Data Structures and Algorithms

Stacks

What is a stack?

- A stack is a Last In, First Out (LIFO) data structure
- Anything added to the stack goes on the “top” of the stack
- Anything removed from the stack is taken from the “top” of the stack
- Things are removed in the reverse order from that in which they were inserted

Constructing a stack

- To use stacks, you need

```
import java.util.*;
```

- There is just one stack constructor:

```
Stack<E> stack = new Stack<E>();
```

- E is the type of element (for example, String) that you intend to put on the stack
  - To get the old (pre-Java 5) behavior, where objects of any kind can be put on the stack, use **<?>**

Fundamental stack operations

stack.push(object)

- Adds the object to the top of the stack; the item pushed is also returned as the value of push

object = stack.pop(); // object is of type “E”

- Removes the object at the top of the stack and returns it

object = stack.peek(); // object is of type “E”

- Returns the top object of the stack but does not remove it from the stack

stack.empty()

- Returns true if there is nothing in the stack

Additional stack operation

int i = stack.search(object);

- **Returns the *1-based* position of the element on the stack. That is, the top element is at position 1, the next element is at position 2, and so on.**
- **Returns -1 if the element is not on the stack**

Inheritance vs. composition

- **Inheritance**: **class X extends class Y { … }**
  - **X inherits all the (non-private) methods and variables of Y**
  - **This is appropriate if X *is a kind of* Y, but not otherwise**
    - **Because you get *all* of Y, whether it's appropriate for X or not**
  - **Inheritance is often overused**
- **Composition: class X { Y myYvariable; … }**
  - **To make a method, say int m(int a), of class Y available to objects of class X, you use delegation:**
    class X { …; int m(int a) { return myYVariable.m(a); } … }
  - **Similarly, class X can have getters and setters that refer to variables of the object myYVariable**
  - **Composition is appropriate if class X *uses* class Y, but isn't *a kind of* class Y**
- **If in doubt, use composition rather than inheritance**

Some uses of stacks

- **Stacks are used for:**
  - **Any sort of nesting (such as parentheses)**
  - **Evaluating arithmetic expressions (and other sorts of expression)**
  - **Implementing function or method calls**
  - **Keeping track of previous choices (as in backtracking)**
  - **Keeping track of choices yet to be made (as in creating a maze)**

A balancing act

- **([]({()}[()])) is balanced; ([]({()}[())]) is not**
- **Simple counting is not enough to check balance**
- **You can do it with a stack: going left to right,**
  - **If you see a (, [, or {, push it on the stack**
  - **If you see a),], or }, pop the stack and check whether you got the corresponding (, [, or {**
  - **When you reach the end, check that the stack is empty**

Exercise

- **Check whether the following given expression consists of balanced parentheses using a stack data structure. State all the states of the stack through out the algorithm.**
  - **array e{{1,5,6},{5,5},2,(45+50),x[6],getRadians(y[5])}**
  - **Math.Ceil(Convert(Math.Round(2*(60-[10-50*5-{60-80}]))))**

Expression evaluation

- Almost all higher-level languages let you evaluate expressions, such as 3*x+y or m=m+1
- In many languages, = is considered to be an operator
  - Its value is (typically) the value of the left-hand side, after the assignment has occurred
- Situations sometimes arise where you want to evaluate expressions yourself, without using the compiler

Performing calculations

- **To evaluate an expression, such as 1+2*3+4, you need *two* stacks: one for operands (numbers), the other for operators: going left to right,**
  - **If you see a number, push it on the number stack**
  - **If you see an operator,**
    - **While the top of the operator stack holds an operator of equal or higher precedence:**
      - **pop the old operator**
      - **pop the top two values from the number stack and apply the old operator to them**
      - **push the result on the number stack**
    - **push the new operator on the operator stack**
  - **At the end, perform any remaining operations**

Example: 1+2*3+4

- 1 : push 1 on number stack
- + : push + on op stack
- 2 : push 2 on number stack
- * : because * has higher precedence than +, push * onto op stack
- 3 : push 3 onto number stack
- + : because + has lower precedence than *:
  - pop 3, 2, and *
  - compute 2*3=6, and push 6 onto number stack
  - push + onto op stack
- 4 : push 4 onto number stack
- end : pop 4, 6 and +, compute 6+4=10, push 10; pop 10, 1, and +, compute 1+10=11, push 11
- 11 (at the top of the stack) is the answer

Exercise

- **Evaluate the following expressions using stack data structure. Clearly state the steps.**
  - **2+3*5-4/5+1**
  - **4*5*6-2*0/8**

Handling parentheses

- When you see a left parenthesis, **(**, treat it as a low-priority operator, and just put it on the operator stack
- When you see a right parenthesis , **)**, perform all the operations on the operator stack until you reach the corresponding left parenthesis; then remove the left parenthesis

Exercise

- **Evaluate the following expressions using stack data structure. Clearly state the steps.**
  - **2+3*(5-4)/5+1**
  - **4*(5-2*2)*(6-2)-16/8**
  - **(2-1)/2*(2*(5+8-2))**

Handling variables

- There are two ways to handle variables in an expression:
  - When you encounter the variable, look up its value, and put its value on the operand (number) stack
    - This simplifies working with the stack, since everything on it is a number
  - When you encounter a variable, put the variable itself on the stack; only look up its value later, when you need it
    - This allows you to have embedded assignments, such as 12 + (x = 5) * x

Bugs?

- The expression may be ill-formed:

  2 + 3 +

  - When you go to evaluate the second +, there won't be two numbers on the stack

  1 2 + 3

  - When you are done evaluating the expression, you have more than one number on the stack

  (2 + 3

  - You have an unmatched (on the stack

  2 + 3)

  - You can't find a matching (on the stack

- The expression may use a variable that has not been assigned a value

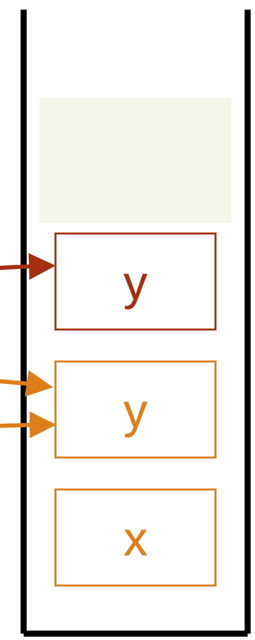
Types of storage

- **In almost all languages (including Java), data is stored in two different ways:**
  - **Temporary variables—parameters and local variables of a method—are stored in a *stack***
    - **These values are popped off the stack when the method returns**
    - **The value returned from a method is also temporary, and is put on the stack when the method returns, and removed again by the calling program**
  - **More permanent variables—objects and their instance variables and class variables—are kept in a *heap***
    - **They remain on the heap until they are "freed" by the programmer (C, C++) or garbage collected (Java)**

Stacks in Java

- **Stacks are used for local variables (including parameters)**

```
void methodA() {
    int x, y; // puts x, y on stack
    y = 0;
    methodB();
    y++;
}
void methodB() {
    int y, z; // puts y, z on stack
    y = 5;
    return; // removes y, z
}
```



Supporting recursion

```
static int factorial(int n) {
    if (n <= 1) return 1;
    else return n * factorial(n - 1);
}
```

- If you call **x = factorial(3)**, this enters the factorial method with **n=3** on the stack
- | **factorial** calls itself, putting **n=2** on the stack
- | | **factorial** calls itself, putting **n=1** on the stack
- | | **factorial** returns **1**
- | **factorial** has **n=2**, computes and returns **2*1 = 2**
- **factorial** has **n=3**, computes and returns **3*2 = 6**

Factorial - 1

```
x = factorial(3)
```

3 is put on stack as n

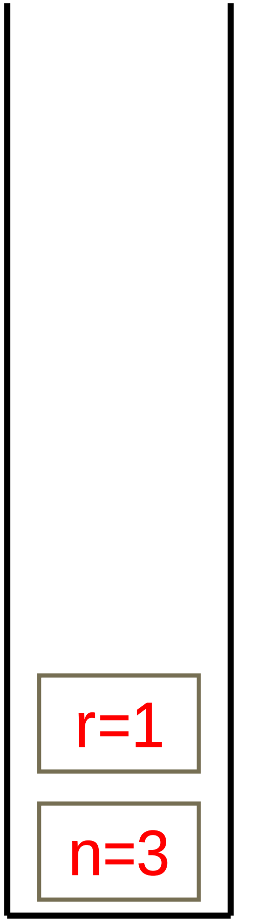
```
static int factorial(int n) { //n=3
    int r = 1;
    if (n <= 1) return r;
    else {
        r = n * factorial(n - 1);
        return r;
    }
}
```

r is put on stack with value 1

All references to r use this r

All references to n use this n

Now we recur with 2...



Factorial - 2

- `r = n * factorial(n - 1);`

2 is put on stack as n

- ```
  static int factorial(int n) {//n=2
        int r = 1;
        if (n <= 1) return r;
        else {
              r = n * factorial(n - 1);
              return r;
        }
  }
  ```

r is put on stack with value 1

Now using this r

And this n

r=1
n=2
r=1
n=3

Now we recur with 1...
  ```

Factorial - 3

```
r = n * factorial(n - 1);
```

1 is put on stack as n

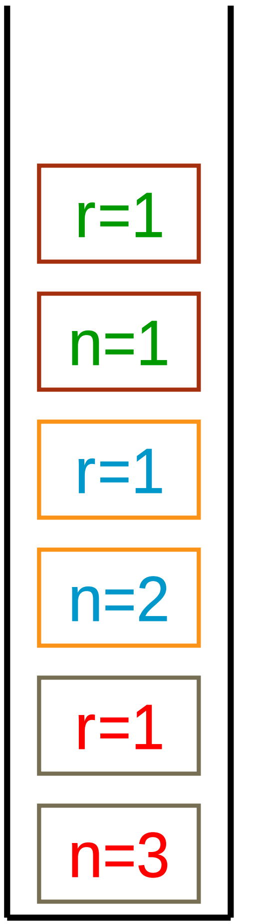
```
static int factorial(int n) {
    int r = 1;
    if (n <= 1) return r;
    else {
        r = n * factorial(n - 1);
        return r;
    }
}
```

Now using this r

r is put on stack with value 1

And this n

Now we pop r and n off the stack and return 1 as factorial(1)



Factorial - 4

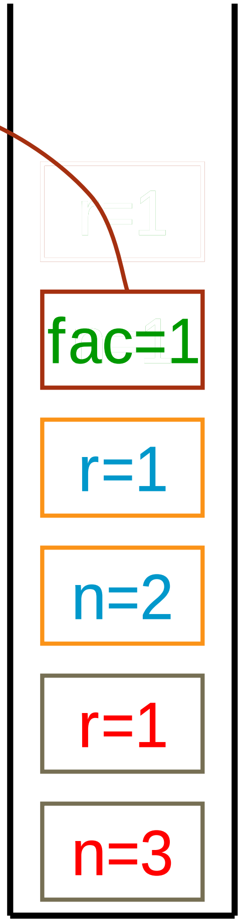
- r = n * factorial(n - 1);

- static int factorial(int n) {

```
static int factorial(int n) {
    int r = 1;
    if (n <= 1) return r;
    else {
        r = n * factorial(n - 1);
        return r;
    }
}
```

Now using this r

And this n



Now we pop r and n off the stack and return 1 as factorial(1)

Factorial - 5

- `r = n * factorial(n - 1);`

- ```
  static int factorial(int n) {
        int r = 1;
        if (n <= 1) return r;
        else {
              r = n * factorial(n - 1);
              return r;
        }
  }
  ```

1

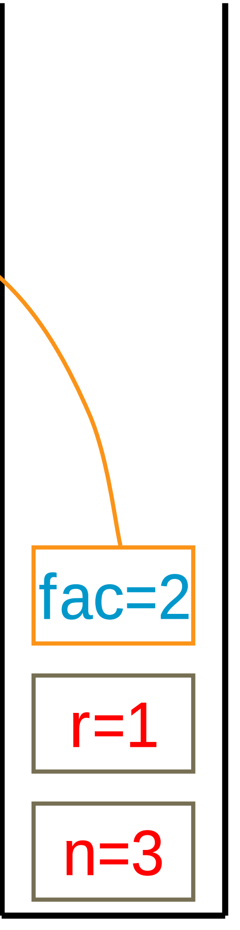
2 * 1 is 2;

Pop r and n;

Return 2

Now using this r

And this n


  ```

Factorial - 6

- x = factorial(3)

- static int factorial(int n) {

```
    int r = 1;
    if (n <= 1) return r;
    else {
        r = n * factorial(n - 1);
        return r;
    }
}
```

2

3 * 2 is 6;

Pop r and n;

Return 6

Now using this r

r=1

And this n

fac=6

Stack frames

- Rather than pop variables off the stack one at a time, they are usually organized into **stack frames**
- Each frame provides a set of variables and their values
- This allows variables to be popped off all at once
- There are several different ways stack frames can be implemented