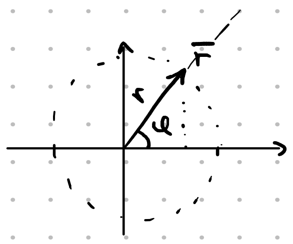
Р3 Хомяк Виктории по теормеху.
Первое задание, 3 сем

1. Кинематика точки

№1.18 $\rho=\rho(t)$ $\varphi=\varphi(t)$ Док. $\rho^2\omega = const \Rightarrow \bar{W} \parallel \bar{r}$

$$\bar{r} = \begin{bmatrix} \rho\cos\varphi \\ \rho\sin\varphi \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$



Рассмотрим проекцию ускорения на $\bar{e}_\rho$ и $\bar{e}_\varphi$

$$\frac{\partial \bar{r}}{\partial \rho} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} \Rightarrow h_\rho = 1 \quad ; \quad v_\rho = \dot{\rho}$$

$$\frac{\partial \bar{r}}{\partial \varphi} = \begin{pmatrix} -\rho\sin\varphi \\ \rho\cos\varphi \end{pmatrix} \Rightarrow h_\rho = \rho \ ; \ v_\varphi = \rho\dot{\varphi} \qquad \Rightarrow v^2 = \dot{\rho}^2 + \rho^2\dot{\varphi}^2$$

$$W_\rho = \frac{d}{dt}(\dot{\rho}) - \rho\dot{\varphi}^2 = \ddot{\rho} - \rho\dot{\varphi}^2$$

$$W_\varphi = \frac{1}{\rho}\left[\frac{d}{dt}\left(\rho^2\omega\right)\right] = 0$$

$$\begin{matrix} W_\rho \neq 0 \\ W_\varphi = 0 \end{matrix} \Rightarrow \bar{W} \parallel \bar{\rho} \parallel \bar{r} \Rightarrow \underline{\bar{W} \parallel \bar{r}} \text{ ч.т.д.}$$

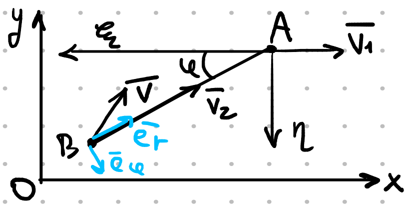
№1.25 Дано: $\bar{W} = a(\bar{v}\times\bar{r})$, $a = const > 0$ Найти: $\rho = \rho(\bar{r}, \bar{v})$

$\bar{W} = a(\bar{v}\times\bar{r}) \Rightarrow \bar{W} \perp \bar{v}, \bar{r} \Rightarrow$

$$\Rightarrow \bar{W} = \bar{W}_n = \frac{v^2}{\rho}\bar{n} = a(\bar{v}\times\bar{r}) \Rightarrow$$

$$\Rightarrow \text{Ответ: } \rho = \frac{v^2}{a|\bar{v}\times\bar{r}|}$$

№1.31



Найти: $AB = r(\varphi)$, $\varphi_0 \neq 0$

$$V_r = \dot{r} = V_1\cos\varphi - V_2 = \frac{dr}{dt}$$

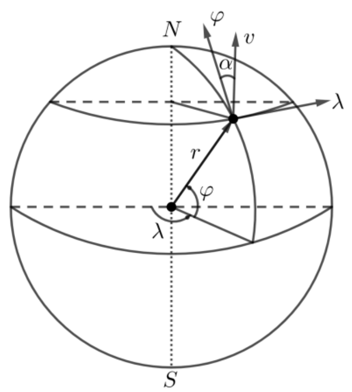
$$V_\varphi = r\dot{\varphi} = -V_1\sin\varphi = r\cdot\frac{d\varphi}{dt}$$

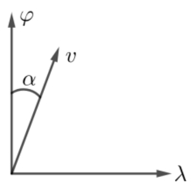
$$\frac{dr}{r} = \frac{V_1\cos\varphi - V_2}{-V_1\sin\varphi}d\varphi \Rightarrow$$

$$\Rightarrow \ln r - \ln c = \frac{V_2}{V_1}\ln\frac{1-\cos\varphi}{\sin\varphi} - \ln\sin\varphi = \frac{V_2}{V_1}\ln tg\frac{\varphi}{2} - \ln\sin\varphi$$

$$\Rightarrow \text{Ответ: } \frac{r}{r_0} = \frac{\sin\varphi_0}{\sin\varphi}\left(\frac{tg\,\varphi/2}{tg\,\varphi_0/2}\right)^{V_2/V_1}$$

№T1





$$\bar{r} = \begin{bmatrix} r\cos\varphi\cos\lambda \\ r\cos\varphi\sin\lambda \\ r\sin\varphi \end{bmatrix}$$

$V = const$, $\alpha = \widehat{\varphi, V} = const$

Найти: ур-ие кривой; W; $|W|$; ρ

$$\begin{matrix} V_\varphi = V\cos\alpha = \frac{d\varphi}{dt}r \\ V_\lambda = V\sin\alpha = \frac{d\lambda}{dt}r\cos\varphi \end{matrix} \Rightarrow \begin{cases} \frac{d\varphi}{dt} = \frac{V\cos\alpha}{r} \\ \frac{d\lambda}{dt} = \frac{V\sin\alpha}{r\cos\varphi} \end{cases} \Rightarrow \frac{d\varphi}{d\lambda} = \frac{\cos\alpha\cos\varphi}{\sin\alpha}$$

$$\int\frac{d\varphi}{\cos\varphi} = \int\frac{\cos\alpha}{\sin\alpha}d\lambda \Rightarrow \ln\frac{tg(\varphi/2+\pi/4)}{tg(\varphi_0/2+\pi/4)} = ctg\,\alpha(\lambda-\lambda_0)$$

$$\Rightarrow \underline{\text{ур-ие кривой: } tg\left(\varphi/2+\frac{\pi}{4}\right) = tg\left(\frac{\varphi_0}{2}+\frac{\pi}{4}\right)\cdot\exp\left(ctg\,\alpha(\lambda-\lambda_0)\right)}$$

Из геом. соображ.: $H_r = 1$; $H_\varphi = r$; $H_\lambda = r\cos\varphi$

$$V_r = \dot{r}\ ; \ V_\varphi = r\dot{\varphi}\ ; \ V_\lambda = \dot{\lambda}r\cos\varphi$$

$$V^2 = \dot{r}^2 + r^2\dot{\varphi}^2 + r^2\cos^2\varphi\,\dot{\lambda}^2$$

$$W_r = \frac{1}{H_r}\left[\left(\frac{v^2}{2}\right)^{\cdot}_{,\dot r} - \left(\frac{v^2}{2}\right)_{,r}\right] = \ddot r - r\dot\varphi^2 - r\cos^2\varphi\cdot\dot\lambda^2 = -r\frac{v^2\cos^2\alpha}{r^2} - r\cdot\frac{v^2\sin^2\alpha}{r^2\cos^2\varphi}\cdot\cos^2\varphi = -\frac{v^2}{r}$$

$$W_\varphi = \frac{1}{H_\varphi}\left[r^2\ddot\varphi + 2r\dot r\dot\varphi + r^2\dot\lambda^2\cos\varphi\sin\varphi\right] = r\ddot\varphi + 2\dot r\dot\varphi + r\dot\lambda^2\cos\varphi\sin\varphi = \frac{v^2\sin^2\alpha\cdot \mathrm{tg}\,\varphi}{r}$$

$$W_\lambda = \frac{r^2}{r\cos\varphi}\left[\ddot\lambda\cos^2\varphi - 2\cos\varphi\sin\varphi\,\dot\varphi\dot\lambda\right] = r(\ddot\lambda\cos\varphi - 2\sin\varphi\,\dot\varphi\dot\lambda) = \frac{1}{2r}v^2\mathrm{tg}\,\varphi\sin 2\varphi - \frac{1}{r}v^2\mathrm{tg}\,\varphi\sin 2\alpha = -\frac{v^2\,\mathrm{tg}\,\varphi\sin 2\alpha}{2r}$$

$$W = \sqrt{\frac{v^4}{r^2} + \frac{v^4\sin^4\alpha\,\mathrm{tg}^2\varphi}{r^2} + \frac{v^4\mathrm{tg}^2\varphi\sin^2 2\alpha}{4r^2}} = \frac{v^2}{r}\sqrt{1+\sin^2\alpha\,\mathrm{tg}^2\varphi}$$

$$v = \mathrm{const} \to W = W_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{r}{\sqrt{1+\sin^2\alpha + \mathrm{tg}^2\varphi}}$$

Ответ: ур-ие: $\mathrm{tg}\left(\frac{\varphi}{2}+\frac{\pi}{4}\right) = \mathrm{tg}\left(\frac{\varphi_0}{2}+\frac{\pi}{4}\right)\cdot\exp\left(\mathrm{ctg}\,\alpha(\lambda-\lambda_0)\right)$;

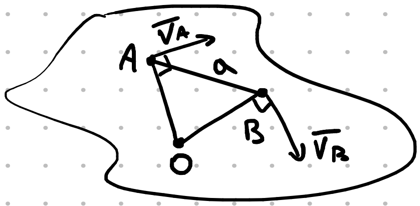
$W_r = -\frac{v^2}{r}$; $W_\varphi = \frac{v^2\sin^2\alpha\cdot\mathrm{tg}\,\varphi}{r}$; $W_\lambda = -\frac{v^2\,\mathrm{tg}\,\varphi\sin 2\alpha}{2r}$;

$W = \frac{v^2}{r}\sqrt{1+\sin^2\alpha + \mathrm{tg}^2\varphi}$; $\rho = \frac{r}{\sqrt{1+\sin^2\alpha+\mathrm{tg}^2\varphi}}$

2. Кинематика твердого тела

2.1. Плоскопараллельное движ.

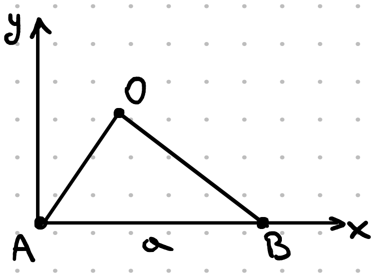
№3.2 Найти: геом. место возможных полож. мгн. центра скорост., если $\lambda \neq 1$; $\lambda = \frac{V_A}{V_B}$; $|AB| = a$



Пусть O – мгн. ц. скорост.
Тогда $\overline{OA} \perp \overline{V_A}$; $\overline{OB} \perp \overline{V_B}$

$$\frac{OA}{OB} = \frac{V_A}{V_B} = \lambda$$

Перейдем в СО, где $A(0,0)$; $B(a,0)$



$$\begin{cases} x^2+y^2 = AO^2 = \lambda^2 \cdot BO^2 \\ (a-x)^2 + y^2 = BO^2 \end{cases}$$

$$x^2+y^2 = \lambda^2 y^2 + \lambda^2 x^2 - 2\lambda^2 a x + \lambda^2 a^2$$

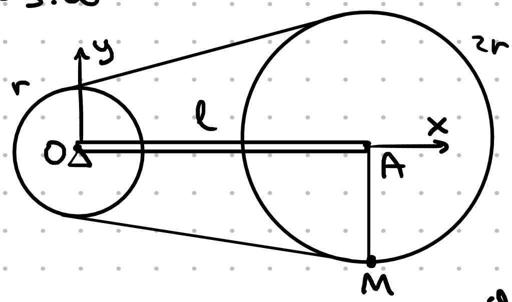
$$x^2(1-\lambda^2) + 2\lambda a^2 x + y^2(1-\lambda^2) = \lambda^2 a^2$$

$$x^2 + \frac{2\lambda^2 a x}{1-\lambda^2} + \frac{\lambda^4 a^2}{(1-\lambda^2)^2} + y^2 = \frac{\lambda^2 a^2}{1-\lambda^2} + \frac{\lambda^4 a^2}{(1-\lambda^2)^2}$$

$$\Rightarrow \left(x + \frac{\lambda^2 a}{1-\lambda^2}\right)^2 + y^2 = \left(\frac{\lambda a}{1-\lambda^2}\right)^2$$

Ответ: $\left(x + \frac{\lambda^2 a}{1-\lambda^2}\right)^2 + y^2 = \left(\frac{\lambda a}{1-\lambda^2}\right)^2$

№3.20



Приводится: l, ω_0, ε_0
r, $R = 2r$,
$AM \perp OA$

Найти: ω_M, V_M?

$$\overline{V_A} = \cancel{\overline{V_0}}^{\,0} + \overline{\omega_0} \times \overline{OA} = \begin{pmatrix}0\\0\\\omega_0\end{pmatrix} \times \begin{pmatrix}l\\0\\0\end{pmatrix} = \begin{pmatrix}0\\\omega_0 l\\0\end{pmatrix}$$

Путь пройденный цепью за время t:

$$a_1 = \omega_0 r t = a_2 = \omega_A 2 r t \Rightarrow \omega_A = \frac{\omega_0}{2}$$

$$\overline{V_M} = \overline{V_A} + \overline{\omega_A} \cdot \overline{AM} = \begin{pmatrix}0\\\omega_0 l\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\omega_0/2\end{pmatrix} \times \begin{pmatrix}0\\-2r\\0\end{pmatrix} = \begin{pmatrix}\omega_0 r\\\omega_0 l\\0\end{pmatrix} \Rightarrow V_M = \omega_0 \sqrt{r^2 + l^2}$$

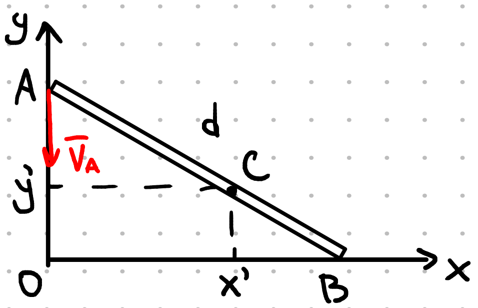
$$\overline{W_A} = \cancel{\overline{W_0}}^{\,0} + \overline{\varepsilon_0} \times \overline{OA} - \omega_0^2 \overline{OA} = \begin{pmatrix}0\\0\\\varepsilon_0\end{pmatrix}\begin{pmatrix}l\\0\\0\end{pmatrix} - \begin{pmatrix}\omega_0^2 l\\0\\0\end{pmatrix} = \begin{pmatrix}-\omega_0^2 l\\\varepsilon_0 l\\0\end{pmatrix}; \quad \overline{\varepsilon_A} = \frac{d\omega_A}{dt} = \frac{d\omega_0}{dt} \cdot \frac{1}{2} = \frac{\varepsilon_0}{2}$$

$$\overline{W_M} = \overline{W_A} + \overline{\varepsilon_A} \cdot \overline{AM} - \omega_A^2 \overline{AM} = \begin{pmatrix}-\omega_0^2 l\\\varepsilon_0 l\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\varepsilon_0/2\end{pmatrix}\begin{pmatrix}0\\-2r\\0\end{pmatrix} + \begin{pmatrix}0\\\omega_0^2 r/2\\0\end{pmatrix} = \begin{pmatrix}\varepsilon_0 r - \omega_0^2 l\\\varepsilon_0 l + \omega_0^2 r/2\\0\end{pmatrix}$$

$$\Rightarrow W_M = \sqrt{(\varepsilon_0 r - \omega_0^2 l)^2 + \left(\varepsilon_0 l + \frac{\omega_0^2 r}{2}\right)^2}$$

Ответ: $V_M = \omega_0 \sqrt{r^2 + l^2}$; $W_M = \sqrt{(\varepsilon_0 r - \omega_0^2 l)^2 + \left(\varepsilon_0 l + \frac{\omega_0^2 r}{2}\right)^2}$

№3.21



$V_A = const$; $AB = d$

Показать, что $W_C \perp Oy$ и $W_C \propto \frac{1}{(\rho(C, O_y))^3}$

$\forall C$ – т. стержня

$$\bar V_B = \bar V_A + \bar\omega \times \overline{AB} = \begin{pmatrix}0\\-V_A\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\omega\end{pmatrix} \times \begin{pmatrix}x\\-y\\0\end{pmatrix} = \begin{pmatrix}y\omega\\\omega x - V_A\\0\end{pmatrix} = \begin{pmatrix}V_B\\0\\0\end{pmatrix}$$

$$\begin{cases} V_B = y\omega \\ V_A = x\omega \end{cases} \Rightarrow \omega = \frac{V_A}{x},\ V_B = \frac{y}{x}V_A \Rightarrow \bar V_B = \begin{pmatrix}\frac{y}{x}V_A\\0\\0\end{pmatrix}$$

$$\bar W_B = \cancel{\bar W_A}^{0} + \bar\varepsilon \times \overline{AB} - \omega^2 \overline{AB} = \begin{pmatrix}0\\0\\\varepsilon\end{pmatrix} \times \begin{pmatrix}x\\-y\\0\end{pmatrix} + \begin{pmatrix}-\omega^2 x\\\omega^2 y\\0\end{pmatrix} = \begin{pmatrix}y\varepsilon - \omega^2 x\\x\varepsilon + \omega^2 y\\0\end{pmatrix} = \begin{pmatrix}W_B\\0\\0\end{pmatrix}$$

$$\dot V_B = \frac{d\left(\frac{y}{x}V_A\right)}{dt} = V_A\left(\frac{V_A}{x} - \frac{y V_B}{x^2}\right) = \frac{V_A^2}{x}\left(1 + \frac{y^2}{x^2}\right) = \frac{V_A^2}{x^3} d^2$$

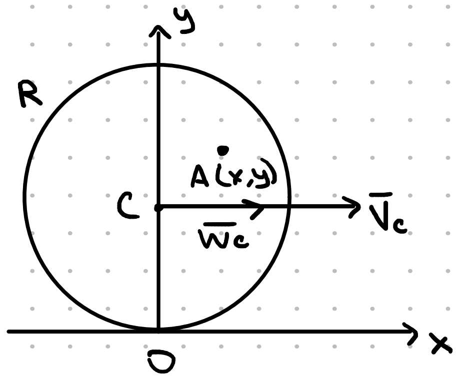
$$\begin{cases} y\varepsilon = -\frac{V_A^2}{x} = -\frac{V_A^2 d^2}{x^3} \\ x\varepsilon = -\frac{y V_A^2}{x^2} \end{cases} \Rightarrow \varepsilon = -\frac{y V_A^2}{x^3}$$

$$\bar W_C = \bar\varepsilon \times \overline{AC} - \omega^2 \overline{AC} = \begin{pmatrix}0\\0\\-y\frac{V_A^2}{x^3}\end{pmatrix}\begin{pmatrix}x'\\y'-y\\0\end{pmatrix} + \begin{pmatrix}-\omega^2 x'\\\omega^2 (y-y')\\0\end{pmatrix} = \begin{pmatrix}-\frac{V_A^2 x'}{x^2} + \frac{V_A^2 y}{x^3}(y'-y)\\-\frac{V_A^2}{x^2}(y'-y) - y V_A^2 \cdot \frac{x'}{x^3}\\0\end{pmatrix} = \frac{V_A^2}{x^2}\begin{pmatrix}-x' - \frac{(y-y')^2}{x'}\\-(y'-y) - (y-y')\\0\end{pmatrix} =$$

$$= \frac{V_A^2}{x^2}\begin{pmatrix}\frac{-x'^2 - (y-y')^2}{x'}\\0\\0\end{pmatrix} = -V_A^2\begin{pmatrix}\underbrace{\frac{d'^2}{x'^3}\ \frac{(y-y')^2}{y^2}}_{=\,const}\\0\\0\end{pmatrix} = -\begin{pmatrix}\frac{k \cdot V_A^2}{x'^3}\\0\\0\end{pmatrix} \Rightarrow$$

$$\Rightarrow \bar W_C \perp Oy;\ \bar W_C \propto \frac{1}{x'^3} \quad \text{— ч.т.д.}$$

№3.25



$R, V_C, W_C, A(x,y)$ $x \neq 0$ $y \neq 0$

Найти: $W_{An}, W_{A\tau}$

$$\bar V_O = \bar V_C + \bar\omega \times \overline{CO} = \bar 0 \Rightarrow \bar V_C = \bar\omega \times \overline{OC}$$

$$\begin{pmatrix}V_C\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\\omega\end{pmatrix}\begin{pmatrix}0\\R\\0\end{pmatrix} = \begin{pmatrix}-\omega R\\0\\0\end{pmatrix} \Rightarrow \omega = \frac{-V_C}{R};\ V_C = -\omega R$$

$$\varepsilon = \dot\omega = -\frac{W_C}{R}$$

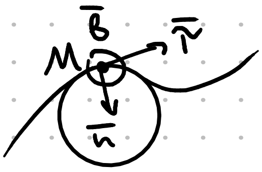
$$\bar V_A = \bar V_C + \bar\omega \cdot \overline{CA} = \begin{pmatrix}V_C\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-V_C/R\end{pmatrix}\cdot\begin{pmatrix}x\\y-R\\0\end{pmatrix} = \begin{pmatrix}V_C y/R\\V_C x/R\\0\end{pmatrix} = V_A\begin{pmatrix}\frac{y}{\sqrt{x^2+y^2}}\\\frac{x}{\sqrt{x^2+y^2}}\end{pmatrix} = V_A\bar\tau$$

$$\bar W_A = \bar W_C + \bar\varepsilon \times \overline{CA} + \bar\omega[\bar\omega \times \overline{CA}] = \begin{pmatrix}W_C\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-W_C/R\end{pmatrix}\begin{pmatrix}x\\y-R\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-\frac{V_C}{R}\end{pmatrix}\begin{pmatrix}V_C y/R\\V_C x/R\\0\end{pmatrix} =$$

$$= \begin{pmatrix}W_C y/R\\W_C x/R\\0\end{pmatrix} + \begin{pmatrix}V_C^2 x/R^2\\V_C^2 y/R^2\\0\end{pmatrix} = \frac{W_C}{R}\sqrt{x^2+y^2}\,\bar\tau + \frac{V_C^2}{R^2}\sqrt{x^2+y^2}\,\bar n = W_{A\tau}\bar\tau + W_{An}\bar n$$

Ответ: $W_{A\tau} = \frac{W_C}{R}\sqrt{x^2+y^2}$, $W_{An} = \frac{V_C^2}{R^2}\sqrt{x^2+y^2}$

№3.36 $\bar V(t), \rho(t)$ Найти: ω, ε трехгранника $(\bar\tau, \bar n, \bar b)$



$\bar V = \omega \bar b \times \rho \bar n$; $\bar\omega = \omega \bar b$

$\omega[\bar b \times n] = \frac{\bar V}{\rho} \;\; |\cdot \bar\tau$

$\omega = \frac{(V,\tau)}{\rho} \;\; |\cdot \bar b$

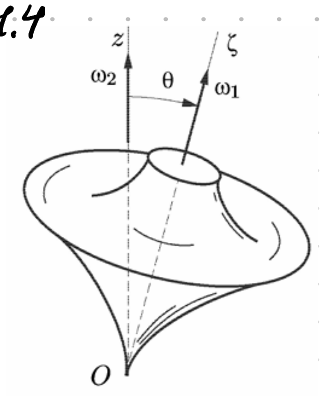
$\bar\omega = \frac{(\bar V,\tau)}{\rho}\bar b$

$$\bar\varepsilon = \dot{\bar\omega} = [((\dot V,\tau)+(V,\dot\tau))\rho - (V,\tau)\dot\rho]\frac{\bar b}{\rho^2} = \bar b\left(\frac{(\dot V,\tau)\rho - (V,\tau)\dot\rho}{\rho^2}\right) = \bar b\left(\frac{(\dot V,\tau)}{\rho} - \frac{(V,\tau)\dot\rho}{\rho^2}\right)$$

Ответ: $\bar\omega = \frac{(\bar V,\tau)}{\rho}\bar b$; $\bar\varepsilon = \bar b\left(\frac{(\dot V,\tau)}{\rho} - \frac{(V,\tau)\dot\rho}{\rho^2}\right)$

2.2. Пространственное движение

№4.4



Дано: $\omega_1, \omega_2, \Theta$; Найти: ω, ε отн. Oz

$\bar\omega = \bar\omega_1 + \bar\omega_2$

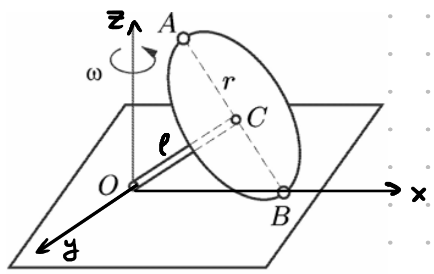
$\Rightarrow \omega = |\bar\omega| = \sqrt{\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\cos\Theta}$

$\bar\varepsilon = \cancel{\bar\varepsilon_1}^{\bar 0} + \cancel{\bar\varepsilon_2}^{\bar 0} + \bar\omega_1 \times \bar\omega_2 = \bar\omega_1 \times \bar\omega_2$

$\Rightarrow \varepsilon = |\bar\varepsilon| = \omega_1\omega_2\sin\Theta$

Ответ. $\omega = \sqrt{\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\cos\Theta}$; $\varepsilon = \omega_1\omega_2\sin\Theta$

№4.10



Дано: r, $l = r\sqrt3$, ε, ω стержня Найти: $\omega_д$, $\varepsilon_д$, W_A, W_B

1) $V_B = \bar 0 = \bar V_C + \bar\omega_{диск} \times \overline{CB} = \bar\omega_0 \times \overline{OC} + \bar\omega_{диск} \times \overline{CB}$

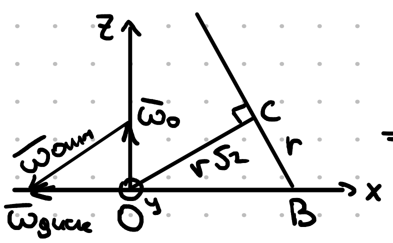
$\bar V_C = \bar V_0 + \bar\omega_0 \times \overline{OC}$

$\Rightarrow \bar\omega_0 \times \overline{OC} = \bar\omega_{диск} \times \overline{BC}$

2) $\bar\omega_{диск} = \bar\omega_0 + \bar\omega_{отн}$; $\cancel{\bar V_B}^0 = \cancel{\bar V_0}^0 + \omega_{диск} \times \overline{OB} \Rightarrow \omega_{диск} \parallel \overline{OB}$

$\bar\omega_{отн} \parallel \overline{OC}$, $\bar\omega_{диск} \parallel \overline{OB}$, $\bar\omega_0 \parallel Oz$

$\omega_{диск} = \omega_0 \cdot ctg30 = \sqrt3\omega_0$; $\omega_{отн} = \frac{\omega_0}{\sin30} = 2\omega_0$



$\Rightarrow \angle COB = arctg\frac{\sqrt3}{3} = 30°$

$\angle(\omega_{отн}, \omega_{диск}) = \angle COB = 30°$

3) $\bar\varepsilon_{диск} = \bar\varepsilon_0 + \bar\varepsilon_{отн} + \bar\omega_0 \times \bar\omega_{отн}$; $\varepsilon_{отн} = \dot\omega_{отн} = 2\dot\omega_0 = 2\varepsilon_0$

$$\bar\varepsilon_{диск} = \begin{bmatrix} 0 \\ 0 \\ \varepsilon_0 \end{bmatrix} + \begin{bmatrix} -2\varepsilon_0\cos30 \\ 0 \\ -2\varepsilon_0\sin30 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_0 \end{bmatrix} \times \begin{bmatrix} -2\omega_0\cos30 \\ 0 \\ -2\omega_0\sin30 \end{bmatrix} = \begin{bmatrix} \sqrt3\varepsilon_0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -\sqrt3\omega_0^2 \\ 0 \end{bmatrix} = -\begin{bmatrix} \sqrt3\varepsilon_0 \\ \sqrt3\omega_0^2 \\ 0 \end{bmatrix}$$

$\Rightarrow \varepsilon_{диск} = \sqrt{3(\varepsilon_0^2 + 3\omega_0^4)}$

4) $\bar W_A = \cancel{\bar W_0}^0 + \bar\varepsilon_{диск} \times \overline{OA} + \bar\omega_{диск} \times [\bar\omega_{диск} \times \overline{OA}]$

$$\overline{OA} = \overline{OC} + \overline{OA} = \begin{bmatrix} l\cos30 \\ 0 \\ l\sin30 \end{bmatrix} + \begin{bmatrix} -r\sin30 \\ 0 \\ r\cos30 \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ \sqrt{3}r \end{bmatrix}$$

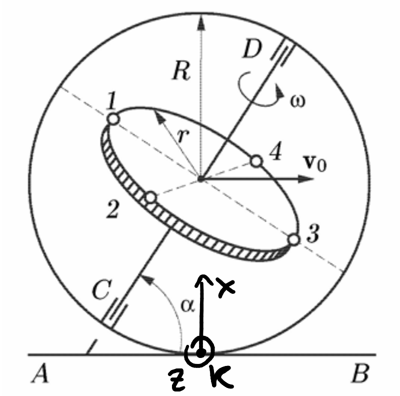
$$\overline{W}_A = -\begin{bmatrix} \sqrt{3}\varepsilon_0 \\ \sqrt{3}\omega_0^2 \\ 0 \end{bmatrix} \times \begin{bmatrix} r \\ 0 \\ \sqrt{3}r \end{bmatrix} + \begin{bmatrix} -\sqrt{3}\omega_0 \\ 0 \\ 0 \end{bmatrix} \times \left[\begin{pmatrix} -\sqrt{3}\omega_0 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ \sqrt{3}r \end{pmatrix} \right] = \begin{pmatrix} -3r\omega_0^2 \\ 3r\varepsilon_0 \\ \sqrt{3}r\omega_0^2 \end{pmatrix} + \begin{pmatrix} -\sqrt{3}\omega_0 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3\omega_0 r \\ 0 \end{pmatrix} =$$

$$= \begin{bmatrix} -3r\omega_0^2 \\ 3r\varepsilon_0 \\ -2\sqrt{3}r\omega_0^2 \end{bmatrix} \Rightarrow W_A = \sqrt{9r^2\omega_0^4 + 9r^2\varepsilon_0^2 + 12r^2\omega_0^4} = r\sqrt{9\varepsilon_0^2 + 21\omega^4}$$

$$\overline{W}_B = \cancel{\overline{W}_0}^{\,0} + \overline{\varepsilon}_{абс} \times \overline{OB} + \overline{\omega}_{абс} \times [\overline{\omega}_{абс} \times \overline{OB}] = \begin{bmatrix} -\sqrt{3}\varepsilon_0 \\ -\sqrt{3}\omega_0^2 \\ 0 \end{bmatrix} \times \begin{bmatrix} 2r \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2\sqrt{3}r\omega_0^2 \end{bmatrix};\ W_B = 2\sqrt{3}r\omega_0^2$$

Ответ: $\omega_g = \sqrt{3}\omega_0$; $\varepsilon_g = \sqrt{3(\varepsilon_0^2 + \omega_0^4)}$;
$W_A = r\sqrt{9\varepsilon_0^2 + 21\omega^4}$; $W_B = 2\sqrt{3}r\omega_0^2$

№4.12 (для 1 т.)



Дано: $R, V_0, \omega, r, \alpha$; Найти: V_1, W_1

$\overline{\omega}_{отн} = \overline{\omega}$; $\overline{\omega}_{пер} = \overline{\omega}_{обруч} = \omega_0$; $\overline{V}_K = \overline{0}$

$\overline{V}_K = \overline{V}_0 + \overline{\omega}_0 \times \overline{OK} = \overline{0} \to \overline{V}_0 = \overline{V} = \overline{\omega}_0 \times \overline{KO}$

$$\begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega_0 \end{pmatrix} \times \begin{pmatrix} 0 \\ R \\ 0 \end{pmatrix} = \begin{pmatrix} -\omega_0 R \\ 0 \\ 0 \end{pmatrix} \Rightarrow \omega_0 = -\frac{V}{R}$$

$$\overline{\omega}_{абс} = \overline{\omega}_{пер} + \overline{\omega}_{отн} = \overline{\omega} + \overline{\omega}_0 = \begin{pmatrix} \omega\cos\alpha \\ \omega\sin\alpha \\ -V/R \end{pmatrix}$$

$$\overline{V}_1 = \overline{V}_0 + \overline{\omega}_{абс} \times \overline{O1} = \begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega\cos\alpha \\ \omega\sin\alpha \\ -V/R \end{pmatrix} \times \begin{pmatrix} -r\sin\alpha \\ r\cos\alpha \\ 0 \end{pmatrix} = \begin{pmatrix} V + V\frac{r}{R}\cos\alpha \\ \frac{Vr}{R}\sin\alpha \\ \omega r \end{pmatrix}$$

$$\Rightarrow V_1 = \sqrt{V^2 + 2V^2\frac{r}{R}\cos\alpha + \frac{V^2r^2}{R^2} + \omega^2r^2}$$

$$\overline{\varepsilon}_{абс} = \overline{\omega}_{пер} \times \overline{\omega}_{отн} + \cancel{\overline{\varepsilon}_{пер}}^{\,0} + \cancel{\overline{\varepsilon}_0}^{\,0} = \begin{pmatrix} 0 \\ 0 \\ -V/R \end{pmatrix} \times \begin{pmatrix} \omega\cos\alpha \\ \omega\sin\alpha \\ 0 \end{pmatrix} = \begin{pmatrix} V/R \cdot \omega\sin\alpha \\ -V/R \cdot \omega\cos\alpha \\ 0 \end{pmatrix}$$

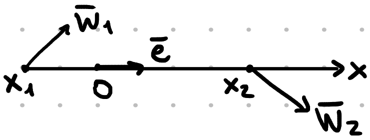
$$\overline{W}_1 = \cancel{\overline{W}_0}^{\,0} + \overline{\varepsilon}_{абс} \times \overline{O1} + \overline{\omega}_{абс} \times [\overline{\omega}_{абс} \times \overline{O1}] = \begin{pmatrix} V/R\sin\alpha \\ -V/R\cos\alpha \\ 0 \end{pmatrix} \times \begin{pmatrix} -r\sin\alpha \\ r\cos\alpha \\ 0 \end{pmatrix} +$$

$$+ \begin{pmatrix} \omega\cos\alpha \\ \omega\sin\alpha \\ -V/R \end{pmatrix} \times \left[\begin{pmatrix} \omega\cos\alpha \\ \omega\sin\alpha \\ -V/R \end{pmatrix} \times \begin{pmatrix} -r\sin\alpha \\ r\cos\alpha \\ 0 \end{pmatrix} \right] = \begin{pmatrix} \omega\cos\alpha \\ \omega\sin\alpha \\ -V/R \end{pmatrix} \times \begin{pmatrix} V/R\, r\cos\alpha \\ V/R \cdot r\sin\alpha \\ \omega r \end{pmatrix} = \begin{pmatrix} \omega^2 r\sin\alpha + V^2/R^2\, r\sin\alpha \\ -\omega^2 r\cos\alpha - V^2/R^2\, r\cos\alpha \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} W_1\sin\alpha \\ W_1\cos\alpha \\ 0 \end{pmatrix} \Rightarrow W_1 = \sqrt{\left(\omega^2 r + \frac{V^2}{R^2}r\right)^2\sin^2\alpha + \left(\omega^2 r + \frac{V^2}{R^2}r\right)^2\cos^2\alpha} = \omega^2 r + \frac{V^2}{R^2}r = r\left(\omega^2 + \frac{V^2}{R^2}\right)$$

Ответ: $V_1 = \sqrt{V^2 + 2V^2\frac{r}{R}\cos\alpha + \left(\frac{Vr}{R}\right)^2 + \omega^2r^2}$; $W_1 = r\left(\omega^2 + \frac{V^2}{R^2}\right)$

№4.23 Дано: x_1, x_2, W_1, W_2 Найти: W_x



$$\overline{W}_2 = \overline{W}_1 + \overline{\varepsilon}\times\overline{x_1x_2} + \overline{\omega}[\overline{\omega}\times\overline{x_1x_2}]$$

$$\begin{cases} \overline{W}_2 = \overline{W}_1 + (x_2-x_1)\overline{\varepsilon}\times\overline{e} + (x_2-x_1)\overline{\omega}\times[\overline{\omega}\times\overline{e}] \\ \overline{W}_x = \overline{W}_1 + (x-x_1)\overline{\varepsilon}\times\overline{e} + (x-x_1)\overline{\omega}\times[\overline{\omega}\times\overline{e}] \\ \overline{W}_2 = \overline{W}_x + (x_2-x)\overline{\varepsilon}\times\overline{e} + (x_2-x)\overline{\omega}\times[\overline{\omega}\times\overline{e}] \end{cases}$$

$$\begin{cases} \overline{W}_x(x_2-x) = \overline{W}_1(x_2-x) + (x-x_1)(x_2-x)[\overline{\varepsilon}\times\overline{e} + \overline{\omega}\times[\overline{\omega}\times\overline{e}]] \\ \overline{W}_2(x-x_1) = \overline{W}_x(x-x_1) + (x-x_1)(x_2-x)[\overline{\varepsilon}\times\overline{e} + \overline{\omega}\times[\overline{\omega}\times\overline{e}]] \end{cases}$$

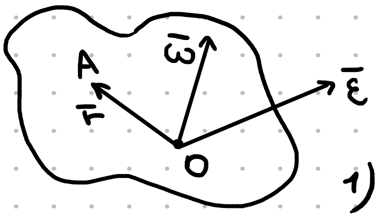
$$\overline{W}_x(x_2-x+x-x_1) = \overline{W}_1(x_2-x) + \overline{W}_2(x-x_1)$$

$$\Rightarrow \overline{W}_x = \overline{W}_1\frac{x_2-x}{x_2-x_1} + \overline{W}_2\frac{x-x_1}{x_2-x_1}$$

Ответ: $\overline{W}_x = \overline{W}_1\frac{x_2-x}{x_2-x_1} + \overline{W}_2\frac{x-x_1}{x_2-x_1}$

№4.30 Дано: ε, ω Д-ать: $\overline{W}_{вр} = \overline{W}_{кас}$; $\overline{W}_{ос} = \overline{W}_{норм}$

тогда если т. в пл. ε, ω



$$\begin{cases} \overline{W} = \cancel{\overline{W}_0} + \overline{\varepsilon}\times\overline{r} + \overline{\omega}\times[\overline{\omega}\times\overline{r}] = \overline{W}_{вр} + \overline{W}_{ос} \\ \overline{W} = \dot{v}\overline{\tau} + v^2/r\,\overline{n} = \overline{W}_{кас} + \overline{W}_{норм} \end{cases}$$

1) Пусть $\overline{\varepsilon}\times\overline{r} = \dot{v}\overline{\tau}$; $\overline{\omega}\times[\overline{\omega}\times\overline{r}] = \frac{v^2}{r}\overline{n}$

$\overline{\tau} = \frac{\overline{v}}{v} = \frac{\overline{\omega}\times\overline{r}}{|\overline{\omega}\times\overline{r}|} \Rightarrow \overline{\omega}\perp\overline{\tau},\ \overline{r}\perp\overline{\tau} \Rightarrow (\overline{\omega}\cdot[\overline{\varepsilon}\times\overline{r}]) - \dot{v}(\overline{\omega}\cdot\overline{\tau}) = 0$

Смеш. произв. $(\overline{\omega}\cdot[\overline{\varepsilon}\times\overline{r}]) = 0 \Rightarrow \overline{\omega}, \overline{\varepsilon}, \overline{r}$ лежат в одной пл.

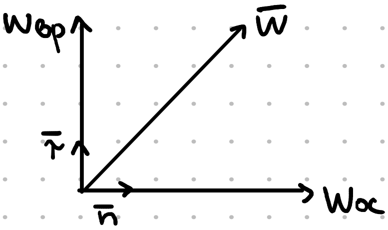
ч.т.д

2) Пусть $\overline{\omega}, \overline{\varepsilon}, \overline{r}$ лежат в одной плоскости

$\overline{\omega}\times[\overline{\omega}\times\overline{r}] \perp \overline{\omega}\times\overline{r} = \lambda[\overline{\varepsilon}\times\overline{r}] \Rightarrow \overline{W}_{вр} \perp \overline{W}_{ос}$.

$\overline{\tau} = \frac{\overline{\omega}\times\overline{r}}{|\overline{\omega}\times\overline{r}|} \parallel \overline{\varepsilon}\times\overline{r} = \overline{W}_{вр}$; $\overline{W} = \overline{W}_{кас} + \overline{W}_{норм} \Rightarrow \overline{W}_{норм}$ лежит в пл. $\overline{W}$ и $\overline{W}_{кас}$

$\overline{W}_{норм} \perp \overline{W}_{кас} \Rightarrow \overline{W}_{норм} \perp \overline{W}_{ос}$

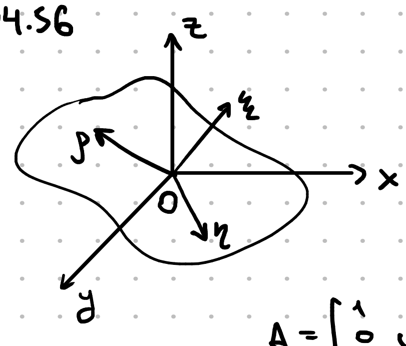


Из геом. сообр: $|\overline{W}_{вр}| = |\overline{W}_{кас}|$, $|\overline{W}_{ос}| = |\overline{W}_{норм}|$

$\Rightarrow$ $\overline{W}_{вр} = \overline{W}_{кас}$; $\overline{W}_{ос} = \overline{W}_{норм}$

ч.т.д

№4.56



Найдём собств. вект. для матр. $A(t)$

$A\overline{u} = \lambda\overline{u}$: $A\overline{u}_1 = \overline{u}_1$; $A\overline{u}_2 = \overline{u}_2\cos\varphi + \overline{u}_3\sin\varphi$;

$A\overline{u}_3 = \overline{u}_3\cos\varphi - \overline{u}_2\sin\varphi$

Направим оси по $\overline{u}_1, \overline{u}_2, \overline{u}_3$:

$$\overline{u}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad \overline{u}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \overline{u}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$A = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi\end{bmatrix}$$ — получили матр. плоского поворота вокруг $\overline{u}_1$ на φ

ч.т.д

2.3 Кватернионы

№4.66 Дано: ψ, θ, φ Найти: угол поворота, напр. косинусы

Посл. т. зр:

$$\Lambda_1 = \cos\frac{\psi}{2} + \bar{i}_3 \sin\frac{\psi}{2}$$
$$\Lambda_2 = \cos\theta/2 + \bar{i}_1 \sin\theta/2$$
$$\Lambda_3 = \cos\varphi/2 + \bar{i}_3 \sin\varphi/2$$

$$\Lambda \circ M = \lambda_0 \mu_0 - \bar{\lambda}\cdot\bar{\mu} + \lambda_0 \bar{\mu} + \mu_0 \bar{\lambda} + \bar{\lambda} \times \bar{\mu}$$
$$\Lambda_1 \circ \Lambda_2 = \cos\frac{\psi}{2}\cdot\cos\theta/2 - (\bar{i}_3 \cdot \bar{i}_1)\sin\frac{\psi}{2}\sin\frac{\theta}{2} + \cos\frac{\psi}{2}\sin\frac{\theta}{2}\,\bar{i}_1 + (\bar{i}_3 \times \bar{i}_1)\sin\frac{\psi}{2}\sin\frac{\theta}{2}$$

$$\Lambda = \Lambda_1 \circ \Lambda_2 \circ \Lambda_3 = (\cos\psi/2 + \bar{i}_3 \sin\psi/2)(\cos\theta/2 \cos\varphi/2 + \bar{i}_3 \cos\theta/2 \sin\varphi/2 + \bar{i}_1 \sin\theta/2 \cos\varphi/2 - \bar{i}_2 \sin\theta/2 \sin\varphi/2) =$$
$$= \cos\frac{\psi}{2}\cos\theta/2\cos\varphi/2 - \sin\psi/2 \cos\theta/2 \sin\varphi/2 + \bar{i}_3 \cos\frac{\psi}{2}\cos\theta/2 \sin\varphi/2 + \bar{i}_3 \sin\psi/2 \cos\theta/2 \cos\varphi/2 +$$
$$+ \bar{i}_1 \cos\psi/2 \sin\theta/2 \cos\varphi/2 - \bar{i}_2 \cos\psi/2 \sin\theta/2 \sin\varphi/2 + \bar{i}_2 \sin\psi/2 \sin\theta/2 \cos\varphi/2 + \bar{i}_1 \sin\psi/2 \sin\theta/2 \sin\varphi/2 =$$
$$= \cos\theta/2 \cos\left(\frac{\psi+\varphi}{2}\right) + \bar{i}_1 \sin\theta/2 \cos\left(\frac{\psi-\varphi}{2}\right) + \bar{i}_2 \sin\theta/2 \sin\frac{\psi-\varphi}{2} + \bar{i}_3 \cos\theta/2 \sin\frac{\psi+\varphi}{2}$$

$$\sqrt{\sin^2\theta/2 \cos^2\frac{\psi-\varphi}{2} + \sin^2\theta/2 \sin^2\frac{\psi-\varphi}{2} + \cos^2\theta/2 \sin^2\frac{\psi+\varphi}{2}} = \sqrt{\sin^2\theta/2 + \cos^2\theta/2 \sin^2\frac{\psi+\varphi}{2}} =$$
$$= \sqrt{1 - \cos^2\theta/2 + \cos^2\theta/2 \sin^2\frac{\psi+\varphi}{2}} = \sqrt{1 - \cos^2\frac{\theta}{2}\cos^2\frac{\psi+\varphi}{2}}$$

Ответ: угол поворота $\alpha = 2\arccos\left(\cos\theta/2 \cdot \cos\frac{\psi+\varphi}{2}\right)$;

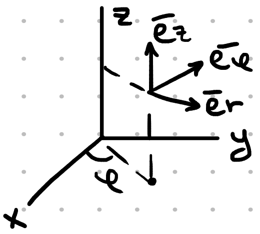
напр. косинусы: $\dfrac{\sin\theta/2 \cos\frac{\psi-\varphi}{2}}{\sqrt{1-\cos^2\theta/2 \cos^2\frac{\psi+\varphi}{2}}}$, $\dfrac{\sin\theta/2 \sin\frac{\psi-\varphi}{2}}{\sqrt{1-\cos^2\theta/2 \cos^2\frac{\psi+\varphi}{2}}}$, $\dfrac{\cos\theta/2 \sin\frac{\psi+\varphi}{2}}{\sqrt{1-\cos^2\theta/2 \cos^2\frac{\psi+\varphi}{2}}}$

Сдано.

Хамаш Виктория
БО1-302

3. Сложное движение точки и тв. тела

№2.9 Дано: $\bar\omega_e=\bar e_z\omega_e$, $\bar\varepsilon_e=\bar e_z\varepsilon_e$, $\bar W_{абс}=0$ Найти: $\bar W_{отн}$, $\bar V_{отн}$



$$\bar W_{абс}=\bar W_{отн}+\bar W_{пер}+\bar W_{кор}=\bar 0 \quad (1)$$

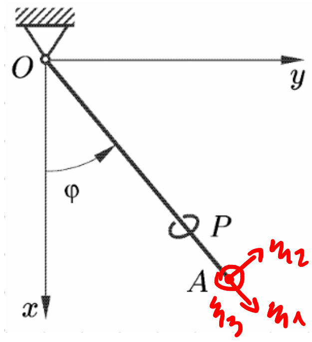
$$\bar W_{пер}=\bar\varepsilon\times\bar\rho+\bar\omega[\bar\omega\times\bar\rho]=\begin{pmatrix}0\\0\\\varepsilon\end{pmatrix}\times\begin{pmatrix}r\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\begin{pmatrix}r\\0\\0\end{pmatrix}\right]=$$

$$=\begin{pmatrix}0\\\varepsilon r\\0\end{pmatrix}+\begin{pmatrix}-\omega^2 r\\0\\0\end{pmatrix}=\begin{pmatrix}-\omega^2 r\\\varepsilon r\\0\end{pmatrix};\quad \bar W_{кор}=2\bar\omega\times\bar V=2\begin{pmatrix}0\\\omega v\\0\end{pmatrix}$$

$$(1)\Rightarrow\begin{pmatrix}W_{отн}\\0\\0\end{pmatrix}+\begin{pmatrix}-\omega^2 r\\\varepsilon r\\0\end{pmatrix}+\begin{pmatrix}0\\2\omega v\\0\end{pmatrix}=\bar 0\Rightarrow \begin{matrix}\bar W_{отн}=\omega_e^2 r\,\bar e_r;\\ \bar V_{отн}=-\dfrac{\varepsilon r}{2\omega_e}\cdot\bar e_r\end{matrix}$$

Ответ: $\bar W_{отн}=\omega_e^2 r\,\bar e_r$; $\bar V_{отн}=-\dfrac{\varepsilon r}{2\omega_e}\cdot\bar e_r$

№2.16



$\varphi=\varphi_0\sin\omega t$; $OP=\frac{at^2}{2}$; Найти: $\bar V_a$, $\bar W_a$

$$\bar V_a=\bar V_r+\bar V_e;\quad \bar V_r=\begin{pmatrix}at\\0\\0\end{pmatrix};\quad \bar V_e=\bar\omega\times\overline{OP}=\begin{pmatrix}0\\0\\\dot\varphi\end{pmatrix}\times\begin{pmatrix}OP\\0\\0\end{pmatrix}=\begin{pmatrix}0\\\dot\varphi\cdot OP\\0\end{pmatrix}$$

$$\bar V_a=\begin{pmatrix}at\\1/2\,at^2\dot\varphi\\0\end{pmatrix}\Rightarrow V_A=\sqrt{a^2t^2+\tfrac14 a^2t^4\dot\varphi^2}$$

$$V_A=\frac{at}{2}\sqrt{4+\varphi_0^2\omega^2\cos^2\omega t}$$

$$\bar W_a=\bar W_r+\bar W_e+\bar W_к;\quad \bar W_r=\begin{pmatrix}a\\0\\0\end{pmatrix}$$

$$\bar W_e=\cancel{\bar W_0}^{\,0}+\bar\varepsilon\times\overline{OP}+\bar\omega_e\times[\bar\omega_e\times\overline{OP}]=\begin{pmatrix}0\\0\\\ddot\varphi\end{pmatrix}\times\begin{pmatrix}OP\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\dot\varphi\end{pmatrix}\times\begin{pmatrix}0\\OP\dot\varphi\\0\end{pmatrix}=\begin{pmatrix}-\dot\varphi^2\cdot OP\\\ddot\varphi\cdot OP\\0\end{pmatrix}$$

$$\bar W_к=2\bar\omega\times\bar V_r=2\begin{pmatrix}0\\0\\\dot\varphi\end{pmatrix}\times\begin{pmatrix}at\\0\\0\end{pmatrix}=\begin{pmatrix}0\\2\dot\varphi at\\0\end{pmatrix}$$

$$\bar W_a=\begin{pmatrix}a-\dot\varphi^2\cdot OP\\\ddot\varphi\cdot OP+2\dot\varphi at\\0\end{pmatrix}$$

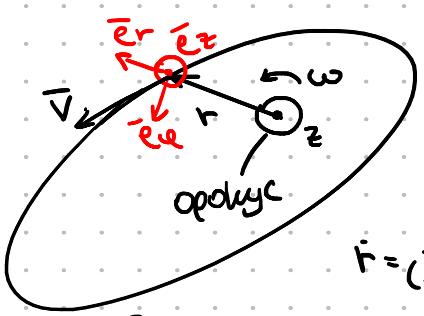
$$\Rightarrow \bar W_a^2=\frac{a^2}{4}\left((2-(\varphi_0\omega t\cos\omega t)^2)^2+(\varphi_0\omega t(4\cos\omega t-\omega t\sin\omega t))^2\right)$$

$$\bar W_a=\frac a2\sqrt{(2-(\varphi_0\omega t\cos\omega t))^2+\varphi_0^2\omega^2t^2(4\cos\omega t-\omega t\sin\omega t)^2}$$

Ответ: $V_A=\dfrac{at}{2}\sqrt{4+\varphi_0^2\omega^2\cos^2\omega t}$;

$\bar W_a=\dfrac a2\sqrt{(2-(\varphi_0\omega t\cos\omega t))^2+\varphi_0^2\omega^2t^2(4\cos\omega t-\omega t\sin\omega t)^2}$

№2.38



Дано: $r=\dfrac{p}{1+e\cos\varphi}$, ω, $r^2\dot\varphi=C$

Найти: $V(r)$, $W(r)$

$$\bar V_{абс}=\bar V_{отн}+\bar V_{пер};\quad \bar V_{пер}=\bar\omega\times\bar r=\begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\begin{pmatrix}r\\0\\0\end{pmatrix}=\begin{pmatrix}0\\\omega r\\0\end{pmatrix}$$

$$\bar V_{отн}=\dot r\,\bar e_r+r\dot\varphi\,\bar e_\varphi=\begin{pmatrix}\dot r\\C/r\\0\end{pmatrix}$$

$$\dot r=\frac{pe}{(1+e\cos\varphi)^2}\sin\varphi\cdot\dot\varphi=\frac{\cancel{p}e\sin\varphi}{\cancel{(1+e\cos\varphi)^2}}\cdot\frac{C}{p^{\cancel{2}}}\cancel{(1+e\cos\varphi)^2}=\frac{Ce\sin\varphi}{p}$$

$$\cos\varphi=\frac{\frac pr-1}{e};\quad \sin\varphi=\frac{\pm\sqrt{e^2-(p/r-1)^2}}{e}\Rightarrow \dot r=\pm C\sqrt{\frac{e^2}{p^2}-\left(\frac1r-\frac1p\right)^2}$$

$$\Rightarrow \bar V_{абс} = \begin{pmatrix} \dot r \\ c/r + \omega r \\ 0 \end{pmatrix} \Rightarrow V(r) = \sqrt{\left(\frac{c}{r}+\omega r\right)^2 + c^2\left(\frac{e^2}{p^2} - \left(\frac1r - \frac1p\right)^2\right)}$$

$$\bar W_{абс} = \bar W_{отн} + \bar W_{пер} + \bar W_{кор}$$

$$\bar W_{пер} = \bar\omega[\bar\omega\times\bar r] = \begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\begin{pmatrix}0\\\omega r\\0\end{pmatrix} = \begin{pmatrix}-\omega^2 r\\0\\0\end{pmatrix}$$

$$\bar W_{кор} = 2\bar\omega\times\bar V_{отн} = \begin{pmatrix}0\\0\\2\omega\end{pmatrix}\times\begin{pmatrix}\dot r\\c/r\\0\end{pmatrix} = \begin{pmatrix}-2\omega c/r\\2\omega\dot r\\0\end{pmatrix}$$

$$\bar W_{отн} = (\ddot r - r\dot\varphi^2)\bar e_r + (\cancel{2\dot r\dot\varphi + r\ddot\varphi}^{\,0})\bar e_\varphi = \left(\ddot r - \frac{c^2}{r^3}\right)\bar e_r$$

$$\ddot r = (\dot r)' = \frac{ce}{p}\cos\varphi\cdot\dot\varphi = \frac{ce}{p}\left(\frac{p}{r}-1\right)\cdot\frac{c}{r^2} = \frac{c^2}{r^2}\left(\frac1r - \frac1p\right)$$

$$\bar W_{абс} = \begin{pmatrix} c^2/r^2(\cancel{1/r} - 1/p) - \cancel{c^2/r^3} - \omega^2 r - 2c\omega/r \\ 2\omega\dot r \\ 0 \end{pmatrix}$$

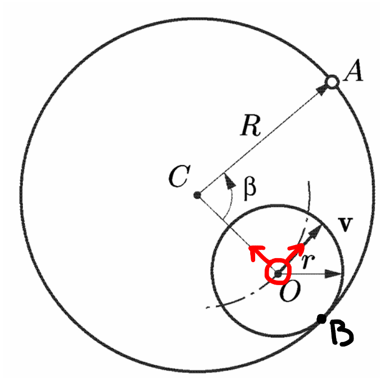
$$\Rightarrow W(r) = \sqrt{\left(\frac{c^2}{r^2p} + \omega^2 c - \frac{2\omega c}{r}\right)^2 + 4\omega^2c^2\left(\frac{e^2}{p^2} - \left(\frac1r-\frac1p\right)^2\right)}$$

Ответ: $V(r) = \sqrt{\left(\frac{c}{r}+\omega r\right)^2 + c^2\left(\frac{e^2}{p^2} - \left(\frac1r - \frac1p\right)^2\right)}$;

$W(r) = \sqrt{\left(\frac{c^2}{r^2p} + \omega^2 c - \frac{2\omega c}{r}\right)^2 + 4\omega^2c^2\left(\frac{e^2}{p^2} - \left(\frac1r-\frac1p\right)^2\right)}$

Д3.30

Дано: $r, R, V_0 = const, \beta$ Найти: V_A^r, W_A^r



$$\bar V_A = \bar V_{Ae} + \bar V_{Ar} = \bar 0 \Rightarrow \bar V_{Ae} = -\bar V_{Ar} = \cancel{\bar V_B}^{\,0} + \bar\omega\times\bar{BA}$$

$$\omega = V/r,\ AB = 2R\sin\beta/2 \Rightarrow \underline{\bar V_{Ar} = 2V\frac{R}{r}\sin\beta/2}$$

$$\bar W_A = \bar W_e + \bar W_r + \bar W_k = \bar 0$$

$$\bar W_e = \bar W_B + \bar\varepsilon\times\bar{BA} - \omega^2\bar{BA}$$

$$\bar W_k = 2\bar\omega\times\bar V_{Ar} = -2\bar\omega\times(\bar\omega\times\bar{BA}) = -2\bar\omega[\cancel{\bar\omega\cdot\bar{BA}}]^{0} + 2\omega^2\bar{BA}$$

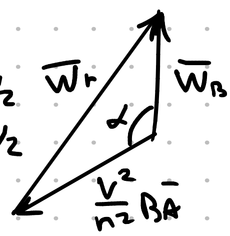
$$\bar W_r = -\bar W_e - \bar W_k = -(\bar W_B + V^2/r^2\,\bar{BA})$$

$$\bar W_B = \bar W_O - \omega^2\bar{OB},\ \bar W_O = -\frac{V^2}{R-r}\cdot\frac{\bar{OB}}{OB}$$

$$\Rightarrow \bar W_B = \bar{BO}\cdot V^2\left(\frac{1}{(R-r)r} + \frac{1}{r^2}\right) = \frac{RV^2}{(R-r)r^2}\bar{BO}$$

$\alpha = 90 + \beta/2$

$s = \cos\alpha = -\sin\beta/2$

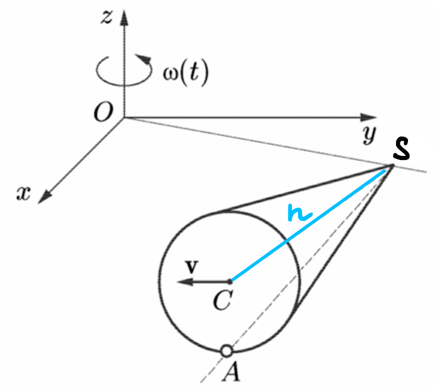


$$\bar W_r^2 = \left(\frac{RV^2}{(R-r)r^2}BO\right)^2 + \left(\frac{V^2}{r^2}\cdot BA\right)^2 + 2\sin\frac{\beta}{2}\frac{V^2}{r^2}BA\cdot\frac{Rr}{(R-r)r^2}V^2 =$$

$$= \frac{R^2V^4}{r^4(R-r)^2}\left(r^2 + 4(R-r)^2s^2 + 4s^2(R-r)r\right)$$

$\Rightarrow$ Ответ: $\bar V_{Ar} = 2V\frac{R}{r}\sin\beta/2$; $\bar W_r = \frac{RV^2}{r^2(R-r)}\sqrt{r^2 + 4R(R-r)\sin^2\beta/2}$

№4.25



Дано: $\bar{\omega}(t)$, $SC = h$, $\angle CSA = \beta$, $\bar{V}_{отн} = \bar{V}$, $|V| = const$

Найти: $\omega_{абс}$, $\varepsilon_{абс}$

1) SA – неподвиж. ось, т.к. скольжения нет

$\Rightarrow \bar{\omega}_{отн} \parallel SA$

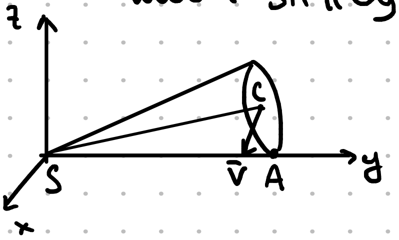
$$\bar{V}_e = \cancel{\bar{V}_S}^{0} + \bar{\omega}_{отн} \times \bar{SC} = \begin{pmatrix} 0 \\ \omega_{отн} \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ h\cos\beta \\ h\sin\beta \end{pmatrix} = \begin{pmatrix} \omega_{отн} h \sin\beta \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \omega_{отн} = \frac{V}{h\sin\beta}$$

$$\bar{\omega}_{абс} = \bar{\omega}_{пер} + \bar{\omega}_{отн} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} + \begin{pmatrix} 0 \\ V/h\sin\beta \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ V/h\sin\beta \\ \omega \end{pmatrix}$$

$$\Rightarrow \omega_{абс} = \sqrt{\omega^2 + V^2/h^2\sin^2\beta}$$

Расположим конус так, чтобы $\bar{SA} \parallel Oy$



2) $\bar{\varepsilon}_{абс} = \dot{\bar{\omega}} + \cancel{\bar{\varepsilon}_{отн}}^{0} + \bar{\omega} \times \bar{\omega}_{отн}$

$$\bar{\omega} \times \bar{\omega}_{отн} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ V/h\sin\beta \\ 0 \end{pmatrix} = \begin{pmatrix} -\omega V/h\sin\beta \\ 0 \\ 0 \end{pmatrix}$$

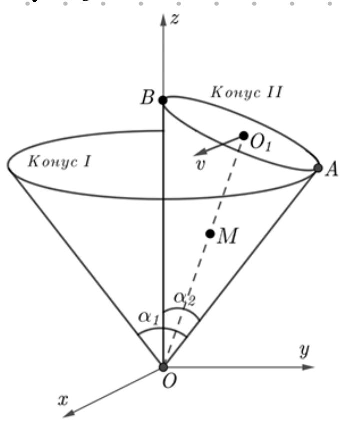
$$\bar{\omega}_{отн} = \omega_z^{отн} \cdot \bar{e}_z + \omega_{sc} \cdot \frac{\bar{SC}}{SC}, \quad \omega_z = \bar{\omega}_{отн} \cdot tg\beta, \quad \omega_{sc} = \frac{\omega_{отн}}{\cos\beta}$$

$$\bar{\varepsilon}_{отн} = \cancel{\dot{\bar{\varepsilon}}_z}^{0} + \cancel{\dot{\bar{\varepsilon}}_{sc}}^{0} + \bar{\omega}_z \times \bar{\omega}_{sc} = \begin{pmatrix} 0 \\ 0 \\ \omega_{отн} tg\beta \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_{отн} \\ \omega_{отн} tg\beta \end{pmatrix} = \begin{pmatrix} \omega_{отн}^2 tg\beta \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{V^2}{h^2} \cdot \frac{1}{\sin\beta\cos\beta} \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \bar{\varepsilon}_{абс} = \begin{pmatrix} \frac{V^2}{h^2} \cdot \frac{1}{\sin\beta\cos\beta} - \frac{\omega V}{h} \cdot \frac{1}{\sin\beta} \\ 0 \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} \frac{V}{h\sin\beta}\left(\frac{V}{h\cos\beta} - \omega\right) \\ 0 \\ \dot{\omega} \end{pmatrix}.$$

Ответ: $\omega_{абс} = \sqrt{\omega^2 + V^2/h^2\sin^2\beta}$; $\varepsilon_{абс} = \sqrt{\dot{\omega}^2 + \frac{V^2}{h^2\sin^2\beta}\left(\frac{V}{h\cos\beta} - \omega\right)^2}$

№75



Дано: $\alpha_2 = 45°$, $\alpha_1 = 90°$, $OO_1 = \ell$, $|V| = const$, $OM = f(t)$

Найти: $V_M^{абс}$, $W_M^{абс}$, когда $OM = MO_1$

1) Пусть в момент t BO_1 пересекает Oy.

$$\bar{V}_M = \bar{V}_{пер} + \bar{V}_{отн} = \bar{V}_{пер} + f'(t) \cdot \frac{\bar{OO_1}}{\ell}$$

OB – неподвиж. ось $\Rightarrow \bar{\omega} \parallel \bar{OB}$

$$\bar{V}_{O_1} = \cancel{\bar{V}_O}^{0} + \bar{\omega} \times \bar{OO_1}$$

$$\begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ \ell\sin 45/2 \\ \ell\cos 45/2 \end{pmatrix} = \begin{pmatrix} \omega\ell\sin 45/2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \omega = \frac{V}{\ell\sin 45/2}$$

$$\bar{V}_M^{пер} = \cancel{\bar{V}_O}^{0} + \bar{\omega} \times \bar{OM} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ \ell/2 \sin 45/2 \\ \ell/2 \cos 45/2 \end{pmatrix} = \begin{pmatrix} \frac{\omega\ell}{2} \cdot \sqrt{\frac{1-\cos 45}{2}} \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \bar{V}_M = \begin{pmatrix} 0 \\ f'(t) \cdot \sin 45/2 \\ f'(t)\cos 45/2 \end{pmatrix} + \bar{V}_M^{пер} = \begin{pmatrix} V/2 \\ f'(t)\sin 45/2 \\ f'(t)\cos 45/2 \end{pmatrix} \Rightarrow V_M = \sqrt{\frac{V^2}{4} + f'(t)^2}$$

2) $\bar{W}_M = \bar{W}_M^{отн} + \bar{W}_M^{пер} + \bar{W}_M^{кор}$

$$\bar{W}_M^{кор} = 2\bar{\omega} \cdot \bar{V}_M^{отн} = 2\begin{pmatrix} 0 \\ 0 \\ V/\ell\sqrt{\frac{1-\cos 45}{2}} \end{pmatrix} \times \begin{pmatrix} 0 \\ f'(t)\sin 45/2 \\ f'(t)\cos 45/2 \end{pmatrix} = \begin{pmatrix} 2 \cdot V/\ell \cdot f'(t) \\ 0 \\ 0 \end{pmatrix}$$

$$\overline{W}_{отн} = \begin{pmatrix} 0 \\ f''(t)\sin 45/2 \\ f''(t)\cos 45/2 \end{pmatrix} \quad ; \quad \bar{\varepsilon} = \frac{d\bar{\omega}}{dt} = \frac{d\omega \cdot \bar{e}_z}{dt} = 0$$

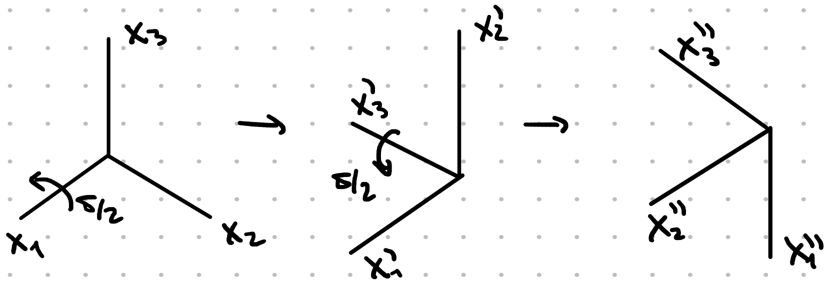
$$\overline{W}_{пер} = \bar{\omega} \times [\bar{\omega} \times \overline{OM}] = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} \frac{\omega l}{2}\sin 45/2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \omega^2 l/2 \sin 45/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{V^2}{2l} \cdot \frac{1}{\sin 45/2} \\ 0 \end{pmatrix}$$

$$\Rightarrow \overline{W}_M = \begin{pmatrix} 2\frac{V}{l} f'(t) \\ f''(t)\sin 45/2 + \frac{V^2}{2l} \cdot \frac{1}{\sin 45/2} \\ f''(t)\cos 45/2 \end{pmatrix}$$

Ответ: $V_M = \sqrt{\frac{V^2}{4} + f'(t)^2}$; $W_M = \sqrt{4\frac{V^2}{l^2} f'^2(t) + f''(t)^2 + \frac{V^4}{l^2} \cdot \frac{1}{2-\sqrt{2}}}$

сдано.

№T2



Повороты вокруг $\bar{i}_3$: $\psi = 0$; $\bar{i}_1'$: $\theta = 90^\circ$; $\bar{i}_3''$: $\varphi = 90^\circ$

$$\Lambda = \Lambda_\theta \circ \Lambda_\varphi = (\cos \pi/4 + \bar{i} \sin \pi/4) \circ (\cos \pi/4 - \bar{k} \sin \pi/4) = \frac{1}{2}(1+\bar{i}) \circ (1-\bar{k}) = \frac{1}{2}(1-\bar{k}+\bar{i}+\bar{j})$$

$$\Rightarrow \alpha = 2 \arccos \lambda_0 = 60^\circ \Rightarrow \bar{e} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

№4.70

№4.85

№T3

№T4