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# ME 3322A: Thermodynamics: Fall 2014
## Homework Set # 7
## Due Date: October 21, 2014

| | Problem # in Textbook | | Answer |
|---|---|---|---|
| | **7th Ed.** | **8th Ed.** | |
| 1 | 5.21 | 5.21 | $T_c$=360 K |
| 2 | 5.33 | 5.34 | 0.163 MW |
| 3 | 5.36 | 5.37 | T>750 K |
| 4 | 5.37 | 5.38 | b) 0.5 |
| 5 | 5.43 | 5.43 | |
| 6 | 5.54 | 5.54 | 1.7 |

## PROBLEM 5.21

**KNOWN:** Data are provided for a reversible power cycle operating between hot and cold reservoirs at temperatures $T_H$ and $T_C$.

**FIND:** Determine the energy rejected, in kJ, and $T_C$, in K.

**ANALYSIS:** The thermal efficiency is given. Thus, since $Q_H$ is also given

$$\underset{0.4}{\eta} = \frac{W_{cycle}}{Q_H} = 1 - \frac{Q_C}{Q_H} \Rightarrow \frac{Q_C}{Q_H} = 0.6 \Rightarrow Q_C = 0.6\,(50\,kJ) = 30\,kJ$$

For every reversible cycle operating between reservoirs at $T_H$, $T_C$,

(Eq. 5.7): $$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \Rightarrow T_C = \frac{Q_C}{Q_H} T_H = 0.6\,(600\,K) = 360\,K$$

CHECK: $\eta_{rev} = 1 - \frac{T_C}{T_H} = 1 - \frac{360}{600} = 0.4$

## PROBLEM 5.22

**KNOWN:** A power cycle operates between hot and cold reservoirs at 602 °C and 112 °C, respectively.

**FIND:** Determine the maximum theoretical thermal efficiency for any such cycle.

**ANALYSIS:** The maximum theoretical thermal efficiency is given by Eq. 5.9.

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{(112+273)}{(602+273)} = 1 - \frac{385\,K}{875\,K} = 0.56\ (56\%)$$

## PROBLEM 5.23

**KNOWN:** Data are provided for a reversible power cycle operating as in Fig. 5.5: $T_C = 40\,°F$, $W_{cycle} = 3\,Q_C$.

**FIND:** Determine $\eta$ and $T_H$.

**ANALYSIS:** (a) $W_{cycle} = Q_H - Q_C \Rightarrow Q_H = W_{cycle} + Q_C = 3Q_C + Q_C = 4Q_C$

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{3Q_C}{4Q_C} = 0.75\ (75\%)$$

(b) Since the power cycle operates reversibly,

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} \Rightarrow 0.75 = 1 - \frac{(40+460)}{T_H}$$

Solving $T_H = 2000\,°R\ (1540\,°F)$

## PROBLEM 5.24

**KNOWN:** A reversible power cycle has the same thermal efficiency for each of two sets of conditions: $T_H = T$, $T_C = 500\,K$ and $T_H = 2000\,K$, $T_C = 1000\,K$.

**FIND:** Determine T.

**ANALYSIS:** $\eta = 1 - \frac{500\,K}{T}$ and $\eta = 1 - \frac{1000\,K}{2000\,K} \Rightarrow T = 1000\,K$

# PROBLEM 5.34

**KNOWN:** A power cycle operates between hot and cold reservoirs at 500 K and 310 K, respectively. The power developed is provided: 0.1 MW.

**FIND:** Determine the minimum theoretical rate at which energy is rejected by heat transfer to the cold reservoir, in MW.

**ANALYSIS:** At steady state the energy rate balance is $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$.

$$\Rightarrow \dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C \quad (1)$$

We know that

$$\eta \leq \eta_{MAX} = \left(1 - \frac{T_C}{T_H}\right) \quad \text{where} \quad \eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H}$$

$$\therefore \quad \frac{\dot{W}_{cycle}}{\dot{Q}_H} \leq \left(1 - \frac{T_C}{T_H}\right). \text{ With Eq. (1), } \frac{\dot{W}_{cycle}}{[\dot{W}_{cycle} + \dot{Q}_C]} \leq \left(1 - \frac{T_C}{T_H}\right).$$

Finally,

$$\frac{\dot{W}_{cycle}[T_C/T_H]}{\left[1 - \frac{T_C}{T_H}\right]} \leq \dot{Q}_C \quad (2)$$

With known values, we get

$$\frac{0.1\,\text{MW}[310/500]}{\left[1 - \frac{310}{500}\right]} \leq \dot{Q}_C$$

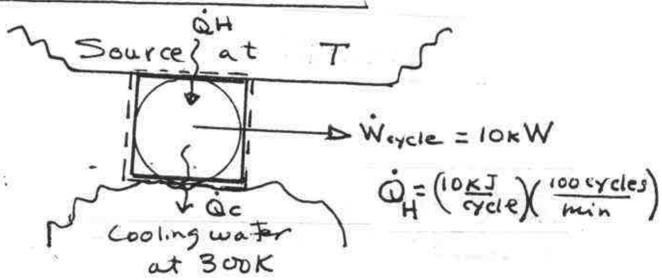
$$0.163\,\text{MW} \leq \dot{Q}_C \quad \Rightarrow \quad (\dot{Q}_C)_{MIN} = 0.163\,\text{MW} \quad \leftarrow$$

PROBLEM 5.36

KNOWN: Steady-state operating data are provided for a power cycle receiving energy by heat transfer from a source at temperature T and rejecting energy by heat transfer to cooling water at 300 K.

FIND: Determine the minimum theoretical value for T, in K.

SCHEMATIC & GIVEN DATA:




ENGR. MODEL

1. The system shown in the sketch undergoes a power cycle.
2. The data provided are for steady-state operation.
3. The source and cooling water play the roles of hot and cold reservoirs, respectively.

ANALYSIS:

The power developed must be less than, or equal to, the power developed by a reversible power cycle operating between thermal reservoirs at the specified temperatures. That is, with Eq. 5.9

$$\eta \le \eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{T}$$

where

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{10 kW}{\left(10 \frac{kJ}{cycle}\right)\left(100 \frac{cycles}{min}\right)\left(\frac{1 min}{60 s}\right)} \left| \frac{1 kJ/s}{1 kW} \right|$$

$$= 0.6$$

$$\therefore \quad 0.6 \le 1 - \frac{300}{T}$$

$$\Rightarrow T \ge 750 K$$

The minimum theoretical value of T is 750 K.

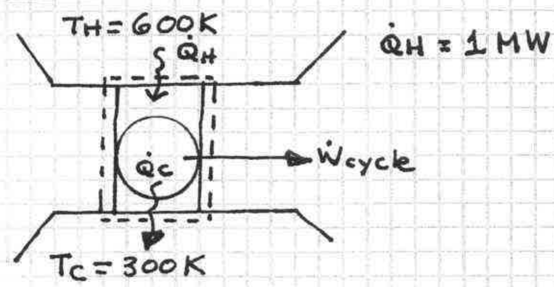
## PROBLEM 5.37

A power cycle operates between hot and cold reservoirs at 600 K and 300 K, respectively. At steady state the cycle develops a power output of 0.45 MW while receiving energy by heat transfer from the hot reservoir at the rate of 1 MW.

(a) Determine the thermal efficiency and the rate at which energy is rejected by heat transfer to the cold reservoir, in MW.

(b) Compare the results of part (a) with those of a reversible power cycle operating between these reservoirs and receiving the same rate of heat transfer from the hot reservoir.

SCHEMATIC & GIVEN DATA:




ANALYSIS: (a) $\dot{Q}_H = 1\,MW$, $\dot{W}_{cycle} = 0.45\,MW$. The actual thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = 0.45 \longleftarrow$$

Energy rate balance: $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$

$$\Rightarrow \dot{Q}_C = \dot{Q}_H - \dot{W}_{cycle} = 0.55\,MW \longleftarrow$$

(b) $$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{300K}{600K} = 0.5 \longleftarrow$$

$$\therefore \quad \dot{W}_{cycle} = \eta_{MAX}\,\dot{Q}_H = 0.5\,(1MW) = 0.5\,MW$$

An energy rate balance gives (as in part (a))

$$\dot{Q}_C = \dot{Q}_H - \dot{W}_{cycle}$$

1 $$= 1MW - 0.5MW = 0.5\,MW \longleftarrow$$

1. From the results of parts (a) and (b) we see that the actual cycle produces less power and discharges more energy to the cold reservoir by heat transfer than the reversible cycle.

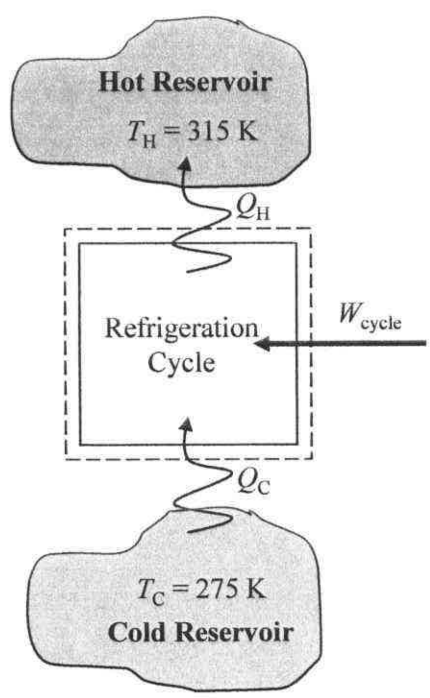
PROBLEM 5.43

5.43 A refrigeration cycle operating between two reservoirs receives energy $Q_C$ from a cold reservoir at $T_C = 275$ K and rejects energy $Q_H$ to a hot reservoir at $T_H = 315$ K. For each of the following cases determine whether the cycle operates *reversibly*, operates *irreversibly*, or is *impossible*:
(a) $Q_C = 1000$ kJ, $W_{cycle} = 80$ kJ.
(b) $Q_C = 1200$ kJ, $Q_H = 2000$ kJ.
(c) $Q_H = 1575$ kJ, $W_{cycle} = 200$ kJ.
(d) $\beta = 6$.

**KNOWN:** A refrigeration cycle operates between two reservoirs with specified temperatures.

**FIND:** Whether each of four cycles operates *reversibly*, operates *irreversibly*, or is *impossible*.

**SCHEMATIC AND GIVEN DATA:**




**ENGINEERING MODEL:**
1. The control volume defined by the dashed line on the accompanying diagram undergoes a refrigeration cycle.

**ANALYSIS:**
The maximum coefficient of performance for a refrigeration cycle operating between two reservoirs is

$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{275\text{ K}}{315\text{ K} - 275\text{ K}} = 6.875$$

PROBLEM 5.43 (Continued)

Coefficient of performance for any refrigeration cycle is

$$\beta = \frac{Q_C}{W_{cycle}}$$

(a) Given $Q_C = 1000$ kJ, $W_{cycle} = 80$ kJ, the coefficient of performance determined using these energy data is

$$\beta = \frac{1000 \text{ kJ}}{80 \text{ kJ}} = 12.5$$

**Since $\beta = 12.5 > \beta_{max} = 6.875$, the cycle is impossible.**

(b) Given $Q_C = 1200$ kJ, $Q_H = 2000$ kJ, cycle work can be determined from

$$W_{cycle} = Q_H - Q_C = 2000 \text{ kJ} - 1200 \text{ kJ} = 800 \text{ kJ}$$

The coefficient of performance determined using these energy data is

$$\beta = \frac{1200 \text{ kJ}}{800 \text{ kJ}} = 1.5$$

**Since $\beta = 1.5 < \beta_{max} = 6.875$, the cycle operates irreversibly.**

(c) Given $Q_H = 1575$ kJ, $W_{cycle} = 200$ kJ, heat transfer associated with the cold reservoir can be determined from

$$W_{cycle} = Q_H - Q_C \rightarrow Q_C = Q_H - W_{cycle} = 1575 \text{ kJ} - 200 \text{ kJ} = 1375 \text{ kJ}$$

The coefficient of performance determined using these energy data is

$$\beta = \frac{1375 \text{ kJ}}{200 \text{ kJ}} = 6.875$$

**Since $\beta = \beta_{max} = 6.875$, the cycle operates reversibly.**

(d) Given $\beta = 6$, the cycle is irreversible.

**Since $\beta = 6 < \beta_{max} = 6.875$, the cycle operates irreversibly.**

# PROBLEM 5.54

Data are provided for two reversible refrigeration cycles. One cycle operates between hot and cold reservoirs at 27°C and −8°C, respectively. The other cycle operates between the same hot reservoir at 27°C and a cold reservoir at −28°C. If each refrigerator removes the same amount of energy by heat transfer from its cold reservoir, determine the ratio of the net work input values of the two cycles.

## ANALYSIS:

Refrigeration cycle #1: $\dfrac{Q}{W_{cycle,1}} = \dfrac{T_C}{T_H - T_C} = \dfrac{265\,K}{35\,K} = 7.57$

Refrigeration cycle #2: $\dfrac{Q}{W_{cycle,2}} = \dfrac{T_C'}{T_H - T_C} = \dfrac{245\,K}{55\,K} = 4.45$

$$\Rightarrow \frac{Q/W_{cycle,1}}{Q/W_{cycle,2}} = \frac{7.57}{4.45} \Rightarrow \frac{W_{cycle,2}}{W_{cycle,1}} = 1.7$$