Hashing

Searching

- Consider the problem of searching an array for a given value
  - If the array is not sorted, the search requires O(n) time
    - If the value isn't there, we need to search all n elements
    - If the value is there, we search n/2 elements on average
  - If the array is sorted, we can do a binary search
    - A binary search requires O(log n) time
    - About equally fast whether the element is found or not
  - It doesn't seem like we could do much better
    - How about an O(1), that is, constant time search?
    - We can do it *if* the array is organized in a particular way

Hashing

- Suppose we were to come up with a "**magic function**" that, given a value to search for, would tell us exactly where in the array to look
  - If it's in that location, it's in the array
  - If it's not in that location, it's not in the array
- This function would have no other purpose
- If we look at the function's inputs and outputs, they probably won't "make sense"
- This function is called a hash function because it "makes hash" of its inputs

Example (ideal) hash function

- Suppose our hash function gave us the following values:

  hashCode("apple") = 5
  hashCode("watermelon") = 3
  hashCode("grapes") = 8
  hashCode("cantaloupe") = 7
  hashCode("kiwi") = 0
  hashCode("strawberry") = 9
  hashCode("mango") = 6
  hashCode("banana") = 2

Why hash tables?

- We don't (usually) use hash tables just to see if something is there or not—instead, we put key/value pairs into the table
  - We use a *key* to find a place in the table
  - The *value* holds the information we are actually interested in

Finding the hash function

- How can we come up with this magic function?
- In general, we cannot--there is no such magic function
  - In a few specific cases, where all the possible values are known in advance, it has been possible to compute a perfect hash function
- What is the next best thing?
  - A perfect hash function would tell us exactly where to look
  - In general, the best we can do is a function that tells us where to *start* looking!

Example imperfect hash function

- Suppose our hash function gave us the following values:
  - hash("apple") = 5
    hash("watermelon") = 3
    hash("grapes") = 8
    hash("cantaloupe") = 7
    hash("kiwi") = 0
    hash("strawberry") = 9
    hash("mango") = 6
    hash("banana") = 2
    hash("honeydew") = 6

• Now what?

Collisions

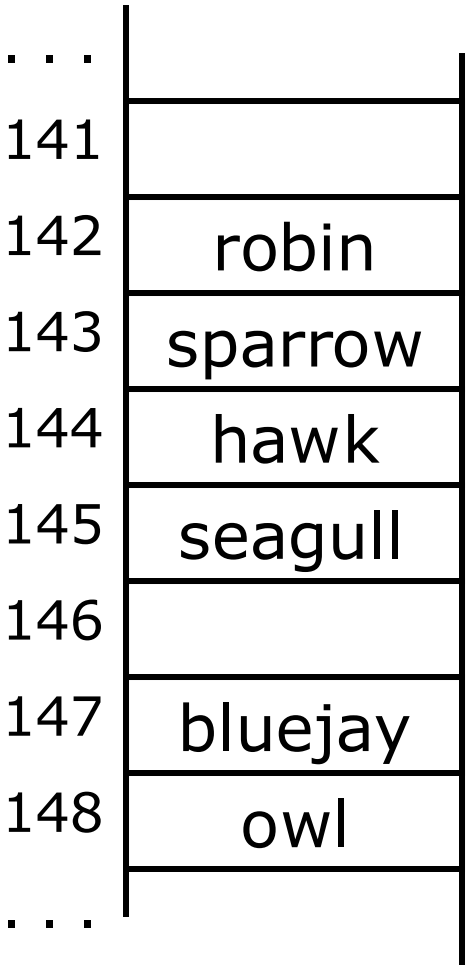
- When two values hash to the same array location, this is called a collision
- Collisions are normally treated as “first come, first served”—the first value that hashes to the location gets it
- We have to find something to do with the second and subsequent values that hash to this same location

Handling collisions

- What can we do when two different values attempt to occupy the same place in an array?
  - **Solution #1:** Search from there for an empty location
    - Can stop searching when we find the value *or* an empty location
    - Search must be end-around
  - **Solution #2:** Use a second hash function
    - ...and a third, and a fourth, and a fifth, ...
  - **Solution #3:** Use the array location as the header of a linked list of values that hash to this location
- All these solutions work, provided:
  - We use the same technique to *add* things to the array as we use to *search* for things in the array

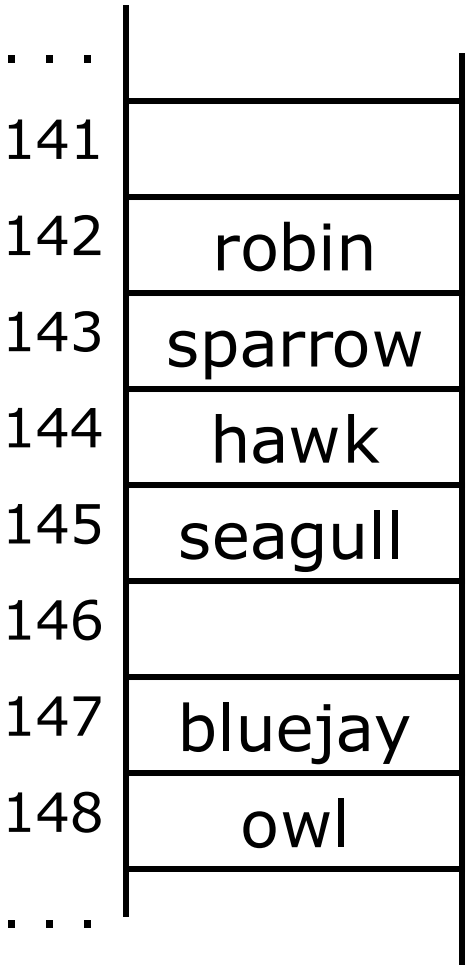
Insertion, I

- Suppose you want to add seagull to this hash table
- Also suppose:
  - hashCode(seagull) = 143
  - table[143] is not empty
  - table[143] != seagull
  - table[144] is not empty
  - table[144] != seagull
  - table[145] is empty
- Therefore, put seagull at location 145



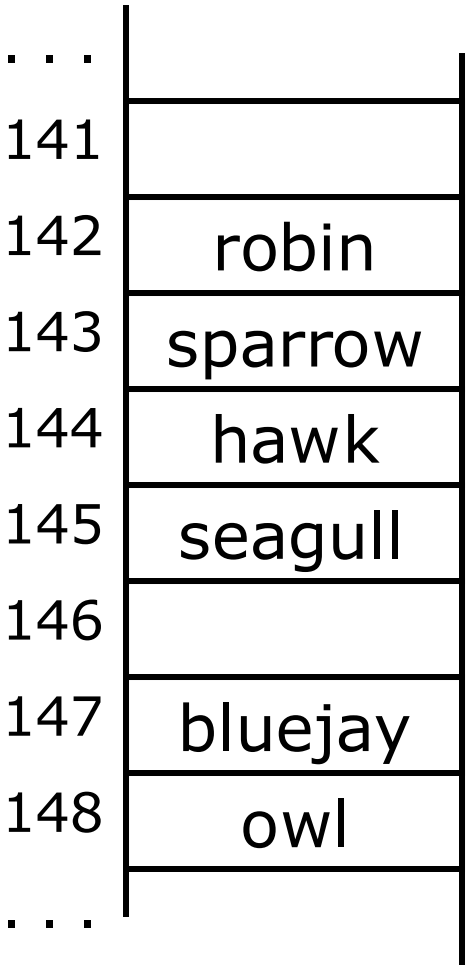
Searching, I

- Suppose you want to look up seagull in this hash table
- Also suppose:
  - hashCode(seagull) = 143
  - table[143] is not empty
  - table[143] != seagull
  - table[144] is not empty
  - table[144] != seagull
  - table[145] is not empty
  - table[145] == seagull !
- We found seagull at location 145



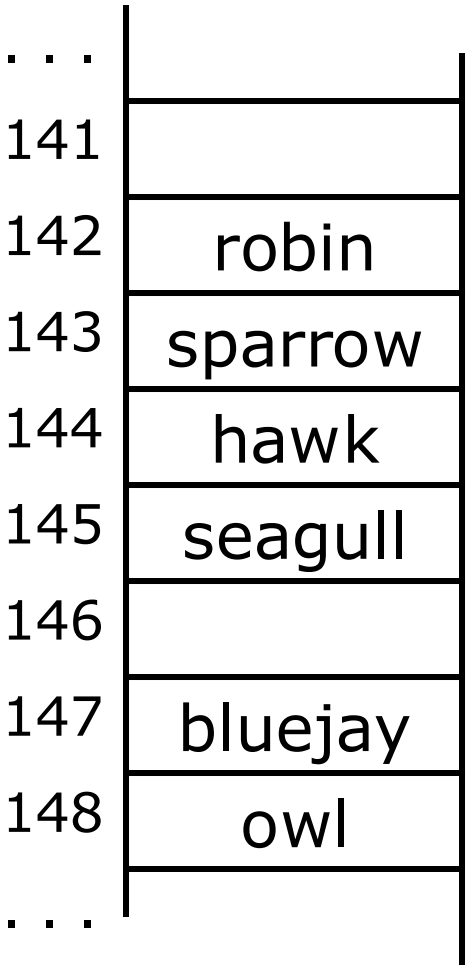
Searching, II

- Suppose you want to look up `cow` in this hash table
- Also suppose:
  - `hashCode(cow) = 144`
  - `table[144]` is not empty
  - `table[144] != cow`
  - `table[145]` is not empty
  - `table[145] != cow`
  - `table[146]` is empty
- If `cow` were in the table, we should have found it by now
- Therefore, it isn't here



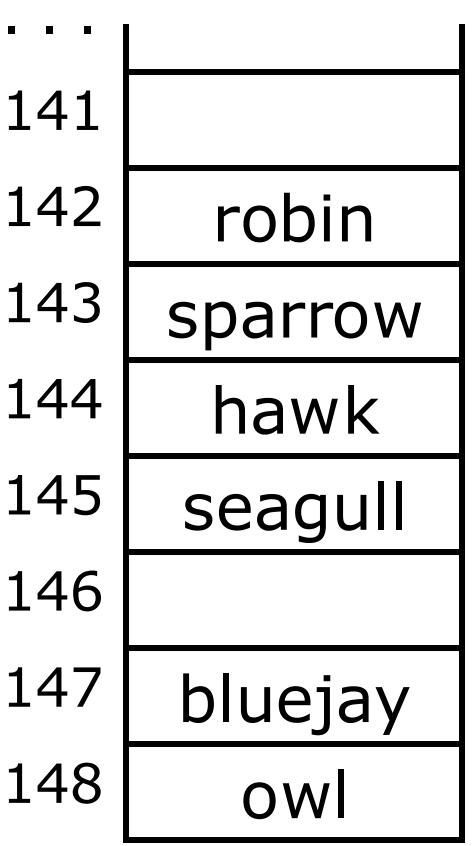
Insertion, II

- Suppose you want to add hawk to this hash table
- Also suppose
  - hashCode(hawk) = 143
  - table[143] is not empty
  - table[143] != hawk
  - table[144] is not empty
  - table[144] == hawk
- hawk is already in the table, so do nothing



Insertion, III

- Suppose:
  - You want to add cardinal to this hash table
  - hashCode(cardinal) = 147
  - The last location is 148
  - 147 and 148 are occupied
- Solution:
  - Treat the table as circular; after 148 comes 0
  - Hence, cardinal goes in location 0 (or 1, or 2, or ...)



Clustering

- One problem with the above technique is the tendency to form “clusters”
- A cluster is a group of items not containing any open slots
- The bigger a cluster gets, the more likely it is that new values will hash into the cluster, and make it ever bigger
- Clusters cause efficiency to degrade
- Here is a *non*-solution: instead of stepping one ahead, step n locations ahead
  - The clusters are still there, they’re just harder to see
  - Unless n and the table size are mutually prime, some table locations are never checked

Efficiency

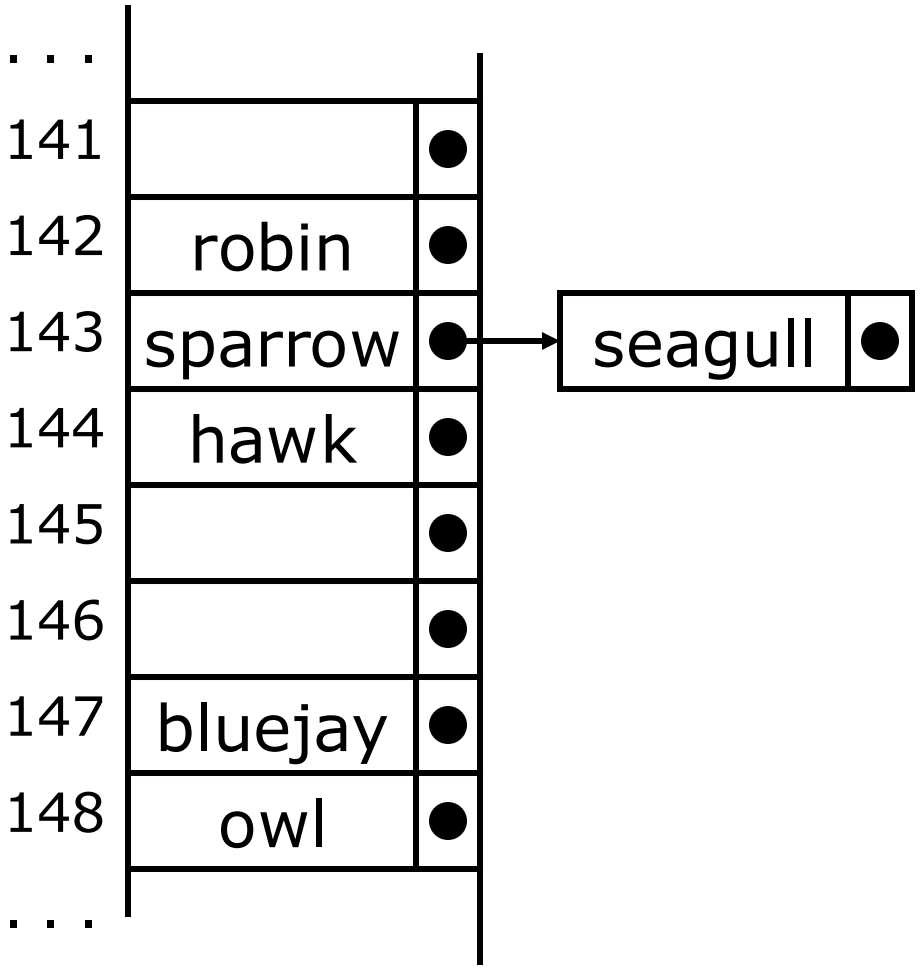
- Hash tables are actually surprisingly efficient
- Until the table is about 70% full, the number of probes (places looked at in the table) is typically only 2 or 3
- Sophisticated mathematical analysis is required to *prove* that the expected cost of inserting into a hash table, or looking something up in the hash table, is O(1)
- Even if the table is nearly full (leading to long searches), efficiency is usually still quite high

Solution #2: Rehashing

- In the event of a collision, another approach is to rehash: compute another hash function
  - Since we may need to rehash many times, we need an easily computable sequence of functions
- Simple example: in the case of hashing Strings, we might take the previous hash code and add the length of the String to it
  - Probably better if the length of the string was not a component in computing the original hash function
- Possibly better yet: add the length of the String plus the number of probes made so far
  - Problem: are we sure we will look at every location in the array?
- Rehashing is a fairly uncommon approach.

Solution #3: Bucket hashing

- The previous solutions used open hashing: all entries went into a "flat" (unstructured) array
- Another solution is to make each array location the header of a linked list of values that hash to that location



The hashCode function

- public int hashCode() is defined in Object
- Like equals, the default implementation of hashCode just uses the address of the object—probably not what you want for your own objects
- You can override hashCode for your own objects
- As you might expect, String overrides hashCode with a version appropriate for strings
- Note that the supplied hashCode method *does not know the size of your array*—you have to adjust the returned int value yourself

Writing your own hashCode method

- A hashCode method *must:*
  - Return a value that is (or can be converted to) a legal array index
  - Always return the same value for the same input
    - It can't use random numbers, or the time of day
  - Return the same value for *equal* inputs
    - Must be consistent with your equals method
- It does *not* need to return different values for different inputs
- A *good* hashCode method *should:*
  - Be efficient to compute
  - Give a uniform distribution of array indices
  - *Not* assign similar numbers to similar input values

Other considerations

- The hash table might fill up; we need to be prepared for that
  - Not a problem for a bucket hash, of course
- You cannot delete items from an open hash table
  - This would create empty slots that might prevent you from finding items that hash before the slot but end up after it
  - Again, not a problem for a bucket hash

Some Odd Examples

- The code below computes a modular hash function for a String s, where R is a small prime integer.

```
int hash = 0;
for (int i = 0; i < s.length(); i++)
    hash = (R * hash + s.charAt(i)) % M;
```

Hash tables in Java

- Java provides HashSet, Hashtable and HashMap
- HashSet is a set; things are in it, or they aren't
- Hashtable and HashMap are maps: they associate *keys* with *values*
- Hashtable is synchronized; it can be accessed safely from multiple threads
  - Hashtable uses an open hash, and has a rehash method, to increase the size of the table
- HashMap is newer, faster, and usually better, but it is not synchronized
  - HashMap uses a bucket hash, and has a remove method

Hash table operations

- HashSet, Hashtable and HashMap are in java.util
- All have no-argument constructors, as well as constructors that take an integer table size
- HashSet has methods add, contains, remove, iterator, etc.
- Hashtable and HashMap have these methods:
  - public Object put(Object key, Object value)
    - (Returns the previous value for this key, or null)
  - public Object get(Object key)
  - public void clear()
  - public Set keySet()
    - Dynamically reflects changes in the hash table
  - ...and many others