Problem 8.3

8.3

$\omega_1 = 3000\,(2\pi)$ rad/sec $= \frac{\pi c}{\ell} = \frac{\pi c}{2}$

$c = (6000\,\pi \times 2/\pi) = 12\,000$ m/s

$\omega_3 = 3\pi c/\ell = 3\,\omega_1 = 9000$ Hz

$c_{original} = (P/\rho)^{1/2} = 12000$ m/s

$c_{new} = (1.2\,P/\rho)^{1/2} = 1.0954\,(P/\rho)^{1/2} = 1.0954\ c_{original}$

$\therefore\ \omega_1$ and ω_3 are increased by 9.54%

Problem 8.8

8.8

$$w(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{\ell}\left[C_n \cos\frac{nc\pi t}{\ell} + D_n \sin\frac{nc\pi t}{\ell}\right]$$

where

$$C_n = \frac{2}{\ell}\int_0^\ell w_0(x)\sin\frac{n\pi x}{\ell}\,dx\ , \qquad D_n = \frac{2}{\pi c n}\int_0^\ell \dot{w}_0(x)\sin\frac{n\pi x}{\ell}\,dx$$

Since $w_0(x) = w(x,0) = 0$, $\quad C_n = 0$

$$D_n = \frac{2}{\pi c n}\left[\int_0^{\ell/2}\frac{2ax}{\ell}\sin\frac{n\pi x}{\ell}\,dx + \int_{\ell/2}^{\ell} 2a\left(1-\frac{x}{\ell}\right)\sin\frac{n\pi x}{\ell}\,dx\right]$$

$$= \frac{8a\ell}{\pi^3 c n^3}\sin\frac{n\pi}{2} = \begin{cases} 0 & \text{if } n \text{ is even} \\ (-1)^{\frac{n-1}{2}}\dfrac{8a\ell}{\pi^3 n^3 c} & \text{if } n \text{ is odd}\end{cases}$$

$$w(x,t) = \frac{8a\ell}{\pi^3 c}\sum_{n=1,3,5,\ldots}^{\infty}\frac{(-1)^{\frac{n-1}{2}}}{n^3}\sin\frac{n\pi x}{\ell}\sin\frac{n\pi c t}{\ell}$$

Problem 8.16

8.16

$$u(x,t) = \left(\underset{\sim}{A} \cos \frac{\omega x}{c} + \underset{\sim}{B} \sin \frac{\omega x}{c} \right)\left(C \cos \omega t + D \sin \omega t \right)$$

$$\frac{\partial u}{\partial x} = \frac{\omega}{c}\left(- \underset{\sim}{A} \sin \frac{\omega x}{c} + \underset{\sim}{B} \cos \frac{\omega x}{c} \right)\left(C \cos \omega t + D \sin \omega t \right)$$

$$\frac{\partial u}{\partial x}(0,t) = 0 \Rightarrow \underset{\sim}{B} = 0$$

$$\frac{\partial u}{\partial x}(\ell,t) = 0 \Rightarrow -\frac{\omega}{c} \underset{\sim}{A} \sin \frac{\omega \ell}{c} \left(C \cos \omega t + D \sin \omega t \right) =$$

$$\Rightarrow \sin \frac{\omega \ell}{c} = 0 \quad ; \quad \frac{\omega_n \ell}{c} = n\pi$$

$$\omega_n = \frac{n\pi c}{\ell} = \frac{n\pi}{\ell}\sqrt{\frac{E}{\rho}}$$

$$u(x,t) = \sum_{n=1}^{\infty} \cos \frac{n\pi x}{\ell} \left[C_n \cos \frac{n\pi c t}{\ell} + D_n \sin \frac{n\pi c t}{\ell} \right]$$

where $C_n = \frac{2}{\ell} \int_0^{\ell} u_0(x) \cos \frac{n\pi x}{\ell}\, dx$

and $D_n = \frac{2}{n\pi c} \int_0^{\ell} \dot{u}_0(x) \cos \frac{n\pi x}{\ell}\, dx$