The standard model

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Particles of the standard model

	(T,T^3)	Y
ν_Ls	$\left(\frac{1}{2},+\frac{1}{2}\right)$	$-\frac{1}{2}$
e_L, μ_L, τ_L	$\left(\frac{1}{2},-\frac{1}{2}\right)$	$-\frac{1}{2}$
e_R, μ_R, τ_R	(0,0)	-1
u_L, c_L, t_L	$\left(\frac{1}{2},+\frac{1}{2}\right)$	$\frac{1}{6}$
d_L, s_L, t_L	$\left(\frac{1}{2},-\frac{1}{2}\right)$	$\frac{1}{6}$
u_R, c_R, t_R	(0,0)	$\frac{2}{3}$
d_R, s_R, b_R	(0,0)	$-\frac{1}{3}$
h	$(\frac{1}{2},-\frac{1}{2})$	$\frac{1}{2}$
W^+	$(1,+1)$	0
W^-	$(1,-1)$	0
Z^0	(–, 0)	0
γ	(–, 0)	0

Generations		
e 0.5×10^{-3} GeV	μ 0.106 GeV	τ 1.77 GeV
u 3×10^{-3} GeV	c 1.3 GeV	t 171 GeV
d 6×10^{-3} GeV	s 0.1 GeV	b 4.2 GeV

$g(M_Z)$	0.6297
$g'(M_Z)$	0.3434
$\sin^2\theta_w(M_Z)$	0.231
m_W	80.4 GeV
m_Z	91.2 GeV
m_H	>114 GeV
v	246 GeV

Note that the gauge fields Z^0 and γ are mixtures of $SU(2)_L$ and $U(1)_Y$ generators, and are a mixture of the trivial and fundamental representation of the weak charge. Both belong to the $T^3=0$. This is analogous to how $|\uparrow\downarrow\rangle$ has $m_z=0$ but is a combination of $\vec{S}=1$ and $\vec{S}=0$ representations. The electric charge is defined by:

$$\boxed{Q=T^3+Y}$$

-U^i stands for $\{u,c,t\}$ -D^i stands for $\{d,s,b\}$ -Q is any quark
-f is any fermion - L is any Lepton -$\{A^1,A^2,A^3\}$ are the $SU(2)_L$ generators
-B is the $U(1)_Y$ generator -X_L, X_R mean left- and right-handed parts of X

The Lagrangian

$$\mathcal{L}_{\text{SM}}=\mathcal{L}_{\text{gauge, kin}}+\mathcal{L}_{\text{fermion, kin}}+\mathcal{L}_{\text{Yukawa}}+\mathcal{L}_{\text{Higgs}} \tag{1}$$

The gauge fields

The gauge fields associated with the $SU(2)_L$ transformation in standard form are A^i_μ, i=1, 2 or 3 when the commutation relations for the associated generators T^i are $[T^i,T^j]=i\varepsilon_{ijk}T^k$. The gauge field for the $U(1)_Y$ field is B_μ. Rewriting in terms of the physical gauge fields we have for the $W^\pm$:

$$W^\pm=\frac{1}{\sqrt{2}}\left(A^1\pm iA^2\right)$$

The gauge fields Z^0 and γ are mixtures of A^3 and B.

$$\begin{pmatrix}Z^0\\A\end{pmatrix}=\begin{pmatrix}\cos\theta_w & -\sin\theta_w\\ \sin\theta_w & \cos\theta_w\end{pmatrix}\begin{pmatrix}A^3\\B\end{pmatrix}$$

Define the field strength tensors $A_{\mu\nu}$ for $SU(2)_L$, $B_{\mu\nu}$ for the U(1) gauge fields, and $G_{\mu\nu}$ for the $SU(3)_{\text{color}}$ as follows:

$$A^a_{\mu\nu}=2\partial_{[\mu}A^a_{\nu]}+ig\varepsilon^{abc}A^b_\mu A^c_\nu,\qquad G^a_{\mu\nu}=2\partial_{[\mu}G^a_{\nu]}+ig_s f^{abc}G^b_\mu G^c_\nu,\qquad B_{\mu\nu}=2\partial_{[\mu}B_{\nu]}$$

The gauge piece of the Lagrangian is given by

$$\mathcal{L}_{\text{gauge}}=\frac{1}{2g^2}\text{Tr}(A^a_{\mu\nu}T^a)^2+\frac{1}{2g_s^2}\text{Tr}(G^a_{\mu\nu}\lambda^a)^2+\frac{1}{2g'^2}B_{\mu\nu}B^{\mu\nu}$$

The fermion kinetic term

The kinetic term for the Lagrangian can be simply written in terms of the covariant derivative:

$$\mathcal{L}_{\text{fermion, kinetic}}=\sum_f\bar{f}i\not{D}f \tag{2}$$

where the gauge-covariant derivative acting on fermion fields is given by

$$\begin{aligned}D_\mu&=\partial_\mu-igA^a_\mu T^a-ig'YB_\mu\\&=\partial_\mu-ig(W^+_\mu T^++W^-_\mu T^-)-i\frac{1}{\sqrt{g^2+(g')^2}}Z_\mu(g^2T^3-g'^2Y)-i\frac{gg'}{\sqrt{g^2+g'^2}}A_\mu(T^3+Y).\end{aligned}$$

Using the standard definitions

$$\cos\theta_w=\frac{g}{\sqrt{g^2+g'^2}},\qquad \sin\theta_w=\frac{g'}{\sqrt{g^2+g'^2}},\qquad e=\frac{gg'}{\sqrt{g^2+g'^2}}$$

allows us to write the covariant derivitave as

$$D_\mu=\partial_\mu-ig(W^+_\mu T^++W^-_\mu T^-)-i\frac{g}{\cos\theta_w}Z_\mu(T^3-Q\sin^2\theta_w)-ieA_\mu Q.$$

Replacing the gauge-covariant derivatives with gauge fields, we can more transparently see the interactions in the fermion sector:

$$\mathcal{L}_{\text{fermion, kinetic}}=\sum_f\bar{f}i\not{D}f=\left(\sum_f\bar{f}i\not{\partial}f\right)+g(W^+_\mu J^\mu_{W^+}+W^-_\mu J^\mu_{W^-}+Z^0_\mu J^\mu_{Z^0})+eA_\mu J^\mu_\gamma$$

where the pure $SU(2)_L$ currents are defined by

$$J^\mu_{W^+}=\frac{1}{\sqrt{2}}\left(\bar{\nu}^i_L\gamma^\mu L^i_L+\bar{U}^i_L\gamma^\mu V_{ij}D^j_L\right)$$
$$J^\mu_{W^-}=\frac{1}{\sqrt{2}}\left(\bar{L}^i_L\gamma^\mu\nu^i_L+\bar{D}^i_L\gamma^\mu V^\dagger_{ij}U^j_L\right)$$

(V is defined in CKM matrix) and the mixed $SU(2)_L$ and $U(1)_Y$ currents are

$$J^\mu_{Z^0}=\frac{1}{\cos\theta_w}\left(\sum_{\text{fermions}}\bar{f}_L\gamma^\mu(T^3_{f_L}-Q_f\sin^2\theta_w)f_L+\bar{f}_R(0-Q_f\sin^2\theta_w)f_R\right)$$
$$J^\mu_\gamma=\sum_{\text{fermions}}Q_f\bar{f}\gamma^\mu f=-\bar{e}\gamma^\mu e+\frac{2}{3}\bar{U}\gamma^\mu U-\frac{1}{3}\bar{D}\gamma^\mu D$$

Yukawa couplings

The coupling of the fermions and the Higgs take the form

$$\mathcal{L}_{\text{Yukawa}}=-\sum_{\text{Leptons}}\lambda_{L^i}(\bar{L}^i_L\cdot\Phi)L^i_R+\text{h.c.}+\sum_{\text{Gens }i}\left(-\lambda_{D_i}(\bar{Q}^i_L\cdot\Phi)D^i_R-\lambda_{U_i}\epsilon^{ab}\bar{Q}_{La}\Phi^\dagger_b u_R+\text{h.c.}\right).$$

Expanding out the Higgs field gives us the mass terms and the Higgs-fermion interaction terms:

$$\begin{aligned}\sum_{\text{Leptons}}\lambda_{L^i}(\bar{L}^i_L\cdot\Phi)L^i_R&=\frac{1}{\sqrt{2}}\lambda_e\bar{e}_L(v+h)e_R+\frac{1}{\sqrt{2}}\lambda_\mu\bar{\mu}_L(v+h)\mu_R+\frac{1}{\sqrt{2}}\lambda_\tau\bar{\tau}_L(v+h)\tau_R\\&=m_e\bar{e}_L\left(1+\frac{h}{v}\right)e_R+m_\mu\bar{\mu}_L\left(1+\frac{h}{v}\right)\mu_R+m_\tau\bar{\tau}_L\left(1+\frac{h}{v}\right)\tau_R\\ \sum_{\text{Gens }i}\lambda_{D_i}(\bar{Q}^i_L\cdot\Phi)D^i_R&=\frac{v}{\sqrt{2}}\lambda_d\bar{d}_Ld_R\left(1+\frac{h}{v}\right)+\frac{v}{\sqrt{2}}\lambda_s\bar{s}_Ls_R\left(1+\frac{h}{v}\right)+\frac{v}{\sqrt{2}}\lambda_b\bar{b}_Lb_R\left(1+\frac{h}{v}\right)\\&=m_d\bar{d}_Ld_R\left(1+\frac{h}{v}\right)+m_s\bar{s}_Ls_R\left(1+\frac{h}{v}\right)+m_b\bar{b}_Lb_R\left(1+\frac{h}{v}\right)\end{aligned}$$

with similar results for the up type quarks. Note that

$$m_f\equiv\frac{v}{\sqrt{2}}\lambda_f$$

The Higgs sector

The simple Higgs model uses the standard Φ^4 potential, where Φ is the Higgs $SU(2)_L$ doublet:

$$\mathcal{L}_{\text{Higgs}}=|D_\mu\Phi|^2+\mu^2|\Phi|^2-\lambda|\Phi|^4.$$

The symmetry is broken by choosing

$$\Phi=\frac{U(x)}{\sqrt{2}}\begin{pmatrix}0\\v+h(x)\end{pmatrix}.$$

$U(x)$ is a gauge degree of freedom (the Goldstone Boson) and is eliminated in the unitary gauge. The only physical degree of freedom is $h(x)$ which has $(T,T^3)=(\frac{1}{2},-\frac{1}{2})$ and (setting $Q=0$) has hypercharge $Y=\frac{1}{2}$.

These quantum numbers imply (in the unitary gauge)

$$\mathcal{L}_{\text{Higgs}}=\frac{1}{2}\partial_\mu h\partial^\mu h+m_W^2\left(1+\frac{h}{v}\right)^2W^-_\mu W^{+\mu}+\frac{1}{2}m_Z^2\left(1+\frac{h}{v}\right)^2Z_\mu Z^\mu-\frac{1}{2}m_h^2h^2-\frac{m_h^2}{2v}h^3-\frac{1}{8}\left(\frac{m_h}{v}\right)^2h$$

where we have eliminated μ and λ in favour of the Higgs mass m_H and the Higgs VEV via the relationships

$$m_H=\sqrt{2}\mu,\qquad v=\sqrt{\frac{\mu^2}{\lambda}},\qquad m_W=g\frac{v}{2},\qquad m_Z=\frac{v}{2}\sqrt{g^2+g'^2}.$$

CKM matrix

$$V_{\text{CKM}}=\begin{pmatrix}c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta_{13}}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{23}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13}\end{pmatrix},\qquad c_{ij}=\cos\theta_{ij},\quad s_{ij}=\sin\theta_{ij}$$

Experimentally, the best determination has been

Quantity	$\sin\theta_{12}$	$\sin\theta_{13}$	$\sin\theta_{23}$	δ_{13}
Value	0.2243 ± 0.0016	0.0037 ± 0.0005	0.0413 ± 0.0015	$60^\circ\pm14^\circ$

Quark phases are all defined so all angles lie in the first quadrant. The above is sufficient for obtaining the cosines as well.

Feynman diagrams