Homework Set 4

Due date: March 9, 2016 at the beginning of class

Problem 1: Calculate the wavelength (λ) in μm for maximum blackbody hemispherical spectral emissive power ($E_{\lambda,b}$) at T = 750, 1500, 3000, and 5777 K.

$$(\lambda T)_{\max} = C_3 = 2898 \ \ \mu\text{m}\cdot\text{K}$$
$$\lambda = \frac{2898 \ \ \mu\text{m}\cdot\text{K}}{T}$$

Temperature, K	Wavelength, μm
750	3.8640
1500	1.9320
3000	0.9660
5777	0.5016

Problem 2: Calculate the temperature of a blackbody if its maximum hemispherical spectral emissive power ($E_{\lambda,b}$) corresponds to λ = 0.1, 1, 5, and 10 μm.

$$(\lambda T)_{\max} = C_3 = 2898 \ \ \mu\text{m}\cdot\text{K}$$
$$T = \frac{2898 \ \ \mu\text{m}\cdot\text{K}}{\lambda}$$

Wavelength, μm	Temperature, K
0.1	28980
1	2898
5	579.6
10	289.8

Problem 3: Calculate the blackbody hemispherical spectral intensity ($I_{\lambda,b}$) for a blackbody at T = 5780 K at 2 μm.

$$I_{b,\lambda} = \frac{C_1}{\pi\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]} = 1.5056\times10^6 \ \text{W}\cdot\text{m}^{-2}\cdot\text{sr}^{-1}$$

Problem 4: Calculate the blackbody hemispherical spectral emissive power ($E_{\lambda,b}$) and the blackbody spectral intensity ($I_{\lambda,\text{b}}$) for T = 1000 K and λ= 20 μm.

$$E_{b,\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]} = 111.01 \ \text{W}\cdot\text{m}^{-2}$$

Problem 5: Calculate the wavelength of maximum emission from a blackbody at room temperature (assume 22 °C).

$$\lambda = \frac{2898\ \ \mu\text{m} \cdot \text{K}}{T} = 9.8187\mu\text{m}$$

Problem 6: Calculate the temperature of a blackbody that radiates its maximum hemispherical spectral emissive power at the center of the visible spectrum (0.4-0.7 μm).

$$T = \frac{2898\ \ \mu\text{m} \cdot \text{K}}{\lambda} = 5269\ \text{K}$$

Problem 7: Calculate the blackbody hemispherical total intensity ($I_{\lambda,b}$) at T = 1000 and 5780 K.

$$I_b = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi} \Rightarrow I_b\left(T = 1000\ \text{K}\right) = 18048\ \text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1}$$
$$I_b = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi} \Rightarrow I_b\left(T = 5780\ \text{K}\right) = 2.0144 \times 10^7\ \ \text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1}$$

Problem 8: Calculate the surface temperature of a blackbody radiating with a hemispherical total emissive power, E_b = 10 kW/m^2.

$$E_b = \sigma T^4 \Rightarrow T = \sqrt[4]{\frac{E_b}{\sigma}} = 364.4\ \text{K}$$

Problem 9: Calculate the hemispherical total emissive power outside the atmosphere of Mercury. Assume the radius of the sun is 6.9599×10^8 m; the distance between the sun and Mercury is 6.98 × 10^{10} m; and the surface temperature of the sun is 5780 K.

For a control volume at the surface of the sun and orbit of mercury $\dot{E}_\text{in} = \dot{E}_\text{out}$

$$\dot{E}_\text{in} = A_\text{sun}\sigma T_\text{sun}^4 = 4\pi r_\text{sun}^2 \sigma T_\text{sun}^4 = q_\text{in} = q_\text{out} = q''_\text{mercury} 4\pi\left(r_\text{sun} + d_\text{mercury}\right)^2$$
$$q''_\text{mercury} = \frac{r_\text{sun}^2 \sigma T_\text{sun}^4}{\left(r_\text{sun} + d_\text{mercury}\right)^2} = 6168.4\ \text{W} \cdot \text{m}^{-2}$$

Or in terms of solid angle:

$$I_\text{sun} = \frac{\sigma T_\text{sun}^4}{\pi}$$

Solid angle of the sun compared to Mercury’s orbit:

$$\sin\theta_\text{sun} = \frac{r_\text{sun}}{d_\text{sun} + r_\text{sun}}$$

$$q''_\text{mercury} = \int_0^{2\pi}\int_0^{0.01} I_\text{sun}\cos\theta\sin\theta d\theta d\phi = \pi I_\text{sun}\sin^2\theta\Big|_0^{\theta_{sun}} = \pi I_\text{sun}\frac{r_\text{sun}^2}{\left(r_\text{sun} + d_\text{sun}\right)^2} = \sigma T_\text{sun}^4 \frac{r_\text{sun}^2}{\left(r_\text{sun} + d_\text{sun}\right)^2} = 6168.4\ \text{W} \cdot \text{m}^{-2}$$

Problem 10: The surface of a distant star has an effective blackbody temperature of 8000 K. What percentage of the radiant emission of the sun lies in the visible range (0.4-0.7 μm)? What percentage is in the ultraviolet (0.2-0.4 μm)? What percentage is in the infrared (0.7-1000 μm)?

$$F_{0-0.2\mu\text{m}\cdot 8000\ \text{K}} = 0.019718$$
$$F_{0-0.4\mu\text{m}\cdot 8000\ \text{K}} = 0.3181$$
$$F_{0-0.7\mu\text{m}\cdot 8000\ \text{K}} = 0.7010$$
$$F_{0-1000\mu\text{m}\cdot 8000\ \text{K}} = 1$$
$$F_{0.4-0.7\mu\text{m}} = F_{0-0.7\mu\text{m}\cdot 8000\ \text{K}} - F_{0-0.4\mu\text{m}\cdot 8000\ \text{K}} = 0.3829$$
$$F_{0.2-0.4\mu\text{m}} = F_{0-0.4\mu\text{m}\cdot 8000\ \text{K}} - F_{0-0.2\mu\text{m}\cdot 8000\ \text{K}} = 0.2984$$
$$F_{0.7-1000\mu\text{m}} = F_{0-0.1000\mu\text{m}\cdot 8000\ \text{K}} - F_{0-0.7\mu\text{m}\cdot 8000\ \text{K}} = 0.2990$$