

1 Blatt 8

1.1 Aufgabe 21c

Consider a cut (S, T) . Then we have:

$$\begin{aligned}\tilde{c}(S, T) &= \sum_{\substack{(u,v) \in \\ S \times T \cap E}} (M \cdot c(u, v) + 1) \\ &= M \cdot \underbrace{\sum_{\substack{(u,v) \in \\ S \times T \cap E}} c(u, v)}_{c(S, T)} + \underbrace{|S \times T \cap E|}_{\# \text{ edges from } S \text{ to } T}\end{aligned}$$

Let f be a circulating flow. We have: $\forall U \subset V$:

$$\sum_{\substack{(u,v) \in \\ (U \times V) \cap E}} f(u, v) = \sum_{\substack{(v,u) \in \\ (\bar{U}, U) \cap E}} f(v, u)$$

$$f(U, \bar{U}) = f(\bar{U}, U)$$

Induction on $|U|$:

(IB) $|U| = 0 \checkmark$
 $|U| = 1$: Wlog $x \in U$: $f(u, \bar{U}) = f(\bar{U}, x)$

(IS) $|U'| = |U| + 1$: Wlog set $U' = U \cup \{v\}$: $v \notin U$
 by **(IH)**:

$$f(U, \bar{U}) = f(\bar{U}, U)$$

$$f(U', \bar{U}') = f(U, \bar{U}) - f(U, v) + f(v, \bar{U})$$

$$f(\bar{U}')$$

\Rightarrow We know:

$$f(U, \bar{U}) = f(\bar{U}, U)$$