1 Sheet 10

Let $G = (V, E)$ be a graph with a perfect matching m. $|M| = \frac{n}{2}$

$$\forall u \in V : \exists! v \in V \setminus \{u\} : \{u, v\} \in M \tag{1}$$

If Paula chooses an edge $\{u_1, v_1\} \in M$ in the first round

$\overset{(1)}{\Rightarrow} \forall u_2 \in V \setminus \{u_1, v_1\} : \{v_1, u_2\} \notin M$

$\Rightarrow$ Paul chooses edge $\{v_2, u_2\}, u_2 \in V \setminus \{u_1, v_1\} : \{v_1, u_2\} \notin M$

$\overset{(2)}{\Rightarrow}$ Paula can choose an edge $\{u_2, v_2\}$ with $v_2 \in V \setminus \{?\} : \{u_2, v_2\} \in M$

In round $\frac{n}{2} =: N$ Paula can choose an edge $\{u_N, v_N\} \in M$.

$V \setminus \{u_1, u_2, \ldots, u_N, v_1, \ldots, v_N\} = \emptyset$

$\Rightarrow$ Paul cannot choose any edge