Sheet 12

Exercise 32

Show: $\exists W \in \Sigma^+, i, j \in \mathbb{N} : W^i = X, W^j = Y.$

Proof: $S := XY = YX \Rightarrow X \sqsubset S \wedge Y \sqsubset S \wedge X \sqsubset S \wedge Y \sqsubset S$
$\Rightarrow (X \sqsubset Y \vee Y \sqsubset X)$

Case 1: $X \sqsubset Y \wedge Y \sqsupset X$
$X = AB \Rightarrow Y = AB \Rightarrow W = X = Y \ i, j = 1$

Case 2: $X \sqsubset Y \wedge X \sqsupset Y$
$X = AB \Rightarrow Y = ABCAB$
$XY = ABABCAB \wedge YX = ABCABAB$
$\Rightarrow C = AB, \ W = AB, \ i = 1, \ j \geq 2$

$\square$

Exercise 33

$P = babacabab$

$\sigma(x) = \max\{k | P_k \sqsupset x\}$

$\delta(q, a) = \sigma(P_q a)$

δ	0	1	2	3	4	5	6	7	8	9
a	0	2	0	4	0	6	0	8	0	4
b	1	1	3	1	3	1	7	1	9	1
c	0	0	0	0	5	0	0	0	0	0

$$
\begin{aligned}
\delta(0,a) &= \sigma(P_0 a) \\
\delta(1,a) &= \sigma(P_1 a) \\
&= \sigma(bc) \\
\delta(2,a) &= \sigma(P_2 a) \\
&= \sigma(bac) \\
\delta(3,a) &= \sigma(babc) \\
&\dots
\end{aligned}
$$

Exercise 34

Let $P \in \Sigma^* \cup \{\Diamond\},\ T \in \Sigma^*$

(1) Split P into $k \in \mathbb{N}$ substrings $P^{(1)}, \dots, P^{(k)}$ by using $\Diamond$ as split symbol.

(2) Construct a string matching automaton M_i for every $P^{(i)}, i \in [k]$

(3) Concatenate automaton M_i with $M_{i+1}(i \in [k-1])$ by merging the final state q of M_i with the initial state t of M_{i-1} by removing all transitions from q and keeping all transitions of t.

(4) Choose the initial state of M_1 as the initial state of the constructed automaton and the final state of M_k as final state.

Proof: by induction on k

(IB) $k = 1 \Rightarrow M = M_1 \checkmark$

(IS) $k \to k+1$.

$$P = P^{(1)} \Diamond P^{(2)} \Diamond \dots \Diamond P^{(k)} \Diamond P^{(k+1)}$$

Assume the automaton for $P^{(1)} \Diamond \dots \Diamond P^{(k)}$ works correct.
Show the automaton for $P^{(1)} \Diamond \dots \Diamond P^{(k)} \Diamond P^{(k+1)}$ works correct.
I.e. show that $M = \underbrace{M_1 \circ \dots \circ M_k}_{\text{corr}} \circ M_{k+1}$ works correct.

Show: If τ contains an occurance of P, then M reaches the final state. δ/ω it doesnt $\square$