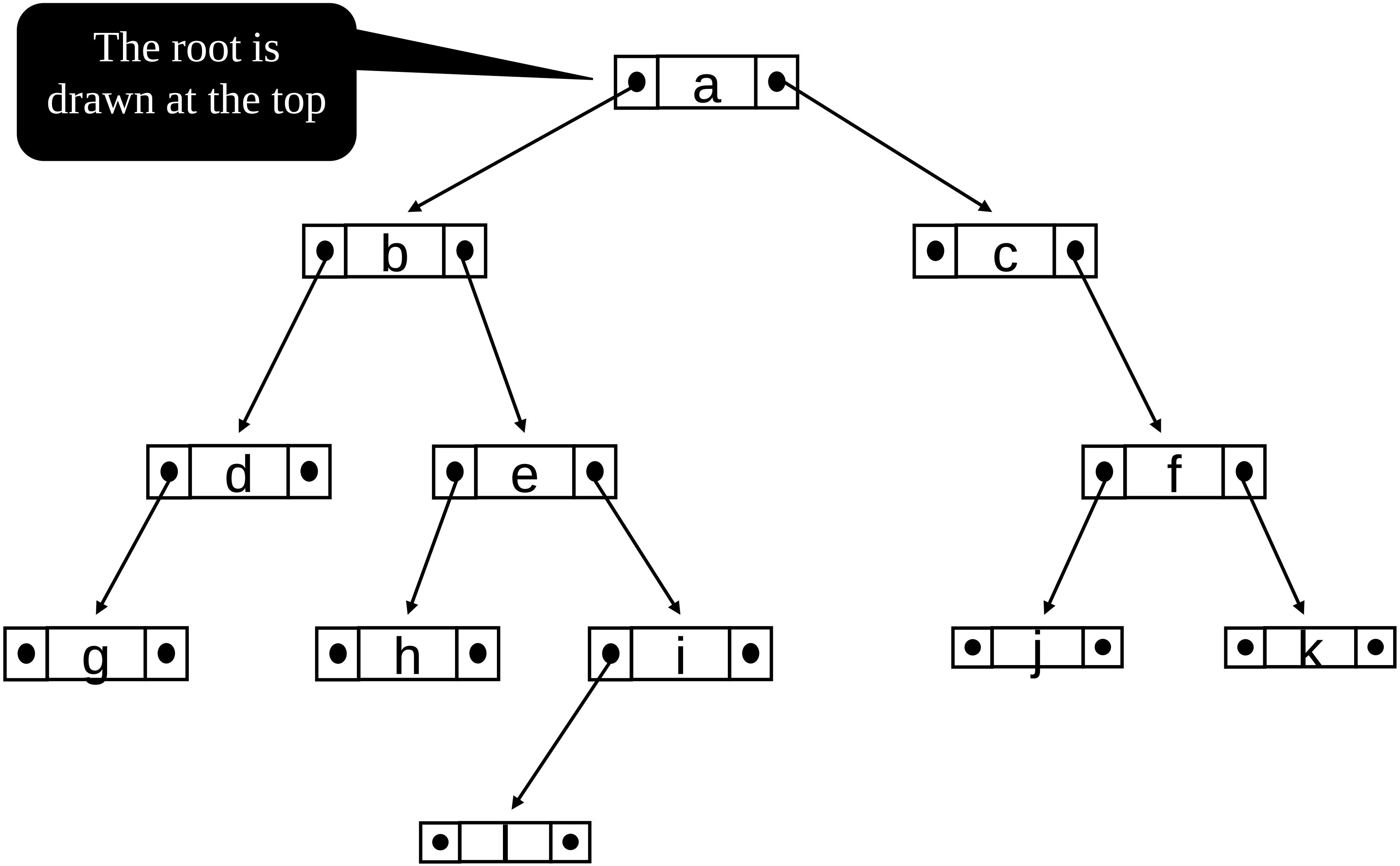
Binary Trees

Parts of a binary tree

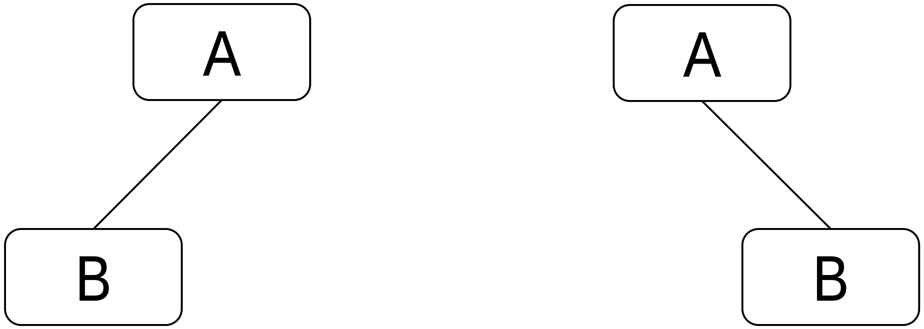
- A binary tree is composed of zero or more nodes
  - In Java, a reference to a binary tree may be `null`
- Each node contains:
  - A value (some sort of data item)
  - A reference or pointer to a left child (may be `null`), and
  - A reference or pointer to a right child (may be `null`)
- A binary tree may be *empty* (contain no nodes)
- If not empty, a binary tree has a root node
  - Every node in the binary tree is reachable from the root node by a *unique* path
- A node with no left child and no right child is called a leaf
  - In some binary trees, only the leaves contain a value

Picture of a binary tree



Left ≠ Right

- The following two binary trees are *different:*

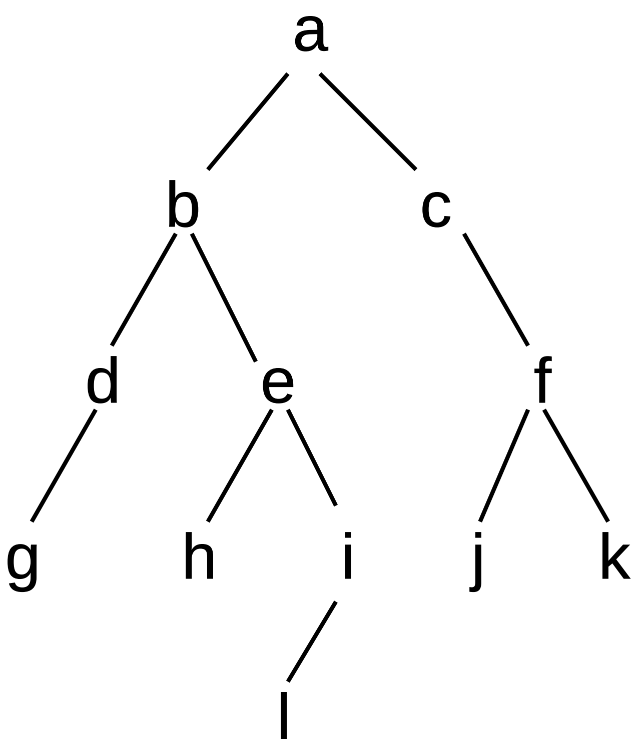


- In the first binary tree, node A has a left child but no right child; in the second, node A has a right child but no left child
- Put another way: Left and right are *not* relative terms

More terminology

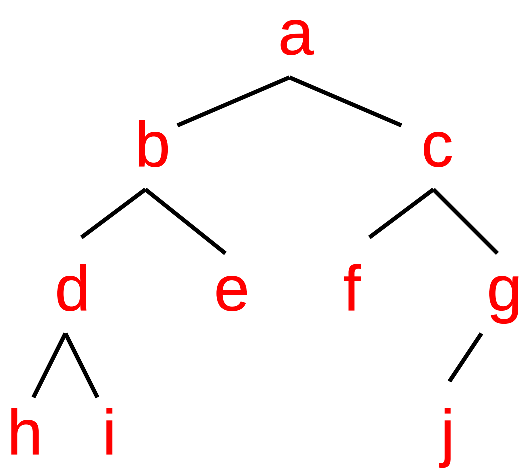
- Node A is the parent of node B if node B is a child of A
- Node A is an ancestor of node B if A is a parent of B, or if some child of A is an ancestor of B
  - In less formal terms, A is an ancestor of B if B is a child of A, or a child of a child of A, or a child of a child of a child of A, etc.
- Node B is a descendant of A if A is an ancestor of B
- Nodes A and B are siblings if they have the same parent

Size and depth

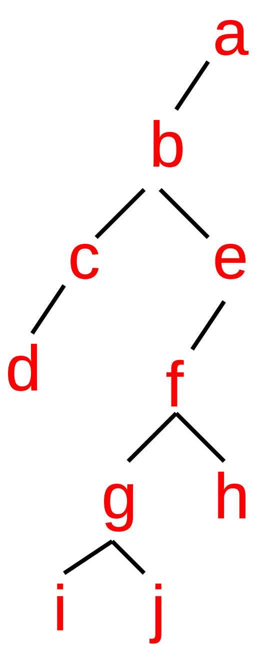


- The size of a binary tree is the number of nodes in it
  - This tree has size 12
- The depth of a node is its distance from the root
  - a is at depth zero
  - e is at depth 2
- The **depth of a binary** tree is the depth of its deepest node
  - This tree has depth 4

Balance



A balanced binary tree

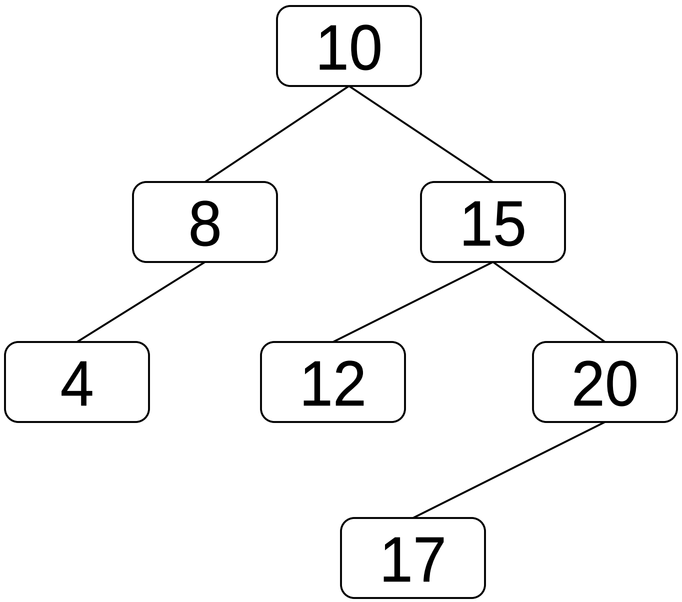


An unbalanced binary tree

- A binary tree is balanced if every level above the lowest is "full" (contains 2^n nodes where n is the depth)
- In most applications, a reasonably balanced binary tree is desirable

Sorted binary trees

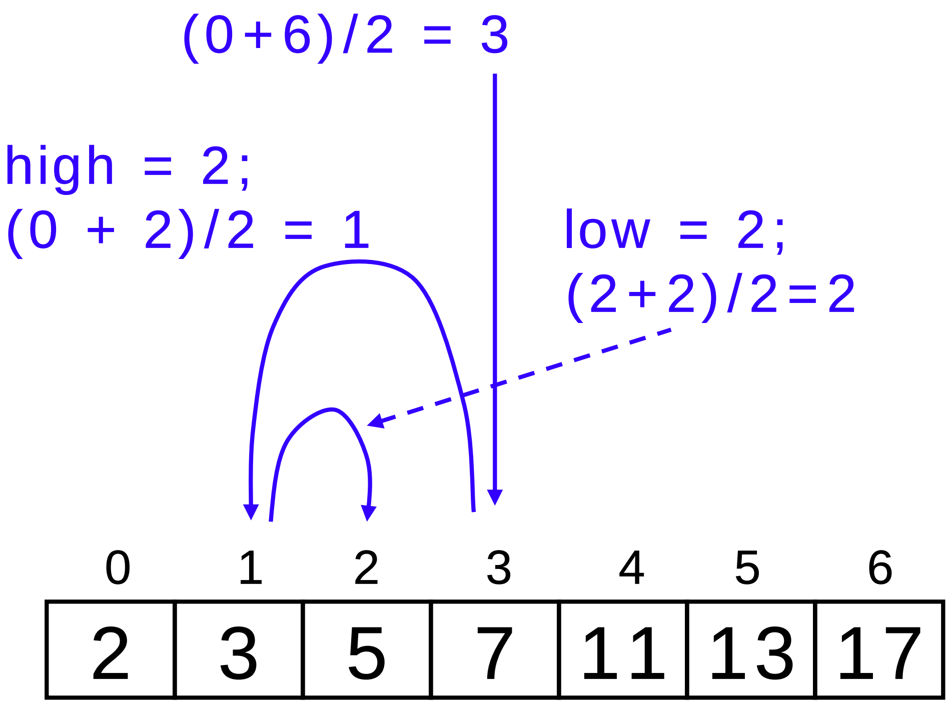
- A binary tree is sorted if every node in the tree is larger than (or equal to) its left descendants, and smaller than (or equal to) its right descendants



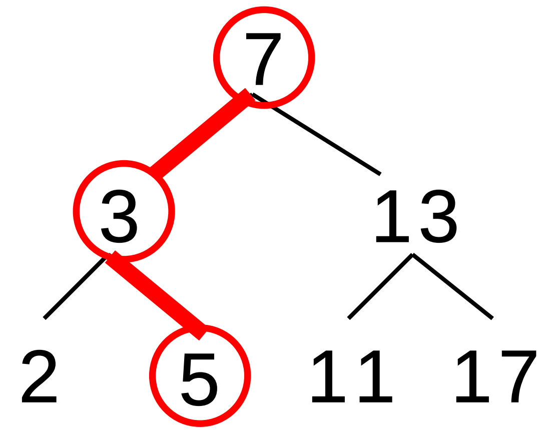
Binary search in a sorted array

- Look at array location `(low + high)/2`

Searching for 5:



Using a binary search tree



Tree traversals

- A binary tree is defined recursively: it consists of a root, a left subtree, and a right subtree
- To traverse (or walk) the binary tree is to visit each node in the binary tree exactly once
- Tree traversals are naturally recursive
- Since a binary tree has three “parts,” there are six possible ways to traverse the binary tree:
  - root, left, right
  - left, root, right
  - left, right, root
  - root, right, left
  - right, root, left
  - right, left, root

Preorder traversal

- In preorder, the root is visited *first*
- Here's a preorder traversal to print out all the elements in the binary tree:

```
public void preorderPrint(BinaryTree bt) {
    if (bt == null) return;
    System.out.println(bt.value);
    preorderPrint(bt.leftChild);
    preorderPrint(bt.rightChild);
}
```

Inorder traversal

- In inorder, the root is visited *in the middle*
- Here's an inorder traversal to print out all the elements in the binary tree:

```
public void inorderPrint(BinaryTree bt) {
    if (bt == null) return;
    inorderPrint(bt.leftChild);
    System.out.println(bt.value);
    inorderPrint(bt.rightChild);
}
```

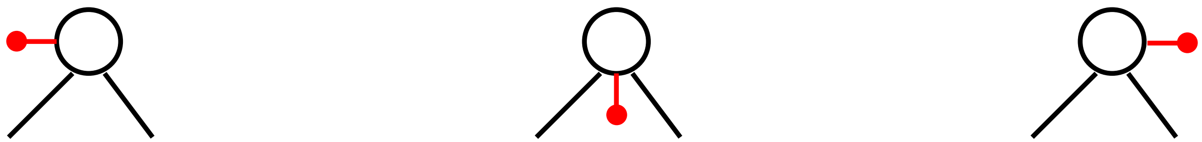
Postorder traversal

- In postorder, the root is visited *last*
- Here's a postorder traversal to print out all the elements in the binary tree:

```
public void postorderPrint(BinaryTree bt) {
    if (bt == null) return;
    postorderPrint(bt.leftChild);
    postorderPrint(bt.rightChild);
    System.out.println(bt.value);
}
```

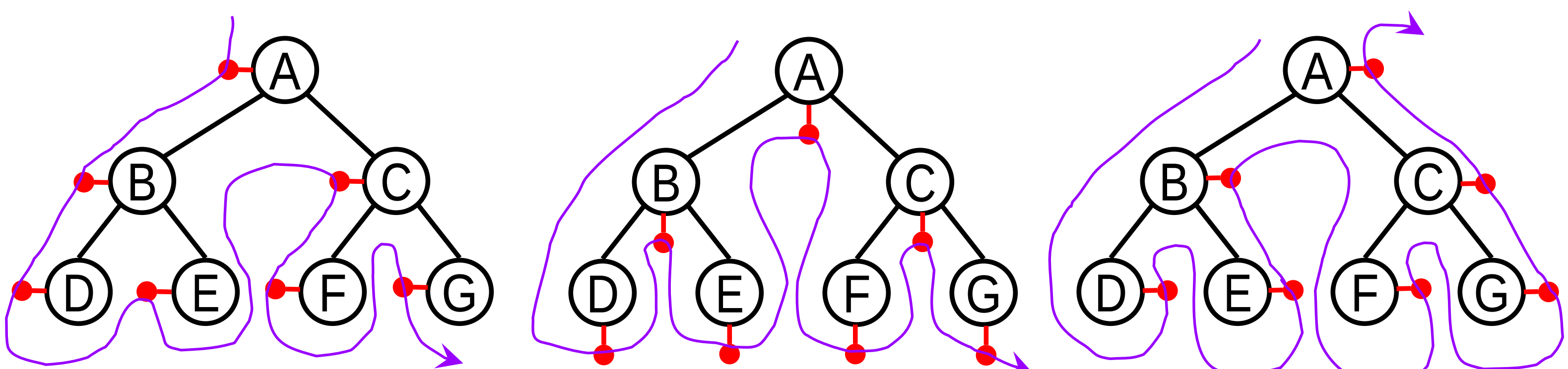
Tree traversals using "flags"

- The order in which the nodes are visited during a tree traversal can be easily determined by imagining there is a "flag" attached to each node, as follows:



preorder　　　　inorder　　　　postorder

- To traverse the tree, collect the flags:



A B D E C F G　　　　D B E A F C G　　　　D E B F G C A

Copying a binary tree

- In postorder, the root is visited *last*
- Here's a postorder traversal to make a complete copy of a given binary tree:

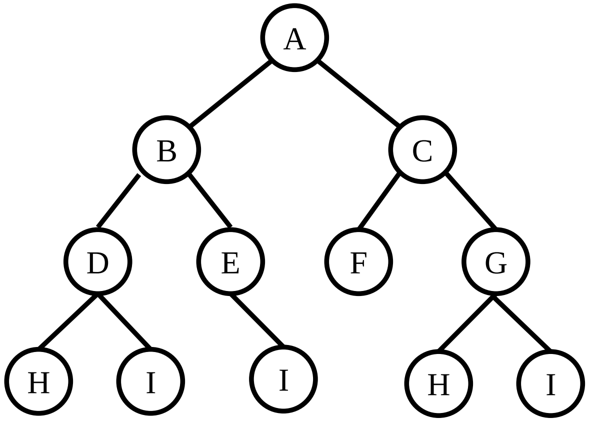
```
public BinaryTree copyTree(BinaryTree bt) {
    if (bt == null) return null;
    BinaryTree left = copyTree(bt.leftChild);
    BinaryTree right = copyTree(bt.rightChild);
    return new BinaryTree(bt.value, left, right);
}
```

Other traversals

- The other traversals are the reverse of these three standard ones
  - That is, the right subtree is traversed before the left subtree is traversed
- Reverse preorder: root, right subtree, left subtree
- Reverse inorder: right subtree, root, left subtree
- Reverse postorder: right subtree, left subtree, root

Exercise

- Traverse the following trees in reverse preorder and print the values.



The End