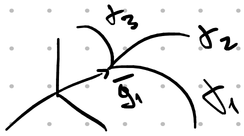
$\bar r(q)$ $\quad q = \begin{bmatrix} q^1 \\ q^2 \\ q^3 \end{bmatrix}$ 03.09.2024



$\bar r_{,k} = \bar g_k \qquad H_k = |\bar g_k|$

$\bar e_a = \dfrac{\bar g_a}{H_a}$ — орт

$\bar V = \dot q^i \bar g_i$

$\bar V = \sum H_i \dot q^i \bar e_i$

$V^2 = g_i g_j \dot q^i \dot q^j = g_{ij} \dot q^i \dot q^j$

$V^2 = \sum H_i H_j \langle \bar e_i, \bar e_j \rangle \dot q^i \dot q^j$

Если δ_{ij} =>

$$\boxed{V^2 = \sum H_i^2 \dot q^{i2}} \quad (1)$$

$$\bar W \cdot \bar g_k = \ddot{\bar r} \cdot \bar r_{,k} = (\dot{\bar r}, \bar r_{,k})^{\cdot} - \dot{\bar r}\, \dot{\bar r}_{,k} \qquad \dot{\bar r}_{,k} = \frac{\partial \dot{\bar r}}{\partial q^k}$$

$$\underbrace{\frac{d}{dt}\frac{\partial \bar r}{\partial q_k}}_{r_{,jk}\dot q^j} = \frac{\partial}{\partial q^k}\frac{d \bar r}{dt}, \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \dot q^i \frac{\partial}{\partial q^i}$$

$$\parallel \quad \underbrace{\frac{\partial}{\partial q^k} r_{,j}(q)\dot q^j}_{r_{,jk}\dot q^j}$$

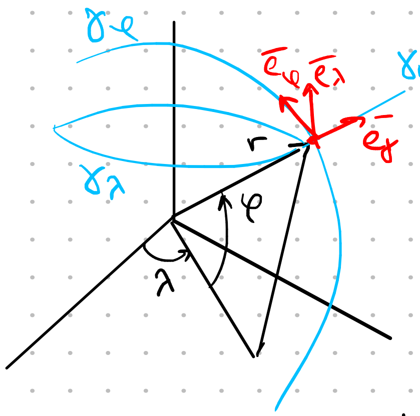
$\dot{\bar r} \cdot \dot{\bar r}_{,k} = (V^2/2)_{,k} \qquad V^2 = \dot{\bar r}\cdot\dot{\bar r}$

$\Rightarrow \underbrace{\dot{\bar r}_{,k}}_{\frac{\partial \dot{\bar r}}{\partial q_k} = \frac{d}{dt}\frac{\partial \bar r}{\partial q^k}} = \dot{\bar r}_{,k} \Rightarrow \dot{\bar r}\cdot\dot{\bar r}_{,\dot k} = \dot{\bar r}\cdot\dot{\bar r}_{,k} = (V^2/2)_{,k}$

$$\boxed{\bar W \cdot \bar g_k = \frac{d}{dt}(V^2/2)_{,\dot k} - (V^2/2)_{,k}}$$

$$\boxed{\bar W \cdot \bar e_a = \frac{1}{H_a}\left[\frac{d}{dt}(V^2/2)_{,\dot a} - (V^2/2)_{,a}\right]}$$

оператор Эйлера–Лагранжа



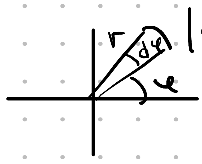
$q^1 = r \quad q^2 = \lambda \quad q^3 = \varphi$

$\bar r = \begin{bmatrix} r\cos\varphi\cos\lambda \\ r\cos\varphi\cos\lambda \\ r\sin\varphi \end{bmatrix} \qquad e_i e_j = \delta_{ij}$ — ОНБ

$\bar V = \dot q^i \bar g_i = \sum \underbrace{H_i \dot q^i}_{?} \bar e_i$

$H_k = |\bar r_{,k}|$

$\bar r_{,r} = \begin{bmatrix} \cos\varphi\cos\lambda \\ \cos\varphi\cos\lambda \\ \sin\varphi \end{bmatrix} \Rightarrow H_r = 1 \quad H_\lambda = r\cos\varphi \quad H_\varphi = r$



$|d\bar r_\varphi| \simeq \underbrace{r}_{H_\varphi} d\varphi$

$dq^a \to d\bar r_a = \bar r_{,a} dq^a$

$|d\bar r_a| \simeq \underbrace{|\bar r_{,a}|}_{H_a} dq^a$

$V^2/2 \overset{(1)}{=} \frac{1}{2}\dot r^2 + r^2\cos^2\varphi + r^2\dot\varphi^2$

$V^2 = \sum H_i^2 \dot q^{i2}$ при ОНБ

$W_k = \bar W \cdot \bar e_k$

$(V^2/2)_{,\dot r} = \dot r \quad ; \quad \frac{d}{dt}(V^2/2)_{,\dot r} = \ddot r$

$(V^2/2)_{,r} = r(\dot\lambda^2\cos^2\varphi + \dot\varphi^2) \overset{(2)}{\Rightarrow}$

$\Rightarrow W_r = \ddot r - r(\dot\lambda^2\cos^2\varphi + \dot\varphi^2)$

W_λ, W_φ — аналог.

2-ой закон Ньютона в ковар. форме

$m\bar{w} = \bar{F} \quad | \cdot \bar{g}_\alpha$

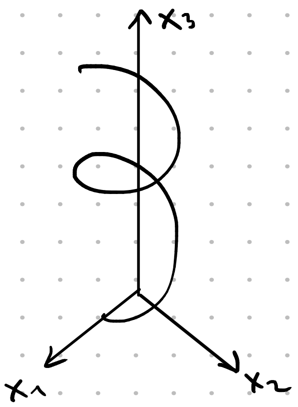
$\frac{d}{dt}\underbrace{(mv^2/2)}_{T}{}_{,\dot{\alpha}} - \underbrace{(mv^2/2)}_{T}{}_{,\alpha} = \bar{F}\bar{g}_\alpha = Q_\alpha$

T – кинет. энергия

$\boxed{\frac{d}{dt}T_{,\dot{\alpha}} - T_{,\alpha} = Q_\alpha}$ – ОНБ

$m\bar{w} = \bar{F} \quad | \cdot e_k \quad \Rightarrow \quad \frac{1}{H_\alpha}\left[\frac{d}{dt}T_{,\dot{\alpha}} - T_{,\alpha}\right] = \bar{F}\cdot\bar{e}_\alpha$ – не ОНБ

Движение по винтовой линии



$\begin{cases} x^1 = a\cos\omega t \\ x^2 = a\sin\omega t \\ x^3 = bt \end{cases}$

w_τ, w_n, ρ – ? радиус кривизны

$\bar{v} = \dot{\bar{r}} = \begin{bmatrix} -a\omega\sin\omega t \\ a\omega\cos\omega t \\ b \end{bmatrix}$

$|\bar{v}| = v = \sqrt{a^2\omega^2 + b^2} = const$

$\bar{w} = w_\tau\bar{\tau} + w_n\bar{n} = \dot{v}\bar{\tau} + \frac{v^2}{\rho}\bar{n}$

$v = const \Rightarrow w_\tau = 0$

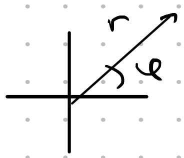
$\dot{\bar{v}} = \bar{w} = \begin{bmatrix} -a\omega^2\cos\omega t \\ -a\omega^2\sin\omega t \\ 0 \end{bmatrix} = \bar{w}_n \rightarrow w_n = a\omega^2$

$\rho = \frac{v^2}{w} = \frac{a^2\omega^2 + b^2}{a\omega^2}$

Пр.1

Дано:

$\left.\begin{array}{l} v_r = \frac{a}{r^2} \\ v_\varphi = \frac{b}{r} \end{array}\right\}$ комп. скорости, $a, b = const$



$v^2 = \sum H_i^2 \cdot \dot{q}^{i\,2}$

Найти:

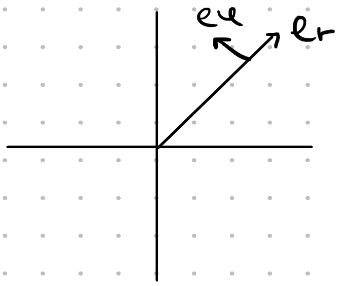
$r(\varphi)$

$w_r(r)$

$w_\varphi(r)$

$r(0) = r_0$

$\varphi(0) = \varphi_0$



$v^2 = \dot{r}^2 + r^2\dot{\varphi}^2$

$\begin{cases} v_r = \dot{r} = a/r^2 \\ v_\varphi = r\dot{\varphi} = b/r \end{cases} \rightarrow$ делим

$\frac{1}{\cancel{r}}\frac{dr}{d\varphi} = \frac{a}{b\cancel{r}} \Rightarrow r - r_0 = \frac{a}{b}(\varphi - \varphi_0)$ ответ

$w_r = \ddot{r} - r\dot{\varphi}^2 \leftarrow \ddot{r} = -2a\dot{r}/r^3 = -2a^2/r^5$

$w_\varphi = \frac{1}{r}\frac{d}{dt}\underbrace{(r^2\dot{\varphi})}_{b} = b \leftarrow \dot{\varphi} = b/r^2$

$\bar{k} = \frac{\bar{n}}{\rho}; \quad \frac{1}{\rho} = k$

$v^2/2 = \frac{1}{2}g_{ij}(q)\dot{q}^i\dot{q}^j \qquad g_{ij} = g_{ji}$

$\bar{g}_i \cdot \bar{g}_i = \bar{r}_{,i} \cdot \bar{r}_{,j} \qquad (v^2/2)_{,\dot{k}} = g_{ki}\dot{q}^i$

Пр.2 №1.40

$\dot{q} = const$

$v = const$

вектор кривизны нормален к поверх (геодезические линии)

$\bar{r}_{,2}, \bar{r}_{,2}$ – кас. векторы необходимо

$\Rightarrow \bar{w}\cdot\bar{r}_{,2} = 0, \quad \bar{w}\cdot\bar{r}_{,3} = 0 \Rightarrow \begin{cases} \frac{d}{dt}(v^2/2)_{,\dot{q}^2} - (v^2/2)_{,q^2} = 0 \\ \frac{d}{dt}(v^2/2)_{,\dot{q}^2} - (v^2/2)_{,q^3} = 0 \end{cases}$

$(v^2/2)_{,\dot{k}} = g_{ki}\dot{q}^i$

$\frac{d}{dt}(v^2/2)_{,\dot{k}} = g_{ki}\ddot{q}^i + g_{ki,j}\dot{q}^j\dot{q}^i$

$(v^2/2)_{,k} = \frac{1}{2}g_{ij,k}\dot{q}^i\dot{q}^j$