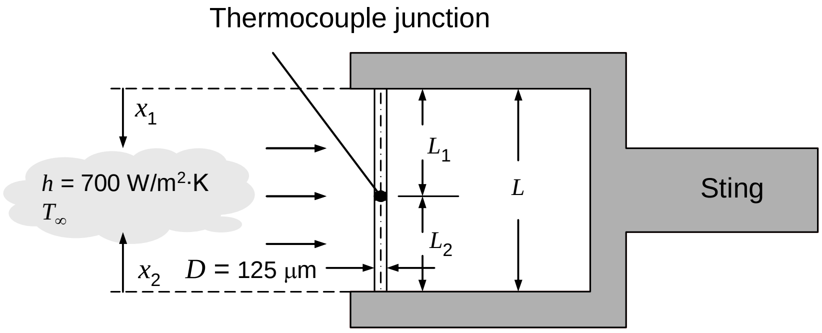
PROBLEM 3.114

KNOWN: Wire diameters associated with a thermocouple junction, value of the convection heat transfer coefficient.

FIND: Minimum wire lengths necessary to ensure the junction temperature is at the gas temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Negligible radiation heat transfer, (3) Constant properties, (4) Infinitely long fin behavior.

PROPERTIES: *Table A-1*, Copper ($\bar{T}$ = 300 K): k = 401 W/m·K; Constantan ($\bar{T}$ = 300 K): k = 23 W/m·K; *Given*, Chromel: k = 19 W/m·K; Alumel: k = 29 W/m·K.

ANALYSIS: To ensure the junction temperature is at the gas temperature (that is, the junction temperature is not influenced by the sting temperature) we require the two wires to behave as infinitely long fins. From Example 3.9, Comment 1, the requirement is,

$$L_1 \geq 4.6\left(\frac{k_1 A_c}{hP}\right)^{1/2}; L_2 \geq 4.6\left(\frac{k_2 A_c}{hP}\right)^{1/2}$$

where $L = L_1 + L_2$. With $A_c = \pi D^2/4 = \pi \times (125 \times 10^{-6}\text{m})^2/4 = 12.27 \times 10^{-9}\text{m}^2$ and $P = \pi D = \pi \times 125 \times 10^{-6}$ m $= 393 \times 10^{-6}$ m, we may calculate the following values of L_1, L_2, and L.

Material	L_1 (mm)	L_2 (mm)		
(1) Copper	19.5	-		
(2) Constantan	-	4.70		
$L = L_1 + L_2$			24.2 mm	<
(1) Chromel	4.24	-		
(2) Alumel	-	5.23		
$L = L_1 + L_2$			9.47 mm	<

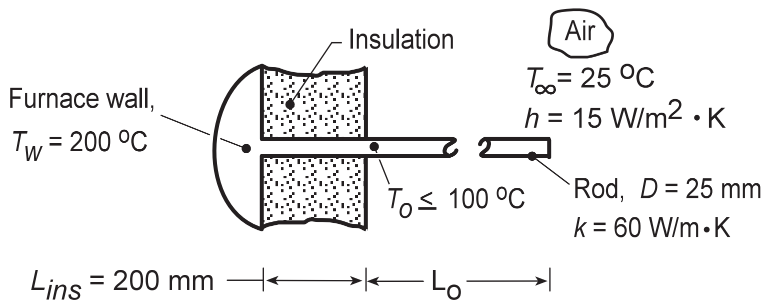
COMMENTS: Use of the chromel-alumel thermocouple junction leads to a substantial reduction in the size of the measurement device, while simultaneously minimizing measurement error associated with conduction along the wires to or from the sting.

PROBLEM 3.122

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \le 100°C$? If not, what design parameters would you change?

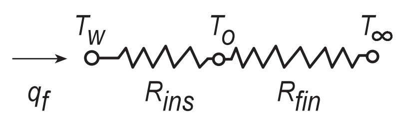
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.81,

$$R_{fin} = \theta_b/q_f = \frac{1}{\left(hPkA_c\right)^{1/2}\tanh\left(mL_o\right)} \qquad (2)$$

$$m = \left(hP/kA_c\right)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}\left(T_w - T_\infty\right) \qquad (6)$$ <

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25°C + \frac{6.298}{6.790 + 6.298}\left(200 - 25\right)°C = 109°C$$ <

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909\times10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi\left(0.025\,m\right)^2/4 = 4.909\times10^{-4}\,m^2$$

$$R_{fin} = 1\Big/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh\left(6.324\times0.200\right) = 6.298\,K/W$$

$$\left(hPkA_c\right) = \left(15\,W/m^2\cdot K\times\pi\left(0.025\,m\right)\times60\,W/m\cdot K\times4.909\times10^{-4}\,m^2\right) = 0.0347\,W^2/K^2$$

Continued...

PROBLEM 3.122 (Cont.)

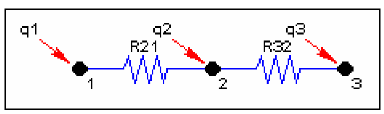
$$m = \left(hP/kA_c\right)^{1/2} = \left(15\,W/m^2 \cdot K \times \pi\left(0.025\,m\right)/60\,W/m \cdot K \times 4.909 \times 10^{-4}\,m^2\right)^{1/2} = 6.324\,m^{-1}$$

Consider the following design changes aimed at reducing $T_o \leq 100°C$. (1) Increasing length of the fin portions: with L_o = 400 and 600 mm, T_o is 102.8°C and 102.3°C, respectively. Hence, increasing L_o will reduce T_o only modestly. (2) Decreasing the thermal conductivity: backsolving the above equation set with T_0 = 100°C, find the required thermal conductivity is k = 14 W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for T_o = 100°C, the required insulation thickness would be L_{ins} = 211 mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by "tack welding" (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit. (6) Using a tube rather than a rod will decrease A_c. For a 3 mm tube wall and 25 mm outside diameter, $A_c = 2.07 \times 10^{-4}\ m^2$, R_{ins} = 16.103 K/W and R_{fin} = 8.61 K/W, yielding T_o = 86°C. (conduction within the air inside the tube is neglected).

COMMENTS: (1) Would replacing the rod by a thick-walled tube provide a practical solution?

(2) The *IHT Thermal Resistance Network Model* and the *Thermal Resistance Tool* for a *fin* with an *adiabatic tip* were used to create a model of the rod. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:



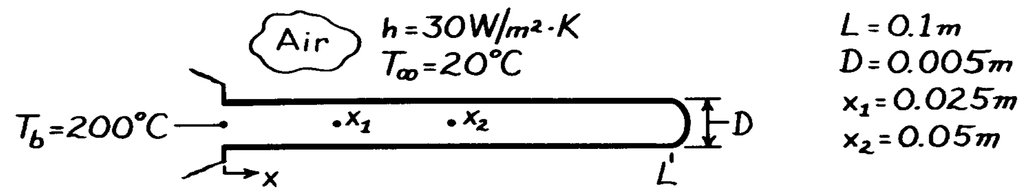

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal
points at which there is no external source of heat. */
T1 = Tw             // Furnace wall temperature, C
//q1 =              // Heat rate, W
T2 = To             // To, beginning of rod exposed length
q2 =  0             // Heat rate, W; node 2; no external heat source
T3 =  Tinf          // Ambient air temperature, C
//q3 =              // Heat rate, W

// Thermal Resistances:
// Rod - conduction resistance
R21 =  Lins / (k * Ac)          // Conduction resistance, K/W
Ac = pi * D^2 / 4               // Cross sectional area of rod, m^2
// Thermal Resistance Tools - Fin with Adiabatic Tip:
R32 =  Rfin                     //  Resistance of fin, K/W
/* Thermal resistance of a fin of uniform cross sectional area Ac, perimeter P, length L, and thermal
conductivity k with an adiabatic tip condition experiencing convection with a fluid at Tinf and coefficient h, */
Rfin = 1/ ( tanh (m*Lo) * (h * P * k * Ac ) ^ (1/2) )          // Case B, Table 3.4
m = sqrt(h*P / (k*Ac))
P = pi * D                      // Perimeter, m
// Other Assigned Variables:
Tw = 200            // Furnace wall temperature, C
k = 60              // Rod thermal conductivity, W/m.K
Lins = 0.200        // Insulated length, m
D = 0.025           // Rod diameter, m
h = 15              // Convection coefficient, W/m^2.K
Tinf = 25           // Ambient air temperature,C
Lo = 0.200          // Exposed length, m
```

PROBLEM 3.130

KNOWN: Length, diameter, base temperature and environmental conditions associated with a brass rod.

FIND: Temperature at specified distances along the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

PROPERTIES: *Table A-1*, Brass $\left(\overline{T} = 110^\circ C\right)$: $k = 133\ W/m \cdot K$.

ANALYSIS: Evaluate first the fin parameter

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{h\pi\ D}{k\pi\ D^2/4}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4 \times 30\ W/m^2 \cdot K}{133\ W/m \cdot K \times 0.005m}\right]^{1/2}$$

$$m = 13.43\ m^{-1}.$$

Hence, m L = (13.43)×0.1 = 1.34 and from the results of Example 3.9, it is advisable not to make the infinite rod approximation. Thus from Table 3.4, the temperature distribution has the form

$$\theta = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL}\theta_b$$

Evaluating the hyperbolic functions, cosh mL = 2.04 and sinh mL = 1.78, and the parameter

$$\frac{h}{mk} = \frac{30\ W/m^2 \cdot K}{13.43m^{-1}\ (133\ W/m \cdot K)} = 0.0168,$$

with θ_b = 180°C the temperature distribution has the form

$$\theta = \frac{\cosh m(L - x) + 0.0168\ \sinh m(L - x)}{2.07}\left(180^\circ C\right).$$

The temperatures at the prescribed locations are tabulated below.

x(m)	cosh m(L-x)	sinh m(L-x)	θ	T(°C)	
x_1 = 0.025	1.55	1.19	136.5	156.5	<
x_2 = 0.05	1.24	0.725	108.9	128.9	<
L = 0.10	1.00	0.00	87.0	107.0	<

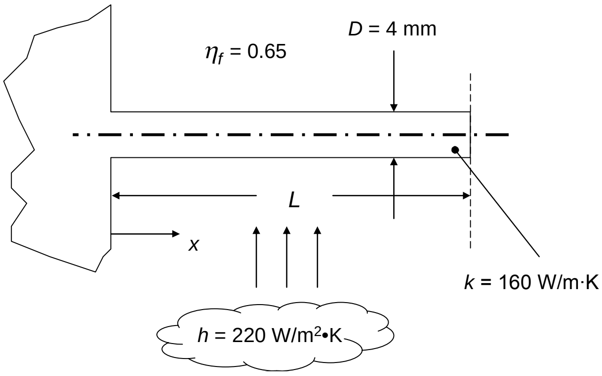
COMMENTS: If the rod were approximated as infinitely long: $T(x_1)$ = 148.7°C, $T(x_2)$ = 112.0°C, and T(L) = 67.0°C. The assumption would therefore result in significant underestimates of the rod temperature.

PROBLEM 3.132

KNOWN: Thermal conductivity and diameter of a pin fin. Value of the heat transfer coefficient and fin efficiency.

FIND: (a) Length of fin, (b) Fin effectiveness.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Negligible radiation heat transfer, (3) Constant properties, (4) Convection from fin tip.

PROPERTIES: *Given*, Aluminum Alloy: k = 160 W/m·K.

ANALYSIS: For an active fin tip, the efficiency may be expressed in terms of the corrected fin length as:

$$\eta_f = \frac{\tanh(mL_c)}{mL_c}$$

where $m = \sqrt{hP/kA_c} = \sqrt{4h/kD} = \sqrt{4 \times 220\text{W/m}^2 \cdot \text{K} / \left(160\text{W/m} \cdot \text{K} \times 4 \times 10^{-3}\text{m}\right)} = 37.1\text{m}^{-1}$

Hence, $\eta_f = 0.65 = \dfrac{\tanh(37.1\text{m}^{-1} \times L_c)}{37.1\text{m}^{-1} \times L_c}$ which may be solved by trial-and-error (or by using *IHT*) to yield L_c = 0.0362 m = 36.2 mm. The fin length is therefore, $L = L_c - D/4$ = 0.0362 m – 0.004m/4 = 0.0352 m = 35.2 mm. <

The fin effectiveness is:

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{M \tanh(mL_c)}{hA_{c,b}\theta_b} = \frac{\sqrt{hPkA_{c,b}}\tanh(mL_c)}{hA_{c,b}} = \frac{2}{\sqrt{hD/k}}\tanh(mL_c)$$

$$= \frac{2}{\sqrt{\dfrac{220\text{W/m}^2 \cdot \text{K} \times 4 \times 10^{-3}\text{m}}{160\text{W/m} \cdot \text{K}}}}\tanh(37.1\text{m}^{-1} \times 36.2 \times 10^{-3}\text{m}) = 23.5$$ <

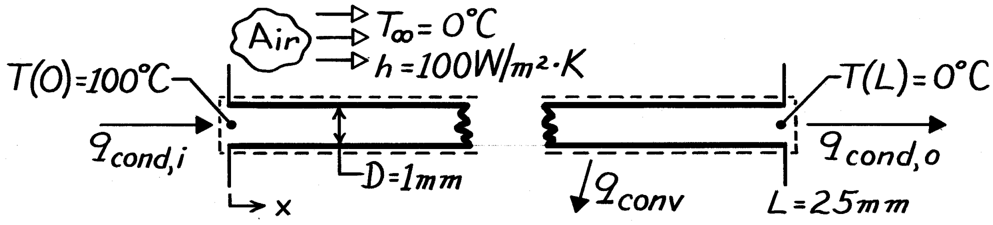
COMMENTS: The values of the fin effectiveness and fin efficiency are independent of the base or fluid temperatures.

PROBLEM 3.137

KNOWN: Dimensions and end temperatures of pin fins.

FIND: (a) Heat transfer by convection from a single fin and (b) Total heat transfer from a 1 m^2 surface with fins mounted on 4mm centers.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rod, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation.

PROPERTIES: *Table A-1*, Copper, pure (323K): $k \approx 400$ W/m·K.

ANALYSIS: (a) By applying conservation of energy to the fin, it follows that

$$q_{conv} = q_{cond,i} - q_{cond,o}$$

where the conduction rates may be evaluated from knowledge of the temperature distribution. The general solution for the temperature distribution is

$$\theta(x) = C_1\, e^{mx} + C_2\, e^{-mx} \qquad \theta \equiv T - T_\infty.$$

The boundary conditions are $\theta(0) \equiv \theta_o = 100°C$ and $\theta(L) = 0$. Hence

$$\theta_o = C_1 + C_2$$

$$0 = C_1\, e^{mL} + C_2\, e^{-mL}$$

Therefore, $\qquad C_2 = C_1\, e^{2mL}$

$$C_1 = \frac{\theta_o}{1-e^{2mL}}, \qquad C_2 = -\frac{\theta_o\, e^{2mL}}{1-e^{2mL}}$$

and the temperature distribution has the form

$$\theta = \frac{\theta_o}{1-e^{2mL}}\left[e^{mx} - e^{2mL-mx}\right].$$

The conduction heat rate can be evaluated by Fourier's law,

$$q_{cond} = -kA_c \frac{d\theta}{dx} = -\frac{kA_c\theta_o}{1-e^{2mL}}\, m\left[e^{mx} + e^{2mL-mx}\right]$$

or, with $m = \left(hP/kA_c\right)^{1/2}$,

$$q_{cond} = -\frac{\theta_o\left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left[e^{mx} + e^{2mL-mx}\right].$$

Continued …

Hence at x = 0,

$$q_{cond,i} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(1+e^{2mL}\right)$$

at x = L

$$q_{cond,o} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(2e^{mL}\right)$$

Evaluating the fin parameters:

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4\times 100\ \text{W/m}^2\cdot\text{K}}{400\ \text{W/m}\cdot\text{K}\times 0.001\text{m}}\right]^{1/2} = 31.62\ \text{m}^{-1}$$

$$\left(hPkA_c\right)^{1/2} = \left[\frac{\pi^2}{4}D^3hk\right]^{1/2} = \left[\frac{\pi^2}{4}\times\left(0.001\text{m}\right)^3\times 100\frac{\text{W}}{\text{m}^2\cdot\text{K}}\times 400\frac{\text{W}}{\text{m}\cdot\text{K}}\right]^{1/2} = 9.93\times 10^{-3}\ \frac{\text{W}}{\text{K}}$$

$$mL = 31.62\ \text{m}^{-1}\times 0.025\text{m} = 0.791, \qquad e^{mL} = 2.204, \qquad e^{2mL} = 4.865$$

The conduction heat rates are

$$q_{cond,i} = \frac{-100\text{K}\left(9.93\times 10^{-3}\ \text{W/K}\right)}{-3.865}\times 5.865 = 1.507\ \text{W}$$

$$q_{cond,o} = \frac{-100\text{K}\left(9.93\times 10^{-3}\ \text{W/K}\right)}{-3.865}\times 4.408 = 1.133\ \text{W}$$

and from the conservation relation,

$$q_{conv} = 1.507\ \text{W} - 1.133\ \text{W} = 0.374\ \text{W}.$$ <

(b) The total heat transfer rate is the heat transfer from N = 250×250 = 62,500 rods and the heat transfer from the remaining (bare) surface ($A = 1\text{m}^2 - NA_c$). Hence,

$$q = N\, q_{cond,i} + hA\theta_o = 62,500\ \left(1.507\ \text{W}\right) + 100\text{W/m}^2\cdot\text{K}\left(0.951\ \text{m}^2\right)\ 100\text{K}$$

$$q = 9.42\times 10^4\ \text{W} + 0.95\times 10^4\ \text{W} = 1.037\times 10^5\ \text{W}.$$

COMMENTS: (1) The fins, which cover only 5% of the surface area, provide for more than 90% of the heat transfer from the surface.

(2) The fin effectiveness, $\varepsilon \equiv q_{cond,i}/hA_c\theta_o$, is ε = 192, and the fin efficiency, $\eta \equiv \left(q_{conv}/h\pi\ DL\theta_o\right)$, is $\eta = 0.48$.

(3) The temperature distribution, $\theta(x)/\theta_o$, and the conduction term, $q_{cond,i}$, could have been obtained directly from Eqs. 3.82 and 3.83, respectively.

(4) Heat transfer by convection from a single fin could also have been obtained from Eq. 3.78.