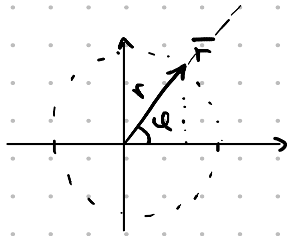
РЗ Хомич Виктории по анмеху.
Первое задание, 3 сем

Неделя 1.

№1.18 $r=r(t)$ $\varphi=\varphi(t)$ Д-ать: $r^2\dot\varphi = const \Rightarrow \bar\omega \parallel \bar r$

$$\bar r = \begin{bmatrix} r\cos\varphi \\ r\sin\varphi\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix} \qquad \bar r\times\bar\omega = \det\begin{pmatrix} \bar i & \bar j & \bar k\\ x & y & 0\\ \ddot x & \ddot y & 0\end{pmatrix} = \begin{pmatrix}0\\0\\x\ddot y-\ddot x y\end{pmatrix} = 0 \Leftrightarrow \bar\omega\parallel\bar r$$



$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi\end{cases}$$

$$\begin{cases} \dot x = \dot r\cos\varphi - r\dot\varphi\sin\varphi \\ \dot y = \dot r\sin\varphi + r\dot\varphi\cos\varphi\end{cases}$$

$$\begin{cases} \ddot x = \ddot r\cos\varphi - \dot r\dot\varphi\sin\varphi - \dot r\dot\varphi\sin\varphi - r\ddot\varphi\sin\varphi - r\dot\varphi^2\cos\varphi \\ \ddot y = \ddot r\sin\varphi + \dot r\dot\varphi\cos\varphi + \dot r\dot\varphi\cos\varphi - r\ddot\varphi\cos\varphi - r\dot\varphi^2\sin\varphi\end{cases}$$

$$x\ddot y - \ddot x y = r\cos\varphi(\ddot r\cancel{\sin\varphi} + \dot r\dot\varphi\cancel{\cos\varphi} + \dot r\dot\varphi\cos\varphi - r\ddot\varphi\cos\varphi - r\dot\varphi^2\sin\varphi)$$
$$- r\sin\varphi(\ddot r\cancel{\cos\varphi} - \dot r\dot\varphi\cancel{\sin\varphi} - \dot r\dot\varphi\sin\varphi - r\ddot\varphi\sin\varphi - r\dot\varphi^2\cos\varphi) =$$
$$= r(2\dot r\dot\varphi + r\ddot\varphi) = (r^2\dot\varphi)$$
$$r^2\dot\varphi = const \Rightarrow x\ddot y - \ddot x y = 0 \Rightarrow \bar\omega\parallel\bar r$$

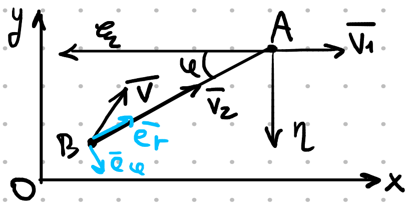
ч.т.д

№1.25 Дано: $\bar w = a(\bar v\times\bar r)$, $a = const > 0$ Найти: $\rho = \rho(\bar r, \bar v)$

$\bar w = a(\bar v\times\bar r) \Rightarrow \bar w\perp\bar v, \bar r \Rightarrow$
$\Rightarrow \bar w = \bar w_n = \frac{v^2}{\rho}\bar n = a(\bar v\times\bar r) \Rightarrow$

$\Rightarrow$ Ответ: $\rho = \dfrac{v^2}{a|\bar v\times\bar r|}$

№1.31



Найти: $AB = r(\varphi)$, $\varphi_0\neq 0$

$V_r = \dot r = V_1\cos\varphi - V_2 = \frac{dr}{dt}$

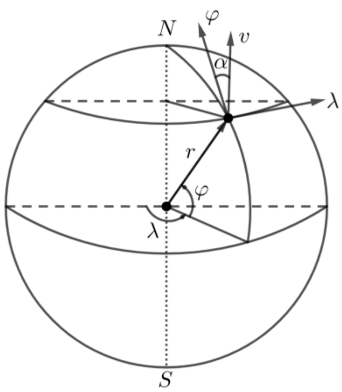
$V_\varphi = r\dot\varphi = -V_1\sin\varphi = r\cdot\frac{d\varphi}{dt}$

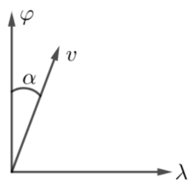
$\frac{dr}{r} = \frac{V_1\cos\varphi - V_2}{-V_1\sin\varphi}d\varphi \Rightarrow$

$\Rightarrow \ln r - \ln c = \frac{V_2}{V_1}\ln\frac{1-\cos\varphi}{\sin\varphi} - \ln\sin\varphi = \frac{V_2}{V_1}\ln tg\frac{\varphi}{2} - \ln\sin\varphi$

$\Rightarrow$ Ответ: $\dfrac{r}{r_0} = \dfrac{\sin\varphi_0}{\sin\varphi}\left(\dfrac{tg\,\varphi/2}{tg\,\varphi_0/2}\right)^{V_2/V_1}$

№Т1





$\bar r = \begin{bmatrix} r\cos\varphi\cos\lambda \\ r\cos\varphi\sin\lambda \\ r\sin\varphi\end{bmatrix}$ $V = const$, $\alpha = \varphi; V = const$

Найти: ур-ие кривой; w_i; $|w|$; ρ

$V_\varphi = V\cos\alpha = \frac{d\varphi}{dt}r$
$V_\lambda = V\sin\alpha = \frac{d\lambda}{dt}r\cos\varphi$ $\Rightarrow \begin{cases}\frac{d\varphi}{dt} = \frac{V\cos\alpha}{r} \\ \frac{d\lambda}{dt} = \frac{V\sin\alpha}{r\cos\varphi}\end{cases} \Rightarrow \frac{d\varphi}{d\lambda} = \frac{\cos\alpha\cos\varphi}{\sin\alpha}$

$\int\frac{d\varphi}{\cos\varphi} = \int\frac{\cos\alpha}{\sin\alpha}d\lambda \Rightarrow \ln\frac{tg(\varphi/2 + \pi/4)}{tg(\varphi_0/2 + \pi/4)} = ctg\,\alpha(\lambda - \lambda_0)$

$\Rightarrow$ ур-ие кривой: $tg\left(\varphi/2 + \frac{\pi}{4}\right) = tg\left(\frac{\varphi_0}{2} + \frac{\pi}{4}\right)\cdot\exp\left(ctg\,\alpha(\lambda - \lambda_0)\right)$

Из коэф. Ламэ: $H_r = 1$; $H_\varphi = r$; $H_\lambda = r\cos\varphi$
$V_r = \dot r$; $V_\varphi = r\dot\varphi$; $V_\lambda = \dot\lambda r\cos\varphi$
$V^2 = \dot r^2 + r^2\dot\varphi^2 + r^2\cos^2\varphi\,\dot\lambda^2$

$$W_r = \frac{1}{H_r}\left[\left(\frac{v^2}{2}\right)^{\cdot}_{,\dot r} - \left(\frac{v^2}{2}\right)_{,r}\right] = \ddot r - r\dot\varphi^2 - r\cos^2\varphi\cdot\dot\lambda^2 = -r\,\frac{v^2\cos^2\alpha}{r^2} - r\cdot\frac{v^2\sin^2\alpha}{r^2\cos^2\varphi}\cdot\cos^2\varphi = -\frac{v^2}{r}$$

$$W_\varphi = \frac{1}{H_\varphi}\left[r^2\ddot\varphi + 2r\dot r\dot\varphi + r^2\dot\lambda^2\cos\varphi\sin\varphi\right] = r\ddot\varphi + 2\dot r\dot\varphi + r\,\dot\lambda^2\cos\varphi\sin\varphi = \frac{v^2\sin^2\alpha\cdot tg\,\varphi}{r}$$

$$W_\lambda = \frac{r^2}{r\cos\varphi}\left[\ddot\lambda\cos^2\varphi - 2\cos\varphi\sin\varphi\,\dot\varphi\dot\lambda\right] = r(\ddot\lambda\cos\varphi - 2\sin\varphi\,\dot\varphi\dot\lambda) = \frac{1}{2r}v^2 tg\varphi\sin 2\varphi - \frac{1}{r}v^2 tg\varphi\sin^2\alpha = -\frac{v^2 tg\,\varphi\sin 2\alpha}{2r}$$

$$W = \sqrt{\frac{v^4}{r^2} + \frac{v^4\sin^4\alpha\, tg^2\varphi}{r^2} + \frac{v^4 tg^2\varphi\sin^2 2\alpha}{4r^2}} = \frac{v^2}{r}\sqrt{1+\sin^2\alpha + tg^2\varphi}$$

$$v = const \Rightarrow W = W_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{r}{\sqrt{1+\sin^2\alpha + tg^2\varphi}}$$

Ответ: ур-ие: $tg\left(\varphi/2 + \frac{\pi}{4}\right) = tg\left(\frac{\varphi_0}{2} + \frac{\pi}{4}\right)\cdot \exp\left(ctg\,\alpha(\lambda-\lambda_0)\right)$;

$W_r = -\frac{v^2}{r}$; $W_\varphi = \frac{v^2\sin^2\alpha\cdot tg\,\varphi}{r}$; $W_\lambda = \frac{-v^2 tg\,\varphi\sin 2\alpha}{2r}$;

$W = \frac{v^2}{r}\sqrt{1+\sin^2\alpha + tg^2\varphi}$; $\rho = \frac{r}{\sqrt{1+\sin^2\alpha + tg^2\varphi}}$