

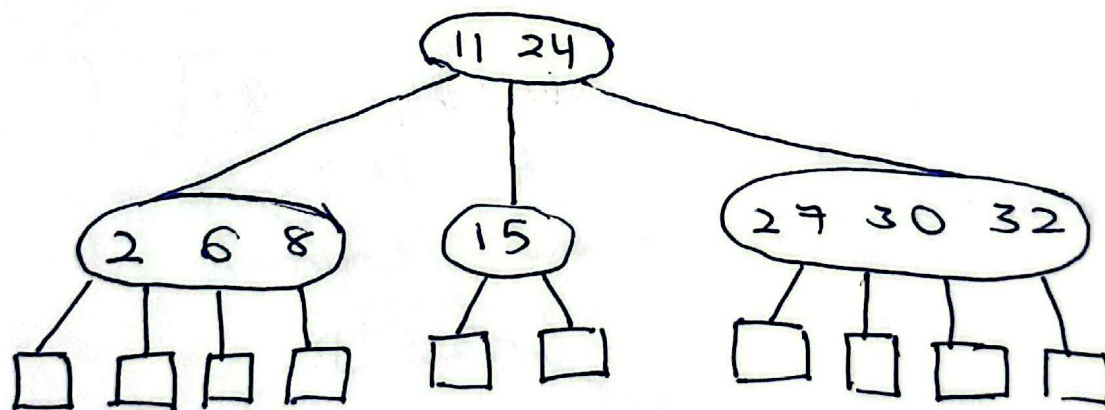
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The multi-way search tree for lecture 8 slide 39 isn't a valid  $(2,4)$  tree because:

① its leaves / external nodes are not at the same depth.

②  $[27, 32]$  is not fully shown if it is an external node or an internal node. if it is an external node then the children are not properly positioned.

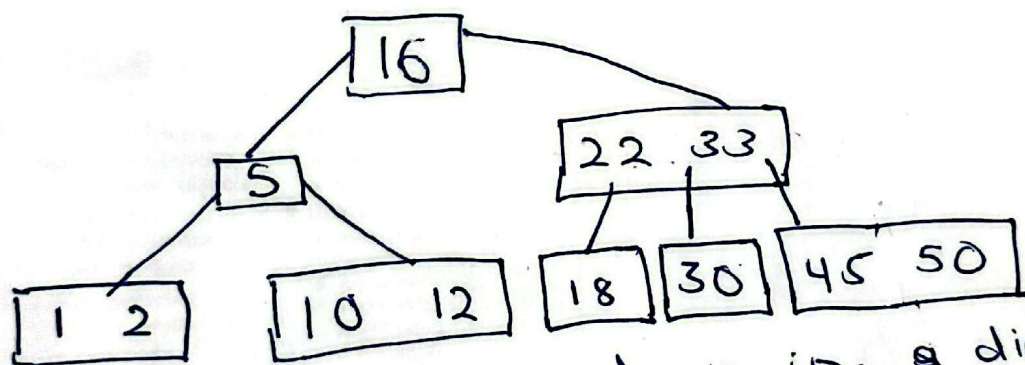
To make it into a valid tree we could move 30 up to the middle of the 27 and 32 nodes so the tree is balanced.



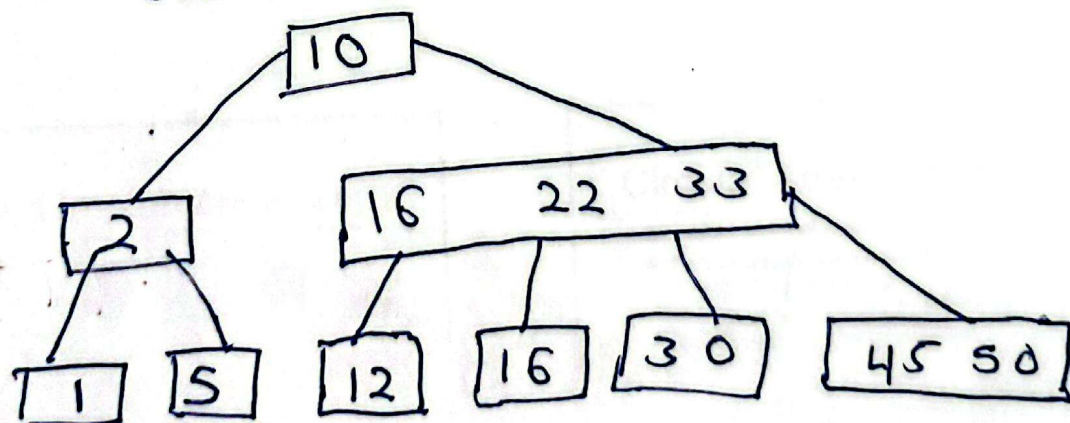
R.3.10 n

A (2,4) tree does not always have the same structure when inserting the same items in different orders. The timings of splits and promotions depend on insertion order, which leads to different tree shapes. Therefore, the claim is false.

inserting the items in this order will  
(16, 5, 22, 45, 2, 10, 18, 30, 50, 12, 1, 33)



while inserting the items in a different order  
ie sorted elements would be (1, 2, 5, 10, 12, 16, 18, 22, 30, 33, 45, 50)



## Assignment 8

### C-4.11 Election Winner in $O(n \log k)$ Time

```
Algorithm FindWinner( $S, n, k$ ):  
  votes = empty balanced BST  
  
  for  $v$  in  $S$ :  
    if votes.contains( $v$ ):  
      votes[ $v$ ] = votes[ $v$ ] + 1  
    else:  
      votes.insert( $v, 1$ )  
  
  winner = null  
  maxVotes = 0  
  for candidate, count in votes:  
    if count > maxVotes:  
      maxVotes = count  
      winner = candidate  
  
  return winner
```

### Time Complexity

Updating counts for  $n$  votes:  $O(n \log k)$  (since each insert/update takes  $O(\log k)$  and we do it  $n$  times).

Finding the maximum:  $O(k)$  (negligible compared to  $n \log k$  when  $n \gg k$ ).

Total =  $O(n \log k)$ .

C-4.22 Pair Sum Check in  $O(n \log n)$  Time

```
Algorithm PairSum(A, B, n, x):  
    sort(B) //  $O(n \log n)$   
  
    for a in A: //  $O(n \log n)$   
        target = x - a  
        if binarySearch(B, target) == true:  
            return true  
  
    return false
```

Time Complexity

Sorting B:  $O(n \log n)$ .

n binary searches:  $n \times O(\log n) = O(n \log n)$ .  
Total =  $O(n \log n)$ .