$$E = f^T A f - 2 f^T b + c$$

$$f = (a_1^n, b_1^n, c_1^n, \ldots a_m^s, b_m^s, c_m^s)^T$$

$$E = E_d + E_s + E_c$$

1 E_d

$$E_d(f_n, f_s) = \sum_{j=1}^{m} \sum_{i \in N} (a_j^n x_i^2 + b_j^n x_i + c_j^n - I_{i,j})^2 + \ldots$$

$$(a_j x_i^2 + b_j x_i + c_j - I_{i,j})^2 = a_j^2 x_i^4 + 2 a_j b_j x_i^3 + 2 a_j c_j x_i^2 - 2 a_j I_{i,j} x_i^2 + b_j^2 x_i^2 + 2 b_j c_j x_i - 2 b_j I_{i,j} x_i + c_j^2 - 2 c_j I_{i,j} + I_{i,j}^2$$

1.1 Quadratic in parameters f

$$\sum_{j=1}^{m} \sum_{i \in N} a_j^2 x_i^4 + 2 a_j b_j x_i^3 + 2 a_j c_j x_i^2 + b_j^2 x_i^2 + 2 b_j c_j x_i + c_j^2$$

Für $j = 1, 2$, non-shadow: $A = \sum_{i \in N} \begin{pmatrix} x_i^4 & 0 & 0 & 0 & 0 & 0 \\ 2x_i^3 & x_i^2 & 0 & 0 & 0 & 0 \\ 2x_i^2 & 2x_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i^4 & 0 & 0 \\ 0 & 0 & 0 & 2x_i^3 & x_i^2 & 0 \\ 0 & 0 & 0 & 2x_i^2 & 2x_i & 1 \end{pmatrix}$

1.2 Linear in parameters f

$$\sum_{j=1}^{m} \sum_{i \in N} -2 a_j I_{i,j} x_i^2 - 2 b_j I_{i,j} x_i - 2 c_j I_{i,j}$$

Für $j = 1, 2$, non-shadow: $b = \sum_{i \in N} \begin{pmatrix} I_{i,1} x_i^2 \\ I_{i,1} x_i \\ I_{i,1} \\ I_{i,2} x_i^2 \\ I_{i,2} x_i \\ I_{i,2} \end{pmatrix}$

2 E_s

$$E_s(f_n, f_s) = \sum_{j=1}^{m} \sum_{i} \left(\frac{\partial^2 f_j(x_i)}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 f_j(x_i)}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 f_j(x_i)}{\partial y^2}\right)^2 + \ldots$$

$$\left(\frac{\partial^2 f_j(x_i)}{\partial x^2}\right)^2 = (2 a_j)^2 = 4 a_j^2$$

$$\frac{\partial f_j(x_i)}{\partial x} = 2 a_j x_i + b_j$$

$$\left(\frac{\partial^2 f_j(x_i)}{\partial x \partial y}\right)^2 = \left(\frac{1}{2}\left(\frac{\partial f_{j+1}(x_i)}{\partial x} - \frac{\partial f_{j-1}(x_i)}{\partial x}\right)\right)^2 = \frac{1}{4}(2 a_{j+1} x_i + b_{j+1} - 2 a_{j-1} x_i - b_{j-1})^2$$

$$\left(\frac{\partial^2 f_j(x_i)}{\partial y^2}\right)^2 = (f_{j+1}(x_i) - 2 f_j(x_i) + f_{j-1}(x_i))^2 = (a_{j+1} x_i^2 + b_{j+1} x_i + c_{j+1} - 2 a_j x_i^2 - 2 b_j x_i - 2 c_j + a_{j-1} x_i^2 + b_{j-1} x_i + c_{j-1})^2$$

$$\sum_i (\frac{\partial^2 f_j(x_i)}{\partial x^2})^2 + 2(\frac{\partial^2 f_j(x_i)}{\partial x \partial y})^2 + (\frac{\partial^2 f_j(x_i)}{\partial y^2})^2$$

$$= \sum_i 2a_{j-1}b_{j-1}x_i^3 - 4a_jb_{j-1}x_i^3 + 2a_{j+1}b_{j-1}x_i^3 - 4a_{j-1}b_jx_i^3 + 8a_jb_jx_i^3 - 4a_{j+1}b_jx_i^3 + 2a_{j-1}b_{j+1}x_i^3 - 4a_jb_{j+1}x_i^3$$

$$+ 2a_{j+1}b_{j+1}x_i^3 + 2.a_{j-1}b_{j-1}x_i - 2.a_{j+1}b_{j-1}x_i - 2.a_{j-1}b_{j+1}x_i + 2.a_{j+1}b_{j+1}x_i + 2a_{j-1}c_{j-1}x_i^2 - 4a_jc_{j-1}x_i^2$$
$$+ 2a_{j+1}c_{j-1}x_i^2 - 4a_{j-1}c_jx_i^2 + 8a_jc_jx_i^2 - 4a_{j+1}c_jx_i^2 + 2a_{j-1}c_{j+1}x_i^2 - 4a_jc_{j+1}x_i^2 + 2a_{j+1}c_{j+1}x_i^2$$
$$+ a_{j-1}^2x_i^4 + 4a_j^2x_i^4 + a_{j+1}^2x_i^4 - 4a_{j-1}a_jx_i^4 + 2a_{j-1}a_{j+1}x_i^4 - 4a_ja_{j+1}x_i^4 + 2.a_{j-1}^2x_i^2 + 2.a_{j+1}^2x_i^2$$
$$- 4.a_{j-1}a_{j+1}x_i^2 + 4a_j^2 + 2b_{j-1}c_{j-1}x_i - 4b_jc_{j-1}x_i + 2b_{j+1}c_{j-1}x_i - 4b_{j-1}c_jx_i + 8b_jc_jx_i - 4b_{j+1}c_jx_i$$
$$+ 2b_{j-1}c_{j+1}x_i - 4b_jc_{j+1}x_i + 2b_{j+1}c_{j+1}x_i + b_{j-1}^2x_i^2 + 4b_j^2x_i^2 + b_{j+1}^2x_i^2 - 4b_{j-1}b_jx_i^2 + 2b_{j-1}b_{j+1}x_i^2$$
$$- 4b_jb_{j+1}x_i^2 + 0.5b_{j-1}^2 + 0.5b_{j+1}^2 - 1.b_{j-1}b_{j+1} + c_{j-1}^2 + 4c_j^2 + c_{j+1}^2 - 4c_{j-1}c_j + 2c_{j-1}c_{j+1} - 4c_jc_{j+1}$$

2.1 Quadratic in parameters f

$$A = \sum_i \begin{pmatrix} x_i^4 + 2x_i^2 & 2x_i^3 + 2x_i & 2x_i^2 & -4x_i^4 & -4x_i^3 & -4x_i^2 & 2x_i^4 - 4x_i^2 & 2x_i^3 - 2x_i & 2x_i^2 \\ 0 & x_i^2 + \frac{1}{2} & 2x_i & 0 & -4x_i^2 & -4x_i & 0 & 2x_i^2 - 1 & 2x_i \\ 0 & 0 & 1 & 0 & 0 & -4 & 0 & 0 & 2 \\ 0 & -4x_i^3 & -4x_i^2 & 4x_i^4 + 4 & 8x_i^3 & 8x_i^2 & -4x_i^4 & -4x_i^3 & -4x_i^2 \\ 0 & 0 & -4x_i & 0 & 4x_i^2 & 8x_i & 0 & -4x_i^2 & -4x_i \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & -4 \\ 0 & 2 - 2x_ix_i^3 & 2x_i^2 & 0 & -4x_i^3 & -4x_i^2 & x_i^4 + 2x_i^2 & 2x_i^3 + 2x_i & 2x_i^2 \\ 0 & 0 & 2x_i & 0 & 0 & -4x_i & 0 & x_i^2 + \frac{1}{2} & 2x_i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

3 E_c

$$E_c(f_n, f_s) = \sum_{j=1}^m \sum_i (\frac{\partial(f_{j,n}(x_i) - f_{j,s}(x_i))}{\partial x})^2 + (\frac{\partial(f_{j,n}(x_i) - f_{j,s}(x_i))}{\partial y})^2$$

$$(\frac{\partial(f_{j,n}(x_i) - f_{j,s}(x_i))}{\partial x})^2 = (2a_{j,n}x_i + b_{j,n} - 2a_{j,s}x_i - b_{j,s})^2$$

$$(\frac{\partial(f_{j,n}(x_i) - f_{j,s}(x_i))}{\partial y})^2 = (\frac{a_{j,n}x_i^2 + b_{j,n}x_i + c_{j,n} - a_{j,s}x_i^2 - b_{j,s}x_i - c_{j,s}}{\partial y})^2$$

$$= \frac{1}{4}(a_{j+1,n}x_i^2 + b_{j+1,n}x_i + c_{j+1,n} - a_{j+1,s}x_i^2 - b_{j+1,s}x_i - c_{j+1,s} - a_{j-1,n}x_i^2 - b_{j-1,n}x_i - c_{j-1,n} + a_{j-1,s}x_i^2 + b_{j-1,s}x_i + c_{j-1,s})^2$$

$$
\begin{aligned}
= {}& 0.25a_{j-1,n}^2 x_i^4 + 0.25a_{j-1,s}^2 x_i^4 + 0.25a_{j+1,n}^2 x_i^4 + 0.25a_{j+1,s}^2 x_i^4 - 0.5a_{j-1,n}a_{j-1,s}x_i^4 - 0.5a_{j-1,n}a_{j+1,n}x_i^4 \\
& + 0.5a_{j-1,s}a_{j+1,n}x_i^4 + 0.5a_{j-1,n}a_{j+1,s}x_i^4 - 0.5a_{j-1,s}a_{j+1,s}x_i^4 - 0.5a_{j+1,n}a_{j+1,s}x_i^4 + 0.5a_{j-1,n}b_{j-1,n}x_i^3 \\
& - 0.5a_{j-1,s}b_{j-1,n}x_i^3 - 0.5a_{j+1,n}b_{j-1,n}x_i^3 + 0.5a_{j+1,s}b_{j-1,n}x_i^3 - 0.5a_{j-1,n}b_{j-1,s}x_i^3 + 0.5a_{j-1,s}b_{j-1,s}x_i^3 \\
& + 0.5a_{j+1,n}b_{j-1,s}x_i^3 - 0.5a_{j+1,s}b_{j-1,s}x_i^3 - 0.5a_{j-1,n}b_{j+1,n}x_i^3 + 0.5a_{j-1,s}b_{j+1,n}x_i^3 + 0.5a_{j+1,n}b_{j+1,n}x_i^3 \\
& - 0.5a_{j+1,s}b_{j+1,n}x_i^3 + 0.5a_{j-1,n}b_{j+1,s}x_i^3 - 0.5a_{j-1,s}b_{j+1,s}x_i^3 - 0.5a_{j+1,n}b_{j+1,s}x_i^3 + 0.5a_{j+1,s}b_{j+1,s}x_i^3 \\
& + 4a_{j,n}^2 x_i^2 + 4a_{j,s}^2 x_i^2 + 0.25b_{j-1,n}^2 x_i^2 + 0.25b_{j-1,s}^2 x_i^2 + 0.25b_{j+1,n}^2 x_i^2 + 0.25b_{j+1,s}^2 x_i^2 - 8a_{j,n}a_{j,s}x_i^2 \\
& - 0.5b_{j-1,n}b_{j-1,s}x_i^2 - 0.5b_{j-1,n}b_{j+1,n}x_i^2 + 0.5b_{j-1,s}b_{j+1,n}x_i^2 + 0.5b_{j-1,n}b_{j+1,s}x_i^2 - 0.5b_{j-1,s}b_{j+1,s}x_i^2 \\
& - 0.5b_{j+1,n}b_{j+1,s}x_i^2 + 0.5a_{j-1,n}c_{j-1,n}x_i^2 - 0.5a_{j-1,s}c_{j-1,n}x_i^2 - 0.5a_{j+1,n}c_{j-1,n}x_i^2 + 0.5a_{j+1,s}c_{j-1,n}x_i^2 \\
& - 0.5a_{j-1,n}c_{j-1,s}x_i^2 + 0.5a_{j-1,s}c_{j-1,s}x_i^2 + 0.5a_{j+1,n}c_{j-1,s}x_i^2 - 0.5a_{j+1,s}c_{j-1,s}x_i^2 - 0.5a_{j-1,n}c_{j+1,n}x_i^2 \\
& + 0.5a_{j-1,s}c_{j+1,n}x_i^2 + 0.5a_{j+1,n}c_{j+1,n}x_i^2 - 0.5a_{j+1,s}c_{j+1,n}x_i^2 + 0.5a_{j-1,n}c_{j+1,s}x_i^2 - 0.5a_{j-1,s}c_{j+1,s}x_i^2 \\
& - 0.5a_{j+1,n}c_{j+1,s}x_i^2 + 0.5a_{j+1,s}c_{j+1,s}x_i^2 + 4a_{j,n}b_{j,n}x_i - 4a_{j,s}b_{j,n}x_i - 4a_{j,n}b_{j,s}x_i + 4a_{j,s}b_{j,s}x_i \\
& + 0.5b_{j-1,n}c_{j-1,n}x_i - 0.5b_{j-1,s}c_{j-1,n}x_i - 0.5b_{j+1,n}c_{j-1,n}x_i + 0.5b_{j+1,s}c_{j-1,n}x_i - 0.5b_{j-1,n}c_{j-1,s}x_i \\
& + 0.5b_{j-1,s}c_{j-1,s}x_i + 0.5b_{j+1,n}c_{j-1,s}x_i - 0.5b_{j+1,s}c_{j-1,s}x_i - 0.5b_{j-1,n}c_{j+1,n}x_i + 0.5b_{j-1,s}c_{j+1,n}x_i \\
& + 0.5b_{j+1,n}c_{j+1,n}x_i - 0.5b_{j+1,s}c_{j+1,n}x_i + 0.5b_{j-1,n}c_{j+1,s}x_i - 0.5b_{j-1,s}c_{j+1,s}x_i - 0.5b_{j+1,n}c_{j+1,s}x_i \\
& + 0.5b_{j+1,s}c_{j+1,s}x_i + b_{j,n}^2 + b_{j,s}^2 + 0.25c_{j-1,n}^2 + 0.25c_{j-1,s}^2 + 0.25c_{j+1,n}^2 + 0.25c_{j+1,s}^2 - 2b_{j,n}b_{j,s} \\
& - 0.5c_{j-1,n}c_{j-1,s} - 0.5c_{j-1,n}c_{j+1,n} + 0.5c_{j-1,s}c_{j+1,n} + 0.5c_{j-1,n}c_{j+1,s} - 0.5c_{j-1,s}c_{j+1,s} - 0.5c_{j+1,n}c_{j+1,s}
\end{aligned}
$$