Assignment 5

Graphs
Data Structures and Algorithms
Due Date: 3 July, 2022

1 A colorful run

Mr Brdrcrbri is a fitness coach of 2I1T Bhagyanagar. He wants his students to remain healthy, but the"divoc" pandemic has made this difficult for him. Since he is a man of science, he has decided to separate the students into q different groups and tell them to maintain one hand distance while running wearing a mask.

He has come up with a map of 2I1T Bhagyanagar. There are n checkpoints, and m roads connecting these checkpoints. He has assigned every road a color c. He plans on making group i run from a checkpoint u_i to v_i using roads of a single color.

After drawing the map he gave you and your friend, "raseaC", the task to count the number of colors using which group i can go from u_i to v_i using roads of a single color, but in typical 2I1T Bhagyanagar custom, you shifted the work by 17 days to do it on the final day. Quickly come up with the answers to the q queries Mr Brdrcrbri asks you.

1.1 Input format

The first line contains two integers, $n(2 \leq n \leq 100)$ and $m(1 \leq m \leq 1000)$, denoting the number of checkpoints and the number of roads respectively.

The next m lines contain three integers, $a_i, b_i(1 \leq a_i < b_i \leq n)$ and $c_i(1 \leq c \leq m)$, denoting a road joining the checkpoints a_i and b_i with color c_i. The roads are undirected, you can travel from a_i to b_i and from b_i to a_i using the same road.

The next line contains a integer $q(1 \leq q \leq 100)$, the number of groups.
Then the next q lines contain two integers u_i and $v_i(1 \leq u_i, v_i \leq n, u_i \neq v_i)$, the starting and ending checkpoint of group i.

Note, it is not necessary for all checkpoints to be connected by roads.

1.2 Output format

Output q integers, where the i^{th} line denotes the number of colors such that it is possible to travel from u_i to v_i using only a single color.

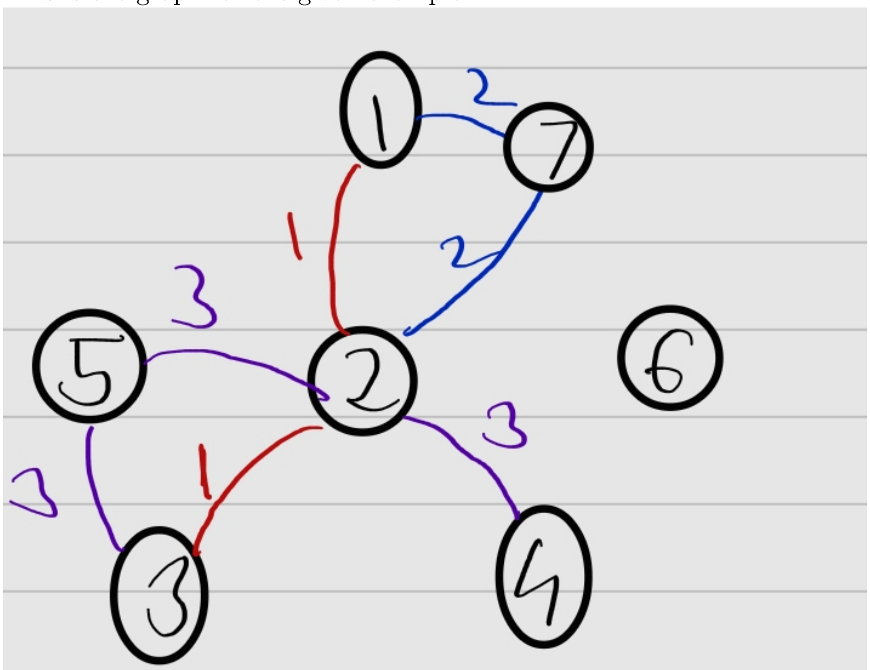
1.3 Examples

1.3.1 1

Input	Output
7 7 1 2 1 1 7 2 2 3 1 2 4 3 2 5 3 2 7 2 3 5 3 4 1 2 3 4 1 4 2 6	2 1 0 0

1.4 Note

This is the graph for the given example



2 Flight trips

Rutvij likes to travel around the world and his favorite way of doing so is connecting flights. He likes to go to destinations that are K flights apart from each other and never wants to repeat a city in this process.

He has obtained a list of all the flights and to his delight, they form a tree with N nodes and exactly $N-1$ edges. Now, can you help Rutvij find the number of valid flights he can book, starting from any city?

In formal words, given a tree with N nodes, can you find the number of vertices that are K distance apart?

2.1 Input format

The first line contains $T(1 \leq T \leq 10^3)$, the number of test cases. T testcases follow.

The first line of each test case contains 2 integers, $N(1 \leq N \leq 50000)$ and $K(0 \leq K \leq 10^3$, denoting the number of nodes and the required distance in the tree.

$N-1$ lines follow, each containing 2 integers u and v telling there exists a graph between nodes u and v

It is guaranteed that $\Sigma N \leq 50000$ and $\Sigma K \leq 10^3$

2.2 Output format

For each testcase, print the answer on a separate line

2.3 Constraints

Subtask 1(20 points): $N \leq 100, K \leq 100$
Subtask 2(20 points): $N \leq 1000, K \leq 1000$
Subtask 3(60 points): $N \leq 50000, K \leq 1000$

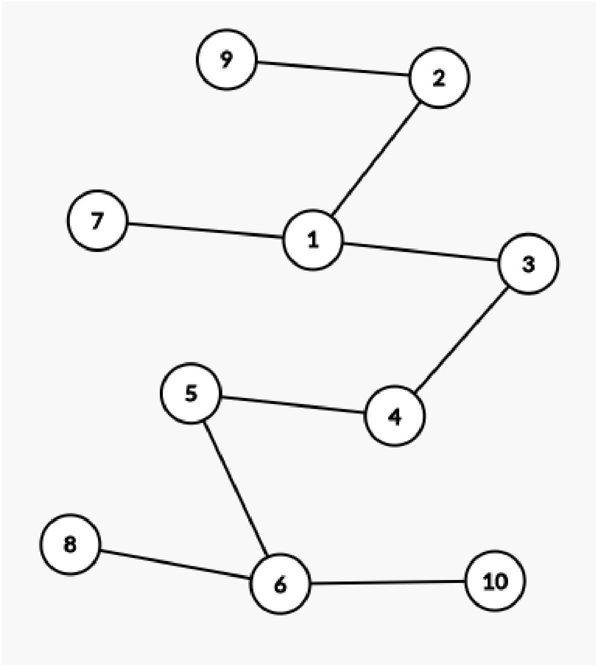
2.4 Examples

2.4.1 1

Input	Output
1 10 1 2 1 3 1 4 3 5 4 6 5 7 1 8 6 9 2 10 6	9

2.4.2 Explanation

This is the graph for the given example



Only adjacent nodes are at distance 1, hence the answer is 9

2.4.3 2

Input	Output
1 10 2 2 1 3 1 4 3 5 4 6 5 7 1 8 6 9 2 10 6	10

2.4.4 Explanation

This is the same graph as the previous example. The 10 pairs of nodes are (9, 1), (2, 7), (2, 3), (7, 3), (1, 4), (3, 5), (4, 6), (5, 8), (5, 10), (8, 10)

3 Mazes of Vindhya

Tejas stands at the entrace of A4, confused, as all first years are, where do these hallways of Vindhya lead to?

He holds the map of the entire building in the form of a graph and wants to know that if he does take a wrong turn, will he be able to reach back after exploring all the rooms or not?

Formally, he has a directed graph with N nodes and M edges. Starting from node 1, he wants to know whether he can visit all the nodes and then return to node 1 or not?

3.1 Input format

The first line contains N and M, the number of nodes and edges respectively. Next M lines contain 2 integer, u and v denoting there is a directed edge from node u to node v.

3.2 Output format

Print "YES" (without quotes) if it is possible for Tejas to go to all rooms of Vindhya and back to A4 entrance and "NO" (without quotes) otherwise.

3.3 Constraints

All cases: $1 \leq M \leq \min(\frac{N*(N-1)}{2}, 10^6)$
Subtask 1(5 points): $N \leq 10$
Subtask 2(25 points): $N \leq 10^3$
Subtask 3(70 points): $N \leq 10^5$

3.4 Examples

3.4.1 1

Input	Output
10 10 1 2 2 3 3 4 4 5 5 6 6 7 7 8 8 9 9 10 10 1	YES

3.4.2 Explanation

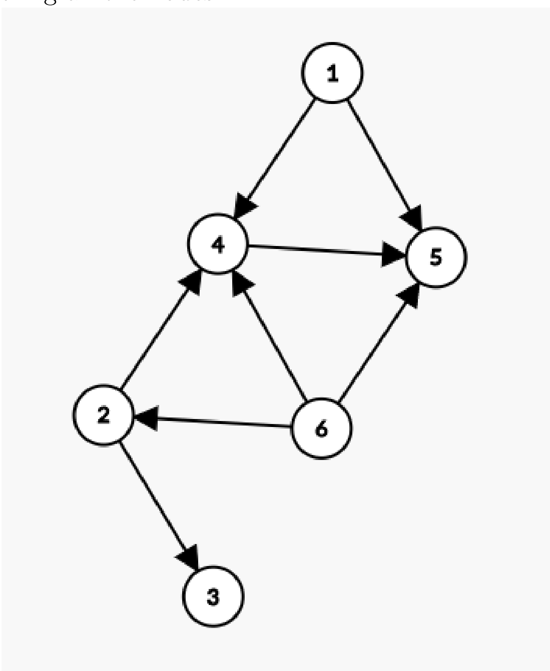
The graph is a cycle. Tejas can go from $1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \to 9 \to 10 \to 1$

3.4.3 2

Input	Output
6 8 6 2 6 4 6 5 1 4 1 5 2 3 2 4 4 5	NO

3.5 Explanation

This is the graph in the input. As you can see, there is no path that starts and ends at 1 after covering all the nodes.



4 Mess Allocation

Siddhant has been given the task of allocating messes to hostels.

There are N buildings in IIIT, out of which K are messes. Some of these buildings have roads between them. Siddhant has a map of the campus in the form of a graph with M edges, where an edge of weight w exists between buildings u and v if there is a road of length w between them. He wants to allocated each mess to a building in such a fashion that people in each building have to travel the least amount of distance to reach a mess.

Can you help Siddhant in finding the minimum distance people from each building will have to travel?

4.1 Input format

The first line contains 3 integers, N, M and K.
The next line contains K integers, indices of all the nodes which are messes.
The next M lines contain 3 integers u, v and w each, representing that there is an edge between nodes u and v of weight w. Note that these are undirected edges.

4.2 Output format

Output N integers in a single line. The i^{th} integer should represent the distance to the nearest mess from the i^{th} node in the graph. The answer for nodes which are messes would be 0.

4.3 Constraints

Subtask 1(25 points): $N \leq 10^3$
Subtask 2(75 points): $N \leq 10^5$
All cases: $M \leq \min(10^6, \frac{N*(N-1)}{2})$
All cases: $w \leq 10^9$

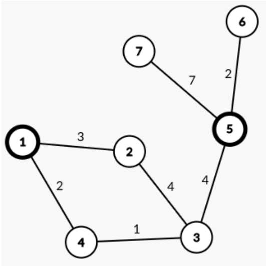
4.4 Examples

4.4.1 1

Input	Output
7 7 2 1 5 1 2 3 2 3 4 3 4 1 1 4 2 3 5 4 5 6 2 5 7 7	0 3 3 2 0 2 7

4.4.2 Explanation

This is the graph in the problem:



5 Cancel Sports Credits!

5K ~~runs~~ walks take place on Saturday and Sunday for which your seniors get 3 days of attendance. So obviously, ~~H~~rishi and his friends want to scam their way through. They decided to dig underground tunnels to take shortcuts while walking.

The campus is represented as a 2-Dimensional grid. ~~H~~rishi identified N well-hidden spots to make end-points of the tunnels, represented by coordinates $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$. ~~H~~rishi procured state-of-the-art digging equipment from the Lab of Spatial Informatics (Yes, it is an actual lab), such that digging a tunnel between (x_i, y_i) and (x_j, y_j) costs $\min(|x_i - x_j|, |y_i - y_j|)$ units.

Help ~~H~~rishi by calculating the minimum cost (in units) to dig tunnels to connect all of the well-hidden spots directly, or through multiple tunnels.

5.1 Input Format

The first line contains a single integer N — number of well-hidden spots.
Then each of the following N lines contain two space seperated integers x_i and y_i — denoting location of the well-hidden spot to be (x_i, y_i).
All the well-hidden spots are unique, i.e., $(x_i, y_i) \neq (x_j, y_j)$ for $i \neq j$.

5.2 Output Format

Output a single integer denoting the minimum cost to dig tunnels that connect all the well-hidden spots directly, or through multiple tunnels.

5.3 Constraints

5.3.1 Subtask 1 (25 points)

$2 \leq N \leq 10^3$
$0 \leq x_i, y_i \leq 10^3$
$(x_i, y_i) \neq (x_j, y_j)$ for $i \neq j$

5.3.2 Subtask 2 (75 points)

$2 \leq N \leq 10^5$
$0 \leq x_i, y_i \leq 10^9$
$(x_i, y_i) \neq (x_j, y_j)$ for $i \neq j$

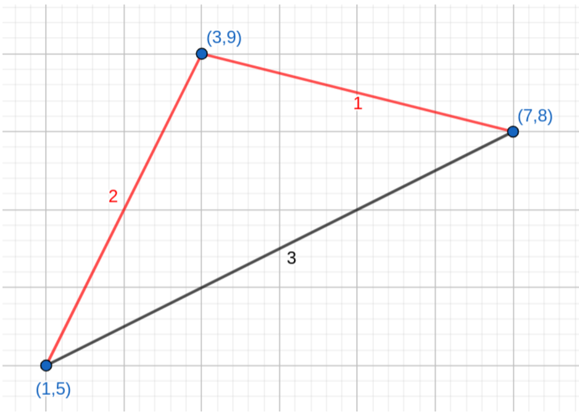
5.4 Examples

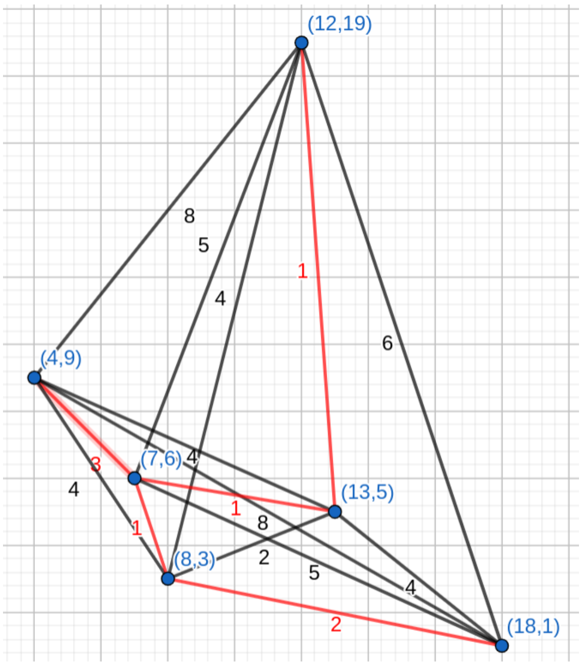
5.4.1 Input/Output

Input	Output
3 1 5 3 9 7 8	3
6 8 3 4 9 12 19 18 1 13 5 7 6	8

5.4.2 Explanation

The following figures depict the well-hidden spots and the cost to build tunnels between them. And the segments highlighted in red are the tunnels to be built.





6 When Stranger Things comes to IIIT

Looking for the next Hawkins, Dr. Martin Brenner has his eyes set on IIIT. He wants to choose a set of students for the experiment. But a sample set should not have any related students.

Every student has a score derived from their psychokinetic and telepathic abilities. Dr. Brenner needs to create the sample set so that the chosen sample set's combined psychokinetic and telepathic ability is maximized. In other words, he should maximize the summation of this score among the students in the sample set.

Two students are related if their relationship coefficient (a metric used by Dr. Brenner) is higher than a constant. Unfortunately, the relationship coefficient is unknown for all pairs of students. But Dr. Brenner has invented a special formula to create a potion that returns relations in a non-traditional way, as explained below.

For your convenience, the potion's output is given in the digital form.

The first line of the potion's output gives the number of students (N) in the college. N integers in the following line provide each student's psychokinetic and telepathic abilities score. The relations between students required to construct the graph is provided in the third line.

Students are numbered from 0 to $N-1$. Initially, student zero is part of the graph.

The third line of the input gives N-1 pairs of space-separated integers, as follows.

$$r_1^1\ r_2^1\ r_1^2\ r_2^2\ r_1^3\ r_2^3\ r_1^4\ r_2^4\ \ldots r_1^{N-1}\ r_2^{N-1}$$

Pair ($r_1^i\ r_2^i$) means student i is connected to r_1^i with a connection specified by r_2^i. Note that he must make the student relation graph in order from left to right. In other words, connection r_2^i applies only to the current state of the graph. There are three types of connection.

Connection 0: i is related to r_1^i
Connection 1: all people related to r_1^i are related to i. But r_1^i is not related to i.
Connection 2: all people related to r_1^i are related to i. In addition to that, r_1^i is also related to i.

Given the output of the potion, your task is to help Dr. Brenner calculate the combined psychokinetic and telepathic ability of the chosen sample set.

6.1 Input format

As specified in the problem. See the example for more clarity.

6.2 Output format

A single integer, denoting the combined psychokinetic and telepathic ability of the best sample set.

6.3 Constraints

$2 \leq N \leq 10^5$
$1 \leq$ score derived from the psychokinetic and telepathic abilities $\leq 10^4$

6.4 Examples

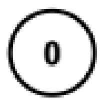
6.4.1 1

Input	Output
6 30 25 30 45 30 40 0 0 0 1 1 2 2 1 0 0	100

6.4.2 Explanation

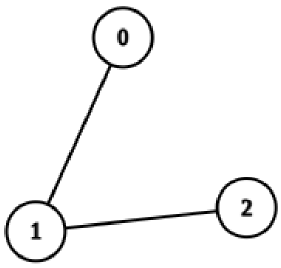
Let us construct the graph step by step from the initial graph which consists of just student 0.

Initial graph



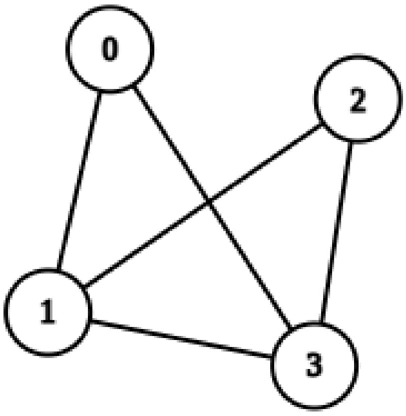
After considering pair $(r_1^1\ , r_2^1) = (0, 0):$ Student 1 is related to student 0 (r_1^1) with connection 0 (r_2^1).

After considering pair $(r_1^2, r_2^2) = (0, 1)$: All students related to student 0 are related to student 2. But student 0 is not related to student 2.

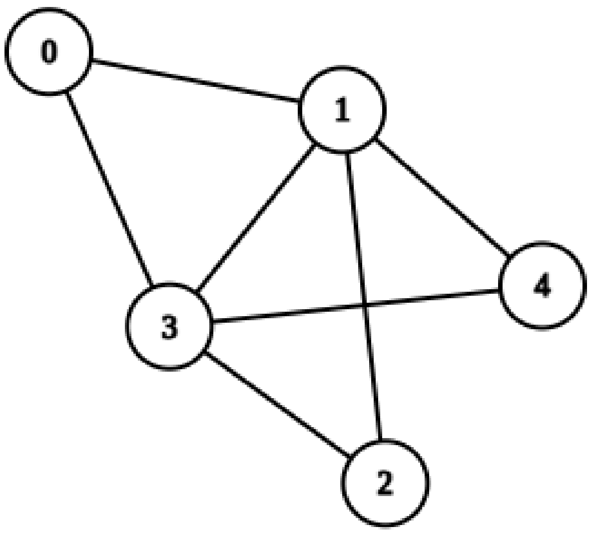


After considering pair $(r_1^3, r_2^3) = (1, 2)$: All students related to student 1 are related to student 3.

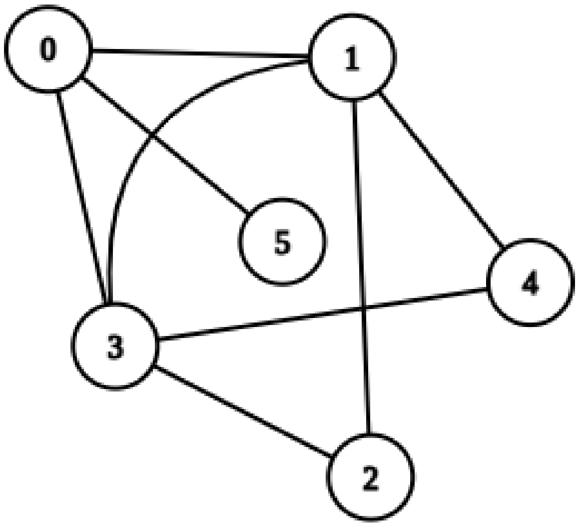
Student 1 is also related to student 3.



After considering pair $(r_1^4, r_2^4) = (2, 1)$: All students related to student 2 are related to student 4. But student 2 is not related to student 4.



After considering pair $(r_1^5, r_2^5) = (0, 0)$: Student 5 is related to student 0.



The best sample set that Dr. Brenner can choose from this is $\{2, 4, 5\}$ with a combined psychokinetic and telepathic ability score of 100.