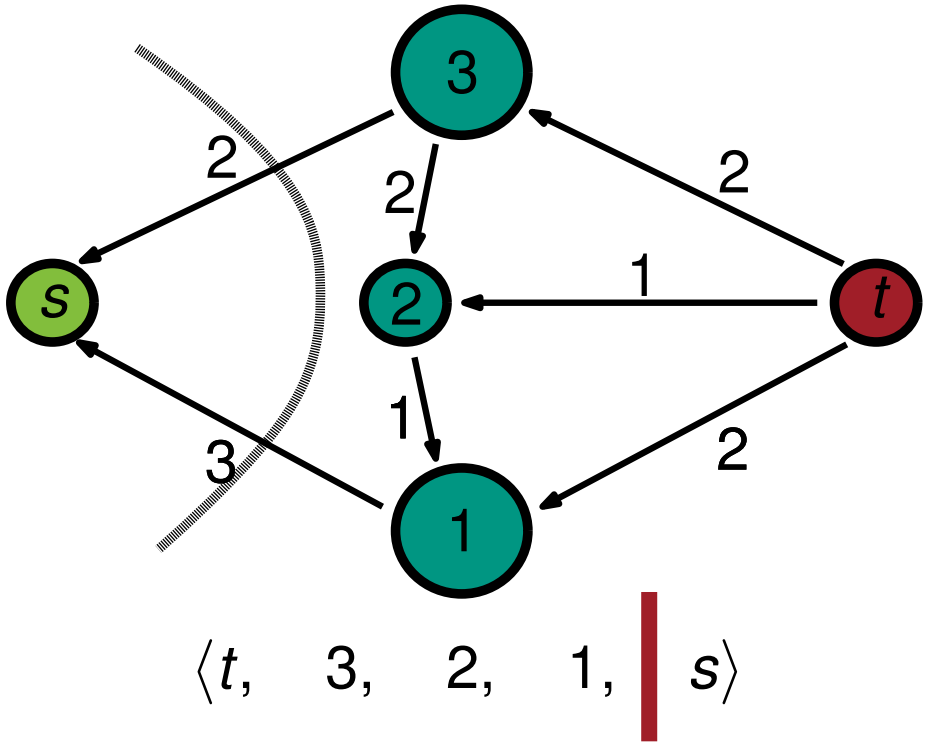
Flow Graph



Picard-Queryanne DAC



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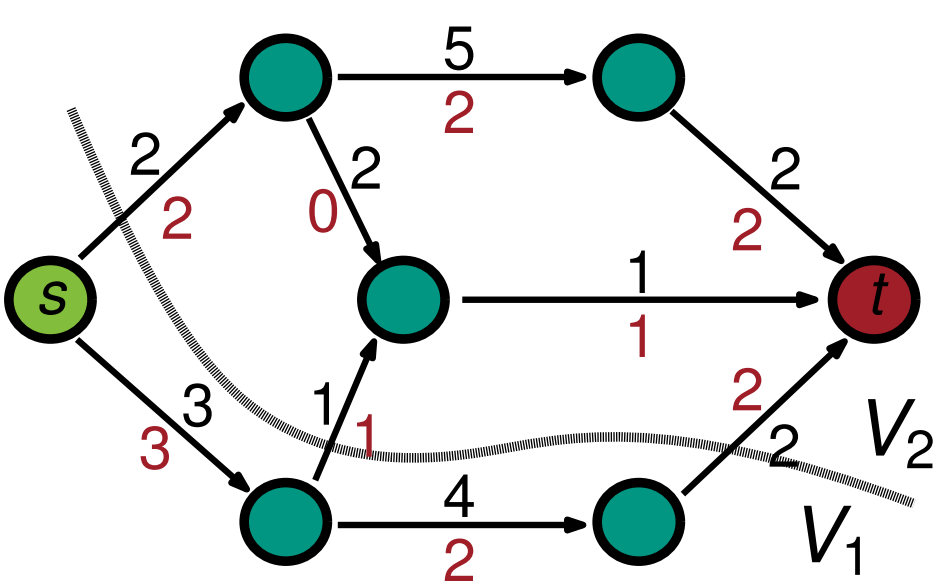

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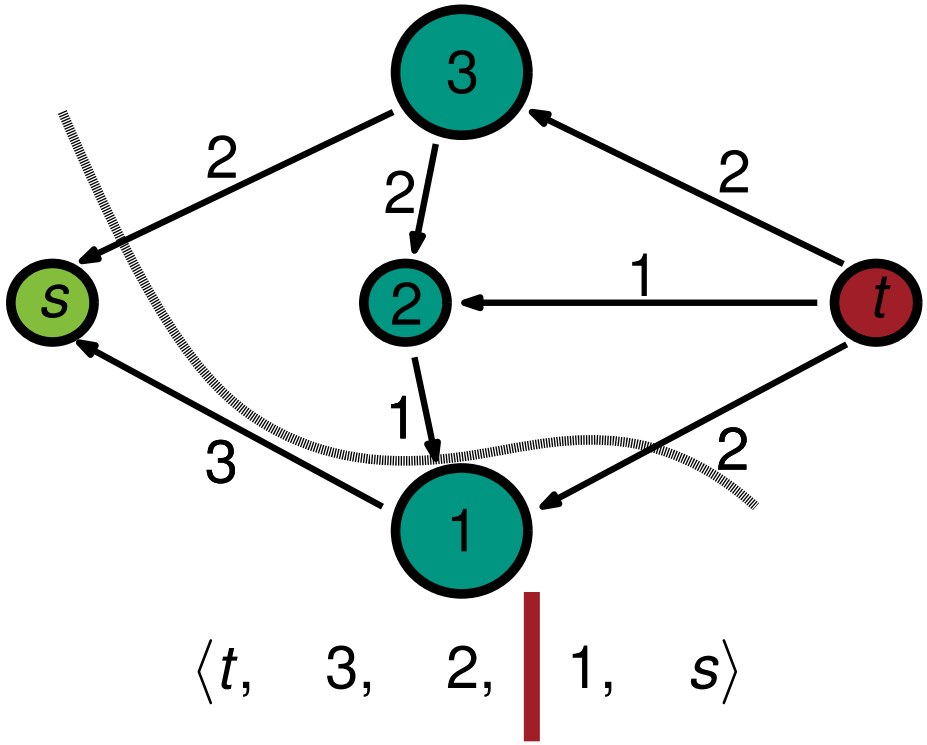

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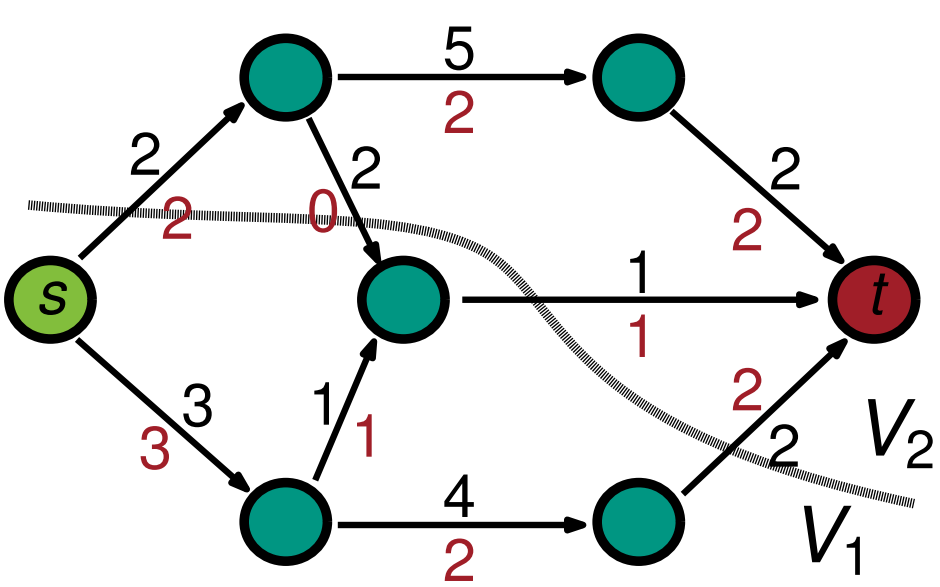
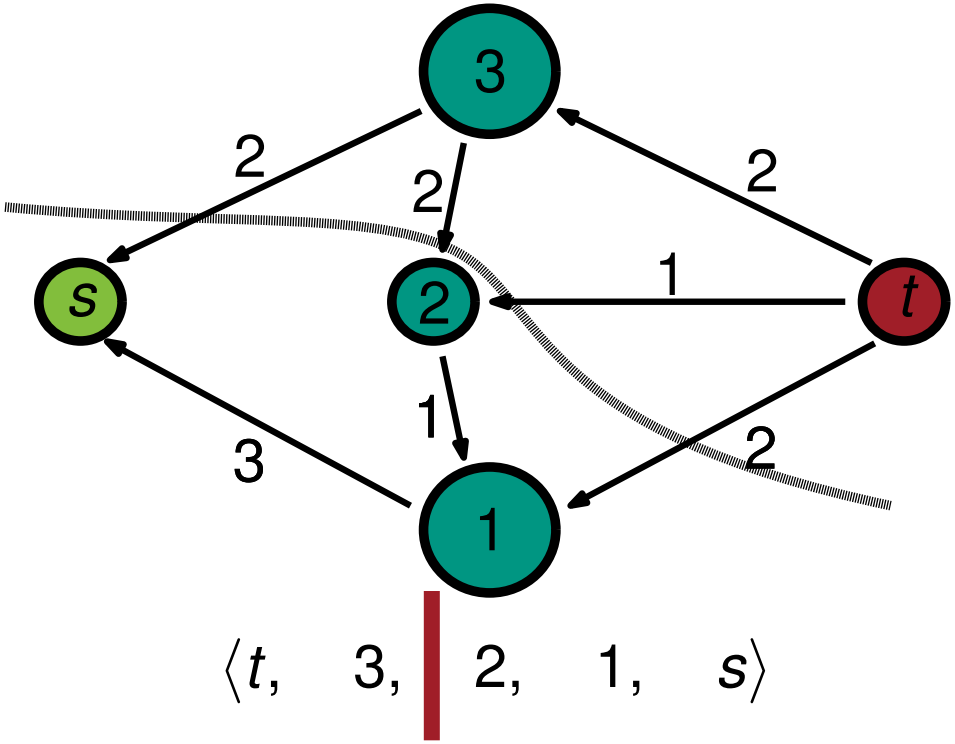


Picard-Queryanne DAC



Sweep through **reverse** topological order

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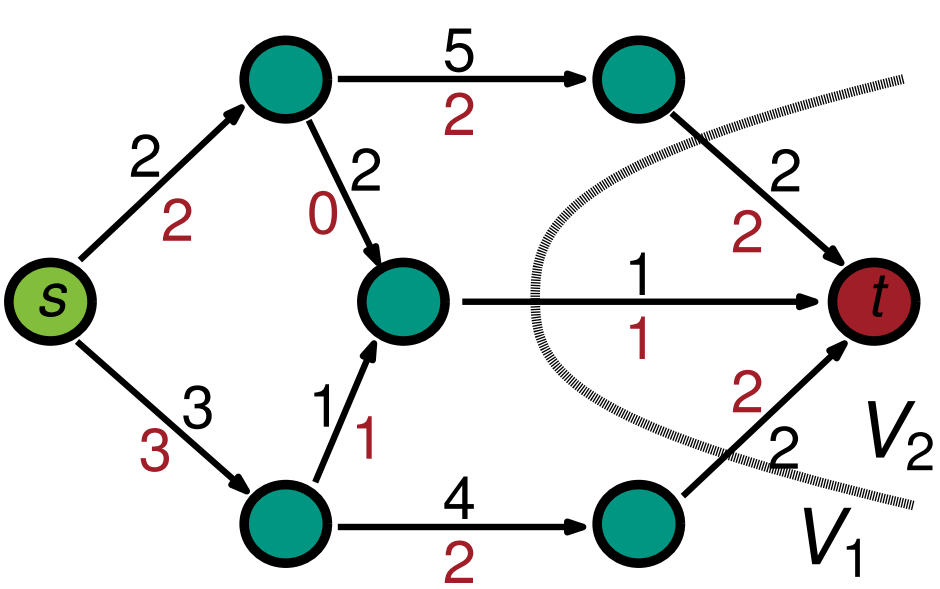

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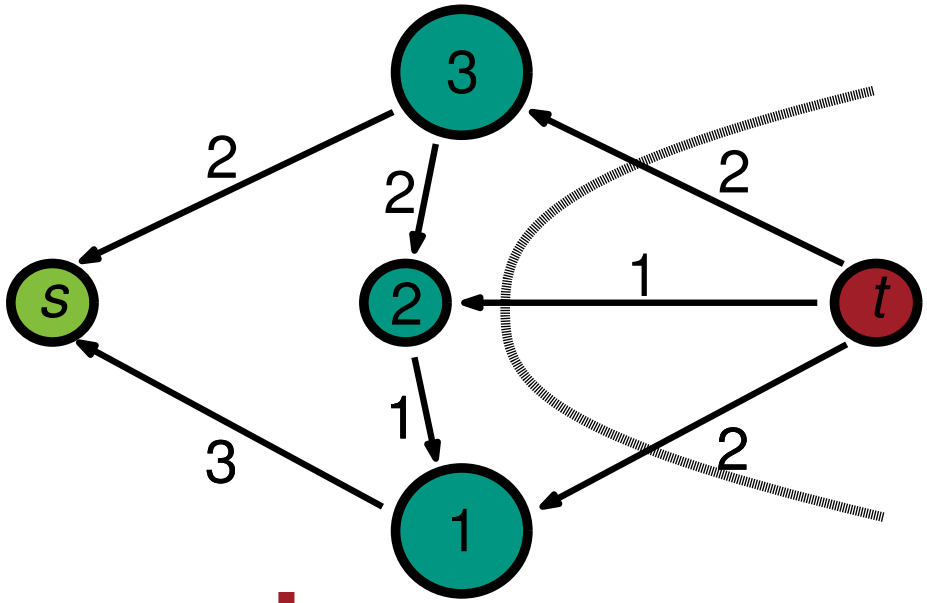

$\langle t, \mid 3, \quad 2, \quad 1, \quad s \rangle$

Sweep through **reverse** topological order

Integration into KaHyPar

- KaHyPar is a *n*-level hypergraph partitioner

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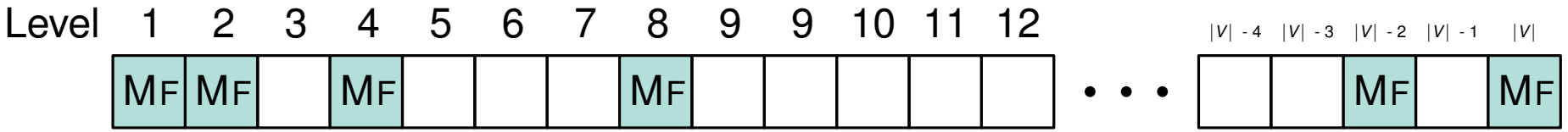
Integration into KaHyPar

- KaHyPar is a *n*-level hypergraph partitioner

Flow Execution Policy

Exponential: Execute *Max-Flow-Min-Cut* computations (MF) on each level i with $i = 2^j$



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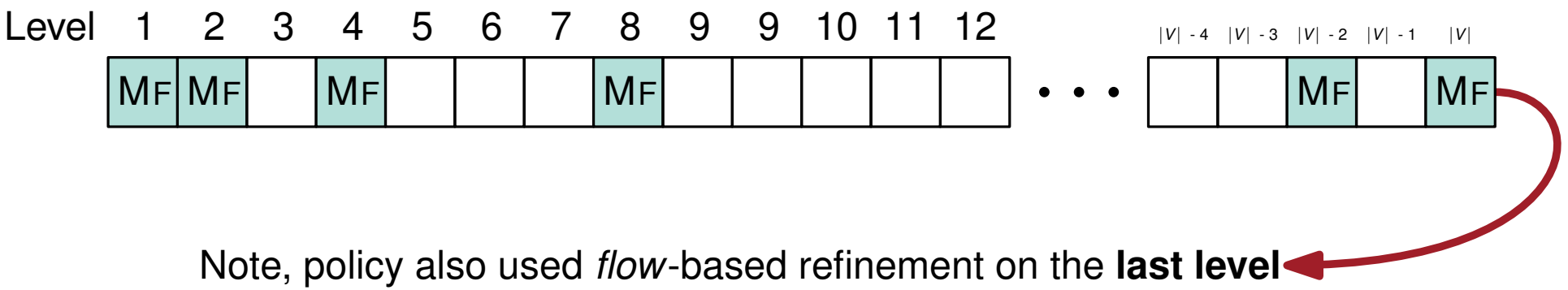

Note, policy also used *flow*-based refinement on the **last level**

Integration into KaHyPar

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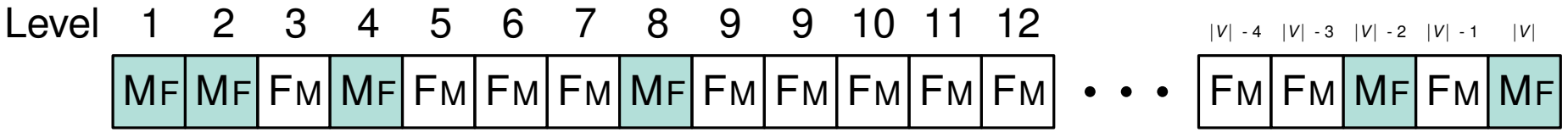
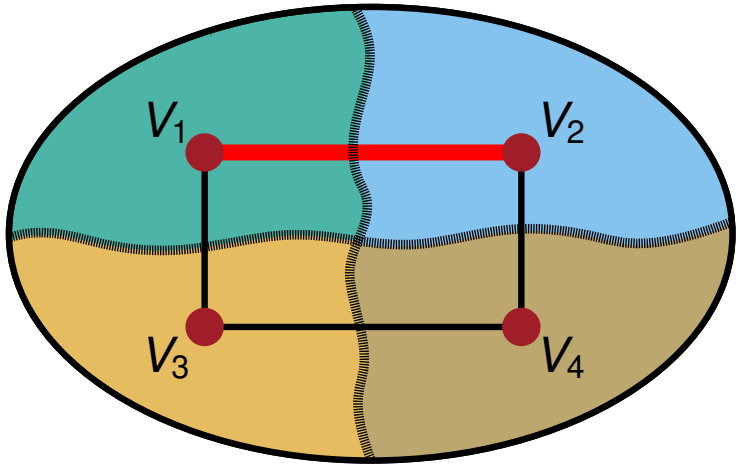
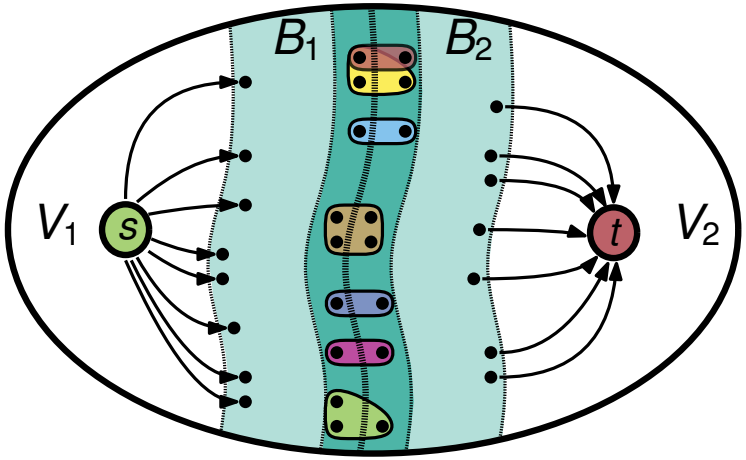


Speed-Up Heuristics



Active Block Scheduling



Adaptive Flow Iterations

(R1) Skip bipartitions with small cuts (e.g. ≤ 10)

(R2) Incorporate improvement history

(R3) Break, if nothing changed

Experimental Setup

System

- Intel Xeon E5-2670 Octa-Core (2.6 GHz)
- 64 GB Main Memory
- 20 MB L3-Cache, 8 $\times$ 256 KB L2-Cache

Flow Algorithms

- EDMONDKARP
- GOLDBERGTARJAN
- BOYKOVKOLMOGOROV
- IBFS (fastest in our experiments)

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Own Implementations

- BOYKOVKOLMOGOROV
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Third-Party Implementations

Experimental Setup

Benchmarks

- Parameter Tuning Benchmark Set (25 Hypergraphs)
- Benchmark Subset (165 Hypergraphs)
- Full Benchmark Set (488 Hypergraphs)

Benchmark Type

- DAC (10 Hypergraphs)
- ISPD98 (18 Hypergraphs)
- PRIMAL (92 Hypergraphs)
- LITERAL (92 Hypergraphs)
- DUAL (92 Hypergraphs)
- SPM (184 Hypergraphs)

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VLSI Design

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SAT Formulas

- SPM (184 Hypergraphs)

Sparse Matrices

Experimental Setup

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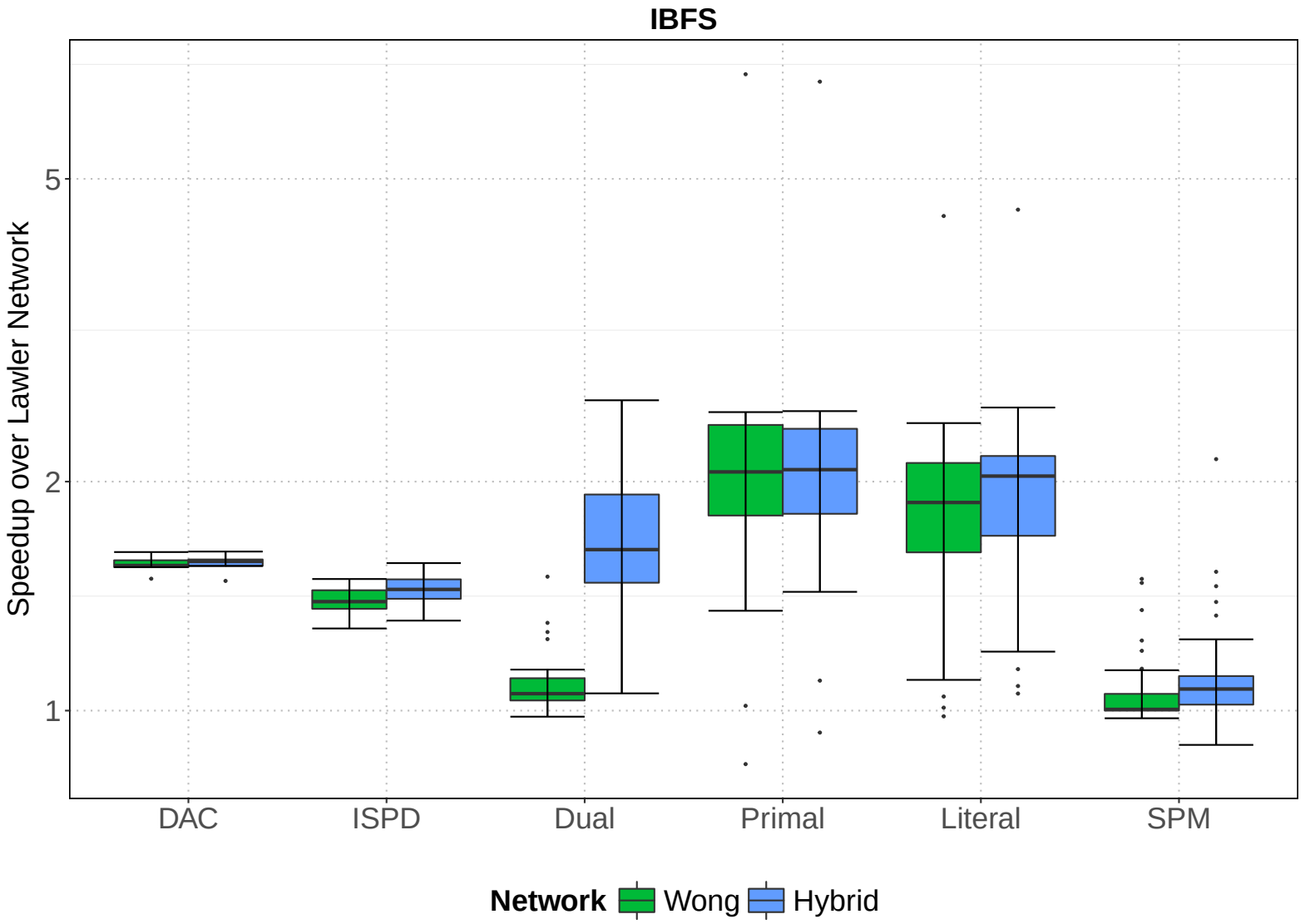
Methodology

- $\varepsilon = 3\%$
- $k \in \{2, 4, 8, 16, 32, 64, 128\}$
- 10 seeds

Flow Networks



Flow Configuration

- +/− F = Flow-based refinement
- +/− M = Most Balanced Minimum Cut
- +/− FM = FM Heuristic

Config. α'	(+F,-M,-FM) Avg [%]	$t[s]$	(+F,+M,-FM) Avg [%]	$t[s]$	(+F,+M,+FM) Avg [%]	$t[s]$
1	−6.09	12.91	−5.60	13.40	0.23	15.16
2	−3.19	15.75	−2.07	16.74	0.73	17.51
4	−1.82	20.37	−0.19	21.88	1.21	21.53
8	−0.85	28.49	0.98	30.67	1.71	28.68
16	−0.19	43.32	1.75	46.66	2.21	41.31
Ref.	(-F,-M,+FM)		6373.88	13.73		

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Flows with FM are faster than flows on its own for large α'

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Speed-Up Heuristics

Algorithm	Avg [%]	Min [%]	$t_{\text{flow}}[s]$	$t[s]$
KaHyPar-CA	7077.20	6820.17	-	29.26
KaHyPar-MF	2.47	2.12	43.04	72.30
KaHyPar-MF$_{(R1)}$	2.41	2.06	33.89	63.15
KaHyPar-MF$_{(R1,R2)}$	2.40	2.05	28.52	57.78
KaHyPar-MF$_{(R1,R2,R3)}$	2.41	2.06	21.23	50.49

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Comparable Quality

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Speed-up by a factor of 2

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Slow-down compared to KaHyPar-CA by factor of 1.72

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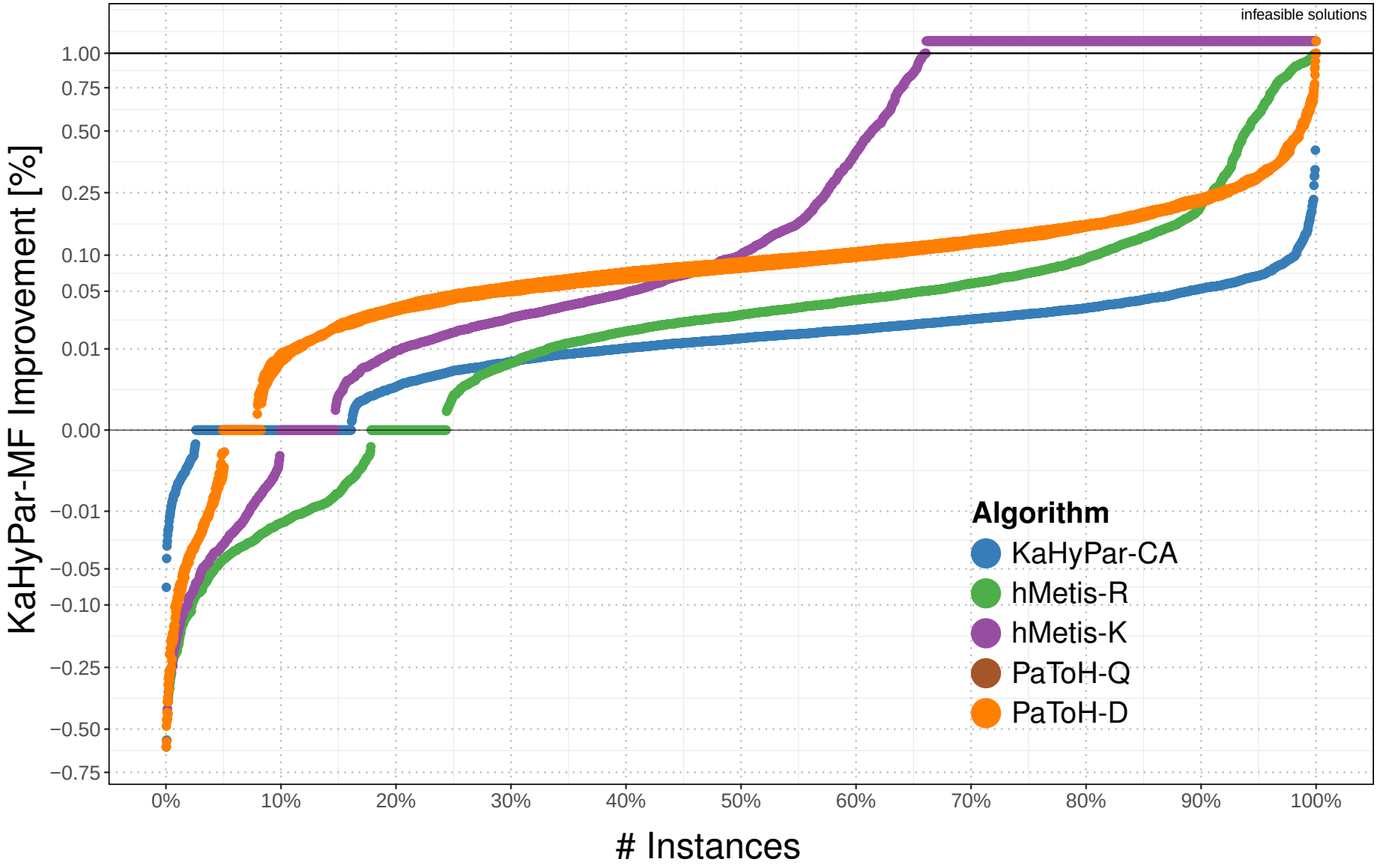
Quality - Full Benchmark Set



Running Time - Full Benchmark Set

Algorithm	$t[s]$
KaHyPar-MF	55.67
KaHyPar-CA	31.05
hMetis-R	79.23
hMetis-K	57.86
PaToH-Q	5.89
PaToH-D	1.22

Running Time - Full Benchmark Set

Algorithm	$t[s]$
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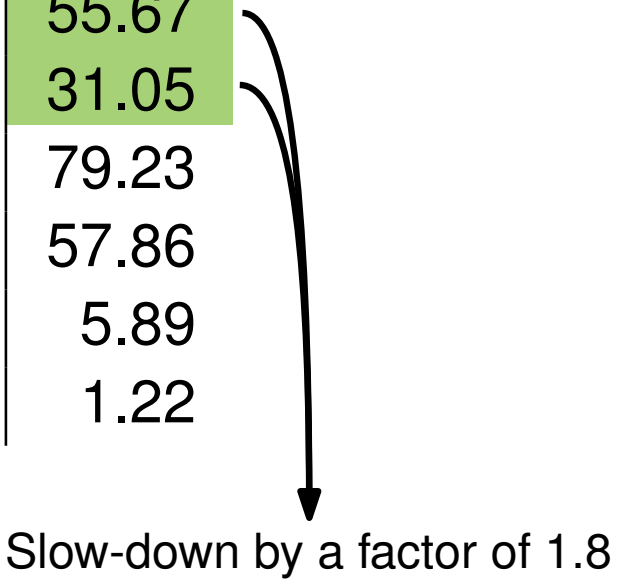
PaToH-Q	5.89
PaToH-D	1.22



Running Time - Full Benchmark Set

Algorithm	$t[s]$
KaHyPar-MF	55.67
KaHyPar-CA	31.05
hMetis-R	79.23
hMetis-K	57.86
PaToH-Q	5.89
PaToH-D	1.22

Comparable running time to hMetis-K

Conclusion

Contributions

- **Generalize framework** of *KaFFPa* [Sanders, Schulz 11]
- **Sparsification techniques** for hypergraph flow networks
- Optimized **flow problem modeling** approach
- Integration of **speedup heuristics**

Conclusion

Contributions

- **Generalize framework** of *KaFFPa* [Sanders, Schulz 11]
- **Sparsification techniques** for hypergraph flow networks
- Optimized **flow problem modeling** approach
- Integration of **speedup heuristics**

Experimental Results

- Best partitions on 73% of 3222 benchmark instances
- Improve quality of *KaHyPar* by 2.5%
- Slowdown by factor of 1.8
- Comparable running time to *hMetis*

References

[Fiduccia, Mattheyses 88]
C. M. Fiduccia and R. M. Mattheyses. A Linear-Time Heuristic for improving Network Partitions. In *Papers on 25 Years of Electronic Design Automation 1988*, ACM, pp.241-247.

[Hu, Moerder 85]
T.C. Hu and K. Multiterminal Flows in a Hypergraph. In VLSI Circuit Layout: Theory and Design. In *IEEE Press 1985*, pp.1-12.

[Lawler 73]
E. Lawler. Cutsets and Partitions of Hypergraphs. *Networks 3 1973*, pp.275-285.

[Liu, Wong 98]
H. Liu and D. Wong. Network Flow Based Multi-way Partitioning with Area and Pin Constraints. In *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems 1998*, pp.50-59.

[Picard, Queyranne 80]
J.-C. Picard and M. Queyranne. On the Structure of all Minimum Cuts in a Network and Applications. In *Combinatorial Optimization II 1980*, pp.8-16.

[Sanders, Schulz 11]
P. Sanders and C. Schulz. Engineering Multilevel Graph Partitioning Algorithms. ESA 2011, vol. 6942, Springer, pp.469-480.

Appendix

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Flow Problem Modeling

KaFFPa vs. *KaHyPar*

	Hypergraphs			Graphs		
α	$\varepsilon = 1\%$	$\varepsilon = 3\%$	$\varepsilon = 5\%$	$\varepsilon = 1\%$	$\varepsilon = 3\%$	$\varepsilon = 5\%$
1	8.3	8.9	8.2	13.3	12.7	11.8
2	8.5	7.1	5.1	12.4	10.1	8.5
4	7.4	4.1	2.8	11.0	7.9	5.7
8	5.4	2.4	1.6	9.4	5.6	4.0
16	3.5	1.3	1.2	7.5	4.2	3.6

Flow Problem Modeling

KaFFPa vs. *KaHyPar*

Test Configuration: (+F,-M,-FM)

25 Hypergraph Instances

15 Graph Instances

	HYPERGRAPHS			GRAPHS		
α	$\varepsilon = 1\%$	$\varepsilon = 3\%$	$\varepsilon = 5\%$	$\varepsilon = 1\%$	$\varepsilon = 3\%$	$\varepsilon = 5\%$
1	8.3	8.9	8.2	13.3	12.7	11.8
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4	7.4	4.1	2.8	11.0	7.9	5.7
8	5.4	2.4	1.6	9.4	5.6	4.0
16	3.5	1.3	1.2	7.5	4.2	3.6

Flow Problem Modeling

KaFFPa vs. *KaHyPar*

Graphs profit more than hypergraphs from new modeling approach

	HYPERGRAPHS			GRAPHS		
α	$\varepsilon = 1\%$	$\varepsilon = 3\%$	$\varepsilon = 5\%$	$\varepsilon = 1\%$	$\varepsilon = 3\%$	$\varepsilon = 5\%$
1	8.3	8.9	8.2	13.3	12.7	11.8
2	8.5	7.1	5.1	12.4	10.1	8.5
4	7.4	4.1	2.8	11.0	7.9	5.7
8	5.4	2.4	1.6	9.4	5.6	4.0
16	3.5	1.3	1.2	7.5	4.2	3.6

Flow Problem Modeling

KaFFPa vs. *KaHyPar*

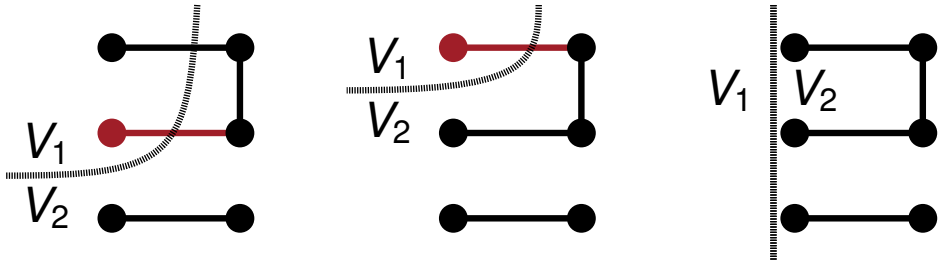
The smaller the flow problem the bigger the improvement

	HYPERGRAPHS			GRAPHS		
α	$\varepsilon = 1\%$	$\varepsilon = 3\%$	$\varepsilon = 5\%$	$\varepsilon = 1\%$	$\varepsilon = 3\%$	$\varepsilon = 5\%$
1	8.3	8.9	8.2	13.3	12.7	11.8
2	8.5	7.1	5.1	12.4	10.1	8.5
4	7.4	4.1	2.8	11.0	7.9	5.7
8	5.4	2.4	1.6	9.4	5.6	4.0
16	3.5	1.3	1.2	7.5	4.2	3.6

FM Algorithm - Disadvantages

- Incorporate only **local** informations about problem structure

FM Algorithm



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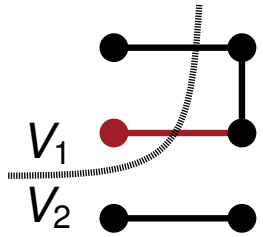
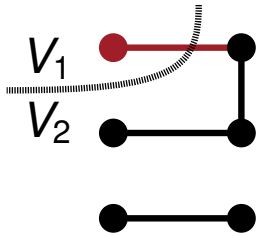
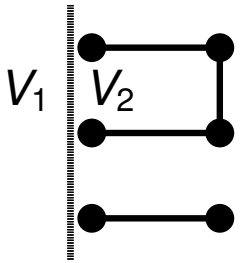
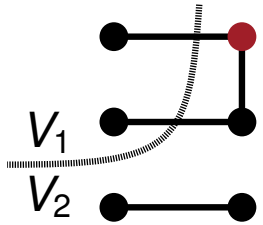
FM Algorithm

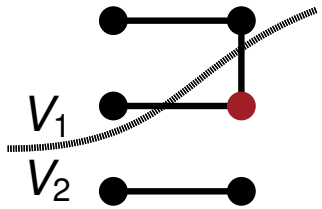


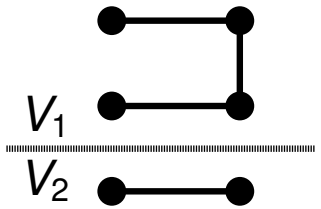




Optimal

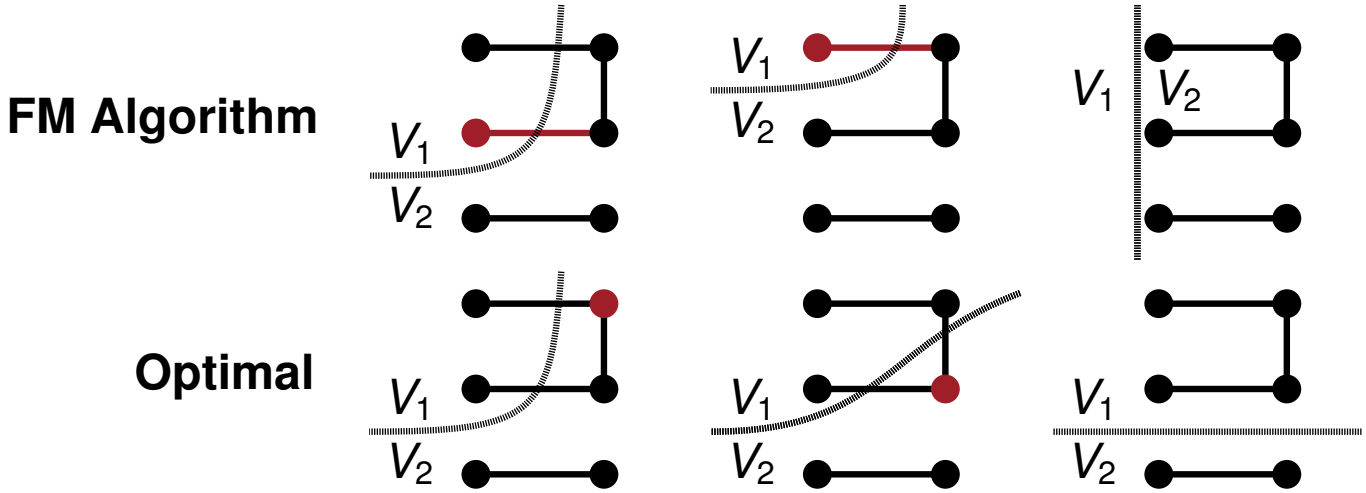




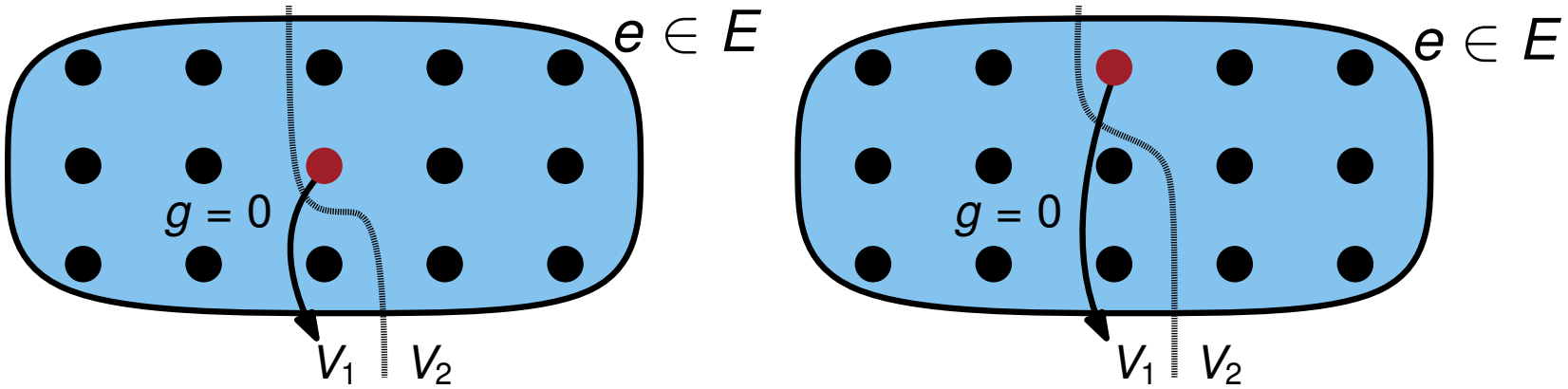


FM Algorithm - Disadvantages

- Incorporate only **local** informations about problem structure

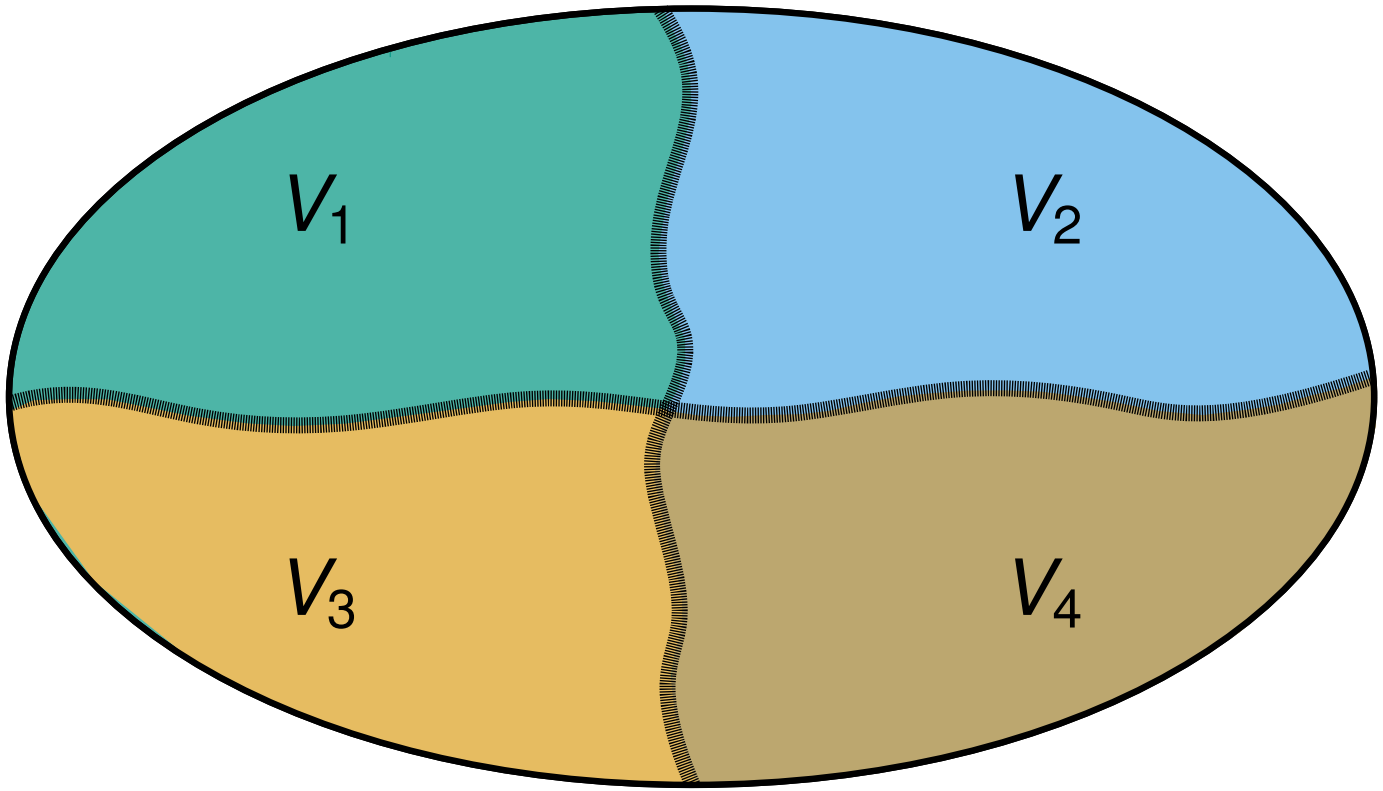


- **Zero-Gain** Moves



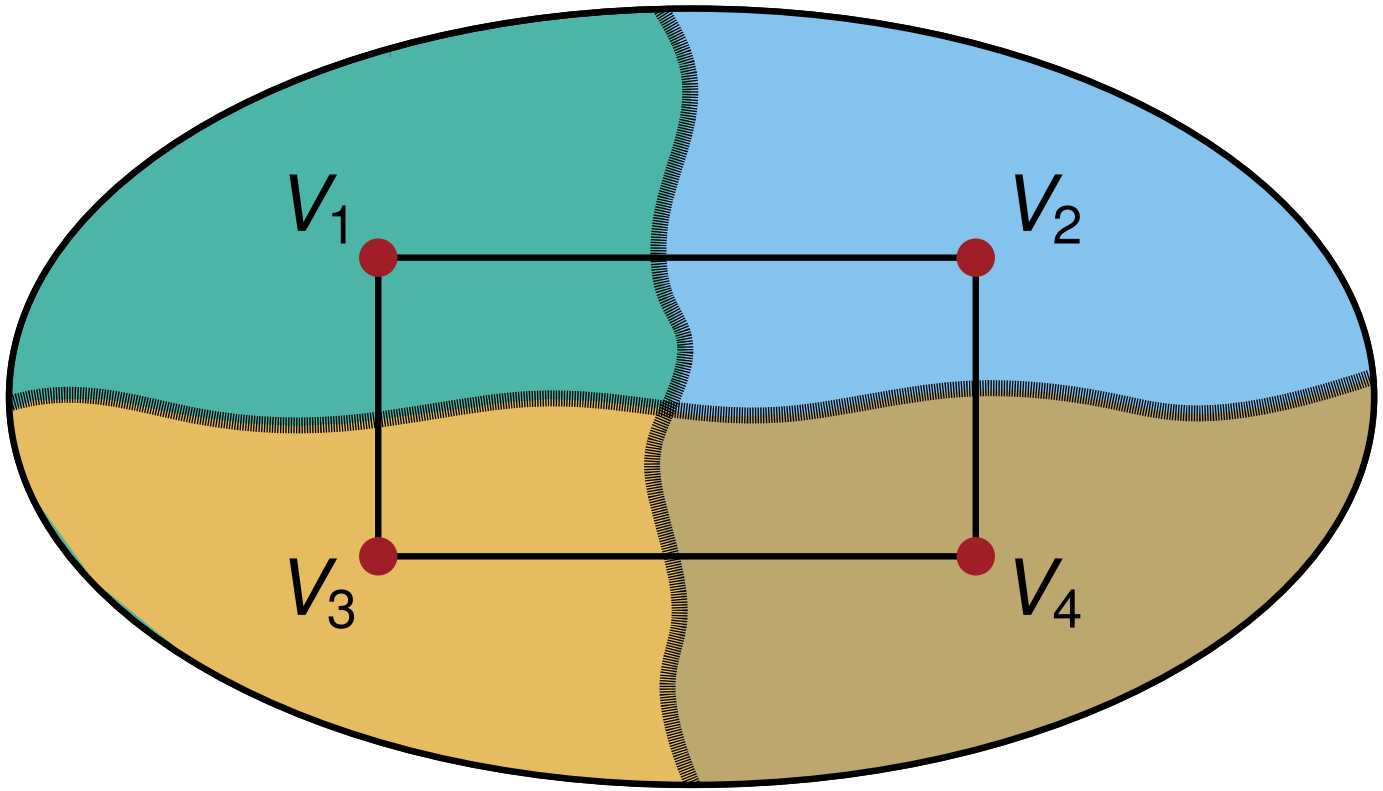
Active Block Scheduling

KaFFPa [Sanders, Schulz 11]



Active Block Scheduling

KaFFPa [Sanders, Schulz 11]



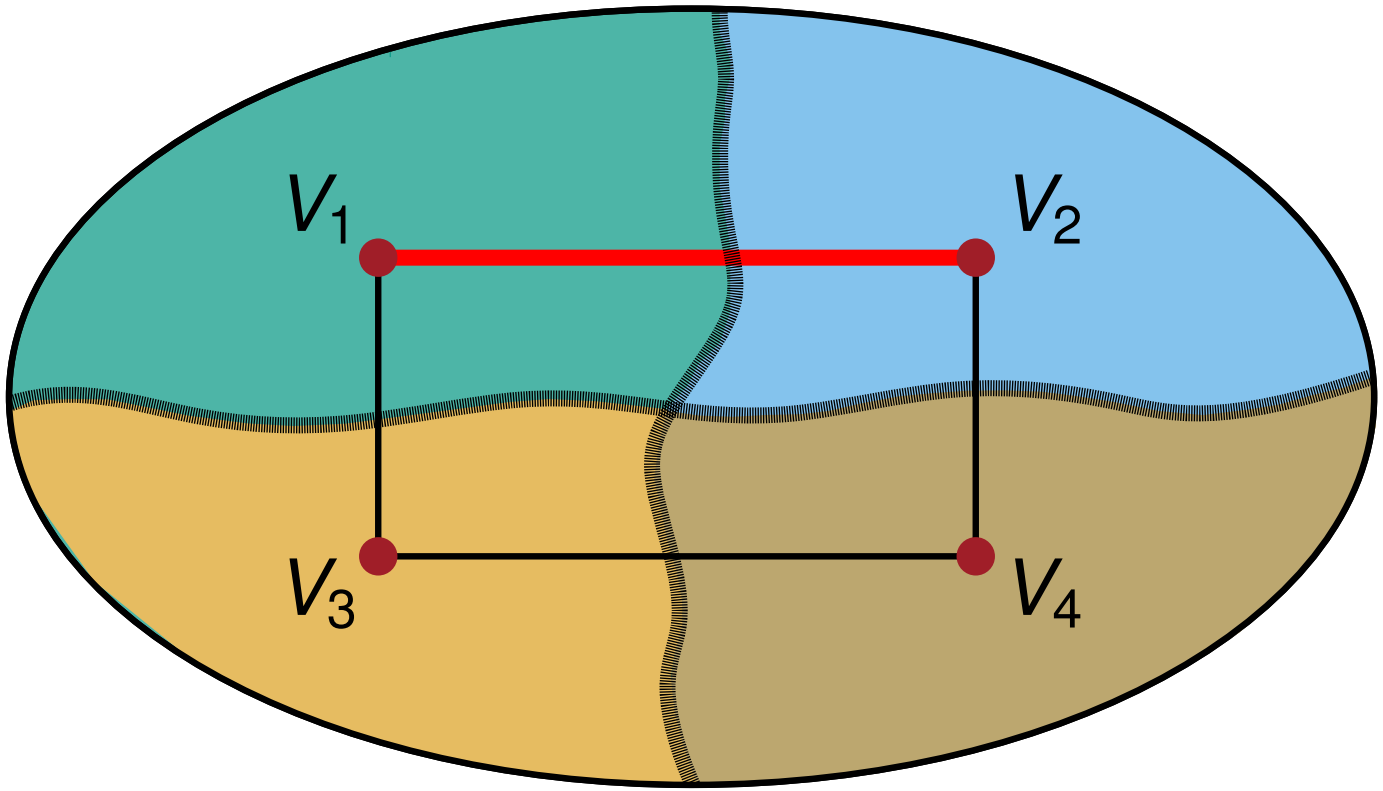
Build Quotient Graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

Round 1

refine(V_1, V_2) = **Improvement!**



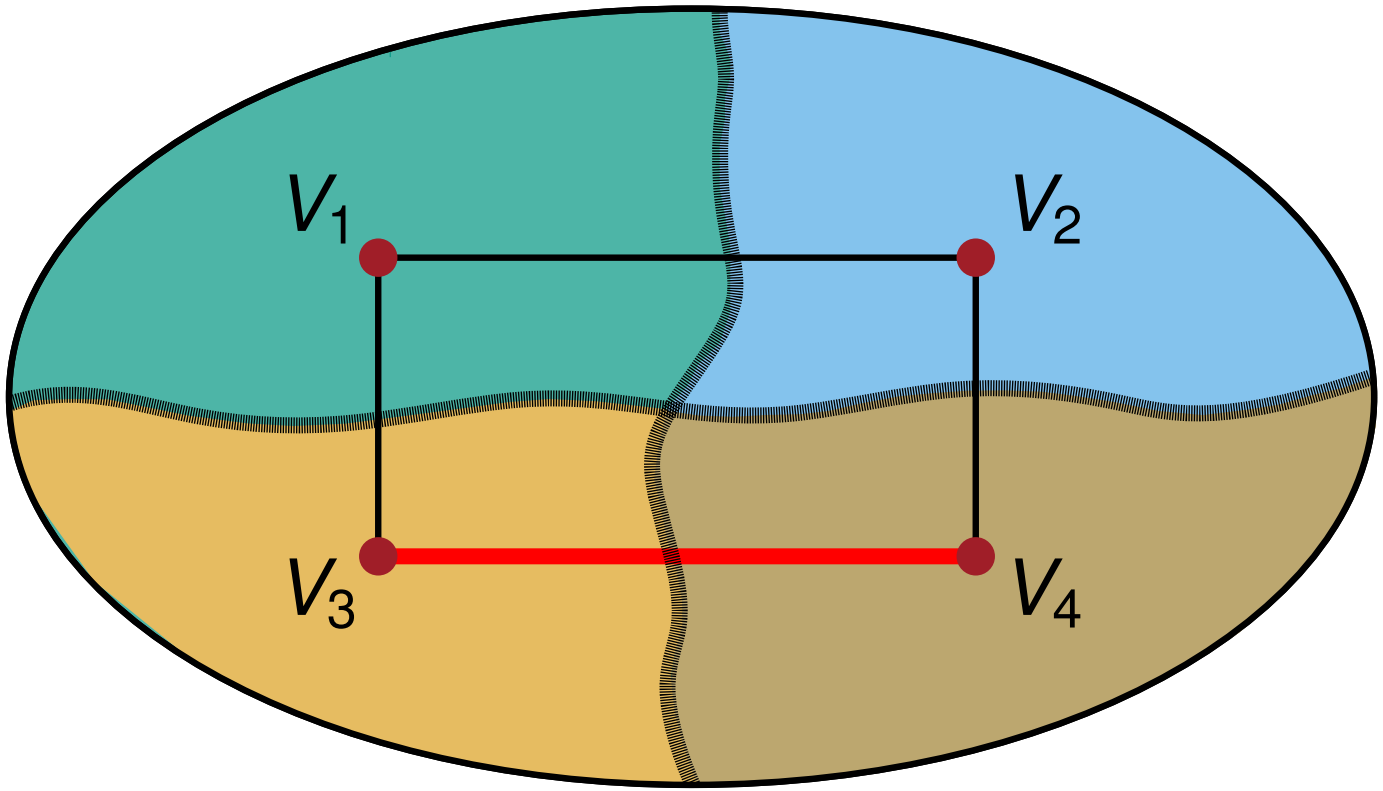
Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

Round 1

refine(V_3, V_4) = **No Improvement!**



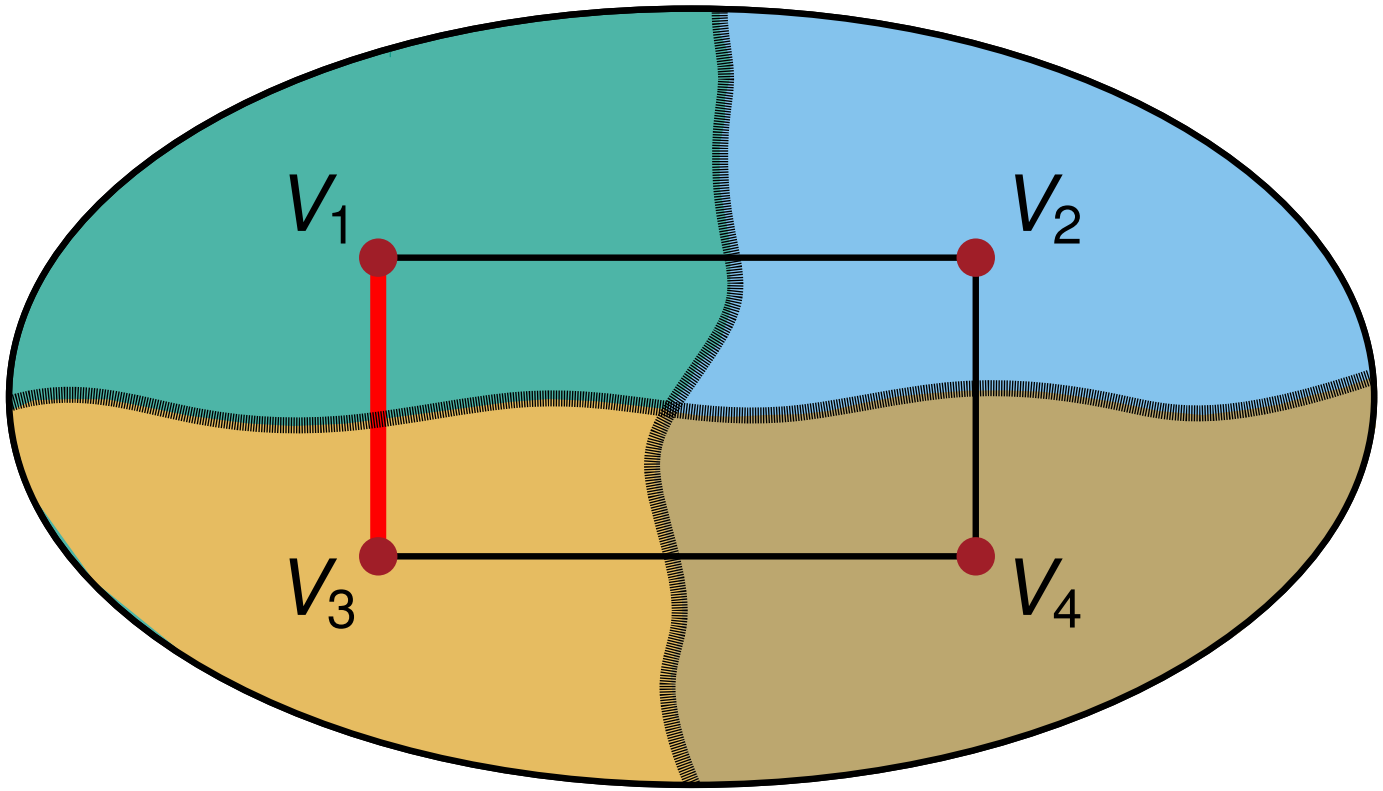
Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

Round 1

refine(V_1, V_3) = **No Improvement!**



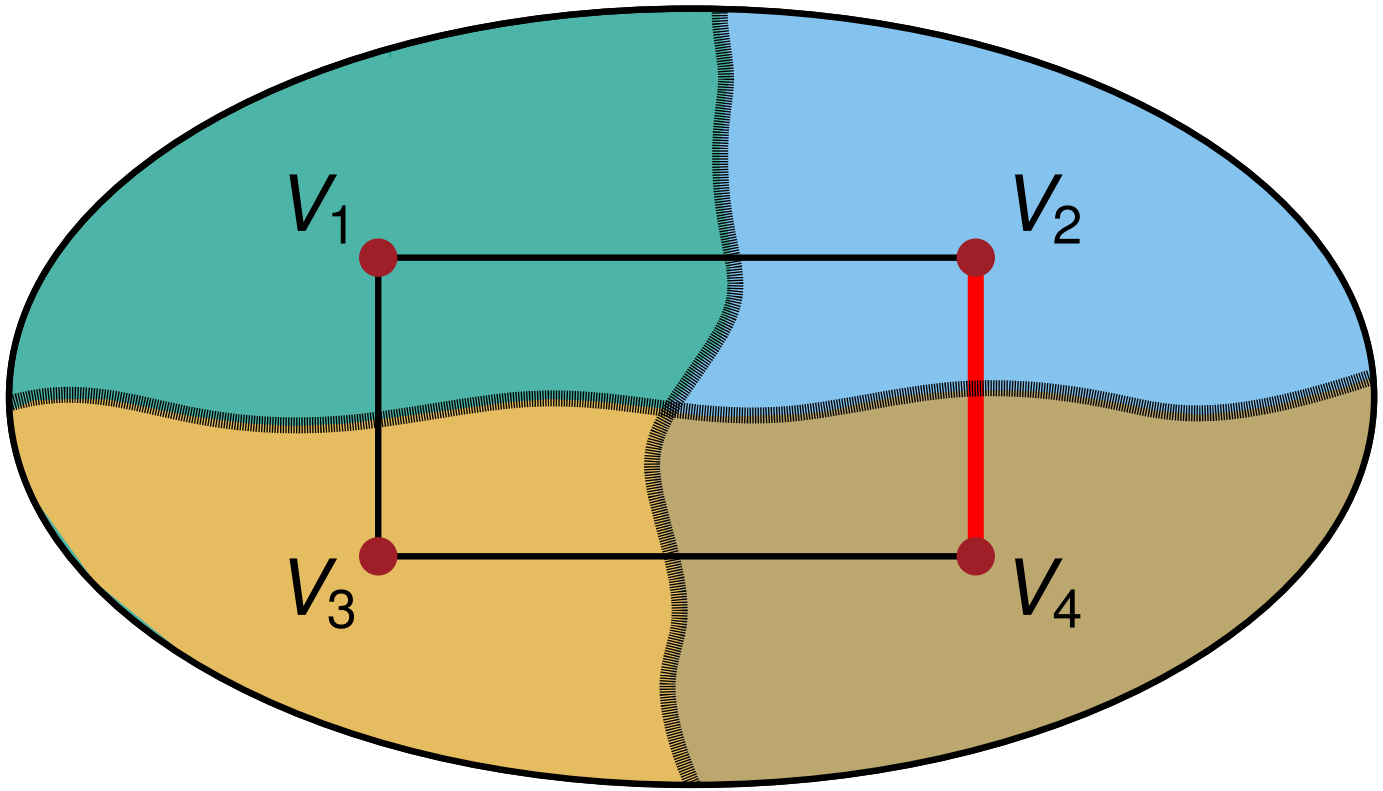
Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

Round 1

refine(V_2, V_4) = **No Improvement!**

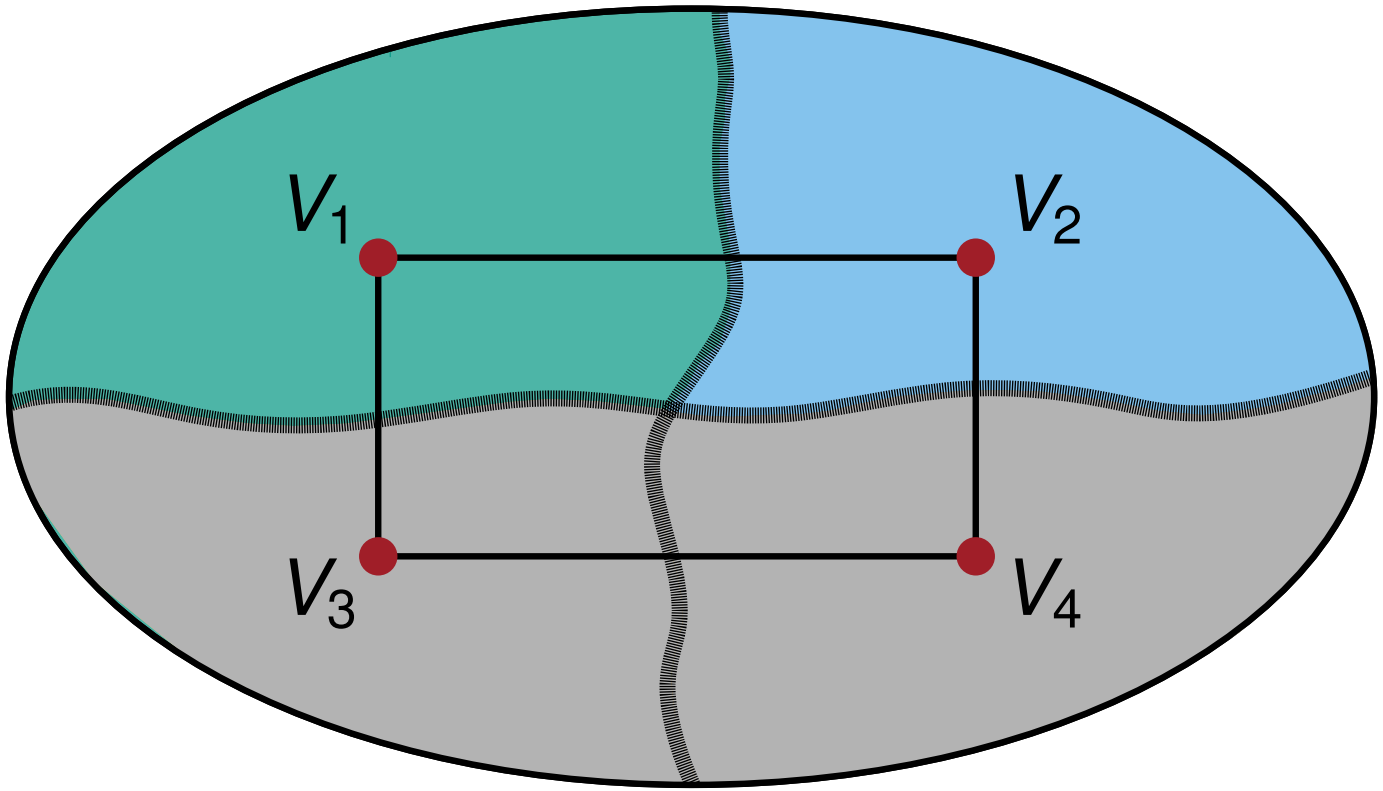


Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

Round 1 Boundary did not change $\Rightarrow$ Mark block as **inactive**



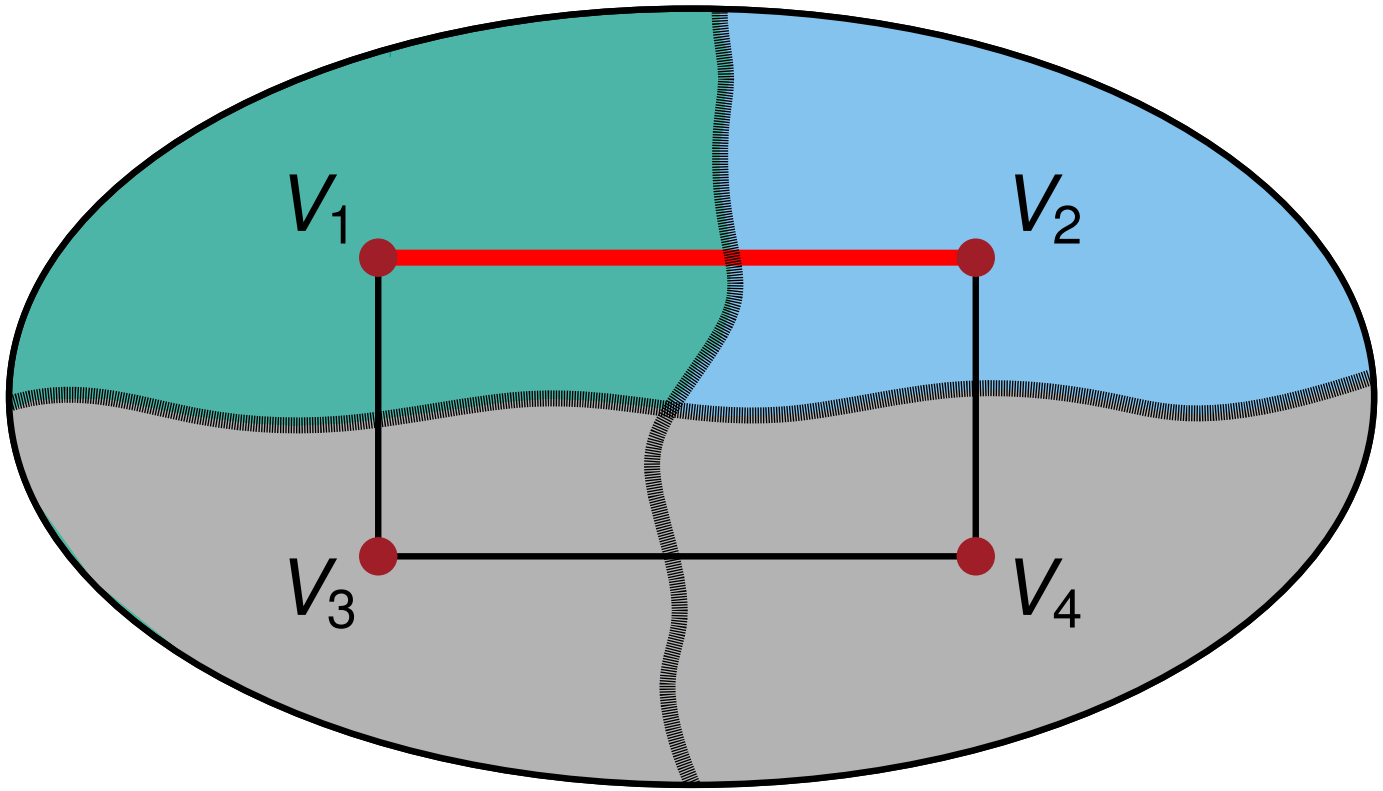
Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

Round 2

refine(V_1, V_2) = **No Improvement!**



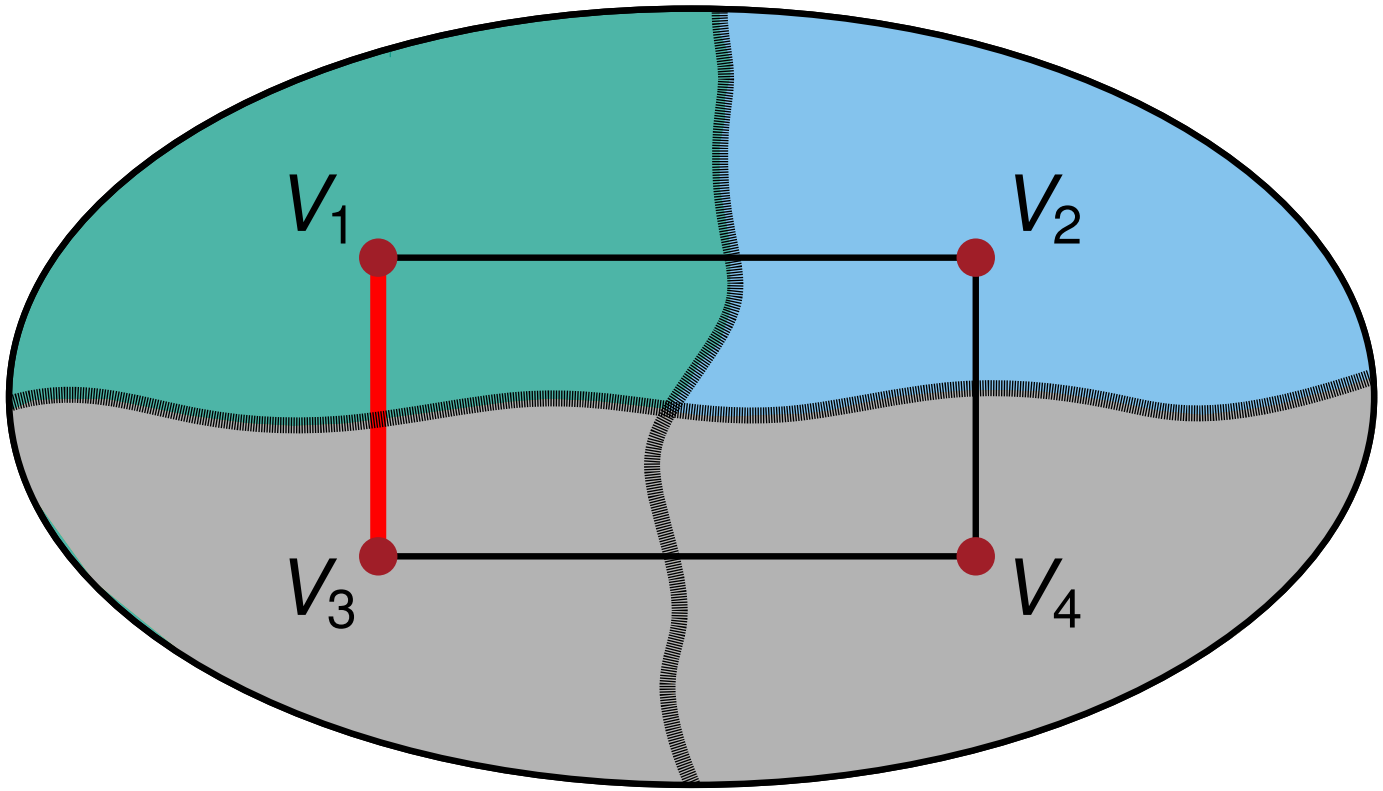
Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

Round 2

refine(V_1, V_3) = **No Improvement!**



Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

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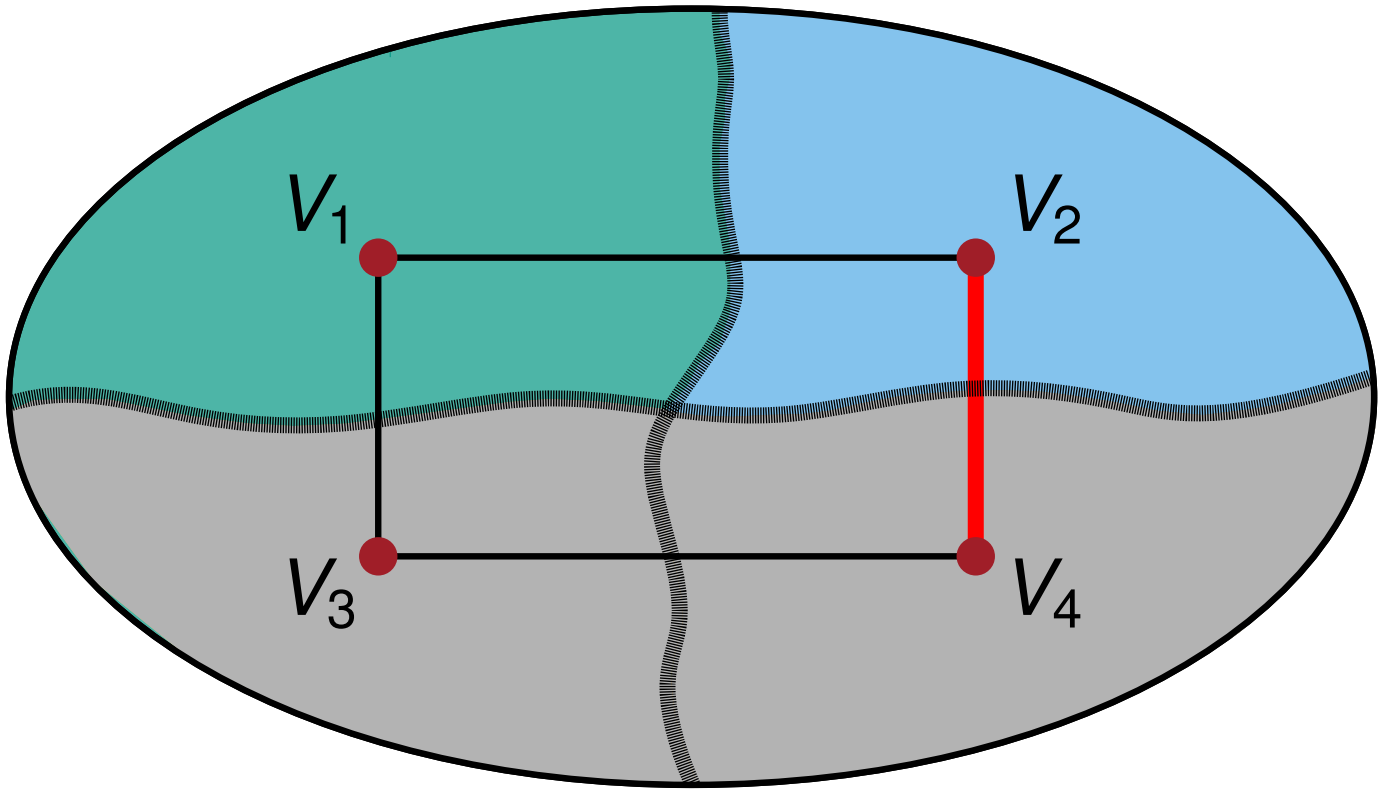
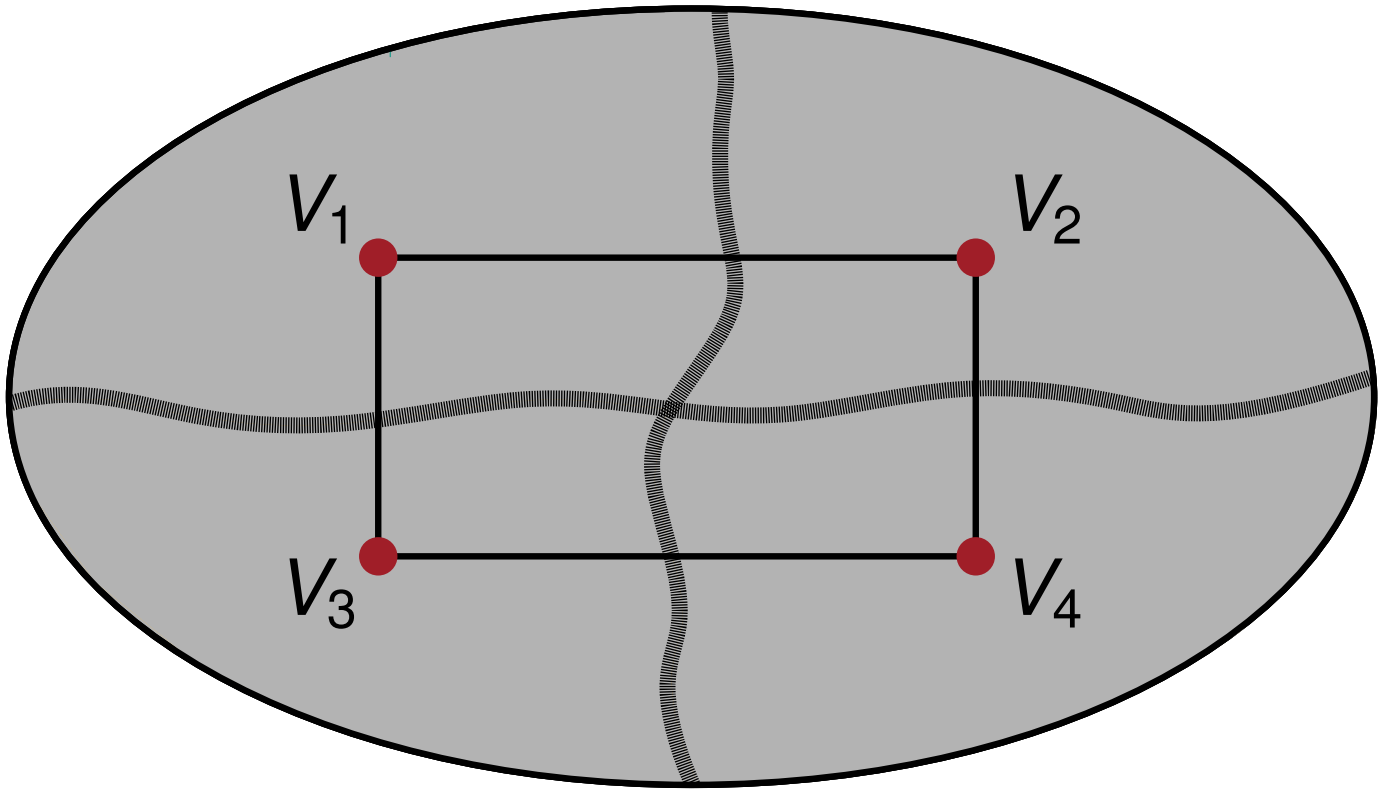


Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

Round 2 Boundary did not change $\Rightarrow$ Mark block as **inactive**

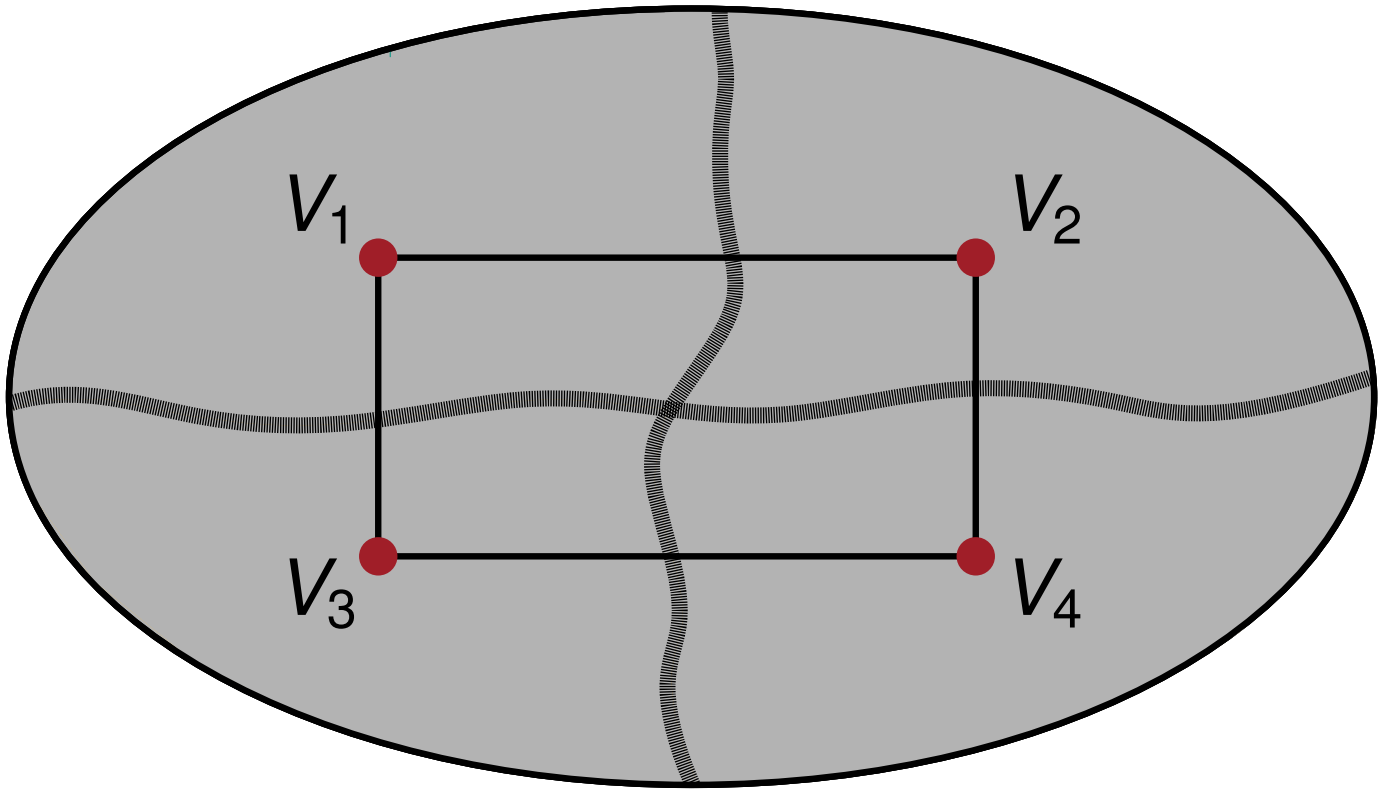


Using 2-way refinement algorithm for **active** blocks of the quotient graph

Active Block Scheduling

KaFFPa [Sanders, Schulz 11]

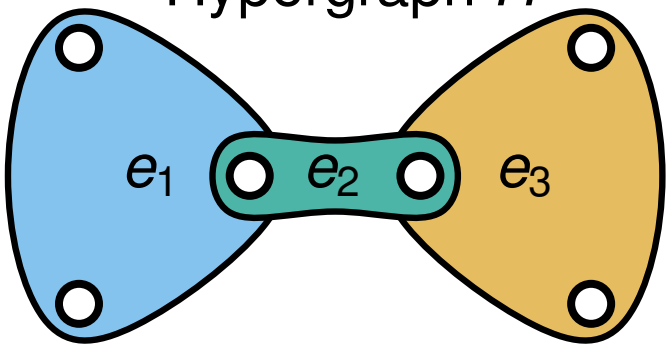
Round 2 All blocks are **inactive** $\Rightarrow$ Algorithm terminates



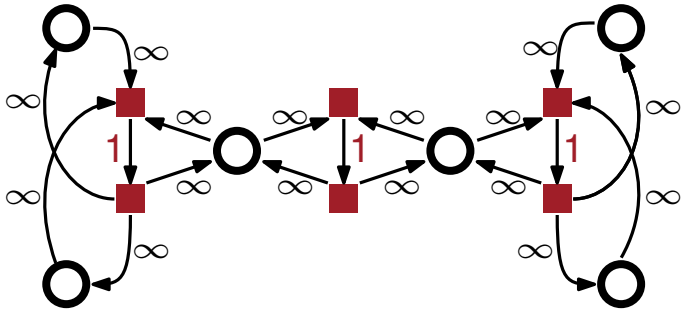
Using 2-way refinement algorithm for **active** blocks of the quotient graph

Hypergraph Flow Network - Graph Edges

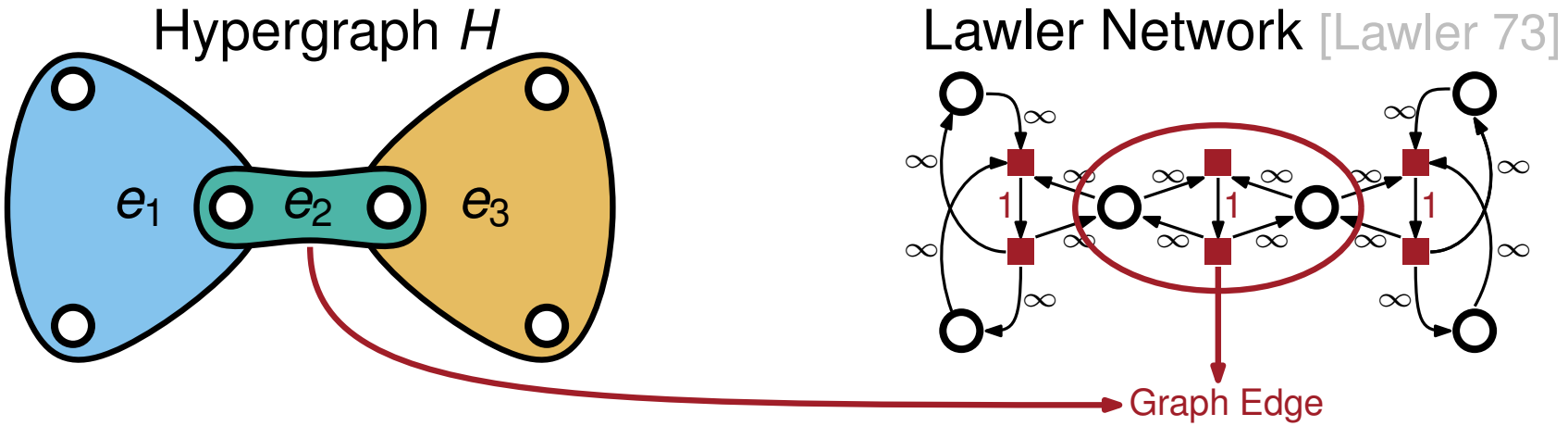
Hypergraph H



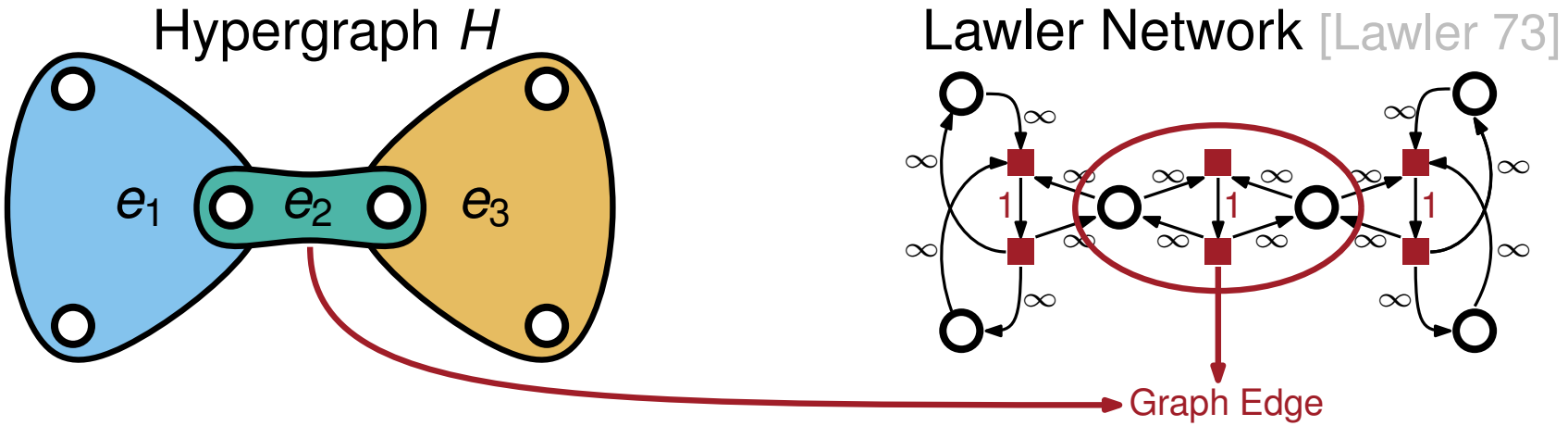
Lawler Network [Lawler 73]

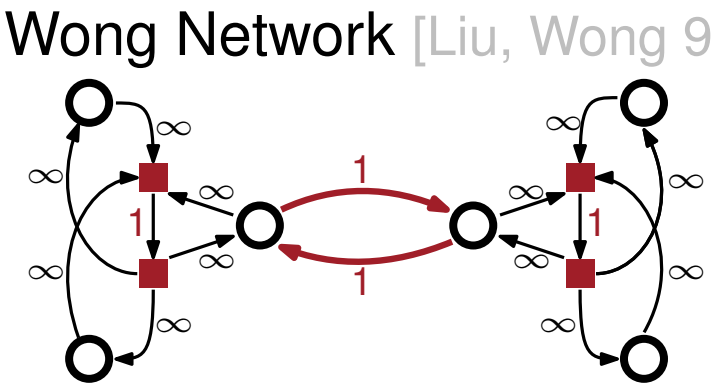


Hypergraph Flow Network - Graph Edges



Hypergraph Flow Network - Graph Edges





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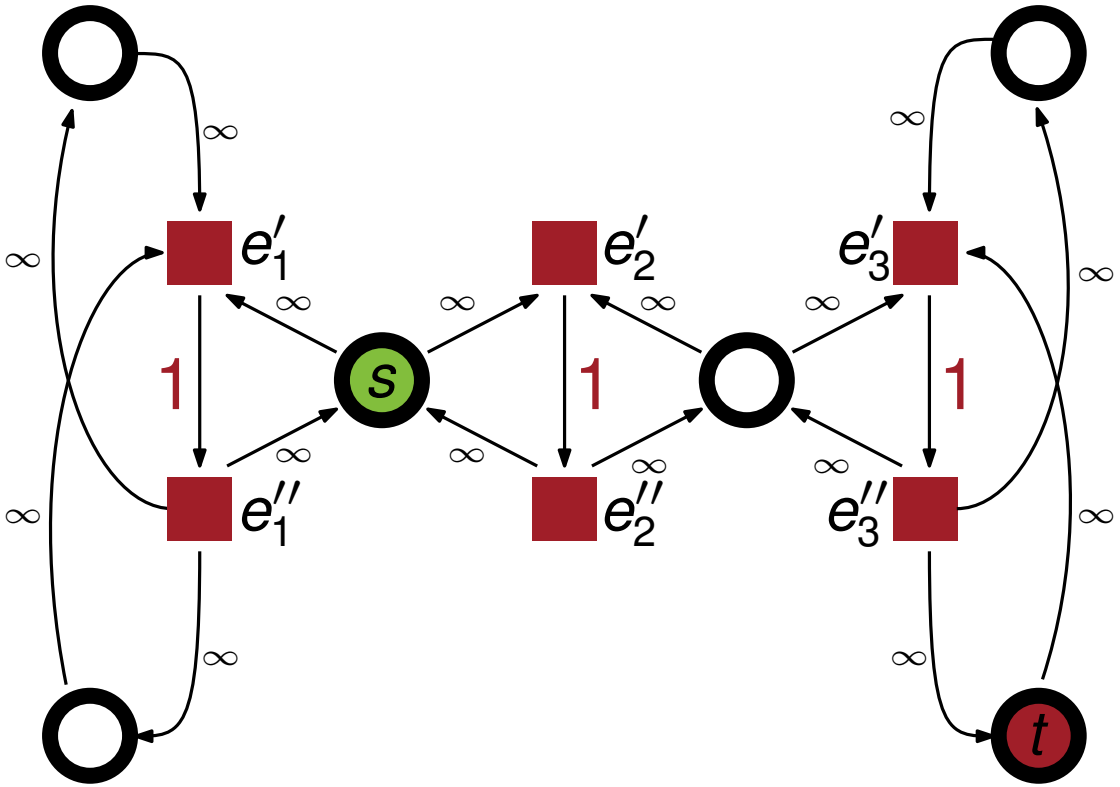
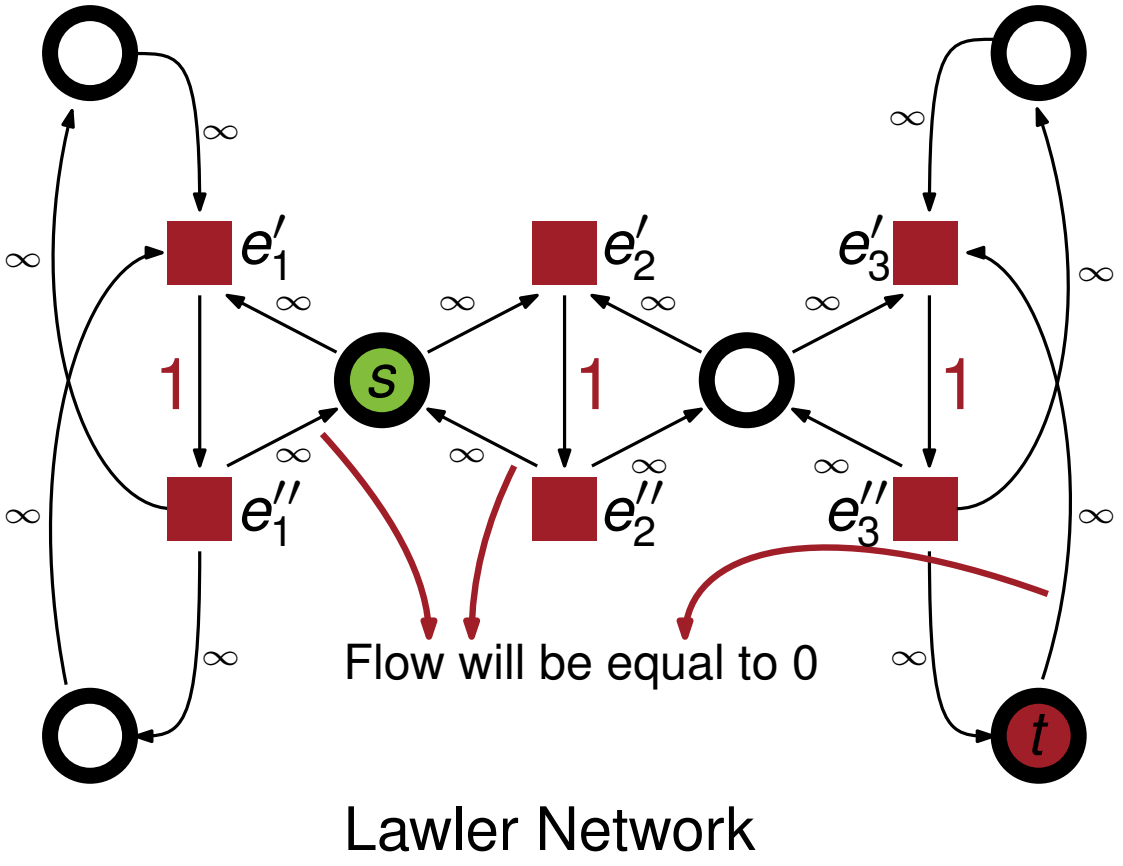
Removing Source and Sink Vertices



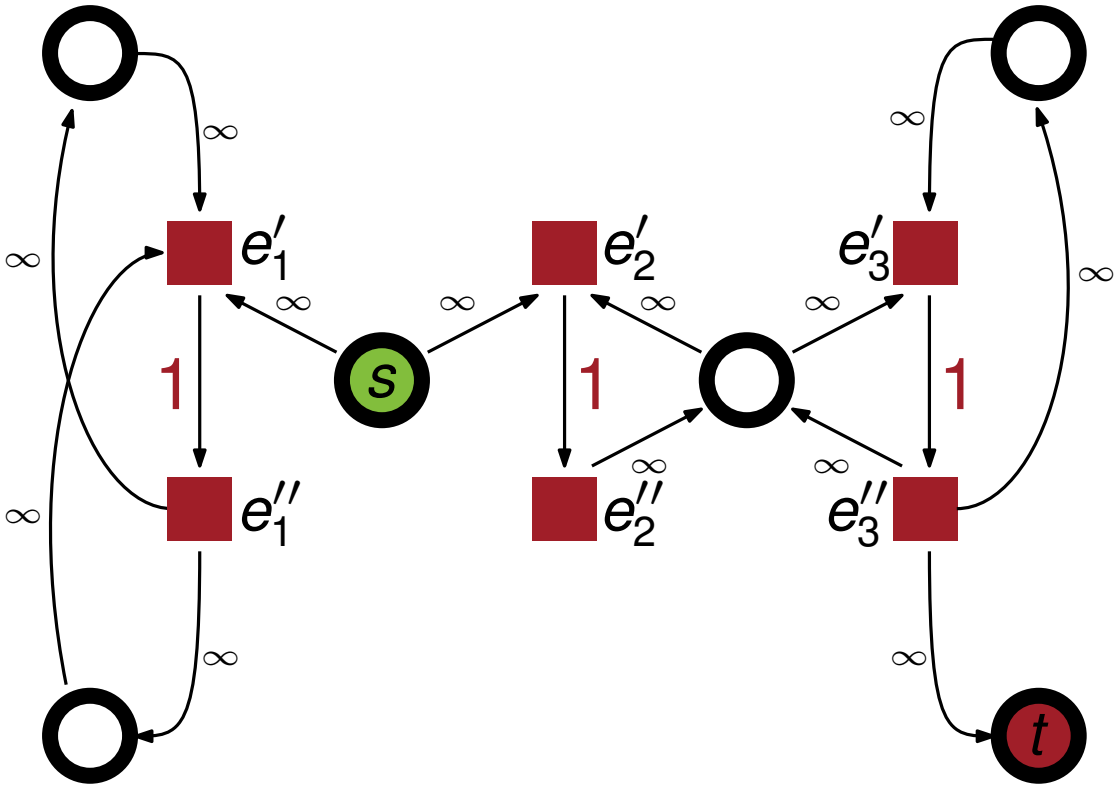
Lawler Network

Removing Source and Sink Vertices



Lawler Network

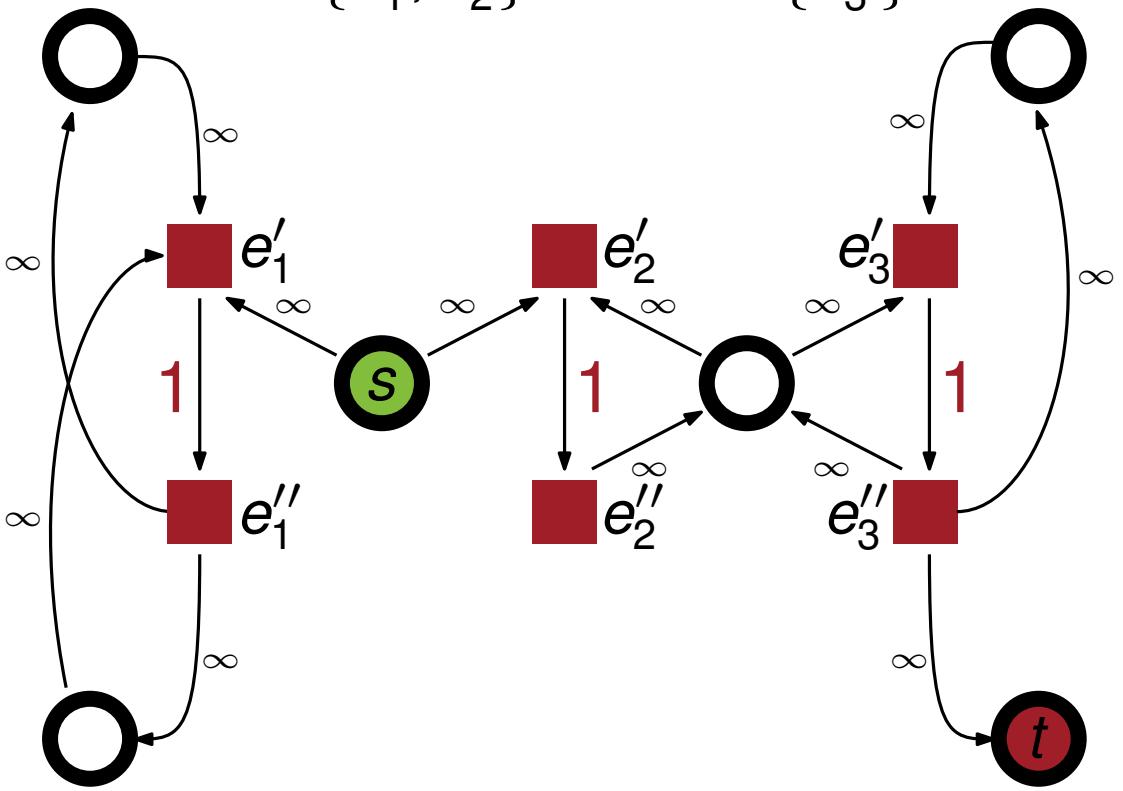
Removing Source and Sink Vertices



Lawler Network

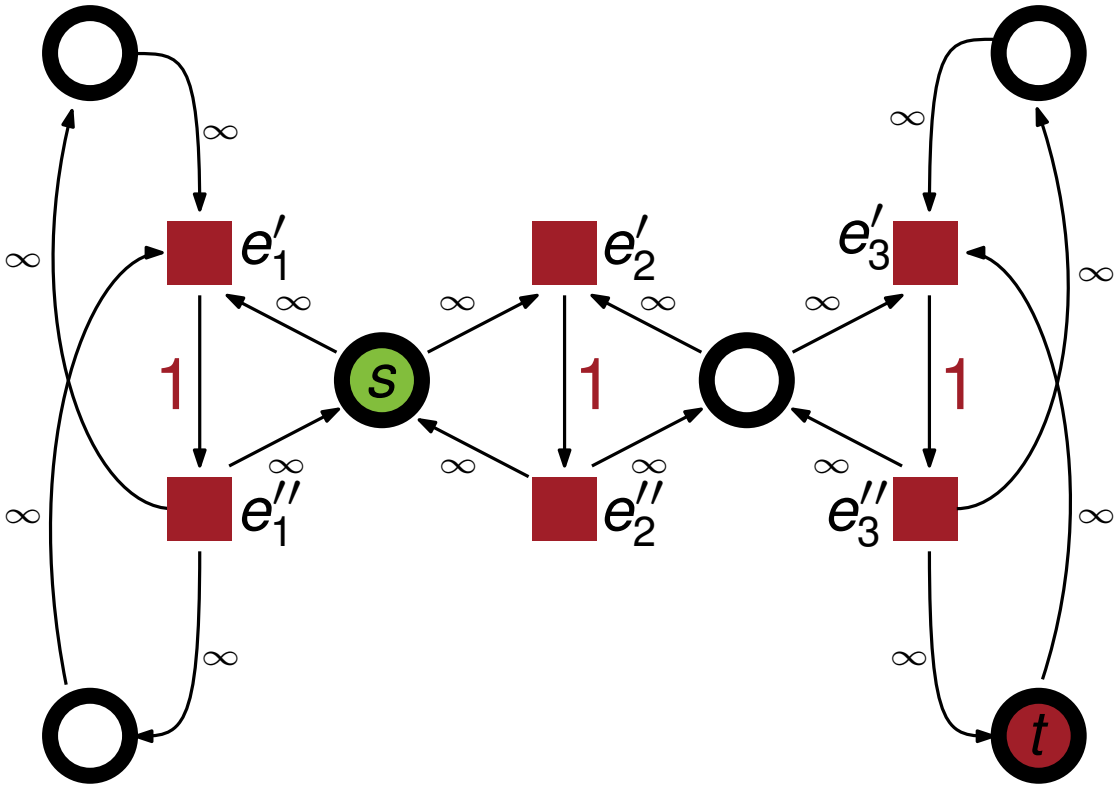
Removing Source and Sink Vertices

Corresponds to *Multi-Source Multi-Sink* problem with
$S = \{e'_1, e'_2\}$ and $T = \{e''_3\}$

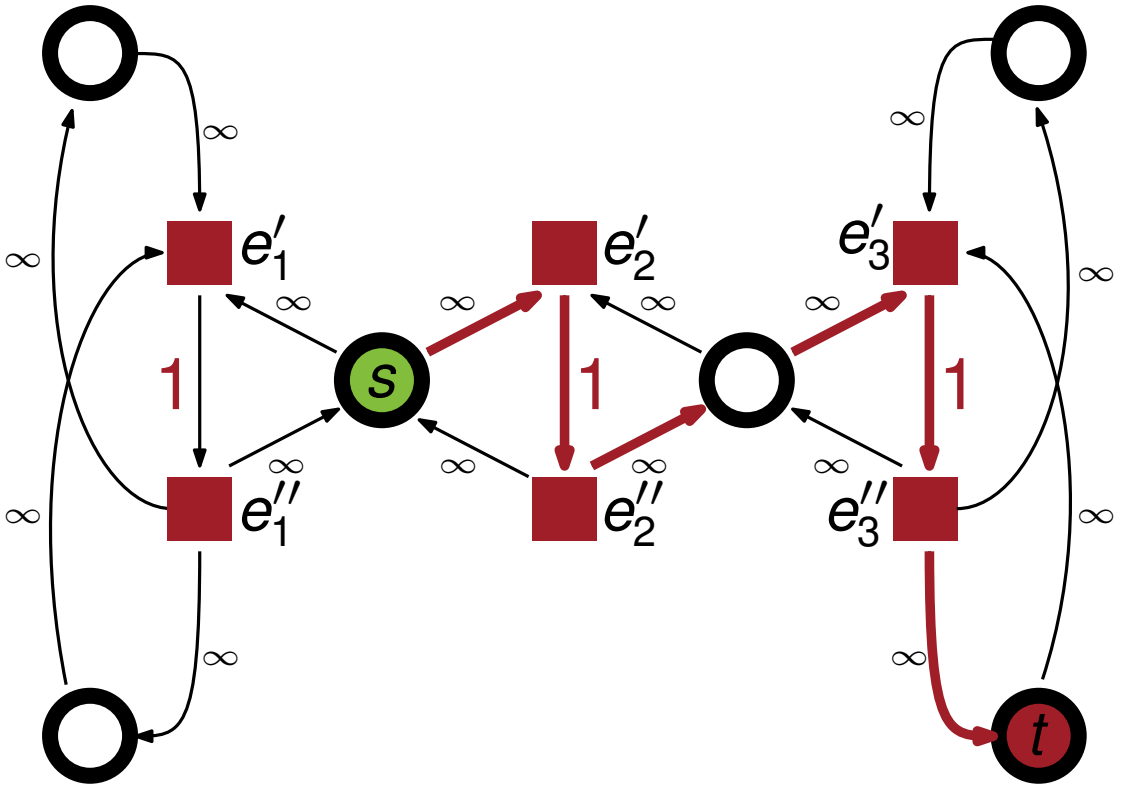


Lawler Network

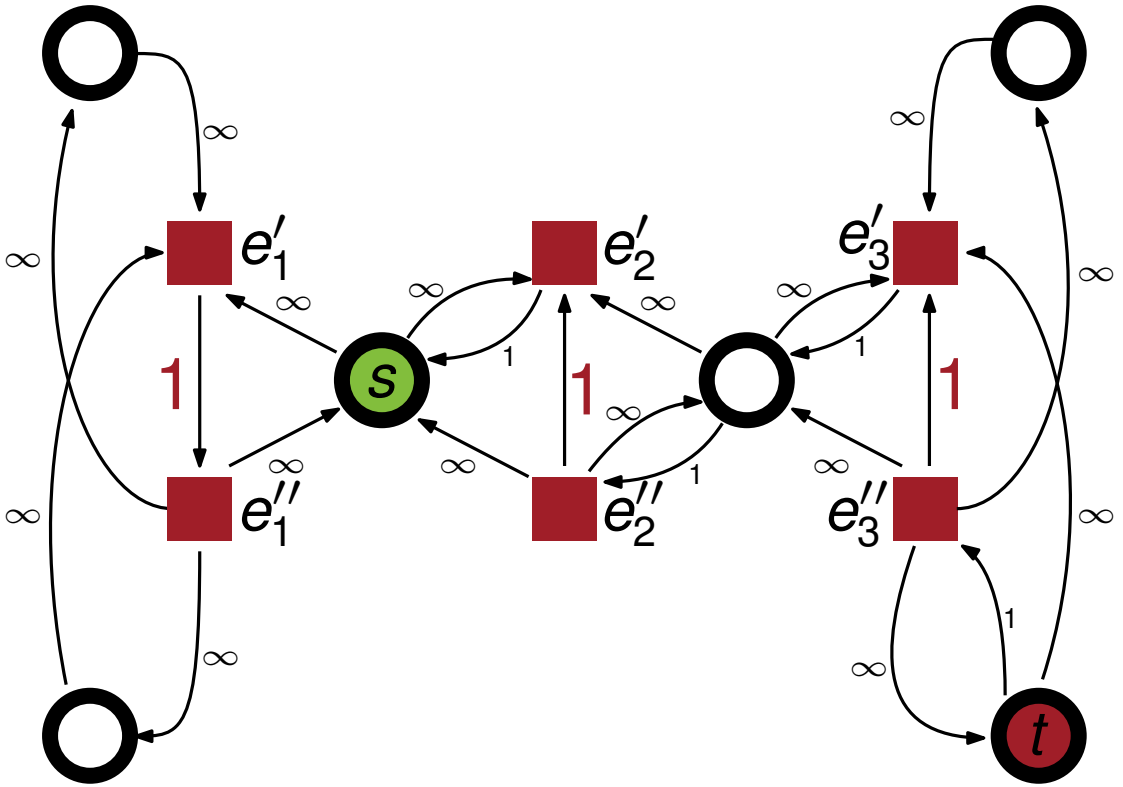
Minimum (s, t)-Bipartition



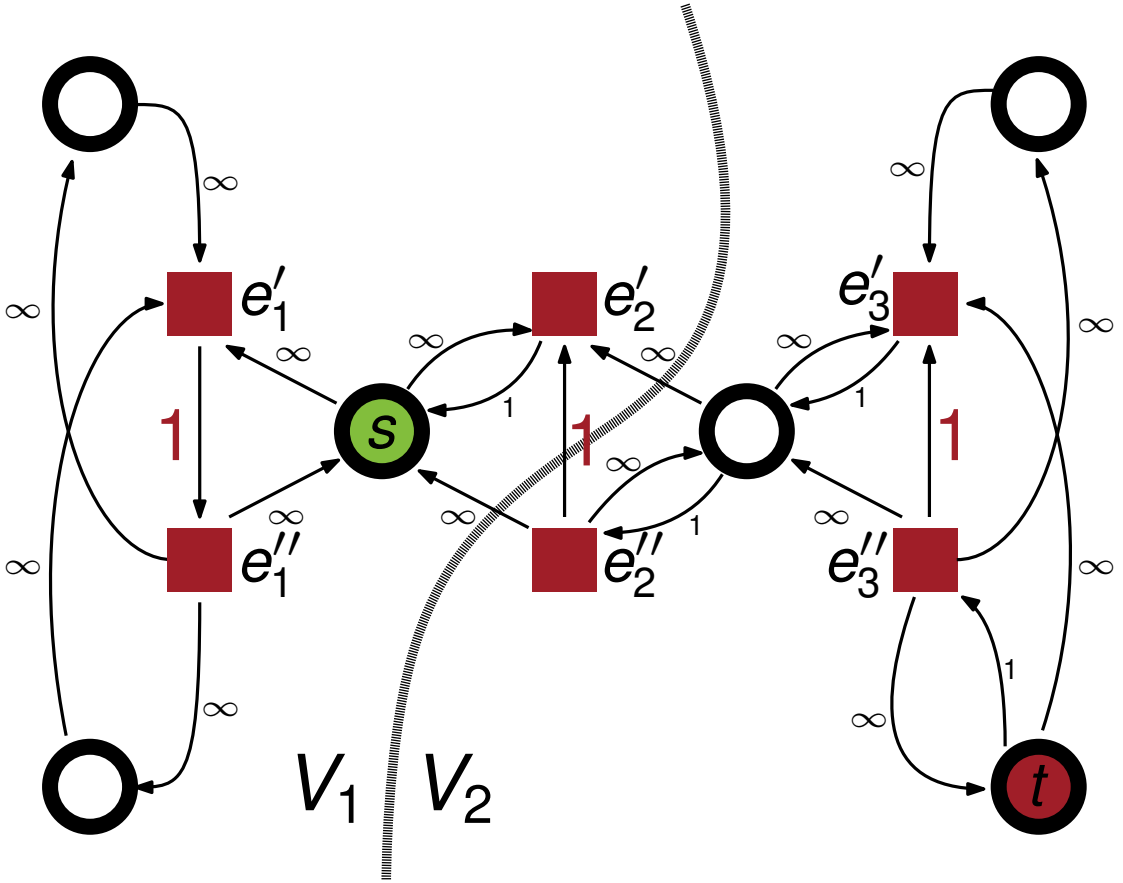
Minimum (s, t)-Bipartition



Minimum (s, t)-Bipartition



Minimum (s, t)-Bipartition



All nodes *reachable* from s are part of V_1 and $V_2 = V \setminus V_1$

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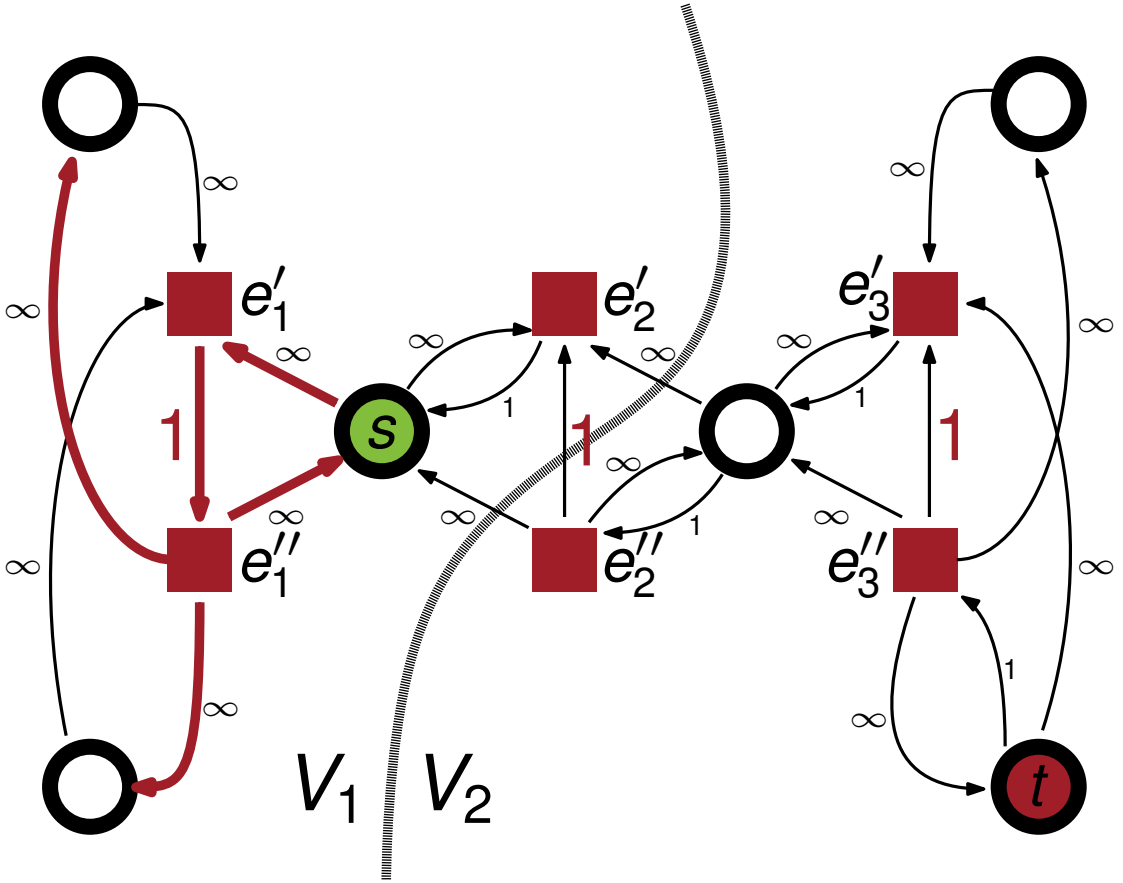
Minimum (s, t)-Bipartition

For each hypernode $v \in V_1$, there exists at least one $e \in I(v)$ with $e'' \in V_1$



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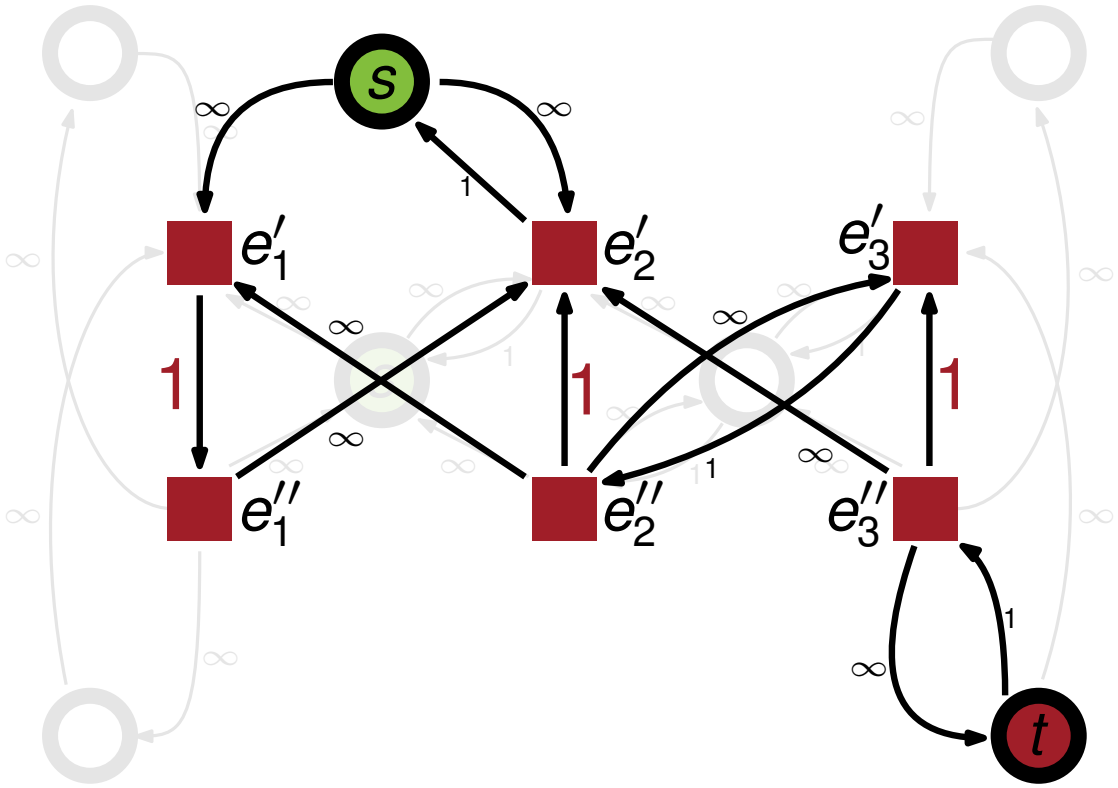
Minimum (s, t)-Bipartition



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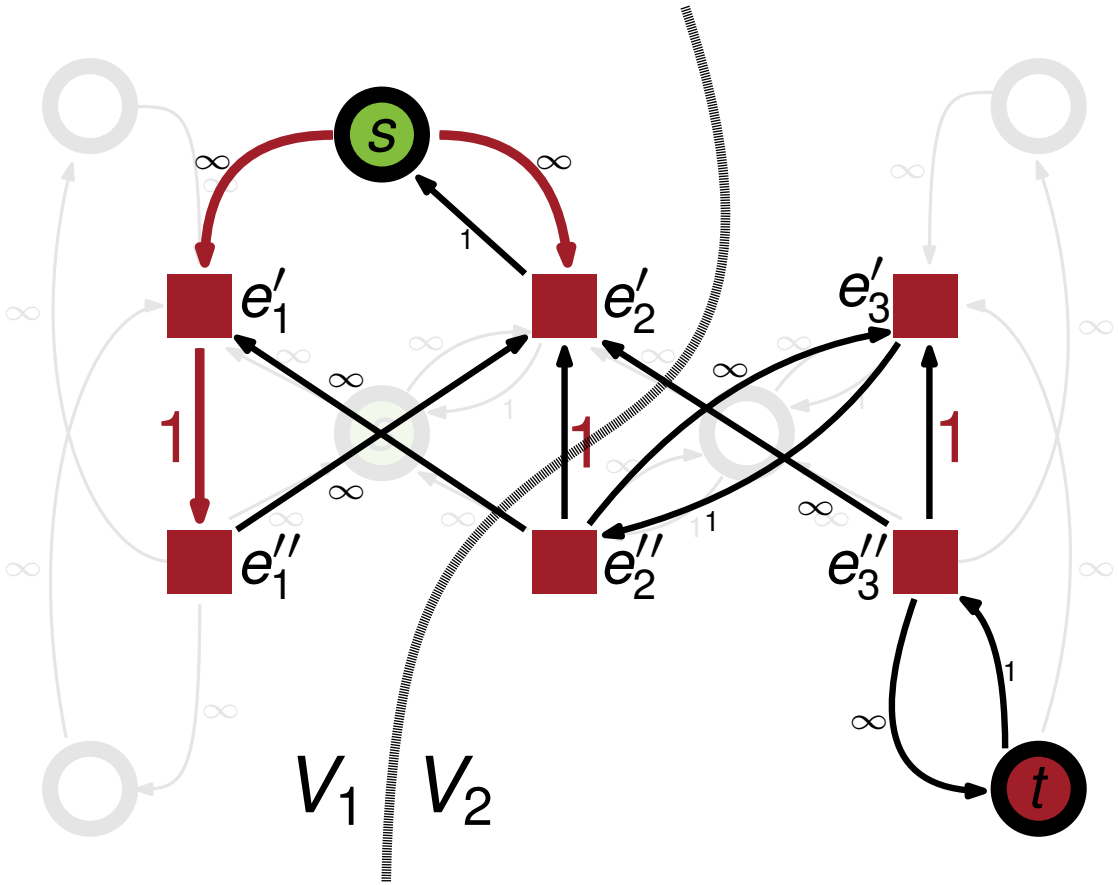
Minimum (s, t)-Bipartition



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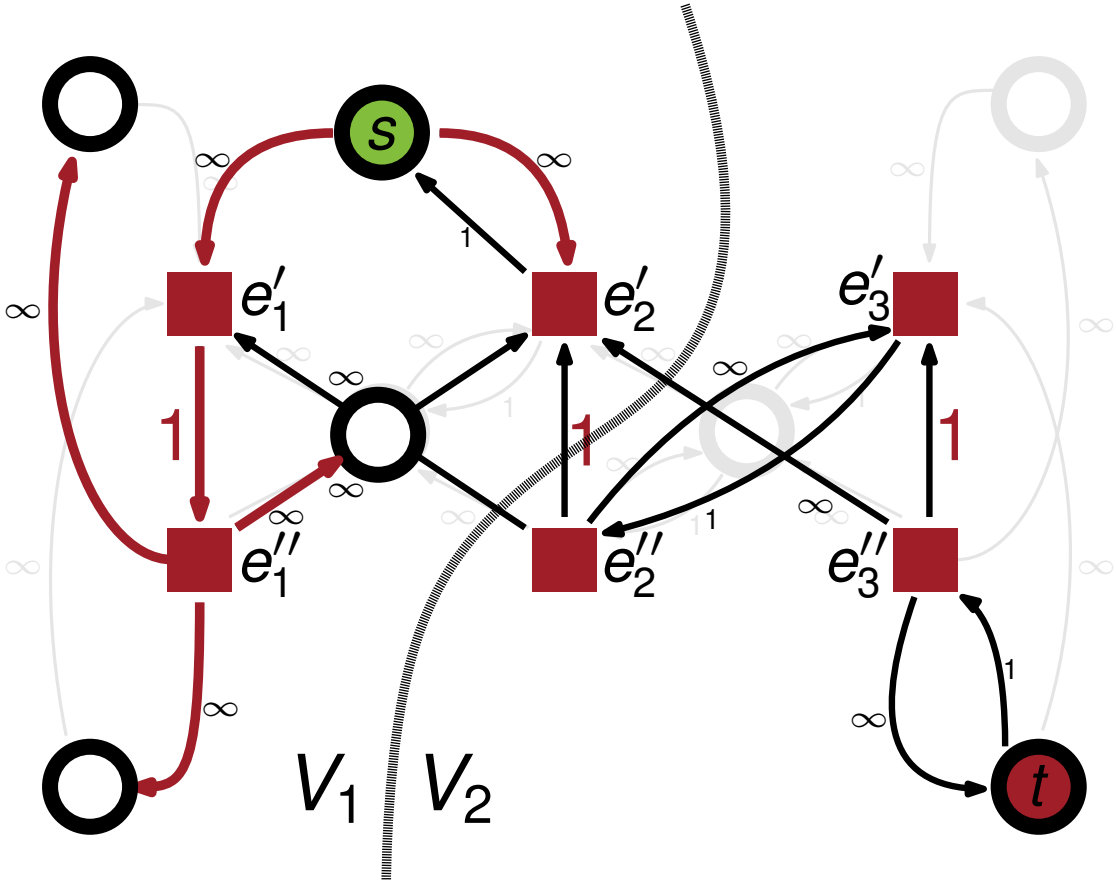
Minimum (s, t)-Bipartition



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Integration into KaHyPar

- KaHyPar is a *n*-level hypergraph partitioner

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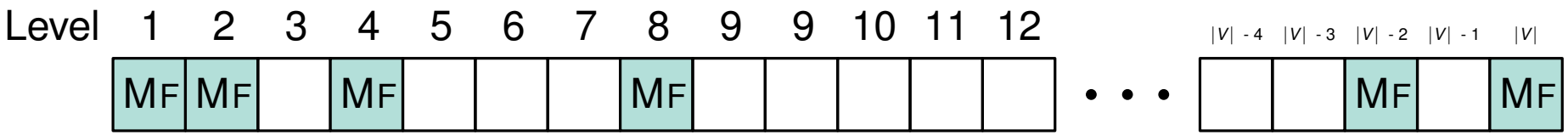
Integration into KaHyPar

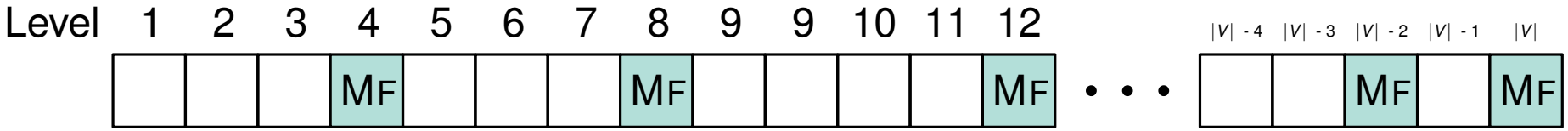
- KaHyPar is a *n*-level hypergraph partitioner

Flow Execution Policies

Exponential: Execute *Max-Flow-Min-Cut* computations (MF) on each level i with $i = 2^j$



Constant: Execute *Max-Flow-Min-Cut* computations (MF) on each level i with $i = \beta \cdot j$



Integration into KaHyPar

- KaHyPar is a *n*-level hypergraph partitioner

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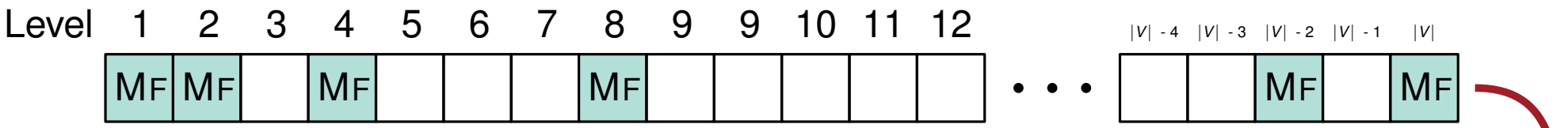

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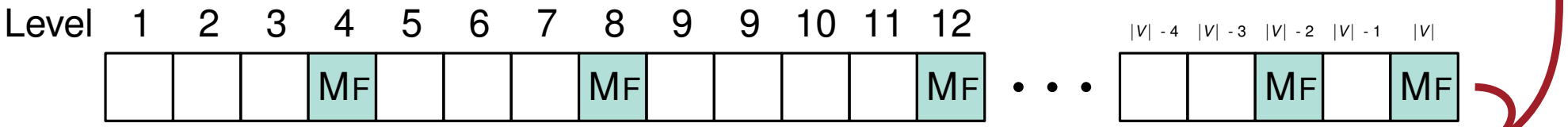

Note, each policy uses *flow*-based refinement on the **last level**

Integration into KaHyPar

- KaHyPar is a *n*-level hypergraph partitioner

Flow Execution Policies

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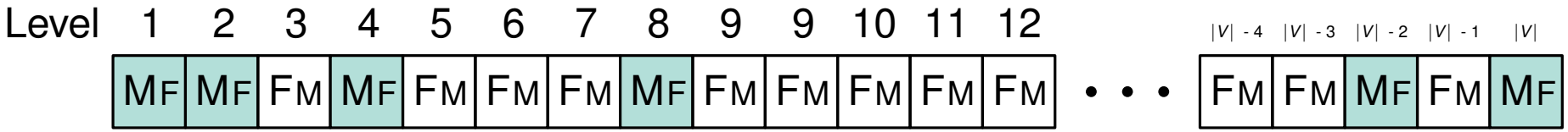

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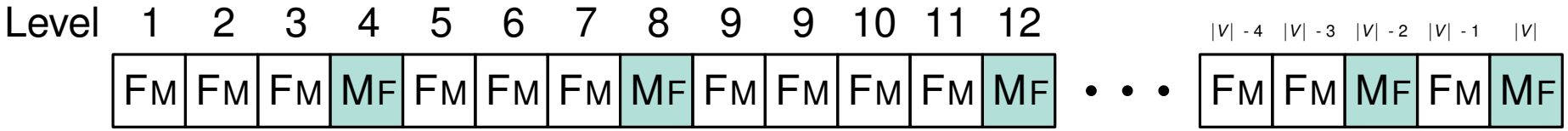

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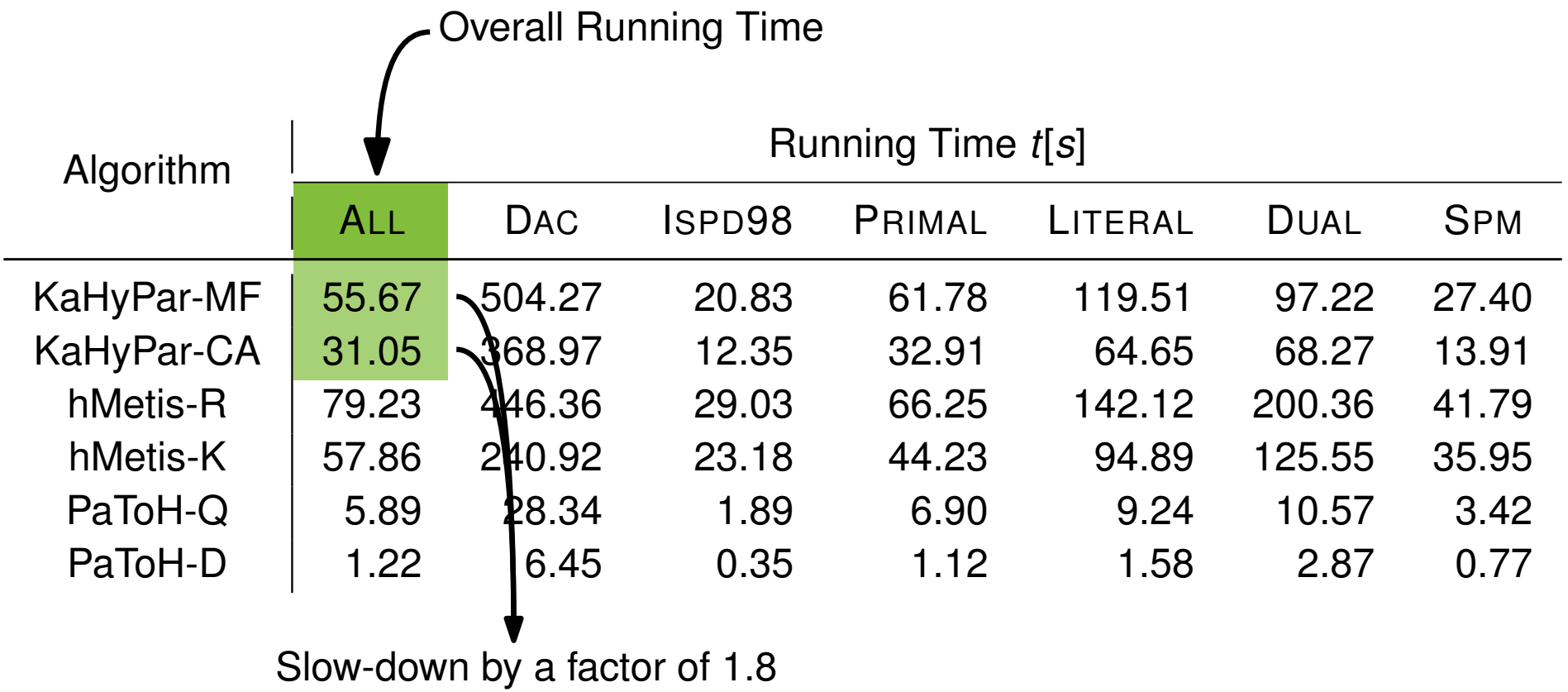
Running Time - Full Benchmark Set

Algorithm	Running Time $t[s]$						
	ALL	DAC	ISPD98	PRIMAL	LITERAL	DUAL	SPM
KaHyPar-MF	55.67	504.27	20.83	61.78	119.51	97.22	27.40
KaHyPar-CA	31.05	368.97	12.35	32.91	64.65	68.27	13.91
hMetis-R	79.23	446.36	29.03	66.25	142.12	200.36	41.79
hMetis-K	57.86	240.92	23.18	44.23	94.89	125.55	35.95
PaToH-Q	5.89	28.34	1.89	6.90	9.24	10.57	3.42
PaToH-D	1.22	6.45	0.35	1.12	1.58	2.87	0.77

Running Time - Full Benchmark Set



Running Time - Full Benchmark Set

Overall Running Time

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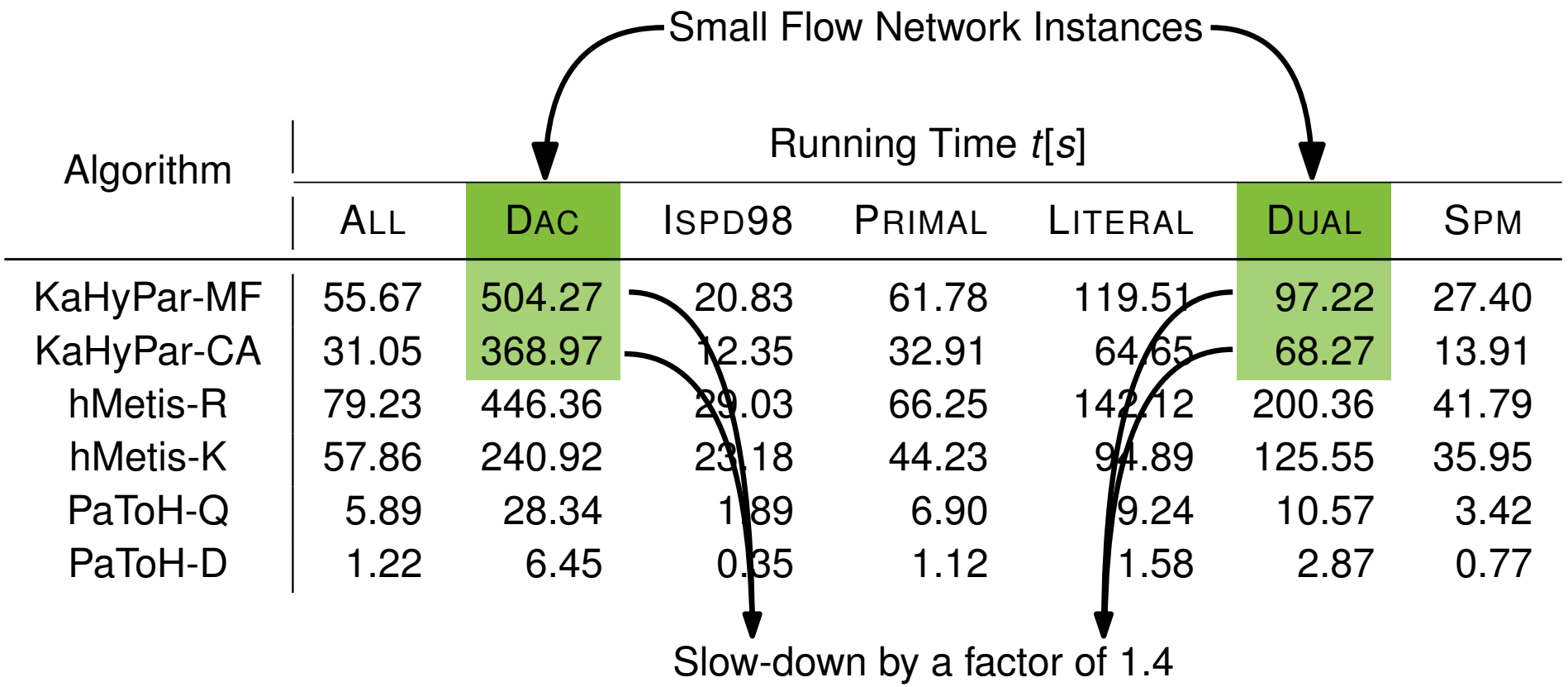
Comparable running time to hMetis-K

Running Time - Full Benchmark Set



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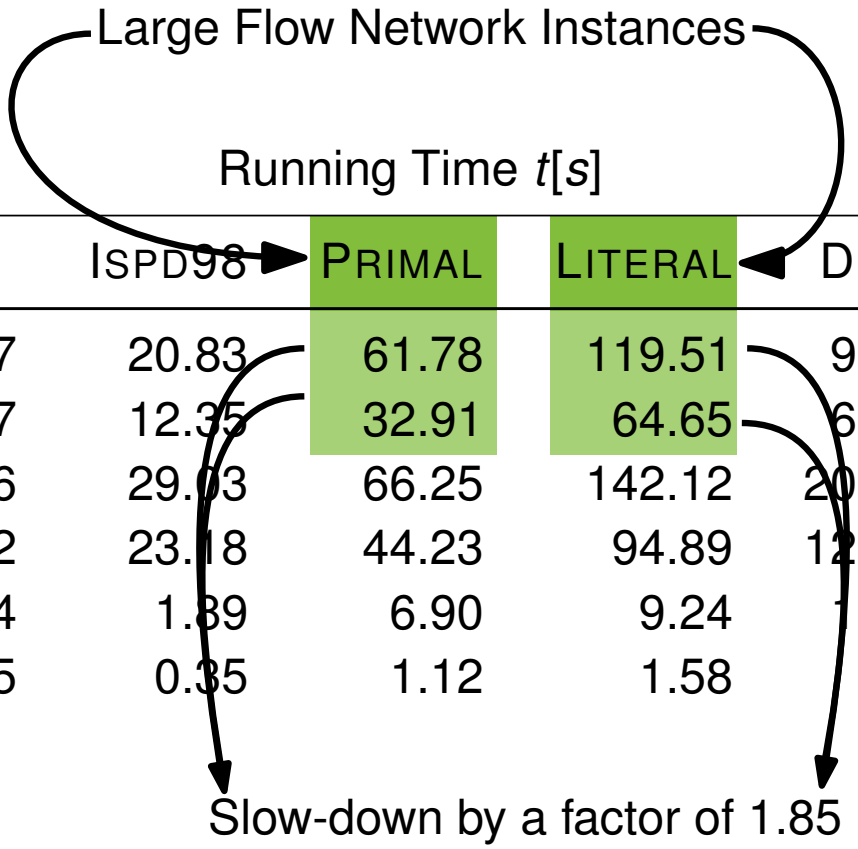

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