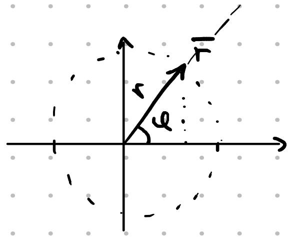
РЗ Хохлов Виктория по анмеху.
Первое задание, 3 сем

1. Кинематика точки

№1.18 $\rho=\rho(t)$ $\varphi=\varphi(t)$ Док-ть: $\rho^2\dot\varphi=const \Rightarrow \bar w \parallel \bar r$

$$\bar r=\begin{bmatrix}\rho\cos\varphi\\ \rho\sin\varphi\end{bmatrix}=\begin{bmatrix}x\\ y\end{bmatrix}$$



Рассмотрим проекцию ускорения на $\bar e_\rho$ и $\bar e_\varphi$

$$\frac{\partial\bar r}{\partial\rho}=\begin{pmatrix}\cos\varphi\\ \sin\varphi\end{pmatrix}\Rightarrow h_\rho=1 \quad ; \quad v_\rho=\dot\rho$$

$$\frac{\partial\bar r}{\partial\varphi}=\begin{pmatrix}-\rho\sin\varphi\\ \rho\cos\varphi\end{pmatrix}\Rightarrow h_\rho=\rho \;;\; v_\varphi=\rho\dot\varphi \qquad \Rightarrow v^2=\dot\rho^2+\rho^2\dot\varphi^2$$

$$w_\rho=\frac{d}{dt}(\dot\rho)-\rho\dot\varphi^2=\ddot\rho-\rho\dot\varphi^2$$

$$w_\varphi=\frac{1}{\rho}\left[\frac{d}{dt}\left(\rho^2\dot\varphi\right)\right]=0$$

$$\begin{matrix}w_\rho\neq 0\\ w_\varphi=0\end{matrix}\Rightarrow \bar w\parallel\bar\rho\parallel\bar r \Rightarrow \underline{\bar w\parallel\bar r} \text{ ч.т.д.}$$

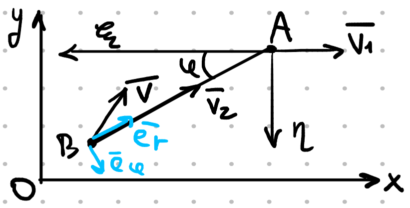
№1.25 Дано: $\bar w=a(\bar v\times\bar r)$, $a=const>0$ Найти: $\rho=\rho(\bar r,\bar v)$

$\bar w=a(\bar v\times\bar r) \Rightarrow \bar w\perp\bar v,\bar r \Rightarrow$

$\Rightarrow \bar w=\bar w_n=\frac{v^2}{\rho}\bar n=a(\bar v\times\bar r)\Rightarrow$

$\Rightarrow$ Ответ: $\rho=\dfrac{v^2}{a|\bar v\times\bar r|}$

№1.31



Найти: $AB=r(\varphi)$, $\varphi_0\neq 0$

$V_r=\dot r=V_1\cos\varphi-V_2=\frac{dr}{dt}$

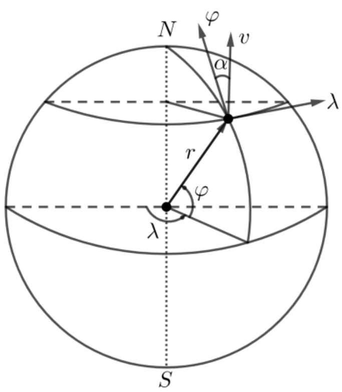
$V_\varphi=r\dot\varphi=-V_1\sin\varphi=r\cdot\frac{d\varphi}{dt}$

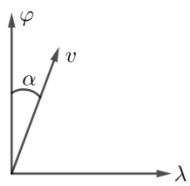
$\frac{dr}{r}=\frac{V_1\cos\varphi-V_2}{-V_1\sin\varphi}d\varphi \Rightarrow$

$\Rightarrow \ln r-\ln c=\frac{V_2}{V_1}\ln\frac{1-\cos\varphi}{\sin\varphi}-\ln\sin\varphi=\frac{V_2}{V_1}\ln tg\frac{\varphi}{2}-\ln\sin\varphi$

$\Rightarrow$ Ответ: $\dfrac{r}{r_0}=\dfrac{\sin\varphi_0}{\sin\varphi}\left(\dfrac{tg\,\varphi/2}{tg\,\varphi_0/2}\right)^{V_2/V_1}$

№Т1





$$\bar r=\begin{bmatrix}r\cos\varphi\cos\lambda\\ r\cos\varphi\sin\lambda\\ r\sin\varphi\end{bmatrix}$$

$V=const$, $\alpha=\widehat{e_\varphi,V}=const$

Найти: ур-ие кривой; w; $|w|$; ρ

$V_\varphi=V\cos\alpha=\frac{d\varphi}{dt}r$

$V_\lambda=V\sin\alpha=\frac{d\lambda}{dt}r\cos\varphi \Rightarrow \begin{cases}\frac{d\varphi}{dt}=\frac{V\cos\alpha}{r}\\ \frac{d\lambda}{dt}=\frac{V\sin\alpha}{r\cos\varphi}\end{cases}\Rightarrow \frac{d\varphi}{d\lambda}=\frac{\cos\alpha\cos\varphi}{\sin\alpha}$

$\int\frac{d\varphi}{\cos\varphi}=\int\frac{\cos\alpha}{\sin\alpha}d\lambda \Rightarrow \ln\frac{tg(\varphi/2+\pi/4)}{tg(\varphi_0/2+\pi/4)}=ctg\,\alpha(\lambda-\lambda_0)$

$\Rightarrow$ ур-ие кривой: $tg\left(\varphi/2+\frac{\pi}{4}\right)=tg\left(\frac{\varphi_0}{2}+\frac{\pi}{4}\right)\cdot\exp\left(ctg\,\alpha(\lambda-\lambda_0)\right)$

Из геом. соображ.: $H_r=1$; $H_\varphi=r$; $H_\lambda=r\cos\varphi$

$V_r=\dot r$; $V_\varphi=r\dot\varphi$; $V_\lambda=\dot\lambda r\cos\varphi$

$V^2=\dot r^2+r^2\dot\varphi^2+r^2\cos^2\varphi\,\dot\lambda^2$

$$W_r = \frac{1}{H_r}\left[\left(\dot{V^2/2}\right)_{,\dot r} - \left(V^2/2\right)_{,r}\right] = \ddot r - r\dot\varphi^2 - r\cos^2\varphi\cdot\dot\lambda^2 = -r\frac{V^2\cos^2\alpha}{r^2} - r\cdot\frac{V^2\sin^2\alpha}{r^2\cos^2\varphi}\cdot\cos^2\varphi = -\frac{V^2}{r}$$

$$W_\varphi = \frac{1}{H_\varphi}\left[r^2\ddot\varphi + 2r\dot r\dot\varphi + r^2\dot\lambda^2\cos\varphi\sin\varphi\right] = r\ddot\varphi + 2\dot r\dot\varphi + r\dot\lambda^2\cos\varphi\sin\varphi = \frac{V^2\sin^2\alpha\cdot tg\,\varphi}{r}$$

$$W_\lambda = \frac{r^2}{r\cos\varphi}\left[\ddot\lambda\cos^2\varphi - 2\cos\varphi\sin\varphi\,\dot\varphi\dot\lambda\right] = r\left(\ddot\lambda\cos\varphi - 2\sin\varphi\,\dot\varphi\dot\lambda\right) = \frac{1}{2r}V^2 tg\varphi\sin 2\varphi - \frac{1}{r}V^2 tg\varphi\sin 2\alpha = -\frac{V^2 tg\,\varphi\sin 2\alpha}{2r}$$

$$W = \sqrt{\frac{V^4}{r^2} + \frac{V^4\sin^4\alpha\, tg^2\varphi}{r^2} + \frac{V^4 tg^2\varphi\sin^2 2\alpha}{4r^2}} = \frac{V^2}{r}\sqrt{1+\sin^2\alpha + tg^2\varphi}$$

$$V = const \rightarrow W = W_n = \frac{V^2}{\rho} \Rightarrow \rho = \frac{r}{\sqrt{1+\sin^2\alpha + tg^2\varphi}}$$

Ответ: ур-ие: $tg(\varphi/2 + \frac{\pi}{4}) = tg(\frac{\varphi_0}{2} + \frac{\pi}{4})\cdot \exp(ctg\,\alpha(\lambda-\lambda_0))$;

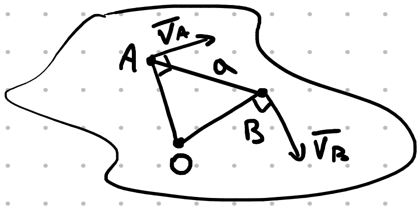
$W_r = -\frac{V^2}{r}$; $W_\varphi = \frac{V^2\sin^2\alpha\cdot tg\varphi}{r}$; $W_\lambda = -\frac{V^2 tg\varphi\sin 2\alpha}{2r}$;

$W = \frac{V^2}{r}\sqrt{1+\sin^2\alpha + tg^2\varphi}$; $\rho = \frac{r}{\sqrt{1+\sin^2\alpha+tg^2\varphi}}$

2. Кинематика твердого тела

2.1. Плоскопараллельное движение.

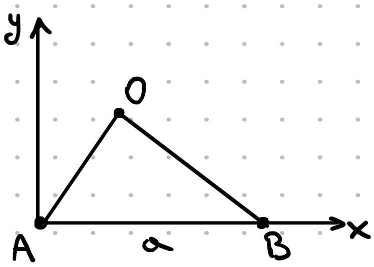
№3.2 Найти: геом. место возможных полож. мгн. центра скоростей, если $\lambda \neq 1$; $\lambda = \frac{V_A}{V_B}$; $|AB| = a$



Пусть O – мгн. ц. скоростей. Тогда $\overline{OA} \perp \overline{V_A}$; $\overline{OB} \perp \overline{V_B}$

$$\frac{OA}{OB} = \frac{V_A}{V_B} = \lambda$$

Перейдем в СО, где $A(0,0)$; $B(a,0)$



$$\begin{cases} x^2 + y^2 = AO^2 = \lambda^2 \cdot BO^2 \\ (a-x)^2 + y^2 = BO^2 \end{cases}$$

$$x^2 + y^2 = \lambda^2 y^2 + \lambda^2 x^2 - 2\lambda^2 a x + \lambda^2 a^2$$

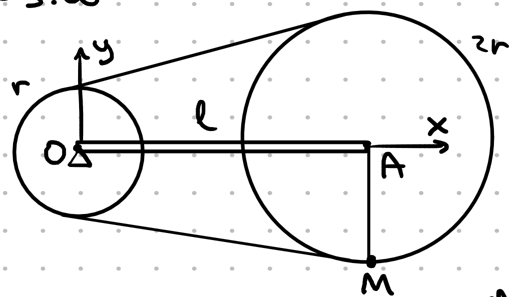
$$x^2(1-\lambda^2) + 2\lambda a^2 x + y^2(1-\lambda^2) = \lambda^2 a^2$$

$$x^2 + \frac{2\lambda^2 a x}{1-\lambda^2} + \frac{\lambda^4 a^2}{(1-\lambda^2)^2} + y^2 = \frac{\lambda^2 a^2}{1-\lambda^2} + \frac{\lambda^4 a^2}{(1-\lambda^2)^2}$$

$$\Rightarrow \left(x + \frac{\lambda^2 a}{1-\lambda^2}\right)^2 + y^2 = \left(\frac{\lambda a}{1-\lambda^2}\right)^2$$

Ответ: $\left(x + \frac{\lambda^2 a}{1-\lambda^2}\right)^2 + y^2 = \left(\frac{\lambda a}{1-\lambda^2}\right)^2$

№3.20



Известно: l, ω_0, ε_0, r, $R = 2r$, $AM \perp OA$

Найти: ω_M, V_M?

$$\bar{V}_A = \cancel{\bar{V}_0}^{\,0} + \bar{\omega}_0 \times \overline{OA} = \begin{pmatrix} 0 \\ 0 \\ \omega_0 \end{pmatrix} \times \begin{pmatrix} l \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \omega_0 l \\ 0 \end{pmatrix}$$

Путь пройденный цепью за время t:

$$a_1 = \omega_0 r t = a_2 = \omega_A 2 r t \Rightarrow \omega_A = \frac{\omega_0}{2}$$

$$\bar{V}_M = \bar{V}_A + \bar{\omega}_A \cdot \overline{AM} = \begin{pmatrix} 0 \\ \omega_0 l \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_0/2 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2r \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_0 r \\ \omega_0 l \\ 0 \end{pmatrix} \Rightarrow V_M = \omega_0 \sqrt{r^2 + l^2}$$

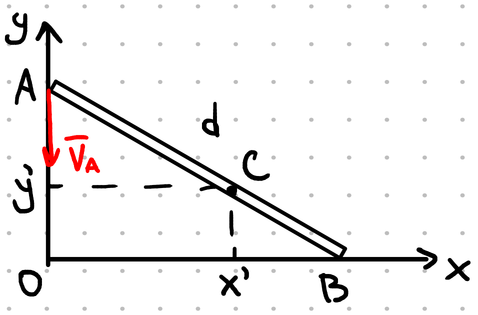
$$\bar{W}_A = \cancel{\bar{W}_0}^{\,0} + \bar{\varepsilon}_0 \times \overline{OA} - \omega_0^2 \overline{OA} = \begin{pmatrix} 0 \\ 0 \\ \varepsilon_0 \end{pmatrix} \begin{pmatrix} l \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \omega_0^2 l \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\omega_0^2 l \\ \varepsilon_0 l \\ 0 \end{pmatrix}; \quad \bar{\varepsilon}_A = \frac{d\omega_A}{dt} = \frac{d\omega_0}{dt} \cdot \frac{1}{2} = \frac{\varepsilon_0}{2}$$

$$\bar{W}_M = \bar{W}_A + \bar{\varepsilon}_A \cdot \overline{AM} - \omega_A^2 \overline{AM} = \begin{pmatrix} -\omega_0^2 l \\ \varepsilon_0 l \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \varepsilon_0/2 \end{pmatrix} \begin{pmatrix} 0 \\ -2r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_0^2 r/2 \\ 0 \end{pmatrix} = \begin{pmatrix} \varepsilon_0 r - \omega_0^2 l \\ \varepsilon_0 l + \omega_0^2 r/2 \\ 0 \end{pmatrix}$$

$$\Rightarrow W_M = \sqrt{(\varepsilon_0 r - \omega_0^2 l)^2 + \left(\varepsilon_0 l + \frac{\omega_0^2 r}{2}\right)^2}$$

Ответ: $V_M = \omega_0 \sqrt{r^2 + l^2}$; $W_M = \sqrt{(\varepsilon_0 r - \omega_0^2 l)^2 + \left(\varepsilon_0 l + \frac{\omega_0^2 r}{2}\right)^2}$

№3.21



$V_A = const$; $AB = d$

Показать, что $W_C \perp Oy$ и $W_C \propto \frac{1}{(\rho(C,O_y))^3}$

$\forall C$ – т. стержня

$$\bar V_B = \bar V_A + \bar\omega \times \overline{AB} = \begin{pmatrix}0\\-V_A\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\begin{pmatrix}x\\-y\\0\end{pmatrix} = \begin{pmatrix}y\omega\\\omega x - V_A\\0\end{pmatrix} = \begin{pmatrix}V_B\\0\\0\end{pmatrix}$$

$$\begin{cases} V_B = y\omega \\ V_A = x\omega \end{cases} \Rightarrow \omega = \frac{V_A}{x}, \; V_B = \frac{y}{x}V_A \Rightarrow \bar V_B = \begin{pmatrix}\frac{y}{x}V_A\\0\\0\end{pmatrix}$$

$$\bar W_B = \cancel{\bar W_A}^{0} + \bar\varepsilon \times \overline{AB} - \omega^2 \overline{AB} = \begin{pmatrix}0\\0\\\varepsilon\end{pmatrix}\times\begin{pmatrix}x\\-y\\0\end{pmatrix} + \begin{pmatrix}-\omega^2 x\\\omega^2 y\\0\end{pmatrix} = \begin{pmatrix}y\varepsilon - \omega^2 x\\x\varepsilon + \omega^2 y\\0\end{pmatrix} = \begin{pmatrix}W_B\\0\\0\end{pmatrix}$$

$$\dot V_B = \frac{d\left(\frac{y}{x}V_A\right)}{dt} = V_A\left(\frac{V_A}{x} - \frac{yV_B}{x^2}\right) = \frac{V_A^2}{x}\left(1 + \frac{y^2}{x^2}\right) = \frac{V_A^2}{x^3}d^2$$

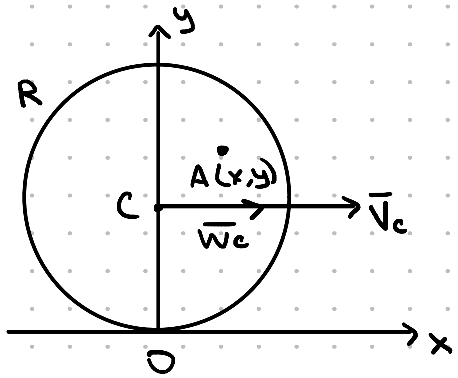
$$\begin{cases} y\varepsilon = -\frac{V_A^2}{x} = -\frac{V_A^2 d^2}{x^3} \\ x\varepsilon = -\frac{yV_A^2}{x^2} \end{cases} \Rightarrow \varepsilon = -\frac{yV_A^2}{x^3}$$

$$\bar W_C = \bar\varepsilon \times \overline{AC} - \omega^2 \overline{AC} = \begin{pmatrix}0\\0\\-y\frac{V_A^2}{x^3}\end{pmatrix}\begin{pmatrix}x'\\y'-y\\0\end{pmatrix} + \begin{pmatrix}-\omega^2 x'\\\omega^2(y-y')\\0\end{pmatrix} = \begin{pmatrix}-\frac{V_A^2 x'}{x^2} + \frac{V_A^2 y}{x^3}(y'-y)\\-\frac{V_A^2}{x^2}(y'-y) - yV_x^2\cdot\frac{x'}{x^3}\\0\end{pmatrix} = \frac{V_A^2}{x^2}\begin{pmatrix}-x' - \frac{(y-y')^2}{x'}\\-(y'-y)-(y-y')\\0\end{pmatrix} =$$

$$= \frac{V_A^2}{x^2}\begin{pmatrix}\frac{-x'^2-(y-y')^2}{x'}\\0\\0\end{pmatrix} = -V_A^2\begin{pmatrix}\underbrace{\frac{d'^2}{x'^3}\frac{(y-y')^2}{y^2}}_{const}\\0\\0\end{pmatrix} = -\begin{pmatrix}\frac{k\cdot V_A^2}{x'^3}\\0\\0\end{pmatrix} \Rightarrow$$

$$\Rightarrow \bar W_C \perp Oy \; ; \; \bar W_C \propto \frac{1}{x'^3}$$ — ч.т.д.

№3.25



$R, V_c, W_c, A(x,y)$ $x \neq 0$ $y \neq 0$

Найти: $W_{An}, W_{A\tau}$

$$\bar V_O = \bar V_C + \bar\omega \times \overline{CO} = \bar 0 \Rightarrow \bar V_C = \bar\omega \times \overline{OC}$$

$$\begin{pmatrix}V_c\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\\omega\end{pmatrix}\begin{pmatrix}0\\R\\0\end{pmatrix} = \begin{pmatrix}-\omega R\\0\\0\end{pmatrix} \Rightarrow \omega = \frac{-V_c}{R} \; ; \; V_c = -\omega R$$

$$\varepsilon = \dot\omega = -\frac{W_c}{R}$$

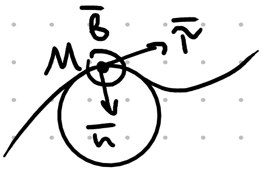
$$\bar V_A = \bar V_c + \bar\omega \cdot \overline{CA} = \begin{pmatrix}V_c\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-V_c/R\end{pmatrix}\cdot\begin{pmatrix}x\\y-R\\0\end{pmatrix} = \begin{pmatrix}V_c y/R\\V_c x/R\\0\end{pmatrix} = V_A\begin{pmatrix}\frac{y}{\sqrt{x^2+y^2}}\\\frac{x}{\sqrt{x^2+y^2}}\end{pmatrix} = V_A\bar\tau$$

$$\bar W_A = \bar W_c + \bar\varepsilon \times \overline{CA} + \bar\omega[\bar\omega \times \overline{CA}] = \begin{pmatrix}W_c\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-\frac{W_c}{R}\end{pmatrix}\begin{pmatrix}x\\y-R\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-\frac{V_c}{R}\end{pmatrix}\begin{pmatrix}V_c y/R\\V_c x/R\\0\end{pmatrix} =$$

$$= \begin{pmatrix}W_c y/R\\W_c x/R\\0\end{pmatrix} + \begin{pmatrix}V_c^2 x/R^2\\V_c^2 y/R^2\\0\end{pmatrix} = \frac{W_c}{R}\sqrt{x^2+y^2}\,\bar\tau + \frac{V_c^2}{R^2}\sqrt{x^2+y^2}\,\bar n = W_{A\tau}\bar\tau + W_{An}\bar n$$

Ответ: $W_{A\tau} = \frac{W_c}{R}\sqrt{x^2+y^2}$, $W_{An} = \frac{V_c^2}{R^2}\sqrt{x^2+y^2}$

№3.36 $\bar V(t), \rho(t)$ Найти: ω, ε трехгранника $(\bar\tau, \bar n, \bar b)$



$\bar V = \omega\bar b \times \rho\bar n$; $\bar\omega = \omega\bar b$

$\omega[\bar b \times n] = \frac{\bar V}{\rho}$ $|\cdot\bar\tau$

$\omega = \frac{(V,\tau)}{\rho}$ $|\cdot\bar b$

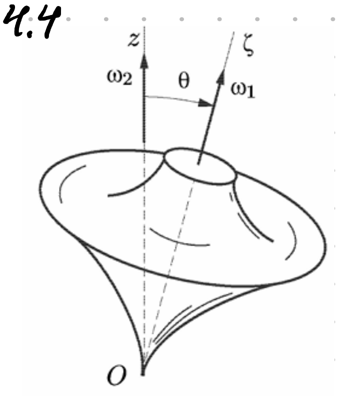
$\bar\omega = \frac{(\bar V,\tau)}{\rho}\bar b$

$$\bar\varepsilon = \dot{\bar\omega} = [((\dot V,\tau)+(V,\dot\tau))\rho - (V,\tau)\dot\rho]\frac{\bar b}{\rho^2} = \bar b\left(\frac{(\dot V,\tau)\rho - (V,\tau)\dot\rho}{\rho^2}\right) = \bar b\left(\frac{(\dot V,\tau)}{\rho} - \frac{(V,\tau)\dot\rho}{\rho^2}\right)$$

Ответ: $\bar\omega = \frac{(\bar V,\tau)}{\rho}\bar b$; $\bar\varepsilon = \bar b\left(\frac{(\dot V,\tau)}{\rho} - \frac{(V,\tau)\dot\rho}{\rho^2}\right)$

2.2. Пространственное движение

№4.4



Дано: $\omega_1, \omega_2, \Theta$; Найти: ω, ε отн. Oz

$\bar\omega = \bar\omega_1 + \bar\omega_2$

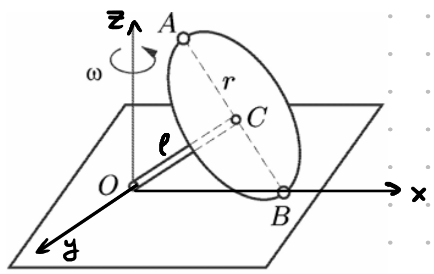
$\Rightarrow \omega = |\bar\omega| = \sqrt{\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\cos\Theta}$

$\bar\varepsilon = \cancel{\bar\varepsilon_1}^{\bar 0} + \cancel{\bar\varepsilon_2}^{\bar 0} + \bar\omega_1 \times \bar\omega_2 = \bar\omega_1 \times \bar\omega_2$

$\Rightarrow \varepsilon = |\bar\varepsilon| = \omega_1\omega_2\sin\Theta$

Ответ. $\omega = \sqrt{\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\cos\Theta}$; $\varepsilon = \omega_1\omega_2\sin\Theta$

№4.10



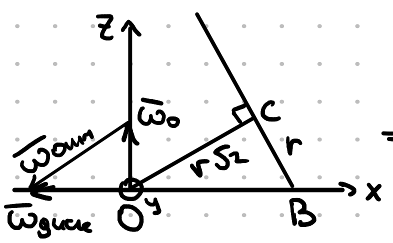
Дано: r, $l = r\sqrt3$, ε, ω (стержня) Найти: $\omega_д$, $\varepsilon_д$, W_A, W_B

1) $V_B = \bar 0 = \bar V_C + \bar\omega_{диск}\times\overline{CB} = \bar\omega_0\times\overline{OC} + \bar\omega_{диск}\times\overline{CB}$

$\bar V_C = \bar V_0 + \bar\omega_0 \times \overline{OC}$

$\Rightarrow \bar\omega_0\times\overline{OC} = \bar\omega_{диск}\times\overline{BC}$

2) $\bar\omega_{диск} = \bar\omega_0 + \bar\omega_{отн}$; $\cancel{\bar V_B}^{0} = \cancel{\bar V_0}^{0} + \omega_{диск}\times\overline{OB} \Rightarrow \omega_{диск} \parallel \overline{OB}$



$\bar\omega_{отн} \parallel \overline{OC}$, $\bar\omega_{диск} \parallel \overline{OB}$, $\bar\omega_0 \parallel Oz$

$\omega_{диск} = \omega_0 \cdot ctg 30 = \sqrt3\omega_0$; $\omega_{отн} = \frac{\omega_0}{\sin 30} = 2\omega_0$

$\Rightarrow \angle COB = arctg\frac{\sqrt3}{3} = 30°$

$\angle(\omega_{отн}, \omega_{диск}) = \angle COB = 30°$

3) $\bar\varepsilon_{диск} = \bar\varepsilon_0 + \bar\varepsilon_{отн} + \bar\omega_0 \times \bar\omega_{отн}$; $\varepsilon_{отн} = \dot\omega_{отн} = 2\dot\omega_0 = 2\varepsilon_0$

$$\bar\varepsilon_{диск} = \begin{bmatrix} 0 \\ 0 \\ \varepsilon_0 \end{bmatrix} + \begin{bmatrix} -2\varepsilon_0\cos 30 \\ 0 \\ -2\varepsilon_0\sin 30 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_0 \end{bmatrix} \times \begin{bmatrix} -2\omega_0\cos 30 \\ 0 \\ -2\omega_0\sin 30 \end{bmatrix} = \begin{bmatrix} \sqrt3\varepsilon_0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -\sqrt3\omega_0^2 \\ 0 \end{bmatrix} = -\begin{bmatrix} \sqrt3\varepsilon_0 \\ \sqrt3\omega_0^2 \\ 0 \end{bmatrix}$$

$\Rightarrow \varepsilon_{диск} = \sqrt{3(\varepsilon_0^2 + 3\omega_0^4)}$

4) $\bar W_A = \cancel{\bar W_0}^{0} + \bar\varepsilon_{диск}\times\overline{OA} + \bar\omega_{диск}\times[\bar\omega_{диск}\times\overline{OA}]$

$$\overline{OA} = \overline{OC} + \overline{OA} = \begin{bmatrix} l\cos 30 \\ 0 \\ l\sin 30\end{bmatrix} + \begin{bmatrix} -r\sin 30 \\ 0 \\ r\cos 30\end{bmatrix} = \begin{bmatrix} r \\ 0 \\ \sqrt{3} r\end{bmatrix}$$

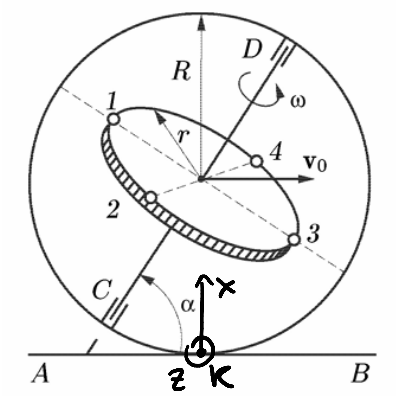
$$\overline{W}_A = -\begin{bmatrix}\sqrt{3}\varepsilon_0\\ \sqrt{3}\omega_0^2\\0\end{bmatrix}\times\begin{bmatrix}r\\0\\\sqrt{3} r\end{bmatrix} + \begin{bmatrix}-\sqrt{3}\omega_0\\0\\0\end{bmatrix}\times\left[\begin{pmatrix}-\sqrt{3}\omega_0\\0\\0\end{pmatrix}\times\begin{pmatrix}r\\0\\\sqrt{3} r\end{pmatrix}\right] = \begin{pmatrix}-3r\omega_0^2\\3r\varepsilon_0\\\sqrt{3} r\omega_0^2\end{pmatrix} + \begin{pmatrix}-\sqrt{3}\omega_0\\0\\0\end{pmatrix}\times\begin{pmatrix}0\\3\omega_0 r\\0\end{pmatrix} =$$

$$= \begin{bmatrix}-3r\omega_0^2\\3r\varepsilon_0\\-2\sqrt{3} r\omega_0^2\end{bmatrix} \Rightarrow W_A = \sqrt{9r^2\omega_0^4+9r^2\varepsilon_0^2+12r^2\omega_0^4} = r\sqrt{9\varepsilon_0^2+21\omega^4}$$

$$\overline{W}_B = \cancel{\overline{W}_O}^{\,0} + \overline{\varepsilon}_{абс}\times\overline{OB} + \overline{\omega}_{абс}\times[\overline{\omega}_{абс}\times\overline{OB}] = \begin{bmatrix}-\sqrt{3}\varepsilon_0\\-\sqrt{3}\omega_0^2\\0\end{bmatrix}\times\begin{bmatrix}2r\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\2\sqrt{3} r\omega_0^2\end{bmatrix};\ W_B = 2\sqrt{3} r\omega_0^2$$

Ответ: $\omega_д = \sqrt{3}\omega_0$; $\varepsilon_д = \sqrt{3(\varepsilon_0^2+\omega_0^4)}$;
$W_A = r\sqrt{9\varepsilon_0^2+21\omega^4}$; $W_B = 2\sqrt{3} r\omega_0^2$

№4.12 (для 1 т.)



Дано: $R, V_0, \omega, r, \alpha$; Найти: V_1, W_1

$\overline{\omega}_{отн} = \overline{\omega}$; $\overline{\omega}_{пер} = \overline{\omega}_{обруч} = \omega_0$; $\overline{V}_K = \overline{0}$

$\overline{V}_K = \overline{V}_O + \overline{\omega}_0\times\overline{OK} = \overline{0} \Rightarrow \overline{V}_O = \overline{V} = \overline{\omega}_0\times\overline{KO}$

$$\begin{pmatrix}V\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\\omega_0\end{pmatrix}\times\begin{pmatrix}0\\R\\0\end{pmatrix} = \begin{pmatrix}-\omega_0 R\\0\\0\end{pmatrix} \Rightarrow \omega_0 = -\frac{V}{R}$$

$$\overline{\omega}_{абс} = \overline{\omega}_{пер} + \overline{\omega}_{отн} = \overline{\omega} + \overline{\omega}_0 = \begin{pmatrix}\omega\cos\alpha\\\omega\sin\alpha\\-V/R\end{pmatrix}$$

$$\overline{V}_1 = \overline{V}_O + \overline{\omega}_{абс}\times\overline{O1} = \begin{pmatrix}V\\0\\0\end{pmatrix} + \begin{pmatrix}\omega\cos\alpha\\\omega\sin\alpha\\-V/R\end{pmatrix}\times\begin{pmatrix}-r\sin\alpha\\r\cos\alpha\\0\end{pmatrix} = \begin{pmatrix}V+V\frac{r}{R}\cos\alpha\\\frac{Vr}{R}\sin\alpha\\\omega r\end{pmatrix}$$

$$\Rightarrow V_1 = \sqrt{V^2+2V^2\frac{r}{R}\cos\alpha + \frac{V^2r^2}{R^2}+\omega^2 r^2}$$

$$\overline{\varepsilon}_{абс} = \overline{\omega}_{пер}\times\overline{\omega}_{отн} + \cancel{\overline{\varepsilon}_{пер}}^{\,0} + \cancel{\overline{\varepsilon}_0}^{\,0} = \begin{pmatrix}0\\0\\-V/R\end{pmatrix}\times\begin{pmatrix}\omega\cos\alpha\\\omega\sin\alpha\\0\end{pmatrix} = \begin{pmatrix}V/R\cdot\omega\sin\alpha\\-V/R\cdot\omega\cos\alpha\\0\end{pmatrix}$$

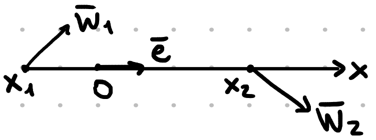
$$\overline{W}_1 = \cancel{\overline{W}_O}^{\,0} + \overline{\varepsilon}_{абс}\times\overline{O1} + \overline{\omega}_{абс}\times[\overline{\omega}_{абс}\times\overline{O1}] = \begin{pmatrix}V/R\sin\alpha\\-V/R\cos\alpha\\0\end{pmatrix}\times\begin{pmatrix}-r\sin\alpha\\r\cos\alpha\\0\end{pmatrix} +$$

$$+\begin{pmatrix}\omega\cos\alpha\\\omega\sin\alpha\\-V/R\end{pmatrix}\times\left[\begin{pmatrix}\omega\cos\alpha\\\omega\sin\alpha\\-V/R\end{pmatrix}\times\begin{pmatrix}-r\sin\alpha\\r\cos\alpha\\0\end{pmatrix}\right] = \begin{pmatrix}\omega\cos\alpha\\\omega\sin\alpha\\-V/R\end{pmatrix}\times\begin{pmatrix}V/R\,r\cos\alpha\\V/R\cdot r\sin\alpha\\\omega r\end{pmatrix} = \begin{pmatrix}\omega^2 r\sin\alpha + V^2/R^2\, r\sin\alpha\\-\omega^2 r\cos\alpha - V^2/R^2\, r\cos\alpha\\0\end{pmatrix} =$$

$$= \begin{pmatrix}W_1\sin\alpha\\W_1\cos\alpha\\0\end{pmatrix} \Rightarrow W_1 = \sqrt{\left(\omega^2 r+\frac{V^2}{R^2}r\right)^2\sin^2\alpha + \left(\omega^2 r+\frac{V^2}{R^2}r\right)^2\cos^2\alpha} = \omega^2 r + \frac{V^2}{R^2}r = r\left(\omega^2+\frac{V^2}{R^2}\right)$$

Ответ: $V_1 = \sqrt{V^2+2V^2\frac{r}{R}\cos\alpha + \left(\frac{Vr}{R}\right)^2+\omega^2r^2}$; $W_1 = r\left(\omega^2+\frac{V^2}{R^2}\right)$

№4.23 Дано: x_1, x_2, W_1, W_2 Найти: W_x



$$\overline{W}_2 = \overline{W}_1 + \overline{\varepsilon} \times \overline{x_1x_2} + \overline{\omega}[\overline{\omega} \times \overline{x_1x_2}]$$

$$\begin{cases} \overline{W}_2 = \overline{W}_1 + (x_2 - x_1)\,\overline{\varepsilon}\times\overline{e} + (x_2-x_1)\,\overline{\omega}\times[\overline{\omega}\times\overline{e}] \\ \overline{W}_x = \overline{W}_1 + (x - x_1)\,\overline{\varepsilon}\times\overline{e} + (x-x_1)\,\overline{\omega}\times[\overline{\omega}\times\overline{e}] \\ \overline{W}_2 = \overline{W}_x + (x_2 - x)\,\overline{\varepsilon}\times\overline{e} + (x_2-x)\,\overline{\omega}\times[\overline{\omega}\times\overline{e}] \end{cases}$$

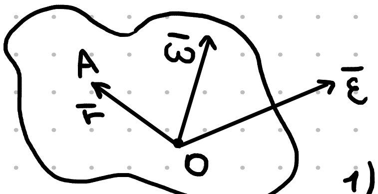
$$\begin{cases} \overline{W}_x(x_2 - x) = \overline{W}_1(x_2 - x) + (x-x_1)(x_2-x)[\overline{\varepsilon}\times\overline{e} + \overline{\omega}\times[\overline{\omega}\times\overline{e}]] \\ \overline{W}_2(x - x_1) = \overline{W}_x(x-x_1) + (x-x_1)(x_2-x)[\overline{\varepsilon}\times\overline{e} + \overline{\omega}\times[\overline{\omega}\times\overline{e}]] \end{cases}$$

$$\overline{W}_x(x_2 - x + x - x_1) = \overline{W}_1(x_2 - x) + \overline{W}_2(x - x_1)$$

$$\Rightarrow \overline{W}_x = \overline{W}_1 \frac{x_2 - x}{x_2 - x_1} + \overline{W}_2 \frac{x - x_1}{x_2 - x_1}$$

Ответ: $\overline{W}_x = \overline{W}_1 \frac{x_2 - x}{x_2 - x_1} + \overline{W}_2 \frac{x - x_1}{x_2 - x_1}$

№4.30 Дано: ε, ω Д-ать: $\overline{W}_{вр} = \overline{W}_{кас}$; $\overline{W}_{ос} = \overline{W}_{норм}$ только если т. в пл. ε, ω



$$\begin{cases} \overline{W} = \cancel{\overline{W}_0} + \overline{\varepsilon}\times\overline{r} + \overline{\omega}\times[\overline{\omega}\times\overline{r}] = \overline{W}_{вр} + \overline{W}_{ос} \\ \overline{W} = \dot{v}\overline{\tau} + v^2/r\,\overline{n} = \overline{W}_{кас} + \overline{W}_{норм} \end{cases}$$

1) Пусть $\overline{\varepsilon}\times\overline{r} = \dot{v}\overline{\tau}$; $\overline{\omega}\times[\overline{\omega}\times\overline{r}] = \frac{v^2}{r}\overline{n}$

$$\overline{\tau} = \frac{\overline{v}}{v} = \frac{\overline{\omega}\times\overline{r}}{|\overline{\omega}\times\overline{r}|} \Rightarrow \overline{\omega}\perp\overline{\tau},\ \overline{r}\perp\overline{\tau} \Rightarrow (\overline{\omega}\cdot[\overline{\varepsilon}\times\overline{r}]) - \dot{v}(\overline{\omega}\cdot\overline{\tau}) = 0$$

Смеш. произв. $(\overline{\omega}\cdot[\overline{\varepsilon}\times\overline{r}]) = 0 \Rightarrow \overline{\omega}, \overline{\varepsilon}, \overline{r}$ лежат в одной пл.

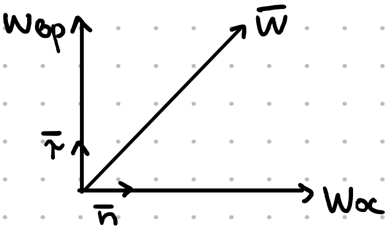
ч.т.д.

2) Пусть $\overline{\omega}, \overline{\varepsilon}, \overline{r}$ лежат в одной плоскости

$\overline{\omega}\times[\overline{\omega}\times\overline{r}] \perp \overline{\omega}\times\overline{r} = \lambda[\overline{\varepsilon}\times\overline{r}] \Rightarrow \overline{W}_{вр} \perp \overline{W}_{ос}$.

$\overline{\tau} = \frac{\overline{\omega}\times\overline{r}}{|\overline{\omega}\times\overline{r}|} \parallel \overline{\varepsilon}\times\overline{r} = \overline{W}_{вр}$; $\overline{W} = \overline{W}_{кас} + \overline{W}_{норм} \Rightarrow \overline{W}_{норм}$ лежит в пл. $\overline{W}$ и $\overline{W}_{кас}$

$\overline{W}_{норм} \perp \overline{W}_{кас} \Rightarrow \overline{W}_{норм} \perp \overline{W}_{ос}$

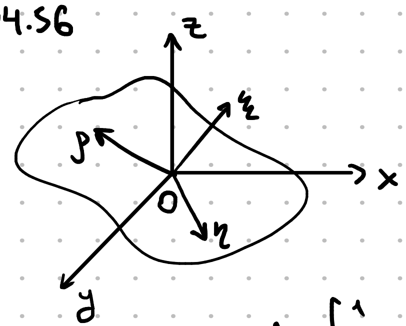


Из геом. сообр: $|\overline{W}_{вр}| = |\overline{W}_{кас}|$, $|\overline{W}_{ос}| = |\overline{W}_{норм}|$

$\Rightarrow$ $\overline{W}_{вр} = \overline{W}_{кас}$; $\overline{W}_{ос} = \overline{W}_{норм}$

ч.т.д.

№4.56



Найдём собств. вект. для матр. $A(t)$

$A\overline{u} = \lambda\overline{u}$: $A\overline{u}_1 = \overline{u}_1$; $A\overline{u}_2 = \overline{u}_2\cos\varphi + \overline{u}_3\sin\varphi$;
$A\overline{u}_3 = \overline{u}_3\cos\varphi - \overline{u}_2\sin\varphi$

Направим оси по $\overline{u}_1, \overline{u}_2, \overline{u}_3$:

$$\overline{u}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}\quad \overline{u}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}\quad \overline{u}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$A = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi\end{bmatrix}$$ — получили матр. плоского поворота вокруг $\overline{u}_1$ на φ

ч.т.д.

2.3 Кватернионы

№4.66 Дано: ψ, θ, φ Найти: угол поворота, напр. косинусы

Посл. т. зр:

$$\Lambda_1 = \cos\frac{\psi}{2} + \bar{i}_3 \sin\frac{\psi}{2}$$
$$\Lambda_2 = \cos\theta/2 + \bar{i}_1 \sin\theta/2$$
$$\Lambda_3 = \cos\varphi/2 + \bar{i}_3 \sin\varphi/2$$

$$\Lambda \circ M = \lambda_0 \mu_0 - \bar{\lambda}\cdot\bar{\mu} + \lambda_0 \bar{\mu} + \mu_0 \bar{\lambda} + \bar{\lambda}\times\bar{\mu}$$
$$\Lambda_1 \circ \Lambda_2 = \cos\frac{\psi}{2}\cdot\cos\theta/2 - (\bar{i}_3\cdot\bar{i}_1)\sin\frac{\psi}{2}\sin\frac{\theta}{2} + \cos\frac{\psi}{2}\sin\frac{\theta}{2}\bar{i}_1 + (\bar{i}_3\times\bar{i}_1)\sin\frac{\psi}{2}\sin\frac{\theta}{2}$$

$$\Lambda = \Lambda_1\circ\Lambda_2\circ\Lambda_3 = (\cos\psi/2 + \bar{i}_3\sin\psi/2)(\cos\theta/2\cos\varphi/2 + \bar{i}_3\cos\theta/2\sin\varphi/2 + \bar{i}_1\sin\theta/2\cos\varphi/2 - \bar{i}_2\sin\theta/2\sin\varphi/2) =$$
$$= \cos\frac{\psi}{2}\cos\theta/2\cos\varphi/2 - \sin\psi/2\cos\theta/2\sin\varphi/2 + \bar{i}_3\cos\frac{\psi}{2}\cos\theta/2\sin\varphi/2 + \bar{i}_3\sin\psi/2\cos\theta/2\cos\varphi/2 +$$
$$+ \bar{i}_1\cos\psi/2\sin\theta/2\cos\varphi/2 - \bar{i}_2\cos\psi/2\sin\theta/2\sin\varphi/2 + \bar{i}_2\sin\psi/2\sin\theta/2\cos\varphi/2 + \bar{i}_1\sin\psi/2\sin\theta/2\sin\varphi/2 =$$
$$= \cos\theta/2\cos\left(\frac{\psi+\varphi}{2}\right) + \bar{i}_1\sin\theta/2\cos\left(\frac{\psi-\varphi}{2}\right) + \bar{i}_2\sin\theta/2\sin\frac{\psi-\varphi}{2} + \bar{i}_3\cos\theta/2\sin\frac{\psi+\varphi}{2}$$

$$\sqrt{\sin^2\theta/2\cos\frac{\psi-\varphi}{2} + \sin^2\theta/2\sin^2\frac{\psi-\varphi}{2} + \cos^2\theta/2\sin^2\frac{\psi+\varphi}{2}} = \sqrt{\sin^2\theta/2 + \cos^2\theta/2\sin^2\frac{\psi+\varphi}{2}} =$$
$$= \sqrt{1-\cos^2\theta/2 + \cos^2\theta/2\sin^2\frac{\psi+\varphi}{2}} = \sqrt{1-\cos^2\frac{\theta}{2}\cos^2\frac{\psi+\varphi}{2}}$$

Ответ: угол поворота $\alpha = 2\arccos\left(\cos\theta/2\cdot\cos\frac{\psi+\varphi}{2}\right)$;

напр. косинусы: $\dfrac{\sin\theta/2\cos\frac{\psi-\varphi}{2}}{\sqrt{1-\cos^2\theta/2\cos^2\frac{\psi+\varphi}{2}}}$, $\dfrac{\sin\theta/2\sin\frac{\psi-\varphi}{2}}{\sqrt{1-\cos^2\theta/2\cos^2\frac{\psi+\varphi}{2}}}$, $\dfrac{\cos\theta/2\sin\frac{\psi+\varphi}{2}}{\sqrt{1-\cos^2\theta/2\cos^2\frac{\psi+\varphi}{2}}}$