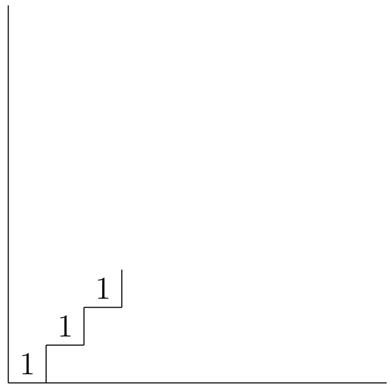
Vorlesung 21.01.2011 - StringMatching

Proof (Lemma 28): Running Time of ComputePrefixFunction is $\mathcal{O}(m)$
Sketch: Potential $\Phi(\text{currentstate}) := k$



Grafik vervollständigen

Always: $k \geq 0$
Initially: $k = 0$
Total increase of k: $\leq m - 2$
$\Rightarrow$ total number of decrease-executions of „$k = \pi(k)$“ $\leq m - 1$
$\Rightarrow \mathcal{O}(m)$ $\square$

Proof (Lemma 29 Sketch): „$<$“ $i \in \pi^*(q) \Rightarrow i \in \pi^u(q),\ u \in \mathbb{N}_0$

(IB) $u = 0 \Rightarrow i = q \checkmark$

(IS) $P_{\pi(i)} \sqsupset P_i \underset{\text{IH}}{\sqsupset} P_q$

„$>$“ Suppose $\exists j \in \{k|P_k \sqsupset P_q\} \setminus \pi^*(q)$. Wlog. j maximal.

$$q \in \{k|P_k \sqsupset P_q\} \cap \pi^*(q) \Rightarrow j < q$$

$$j' = \min\{r \in \pi^*(q) | r > j\}$$

Then

$$\left.\begin{cases} P_j \sqsupset P_q \text{ since } j \in \{k|P_k \sqsupset P_q\} \\ P_j' \sqsupset P_q \text{ since } j' \in \pi^*(q) \text{ and } „<“ \end{cases}\right\} \underset{\text{L25a}}{\Rightarrow} P_j \sqsupset P_{j'}$$

j max, $\pi(j') = j \Rightarrow j \in \pi^*(q) \lightning$ $\square$

Proof (Corollary 9): If $r = \pi(q)$, then $P_r \sqsupset P_q$ and thus $r \geq 1$ implies $p_r = p_q$. By Lemma 30, if $r \geq 1$ then

$$\begin{aligned} r &= 1 + \max\{k \in \pi^*(q-1) | p_{k+1} = p_q\} \\ &= 1 + \max\{k | k \in E_{q-1}\} \text{ and } E_{q-1} \neq \emptyset \end{aligned}$$

If $r = 0$, there is no $k \in \pi^*(q-1)$ for which we can extend P_k to P_{k+1} and get a suffix of P_q. Since then $\pi(q) > 0$. Thus $E_{q-1} = \emptyset$. $\square$

Proof (Corollary 10): $\pi(1) = 0$✓

At the start of each iteration of the for-loop we have $k = \pi(q-1)$. This is maintained as an invariant. The while-loop searches through all values $k \in \pi^*(q-1)$ until one is found for which $p_{k+1} = p_q$. At that point $k = \max\{E_{q-1}\}$, so by Corollary 9 we can set $\pi(q)$ to $k+1$. If no such k is found, $\pi(q)$ is correctly set to 0. $\square$