AIN SHAMS UNIVERSITY
I-Credit Hours Engineering Programs
(i.CHEP)

University of East London

Course Code:	PHM114	Course Name:	Numerical Analysis	Major Task	Date:	**2/7/2023**
Student Name:	**Karim Sherif Louis**			Student ID:	u2498559	

	Mastery ≥90%				Accomplished ≥75 & <90%				Adequate ≥60 & <75%				Inadequate <60%			
	100	96	93	90	89	84	79	75	74	69	64	60	59	40	20	0
	• Write down correctly all used equations and formula • Substitute all given date correctly • Solve to get the required values correctly • Using related SI units • Draw related – sketches, wave forms, items arrangements, • Comment in results				• Write down correctly all used equations and formula • Substitute all given date correctly • Solve to get the required values correctly • Using related SI units • Partially Draw related – sketches, wave forms, items arrangements, • Comment in results				• Write down correctly all used equations and formula • Substitute all given date correctly • Solve to get the required values • Did not use related SI units • Partially Draw related – sketches, wave forms, items arrangements, • Did not comment in results				• Write down partially all used equations and formula • Substitute all given date partially correctly • Solve to get the required values • Using related SI units • Draw related – sketches, wave forms, items arrangements, • Did not comment in results			

1st marker Total	30	1st marker Signature	Mohammed Berg	ASU Agreed Mark	30
2nd Marker Total	30	2nd marker Signature	Islam Samir	UEL Agreed Mark	30

General Comments:

Well done!

Very good, keep going!

UEL Grading System	Agreed	ASU Grading Scale	
% equivalent at UEL	**Mark Range**	**% at ASU**	**Grade**
95% and higher	Yes Yes	97% and higher	A+
82% to less than 95%		93% to less than 97%	A
70% to less than 82%		89% to less than 93%	A-
66% to less than 70%		84% to less than 89%	B+
63% to less than 66%		80% to less than 84%	B
60% to less than 63%		76% to less than 80%	B-
56% to less than 60%		73% to less than 76%	C+
53% to less than 56%		70% to less than 73%	C
50% to less than 53%		67% to less than 70%	C-
45% to less than 50%		64% to less than 67%	D+
40% to less than 45%		60% to less than 64%	D
Less than 40%		Less than 60%	F

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Major Task	Total: 30 marks

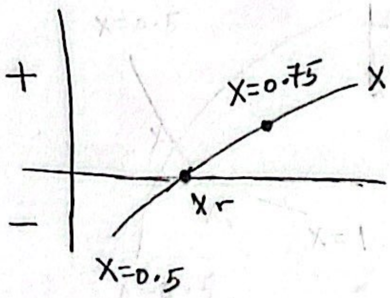
PHM 114: Numerical Analysis

Name: ..Karim..Sherif..Louis.................... **ID.:** 21P0223 **Section:** ..3...

Roots of Equations

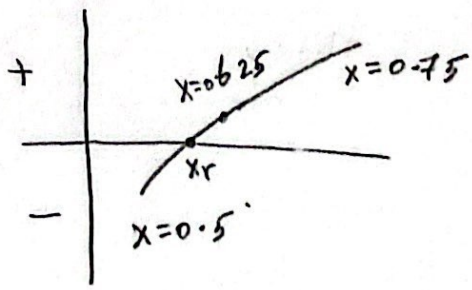
(1) Determine the real roots of: $f(x) = -26 + 82.3x - 88x^2 + 45.4x^3 - 9x^4 + 0.65x^5$ using the bisection method with initial guesses of $x_L = 0.5$ and $x_u = 1.0$.
The accuracy required for the solution is $\varepsilon_a = 10\ \%$.

n	X_L	$f(X_L)$	X_u	$f(X_u)$	X_r	$f(X_r)$	ε_a
1	0.5	−	1	+	0.75	+	
2	0.5	−	0.75	+	0.625	+	20%
3	0.5	−	0.625	+	0.5625	−	11.1%
4	0.5625	−	0.625	+	0.5938	+	5.27%



$$\text{Bisection} = X_r = \frac{X_u + X_L}{2}$$

$$\varepsilon_a = \frac{X_{r_{new}} - X_{r_{old}}}{2} \times 100$$



$$\boxed{X_r = 0.5938 \qquad n=4}$$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: ...Karim...Sherif..Louis.................. **ID.:** 21P0223. **Section:** .3.....

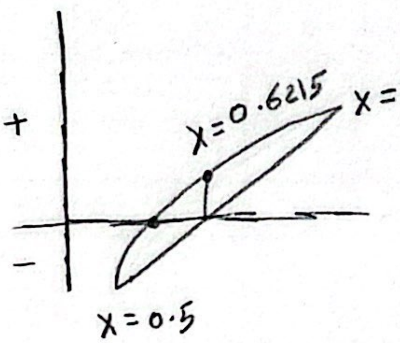
Roots of Equations

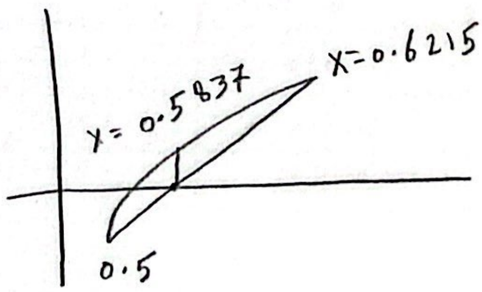
(2) Determine the real roots of: $f(x) = -26 + 82.3x - 88x^2 + 45.4x^3 - 9x^4 + 0.65x^5$ using the false position method with initial guesses of $x_L = 0.5$ and $x_u = 1.0$. The accuracy required for the solution is $\varepsilon_a = 0.1\ \%$.

n	X_L	$f(X_L)$	X_u	$f(X_u)$	X_r	$f(X_r)$	ε_a
1	0.5	-1.7172	1.0	5.35	0.6215	0.7748	
2	0.5	-1.7172	0.6215	0.7748	0.5837	0.0844	6.4 %
3	0.5	-1.7172	0.5837	0.0844	0.5798	0.0092	0.67%
4	0.5	-1.7172	0.5798	0.0092	0.5794	0.0014	0.07%

$$\varepsilon_a = \left| \frac{x_{r\,new} - x_{r\,old}}{x_{r\,new}} \right| \times 100$$

$$\text{False position} = x_r = x^u - f(x^u)\,\frac{x^u - x_L}{f(x^u) - f(x_L)}$$





$$\boxed{X_r = 0.5794 \quad n = 4}$$

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Roots of Equations

(3) Use simple fixed – point iteration to locate the root of: $f(x) = \sin(x^{0.5}) - x$.
Use initial guess of $x_0 = 0.5$ and iterate till the approximate error becomes less than 0.01 %.

let $X = \sin x^{0.5}$

$$g'(x) = \cos x^{0.5}(0.5x^{-0.5}) = \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

$$g'(0.5) = 0.53 < 1$$

n	X_n	X_{n+1}	ε_a
1	0.5	0.6496	
2	0.6496	0.7215	9.9%
3	0.7215	0.7509	3.9%
4	0.7509	0.7621	1.4%
5	0.7621	0.7662	0.5%
6	0.7662	0.7678	0.2%
7	0.7678	0.7683	0.06%
8	0.7683	0.7685	0.024
9	0.7685	0.7686	0.01%

$X_r = 0.7686$

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Roots of Equations

(4) Apply 5 iterations of Newton method to locate the root of: $f(x) = x^3 - 6x^2 + 11x - 6.1$
Use initial guess of $x_0 = 3.5$ and calculate the approximate error.

$$f'(x) = 3x^2 - 12x + 11$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n	x_{n+1}	ε_a
1	3.5	3.1913	
2	3.1913	3.0687	4 %
3	3.0687	3.0473	0.7 %
4	3.0473	3.0467	0.02 %
5	3.0467	3.0467	0 %

$x_r = 3.0467$

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Roots of Equations

(5) Apply 5 iterations of Secant method to locate the root of: $f(x) = x^3 - 6x^2 + 11x - 6.1$
Use $x_{i-1} = 2.5$ and $x_i = 3.5$ and calculate the approximate error.

n	x_{n-1}	$f(x_{n-1})$	x_n	$f(x_n)$	x_{n+1}	$f(x_{n+1})$	ε_a
2	2.5	-0.475	3.5	1.775	2.7111	-0.45157	-
3	3.5	1.775	2.7111	-0.45157	2.8711	-0.310	5.57%
4	2.7111	-0.45157	2.8711	-0.310	3.2215	0.50252	10%
5	2.8711	-0.31	3.2215	0.50252	3.00506	-0.0898	7%
6	3.2215	0.50252	3.00506	-0.0898	3.03796	-0.01791	1.083%

$$x_{n+1} = x_n - f(x_n)\left(\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right)$$

$$x_r = 3.03796$$

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PHM 114: Numerical Analysis

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Roots of Equations

(6) Apply 3 iterations of Newton method to locate the root of: $f(x) = \cos(\sqrt[3]{x}) + x$
Use $x_0 = -0.5$ and calculate the approximate error.

$$X_{n+1} = X_n - \frac{f(x)}{f'(x)} = X_n - \frac{\cos(\sqrt[3]{x}) + x}{-\frac{1}{3}x^{-2/3}\sin(\sqrt[3]{x}) + 1}$$

n	X_n	X_{n+1}	$\varepsilon_a\%$
1	-0.5	-0.646097	
2	-0.646097	-0.64828	0.337%
3	-0.64828	-0.648279	0.00015%

$X_r = -0.648279$

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Name: ..Karim..Sherif..Louis.................. **ID.:** 21P0223. **Section:** .3.....

System of Linear Equations

(7) Apply Gauss Elimination method to find the approximate solution of the system:

$$2x_1 + 10x_2 + 3x_3 = 15$$
$$10x_1 + x_2 + 2x_3 = 13$$
$$5x_1 - 3x_2 + 10x_3 = 12$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 2 & 10 & 3 & 15 \\ 10 & 1 & 2 & 13 \\ 5 & -3 & 10 & 12 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - 5R_1 \\ R_3 - \frac{5}{2}R_1 \end{array} \left[\begin{array}{ccc|c} 2 & 10 & 3 & 15 \\ 0 & -49 & -13 & -62 \\ 0 & -28 & 2.5 & -25.5 \end{array}\right] \qquad -\left(\frac{10}{2}\right) \quad -\left(\frac{5}{2}\right)$$

$$\begin{array}{c} \\ \\ R_3 - \frac{4}{7}R_2 \end{array} \left[\begin{array}{ccc|c} 2 & 10 & 3 & 15 \\ 0 & -49 & -13 & -62 \\ 0 & 0 & \frac{139}{14} & \frac{139}{14} \end{array}\right] \qquad -\left(\frac{-28}{-49}\right)$$

$$\frac{139}{14} X_3 = \frac{139}{14} \qquad \boxed{X_3 = 1}$$

$$-49(X_2) - 13 = -62 \qquad \boxed{X_2 = 1}$$

$$2X_1 + 10 + 3 = 15 \qquad \boxed{X_1 = 1}$$

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PHM 114: Numerical Analysis

Name: ..Karim.sherif.louis...................... **ID.:** 21.P0223. **Section:** ..3...

System of Linear Equations

(8) Apply Jacobi method to find the approximate solution of the system:

$$2x_1 + 10x_2 + 3x_3 = 15$$
$$10x_1 + x_2 + 2x_3 = 13$$
$$5x_1 - 3x_2 + 10x_3 = 12$$

to within an accuracy of 0.01.

$$X_1 = \frac{1}{10}(13 - X_2 - 2X_3)$$

$$X_2 = \frac{1}{10}(15 - 2X_1 - 3X_3)$$

let $X_1 = X_2 = X_3 = 0$

$$X_3 = \frac{1}{10}(12 - 5X_1 + 3X_2)$$

n	X_1	X_2	X_3
0	0	0	0
1	1.3	1.5	1.2
2	0.91	0.88	1
3	1.012	1.018	1.009
4	0.9964	0.9949	0.9994
5	1.00063	1.0009	1.0027
6	0.99983	0.9998	0.99996
7	1	1	1
8	1	1	1

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PHM 114: Numerical Analysis

Name: ..Karim..Sherif...Louis................ **ID.:** 21P0223. **Section:** ..3...

System of Linear Equations

(9) Apply Gauss Seidel method to find the approximate solution of the system:

$$2x_1 + 10x_2 + 3x_3 = 15$$
$$10x_1 + x_2 + 2x_3 = 13$$
$$5x_1 - 3x_2 + 10x_3 = 12$$
to within an accuracy of 0.01.

$$x_1 = \frac{1}{10}(13 - x_2 - 2x_3)$$

$$x_2 = \frac{1}{10}(15 - 2x_1 - 3x_3)$$

$$x_3 = \frac{1}{10}(12 - 5x_1 + 3x_2)$$

let $x_1 = x_2 = x_3 = 0$

n	x_1	x_2	x_3
0	0	0	0
1	1.3	1.24	0.922
2	0.9916	1.0251	1.0117
3	0.9952	0.9975	1.0017
4	0.9999	0.9995	0.9999
5	1	1	1
6	1	1	1

Major Task Total: 30 marks

PHM 114: Numerical Analysis

Name: ...Karim Sherif Louis........... **ID.:** 21P0223.. **Section:** .3.....

Least Squares Regression

(10) Use least-squares regression to fit a straight line to the data:

x	5	6	10	14	16	20	22	28	28	36	38
y	30	22	28	14	22	16	8	8	14	0	4

$n = 11$

$y = ax + b$

$\sum y = a \sum x + nb$

$\sum xy = a \sum x^2 + b \sum x$

$\sum x = 223$

$\sum y = 166$

$\sum xy = 2374$

$\sum x^2 = 5805$

$166 = 223a + 11b$

$2374 = 5805a + 223b$

$a = -0.772$

$b = 30.74$

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Major Task — Total: 30 marks

PHM 114: Numerical Analysis

Name: Karim Sharif Louis .... **ID.:** 2100223 **Section:** 3.....

Least Squares Regression

(11) Use least-squares regression to fit a parabola to the data:

x	2	3	4	7	8	9	5	5
y	9	6	5	10	9	11	2	3

$y = b_0 + b_1 x + b_2 x^2$

$n = 8$

$\Sigma y = n b_0 + b_1 \Sigma x + b_2 \Sigma x^2$

$\Sigma xy = b_0 \Sigma x + b_1 \Sigma x^2 + b_2 \Sigma x^3$

$\Sigma x^2 y = b_0 \Sigma x^2 + b_1 \Sigma x^3 + b_2 \Sigma x^4$

$\Sigma y = 55$

$\Sigma x = 43$

$\Sigma x^2 = 273$

$\Sigma x^3 = 1933$

$\Sigma x^4 = 14661$

$\Sigma xy = 322$

$\Sigma x^2 y = 2252$

$8 b_0 + 43 b_1 + 273 b_2 = 55$

$43 b_0 + 273 b_1 + 1933 b_2 = 322$

$273 b_0 + 1933 b_1 + 14661 b_2 = 2252$

$b_0 = 16.027$

$b_1 = -4.807$

$b_2 = 0.489$

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Major Task Total: 30 marks

PHM 114: Numerical Analysis

Name: ..Karim...Sherif..Louis...................... ID.: .21P0222. Section: 3......

Least Squares Regression

(12) Use least-squares regression to fit a power model to the data:

x	1	3	4	5	7	8	9	11	12	13	15
y	10	23	27.5	31	39	41.5	42.9	46	45.5	46	59

$y = a x^b$

$\ln y = \ln a + b \ln x \longrightarrow Y = A + bX$

$\sum \ln y = An + b \sum \ln y$

$\sum \ln y . \ln y = A \sum \ln x + b \sum \ln^2 x$

$\sum \ln y = 38.887$

$\sum \ln x = 20.473$

$\sum \ln^2 x = 44.495$

$\sum \ln x \ln y = 76.231$

$11A + 20.473b = 38.887$

$20.43A + 44.495b = 76.231$

$A = \ln a = 2.4122$

$b = 0.6034$

$a = 11.1585$

$$\boxed{Y = 11.1585\, X^{0.6034}}$$

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Major Task — Total: 30 marks

PHM 114: Numerical Analysis

Name: ..Karim...Sherif..Louis.................. **ID.:** 21P0223. **Section:** ..3....

Least Squares Regression

(13) Use least-squares regression to fit an exponential model to the data:

x	1	3	4	5	7	8	9	11	12	13	15
y	10	23	27.5	31	39	41.5	42.9	46	45.5	46	59

$y = ae^{bx}$

$\ln y = \ln a + bx \longrightarrow Y = A + bx$ $\quad n = 11$

$\sum \ln y = nA = b\sum x$

$\sum x \ln y = A\sum x + b\sum x^2$

$\sum x = 88$

$\sum \ln y = 38.887$

$\sum x^2 = 904$

$\sum x \ln y = 330.279$

$11A + 88b = 38.887$

$88A + 904b = 330.279$

$A = 2.7678$ $\quad b = 0.0954$

$a = e^{2.7678} = 15.9236$

$$\boxed{y = 15.9236\, e^{0.0959x}}$$

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PHM 114: Numerical Analysis

Name: ..Karım....Sherif...louis.................... **ID.:** 21P0223. **Section:** ...3..

Least Squares Regression

(14) Use least-squares regression to fit the model: $y = \frac{1}{a_0 x + a_1}$ to the data:

x	1	3	4	5	6
y	0.5	2	4	3.5	6

$\frac{1}{y} = a_0 x + a_1$

$\sum \frac{1}{y} = a_0 \sum x + n a_1$

$\sum \frac{x}{y} = a_0 \sum x^2 + a_1 \sum x$

$\frac{1}{y}$	2	0.5	0.25	2/7	1/6
x^2	1	9	16	25	36
x/y	2	3/2	1	10/7	1

$\sum x = 19$

$\sum x^2 = 87$

$\sum \frac{1}{y} = 3.2024$

$\sum \frac{x}{y} = 6.9286$

$19 a_0 + 5 a_1 = 3.2024$

$87 a_0 + 19 a_1 = 6.9286$

$a_0 = 0.3541$

$a_1 = 1.986$

$y = \frac{1}{0.3541 x + 1.986}$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: ..Karim...Sharf...Louis.................. **ID.:** 21P0225 **Section:** .3.....

Least Squares Regression

(15) Use least-squares regression to fit the quadratic model: $A = a_0 + a_1 d^2$ to the data:

Day (d)	10	20	30	40	50
Amount (A)	25	70	380	550	610

$\sum A = n a_0 + a_1 \sum d^2$

$\sum d^2 A = a_0 \sum d2 + a_1 \sum d^4$

d2	100	400	900	1600	2500
d4	10,000	160,000	810,000	2560000	6250000
Ad 2	2500	28000	342000	880000	1525000

$\sum A = 1635$

$\sum d^2 = 5500$

$\sum d4 = 9790000$

$\sum Ad\,2 = 2777500$

$5a_0 + 5500 a_1 = 1635$

$5500 a_0 + 9790000 a_1 = 277500$

$a_0 = \frac{664}{17}$

$a_1 = \frac{89}{340}$

$A = \frac{664}{17} + \frac{89}{340} d2$

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PHM 114: Numerical Analysis

Name: ....Karim...Sherif Louis....................... **ID.:** 2P0222.. **Section:** ..3....

Interpolating Polynomial

(16) Use Newton Interpolation formula to obtain the interpolating polynomial of the data:

x	1	4	6	5
y	0	1.386294	1.791759	1.6094381

then, use it to calculate f(2).

x	y	Δ_1	Δ_2	Δ_3
1	0			
		0.4621		
			0.05188	
4	1.386294			7.875×10^{-3}
		0.2027		
			0.02038	
6	1.791759			
		0.18232		
5	1.6094381			

$$P(x) = (x-1)(0.4621) + (x-1)(x-4)(0.05188) + (x-1)(x-4)(x-6)(7.875\times10^{-3})$$

$$P(x) = 7.875\times10^{-3}x^3 + 0.1383x^2 + 0.43895x - 0.44358$$

$$P(2) = 0.693147$$

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PHM 114: Numerical Analysis

Name: ..Karim..Sherif..Louis................. **ID.:** 21P0223 **Section:** ..3....

Interpolating Polynomial

(17) Use Newton Interpolation formula to obtain the interpolating polynomial of the data:

x	1	2	3	4
y	4	6	10	14

then, use it to calculate f(1.5).

X	Y	D_1	D_2	D_3
1	4			
		2		
2	6		1	
		4		$-\frac{1}{3}$
3	10		0	
		4		
4	14			

$P(x) = 4 + 2(x-1) + (x-1)(x-2) - \frac{1}{3}(x-1)(x-2)(x-3)$

$P(x) = -\frac{1}{3}x^3 + 3x^2 + \frac{14}{3}x + 6$

$P(1.5) = 18.625$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: ..karim..Sherif...louis........................ **ID.:** 2.1P0223 **Section:** 3......

Interpolating Polynomial

(18) Use Lagrange Interpolation formula to obtain the interpolating polynomial of the data:

x	1	2	3	4
y	4	6	10	14

then, use it to calculate f(1.5).

$$P(x) = y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x) + y_4 L_4(x)$$

$$L_1 = \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)} \qquad L_2 = \frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)}$$

$$L_3 = \frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)} \qquad L_4 = \frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)}$$

$$P(x) = \frac{-4}{6}(x-2)(x-3)(x-4) + 3(x-1)(x-3)(x-4) - 5(x-1)(x-2)(x-4) + \frac{7}{3}(x-1)(x-2)(x-3)$$

$$P(x) = -\frac{x^3}{3} + 3x^2 - \frac{13}{3}x + 6$$

$$P(1.5) = 5.5$$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: ..Karim.Sherif.Louis.......................... **ID.:** 21P0.272.. **Section:** 3......

Interpolating Polynomial

(19) Use Lagrange Interpolation formula to obtain the interpolating polynomial of the data:

x	-1	4	1	0
y	-2	43	4	-1

then, use it to calculate f(2).

$P(x) = y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x) + y_4 L_4(x)$

$L_1 = \frac{(x-4)(x-1)x}{(-1-4)(-1-1)(-1)}$ $\quad L_2 = \frac{(x+1)(x-1)(x)}{(4+1)(4-1)(4)}$

$L_3 = \frac{(x+1)(x-4)(x)}{(1+1)(1-4)(1)}$ $\quad L_4 = \frac{(x+1)(x-1)(x-4)}{(1)(-1)(-4)}$

$P(x) = \frac{1}{5}x(x-4)(x-1) + \frac{43}{60}x(x-1)(x+1) - \frac{2}{3}x(x+1)(x-4)$

$- \frac{1}{4}(x+1)(x-1)(x-4)$

$P(x) = 2x^2 + 3x - 1$

$P(2) = 13$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: ..Kazim..Sherif..Louis.............................. **ID.:** 21P0223. **Section:** 3.....

Interpolating Polynomial

(20) Use Lagrange formula to obtain the second order interpolating polynomial of the data:

x	1	4	6	5
y	0	1.386294	1.791759	1.6094381

then, use it to calculate f(2).

$$f(x) = 0 + \frac{(x-1)(x-6)}{(4-1)(4-6)}(1.386294) +$$

$$\frac{(x-1)(x-4)}{(6-1)(6-4)}(1.791759)$$

$$f(x) = -0.05x^2 + 0.72x - 0.67$$

$$f(2) = 0.57$$

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Total: 30 marks

Major Task

PHM 114: Numerical Analysis

Name: ..Karim...Sherif.Louis...................... **ID.:** 2100223. **Section:** ...3..

Interpolating Polynomial

(21) Use Inverse Interpolation to obtain the value of x that corresponds to y=0.85 of the data:

x	0	1	2	3	4	5
y	0	0.5	0.8	0.9	0.941176	0.961538

$X - y = 0 \quad 0.5 \quad 0.8 \quad 0.9 \quad 0.941176 \quad 0.961538$

$Y = x = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

X	Y					
0	0					
		2				
0.5	1		5/3			
		10/3		50/3		
0.8	2		50/3		185.86	
		10		191.593		3590.882
0.9	3		10.193		3678.437	
		24.256		1870.87		
0.941176	4		403.411			
		49.11109				
0.961538	5					

$$f(0.85) = 0 + 2(0.85) + \frac{5}{3}(0.85)(0.85-0.5) - 50/3\,(0.85)(0.85-0.3)(0.85-0.8)$$

$$+ 185.86(0.85)(0.85-0.5)(0.85-0.8)(0.85-0.9)$$

$$+ 3590.6821(0.85)(0.85-0.5)(0.85-0.8)(0.85-0.9)(0.85-0.941176)$$

$$= 2.5490$$

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Major Task	Total: 30 marks

PHM 114: Numerical Analysis

Name: ..Karim.sherif..Louis.Kamel........ **ID.:** 21P0223 **Section:** ..3....

Numerical Integration

(22) Evaluate the integral: $\int_0^3 (1-e^{-x}+8x-4x^2)\,dx$ Using

(i) Analytical method
(ii) Trapezoidal method, **n = 6**
(iii) Simpson method, **n = 6**

(i) $I_{exact} = \left[x + \frac{1}{e^x} + \frac{8x^2}{2} - \frac{4x^3}{3} \right]_0^3 = \frac{1}{e^3} + 2 = \boxed{2.04979}$

(ii) $h = \frac{3-0}{6} = \frac{1}{2}$

$I_{approx} = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\sum_i^{n-1} f(x_i) \right)$

$x_0 = 0$ $\quad x_3 = 1.5$ $\quad x_6 = 3$
$x_1 = 0.5$ $\quad x_4 = 2$
$x_2 = 1$ $\quad x_5 = 2.5$

$I_{approx} = \frac{0.5}{2}\left[0 - 11.0498 + 2\left(3.3935 + 4.6321 + 3.7769 + 0.8647 - 4.0821\right)\right] = \boxed{1.5301}$

(iii) $I_{approx} = \frac{h}{3}\left[f(x_0) + f(x_n) + 4\sum f_{odd} + 2\sum f_{even} \right]$

$= \frac{0.5}{3}\left[0 - 11.0498 + 4\left(3.3935 + 3.7769 - 4.0821\right) + 2\left(4.6321 + 0.8647\right)\right)$

$= \boxed{2.0495}$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: Karim Sherif Louis **ID.:** 21P0223 **Section:** 3

Numerical Integration

(23) Evaluate the integral: $\int_0^{\pi/2} \cos^2(x) \sin^7(x)\,dx$ Using

(i) Analytical method
(ii) Trapezoidal method, n = 6
(iii) Simpson method, n = 6

$\sin^2 x = 1 - \cos^2 x$

(i) $\cos^2 x \sin x \sin^6 x$

$(\sin^2 x)^3 \cos^2 x \cdot \sin x\,dx$

$(1-\cos^2 x)^3 \cdot \cos^2 x \cdot \sin x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$\frac{du}{-\sin x} = dx$

$\int (1-u^2)^3 \cdot u^2 \cancel{\sin x} \frac{du}{-\cancel{\sin x}}$

$-\int (1-3u^2+3u^4-u^6)\,u^2\,du$

$-\int u^2-3u^4+3u^6-u^8\,du$

$-\left[\frac{u^3}{3} - \frac{3u^5}{5} + \frac{3u^7}{7} - \frac{u^9}{9}\right]$

$\left[-\frac{\cos^3 x}{3} + \frac{3\cos^5 x}{5} - \frac{3\cos^7 x}{7} + \frac{\cos^9 x}{9}\right]_0^{\frac{\pi}{2}}$

$\boxed{= \frac{16}{315} = 0.05079}$

(ii) $h = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$

$x_0 = 0$ $x_3 = \frac{\pi}{4}$ $x_6 = \frac{\pi}{2}$

$x_1 = \frac{\pi}{12}$ $x_4 = \frac{\pi}{3}$

$x_2 = \frac{\pi}{6}$ $x_5 = \frac{5}{12}\pi$

$I_{approx} = \frac{h}{2}\left[f(x_0)+f(x_n)+2\sum_1^{n-1}(f x_i)\right]$

$= \frac{\pi/12}{2}\left[0+0+2\left(\overset{1}{7.2587\times10^{-5}} + \overset{2}{5.8594\times10^{-3}} + \overset{3}{0.0442} + \overset{4}{0.0913} + \overset{5}{0.0526}\right)\right]$

$= \boxed{0.05079}$

(iii)

$I_{approx} = \frac{h}{3}\left[f(x_0)+f(x_n)+4\sum f_{odd} + 2\sum f_{even}\right.$

$= \frac{\pi/12}{3}\left[0+0+4\left[7.2587\times10^{-5}+0.0442+0.0526\right]+2\left(5.8594\times10^{-3}+0.0913\right)\right]$

$= 0.05077$

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Major Task — Total: 30 marks

PHM 114: Numerical Analysis

Name: ..Karim..Sherif..Louis.............. **ID.:** 21P0222 **Section:** ...3..

Numerical Solution for solving ODE

(24) Solve the Differential Equation: $\frac{dy}{dx} = (1+x)\sqrt{y}$; y(0)=1 Between x=0 and x=1 Using

(i) Analytical method
(ii) Euler method, **h = 0.25**
(iii) Runge Kutta method, **h = 1** and **h = 0.5**

$y^{-\frac{1}{2}}$ $\frac{y^{\frac{1}{2}}}{\frac{1}{2}}$

(i) $\int \frac{1}{\sqrt{y}}\, dy = \int (1+x)\, dx$

$2y^{\frac{1}{2}} = x + \frac{1}{2}x^2 + c$ $c = 2$

$2y^{\frac{1}{2}} = x + \frac{1}{2}x^2 + 2 \rightarrow y = \left(\frac{x+\frac{1}{2}x^2+2}{2}\right)^2$

$y(0) = 1$

$\boxed{y(1) = 3.0625}$

(ii) $y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	y_{n+1}
0	0	1	1.25
1	0.25	1.25	1.5994
2	0.5	1.5994	2.0737
3	0.75	2.0737	2.7037
4	1	$\boxed{2.7037}$	3.5258

at n= $\boxed{y = 2.7037}$

(iii) at n=0 x = 0 $\boxed{h = 1}$

$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = 1 \times (1) = 1$

$K_2 = 1 \times 1.8371 = 1.8371$

$K_3 = 1 \times 2.0777 = 2.0777$

$K_4 = 3.5088$

$\boxed{y_{n+1} = 3.0563}$

iii $\boxed{h = 0.5}$ 1st iteration

$K_1 = 0.5$
$K_2 = 0.69877$
$K_3 = 0.72602$
$K_4 = 0.98534$

$y(0.5) = 1.72249$

2nd iteration

$K_1 = 0.984327$
$K_2 = 1.302149$
$K_3 = 1.348065$
$K_4 = 1.75230$

$\Delta y = \frac{1}{6}(0.984327 + 1.75230 + 2(1.302149 + 1.348065)) = 1.33451$

$\boxed{y(1) = 3.0620}$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: ..Karim...Sharif...Louis.................. **ID.:** 21P0223. **Section:** ...3...

Numerical Solution for solving ODE

(25) Solve the Differential Equation: $\frac{dy}{dx} = x^2 y - 1.2 y$; $y(0)=1$ Between $x=0$ and $x=1$ Using

- **(i)** Analytical method
- **(ii)** Euler method, **h = 0.25**
- **(iii)** Runge Kutta method, **h = 1 and h = 0.5**

(i) $\frac{dy}{dx} = y(x^2 - 1.2)$

$\int \frac{dy}{y} = \int x^2 - 1.2\, dx$

$\ln y = \frac{x^3}{3} - 1.2x + c \rightarrow c = 0$

$y = e^{\frac{x^3}{3} - 1.2x}$

$y(1) = 0.42035$

(ii) $h = 0.25$

n	x_n	y_n	y_{n+1}
0	0	1	0.7
1	0.25	0.7	0.5004375
2	0.5	0.5004375	0.38196
3	0.75	0.38196	0.32109
4	1	0.3269	

$y(1) = 0.32109$

(iii) $h = 1$

$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = -1.2$

$K_2 = -0.38$

$K_3 = -0.7695$

$K_4 = -0.0461$

$\Delta y = \frac{1}{6}(-1.2 + -0.0461 + 2(-0.38 - 0.7695))$

$= -0.59085$

$y(1) = 0.40915$

$h = 0.5$ $n = 0$

$K_1 = -0.6$

$K_2 = -0.348125$

$K_3 = -0.4553$

$K_4 = -0.25862$

$\Delta y = \frac{1}{6}(-0.6 - 0.25862 + 2(-0.348125 + -0.45553))$

$= -0.427655$

$y = 0.572345$

$h = 0.5$ $n = 1$

$K_1 = -0.27186$

$K_2 = -0.13907$

$K_3 = -0.16026$

$K_4 = -0.04121$

$\Delta y = \frac{1}{6}(-0.27186 - 0.04121 + 2(0.13907 - 0.16026))$

$= -0.151967$

$y(1) = 0.4203725$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: ..Karim...Serif...Louis.............. **ID.:** 21P.0423 **Section:** ...3..

Eigen values and Eigen vector

(26) Use Power method to approximate the highest Eigenvalue and its corresponding Eigenvector of the matrix: $\mathbf{A} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$ Use initial Eigenvector $\mathbf{X_0} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

1st iteration

$$A X_0 \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \to X_1 \begin{pmatrix} 1 \\ -1/3 \end{pmatrix}$$

$$\lambda_1 = \frac{(1)(4.666617) + (-0.33333)(4.66670)}{(1)^2 + (-0.33333)^2} = 5.2$$

2nd iteration

$$A X_1 = \begin{pmatrix} 5 & 1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -0.33333 \end{pmatrix} = \begin{pmatrix} 4.66667 \\ -3.33333 \end{pmatrix} \to X_2 = \begin{pmatrix} 1 \\ -0.71425 \end{pmatrix}$$

$$\lambda_2 = \frac{(1)(4.28572) + (-0.71428)(-3.71428)}{1 + (-0.71428)^2} = 4.39461$$

3rd iteration

$$A X_2 \begin{pmatrix} 5 & 1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -0.71428 \end{pmatrix} = \begin{pmatrix} 4.28572 \\ -3.71428 \end{pmatrix} \to X_3 \begin{pmatrix} 1 \\ -0.86666 \end{pmatrix}$$

$$\lambda_3 = \frac{(1)(4.3333) + (-0.86668)(-3.86667)}{1^2 + (-0.86666)^2} = 4.2$$

$$A X_3 = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}\begin{pmatrix} 1 \\ -0.86666 \end{pmatrix} = \begin{pmatrix} 4.1333 \\ -3.86667 \end{pmatrix}$$

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Major Task **Total: 30 marks**

PHM 114: Numerical Analysis

Name: ...Karim...Sherif...louis............ **ID.:** 21P0223 **Section:** ...3...

Eigen values and Eigen vector

(27) Use Power method to approximate the highest Eigenvalue and its corresponding Eigenvector of the matrix: $A = \begin{bmatrix} -2 & 8 \\ 3 & 8 \end{bmatrix}$ Use initial Eigenvector $X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

1st iteratio

$$A X_0 \begin{bmatrix} -2 & 8 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{pmatrix} 6 \\ 11 \end{pmatrix} \to X_1 \begin{pmatrix} 6/11 \\ 1 \end{pmatrix}$$

$$\lambda_1 = \frac{(6/11)(6.9.91) + (1)(9.6364)}{(6/11)^2 + 1^2} = 40.3312$$

2nd iteration

$$A X_1 \begin{pmatrix} -2 & 8 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} 6/11 \\ 1 \end{pmatrix} = \begin{pmatrix} 6.9.91 \\ 9.6364 \end{pmatrix} \to X_2 = \begin{pmatrix} 0.7170 \\ 1 \end{pmatrix}$$

$$\lambda_2 = \frac{6.5660(0.7170) + (1)(10.131)}{1^2 + 0.717^2} = 9.8137$$

3rd iteration

$$A X_2 = \begin{pmatrix} -2 & 8 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} 0.7170 \\ 1 \end{pmatrix} = \begin{pmatrix} 6.65660 \\ 10.151 \end{pmatrix} \to X_3 \begin{pmatrix} 0.64683 \\ 1 \end{pmatrix}$$

$$\lambda_3 = \frac{0.64683(6.70634) + 1(9.4049}{1^2 + 0.64683^2} = 10.0666$$

$$A X_3 \begin{pmatrix} -2 & 8 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} 0.64683 \\ 1 \end{pmatrix} = \begin{pmatrix} 6.70634 \\ 9.94049 \end{pmatrix}$$

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Major Task **Total: 30 marks**

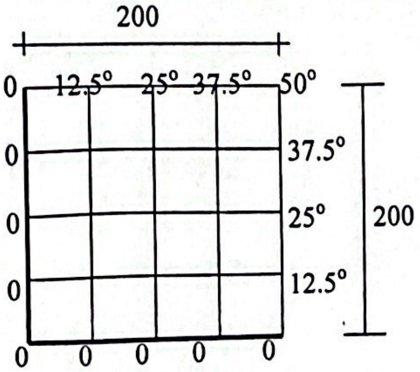
PHM 114: Numerical Analysis

Name: ..Karim....Sherif....Louis....... **ID.:** 21P0223 **Section:** ..3...

Numerical Solution for solving PDE

(28) The temperature of a heated plate (200 mm x 200 mm) is given by: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0.0$

Solve this PDE using the finite difference method with the grid and the boundary conditions shown in the figure:



$h = \frac{200}{4} = 50$ mm

$T_2 = T_4 \quad T_3 = T_7$

$T_6 = T_8$

at T_1 $\quad 0 + T_2 + T_2 + 0 - 4T_1 = 0$

$-4T_1 + 2T_2 = 0 \longrightarrow 1$

at T_2 $\quad T_5 + 0 + T_3 + T_1 - 4T_2 = 0$

$T_1 - 4T_2 + T_3 + T_5 = 0 \rightarrow 2$

at T_3 $\quad 0 + T_6 + T_2 + 12.5 - 4T_3 = 0$

$T_2 - 4T_3 + T_6 = -12.5 \rightarrow 3$

at T_5 $\quad T_2 + T_6 + T_6 + T_2 - 4T_5 = 0$

$2T_2 - 4T_5 + 2T_6 = 0 \rightarrow 4$

at T_6 $\quad T_9 + T_3 + 25 + T_5 - 4T_6 = 0$

$T_3 + T_5 - 4T_6 + T_9 = -25 \rightarrow 5$

at T_9 $\quad T_6 + 37.5 + 37.5 + T_6 - 4T_9 = 0$

$2T_6 - 4T_9 = -75 \rightarrow 6$

$T_1 = 3.1250$

$T_2 = T_4 = 6.25000$

$T_3 = T_7 = 9.3750$

$T_5 = 12.50000$

$T_6 = T_8 = 18.75000$

$T_9 = 28.12500$

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Major Task **Total: 30 marks**

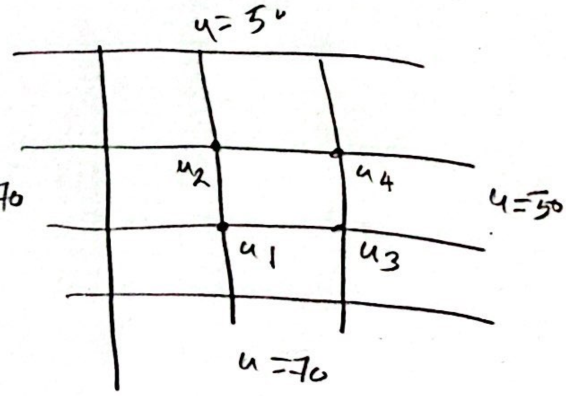
PHM 114: Numerical Analysis

Name: ..Karim..Sherif..Lawis............. **ID.:** 21P0223 **Section:** 3....

Numerical Solution for solving PDE

(29) Solve $\nabla^2 u = 0$, in the region bounded by the four lines: $x = 0$, $x = 3$, $y = 0$ and $y = 3$ with the boundary conditions :

$u = 50$ on the lines $y = 3$ and $x = 3$,
$u = 70$ on the lines $y = 0$ and $x = 0$ Take $h = 1$.



at u_1 $70+u_2+u_3+70-4u_1=0$

$-4u_1+u_2+u_3=-140 \rightarrow 1$

at u_2 $50+u_1+70+u_4-4u_2=0$

$u_1+4u_2+u_4=-120 \rightarrow 2$

at $u_3 \rightarrow u_4+70+u_1+50-4u_3=0$

$u_1-4u_3+u_4=-120 \rightarrow 3$

at u_4 $50+u_3+u_2+50-4u_4=0$

$u_2+u_3-4u_4=-100 \rightarrow 4$

$u_1=65$ $u_2=60$ $u_3=60$ $u_4=55$

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Major Task **Total: 30 marks**

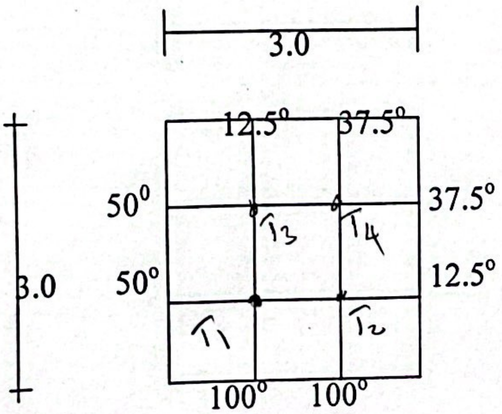
PHM 114: Numerical Analysis

Name: ..Karim...Sherif..Louis.................. **ID.:** 21P0223 **Section:** ..3...

Numerical Solution for solving PDE

(30) The temperature of a heated plate (300 mm x 300 mm) is given by: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0.0$

Solve this PDE using the finite difference method with the grid and the boundary conditions shown in the figure



at T_1 $50 + T_2 + 100 + T_3 - 4T_1 = 0 \rightarrow 1$ $T_2 = 4T_1 - T_3 - 150$

at T_2 $T_1 + 12.5 + T_4 + 100 - 4T_2 = 0 \rightarrow 2$

at T_3 $12.5 + T_1 + 50 + T_4 - 4T_3 = 0 \rightarrow 3$

at T_4 $37.5 + T_2 + T_3 + 37.5 - 4T_4 = 0 \rightarrow 4$

$T_1 + T_4 - 4(4T_1 - T_3 - 150) = -112.5$

$T_1 + T_4 - 16T_1 + 4T_3 + 600 = -112.5$

$-15T_1 + 4T_3 + T_4 = -712.5$ $T_1 - 4T_3 + T_4 = -62.5$

$4T_1 - 4T_4 = 75$

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$T_1 = 61.45833$ $T_3 = 41.86667$ $T_2 = 54.16665$ $T_4 = 42.70833$