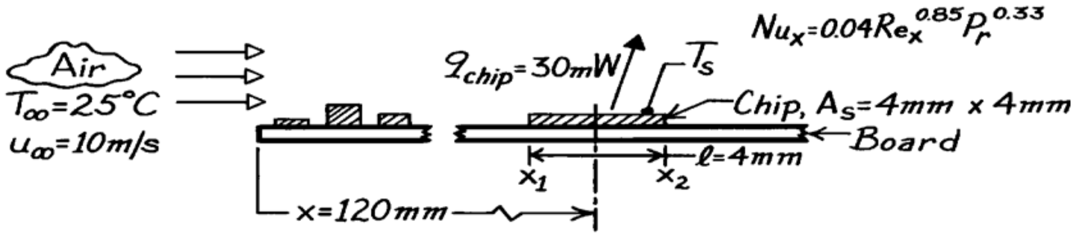
PROBLEM 7.38

KNOWN: Convection correlation for irregular surface due to electronic elements mounted on a circuit board experiencing forced air cooling with prescribed temperature and velocity

FIND: Surface temperature when heat dissipation rate is 30 mW for chip of prescribed area located a specific distance from the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Situation approximates parallel flow over a flat plate with prescribed correlation, (2) Heat rate is from top surface of chip.

PROPERTIES: *Table A-4*, Air (assume $T_s \approx 45°C$, then $\bar{T} = (45 + 25)°C/2 \approx 310$ K, 1 atm): k = 0.027 W/m·K, $\nu = 16.90 \times 10^{-6}$ m^2/s, Pr = 0.706.

ANALYSIS: For the chip upper surface, the heat rate is

$$q_{chip} = \bar{h}_{chip} A_s (T_s - T_\infty) \qquad \text{or} \qquad T_s = T_\infty + q_{chip} / \bar{h}_{chip} A_s$$

Assuming the average convection coefficient over the chip length to be equal to the local value at the center of the chip ($x = x_o$), $\bar{h}_{chip} \approx h_x(x_o)$, where

$$Nu_x = 0.04 Re_x^{0.85} Pr^{0.33}$$

$$Nu_x = 0.04\left(10 \text{ m/s} \times 0.120 \text{ m}/16.90 \times 10^{-6} \text{ m}^2/\text{s}\right)^{0.85} (0.706)^{0.33} = 473.4$$

$$h_x = \frac{Nu_x k}{x_o} = \frac{473.4 \times 0.027 \text{ W/m} \cdot \text{K}}{0.120 \text{ m}} = 107 \text{ W/m}^2 \cdot \text{K}$$

Hence,

$$T_s = 25°C + 30 \times 10^{-3} \text{ W}/107 \text{ W/m}^2 \cdot \text{K} \times \left(4 \times 10^{-3} \text{m}\right)^2 = (25 + 17.5)° \text{C} = 42.5°\text{C}. \quad <$$

COMMENTS: (1) Note that the assumed value of $\bar{T}$ used to evaluate the thermophysical properties was reasonable. (2) We could have evaluated $\bar{h}_{chip}$ by two other approaches. In one case the average coefficient is approximated as the arithmetic mean of local values at the leading and trailing edges of the chip.

$$\bar{h}_{chip} \approx \left[h_{x2}(x_2) + h_{x1}(x_1)\right]/2 = 107 \text{ W/m}^2 \cdot \text{K}.$$

The exact approach is of the form

$$\bar{h}_{chip} \cdot \ell = \bar{h}_{x2} \cdot x_2 - \bar{h}_{x1} \cdot x_1$$

Recognizing that $h_x \sim x^{-0.15}$, it follows that

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x \cdot dx = 1.176 h_x$$

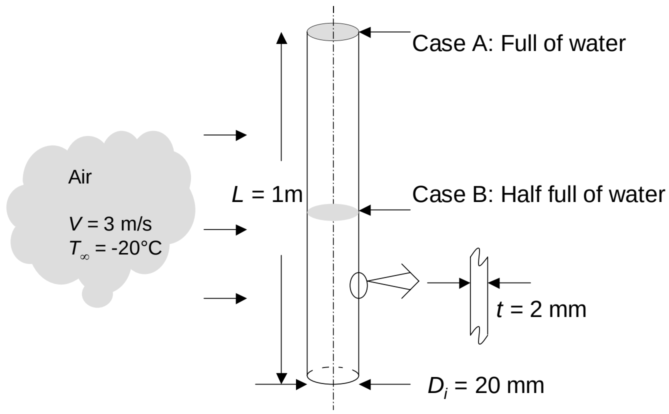
and $\bar{h}_{chip} = 108 \text{ W}/\text{m}^2 \cdot \text{K}$. Why do results for the two approximate methods and the exact method compare so favorably?

PROBLEM 7.48

KNOWN: Dimensions of a vertical copper tube experiencing crossflow. Air velocity and temperature, water temperature inside the tube.

FIND: (a) The heat loss per unit mass from the water (W/kg) when the pipe is full. (b) The heat loss from the water (W/kg) when the pipe is half full.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Tube behaves as an infinite fin, (4) Water is well-mixed, (5) One-dimensional heat transfer, (6) Inside copper wall temperature at water temperature, (7) Negligible heat transfer to/from the gas above the liquid water, (8) Negligible radiation.

PROPERTIES: *Table A.4*, air assumed: (T_f = (0°C - 20°C)/2 = -10°C ≈ 263K, p = 1 atm): $\nu = 12.6\times10^{-6}$ m^2/s, Pr = 0.717, k = 0.0233 W/m·K. Table A.1, copper: (T = 300 K): k_{CU} = 401 W/m·K. Table A.6, water (T = 273 K), ρ = 1000 kg/m^3.

ANALYSIS: For either case, the average convection coefficient about the tube must be evaluated. The Reynolds number, based upon the outer diameter D_o = 20 mm + 4 mm = 24 mm is $Re_D = VD_o/\nu$ = 3 m/s × 24 × 10^{-3} m/12.6 × 10^{-6} m^2/s = 5714. Using Eq. 7.54, the average heat transfer coefficient about the exterior of the tube is

$$h = \frac{0.0233\text{W/m}\cdot\text{K}}{24\times10^{-3}\text{m}}\left\{0.3+\frac{0.62\left(5714^{1/2}\right)0.717^{1/3}}{\left[1+(0.4/0.717)^{2/3}\right]^{1/4}}\left[1+\left(\frac{5714}{282{,}000}\right)^{5/8}\right]^{4/5}\right\} = 38.56\ \text{W/m}^2\cdot\text{K}$$

(a) From Eq. 3.34 heat loss from the water is

$$q = 1\text{m}\times\frac{\left(0-(-20)°\text{C}\right)}{\dfrac{\ln(24/20)}{2\pi\times401\text{W/m}\cdot\text{K}}+\dfrac{1}{\pi\times24\times10^{-3}\text{m}\times38.56\text{W/m}^2\cdot\text{K}}} = 58\text{W}$$

while the mass of water is $M = \pi(D_i^2/4)L\rho = \pi\times(20\times10^{-3}\text{m})^2/4\times1\text{m}\times1000\text{kg/m}^3$ = 0.314 kg. Hence, the heat loss per unit mass of water is

$$q_M = q/M = 58\ \text{W}/0.314\ \text{kg} = 185\ \text{W/kg}. \qquad <$$

Continued…

(b) When the tube is half full, the upper half of the tube will act as a fin. The total heat loss per unit mass will be $q_M = q_{M1} + q_{M2}$ where q_{M1} is the radial heat loss that is the same as in part (a) and q_{M2} is the heat loss to the upper half of the copper tubing, which serves as a fin. From part (a) q_{M1} = 185 W/kg. Assuming an infinite fin and recognizing that the cross-sectional area is associated with the inner and outer diameters of the tubing,

$$
\begin{aligned}
q_{M2} &= \sqrt{hPkA_c}\,\theta_b / M \\
&= \sqrt{38.56\text{W/m}^2\cdot\text{K}\times\pi\times24\times10^{-3}\text{m}\times401\text{W/m}\cdot\text{K}\times\pi\left((24\times10^{-3}\text{m})^2-(20\times10^{-3}\text{m})^2\right)/4} \\
&\times\left(0-(-20)\right)°\text{C/0.157kg} \\
&= 51.3 \text{ W/kg}
\end{aligned}
$$

Therefore, q_M = 185 W/kg + 51.3 W/kg = 236 W/kg <

COMMENTS: (1) The fin effect is significant, and the water in the half-full tube will freeze before the water in the full tube. (2) The temperature distribution in the copper tubing above the water level in the half-full tubing is $\theta/\theta_b = \exp^{-mx}$ where x is a local coordinate with origin at the water level. For this problem,

$$
\begin{aligned}
m &= \sqrt{hP/kA_c} = \sqrt{4hD_o/k(D_o^2-D_i^2)} \\
&= \sqrt{4\times38.56\text{W/m}^2\cdot\text{K}\times24\times10^{-3}\text{m}/401\text{W/m}\cdot\text{K}\left((24\times10^{-3}\text{m})^2-(20\times10^{-3}\text{m})^2\right)} = 7.24\text{ m}^{-1}.
\end{aligned}
$$

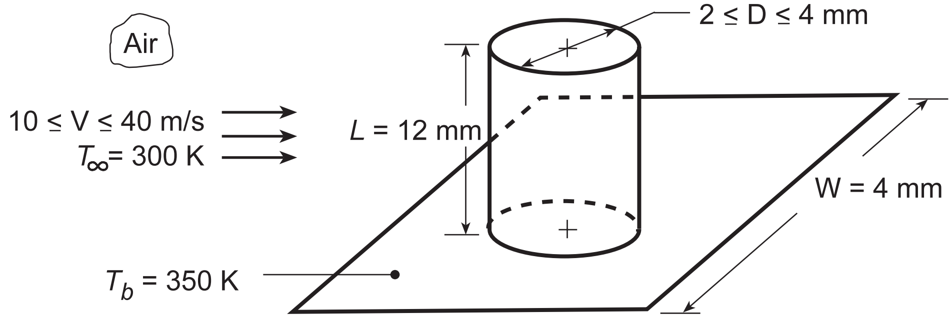
Therefore, over a 0.5 m length above the water surface for part (b), the temperature decreases to $T(x = 0.5\text{ m}) = -20°\text{C} + (20°\text{C})\exp(-7.24\text{m}^{-1}\times0.5\text{m}) = -19.5°\text{C}$. The assumption of an infinitely long fin is reasonable.

PROBLEM 7.54

KNOWN: Dimensions of chip and pin fin. Chip temperature. Free stream velocity and temperature of air coolant.

FIND: (a) Average pin convection coefficient, (b) Pin heat transfer rate, (c) Total heat rate, (d) Effect of velocity and pin diameter on total heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pin, (3) Constant properties, (4) Convection coefficients on pin surface (tip and side) and chip surface correspond to single cylinder in cross flow, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Copper (350 K): k = 399 W/m·K; *Table A.4*, Air ($T_f \approx 325$ K, 1 atm): $\nu = 18.41 \times 10^{-6}\ m^2/s$, k = 0.0282 W/m·K, Pr = 0.704.

ANALYSIS: (a) With V = 10 m/s and D = 0.002 m,

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.002\,m}{18.41 \times 10^{-6}\,m^2/s} = 1087$$

Using the Churchill and Bernstein correlations, Eq. (7.54),

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 16.7$$

$$\overline{h} = \left(\overline{Nu}_D k/D\right) = \left(16.7 \times 0.0282\,W/m \cdot K/0.002\,m\right) = 235\,W/m^2 \cdot K \qquad <$$

(b) For the fin with tip convection and

$$M = \left(\overline{h}\pi Dk\pi D^2/4\right)^{1/2}\theta_b = (\pi/2)\left[235\,W/m^2 \cdot K\,(0.002\,m)^3\,399\,W/m \cdot K\right]^{1/2} 50\,K = 2.15\,W$$

$$m = \left(\overline{h}P/kA_c\right)^{1/2} = \left(4 \times 235\,W/m^2 \cdot K/399\,W/m \cdot K \times 0.002\,m\right)^{1/2} = 34.3\,m^{-1}$$

$$mL = 34.3\,m^{-1}(0.012\,m) = 0.412$$

$$\left(\overline{h}/mk\right) = \left(235\,W/m^2 \cdot K/34.3\,m^{-1} \times 399\,W/m \cdot K\right) = 0.0172.$$

The fin heat rate is

$$q_f = M\frac{\sinh mL + \left(\overline{h}/mk\right)\cosh mL}{\cosh mL + \left(\overline{h}/mk\right)\sinh mL} = 0.868\,W\,. \qquad <$$

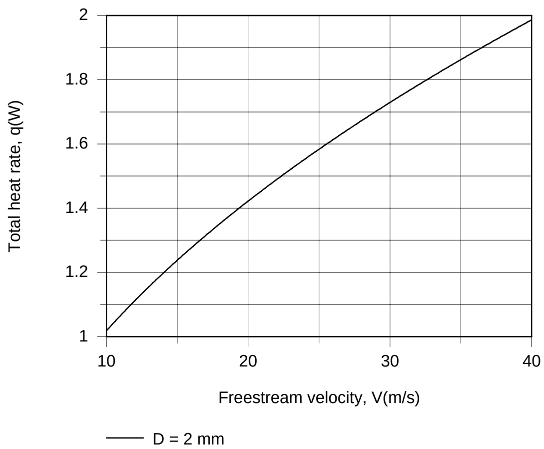
Continued...

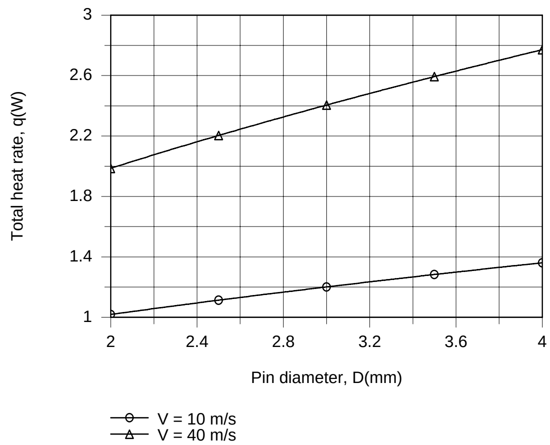
PROBLEM 7.54 (Cont.)

(c) The total heat rate is that from the base and through the fin,

$$q = q_b + q_f = \bar{h}\left(W^2 - \pi D^2/4\right)\theta_b + q_f = \left(0.151 + 0.868\right)W = 1.019\,W \,. \qquad <$$

(d) Using the IHT Extended Surface Model for a Pin Fin with the Correlations Tool Pad for a Cylinder in crossflow and Properties Tool Pad for Air, the following results were generated.





Clearly, there is significant benefit associated with increasing V which increases the convection coefficient and the total heat rate. Although the convection coefficient decreases with increasing D, the increase in the total heat transfer surface area is sufficient to yield an increase in q with increasing D. The maximum heat rate is q = 2.77 W for V = 40 m/s and D = 4 mm.

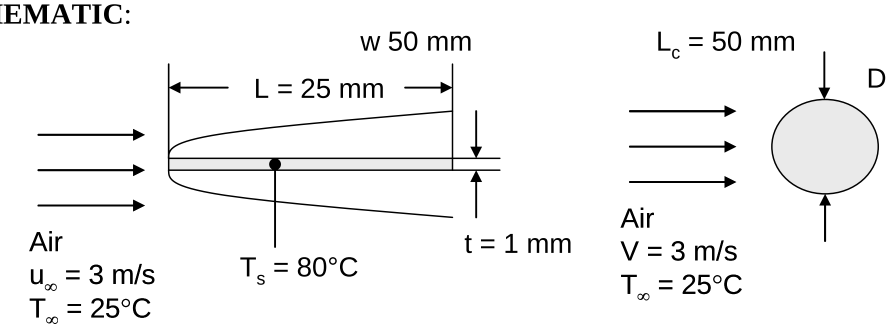
COMMENTS: Radiation effects should be negligible, although tip and base convection coefficients will differ from those calculated in parts (a) and (d).

PROBLEM 7.62

KNOWN: Dimensions of a flat plate in parallel flow. Plate and air temperatures and air velocity. Dimensions of a horizontal cylinder.

FIND: Convective heat loss from top and bottom of the flat plate and from the cylinder.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

PROPERTIES: *Table A.4*, air (T_f = (80°C + 25°C)/2 = 52.5°C ≈ 325K, p = 1 atm): $\nu = 18.4\times10^{-6}$ m^2/s, Pr = 0.704, k = 0.0282 W/m·K.

ANALYSIS: For the plate,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{3\text{m/s}\times 25\times10^{-3}\text{m}}{18.4\times10^{-6}\text{m}^2\text{/s}} = 4076$$

Therefore, the flow is laminar and Eq. 7.30 yields

$$\overline{h} = \frac{k}{L}\left[0.664 Re_L^{1/2}\right]Pr^{1/3} = \frac{0.0282\text{W/m}\cdot\text{K}}{25\times10^{-3}\text{m}}\left[0.664\times 4076^{1/2}\right]0.704^{1/3} = 42.5\ \text{W/m}^2\cdot\text{K}$$

and the convective heat transfer rate from the top and bottom of the flat plate is

$$q = 2wLh(T_s - T_\infty) = 2\times 50\times10^{-3}\text{m}\times 25\times10^{-3}\text{m}\times 42.5\text{W/m}^2\cdot\text{K}(80-25)\text{°C} = 5.84\text{W}$$ <

For the cylinder, $D = \sqrt{\frac{4}{\pi}tL} = \sqrt{\frac{4}{\pi}\times 1\times10^{-3}\text{m}\times 25\times10^{-3}\text{m}} = 0.00564\text{m} = 5.64\text{mm}$ and

$$Re_D = \frac{VD}{\nu} = \frac{3\text{m/s}\times 5.64\times10^{-3}\text{m}}{18.4\times10^{-6}\text{m}^2\text{/s}} = 920$$

Equation 7.54 yields

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62\times 920^{1/2} 0.704^{1/3}}{\left[1+(0.4/0.704)^{2/3}\right]^{1/4}}\left[1+\left(\frac{920}{282{,}000}\right)^{5/8}\right]^{4/5} = 15.3$$

Continued…

and $$\bar{h} = \frac{\overline{Nu}_D k}{D} = \frac{15.3 \times 0.0282\,\text{W/m}\cdot\text{K}}{5.64 \times 10^{-3}\,\text{m}} = 76.5\,\text{W/m}^2\cdot\text{K}$$

Therefore, the heat transfer rate from the cylinder is,

$$q = \pi D L_c \bar{h}(T_s - T_\infty) = \pi \times 5.64 \times 10^{-3}\,\text{m} \times 50 \times 10^{-3}\,\text{m} \times 76.52\,\text{W}/\text{m}^2\cdot\text{K}(80 - 25)°C = 3.73\,\text{W}$$ <

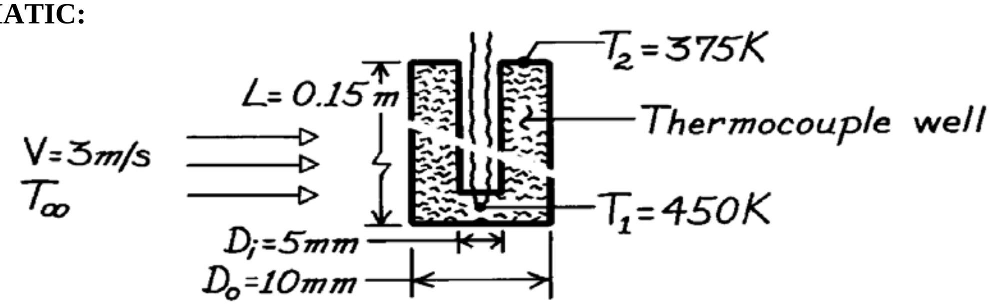
COMMENTS: (1) The heat transfer coefficient associated with the cylinder is 80% greater than that associated with the flat plate. However, for the same volume, the exposed surface area of the cylinder is 65% smaller than that of the flat plate, resulting in an overall smaller heat transfer rate for the cylinder. (2) A trial-and-error solution reveals that a larger cylinder of diameter *D* = 13.6 mm is necessary to transfer the same amount of energy by convection as the flat plate.

PROBLEM 7.69

KNOWN: Dimensions and thermal conductivity of a thermocouple well. Temperatures at well tip and base. Air velocity.

FIND: Air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction along well, (4) Uniform convection coefficient, (5) Negligible radiation.

PROPERTIES: Steel (given): k = 35 W/m·K; Air (given): $\rho = 0.774\ \text{kg/m}^3$, $\mu = 251 \times 10^{-7}\ \text{N·s/m}^2$, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: Applying Equation 3.75 at the well tip (x = L), where $T = T_1$,

$$\frac{T_1 - T_\infty}{T_2 - T_\infty} = \left[\cosh mL + \left(\bar{h}/mk\right)\sinh mL\right]^{-1}$$

$$m = \left(\bar{h}P/kA_c\right)^{1/2} \qquad P = \pi D_o = \pi(0.010\ \text{m}) = 0.0314\ \text{m}$$

$$A_c = (\pi/4)\left(D_o^2 - D_i^2\right) = (\pi/4)\left(0.010^2 - 0.005^2\right)\text{m}^2 = 5.89\times10^{-5}\ \text{m}^2.$$

With $$Re_D = \frac{\rho VD}{\mu} = \frac{0.774\ \text{kg/m}^3\,(3\text{m/s})\,0.01\ \text{m}}{251\times10^{-7}\ \text{N}\cdot\text{s/m}^2} = 925$$

C = 0.51, m = 0.5, n = 0.37 and the Zhukauskas correlation yields

$$\overline{Nu}_D = 0.51 Re_D^{0.5} Pr^{0.37}\left(Pr/Pr_s\right)^{1/4} \approx 0.51(925)^{0.5}(0.686)^{0.37}\times1 = 13.5$$

$$\bar{h} = \overline{Nu}_D\frac{k}{D_o} = 13.5\frac{0.0373\ \text{W/m}\cdot\text{K}}{0.01\ \text{m}} = 50.4\ \text{W/m}^2\cdot\text{K}.$$

Hence

$$m = \left[\frac{\left(50.4\ \text{W/m}^2\cdot\text{K}\right)0.0314\ \text{m}}{\left(35\ \text{W/m}\cdot\text{K}\right)5.89\times10^{-5}\ \text{m}^2}\right]^{1/2} = 27.7\ \text{m}^{-1} \qquad mL = \left(27.7\ \text{m}^{-1}\right)0.15\ \text{m} = 4.15.$$

With

$$\left(\bar{h}/mk\right) = \left(50.4\ \text{W/m}^2\cdot\text{K}\right)/\left(27.7\ \text{m}^{-1}\right)\left(35\ \text{W/m}\cdot\text{K}\right) = 0.0519$$

find $$\frac{T_1 - T_\infty}{T_2 - T_\infty} = \left[31.9 + (0.0519)31.8\right]^{-1} = 0.0298 \qquad T_\infty = 452.2\ \text{K}.$$ <

COMMENTS: Heat conduction along the wall to the base at 375 K is balanced by convection from the air.