Beweis: Floyd-Warshall Lemma 4

a)

By induction on k.

$$k = 0 : \text{A shortest path from } i \to j \text{ without intermediate nodes has weight } w_{ij}$$

$$k > 0 : \text{Let } P \text{ be a shortest path from } i \to j \text{ with intermediate nodes from } [k]$$

P is simple, since edge weights are nonnegative.
If $k \notin P$ then $w(P) = d_{ij}^{(k-1)}$.
If $k \in P$ then $P = P_{ik}P_{kj}$ and
$w(P) = w(P_{ik}) + w(P_{kj}) = d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$

Beweis: Lemma 5 (Shortest Paths and Matrix Multiplication)

By induction on m: $m = 0$: correct.
$m > 0 : \delta_{ij}^{(m)} = \min \left\{ \underbrace{\delta_{ij}^{(m-1)}}_{\text{paths with } \leq m-1 \text{ edges}}, \min_{k \in [n]} \left\{ \underbrace{\delta_{ij}^{(m-1)}}_{\text{paths with } \leq m-1 \text{ edges}} + \underbrace{w_{ij}}_{\text{1edge}} \right\} \right\}$
$=_{w_{ik}=0 \forall k} \min_{k \in [n]} \left\{ \delta_{ik}^{(m-1)} + w_{kj} \right\}$
Since G does not contain negative weight cycles, shortest paths are simple and thus $\delta(i,j) - \delta_{ij}^{(n-1)}$.