# Homework Set 1

Due date: January 27, 2016 after class

**Problem 1:** An infinitely long cylinder with a thermal conductivity of $k_i$ = 5 W/m·K and a radius of $r_i$ = 0.40 m generates heat as a function of the radius at a rate of $\dot{q}_i = 5000[1-r/r_i]$ W·m$^{-3}$. The temperature at the center of the cylinder, $r$ = 0, is $T_{center}$ = 500°C. Natural convection is used to remove heat at the outer surface of the cylinder, resulting in a convective heat transfer coefficient of $h$ = 8.5 W/m$^2$·K. The system is schematically depicted in Figure 1. Assuming steady-state, negligible contact resistance, one-dimensional radial conduction in the cylinder, constant properties, and negligible radiative heat exchange with the surroundings, determine

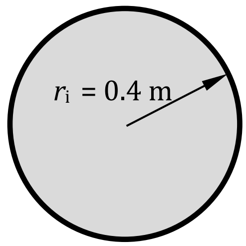


**Figure 1.**

a) The temperature profile as a function of radius.

$$c\rho\cancel{\frac{\partial T}{\partial t}}^{0} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + k\cancel{\frac{\partial^2 T}{\partial z^2}}^{0} + \frac{1}{r^2}\cancel{\frac{\partial^2 T}{\partial \phi^2}}^{0} + \dot{q}$$

$$0 = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \dot{q} \Rightarrow \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{-\dot{q}}{k}$$

$$0 = \frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \dot{q} \Rightarrow \frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-\dot{q}r}{k} = -\frac{5000r}{k} + 5000\frac{r^2}{r_i}$$

$$r\frac{dT}{dr} = -\frac{2500r^2}{k} + \frac{5000}{3k}\frac{r^3}{r_i} + C_1$$

Heat flux at center: $q''_{r=0} = 0 \Rightarrow \left.\frac{dT}{dr}\right|_{r=0} = 0 \Rightarrow 0 = -\frac{2500(0)^2}{k} + \frac{5000}{3k}\frac{(0)^3}{r_i} + C_1 \Rightarrow C_1 = 0$

$$\frac{dT}{dr} = -\frac{2500r}{k} + \frac{5000}{3k}\frac{r^2}{r_i} \Rightarrow T = -\frac{1250r^2}{k} + \frac{5000}{9k}\frac{r^3}{r_i} + C_2$$

$$T(r=0) = -\frac{0}{k} + \frac{5000}{9k}\frac{0}{r_i} + C_2 = 500°C$$

$$T(r) = -\frac{1250r^2}{k} + \frac{5000}{9k}\frac{r^3}{r_i} + 500°C$$

b) The heat flux at $r$ = 0.2 m in W·m$^{-2}$.

$$q''_{r=0.2\text{m}} = -k\left.\frac{dT}{dr}\right|_{r=0.2\text{m}} = 2500r - \frac{5000}{3}\frac{r^2}{r_i} = 333.3\ \text{W}\cdot\text{m}^{-2}$$

c) The temperature at outer surface and outside the boundary layer in °C.

$$T(0.4) = -\frac{1250(0.4\text{m})^2}{k} + \frac{5000}{9k}\frac{(0.4\text{m})^3}{0.4\text{m}} + 500°C = 477.8°\text{C}$$

$$q''_{r=0.4\text{m}} = -k\frac{\partial T}{\partial r}_{r=0.4\text{m}} = -\frac{2500(0.4\text{m})}{k} + \frac{5000}{3k}\frac{(0.4m)^2}{0.4m} = 333.3\ \text{W}\cdot\text{m}^{-2} = h\left[T(r=0.4) - T_\infty\right]$$

$$q''_{r=0.4\text{m}} = h\left[T(r=0.4) - T_\infty\right] \Rightarrow T_\infty = T(r=0.4) - \frac{q''_{r=0.4\text{m}}}{h} = 438.6°\text{C}$$

**Problem 2:** Heat is conducted through a homogeneous wall composed of a material with a temperature-dependent thermal conductivity of $k$ = 0.01921+0.000137$T$ W/m·K[1] (where $T$ is given in °C). The inside convective heat transfer coefficient is $h$ = 8.5 W/m$^2$·K, the outer and inner surface temperatures are 10°C and 40 °C, respectively, and the wall is 100 mm thick. Assume 1-D heat flow and steady state. Determine the heat flux and inner air temperature. List any other relevant assumptions that are required to solve the problem.

Additional Assumptions:

- No heat generation

Heat transfer across the wall:

$$\cancelto{0}{\dot{E}_{\text{st}}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \cancelto{0}{\dot{E}_{\text{gen}}} \Rightarrow \dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{E}_{\text{in}} = A_s q''_{x=0}, \dot{E}_{\text{out}} = A_s q''_{x=L} \Rightarrow q''_{x=0} = q''_{x=L} = q''$$

$$q'' = -k\frac{dT}{dx} \Rightarrow q''dx = -kdT \Rightarrow q''\int_0^L dx = -\int_{T_{\text{s,in}}}^{T_{\text{s,o}}} kdT$$

$$q''L = -\int_{T_{\text{s,in}}}^{T_{\text{s,o}}} \left[0.01921 + 0.000137T\right]dT = \left[0.01921T + \frac{0.000137T^2}{2}\right]_{10°\text{C}}^{40°\text{C}}$$

$$q'' = \frac{1}{L}\left[0.01921T + \frac{0.000137T^2}{2}\right]_{10°\text{C}}^{40°\text{C}} = \frac{1}{0.1\text{ m}}\left[0.01921(40-10) + \frac{0.000137\left(40^2 - 10^2\right)}{2}\right]$$

$$q'' = h\left(T_{\text{in}} - T_{\text{s,in}}\right) \Rightarrow T_{\text{in}} = \frac{q''}{h} + T_{\text{s,in}} = 40.8°\text{C}$$

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[1]H. Manz, P. Loutzenhiser, T. Frank, P.A. Strachan, R. Bundi, G. Maxwell, Series of experiments for empirical validation of solar gain modeling in building energy simulation codes—Experimental setup, test cell characterization, specifications and uncertainty analysis, *Building and Environment*, Volume 41, Issue 12, Pages 1784-1797