Vorlesung 14.01.2011 - StringMatching

Proof (Proof of Lemma 2): Let $x \in \Sigma^*$ and $a \in \Sigma$, $\sigma(xa) =: r$

Case 1 $r = 0, \text{ie}\sigma(xa) = 0 \leq \underbrace{\sigma(x)}_{0} + 1$

Case 2 $r > 0 \Rightarrow P_r \sqsupset xa \Rightarrow P_{r-1} \sqsupset x$

$$r - 1 \leq \sigma(x) \Rightarrow r \leq \sigma(x) + 1$$

□

Proof (Proof of Lemma 3): Let $x \in \Sigma^*, a \in \Sigma$ Show:

$$q = \sigma(x) \Rightarrow \sigma(xa) = \sigma(P_q a)$$

- Let $r = \sigma(xa) \overset{L2}{\Rightarrow} \sigma(xa) = r \leq \underbrace{\sigma(x)}_{q} + 1$, i.e. $r \leq q + 1 = |P_q a|$

- $P_q a \sqsupset xa, P_r \sqsupset xa, |P_r| \leq |P_q a|$

  $$\overset{L1b}{\Rightarrow} P_r \sqsupset P_q a \Rightarrow \sigma(xa) = r \leq \sigma(P_q a)$$

- $P_q a \sqsupset xa$

  $$\overset{Note1d)}{\Rightarrow} \sigma(P_q a) \leq \sigma(xa)$$

  $$\sigma(P_q a) = \sigma(xa)$$

□

Proof (Proof of Theorem 4): Proof by induction on i:

(IB) $i = 0 : \delta^*(\underset{\epsilon}{T_0}) = 0 = \sigma(\overset{\epsilon}{T_0})$✓

(IS) $i \to i+1$: Assume $\delta^*(T_i) = \sigma(T_i)$, for some $i \in \mathbb{N}$
Let $q := \delta^*(T_i), a := t_{i+1}$
Then

$$\begin{aligned}
\delta^*\left(T_{i+1}\right) &= \delta^*(T_i a) \\
&= \delta\big(\delta^*(T_i), a\big) \text{ by def} \\
&= \delta(q, a) \\
&= \sigma(P_q a) \\
&\overset{*}{=} \sigma T_i a \\
&= \sigma(T_{i+1})
\end{aligned}$$

□

Proof (Corollary 5): By Theorem 4, if M_p enters state a, then $\delta^*(P_q) = \sigma(P_q)$. i.e. q is the largest prefix of P which is a suffix of q.

Thus, $q = m \Leftrightarrow P_q = P \sqsupset T_i$ □