

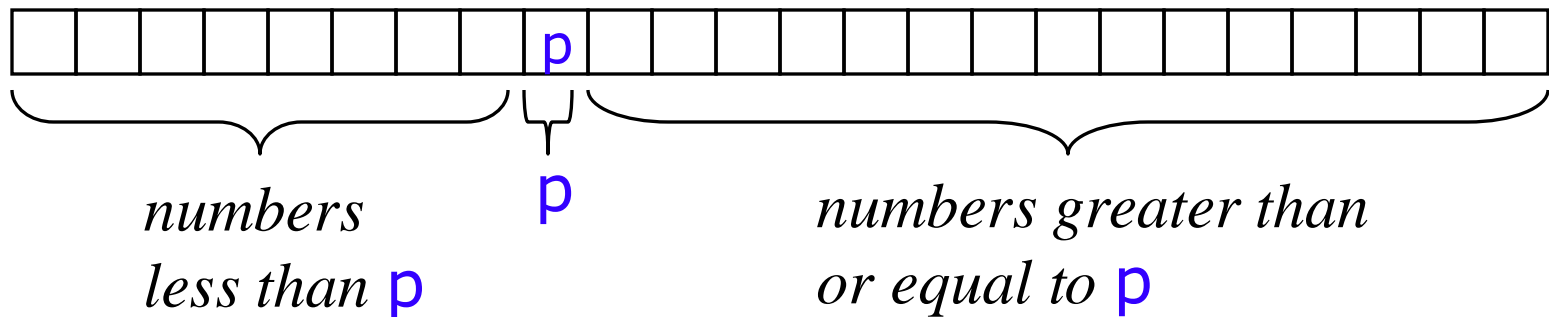
Quicksort

Quicksort I

- To sort $a[\text{left} \dots \text{right}]$:
 1. if $\text{left} < \text{right}$:
 - 1.1. Partition $a[\text{left} \dots \text{right}]$ such that:
 - all $a[\text{left} \dots p-1]$ are less than $a[p]$, and
 - all $a[p+1 \dots \text{right}]$ are $\geq a[p]$
 - 1.2. Quicksort $a[\text{left} \dots p-1]$
 - 1.3. Quicksort $a[p+1 \dots \text{right}]$
 2. Terminate

Partitioning (Quicksort II)

- A key step in the Quicksort algorithm is **partitioning** the array
 - We choose some (any) number **p** in the array to use as a **pivot**
 - We **partition** the array into three parts:



Partitioning II

- Choose an array value (say, the first) to use as the pivot
- Starting from the left end, find the first element that is greater than or equal to the pivot
- Searching backward from the right end, find the first element that is less than the pivot
- Interchange (swap) these two elements
- Repeat, searching from where we left off, until done

Partitioning

- To partition $a[\text{left} \dots \text{right}]$:
 1. Set $\text{pivot} = a[\text{left}]$, $l = \text{left} + 1$, $r = \text{right}$;
 2. while $l < r$, do
 - 2.1. while $l < \text{right} \ \& \ a[l] < \text{pivot}$, set $l = l + 1$
 - 2.2. while $r > \text{left} \ \& \ a[r] \geq \text{pivot}$, set $r = r - 1$
 - 2.3. if $l < r$, swap $a[l]$ and $a[r]$
 3. Set $a[\text{left}] = a[r]$, $a[r] = \text{pivot}$
 4. Terminate

Example of partitioning

- choose pivot: 4 3 6 9 2 4 3 1 2 1 8 9 3 5 6
- search: 4 3 6 9 2 4 3 1 2 1 8 9 3 5 6
- swap: 4 3 3 9 2 4 3 1 2 1 8 9 6 5 6
- search: 4 3 3 9 2 4 3 1 2 1 8 9 6 5 6
- swap: 4 3 3 1 2 4 3 1 2 9 8 9 6 5 6
- search: 4 3 3 1 2 4 3 1 2 9 8 9 6 5 6
- swap: 4 3 3 1 2 2 3 1 4 9 8 9 6 5 6
- search: 4 3 3 1 2 2 3 1 4 9 8 9 6 5 6 (left > right)
- swap with pivot: 1 3 3 1 2 2 3 4 4 9 8 9 6 5 6

The partition method (Java)

```
static int partition(int[] a, int left, int right) {  
    int p = a[left], l = left + 1, r = right;  
    while (l < r) {  
        while (l < right && a[l] < p) l++;  
        while (r > left && a[r] >= p) r--;  
        if (l < r) {  
            int temp = a[l]; a[l] = a[r]; a[r] = temp;  
        }  
    }  
    a[left] = a[r];  
    a[r] = p;  
    return r;  
}
```

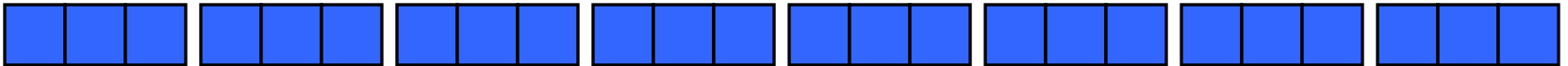
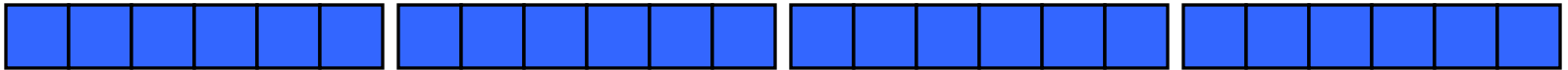
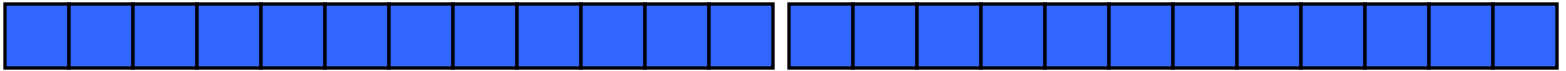
The quicksort method (in Java)

```
static void quicksort(int[] array, int left, int right) {  
    if (left < right) {  
        int p = partition(array, left, right);  
        quicksort(array, left, p - 1);  
        quicksort(array, p + 1, right);  
    }  
}
```


Analysis of quicksort—best case

- Suppose each partition operation divides the array almost exactly in half
- Then the depth of the recursion is $\log_2 n$
 - Because that's how many times we can halve n
- However, there are many recursions!
 - How can we figure this out?
 - We note that
 - Each partition is linear over its subarray
 - All the partitions at one level cover the array

Partitioning at various levels



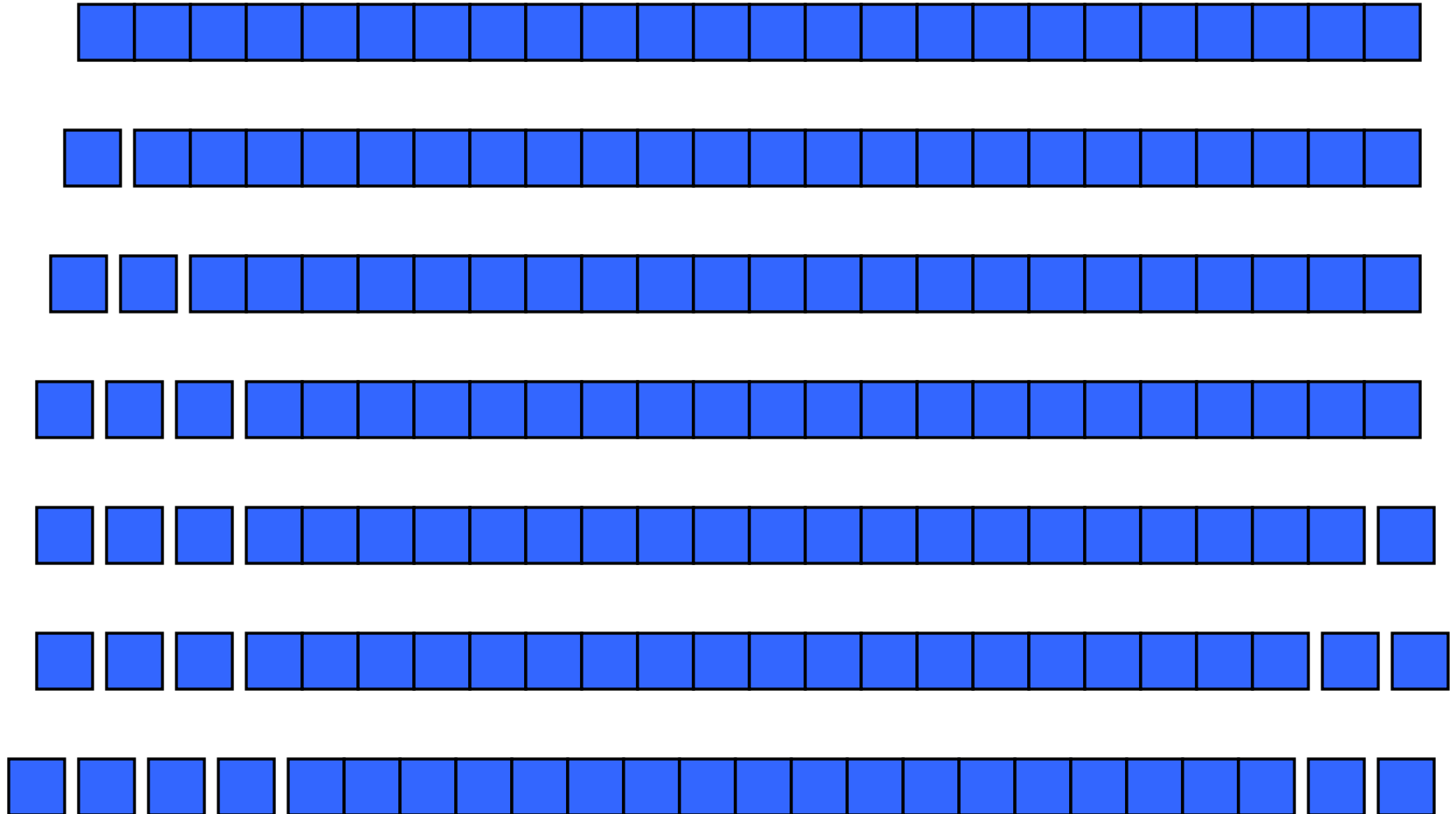
Best case II

- We cut the array size in half each time
- So the depth of the recursion is $\log_2 n$
- At each level of the recursion, all the partitions at that level do work that is linear in n
- $O(\log_2 n) * O(n) = O(n \log_2 n)$
- Hence in the average case, quicksort has time complexity $O(n \log_2 n)$
- What about the worst case?

Worst case

- In the worst case, partitioning always divides the size n array into these three parts:
 - A length one part, containing the pivot itself
 - A length zero part, and
 - A length $n-1$ part, containing everything else
- We don't recur on the zero-length part
- Recurring on the length $n-1$ part requires (in the worst case) recurring to depth $n-1$

Worst case partitioning



Worst case for quicksort

- In the worst case, recursion may be n levels deep (for an array of size n)
- But the partitioning work done at each level is still n
- $O(n) * O(n) = O(n^2)$
- So worst case for Quicksort is $O(n^2)$
- When does this happen?
 - There are many arrangements that *could* make this happen
 - Here are two common cases:
 - When the array is already sorted
 - When the array is *inversely* sorted (sorted in the opposite order)

Typical case for quicksort

- If the array is sorted to begin with, Quicksort is terrible: $O(n^2)$
- It is possible to construct other bad cases
- However, Quicksort is *usually* $O(n \log_2 n)$
- The constants are so good that Quicksort is generally the fastest algorithm known
- Most real-world sorting is done by Quicksort

Tweaking Quicksort

- Almost anything you can try to “improve” Quicksort will actually slow it down
- One *good* tweak is to switch to a different sorting method when the subarrays get small (say, 10 or 12)
 - Quicksort has too much overhead for small array sizes
- For large arrays, it *might* be a good idea to check beforehand if the array is already sorted
 - But there is a better tweak than this

Picking a better pivot

- Before, we picked the *first* element of the subarray to use as a pivot
 - If the array is already sorted, this results in $O(n^2)$ behavior
 - It's no better if we pick the *last* element
- We could do an *optimal* quicksort (guaranteed $O(n \log n)$) if we always picked a pivot value that exactly cuts the array in half
 - Such a value is called a **median**: half of the values in the array are larger, half are smaller
 - The easiest way to find the median is to *sort* the array and pick the value in the middle (!)

Median of three

- Obviously, it doesn't make sense to sort the array in order to find the median to use as a pivot
- Instead, compare just *three* elements of our (sub)array—the first, the last, and the middle
 - Take the *median* (middle value) of these three as pivot
 - It's possible (but not easy) to construct cases which will make this technique $O(n^2)$
- Suppose we rearrange (sort) these three numbers so that the smallest is in the first position, the largest in the last position, and the other in the middle
 - This lets us simplify and speed up the partition loop

Final comments

- Quicksort is the fastest known sorting algorithm
- For optimum efficiency, the pivot must be chosen carefully
- “Median of three” is a good technique for choosing the pivot
- However, no matter what you do, there will be some cases where Quicksort runs in $O(n^2)$ time