Probeklausur

Task 1

function INITSSSP$(G = (V, E), s \in V)$
    for $v \in V$ **do**
        $d[v] = -\infty$
    $d[s] = \infty$

function RELAX$(u, v \in V,$ weight $w)$
    if $d[v] < \min\{d[u], w(u, v)\}$ **then**
        $d[v] = \min\{d[u], w(u, v)\}$

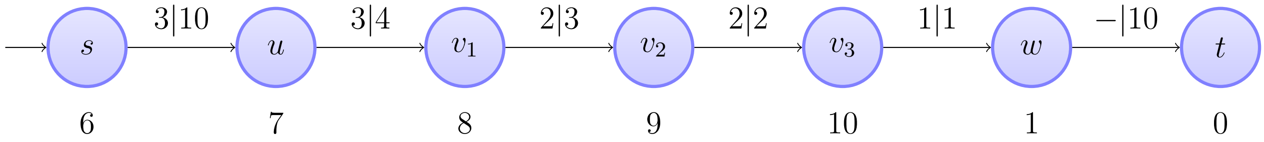
function DIJKSTRA(graph $G = (V, E)$, weights w, node $s \in V$)
    initSSSP(G, s)
    $S = \emptyset,\ Q = V$
    while $Q \neq \emptyset$ **do**
        choose $u \in Q$, so that $d[u]$ is maximum
        $Q = Q \setminus \{u\},\ S = S \cup \{u\}$
        for $v \in N_G[u]$ **do**
            Relax(u, v, w)

Task 2

a)

$\vdots$

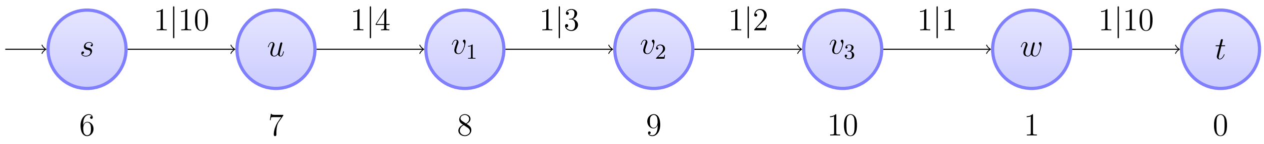
After phase 4:



new $Q = \{w, v_3, v_1\}$

$\vdots$

After phase 8:



new $Q = \emptyset$

b)

#pushes from node	to the right	to the left	total
s	0	0	0
u	1	4	5
v_1	1	3	4
v_2	1	2	3
v_3	1	1	2
w	1	0	1
t	0	0	0

$$\Rightarrow \sum = 15$$

c)

#pushes from node	to the right	to the left	total
s	0	0	0
u	1	k+1	k+2
v_1	1	k	k+1
v_2 $\vdots$	1	k-1	k
v_{k-1}	1	2	3
v	1	1	2
w	1	0	1
t	0	0	0

$$\Rightarrow \sum_{i=1}^{k+2} i = \frac{(k+2)(k+3)}{2}$$

Task 3

Proof:

$$\begin{aligned}
\sum_{e\in E} f(e)\Delta(e) &= \sum_{(u,v)\in V^2} f(u,v)\cdot\Delta(u,v) = \sum_{(u,v)\in V^2} f(v,u)\cdot\big(\pi(v)-\pi(u)\big)\\
&= \sum_{(u,v)\in V^2} f(u,v)\cdot\pi(v) - \sum_{(u,v)\in V^2} f(u,v)\cdot\pi(u)\\
&= \sum_{v\in V}\pi(v)\cdot\sum_{u\in V} f(u,v) - \sum_{u\in V}\pi(u)\cdot\sum_{v\in V} f(u,v)\\
&= \sum_{u\in V\setminus\{s,t\}} \pi(u)\cdot\sum_{v\in V} f(v,u) - \sum_{u\in V\setminus\{s,t\}} \pi(u)\cdot\sum_{v\in V} f(u,v)\\
&\quad + \pi(s)\underbrace{\sum_{v\in V}\big[f(v,s)-f(s,v)\big]}_{-|f|} + \pi(t)\cdot\underbrace{\sum_{v\in V}\big[f(v,t)-f(t,v)\big]}_{|f|} \quad (*)
\end{aligned}$$

□

Task 4

a)

Sketch: Double y nodes; capacity 1 on every edge.

Construct a flow network (G,s,t,c) as follows:

$$\begin{aligned}
&G=(V,Z) \text{ with}\\
&V=\{s,t\}\cup X\cup Z\cup Y^{(1)}\cup Y^{(2)}\\
&Y^{(i)}=\{y^{(i)}|y\in Y\}, i=1,2
\end{aligned}$$

$$\begin{aligned}
E=&\{(s,x)|x\in X\}\cup\{(x,y^{(1)})|(x,y)\in E_N\}\\
&\cup\{(y_1^{(2)},y_2^{(1)})|(y_1,y_2)\in E_N\}\\
&\cup\{(y^{(2)},z)|(y,z)\in E_N\}\cup\{(z,t)|z\in Z\}\\
&\cup\{(y^{(1)},y^{(2)})|y\in Y\},\ c(e)=1\ \forall e\in E
\end{aligned}$$

b)

$\ldots \Leftrightarrow (G,s,t,c)$ yields a flow f with $|f|=|x|$

c)

$\textsc{MaxSucnFlow}$, $\mathcal{O}(\sqrt{|V_N|} \cdot |E_N|)$