1 Sheet 10

1.1 Aufgabe 26

Let $G = (V, E)$ be a graph with a perfect matching m. $|M| = \frac{n}{2}$

$$\forall u \in V : \exists! v \in V \setminus \{u\} : \{u, v\} \in M \tag{1}$$

If Paula chooses an edge $\{u_1, v_1\} \in M$ in the first round

$$\overset{(1)}{\Rightarrow} \forall u_2 \in V \setminus \{u_1, v_1\} : \{v_1, u_2\} \notin M$$

$$\Rightarrow \text{ Paul chooses edge } \{v_2, u_2\}, u_2 \in V \setminus \{u_1, v_1\} : \{v_1, u_2\} \notin M$$

$$\overset{(2)}{\Rightarrow} \text{ Paula can choose an edge } \{u_2, v_2\} \text{ with } v_2 \in V \setminus \{?\} : \{u_2, v_2\} \in M$$

In round $\frac{n}{2} =: N$ Paula can choose an edge $\{u_N, v_N\} \in M$.

$$V \setminus \{u_1, u_2, \ldots, u_N, v_1, \ldots, v_N\} = \emptyset$$

$$\Rightarrow \text{ Paul cannot choose any edge}$$

1.2 Aufgabe 27

Show:

(1) If T is a transversal in A, then the edges corresponding to the entries of T form a matching

(2) If M is a matching in G, then the entries correspond to the edges in M form a transversal.

Proof. (1) Let T be a transv. of A

$$\forall a_{ij} \in T : a_{ij} \neq 0 \Rightarrow \{u_i, v_j\} \in E$$

$$\forall a_{ij}, a_{kl} \in T : i \neq k \text{ (not in the same row)}$$

$$\wedge j \neq l \text{ (not in the same column}$$

$$\Leftrightarrow \text{ no two edges corresponding to positions in } T \text{ are independent.}$$

□