Homework Set 7 Solution

Problem 1. In a particular application involving airflow over a heated surface, the boundary layer temperature distribution may be approximated as:

$$\frac{T-T_s}{T_\infty-T_s}=1-\exp\left[-\mathrm{Pr}\frac{u_\infty y}{\nu}\right]$$

where y is the distance normal to the surface and the Prandtl number, Pr = 0.7, is a dimensionless property. If T_∞ = 400 K, T_s = 300 K, and u_∞/ν = 5000 m^{-1}, determine the surface heat flux.

$$k=0.0263\ \mathrm{W/m\cdot K}$$

$$\frac{\partial T}{\partial y}=-\left(T_\infty-T_s\right)\exp\left[-\mathrm{Pr}\frac{u_\infty y}{\nu}\right]\left(-\mathrm{Pr}\frac{u_\infty}{\nu}\right)$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0}=-100\ \mathrm{K}\left(-0.7\left[5000\ \mathrm{m}^{-1}\right]\right)=350{,}000\ \mathrm{K/m}$$

$$q''=-k\left.\frac{\partial T}{\partial y}\right|_{y=0}=-0.0263\ \mathrm{W/m\cdot K}\cdot 350{,}000\ \mathrm{K/m}=\text{-}9205\ \mathrm{W\cdot m^{-2}}$$

Problem 2. Experimental results for heat transfer over a flat plate with an extremely rough surface were found to be correlated by an expression of the form:

$$Nu_x=0.04\,\mathrm{Re}_x^{0.9}\,\mathrm{Pr}^{1/3}$$

where Nu_x is the local value of the Nusselt number at a position x measured from the leading edge of the plate. Obtain an expression for the ratio of the average heat transfer coefficient $\bar{h}$ to the local coefficient h_x.

$$Nu_x=\frac{hx}{k}=0.04\,\mathrm{Re}_x^{0.9}\,\mathrm{Pr}^{1/3}=0.04\left(\frac{Vx}{\nu}\right)^{0.9}\mathrm{Pr}^{1/3}$$

$$h_x=0.04\left(\frac{V}{\nu}\right)^{0.9}x^{-0.1}\,\mathrm{Pr}^{1/3}\,k$$

$$\bar{h}_x=\frac{1}{x}\int_0^x h_x dx=\frac{0.04\,\mathrm{Pr}^{1/3}\,k\left(\frac{V}{\nu}\right)^{0.9}}{x}\int_0^x x'^{-0.1}\,dx'=\frac{0.04\,\mathrm{Pr}^{1/3}\,k\left(\frac{V}{\nu}\right)^{0.9}}{0.9x^{0.1}}$$

$$\frac{\bar{h}_x}{h_x}=\frac{\dfrac{0.04\,\mathrm{Pr}^{1/3}\,k\left(\frac{V}{\nu}\right)^{0.9}}{0.9x^{0.1}}}{0.04\left(\frac{V}{\nu}\right)^{0.9}x^{-0.1}\,\mathrm{Pr}^{1/3}\,k}=0.9^{-1}=1.11$$