Direct Volume Rendering

Discrete Implementation

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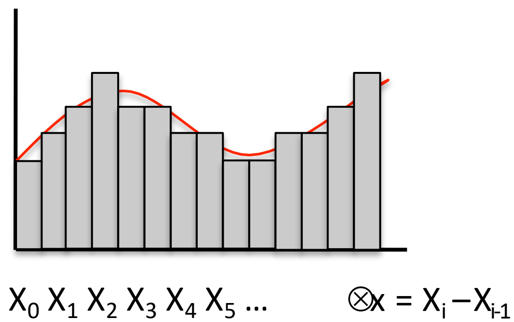
Discrete Implementation

- Numerical integration:



$$\int_0^D h(x)dx = \sum_{i=1}^{i=n} h(x_i)\Delta x$$

(Riemann Sum)

$$e^{-\int_0^D \tau(t)dt} = e^{-\sum_{i=1}^{i=n} \tau(t_i)\Delta t}$$

$$= e^{-\sum_{i=1}^{i=n} \tau(i\Delta x)\Delta x} = \prod e^{-\tau(i\Delta x)\Delta x}$$

$$= \prod_{i=1}^{i=n}(1-\alpha_i)$$ (Remember $1 - e^{-\int_0^D \tau(t)dt} = \alpha$)

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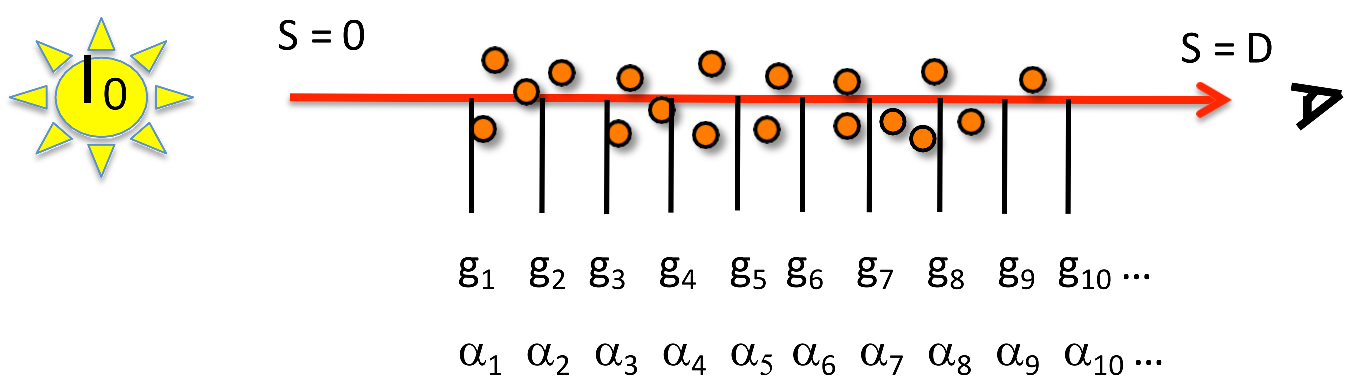
Discrete Implementation

$$\boxed{I(D) = I_0 \times e^{-\int_0^D \tau(t)dt} + \int_0^D g(s)e^{-\int_s^D \tau(t)dt}ds}$$

$$I_0 \prod_{i=1}^{i=n}(1-\alpha_i) \qquad \sum_{i=1}^{i=n} g_i \times \prod_{j=i+1}^{n}(1-\alpha_i)$$

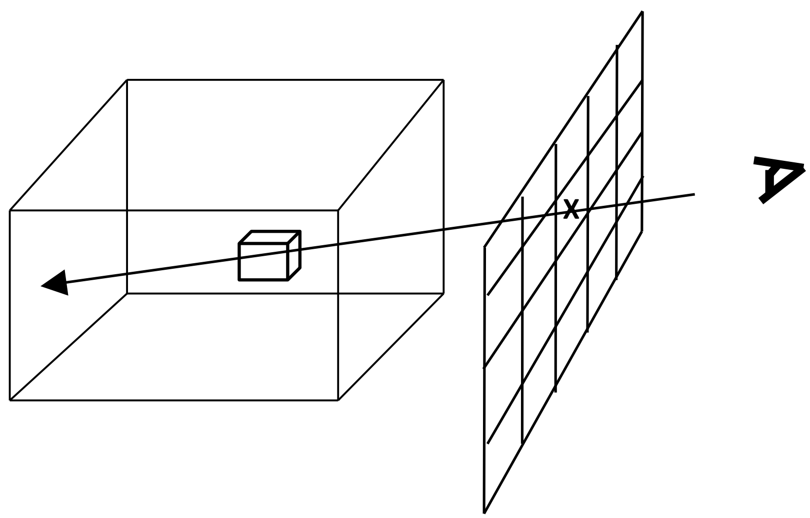
$= g_n + (1-\alpha_n)(g_n-1+(1-\alpha_{n-1})(g_{n-2}+ (1-\alpha_{n-2}(\ldots. (1-\alpha_2)(g_2+(1-\alpha_1)(g_1+I_0)))))\ldots)))))$

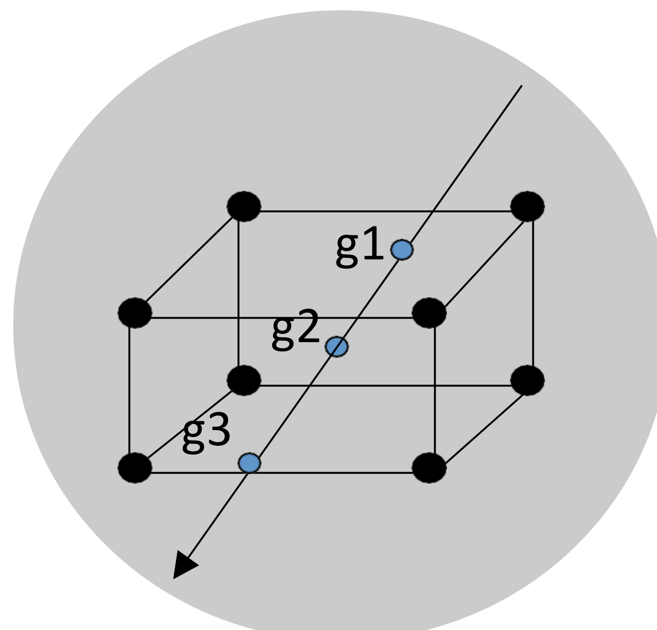
This is called - Back to Front Compositing



Ray Casting Algorithm

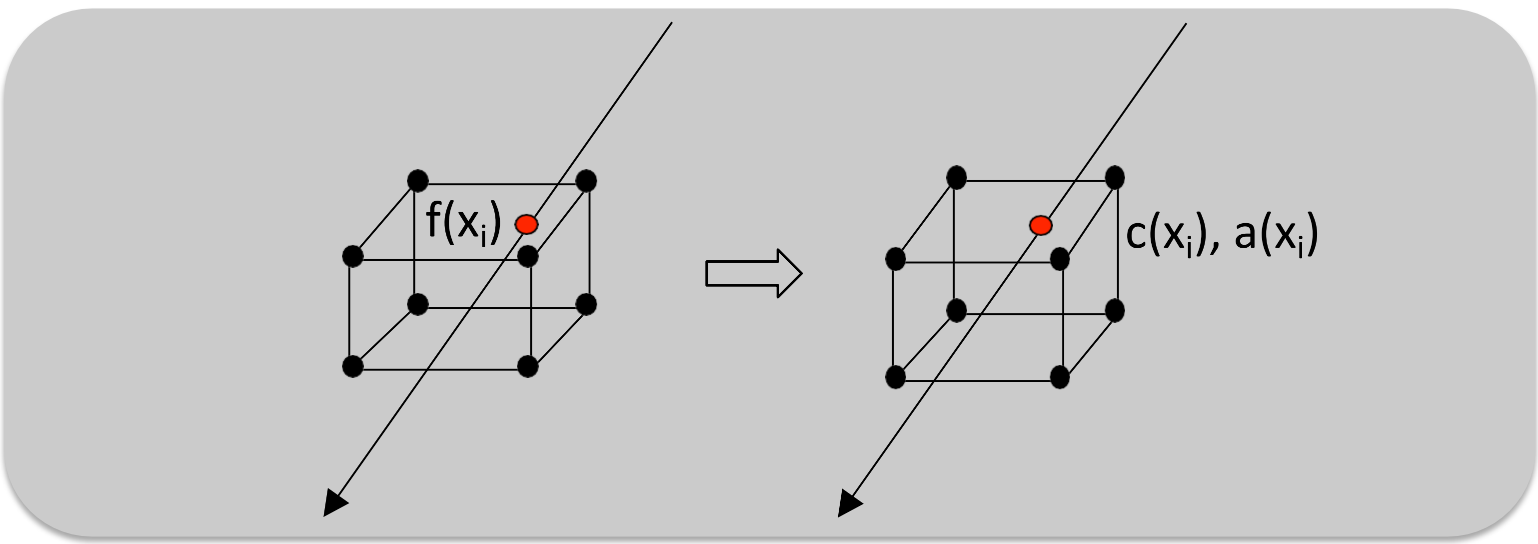
- For each pixel
  - Cast a ray into the volume
  - Linearly interpolate data values from cell (voxel) corners
  - Convert the data values to optical properties (color and opacity)
  - Composite the optical properties
  - Return the final color





Shading and Classificaton

- Shading: computer a color for every sample in the volume
- Classification: compute an opacity for every sample in the volume



- This is often done through a table (transfer function) lookup

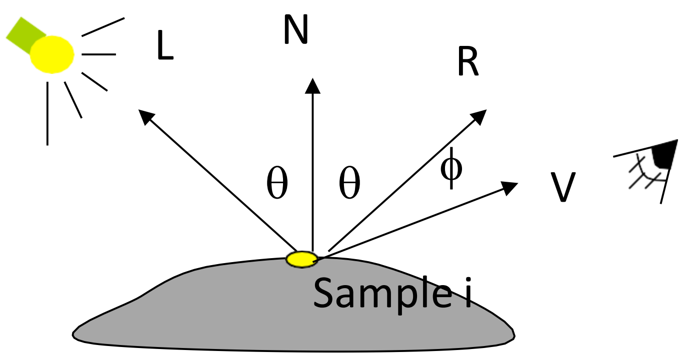
Shading

- Use the Phong illumination model

illumination = ambient + diffuse + specular

$$= \boxed{C(x_i) \times I_a} + \boxed{C(x_i) \times I_d \times (N.L)} + \boxed{C(x_i) \times I_s \times (R.V)^n}$$

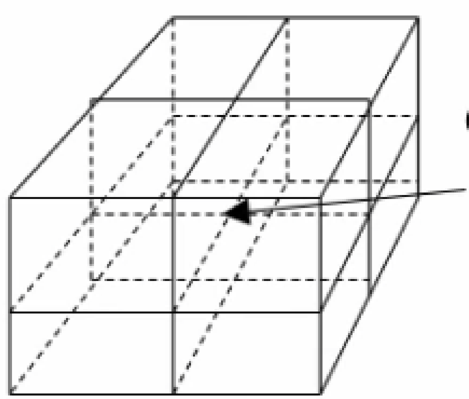
$C(x_i)$: color of sample i
I_a, I_d, I_s: light's ambient, diffuse, and specular colors (usually set as white)
N: normal at sample i
V: vector from sample point to eye
L: light vector, from sample to light source
R: reflection vector of light vector
n: shininess



Normal Estimation

- How to compute the sample normal N?
  - Normal: a vector that is perpendicular to the local surface, which is the gradient of the sample point

1. Compute the gradient G at the cell corners using *central difference*
2. Linearly interpolate the gradients



$$G(x,y,z) = \left(\frac{f(x+1,y,z)-f(x-1,y,z)}{2}, \frac{f(x,y+1,z)-f(x,y-1,z)}{2}, \frac{f(x,y,z+1)-f(x,y,z-1)}{2}\right)$$

Classification

- Classification: mapping from data values to opacities

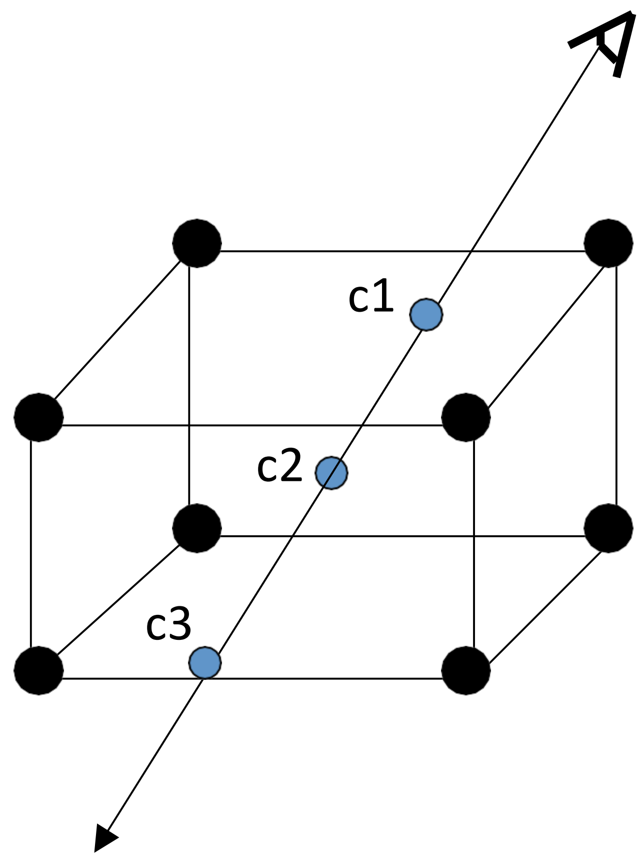
$$\boxed{I(D) = I_0 \times e^{-\int_0^D \tau(t)dt} + \int_0^D g(s) e^{-\int_s^D \tau(t)dt} ds}$$

$$I_0 \prod_{i=1}^{i=n} (1 - \alpha_i) \qquad \sum_{i=1}^{i=n} g_i \times \prod_{j=i+1}^{n} (1 - \alpha_i)$$

  - Region of interest: high opacity
  - Rest: translucent or transparent
- The opacity function, or called transfer function, is given by the user

Ray Sampling

- Sample the volume at discrete points along the ray
- Perform tri-linear interpolation to get the sample values
- Look up the transfer function to get the color and opacity
- Compositing the color/opacity (front-to-back or back-to-front)

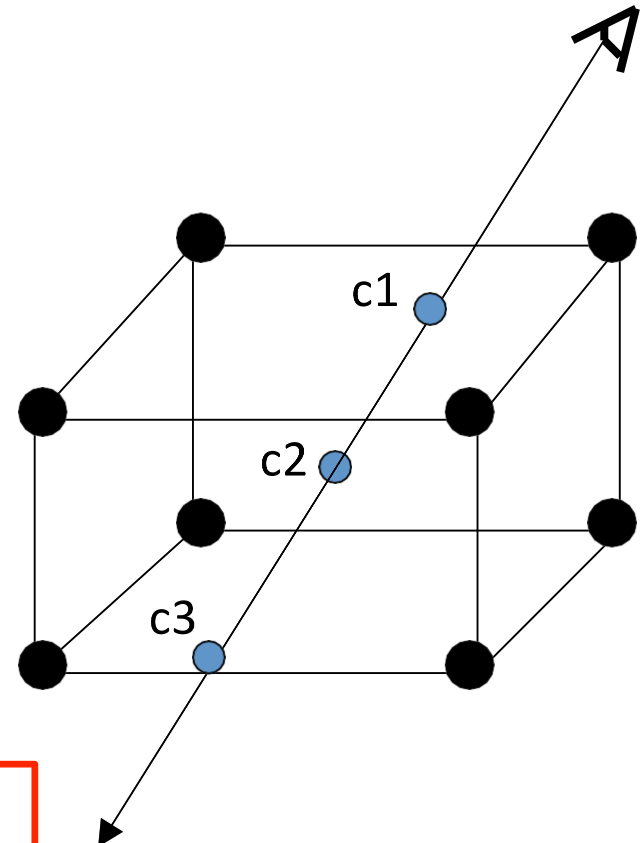


Back-to-Front Compositing

The initial pixel color = Black

Back-to-Front compositing:
use ‘under’ operator

C = C1 ‘under’ background
C = C2 ‘under C
C = C3 ‘under C
…



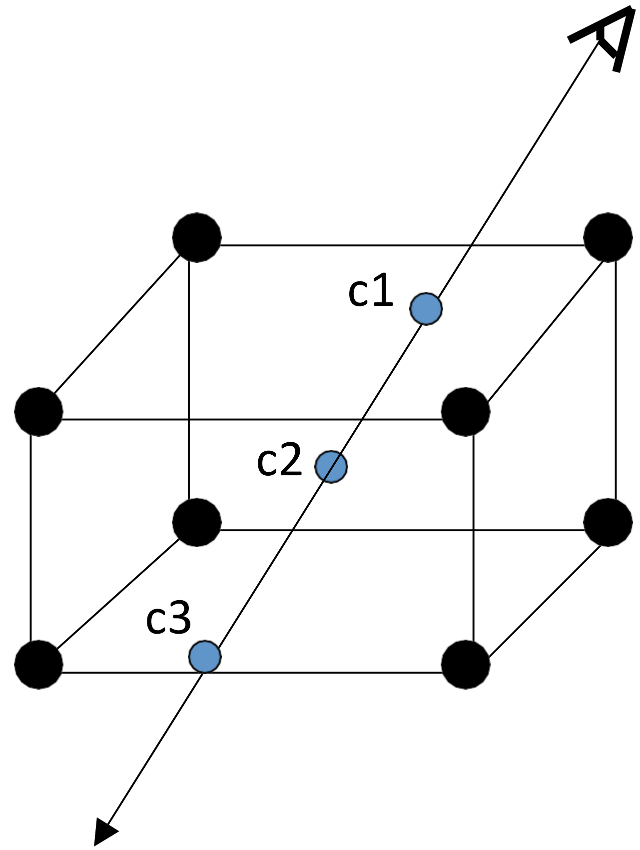
$C_{out} = C_{in} * (1-\alpha(x)) + C(x)*\alpha(x)$
(this is the alpha blending formula)

Front-to-Back Compositing

Front-to-Back compositing:
use 'over' operator

C = background 'over' C1
C = C 'over' C2
C = C 'over' C3
...

$$C_{out} = C_{in} + C(x)\ \alpha(x)\ *(1-\alpha_{in});$$
$$\alpha_{out} = \alpha_{in} + \alpha(x)\ *(1-\alpha_{in})$$



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