**Ultra-narrow high-efficient power splitters and waveguides based on the TE01 Mie-resonant bandgap**

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**Abstract:** The study of Mie resonances brings new freedoms in controlling magnetic and electric fields of light in all-dielectric subwavelength metamaterials. In this paper, ultra-narrow and high-efficient Y and T shapes optical power splitters as well as straight and П shapes waveguides using two rows of 2D germanium rods in the air are designed and simulated, also the position disordering effect on the waveguides are considered. The finite-difference time-domain simulation results for two rows straight and П shapes waveguides with no position disordering at the normalized frequency of show the transmission of 90% and, the transmissions with no disordering for two rows of Y and T shapes power splitters for each output branch at the normalized frequency of are more than 46%. In this paper, the straight and П shapes waveguides with four rows of germanium rods show robustness to position disordering of η = 10%. The proposed ultra-narrow waveguides and power splitters are vital components in the high-density optical integrated circuits.

1. **Introduction:**

With the advancement of photonics technology, the demand for small, compact, low loss, and high-efficient photonic devices is vital. Plasmonics opened a new way for accommodating a few of these demands by controlling light in a small fraction of wavelength using coupling light to the oscillations of conduction-band electrons (plasmon) [1–3]. Due to the free electrons in plasmonic structures (mainly metals), they suffer from high energy dissipation normally at optical frequencies [4,5]. Also, plasmonic particles (such as metal dimers) only support electric dipoles which result in a single polarization coupling [6]. To overcome the energy dissipation of plasmonic structures, photonic crystals which are artificial dielectric structures in periodic arrangements are ideal replacements. Because of the periodic nature of photonic crystals, light is blocked in photonic bandgaps which act as mirrors in all directions that is due to the Bragg effect [7–9]. Creating defects in photonic crystals localize one or more modes in photonic bandgaps that act as guided modes, result in steering the light wave in the defects. Defects are created using removing or modifying the photonic crystals’ elements. The other type of defect is created by putting two different lattices near together which is called topological photonic crystals. This kind of defect localizes a Dirac-shape mode in the band diagram which results in propagation of light through the defect [10–13]. Although photonic crystals are low-loss structures, they suffer from big sizes and can just manipulate light waves with the wavelength in the order of their lattice constants.

The transition from photonic crystals to all-dielectric metamaterials brings a new era in designing high-efficient, low loss, compact, and shrank photonic devices. Because of the high-permittivity nature of dielectric elements in metamaterials, electric and magnetic dipoles are excited ans support orthogonal polarizations [6]. The other merits of high-refractive-index dielectric elements are their low-loss nature, resulting in high-efficient devices and controlling light in subwavelengths with very low dissipation energy [14–16]. There are two scenarios to transit from photonic crystals to metamaterials. The first scenario is decreasing lattice constant which is called *r/a*-scenario (where *r* is the radius of particles and *a* is the lattice constant). The latter is increasing permittivity of the elements that is called *ɛ*-scenario [17]. The goal of both scenarios is to increase light-matter interactions to give more freedom in controlling light waves in subwavelengths. The transition brings the phenomenon of Mie resonance rather than the Bragg effect. Mie resonance is a phenomenon related to the scattering of light by small particles [18–20]. Based on the Mie resonance, the total scattering cross-section of a spherical particle is expressed as [21]

 (1)

where *C*sca and *k* are the scattering cross-section and wavenumber, two coefficients of *a*m and *b*m (where subscript *m* is the order of dipole coefficients) stand for electric and magnetic multipolar modes, respectively. At low frequencies, the coefficients *a*1 and *b*1 correspond to the electric dipole (ED) and magnetic dipole (MD), respectively, are dominant rather than the other coefficients. The ED and MD were shown experimentally in spherical nanoparticles [22–24], cubic nanoparticles [25,26], nanodisks [27,28], and so forth. In dielectric particles with low permeability the magnetic response is week, but in high-permittivity particles, the displacement current results in magnetic response which in turn excites Mie resonances inside the particle [14,18]. Strong Mie resonances appear in high-refractive-index particles in the form of periodic structures such as metasurfaces, that are due to constructive interference of scatterings. In high refractive index all-dielectric 2D metamaterials these Mie resonances appear in the form of bandgaps, showing interesting properties such as immune to position disordering for the TE01 Mie bandgap due to the transition from photonic crystal (*ε* = 4) to metamaterials (*ε* = 25) [29].

In this paper, we have proposed ultra-narrow waveguides and power splitters based on 2D germanium rods in the air in cubic and hexagonal lattices. The proposed waveguides are in straight, and П shapes with just two rows of germanium rods in the air and also, power splitters are in T and Y shapes with two rows of germanium rods in the air. These structures are designed based on the TE01 Mie bandgap that is immune to position disordering. For simulating the structures, the 2D finite-difference time-domain (FDTD) module of Lumerical software is utilized. The paper has been organized into three sections. The second section describes theory and design and the final section concludes the paper.

1. **Theory and design**

A germanium rod with the refractive index of 4.13 has been depicted in Fig. 1, with radius *r* and length *L* (*L* >> *r*) that is considered as a 2D rod. There are two types of resonances in the cylindrical rod. The first one is Mie and the second one is Fabry Perot (FP) resonances [28,30]. The resonances are denoted as  ( and  = 1, 2, …), where  and are mode numbers for Mie and FP resonances, respectively. Concerning the incident polarization of transverse magnetic (TM) or transverse electric (TE), each of the resonances  and  excite, correspondingly. In Fig. 1, an incident plane wave with TE polarization hits the germanium rod to excite  resonances. Due to the homogeneity of the rod in the *z*-direction, we consider 2D simulations and the results show the Mie resonances of , , and , as shown in Fig. 1. The excited , , and  resonances show one, two, and four magnetic dipoles, respectively, which are created by circulating polarization currents inside the rod as represented in the *x*-*y* view of electromagnetic field distributions. As it is obvious from this figure the scattering cross-section in the *y*-direction (parallel to the incident **E**-field direction) has the strongest peak rather than the other peaks, which means putting another adjacent rod in the parallel direction of the **E**-field makes a strong hotspot or coupling between the rods.

In the form of periodic arrangements of 2D Ge rods, the scattering resonance of show the strong couplings between adjacent rods that have been located parallel to the **E**-field direction, rather than the other rods, as shown in Fig. 2. In this figure by exciting TE01 mode which is matched with  Mie resonance, strong couplings of **E**-field in the adjacent rods appear, that is due to the scattering of rods in the **E**-field direction. The electrical field distributions in the first row show strong couplings between rods and the **E**-field direction of the couplings is from right to left that is due to the circulating displacement current inside each particle, as shown in Fig. 2 (a). The magnetic field distributions of the structure show strong localization of magnetic fields in the first row of the structure and represented outward magnetic dipole. The other rows show very low coupling between the rods.

*r*

*L*

**E**

**H**

k

TE

*y*

*z*

*x*



Fig. 1. Left, single Ge rod with the refractive index of 4.13 which is hit with TE polarized plane wave with radius *r* and length *L* (*L* >> *r*) and right shows *x*, *y*-directions, and total scattering cross-section of the germanium rod versus normalized frequency. The electric field (**E**) and magnetic field (**H**) distributions in the *x*-*y* plane have been shown for each scattering peak in the right graph.



Fig. 2. Excitation of TE01 mode by a plane wave upward in a 2D periodic array of Ge rods. (a) shows the electric field, and (b) shows Magnetic field distributions. Incident wave with TE polarization hit the structure from the bottom to the top.

In the periodic structure of 2D Ge rods, in addition to the TE01 mode, TE11 and TE21 modes appear. These TE01, TE11, and TE21 modes are responsible for three bandgaps with the same normalized frequency of , , and  resonances as shown in Fig. 3. The TE01 bandgap that is due to the scattering resonance of  is stronger than the others due to the coupling between adjacent rods. Ref. [29] expresses, by a transition from photonic crystals (ε = 4) to metamaterials (ε = 25) the TE01 bandgap is immune to position disordering of about 40%. By following their definitions, the position of each rod  is and  where , , , and  are the original position of the rods, random variables between -1 and 1, and the strength of the disorder. Based on the definitions, the disorder parameter of *η* is defined as the ratio of *σ* to *a*, where *a* is the lattice constant. As it is obvious from Fig. 3, the TE01 bandgap mode shows the disorder immunity of *η* = 20% for the 2D Ge metamaterials in cubic and hexagonal lattices. Utilizing this bandgap mode gives the freedom to design subwavelength structures in the form of waveguides and power spitters.



TE01

TE11

TE21

TE01

TE11

TE21

Fig. 3. (a) shows the transmission spectrum of cubic structure with *η* = 0, 10, and 20% in black, red, and blue colors, respectively, and (b) shows the transmission spectrum of hexagonal structure for *η* = 0, 10, and 20% in black, red and blue colors, respectively. The inset of (a) and (b) show the unit cell of the Ge rods in cubic and hexagonal structures.

**Waveguides:**

As discussed earlier, the TE01 mode is robust to position disordering as well as strong couplings only happen for the first row of rods and the other rows have a very small portion of light-matter interactions. The normalized center frequency of this bandgap mode is smaller than the other bandgap modes which results in a subwavelength structure. Based on this idea of strong coupling between rods just in the first row, we have proposed two straight and П shapes waveguides with two rows of Ge rods in the air in cubic structure as shown in Fig. 4. This figure shows the structure of waveguides entitled B, C, and D, which A is the pure structure with no defects. (b)–(d) and (f)–(h) depict the straight waveguides with ten, four, and two rows of 2D Ge rods (red color) in the air (blue color) as shown in Fig. 4.



Fig. 4. Structure of the 2D Ge metamaterials waveguides with η = 10% disordering. (a) and (e) show the structures with no defects. (b)–(d) are straight waveguides with ten, four, and two rows of Ge rods in the air. (f)–(h) show the structures of П shape waveguides with ten, four, and two rows of Ge rods in the air. The blue and red parts show the air and Ge rods, respectively.

Fig. 5 shows the transmission spectrums for the structures of A, B, C, and D for straight and П shapes waveguides with different position disordering of *η* = 0, 10, 20%. (a)–(c) and (d)–(f) show transmission spectrums for straight and П shapes waveguides, respectively. (a) and (d), (b) and (e), and (c) and (f) represent transmission spectrums for *η* = 0, 10, 20%, respectively. As obvious from the transmission spectrums, the structures with no defects (black dashed curves) show immunity of TE01 bandgap to position disordering over  (where *a* and *λ* are lattice constant and wavelength, respectively). Creating line defects either straight or П shapes localize a guided mode inside the immune bandgap. For *η* = 0, the structures with no disordering as shown in (a) and (d), the structure B shows a localized mode in the solid red line in the bandgap with slow oscillations that are due to the rows of rods surrounded the straight and П defects. In another word, due to the constructive and destructive interferences between the refractions and reflections of light among the many rods in structure B, the transmission spectrums have a few slow oscillations. Reducing the number of rows to four, the number of reflections and reflections are reduced, results in reducing the number of oscillations as shown in Fig. 5 (a) and (d) in green dot lines. As the number of rows reduces to two, the number of oscillations subsequently reduces as shown in Fig. 5 (a) and (d) in blue dot-dashed lines. As shown in Fig. 5 (a) and (d), the transmission amplitudes are about 90% at the normalized frequency of (solid black circles) for both straight and П shapes waveguides with ten, four, and two rows of Ge rods (B, C, and D structures).



Fig. 5. Transmission spectrums of the structures A, B, C, and D for different position disordering. (a)–(c) show transmission spectrums for straight waveguides with disordering of *η* = 0, 10, 20%, respectively. (d)–(f) show transmission spectrums for П shape waveguides with disordering of *η* = 0, 10, 20%, respectively.

For the straight waveguides with position disordering of *η* = 10%, as shown in Fig. 5 (b), the transmission amplitudes are about 80, 90%, and 75% at the normalized frequency of  for structures B, and C, and D, respectively. For the case of П shape waveguides with position disordering of *η* = 10% the transmission amplitudes for the structures B, C, and D are approximately 90, 90, and 50%, respectively, at the normalized frequency of . As obvious from Fig. 5 (b) and (e), the transmission spectrums for the structure D (just two rows of Ge rods) in dot-dashed blue colors for both straight and П shapes waveguides are 75 and 50% respectively; this decreasing is due to the disordering of the structure, the disordering increases the distance between some rods which weaken the coupling between them and the light wave flee between the rods. Because of the sharp 90˚ bend in П shape waveguides, the transmission for D type structure is also less than the straight D type structure. With increasing of position disordering to *η* = 20%, as shown in Fig. 5 (c) and (f) the transmission amplitudes are the lowest than the structures with position disordering of *η* = 0, and 10%, which is due to the increasing in disordering that reduces coupling between adjacent rods that also results in changing in phases which in turn change the normalized guiding frequencies.



Fig. 6. Electromagnetic power distributions of the structure D for different position disordering. (a)–(c) show power distributions for straight waveguides with disordering of *η* = 0, 10, 20%, respectively. (d)–(f) show power distributions for П waveguides with disordering of *η* = 0, 10, 20%, respectively. The incident wave is a Gaussian wave the with normalized frequency of  injects from bottom to up.

Electromagnetic power distributions at  for the both straight and П shapes waveguides of two rows of rods (C type), show high coupling between adjacent rods in structures when *η* = 0, that results in high confinement of electromagnetic power in the waveguides as shown in Fig. 6 (a) and (d). With increasing disordering to *η* = 10 and 20%, the coupling distance between adjacent rods decreases, consequently light waves flee between rods with increased adjacent distances as obvious from Fig. 6.

**Splitters:**

In this section Y and T power shapes power splitter in hexagonal and cubic structures, respectively, are considered. The structures in Fig. 7 (a) and (e) show the hexagonal and cubic lattices with no defect and with no position disordering (*η* = 0%). In this figure, (b) and (f) show Y and T line defects in hexagonal and cubic structures which are called B shapes, and (c) and (g) structures are power splitters in Y and T shapes with four rows of rods in hexagonal and cubic lattices which are called C shapes. The last (d) and (h) figures show power splitters in Y and T shapes with two rows of rods in hexagonal and cubic lattices which are called D shapes. In this figure the red and blue colors show germanium rods the air, respectively.



Fig. 7. Structure of the 2D Ge metamaterials power splitters with *η* = 0%. (a) and (e) show hexagonal and cubic structures with no defects. (b) and (f) represent Y and T shapes splitters in hexagonal and cubic lattices, respectively. (c) and (d) show Y splitters with four and two rows of Ge rods in a hexagonal lattice, and also (g) and (h) show T splitters with four and two rows of Ge rods in a cubic lattice. The blue and red colors show the air and Ge rods, respectively.

The transmission spectrums at the one output branch of the Y and T shapes power splitters in hexagonal and cubic lattices are shown in Fig. 8 (a) and (b), respectively. As obvious in this figure, the structures with no defects (A) show a strong bandgap in solid black color with different bandwidths for hexagonal and cubic structure. By creating either Y or T defects, a guided mode localizes in the bandgap which has a few oscillations that is due to the reflections and refractions of light between the rods of the whole lattice and these oscillations decrease with reducing the number of rows of the waveguides as shown in the transmission spectra of Fig. 8. The Y shape waveguides with the B and C shapes show the high transmission of 47% at the frequency of  and the D shape show high transmission of 46% at the normalized frequency of . At  the refractions and reflections between rods in structures B and C interference destructively with the guided mode which result in decreasing in transmission amplitudes of 30%, but at  for the structure D with just two rows of rods, there are no those kinds of reflections and refractions, so the transmission amplitude is higher rather than structures B and C as shown in Fig. 8 (a). As obvious from transmission spectrum of T shape power splitters shown in Fig. 8 (b), the transmission amplitudes for B, C and, D structures are the same with the value of 47% at the frequency of . Due to the cubic structure, the reflections and transmissions are not prominent at the frequency of , so the transmission amplitudes are the same for all structures of B, C, and D.



Fig. 8. Transmission spectrum for the Y and T splitters for A, B, C, and D structures. (a) Y and (b) shows T splitters.

The electromagnetic power distributions for Y and T shape power splitters for structure D have been shown in Fig. 9. This figure shows the high confinement of the electromagnetic waves in the waveguides. Due to the structures with no position disordering, there are strong couplings between adjacent rods in both Y and T shape structures, results in high-efficient and small all-dielectric power splitters with just two rows of Ge rods.



Fig. 9. Electromagnetic power distributions for the Y and T splitters for D structure. (a) shows Y, and (b) shows T splitters transmission spectrums at .

**Conclusion:**

In this study we proposed, all-dielectric ultra-narrow high-efficient straight and П shapes waveguides as well as Y and T shapes power splitters using 2D germanium rods in the air. The proposed optical components were designed based on TE01 Mie-resonant bandgap that is immune to position disordering of *η* = 10%. The straight and П shapes waveguides as well as T power splitters contain 2D germanium rods, are in a cubic lattice. The Y shape power splitters contain 2D germanium rods, are in a hexagonal lattice. The transmission results with no position disordering for two rows of Ge rods for waveguides and power splitters at the normalized frequency of  showed 90% and more than 46%, respectively. The transmissions for four rows of straight and П shapes with disordering of *η* = 10% waveguides are about 90% at . The designed components are the best candidates for the optical integrated circuits.

**References**

1. H. Yu, Y. Peng, Y. Yang, and Z. Y. Li, "Plasmon-enhanced light–matter interactions and applications," npj Comput. Mater. **5**(1), 1–14 (2019).

2. S. A. Maier and H. A. Atwater, "Plasmonics: Localization and guiding of electromagnetic energy in metal/dielectric structures," J. Appl. Phys. **98**(1), 11101 (2005).

3. Y. Wang, J. Z. Ou, A. F. Chrimes, B. J. Carey, T. Daeneke, M. M. Y. A. Alsaif, M. Mortazavi, S. Zhuiykov, N. Medhekar, M. Bhaskaran, J. R. Friend, M. S. Strano, and K. Kalantar-Zadeh, "Plasmon resonances of highly doped two-dimensional MoS2," Nano Lett. **15**(2), 883–890 (2015).

4. A. Boltasseva and H. A. Atwater, "Low-loss plasmonic metamaterials," Science (80-. ). **331**(6015), 290–291 (2011).

5. D. García-Lojo, S. Núñez-Sánchez, S. Gómez-Graña, M. Grzelczak, I. Pastoriza-Santos, J. Pérez-Juste, and L. M. Liz-Marzán, "Plasmonic Supercrystals," Acc. Chem. Res. **52**(7), 1855–1864 (2019).

6. R. M. Bakker, D. Permyakov, Y. F. Yu, D. Markovich, R. Paniagua-Domínguez, L. Gonzaga, A. Samusev, Y. Kivshar, B. Lukyanchuk, and A. I. Kuznetsov, "Magnetic and electric hotspots with silicon nanodimers," Nano Lett. **15**(3), 2137–2142 (2015).

7. S. A. Rinne, F. García-Santamaría, and P. V. Braun, "Embedded cavities and waveguides in three-dimensional silicon photonic crystals," Nat. Photonics **2**(1), 52–56 (2008).

8. R. K. Cersonsky, J. Antonaglia, B. D. Dice, and S. C. Glotzer, "The diversity of three-dimensional photonic crystals," Nat. Commun. **12**(1), 1–7 (2021).

9. Y. Liu, H. Wang, J. Ho, R. C. Ng, R. J. H. Ng, V. H. Hall-Chen, E. H. H. Koay, Z. Dong, H. Liu, C. W. Qiu, J. R. Greer, and J. K. W. Yang, "Structural color three-dimensional printing by shrinking photonic crystals," Nat. Commun. **10**(1), 1–8 (2019).

10. L. H. Wu and X. Hu, "Scheme for achieving a topological photonic crystal by using dielectric material," Phys. Rev. Lett. **114**(22), 223901 (2015).

11. Y. Ota, R. Katsumi, K. Watanabe, S. Iwamoto, and Y. Arakawa, "Topological photonic crystal nanocavity laser," Commun. Phys. **1**(1), 1–8 (2018).

12. X. Zhou, Z. K. Lin, W. Lu, Y. Lai, B. Hou, and J. H. Jiang, "Twisted Quadrupole Topological Photonic Crystals," Laser Photonics Rev. **14**(8), 2000010 (2020).

13. E. Sauer, J. P. Vasco, and S. Hughes, "Theory of intrinsic propagation losses in topological edge states of planar photonic crystals," Phys. Rev. Res. **2**(4), 43109 (2020).

14. S. Jahani and Z. Jacob, "All-dielectric metamaterials," Nat. Nanotechnol. **11**(1), 23–36 (2016).

15. H. N. S. Krishnamoorthy, G. Adamo, J. Yin, V. Savinov, N. I. Zheludev, and C. Soci, "Infrared dielectric metamaterials from high refractive index chalcogenides," Nat. Commun. **11**(1), 1–6 (2020).

16. Y. Rao, L. Pan, C. Ouyang, Q. Xu, L. Liu, Y. Li, J. Gu, Z. Tian, J. Han, and W. Zhang, "Asymmetric transmission of linearly polarized waves based on Mie resonance in all-dielectric terahertz metamaterials," Opt. Express **28**(20), 29855 (2020).

17. M. V. Rybin, D. S. Filonov, K. B. Samusev, P. A. Belov, Y. S. Kivshar, and M. F. Limonov, "Phase diagram for the transition from photonic crystals to dielectric metamaterials," Nat. Commun. **6**(1), 1–6 (2015).

18. Q. Zhao, J. Zhou, F. Zhang, and D. Lippens, "Mie resonance-based dielectric metamaterials," Mater. Today **12**(12), 60–69 (2009).

19. W. Chen, Q. Yang, Y. Chen, Y. Chen, and W. Liu, "Global Mie Scattering: Polarization Morphologies and the Underlying Topological Invariant," ACS Omega **5**(23), 14157–14163 (2020).

20. D. A. Bobylev, D. A. Smirnova, and M. A. Gorlach, "Nonlocal response of Mie-resonant dielectric particles," Phys. Rev. B **102**(11), 115110 (2020).

21. C. Bohren and D. Huffman, *Absorption and Scattering of Light by Small Particles* (2008).

22. C. Zhang, Y. Xu, J. Liu, J. Li, J. Xiang, H. Li, J. Li, Q. Dai, S. Lan, and A. E. Miroshnichenko, "Lighting up silicon nanoparticles with Mie resonances," Nat. Commun. **9**(1), 1–7 (2018).

23. T. Okazaki, H. Sugimoto, T. Hinamoto, and M. Fujii, "Color Toning of Mie Resonant Silicon Nanoparticle Color Inks," ACS Appl. Mater. Interfaces **13**(11), 13613–13619 (2021).

24. S. Kruk and Y. Kivshar, "Functional Meta-Optics and Nanophotonics Govern by Mie Resonances," ACS Photonics **4**(11), 2638–2649 (2017).

25. Y. Nagasaki, M. Suzuki, and J. Takahara, "All-Dielectric Dual-Color Pixel with Subwavelength Resolution," Nano Lett. **17**(12), 7500–7506 (2017).

26. K. Sugawa, M. Matsubara, H. Tahara, D. Kanai, J. Honda, J. Yokoyama, K. Kanakubo, H. Ozawa, Y. Watanuki, Y. Kojima, N. Nishimiya, T. Sagara, K. Takase, M. A. Haga, and J. Otsuki, "Mie Resonance-Enhanced Light Absorption of FeS2 Nanocubes in a Near-Infrared Region: Intraparticulate Synergy between Electronic Absorption and Mie Resonances," ACS Appl. Energy Mater. **2**(9), 6472–6483 (2019).

27. G. Grinblat, Y. Li, M. P. Nielsen, R. F. Oulton, and S. A. Maier, "Degenerate Four-Wave Mixing in a Multiresonant Germanium Nanodisk," ACS Photonics **4**(9), 2144–2149 (2017).

28. A. A. Bogdanov, K. L. Koshelev, P. V. Kapitanova, M. V. Rybin, S. A. Gladyshev, Z. F. Sadrieva, K. B. Samusev, Y. S. Kivshar, and M. F. Limonov, "Bound states in the continuum and Fano resonances in the strong mode coupling regime," Adv. Photonics **1**(01), 1 (2019).

29. C. Liu, M. V. Rybin, P. Mao, S. Zhang, and Y. Kivshar, "Disorder-Immune Photonics Based on Mie-Resonant Dielectric Metamaterials," Phys. Rev. Lett. **123**(16), 163901 (2019).

30. D. J. Traviss, M. K. Schmidt, J. Aizpurua, and O. L. Muskens, "Antenna resonances in low aspect ratio semiconductor nanowires," Opt. Express **23**(17), 22771 (2015).