4.20

4.20 A velocity field is given by $u = cx^2$ and $v = cy^2$, where c is a constant. Determine the x and y components of the acceleration. At what point (points) in the flow field is the acceleration zero?

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (cx^2)(2cx) = \underline{\underline{2c^2x^3}}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (cy^2)(2cy) = \underline{\underline{2c^2y^3}}$$

Thus, $\vec{a} = a_x\hat{i} + a_y\hat{j} = 0$ at $\underline{\underline{(x, y) = (0, 0)}}$

4.21

4.21 Determine the acceleration field for a three-dimensional flow with velocity components $u = -x$, $v = 4x^2y^2$, and $w = x - y$.

$u = -x$, $v = 4x^2y^2$, and $w = x - y$ so that

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$= 0 + (-x)(-1) + 4x^2y^2(0) + (x-y)(0) = x$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$= 0 + (-x)(8xy^2) + (4x^2y^2)(8x^2y) + (x-y)(0)$$
$$= -8x^2y^2 + 32x^4y^3 = 8x^2y^2(4x^2y - 1)$$

and

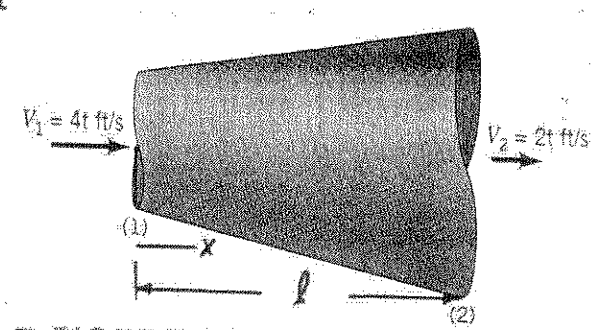
$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$
$$= 0 + (-x)(1) + (4x^2y^2)(-1) + (x-y)(0)$$
$$= -x - 4x^2y^2$$

Thus,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$
$$= \underline{\underline{x\hat{i} + 8x^2y^2(4x^2y-1)\hat{j} - (x + 4x^2y^2)\hat{k}}}$$

4.24

4.24 The velocity of air in the diverging pipe shown in Fig. P4.24 is given by $V_1 = 4t$ ft/s and $V_2 = 2t$ ft/s, where t is in seconds. **(a)** Determine the local acceleration at points (1) and (2). **(b)** Is the average convective acceleration between these two points negative, zero, or positive? Explain.



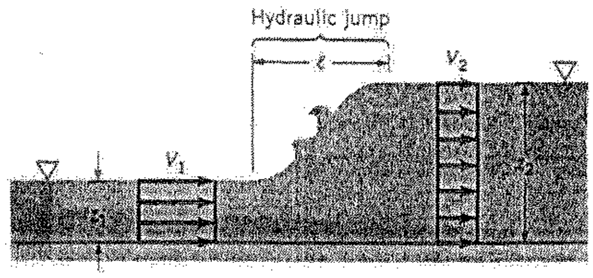
■ FIGURE P4.24

a) $\left.\frac{\partial u}{\partial t}\right|_{(1)} = \underline{\underline{4\,\frac{ft}{s^2}}}$ and $\left.\frac{\partial u}{\partial t}\right|_{(2)} = \underline{\underline{2\,\frac{ft}{s^2}}}$

b) convective acceleration along the pipe $= u\frac{\partial u}{\partial x}$
where $u > 0$. At any time, t, $V_2 < V_1$. Thus, between (1) and (2)
$\frac{\partial u}{\partial x} \approx \frac{V_2 - V_1}{\ell} < 0$
Hence, $u\frac{\partial u}{\partial x} < 0$ or the average convective acceleration is <u>negative</u>.

4.34

4.34 A hydraulic jump is a rather sudden change in depth of a liquid layer as it flows in an open channel as shown in Fig. P4.34 and **Video V10.12**. In a relatively short distance (thickness = ℓ) the liquid depth changes from z_1 to z_2, with a corresponding change in velocity from V_1 to V_2. If V_1 = 1.20 ft/s, V_2 = 0.30 ft/s, and ℓ = 0.02 ft, estimate the average deceleration of the liquid as it flows across the hydraulic jump. How many g's deceleration does this represent?



■ FIGURE P4.34

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} \quad \text{so with } \vec{V} = u(x)\hat{i}\,, \quad \vec{a} = a_x\hat{i} = u\frac{\partial u}{\partial x}\hat{i}$$

Without knowing the actual velocity distribution, $u = u(x)$, the acceleration can be approximated as

$$a_x = u\frac{\partial u}{\partial x} \approx \frac{1}{2}(V_1+V_2)\frac{(V_2-V_1)}{\ell} = \frac{1}{2}(1.20+0.30)\frac{ft}{s}\;\frac{(0.30-1.20)\frac{ft}{s}}{0.02\,ft}$$

$$= \underline{\underline{-33.8\,\frac{ft}{s^2}}}$$

Thus, $$\frac{|a_x|}{g} = \frac{33.8\,\frac{ft}{s^2}}{32.2\,\frac{ft}{s^2}} = \underline{\underline{1.05}}$$

4.64

4.64 In the region just downstream of a sluice gate, the water may develop a reverse flow region as is indicated in Fig. P4.64 and Video V10.9. The velocity profile is assumed to consist of two uniform regions, one with velocity $V_a = 10$ fps and the other with $V_b = 3$ fps. Determine the net flowrate of water across the portion of the control surface at section (2) if the channel is 20 ft wide.

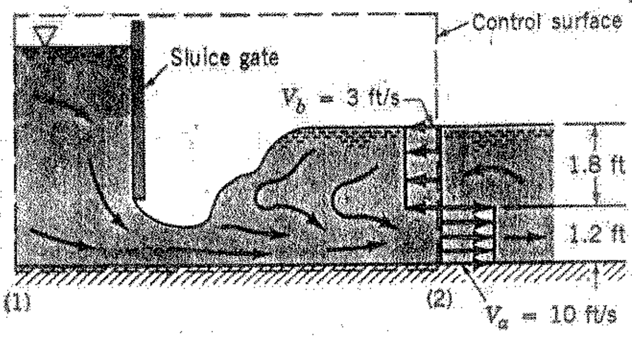


FIGURE P4.64

$$Q = V_a A_a - V_b A_b = (10\tfrac{ft}{s})(1.2ft)(20ft) - (3\tfrac{ft}{s})(1.8ft)(20ft)$$
$$= \underline{\underline{132 \tfrac{ft^3}{s}}}$$

4.65

4.65 At time $t = 0$ the valve on an initially empty (perfect vacuum, $\rho = 0$) tank is opened and air rushes in. If the tank has a volume of $\forall_0$ and the density of air within the tank increases as $\rho = \rho_\infty(1 - e^{-bt})$, where b is a constant, determine the time rate of change of mass within the tank.

For $t \geq 0$, $\rho = \rho_o[1 - e^{-bt}]$ so that M = mass of air in tank
$= \rho \forall_o = \rho_o \forall_o [1 - e^{-bt}]$

Thus, $\frac{dM}{dt} = \underline{\underline{\rho_o \forall_o b\, e^{-bt}}}$

4.68

4.68 A layer of oil flows down a vertical plate as shown in Fig. P4.68 with a velocity of $\mathbf{V} = (V_0/h^2)(2hx - x^2)\hat{\mathbf{j}}$ where V_0 and h are constants. (a) Show that the fluid sticks to the plate and that the shear stress at the edge of the layer ($x = h$) is zero. (b) Determine the flowrate across surface AB. Assume the width of the plate is b. (Note: The velocity profile for laminar flow in a pipe has a similar shape. See Video V6.13)

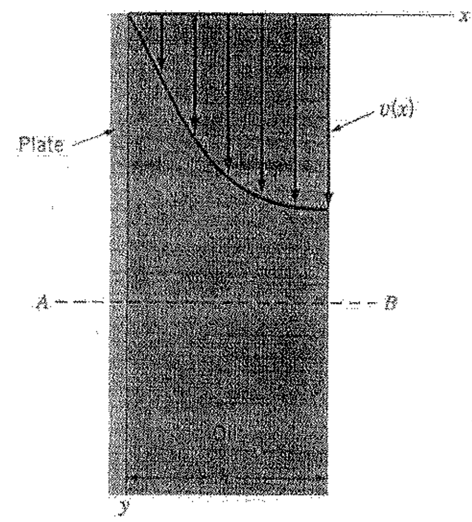


FIGURE P4.68

a) $v = \frac{V_0}{h^2}(2hx - x^2)$

Thus,

$$v\Big|_{x=0} = \frac{V_0}{h^2}(0-0) = 0 \quad \text{and}$$

$$\tau\Big|_{x=h} = \mu \frac{dv}{dx}\Big|_{x=h} = \mu \frac{V_0}{h^2}\left[2h - 2x\right]\Big|_{x=h} = 0$$

Hence, the fluid sticks to the plate and there is no shear stress at the free surface.

b) $$Q_{AB} = \int v\, dA = \int_{x=0}^{x=h} v\, b\, dx = \int_0^h \frac{V_0}{h^2}(2hx - x^2)\, b\, dx$$

or

$$Q_{AB} = \frac{V_0 b}{h^2}\left[hx^2 - \frac{1}{3}x^3\right]_0^h = \underline{\underline{\frac{2}{3}V_0 h b}}$$