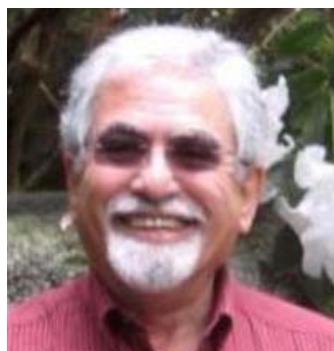


# Network Dilation: A Strategy for Building Families of Parallel Processing Architectures

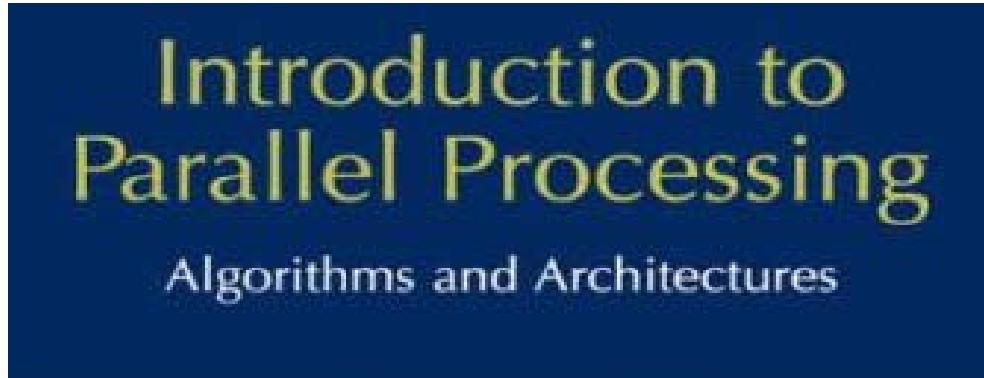


**Behrooz Parhami**

Dept. Electrical & Computer Eng.  
Univ. of California, Santa Barbara

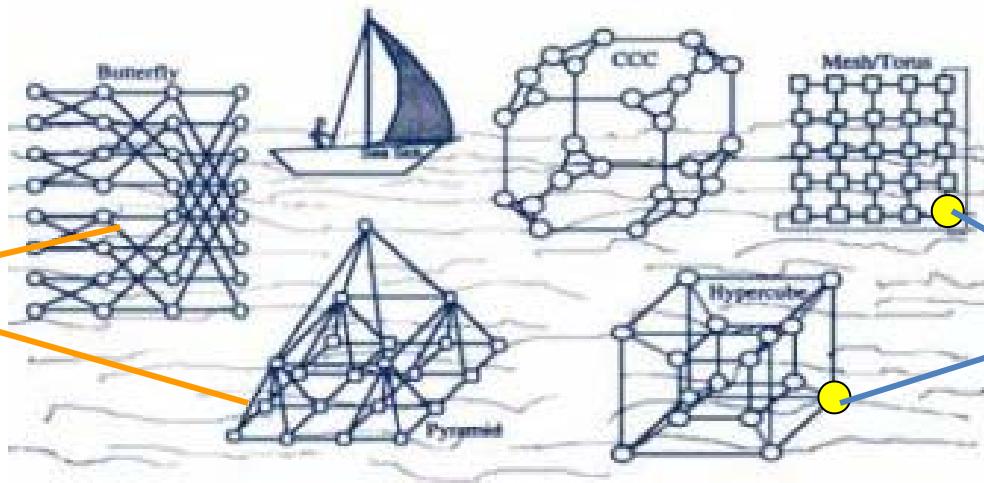
# Parallel Computer Architecture

Parallel computer =  
Nodes +  
Interconnects  
(+ Switches)



B. Parhami,  
Plenum Press,  
1999

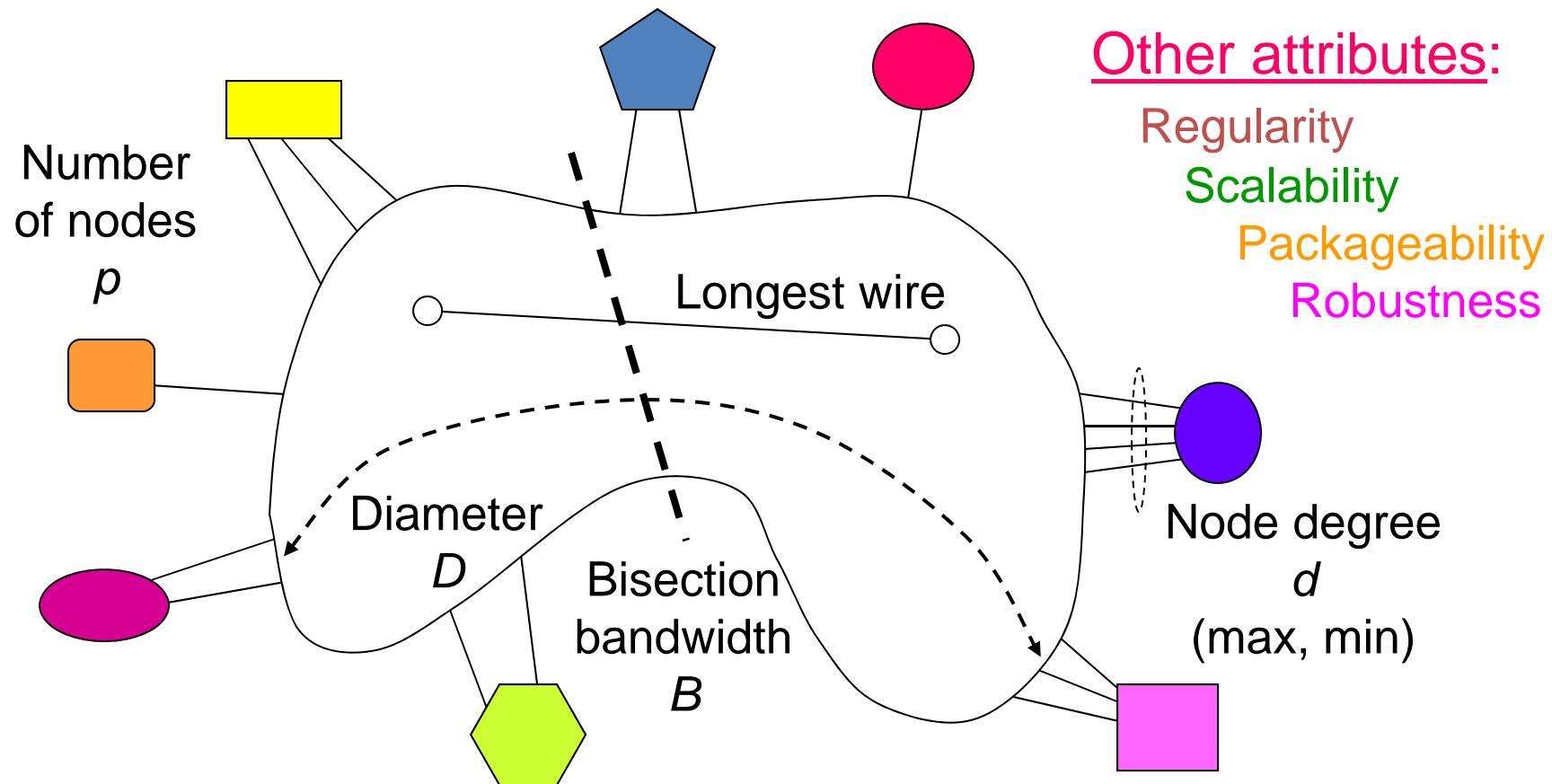
Interconnects,  
communication  
channels,  
or links



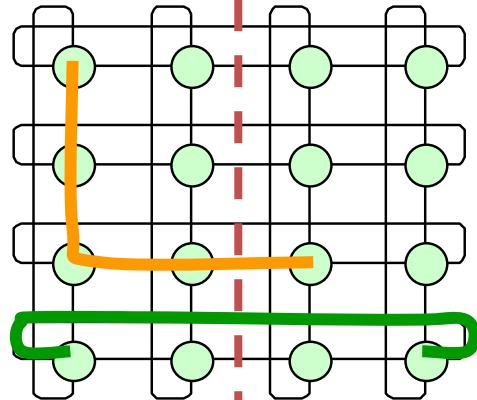
Nodes or  
processors

# Interconnection Networks

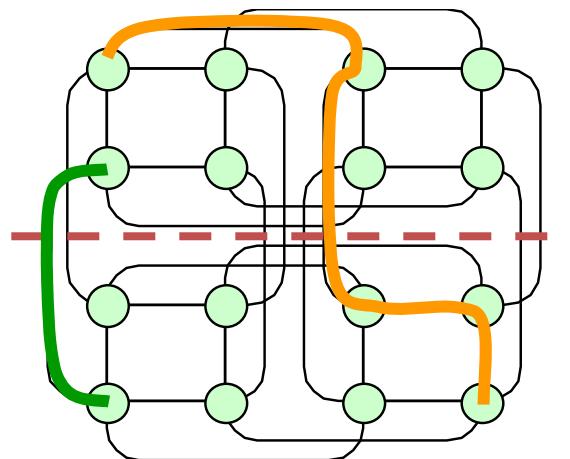
Heterogeneous or homogeneous nodes



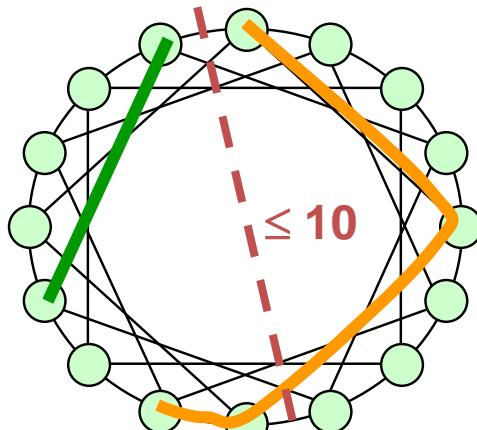
# Four Example Networks



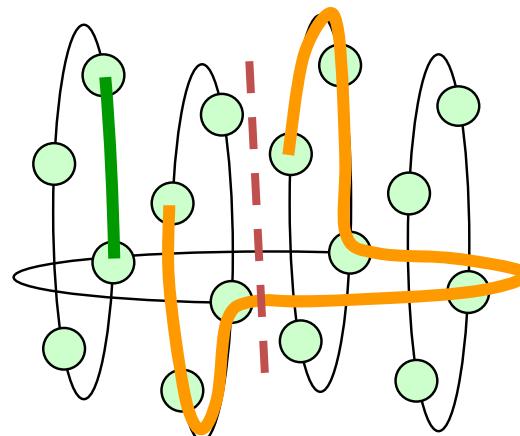
(a) 2D torus



(b) 4D hypercube



(c) Chordal ring



(d) Ring of rings

Nodes  $p = 16$

Degree  $d = 4$

Diameter  $D$

Avg. distance  $\Delta$

Bisection  $B$

Longest wire

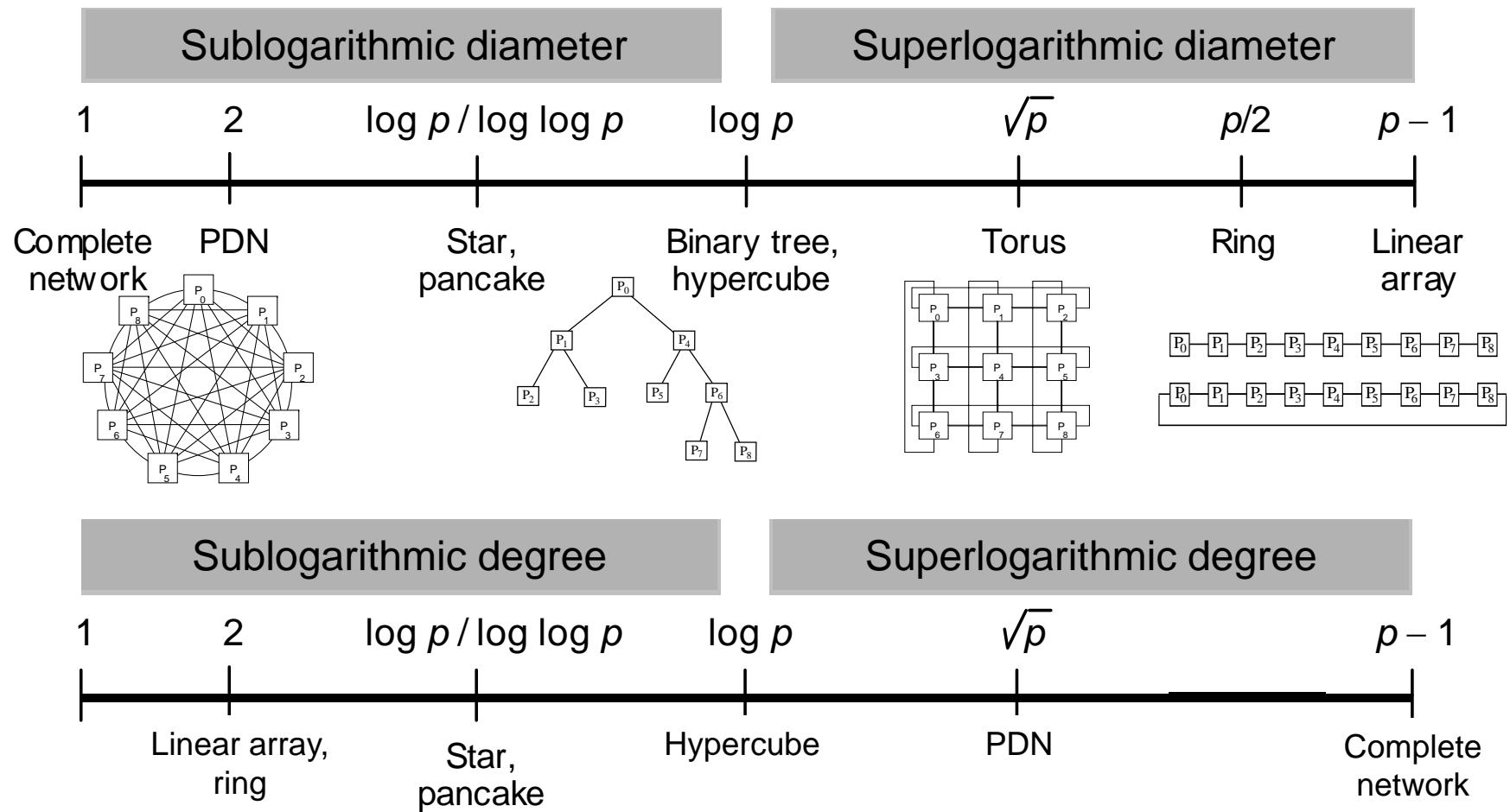
Regularity

Scalability

Packageability

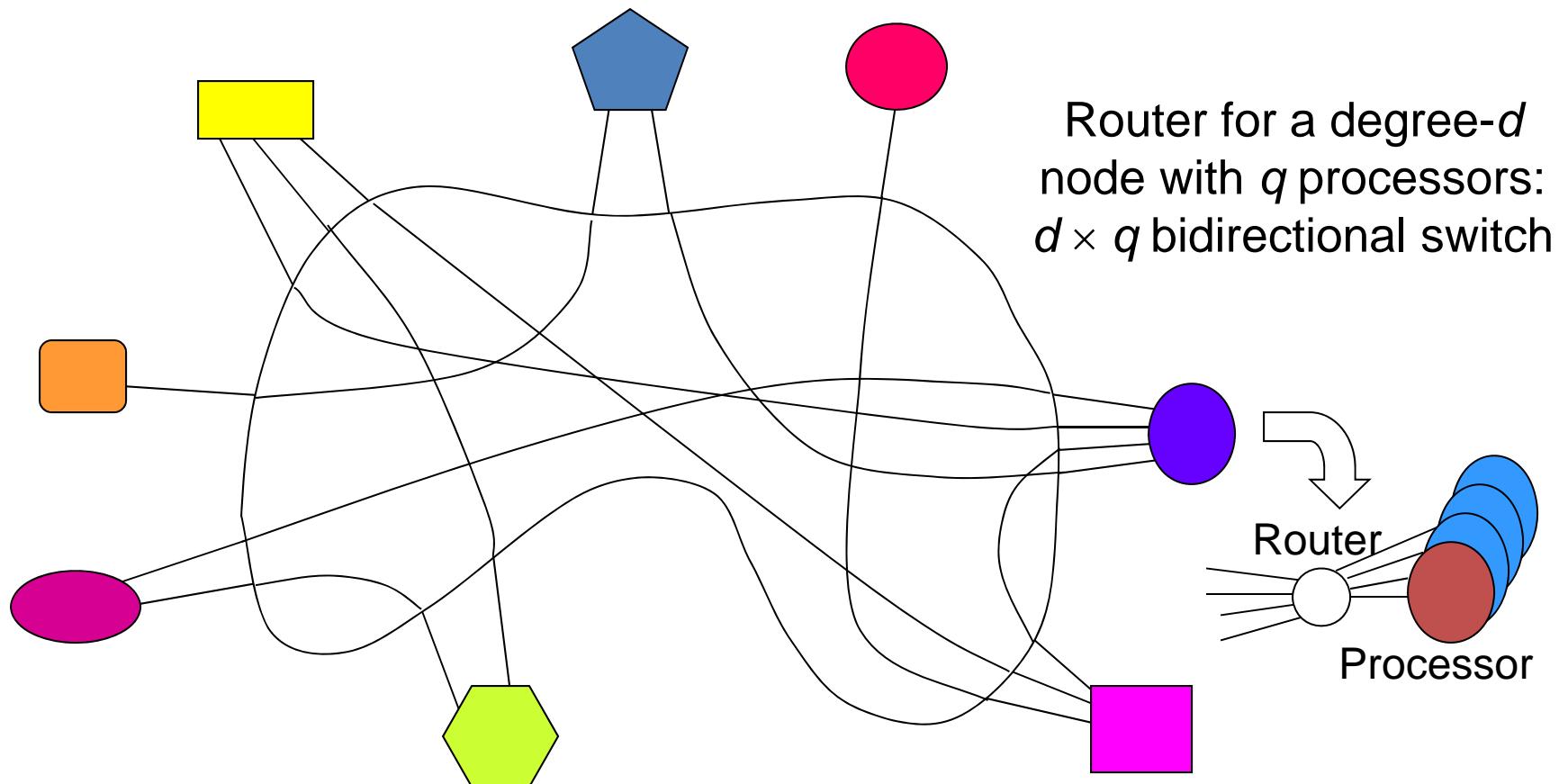
Robustness

# Spectrum of Networks



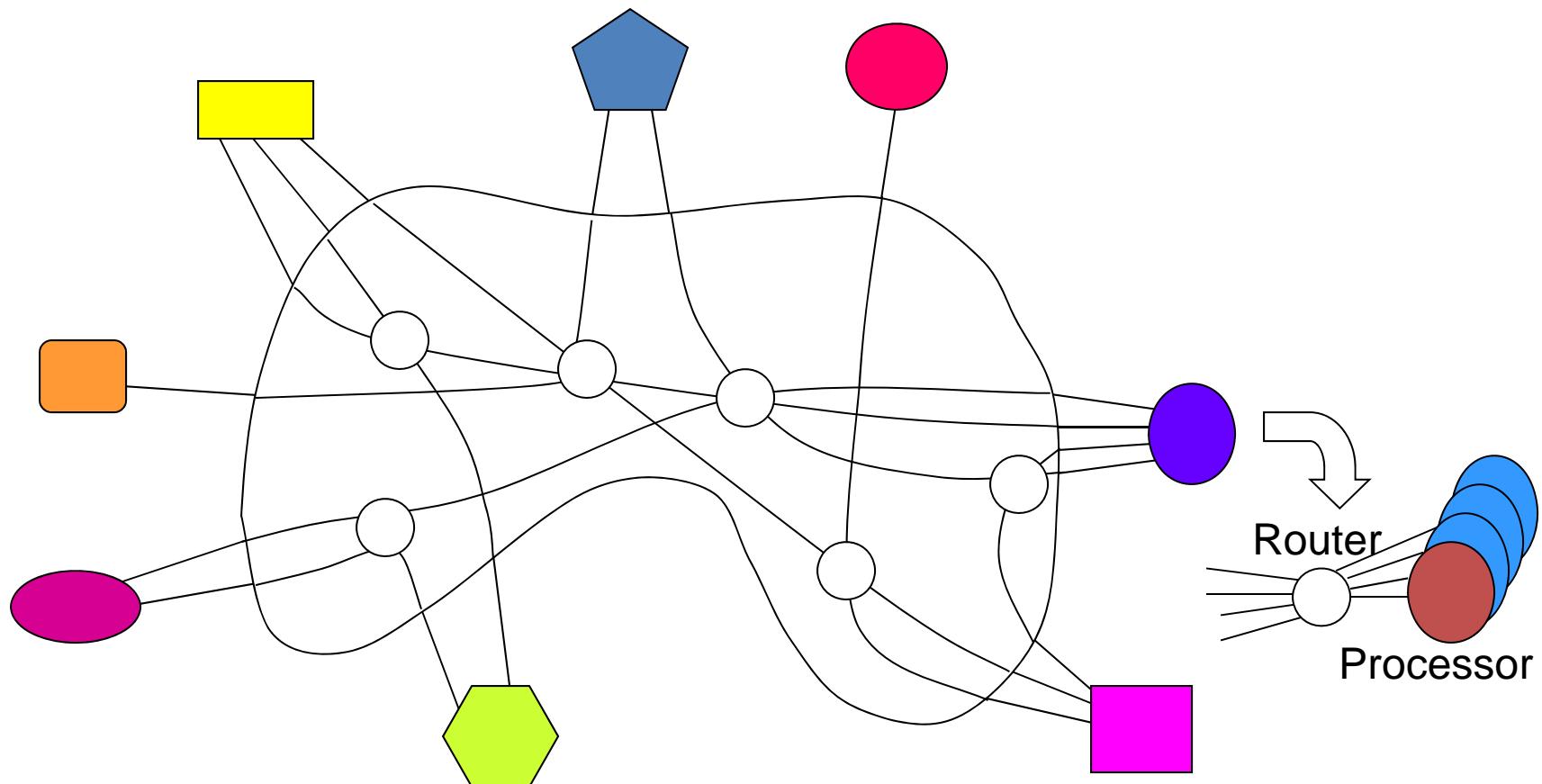
# Direct Networks

Nodes (or associated routers) directly linked to each other

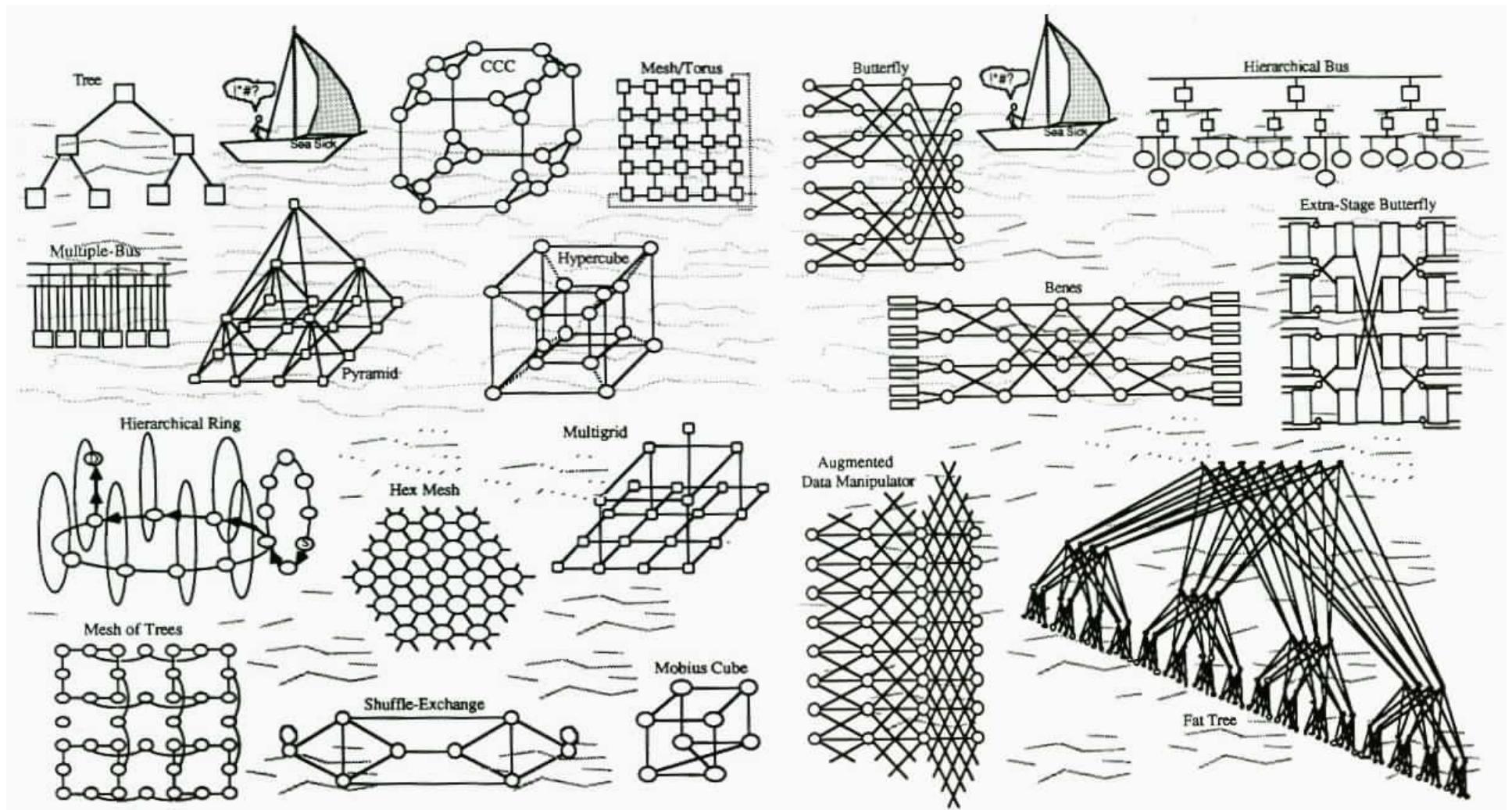


# Indirect Networks

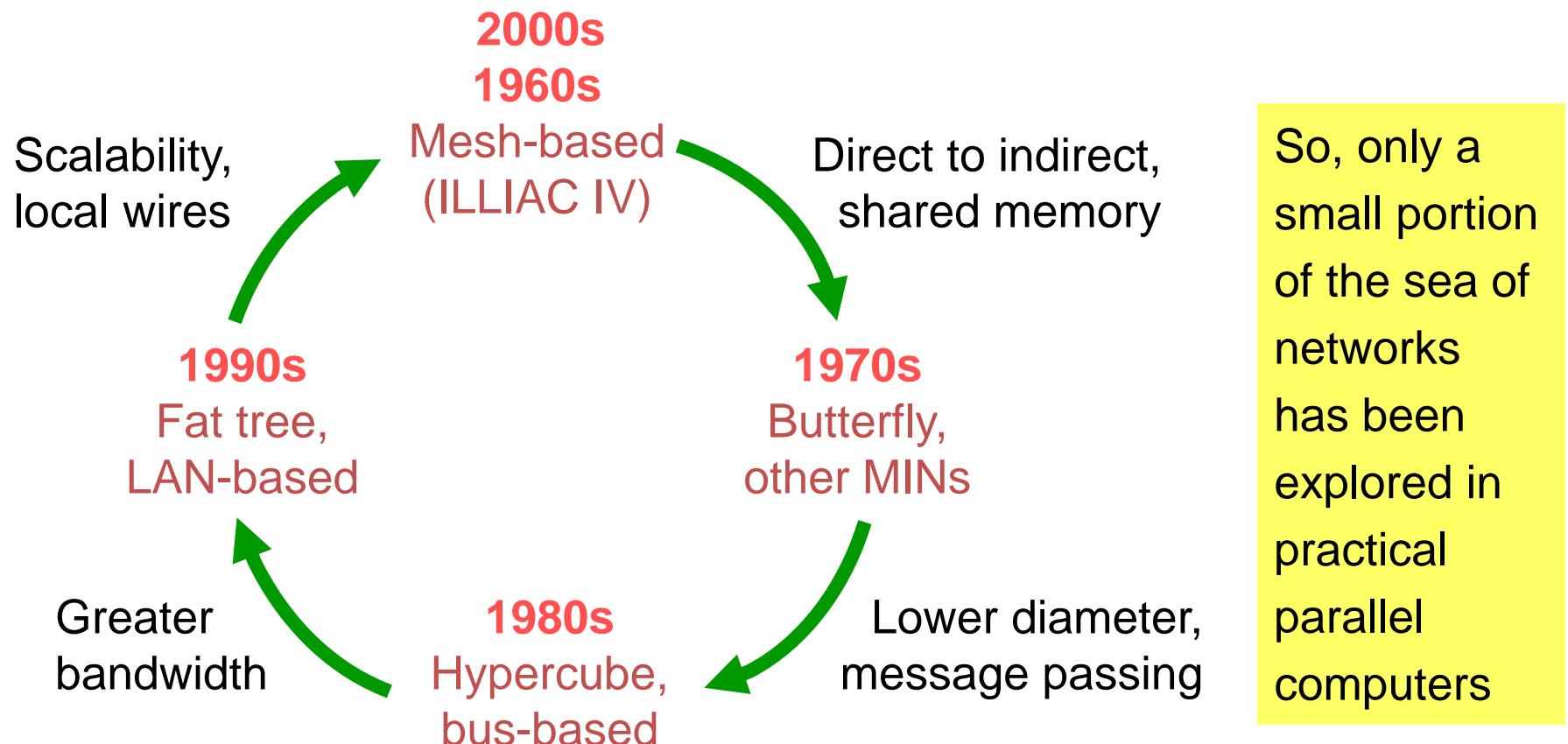
Nodes (or associated routers) linked via intermediate switches



# A Sea of Networks



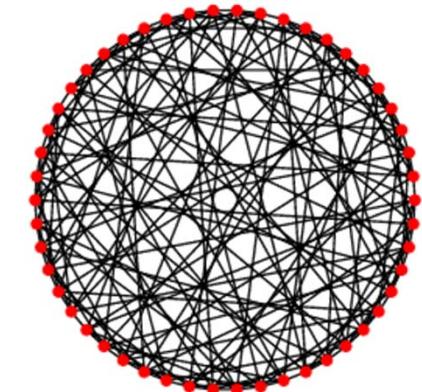
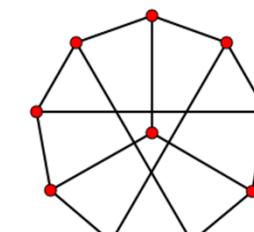
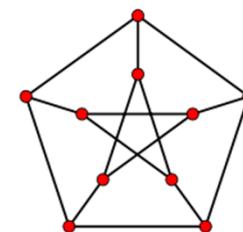
# A Bit of History: Moving Full Circle



# The $(d, D)$ Graph Problem

Suppose you have an unlimited supply of degree- $d$  nodes  
How many can be connected into a network of diameter  $D$ ?

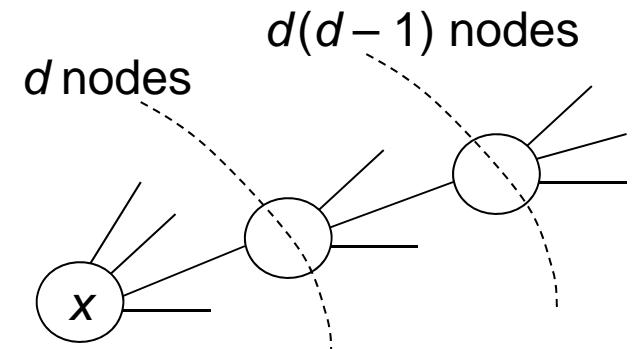
Example 1:  $d = 3, D = 2$ ;  
10-node Petersen graph



Moore bound (undirected graphs)

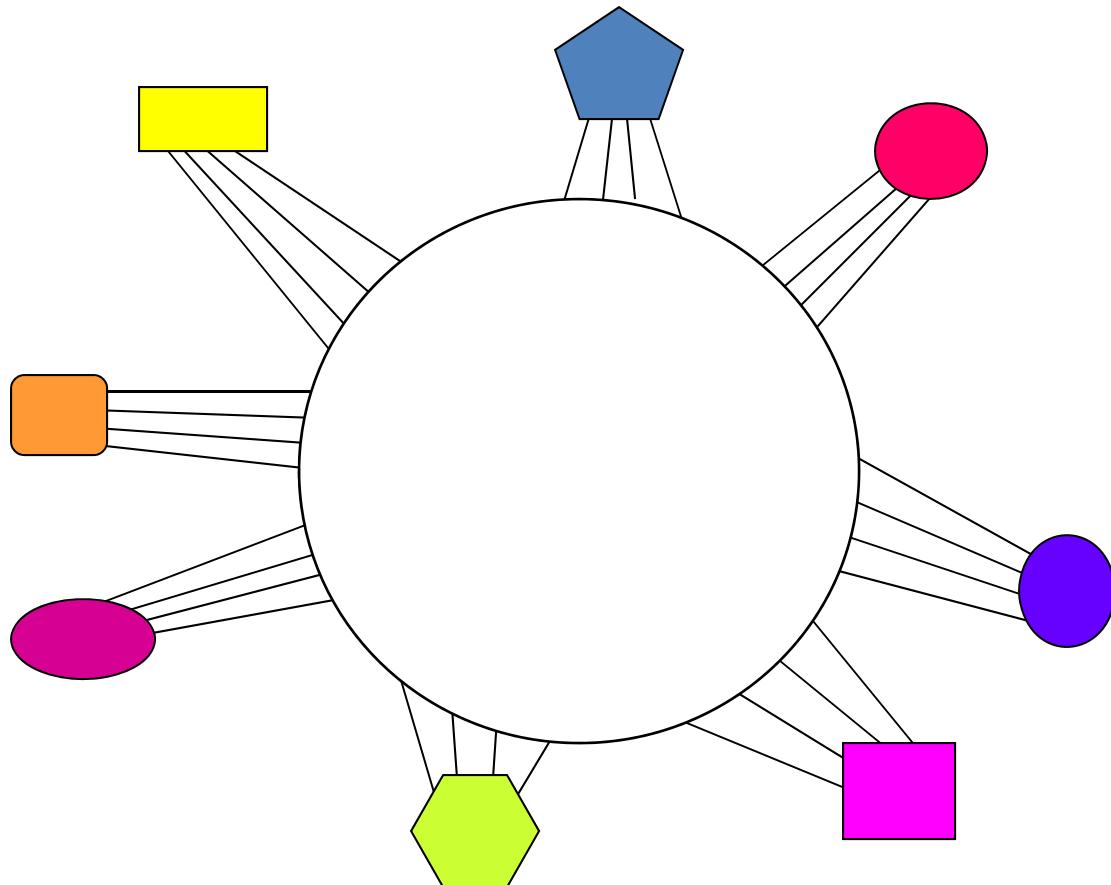
$$\begin{aligned} p &\leq 1 + d + d(d - 1) + \dots + d(d - 1)^{D-1} \\ &= 1 + d[(d - 1)^D - 1]/(d - 2) \end{aligned}$$

Only ring with odd  $p$  and a few other networks match this bound

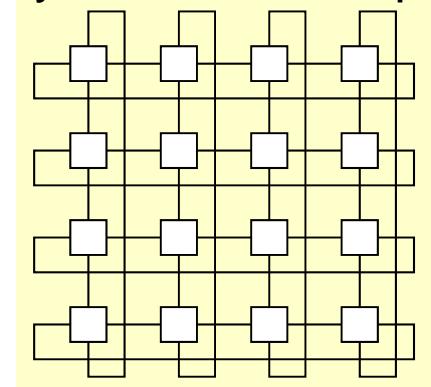


# Symmetric Network

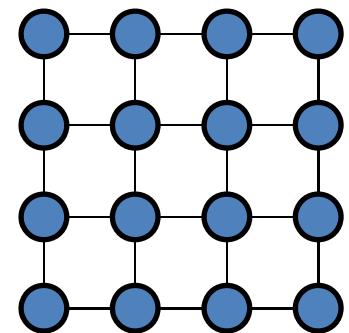
Viewed from any node, it looks the same



Symmetric example

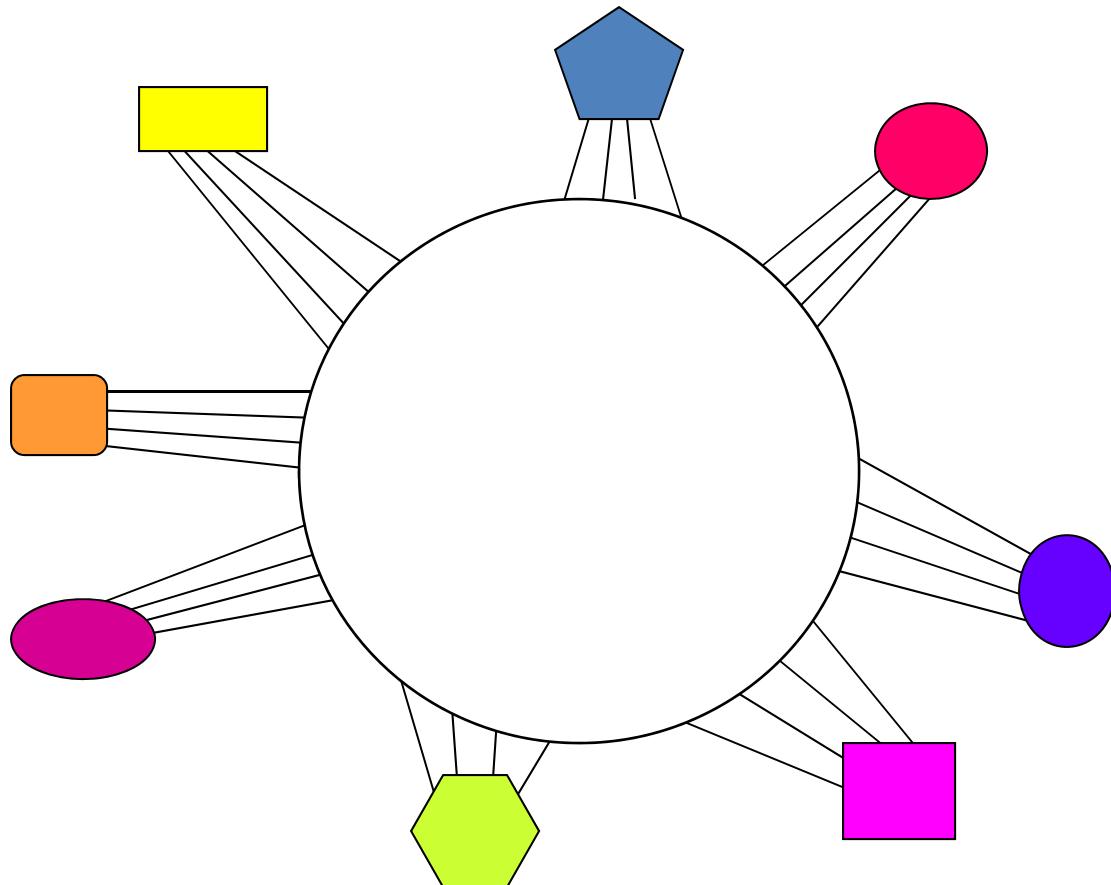


Asymmetric example



# Implications of Symmetry for Networks

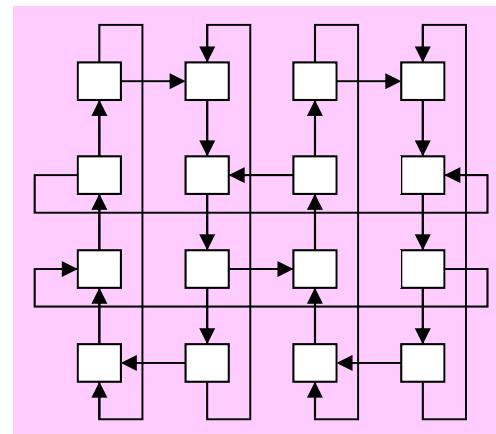
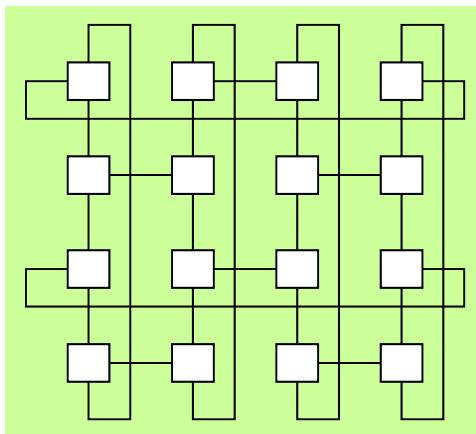
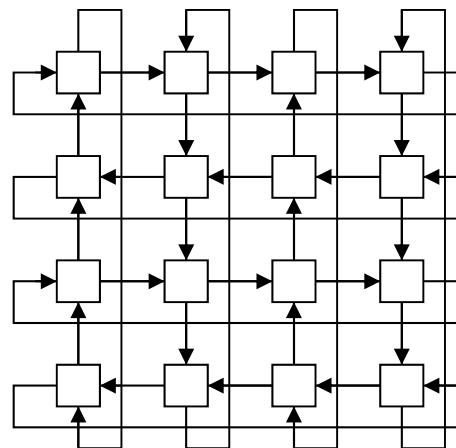
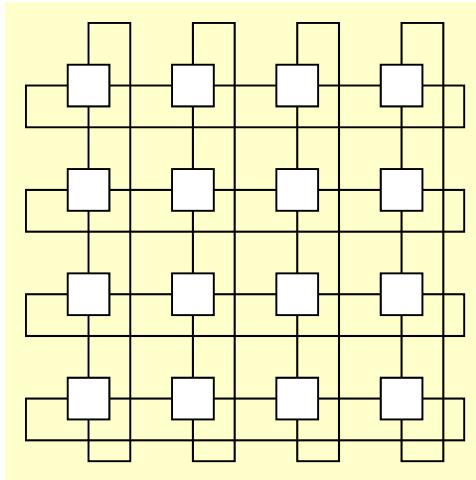
A degree-4 network



- Routing algorithm the same for every node
- No weak spots (critical nodes or links)
- Maximum number of alternate paths feasible
- Derivation and proof of properties easier

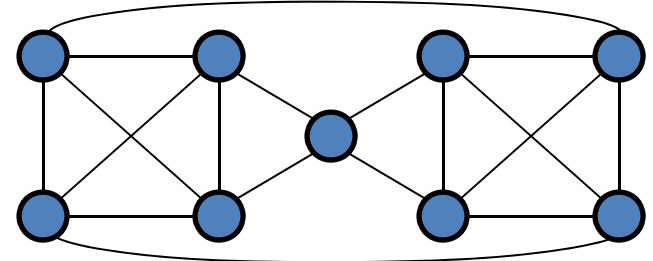
We need to prove a particular topological or routing property for only one node

# A Necessity for Symmetry



Uniform node degree:  
 $d = 4; d_{\text{in}} = d_{\text{out}} = 2$

An asymmetric network  
With uniform node degree

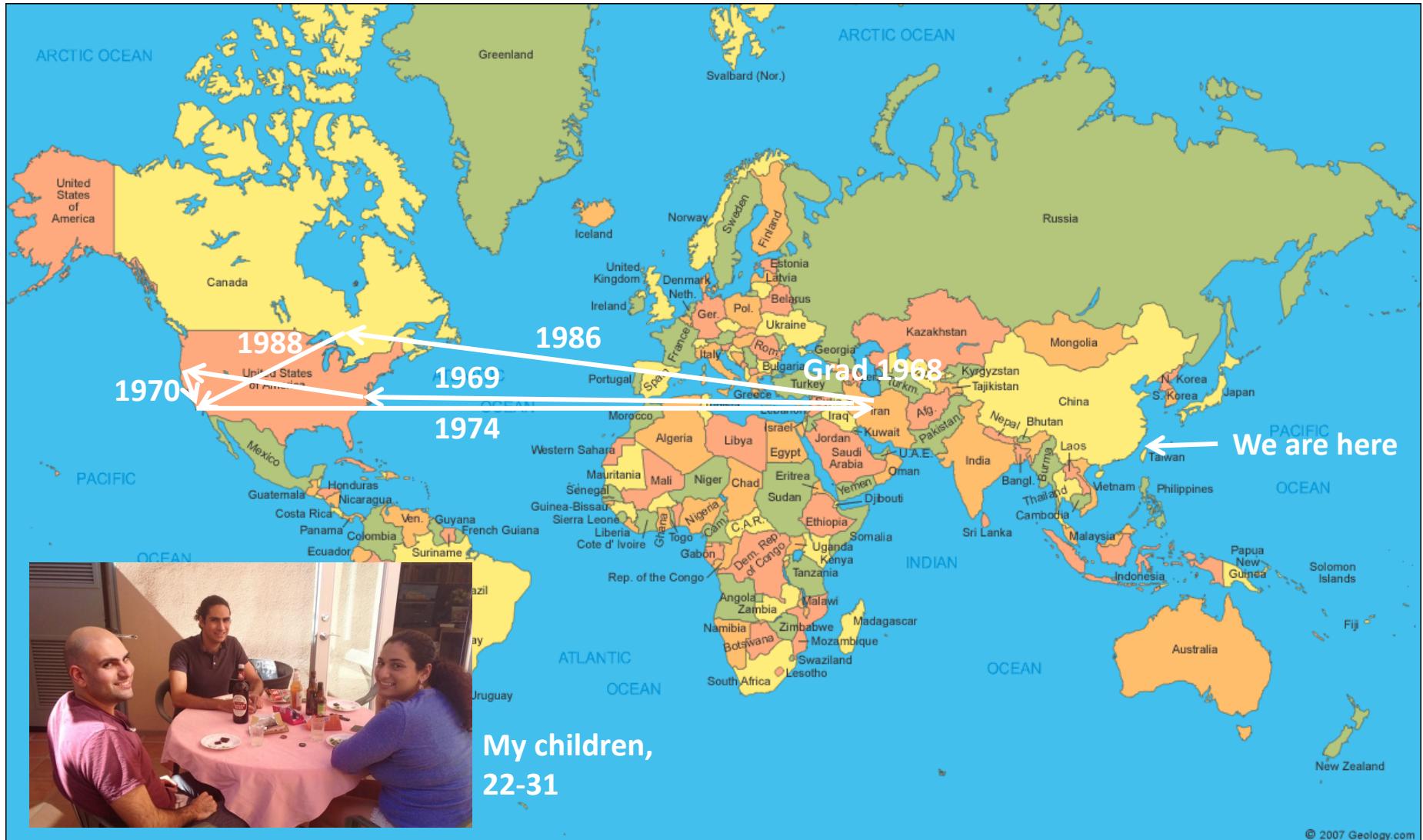


Uniform node degree  
is necessary but not  
sufficient for symmetry

# Interconnection Network Research

- Topologies for connecting processing nodes  
**Devising and assessing new interconnection schemes**
- Routing algorithms and their performance  
**Oblivious / adaptive routing, deadlock avoidance/recovery**
- Layout and packaging of networks  
**Routing of links within / between chips, boards, cabinets**
- Robustness of interconnection networks  
**Reconfiguration capabilities and fault-tolerant routing**
- Networks-on-chip (NoC)  
**Optimal interconnection strategies for systems-on-chip**
- Data-center communication networks  
**Optimized for data-center traffic and energy efficiency**

# My Personal Research History



# The Challenge of Comparing Networks



Liszka et al.: Is an alligator

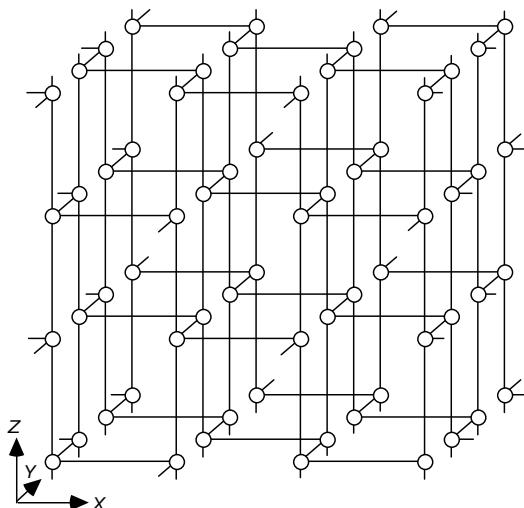
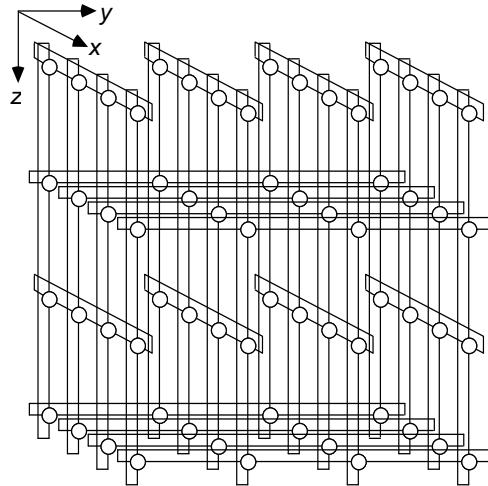


better than an armadillo?

# My Previous Work on Network Families

Systematic pruning

IPL, 1998

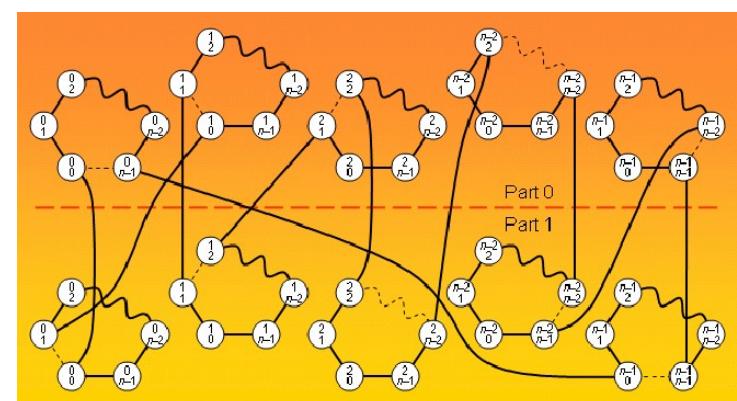
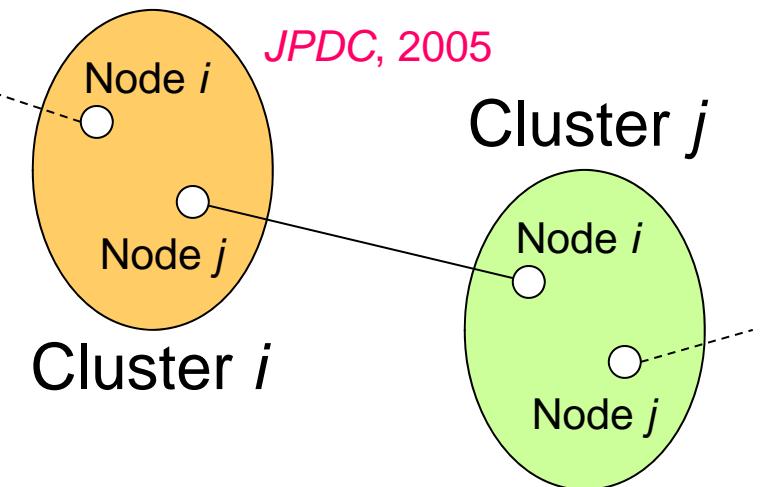


IEEE TPDS,  
2001

Int'l J Comp Math,  
2011

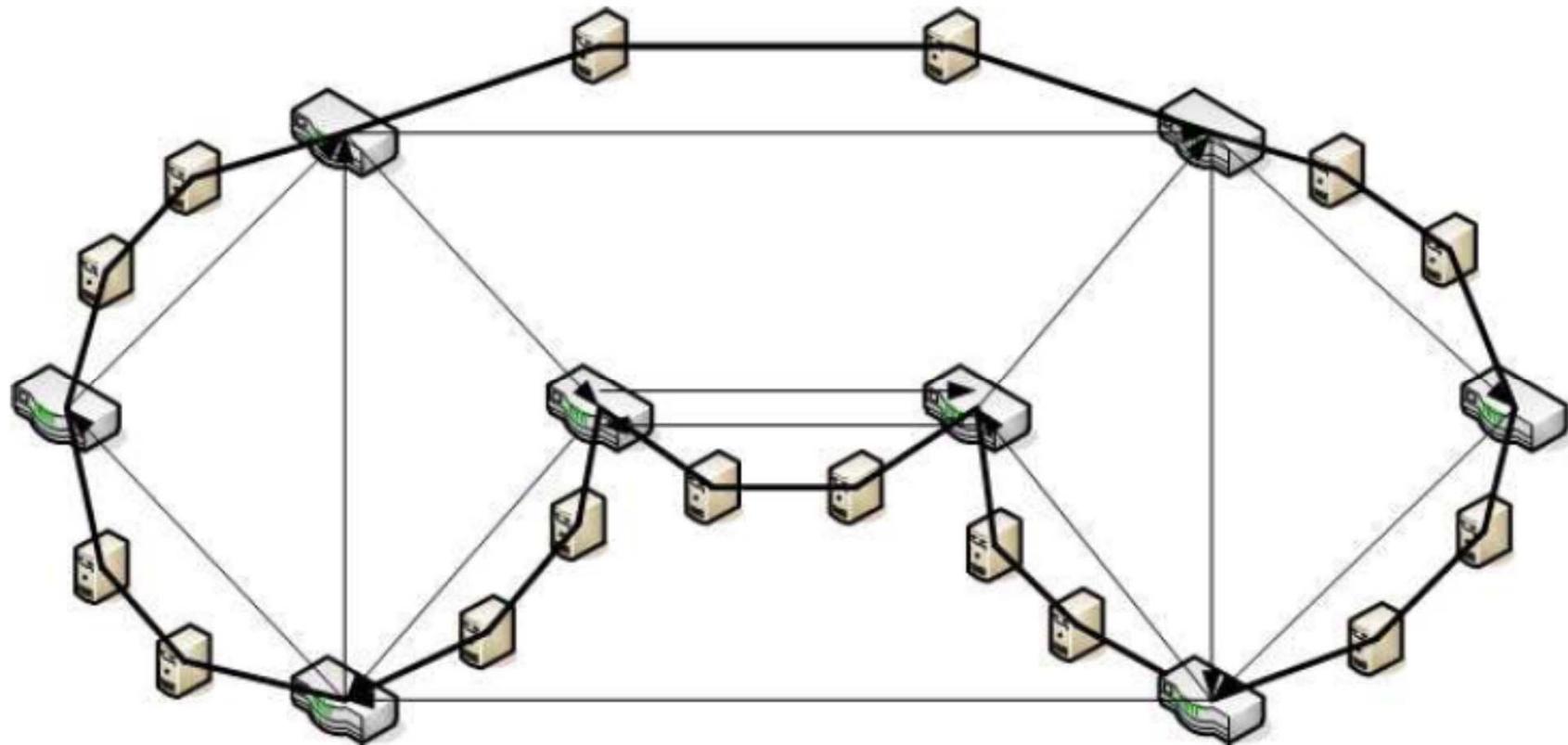
Swapped/OTIS networks  
Biswapped networks

JPDC, 2005



# My Previous Work on Dilated Networks

Dilation along a Hamiltonian path of a de Bruijn network  
(Xiao, Liang, Parhami; *IPL*, 2012)

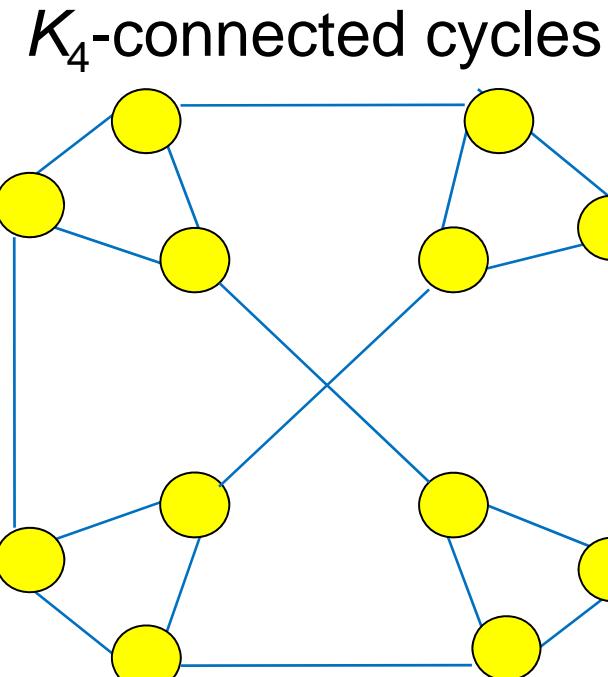
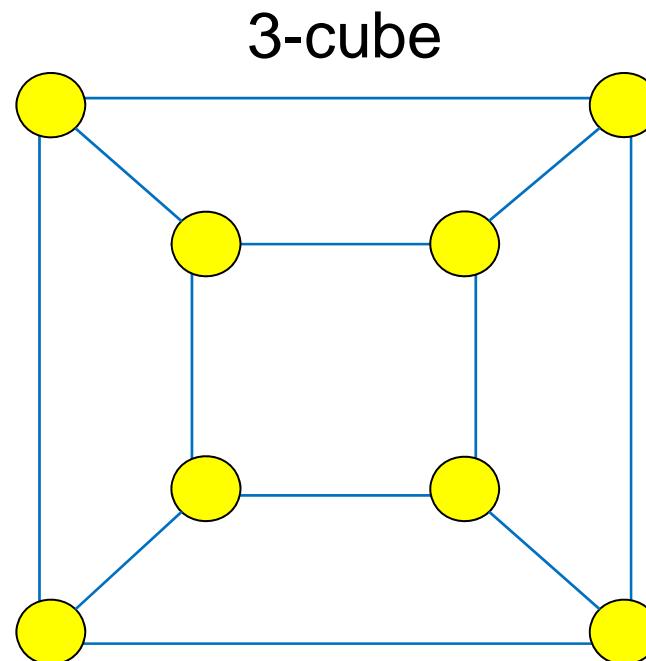


# Switch Networks Used in Examples

Small example networks to illustrate the concepts

3D hypercube = 3-cube (8 nodes,  $d = 3$ ,  $D = 3$ )

$K_4$ -connected cycles (12 nodes,  $d = 3$ ,  $D = 3$ )



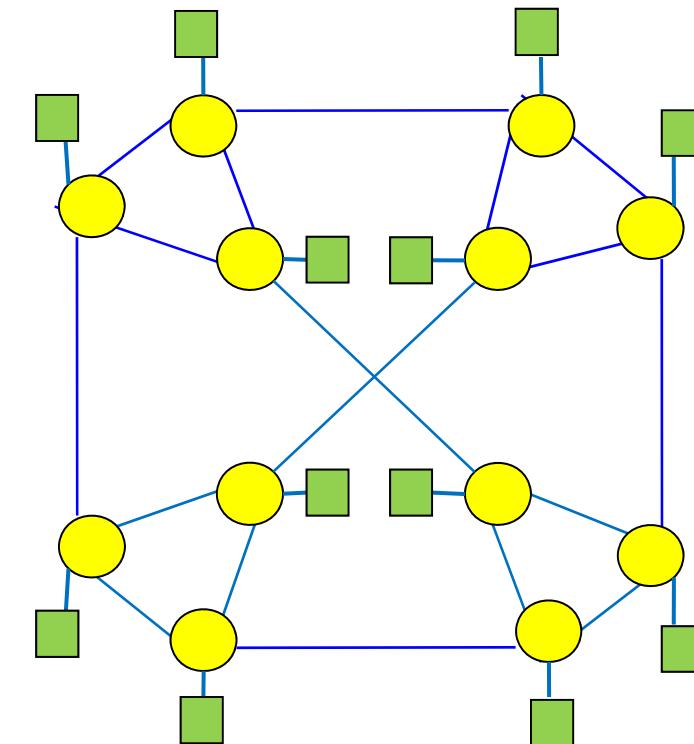
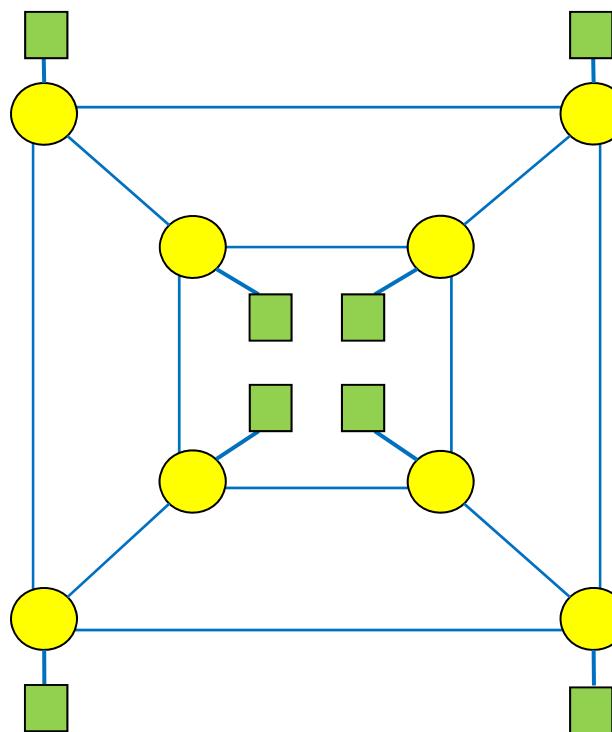
# Simplest Parallel Architectures

One processing node per switch/router node

$D = 2 + \text{switch network diameter}$

$d = 1 + \text{switch network degree}$

Degree-1 processing nodes



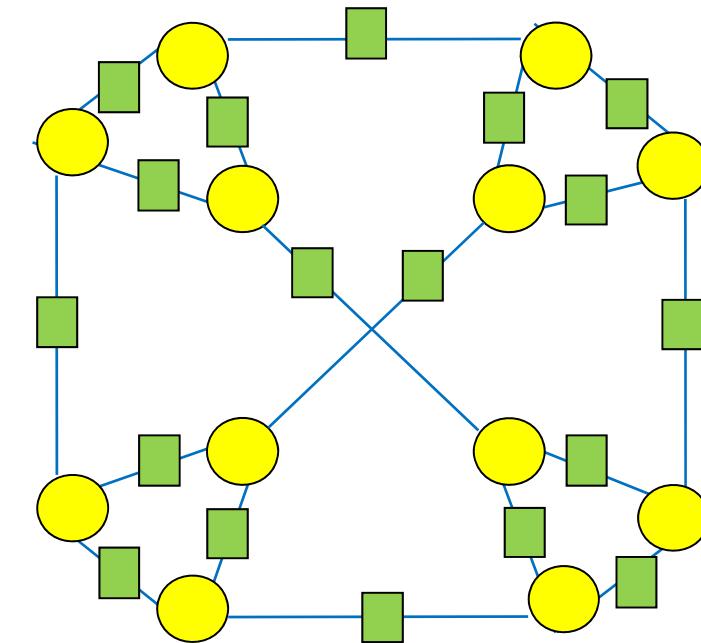
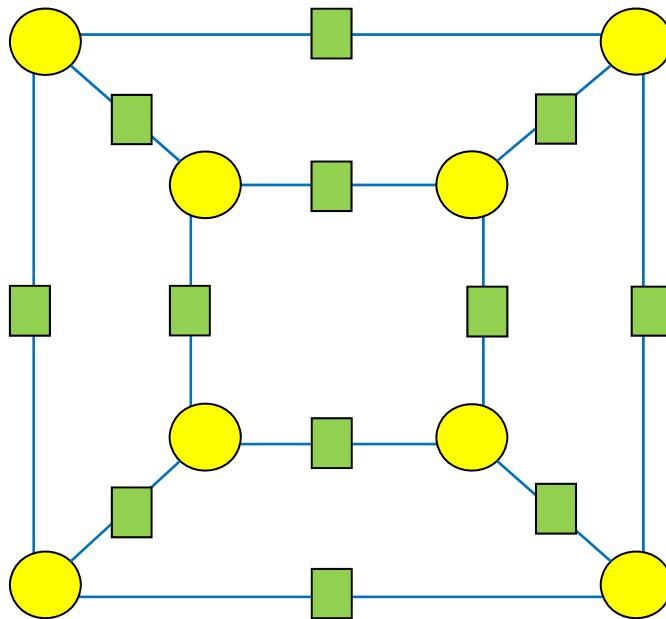
# Alternative Parallel Architectures

One processing node per switch/router link

$D \approx 2 \times$  switch network diameter

$d$  = switch network degree

Degree-2 processing nodes



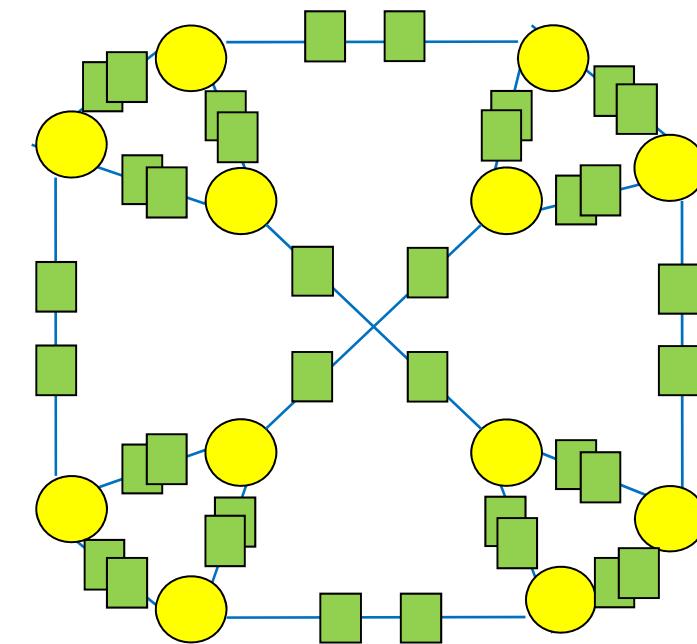
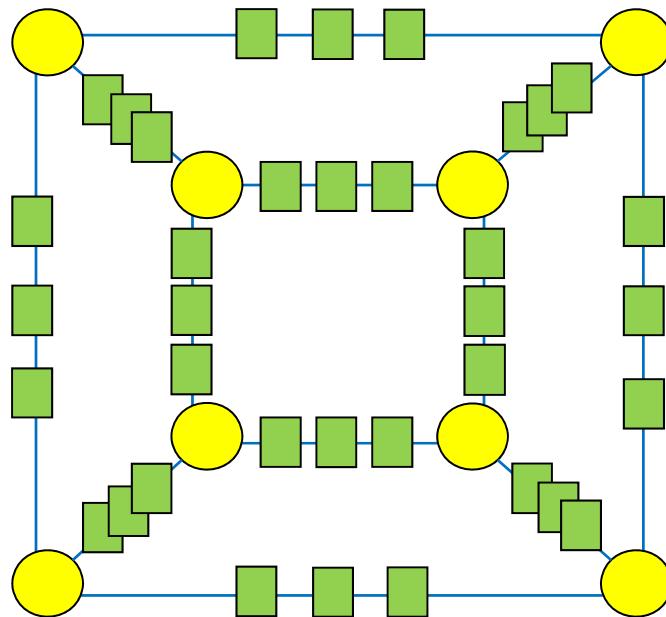
# 3- and 2-Dilated Network Examples

$k$  processing nodes per switch/router link

$$D \approx (k + 1) \times \text{switch network diameter}$$

$d$  = switch network degree

Degree-2 processing nodes



# Diameter of Dilated Networks

The diameter of a  $k$ -dilated network based on a diameter- $D_s$  switch network is bounded as  $(k + 1)D_s \leq D \leq (k + 1)D_s + k$ . Both bounds are tight, in the sense of equality being possible on both sides for suitably chosen networks.

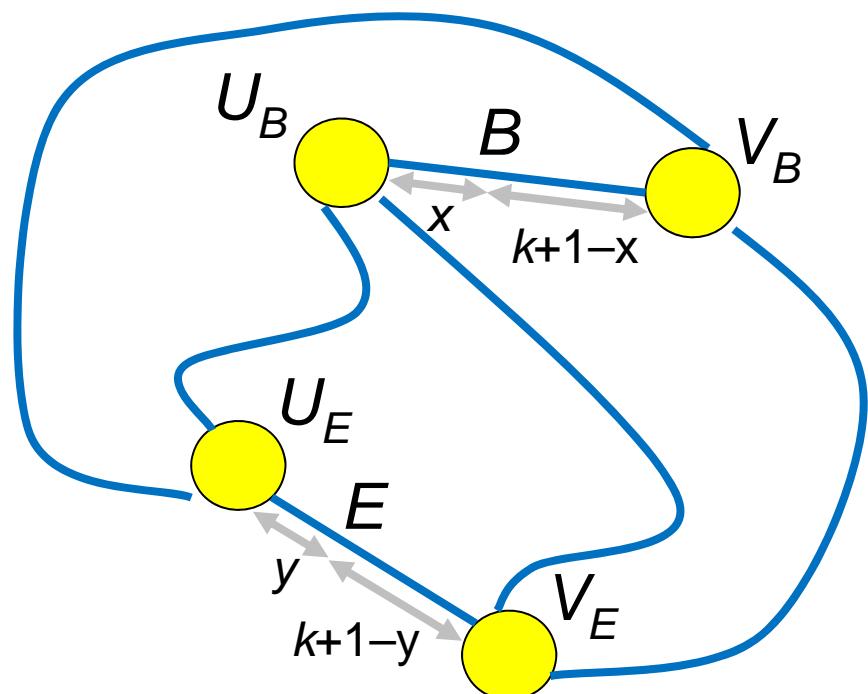
Worst case:

All four  $U_B \rightarrow U_E$ ,  $U_B \rightarrow V_E$ ,  $V_B \rightarrow U_E$ ,  $V_B \rightarrow V_E$  paths are diametral

Best case:

There is a non-diametral switch path (which can be at most one hop shorter than  $D_s$ )

Proof details in my forthcoming  
*Scientia Iranica* paper



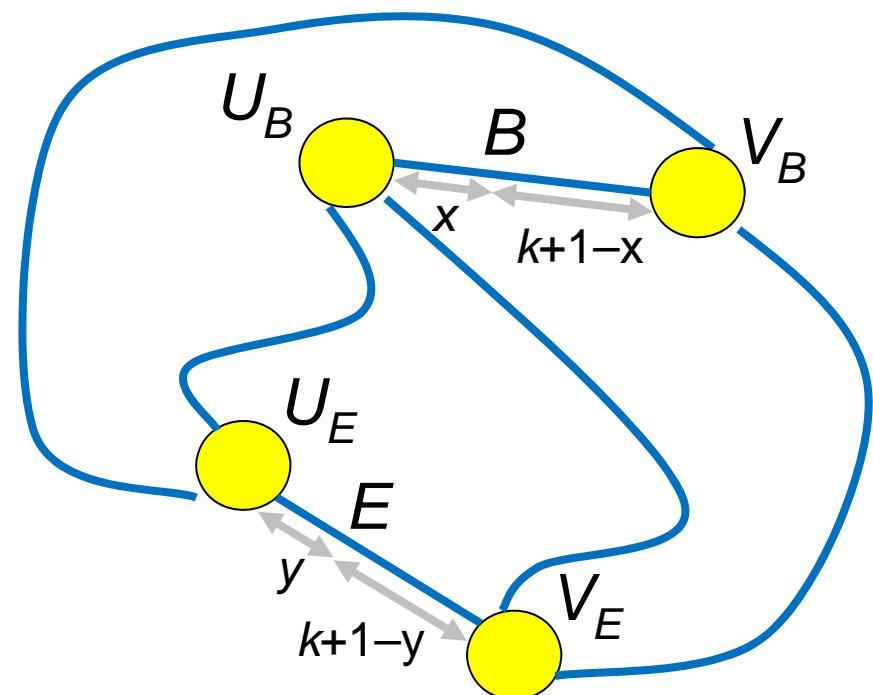
# Average Distance and Bisection Width

The average internode distance of a  $k$ -dilated network based on a switch network with average internode hop distance  $\Delta_s$  is  $\Delta = (k + 1)\Delta_s + k/2 + 1 + (k \bmod 2)/(2k)$ .

Proof details in my forthcoming *Scientia Iranica* paper

The bisection (band)width  $B$  of a dilated network remains the same as the bisection  $B_s$  of the switch network used

Proof details in my forthcoming *Scientia Iranica* paper



# Aggregate Bandwidth and Its Scalability

Network bisection  $B = B_s$  shows lack of scalability

So, unless traffic is mostly local, performance will suffer

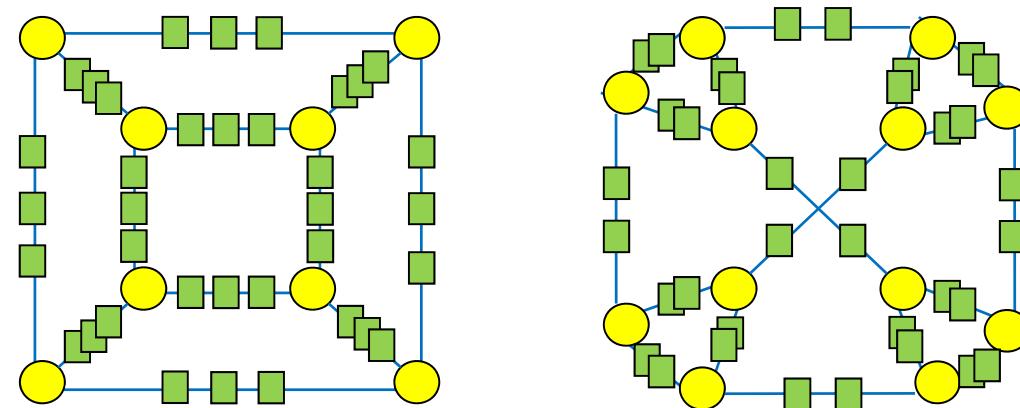
Aggregate bandwidth

$$B_{\text{agg}} = (k + 1)ndb$$

[ $b$  is link bandwidth]

BW scalability ratio

$$\text{BSR} = B_{\text{agg}}/\Delta \approx ndb/\Delta_s$$



BSR is sublinear in the number  $knd/2$  of nodes

For square torus of the same size:  $\text{BSR} = 8(knd/2)^{1/2}b$

For hypercube of the same size:  $\text{BSR} = kndb/2$

# Connectivity and Robustness

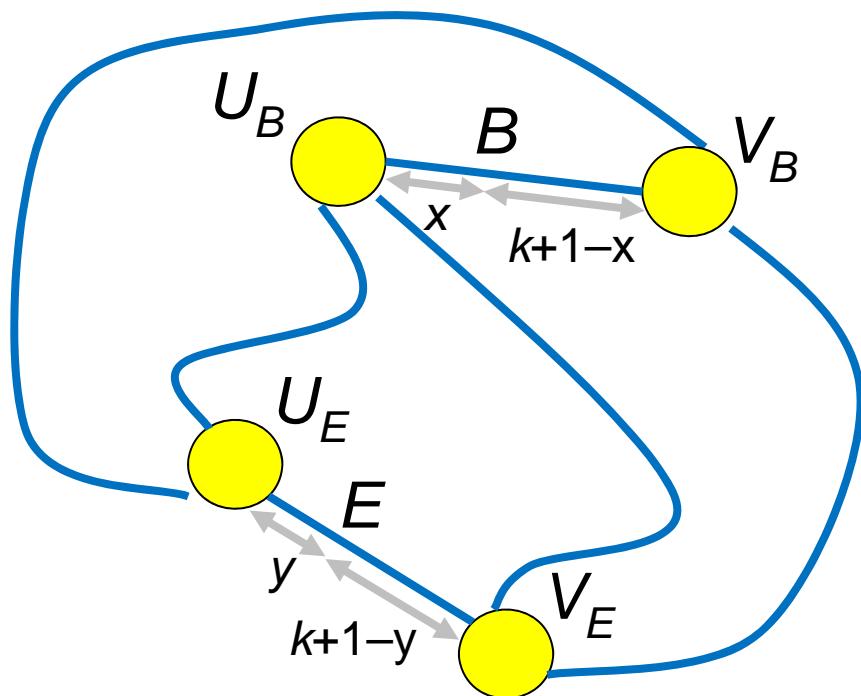
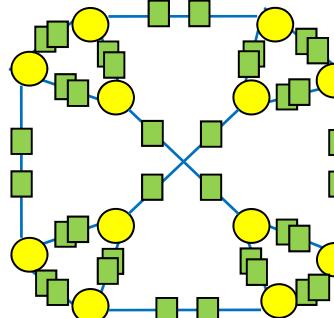
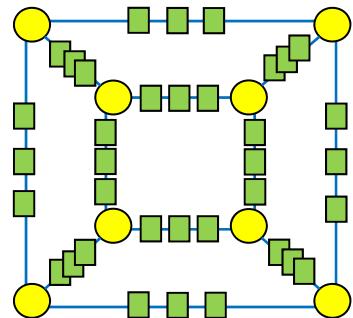
Processing node degree of 2 precludes a connectivity  $> 2$

Connectivity of 2 can be achieved with many switch networks

All we need is for 2 of the 4 paths below to be node-disjoint

Fault diameter  $\leq D + 2$

Wide diameter  $\leq D + 2$



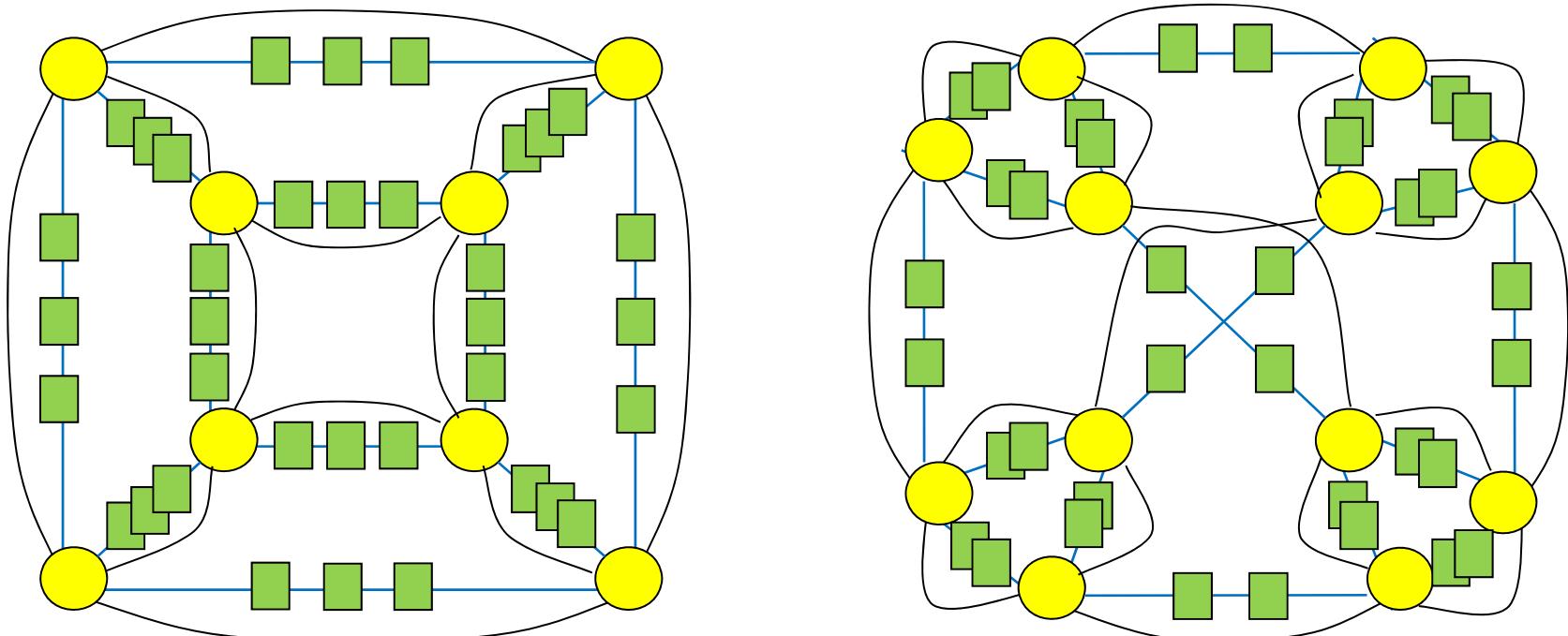
# Superimposed Direct & Dilated Networks

$k$  processing nodes per some switch/router links

$D \approx k + \text{switch network diameter}$

$d = 2 \times \text{switch network degree}$

Degree-2 processing nodes



# Conclusions and Future Work

- A strategy for building families of networks
  - Variation in network size with same switch network
  - Same node architecture and routing used throughout
  - Applicable to many existing or proposed networks
- More network-independent / specific results
  - Improve, assess, and fine-tune the architectures
  - Use simulation to evaluate with realistic workloads
  - Derive scalability bounds, given performance goals
  - Which networks are better for use with dilation?
  - Full, partial, and hybrid schemes for network dilation

# Questions or Comments?

[parhami@ece.ucsb.edu](mailto:parhami@ece.ucsb.edu)

<http://www.ece.ucsb.edu/~parhami/>

