Дз Хамин Виктории Б01-302 по матану.
Задание 1, 3 семестр

I.

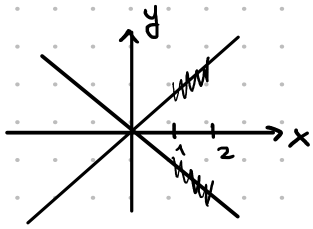
Т1 $x^2=y^2$

а) Сколько $y: \mathbb{R}\to\mathbb{R}$?

Ответ: беск. много

Пример: $f(x)=\begin{cases}x, & x\in X\\ -x, & x\notin X\end{cases}$ $X\in\mathbb{R}$ — таких функций беск. много

б) Сколько непр. $y: \mathbb{R}\to\mathbb{R}$?



Ответ: 4 : $y=x$; $y=-x$; $y=|x|$; $y=-|x|$

в) Сколько непр. $y: \mathbb{R}\to\mathbb{R}$ и $y(1)=1$?

Ответ: 2 : $y=x$; $y=|x|$

г) Сколько непр. $y: [1,2]\to\mathbb{R}$ и $y(1)=1$?

Ответ: 1 : $y=x$

§3

№61 2) Найти u'_x, u'_y в $(0,1,0)$ Проверим: $\cos 0=1$ — верно

$$x\cos y+y\cos u+u\cos x=1$$

$$\cos y\,dx-x\cancel{\sin y\,dy}+\cos u\,dy-y\cancel{\sin u}\,du+\cos x\,du-u\cancel{\sin x\,dx}=0$$

$x=0$
$y=1$
$u=0$

$$\cos 1\,dx+dy+du=0$$

$$du=-\cos 1\,dx-dy \Rightarrow$$ Ответ: $u'_x=-\cos 1$; $u'_y=-1$

№64 2) Найти du в а) $(1,1,1)$ б) $(1,1,-2)$

$$x^3+2y^3+u^3-3xyu+2y-3=0$$

$$3x^2dx+6y^2dy+3u^2du-3xy\,du-3xu\,dy-3yu\,dx+2dy=0$$

$$(3x^2-3yu)dx+(6y^2-3xu+2)dy+(3u^2-3xy)du=0 \quad (1)$$

б) Проверим: $1+2-8+6+2-3=0$

$(1)\to (3+6)dx+(6+6+2)dy+(12-3)du=0$

$9dx+14dy+9du=0$

$du=-dx-\frac{14}{9}dy$

а) Проверим: $1+2+1-3+2-3=0$

$(1)\to 3dy=0 \Rightarrow du$ не сущ.

Ответ: а) в т.$(1,1,1)$ du не сущ.
б) в т.$(1,1,-2)$ $du=-dx-\frac{14}{9}dy$

№69 $f(x-y;\ y-z;\ z-x)=0$; $f(u,v,w)$ — дифф.; $z=z(x,y)$
Найти $dz(x,y)$

$$f'_u(dx-dy)+f'_v(dy-dz)+f'_w(dz-dx)=0$$

$$dx(f'_u-f'_w)+dy(f'_v-f'_u)+dz(f'_w-f'_v)=0$$

$\Rightarrow$ Ответ: $dz=\dfrac{(f'_u-f'_w)dx+(f'_v-f'_u)dy}{f'_v-f'_w}$

№75 $u=u(x,y)$ $v=v(x,y)$ Найти: u'_x, u'_y, v'_x, v'_y в т.$(1,2)$

$$\begin{cases} xe^{u+v}+2uv=1 \\ ye^{u-v}-\frac{u}{1+v}=2x \end{cases} ;\quad x=1;\ y=2;\ u=v=0$$

$$\begin{cases} e^{u+v}+xe^{u+v}(u'_x+v'_x)+2uv'_x+2u'_xv=0 \\ xe^{u+v}(u'_y+v'_y)+2uv'_y+2u'_yv=0 \\ e^{u-v}+ye^{u-v}(u'_y-v'_y)-\frac{u'_y(1+v)-v'_yu}{(1+v)^2}=0 \\ ye^{u-v}(u'_x-v'_x)-\frac{u'_x(1+v)-v'_xu}{(1+v)^2}=2 \end{cases} \quad \begin{cases} 1+u'_x+v'_x=0 \\ u'_y+v'_y=0 \\ 1+2(u'_y-v'_y)-u'_y=0 \\ 2(u'_x-v'_x)-u'_x=2 \end{cases} \quad \begin{cases} u'_x=0 \\ v'_x=-1 \\ u'_y=1/3 \\ v'_y=-1/3 \end{cases}$$

Ответ: $u'_x=0;\ u'_y=1/3;$
$v'_x=-1;\ v'_y=-1/3$

§4

№43 5) Найти: d^2u в т. $(1,-1,-1)$

$u^3+2yu+xy=0$

$3u^2du+2ydu+2udy+xdy+ydx=0$

$3du-2du-2dy+dy-dx=0$

$-du+dy-dx=0 \leftarrow$ в т$(1,-1,-1)$

$\underline{du=dy-dx}$

$6udu^2+3u^2d^2u+2dudy+2yd^2u+2dudy+2ud^2y+dxdy+xd^2y+dxdy+yd^2x=0$

$d^2u(3u^2+2y)+4dudy+d^2y(2u+x)+2dxdy+yd^2x+6udu^2=0$

в т$(1,-1,-1)$: $\begin{smallmatrix}x & y & u\end{smallmatrix}$

$d^2u+4dudy-d^2y+2dxdy-d^2x-6du^2=0$

$d^2u+4dy^2-4dydx-d^2y+2dxdy-d^2x-6(dy-dx)^2=0$

$d^2u+4dy^2-2dydx-d^2y-d^2x-6(dy^2-2dydx+dx^2)=0$

$d^2u-2dy^2-6dydx\cancel{-d^2y}\cancel{-d^2x}-6dx^2=0$

$d^2u=6dx^2+6dxdy+2dy^2$ $\Rightarrow$ Ответ: $d^2u=6dx^2+6dxdy+2dy^2$

№46 1) $f(x+u,y+u)=0$ $u=u(x,y)$ Найти: $d^2u(x,y)$

$f'_1(dx+du)+f'_2(dy+du)=0$

$f'_1dx+f'_1du+f'_2dy+f'_2du=0 \Rightarrow du=-\frac{f'_1dx+f'_2dy}{f'_1+f'_2}$

$f'_1d^2x+(f''_{11}+f''_{12})dx+f'_2d^2y+(f''_{21}+f''_{22})dy+d^2u(f'_1+f'_2)+(f''_{11}+f''_{12}+f''_{21}+f''_{22})du=0$

$d^2u(f'_1+f'_2)+f'_1d^2x+f'_2d^2y+f''_{11}(dx+du)+f''_{12}(dx+du)+(f''_{21}+f''_{22})(dy+du)=0$

$d^2u(f'_1+f'_2)+f'_1d^2x+f'_2d^2y+(f''_{11}+f''_{12})(dx+du)+(f''_{21}+f''_{22})(dy+du)=0$

$dx+du=\frac{f'_1dx+f'_2dx-f'_1dx-f'_2dy}{f'_1+f'_2}=\frac{f'_2(dx-dy)}{f'_1+f'_2}$ $\quad dy+du=\frac{f'_1dy+f'_2dy-f'_1dx-f'_2dy}{f'_1+f'_2}=\frac{f'_1(dy-dx)}{f'_1+f'_2}$

$d^2u+\cancel{\frac{f'_1d^2x+f'_2d^2y}{f'_1+f'_2}}^{\,0}+f'_2(dx-dy)(f''_{11}+f''_{22})-f'_1(dx-dy)(f''_{21}+f''_{22})=0$

$d^2u=(dx-dy)\left(f'_2(f''_{11}+f''_{22})-f'_1(f''_{21}+f''_{22})\right)$

?

II.

§3 N105 Найти: $\frac{\partial(x,y,z)}{\partial(r,\varphi,\psi)}$: $x = r\cos^p\varphi\cos^q\psi$, $y = r\sin^p\varphi\cos^q\psi$, $z = r\sin^q\psi$, $p, q \in \mathbb{N}$

$$\frac{\partial(x,y,z)}{\partial(r,\varphi,\psi)} = \begin{vmatrix} \cos^p\varphi\cdot\cos^q\psi & -rp\cdot\cos^{p-1}\varphi\cdot\sin\varphi\cdot\cos^q\psi & -rq\sin\psi\cos^p\varphi\cos^{q-1}\psi \\ \sin^p\varphi\cos^q\psi & rp\cos\varphi\sin^{p-1}\varphi\cdot\cos^q\psi & -rq\cdot\sin\psi\cdot\cos^{q-1}\psi\sin^p\varphi \\ \sin^q\psi & 0 & rq\cos\psi\sin^{q-1}\psi \end{vmatrix} =$$

$$= \sin^q\psi\left[r^2pq\sin^{p+1}\varphi\cos^{p-1}\varphi\sin\psi\cos^{2q-1}\psi + r^2pq\sin^{p-1}\varphi\cos^{p+1}\varphi\sin\psi\cos^{2q-1}\psi\right] +$$
$$+ rq\cos\psi\sin^{q-1}\psi\left[rp\sin^{p-1}\varphi\cos^{p+1}\varphi\cos^{2q}\psi + rp\sin^{p+1}\varphi\cos^{p-1}\varphi\cos^{2q}\varphi\right] =$$
$$= r^2pq\sin^{p-1}\varphi\cos^{p-1}\varphi\sin^{q+1}\varphi\sin^{q+1}\psi\cos^{2q-1}\psi + r^2pq\sin^{p-1}\varphi\cos^{p-1}\varphi\sin^{q-1}\varphi\cos^{2q+1}\psi =$$
$$= r^2pq\sin^{p-1}\varphi\cos^{p-1}\varphi\sin^{q-1}\psi\cos^{2q-1}\psi$$

$\Rightarrow$ Ответ: $r^2pq(\sin\varphi\cos\varphi)^{p-1}\cdot(\sin\psi)^{q-1}\cdot(\cos\psi)^{2q-1}$

T3. $f: \mathbb{R}^2 \to \mathbb{R}^2$

а) Показать, что $J \neq 0$; отобр не явл. биективным

б) Найти: мн-во значений f

а) $u = e^x\cos y$, $v = e^x\sin y$

$$J = \begin{vmatrix} u'_x & u'_y \\ v'_x & v'_y \end{vmatrix} = \begin{vmatrix} e^x\cos y & -e^x\sin y \\ e^x\sin y & e^x\cos y \end{vmatrix} = e^{2x} > 0$$

при этом отобр. не биективно из-за периодичности:

$u(r,\varphi) = u(r,\varphi+2\pi)$

$v(r,\varphi) = v(r,\varphi+2\pi)$ ч.т.д.

б) $u = \mathrm{Re}\, e^{x+iy}$, $v = \mathrm{Im}\, e^{x+iy}$ $\Rightarrow$ e^z принимает все знач, кроме 0

Ответ: $\mathbb{R}^2 \setminus \{0\}$

T4. Найти: $r'_x, r'_y, \varphi'_x, \varphi'_y$ через r, φ

$x = r\cos\varphi$; $y = r\sin\varphi$

$1 = r'_x\cdot\cos\varphi - r\sin\varphi\cdot\varphi'_x$

$0 = r'_y\cdot\cos\varphi - r\sin\varphi\cdot\varphi'_y$

$0 = r'_x\cdot\sin\varphi + r\cos\varphi\cdot\varphi'_x$

$1 = r'_y\cdot\sin\varphi + r\cos\varphi\cdot\varphi'_y$

$\Rightarrow$

$$\begin{cases} \cos\varphi\, r'_x - r\sin\varphi\cdot\varphi'_x = 1 \\ \sin\varphi\, r'_x + r\cos\varphi\,\varphi'_x = 0 \end{cases} \quad (1)$$

$$\begin{cases} \cos\varphi\, r'_y - r\sin\varphi\cdot\varphi'_y = 0 \\ \sin\varphi\cdot r'_y + r\cos\varphi\cdot\varphi'_y = 1 \end{cases} \quad (2)$$

(1) $\Delta = \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = r\cos^2\varphi + r\sin^2\varphi = r$

$\Delta r = \begin{vmatrix} 1 & -r\sin\varphi \\ 0 & r\cos\varphi \end{vmatrix} = r\cos\varphi$ $\qquad r'_x = \frac{\Delta r}{\Delta} = \cos\varphi$

$\Delta\varphi = \begin{vmatrix} \cos\varphi & 1 \\ \sin\varphi & 0 \end{vmatrix} = -\sin\varphi$ $\qquad \varphi'_x = \frac{\Delta y}{\Delta} = \frac{-\sin\varphi}{r}$

(2) $\Delta = \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = r$

$\Delta r = \begin{vmatrix} 0 & -r\sin\varphi \\ 1 & r\cos\varphi \end{vmatrix} = r\sin\varphi$ $\qquad r'_y = \frac{\Delta r}{\Delta} = \sin\varphi$

$\Delta\varphi = \begin{vmatrix} \cos\varphi & 0 \\ \sin\varphi & 1 \end{vmatrix} = \cos\varphi$ $\qquad \varphi'_y = \frac{\Delta\varphi}{\Delta} = \frac{\cos\varphi}{r}$

Ответ: $r'_x = \cos\varphi$; $r'_y = \sin\varphi$; $\varphi'_x = -\frac{\sin\varphi}{r}$; $\varphi'_y = \frac{\cos\varphi}{r}$

§3

№86 Решить, преобр. к полярным коорд.

$$x\frac{\partial u}{\partial y}-y\frac{\partial u}{\partial x}=0 \qquad \begin{cases} x=r\cos\varphi \\ y=r\sin\varphi \end{cases}$$

(*) $x\frac{\partial u}{\partial y}=y\frac{\partial u}{\partial x}$

$$u(x,y)=\tilde u(r(x,y),\varphi(x,y))$$

$$r(x,y)=\sqrt{x^2+y^2}$$

$$\varphi(x,y)=\operatorname{arctg}\left(\frac{y}{x}\right)+\pi k,\ k\in\mathbb{Z}$$

$$\frac{\partial r}{\partial x}=\frac{2x}{2\sqrt{x^2+y^2}}=\frac{x}{r};\quad \frac{\partial r}{\partial y}=\frac{y}{r}$$

$$\frac{\partial\varphi}{\partial x}=\frac{-y/x^2}{1+y^2/x^2}=-\frac{y}{x^2+y^2}=-\frac{y}{r^2};\quad \frac{\partial\varphi}{\partial y}=\frac{x}{r^2}$$

$$\Rightarrow \frac{\partial u}{\partial x}=\frac{\partial\tilde u}{\partial x}=\frac{\partial\tilde u}{\partial r}\cdot\frac{\partial r}{\partial x}+\frac{\partial\tilde u}{\partial\varphi}\cdot\frac{\partial\varphi}{\partial x}=\frac{x}{r}\cdot\frac{\partial\tilde u}{\partial r}-\frac{y}{r^2}\frac{\partial\tilde u}{\partial\varphi}$$

$$\frac{\partial u}{\partial y}=\frac{\partial\tilde u}{\partial y}=\frac{\partial\tilde u}{\partial r}\cdot\frac{\partial r}{\partial y}+\frac{\partial\tilde u}{\partial\varphi}\cdot\frac{\partial\varphi}{\partial y}=\frac{y}{r}\cdot\frac{\partial\tilde u}{\partial r}+\frac{x}{r^2}\cdot\frac{\partial\tilde u}{\partial\varphi}$$

$$\overset{(*)}{\Rightarrow}\ \frac{xy}{r}\cdot\frac{\partial\tilde u}{\partial r}+\frac{x^2}{r^2}\cdot\frac{\partial\tilde u}{\partial\varphi}-\frac{xy}{r}\cdot\frac{\partial\tilde u}{\partial r}+\frac{y^2}{r^2}\cdot\frac{\partial\tilde u}{\partial\varphi}=0$$

произв. диф. ф-ия

$$\frac{x^2+y^2}{r^2}\cdot\frac{\partial\tilde u}{\partial\varphi}=0\ \Rightarrow\ \frac{\partial\tilde u}{\partial\varphi}=0\ \Rightarrow\ \tilde u(r,\varphi)=f(r)=\tilde f(x^2+y^2)=u(x,y)$$

Ответ: $u=\tilde f(x^2+y^2)$; $\tilde f$ – произв. диф. ф-ия

№88 1) Решить ур-ие, преобр. к u,v

(*) $\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}=0$, $u=x+y$ $v=x-y$

$$z(x,y)=\tilde z(u,v)=\tilde z(x+y,x-y)$$

$$\frac{\partial u}{\partial x}=1\quad \frac{\partial u}{\partial y}=1\quad \frac{\partial v}{\partial x}=1\quad \frac{\partial v}{\partial y}=-1$$

$$\frac{\partial z}{\partial x}=\frac{\partial\tilde z}{\partial x}=\frac{\partial\tilde z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial\tilde z}{\partial v}\cdot\frac{\partial v}{\partial x}=\frac{\partial\tilde z}{\partial u}+\frac{\partial\tilde z}{\partial v};\quad \frac{\partial z}{\partial y}=\frac{\partial\tilde z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial\tilde z}{\partial v}\cdot\frac{\partial v}{\partial y}=\frac{\partial\tilde z}{\partial u}-\frac{\partial\tilde z}{\partial v}$$

$$\overset{(*)}{\Rightarrow}\ \frac{\partial\tilde z}{\partial u}+\frac{\partial\tilde z}{\partial v}-\frac{\partial\tilde z}{\partial u}+\frac{\partial\tilde z}{\partial v}=2\cdot\frac{\partial\tilde z}{\partial v}=0\ \Rightarrow\ \frac{\partial\tilde z}{\partial v}=0\ \Rightarrow\ \tilde z(u,v)=f(u)=f(x+y)=z(x,y)$$

$\Rightarrow$ Ответ: $z(x,y)=f(x+y)$; f–произв. диф. ф-ия.

№91 Преобр. ур-ие, приняв: x – ф-ия; u,v – незав. перем-ые

$$(y-z)\frac{\partial z}{\partial x}+(y+z)\frac{\partial z}{\partial y}=0\ (*)\qquad u=y-z;\ v=y+z$$

$$x=x(u,v)=x(y-z,y+z)$$

$$dx=\frac{\partial x}{\partial u}du+\frac{\partial x}{\partial v}dv=\frac{\partial x}{\partial u}(dy-dz)+\frac{\partial x}{\partial v}(dy+dz)=dy\left(\frac{\partial x}{\partial u}+\frac{\partial x}{\partial v}\right)+dz\left(\frac{\partial x}{\partial v}-\frac{\partial x}{\partial u}\right)$$

$$\Rightarrow dz=\frac{dx}{\frac{\partial x}{\partial v}-\frac{\partial x}{\partial u}}-\frac{dy\left(\frac{\partial x}{\partial u}+\frac{\partial x}{\partial v}\right)}{\frac{\partial x}{\partial v}-\frac{\partial x}{\partial u}};\quad \frac{\partial z}{\partial x}=\frac{1}{\frac{\partial x}{\partial v}-\frac{\partial x}{\partial u}};\quad \frac{\partial z}{\partial y}=\frac{\frac{\partial x}{\partial u}+\frac{\partial x}{\partial v}}{\frac{\partial x}{\partial u}-\frac{\partial x}{\partial v}}$$

$$\overset{(*)}{\Rightarrow}\ u\frac{\partial z}{\partial x}+v\frac{\partial z}{\partial y}=0\qquad \frac{1}{\frac{\partial x}{\partial v}-\frac{\partial x}{\partial u}}\left(u-v\left(\frac{\partial x}{\partial u}+\frac{\partial x}{\partial v}\right)\right)=0\qquad \frac{u}{v}=\frac{\partial x}{\partial u}+\frac{\partial x}{\partial v}$$

$\Rightarrow$ Ответ: $\frac{u}{v}=\frac{\partial x}{\partial u}+\frac{\partial x}{\partial v}$

§4

№51 2) Преобр. к полярным коорд.

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

$x = r\cos\varphi$ $\quad r = \sqrt{x^2+y^2}$

$y = r\sin\varphi$ $\quad \varphi = \operatorname{arctg}\left(\frac{y}{x}\right) + \pi k, \ k \in \mathbb{Z}$

$u(x,y) = v(r,\varphi)$

$\frac{\partial r}{\partial x} = \frac{x}{r}$; $\frac{\partial r}{\partial y} = \frac{y}{r}$; $\frac{\partial \varphi}{\partial x} = \frac{x}{r^2}$; $\frac{\partial \varphi}{\partial y} = -\frac{y}{r^2}$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial v}{\partial \varphi} \cdot \frac{\partial \varphi}{\partial x} = \frac{\partial v}{\partial r} \cdot \frac{x}{r} + \frac{\partial v}{\partial \varphi} \cdot \frac{x}{r^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial r} \cdot \frac{x}{r^2} - \frac{\partial v}{\partial \varphi} \cdot \frac{y}{r^2}$$

$$\frac{\partial^2 u}{\partial x^2} =$$

№52 1)

III.

T5. В стац. т. кв. форма 2 диф. положит. полуопр.

а) Может ли она быть лок. мин?

Ответ: да, например 5.9.

б) —//— лок. макс?

Ответ: нет

в) —//— не быть т. лок. экстр?

Ответ: да, например для отр. опр. 5.9.

§5

№2 2) Иссл. на экстр.

$u = 3x^2y + y^3 - 12x - 15y + 3$

$u'_x = 6yx - 12 = 0 \Rightarrow yx = 2 \quad y = \frac{2}{x}$

$u'_y = 3x^2 + 3y^2 - 15 = 0 \Rightarrow x^2 + y^2 = 5 \quad x^2 + \frac{4}{x^2} = 5 \quad x^4 - 5x^2 + 4 = 0 \quad x = \pm 1; x = \pm 2$

$y = \pm 2; y = \pm 1$

$u''_{xx} = 6y$

$u''_{yy} = 6y \Rightarrow d^2u = 6y\,dx^2 + 6y\,dy^2 + 12x\,dx\,dy$

$u''_{xy} = 6x \qquad \frac{d^2u}{6} = y\,dx^2 + y\,dy^2 + 2x\,dx\,dy$

в т. $(1,2)$ $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ — полож. опр., лок. мин.

в т. $(2,1)$ $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ — неопр; не лок. экстр.

в т. $(-1,-2)$ $\begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}$ — отр. опр; лок. макс.

в т. $(-2,-1)$ $\begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix}$ — неопр.; не лок. экстр.

Ответ: $(-1,-2)$ − лок. макс

$(1,2)$ − лок. мин

№9 Найти стац. точки и иссл. на экстр.

$u = x^4 + y^4 - 2x^2$

$u'_x = 4x^3 - 4x = 0 \Rightarrow x = 0; \pm 1$

$u'_y = 4y^3 = 0 \qquad y = 0 \Rightarrow (0,0); (1,0); (-1;0)$ — стац. точки

$u''_{xx} = 12x^2 - 4$

$u''_{yy} = 12y^2 \Rightarrow d^2u = (12x^2 - 4)dx^2 + 12y^2\,dy^2 \quad \begin{pmatrix} 12x^2-4 & 0 \\ 0 & 12y^2 \end{pmatrix} \quad \Delta_1 = 12x^2 - 4$

$u''_{xy} = 0 \qquad \Delta_2 = 12y^2(12x^2 - 4)$

в т. $(0,0)$ $\Delta_1 < 0 \quad \Delta_2 = 0$ — отр. полуопр.

$u(\Delta x, \Delta y) - u(0,0) = \Delta x^4 + \Delta y^4 - 2\Delta x^2 = \Delta x^2(\Delta x^2 - 2) + \Delta y^4$

при $\Delta x = 0 \quad \Delta y \neq 0 \quad > 0$

при $0 < \Delta x < \sqrt{2} \quad \Delta y = 0 \quad < 0$

$\Rightarrow$ экстр. нет

в т. $(\pm 1, 0)$ $\Delta_1 > 0 \quad \Delta_2 = 0$ — полож. полуопр.

$u(\pm 1 + \Delta x, \Delta y) - u(\pm 1, 0) = (\pm 1 + \Delta x)^4 + \Delta y^4 - 2(\pm 1 + \Delta x)^2 + 1 = (\pm 1 + \Delta x)^2((\pm 1 + \Delta x)^2 - 2) + \Delta y^4 + 1 =$

$= (\Delta x^2 \pm 2\Delta x + 1)(\Delta x^2 \pm 2\Delta x - 1) + \Delta y^4 + 1 = \Delta x^4 \pm 2\Delta x^3 - \Delta x^2 \pm 2\Delta x^3 + 4\Delta x^2 \cancel{\mp 2\Delta x} + \Delta x^2 \cancel{\pm 2\Delta x} - 1 + \Delta y^2 + 1 =$

$= \Delta x^4 \pm 4\Delta x^3 + 4\Delta x^2 + \Delta y^4 = \Delta x^2(\Delta x^2 - 2) + \Delta y^4 > 0 \Rightarrow \min$

Ответ: лок. мин. в $(\pm 1, 0)$

№13 1) $u = x^2 + y^2 + (z+1)^2 - xy + x$

$u'_x = 2x - y + 1 = 0$
$u'_y = 2y - x = 0 \quad \Rightarrow \quad x = -2/3,\ y = -1/3,\ z = -1 \qquad (-2/3; -1/3; -1)$
$u'_z = 2z + 2 = 0$

$u''_{xx} = 2 \qquad u''_{yy} = 2 \qquad d^2u = 2dx^2 + 2dy^2 + 2dz^2 - 2dxdy$
$u''_{xy} = -1 \qquad u''_{yz} = 0$
$u''_{xz} = 0 \qquad u''_{zz} = 2$

в т. $(-2/3; -1/3; -1)$: $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ $\Delta_1 > 0,\ \Delta_2 > 0,\ \Delta_3 > 0 \Rightarrow$ лок. мин.

Ответ: лок. мин. в $(-2/3; -1/3; -1)$

№18 1) Иссл. неявную ф-ию, зад. ур-ием

$x^2 + y^2 + u^2 + 2x - 2y + 4u - 3 = 0$

$\cancel{2}x\,dx + \cancel{2}y\,dy + \cancel{2}u\,du + \cancel{2}dx - \cancel{2}dy + \overset{2}{\cancel{4}}du = 0$

$du(u+2) + dx(x+1) + dy(y-1) = 0$

$du = -\dfrac{dx(x+1) + dy(y-1)}{u+2} = 0 \Rightarrow x = -1;\ y = 1$

$1 \cancel{+x} + u^2 \cancel{-2} - 2 + 4u - 3 = 0$

$u^2 + 4u - 5 = 0 \Rightarrow u = 1, -5$

$d^2u(u+2) + \overset{0}{\cancel{du^2}} + dx^2 + dy^2 = 0$

$d^2u = -\dfrac{dx^2 + dy^2}{u+2}$ в т. $(-1, 1, 1)$ $d^2u = -\dfrac{dx^2}{3} - \dfrac{dy^2}{3}$ – отр. опр. $\Rightarrow$ max

в т. $(-1, 1, -5)$ $d^2u = \dfrac{dx^2}{3} + \dfrac{dy^2}{3}$ – полож. опр $\Rightarrow$ min

Ответ: $(-1, 1, 1)$ – лок. макс.
$(-1, 1, -5)$ – лок. мин.

№19 1) Найти усл. экстр. ф-ии $u = f(x,y)$

$u = xy$; $x + y - 1 = 0$

$y = 1 - x \Rightarrow f(x,y) = x(1-x) = x - x^2 = f_0(x)$

$f_0(x) = -x^2 + x = -(x^2 - x + \frac{1}{4} - \frac{1}{4}) = -(x - \frac{1}{2})^2 + \frac{1}{4}$

$f_0(x)$ принимает наиб. знач. в $x = 1/2$, других экстр. нет.
$\Rightarrow$ лок. макс. $f(x,y)$ в $x = 1/2$; $y = 1 - 1/2 = 1/2$

Ответ: $(1/2, 1/2)$ – лок. макс

№21 2) $u = 1 - 4x - 8y$; $x^2 - 8y^2 - 8 = 0$

$\mathcal{L} = 1 - 4x - 8y + \lambda(x^2 - 8y^2 - 8)$

$\begin{cases} \mathcal{L}'_x = -4 + 2\lambda x = 0 \\ \mathcal{L}'_y = -8 - 16\lambda y = 0 \\ x^2 - 8y^2 - 8 = 0 \end{cases}$ $\quad \lambda x = 2,\ \lambda y = -\frac{1}{2},\ \left(\frac{2}{\lambda}\right)^2 - 8\left(\frac{1}{2\lambda}\right)^2 - 8 = 0$

$\frac{4}{\lambda^2} - \frac{2}{\lambda^2} - 8 = 0 \quad \frac{2}{\lambda^2} = 8 \quad \lambda = \pm 1/2$

$\Rightarrow x = \pm 4;\ y = \mp 1$

$\mathcal{L}''_{xx} = 2\lambda$; $\mathcal{L}''_{yy} = -16\lambda$; $\mathcal{L}''_{xy} = 0$; $d^2\mathcal{L} = \pm dx^2 \mp 8dy^2$ $\begin{pmatrix} \pm 1 & 0 \\ 0 & \mp 8 \end{pmatrix}$ $\Delta_1 > 0,\ \Delta_2 < 0$ или $\Delta_1 < 0,\ \Delta_2 < 0$
неопр. форма

$2x\,dx - 16y\,dy = 0$; $dx = 8\frac{y}{x}dy$

при $\lambda = 1/2$; $y/x = -1/4$
$d^2\mathcal{L} = 4dy^2 - 8dy^2 = -4dy^2$ – отр.
$(4, -1)$ – лок. макс.

при $\lambda = -1/2$; $y/x = -1/4$
$d^2\mathcal{L} = -4dy^2 + 8dy^2 = 4dy^2$ полож.
$(-4, 1)$ – лок. мин

Ответ: $(4, -1)$ – лок. макс.
$(-4, 1)$ – лок. мин.

№25 б) $u = x - y + 2z$; $x^2+y^2+2z^2=16$

$\mathcal{L} = x - y + 2z + \lambda(x^2+y^2+2z^2-16)$

$$\begin{cases} \mathcal{L}'_x = 1+2\lambda x = 0 & x = -\frac{1}{2\lambda} \\ \mathcal{L}'_y = -1+2\lambda y = 0 & y = \frac{1}{2\lambda} \\ \mathcal{L}'_z = 2+4\lambda z = 0 & z = -\frac{1}{2\lambda} \\ x^2+y^2+2z^2=16 \end{cases}$$

$\frac{1}{4\lambda^2}+\frac{1}{4\lambda^2}+\frac{2}{4\lambda^2}=16$ $\quad\lambda^2=\frac{1}{16}$

$\lambda = \pm\frac{1}{4} \Rightarrow x = \mp 2$; $y = \pm 2$; $z = \mp 2$

$\mathcal{L}''_{xx} = 2\lambda$ $\quad\mathcal{L}''_{yy} = 2\lambda$ $\quad\mathcal{L}''_{zz} = 4\lambda$

$\mathcal{L}''_{xy} = 0$ $\quad\mathcal{L}''_{yz} = 0$ $\quad\mathcal{L}''_{xz} = 0$

$\lambda = 1/4$

$d^2\mathcal{L} = \frac{1}{2}(dx^2+dy^2+dz^2)$ – полож. опр; лок. мин.

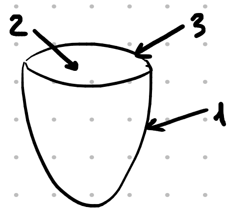
$\lambda = -1/4$

$d^2\mathcal{L} = -1/2\,(dx^2+dy^2+dz^2)$ – отр. опр; лок. макс.

Ответ: $(-2,2,-2)$ – лок. мин.
$(2,-2,2)$ – лок. макс.

№31 3) Найти наиб. и наим. знач. u на мн-ве

$u = x+y+z \quad x^2+y^2 \le z \le 1$



1) $u = x+y+z \quad x^2+y^2=z \quad x^2+y^2<1$

$u = x+y+x^2+y^2$

$u'_x = 1+2x = 0$
$u'_y = 1+2y = 0$ $\Rightarrow$ стац.т. $(-1/2;-1/2)$ $\quad u = -1/2$

2) $u = x+y+z \quad x^2+y^2<z \quad z=1$

$u = x+y+1$

стац. точки: $\varnothing$

3) $u = x+y+z \quad x^2+y^2=z \quad z=1$

$u = x+y+1$; $x^2+y^2=1$

$\mathcal{L} = x+y+1+\lambda(x^2+y^2-1)$

$$\begin{cases} \mathcal{L}'_x = 1+2\lambda x = 0 \\ \mathcal{L}'_y = 1+2\lambda y = 0 \\ x^2+y^2=1 \end{cases}$$

$x = -\frac{1}{2\lambda} = y$

$\frac{1}{2\lambda^2}=1 \quad \lambda^2=\frac{1}{2} \quad \lambda=\pm\frac{1}{\sqrt{2}} \quad x=y=\mp\frac{1}{\sqrt{2}}$

$u(\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}}) = 1+\sqrt{2} \quad u(-\frac{1}{\sqrt{2}};-\frac{1}{\sqrt{2}}) = 1-\sqrt{2}$

$\Rightarrow$ Ответ: $\max u = 1+\sqrt{2}$
$\min u = -1/2$