Blatt 2

Beweis:

„$\Leftarrow$“:(IH) After pass i of the outer loop, it holds

$$d[v_j] < \infty \quad \forall j \leq i$$

Assumption: $s \rightsquigarrow_G v = (v_0 =: s, v_1, \dots, v_k =: v)$
(IB) $i = 0$: $d[v_i] = d[s] = 0 < \infty$
(IS) $i > 0, i-1 \to i$: We know, after the $i-1$th pass: $d[v_0] < \infty \quad \forall j \leq i-1$
In the i-th pass: (v_{i-1}, v_i) is relaxed
$\Rightarrow$ after relaxing $d[v_i] \leq d[v_{i-1}] + w(v_{i-1}, v_i) \leq \infty$

„$\Rightarrow$“: We know $d[v] < \infty$. Show: $s \rightsquigarrow_G v$
$d[v] < \infty \Rightarrow v = s \quad$ or $\quad v \neq s$ and
There was a Relax-operation on an edge $(v_1, v) \in E$, which updated $d[v] \Rightarrow d[v_1] < \infty$
$\Rightarrow$ either $v_1 = s \Rightarrow (s = v_1, v)$ is a path from s to v in G or $v_1 \neq s$ and there was a Relax-operation on an edge $(v_2, v_1) \in E$, which updated $d[v_1] \Rightarrow d[v_2] < \infty$
$\Rightarrow$ either $v_2 = s \Rightarrow \dots$ or $\dots$
After at most $|V| - 1$ iterations of this argumentation we get that there exists a Relax-operation or an edge $(s = v_k, v_{k-1}) \in E$ which updated $d[v_{k-1}] = d[v_k] < \infty$
$\Rightarrow v_k = s \Rightarrow (s = v_k, v_{k-1}, \dots, v_1, v)$ is a path in G from s to v.