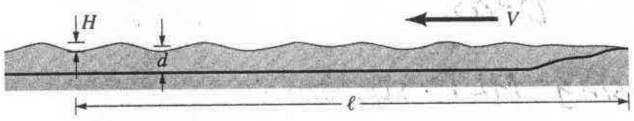
7.7

7.7 It is desired to determine the wave height when wind blows across a lake. The wave height, H, is assumed to be a function of the wind speed, V, the water density, ρ, the air density, ρ_a, the water depth, d, the distance from the shore, ℓ, and the acceleration of gravity, g, as shown in Fig. P7.7. Use d, V, and ρ as repeating variables to determine a suitable set of pi terms that could be used to describe this problem.



■ FIGURE P7.7

$$H = f(V, \rho, \rho_a, d, \ell, g)$$

$H \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \rho_a \doteq FL^{-4}T^{2}$ $\quad d \doteq L$ $\quad \ell \doteq L$ $\quad g \doteq LT^{-2}$

From the pi theorem, $7-3=4$ pi terms required. Use d, V, and ρ as repeating variables. Thus,

$$\Pi_1 = H d^a V^b \rho^c$$

and

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c = F^0L^0T^0$$

so that

$$c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$
$$-b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_1 = \frac{H}{d}$$

which is obviously dimensionless.

For Π_2:

$$\Pi_2 = \rho_a d^a V^b \rho^c$$

and

$$(FL^{-4}T^2)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c = F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F\text{)}$$
$$-4 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$
$$2 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=0$, $b=0$, $c=-1$ so that

$$\Pi_2 = \frac{\rho_a}{\rho}$$

which is obviously dimensionless.

(con't)

7.7	(con't)

For Π_3:

$$\Pi_3 = \ell\, d^a V^b \rho^c$$

and as for Π_1, $a=-1$, $b=0$, $c=0$ so that

$$\Pi_3 = \frac{\ell}{d}$$

For Π_4:

$$\Pi_4 = g\, d^a V^b \rho^c$$

$$(LT^{-2})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c = F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F\text{)}$$

$$1 + a + b - 4c = 0 \qquad \text{(for } L\text{)}$$

$$-2 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=$, $b=-2$, $c=0$, and therefore

$$\Pi_4 = \frac{gd}{V^2}$$

Check dimensions:

$$\frac{gd}{V^2} = \frac{(LT^{-2})(L)}{(LT^{-1})^2} \doteq L^0T^0 \qquad \therefore \text{ ok}$$

Thus,

$$\underline{\underline{\frac{H}{d} = \phi\left(\frac{\rho_a}{\rho}, \frac{\ell}{d}, \frac{gd}{V^2}\right)}}$$

7.23

7.23 A cylinder with a diameter, D, floats upright in a liquid as shown in Fig. P7.23. When the cylinder is displaced slightly along its vertical axis it will oscillate about its equilibrium position with a frequency, ω. Assume that this frequency is a function of the diameter, D, the mass of the cylinder, m, and the specific weight, γ, of the liquid. Determine, with the aid of dimensional analysis, how the frequency is related to these variables. If the mass of the cylinder were increased, would the frequency increase or decrease?

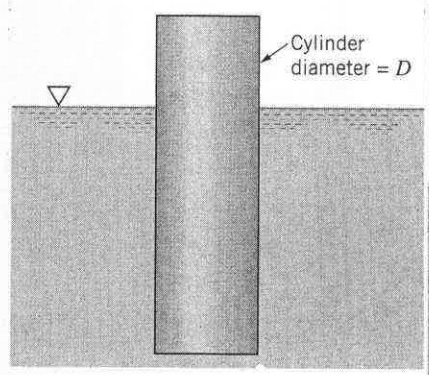


FIGURE P7.23

$$\omega = f(D, m, \gamma)$$

$\omega \doteq T^{-1}$ $\quad D \doteq L$ $\quad m \doteq FL^{-1}T^{2}$ $\quad \gamma = FL^{-3}$

From the pi theorem, $4-3 = 1$ pi term required.

By inspection:

$$\Pi_1 = \frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{FL^{-1}T^{2}}{FL^{-3}}} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{M}{ML^{-2}T^{-2}}} \doteq M^0L^0T^0 \quad \therefore OK$$

Since there is only 1 pi term, it follows that

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} = C$$

where C is a constant. Thus,

$$\underline{\underline{\omega = CD\sqrt{\frac{\gamma}{m}}}}$$

From this result it follows that if m is increased ω will <u>decrease</u>.

7.29

7.29 Assume that the drag, $\mathfrak{D}$, on an aircraft flying at supersonic speeds is a function of its velocity, V, fluid density, ρ, speed of sound, c, and a series of lengths, $\ell_1, \ldots, \ell_i$, which describe the geometry of the aircraft. Develop a set of pi terms that could be used to investigate experimentally how the drag is affected by the various factors listed. Form the pi terms by inspection.

$$\mathfrak{D} = f(V, \rho, c, \ell_1, \ldots \ell_i)$$

$\mathfrak{D} \doteq F \quad V = LT^{-1} \quad \rho \doteq FL^{-4}T^2 \quad c \doteq LT^{-1}$ all lengths, $\ell_i \doteq L$

From the pi theorem, $(4+i)-3 = 1+i$ pi terms required, where i is the number of length terms ($i = 1, 2, 3,$ etc.).

By inspection, for Π_1 (containing $\mathfrak{D}$):

$$\Pi_1 = \frac{\mathfrak{D}}{\rho V^2 \ell_1^2} \doteq \frac{F}{(FL^{-4}T^2)(LT^{-1})^2(L)^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\mathfrak{D}}{\rho V^2 \ell_1^2} \doteq \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2(L)^2} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2 (containing c):

$$\Pi_2 = \frac{c}{V} \quad \text{or} \quad \frac{V}{c}$$

and both are obviously dimensionless.

For all other pi terms containing ℓ_i

$$\Pi_i = \frac{\ell_i}{\ell_1}$$

and these terms involving the ℓ's are obviously dimensionless.

Thus,

$$\frac{\mathfrak{D}}{\rho V^2 \ell_1^2} = \phi\left(\frac{V}{c}, \frac{\ell_i}{\ell_1}\right)$$

where $\frac{\ell_i}{\ell_1}$ is a series of pi terms, $\frac{\ell_2}{\ell_1}, \frac{\ell_3}{\ell_1}$, etc.

7.19

7.32

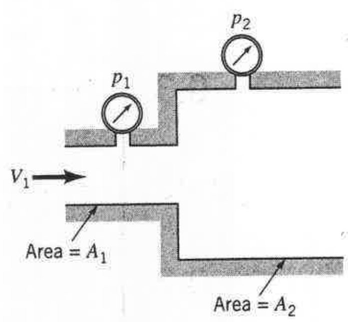
7.32 The pressure rise, $\Delta p = p_2 - p_1$, across the abrupt expansion of Fig. P7.32 through which a liquid is flowing can be expressed as

$$\Delta p = f(A_1, A_2, \rho, V_1)$$

where A_1 and A_2 are the upstream and downstream cross-sectional areas, respectively, ρ is the fluid density, and V_1 is the upstream velocity. Some experimental data obtained with $A_2 = 1.25\ \text{ft}^2$, $V_1 = 5.00$ ft/s, and using water with $\rho = 1.94\ \text{slugs/ft}^3$ are given in the following table:

A_1 (ft^2)	0.10	0.25	0.37	0.52	0.61
Δp (lb/ft^2)	3.25	7.85	10.3	11.6	12.3

Plot the results of these tests using suitable dimensionless parameters. With the aid of a standard curve fitting program determine a general equation for Δp and use this equation to predict Δp for water flowing through an abrupt expansion with an area ratio $A_1/A_2 = 0.35$ at a velocity $V_1 = 3.75$ ft/s.



■ FIGURE P7.32

$\Delta p \doteq FL^{-2} \quad A_1 \doteq L^2 \quad A_2 \doteq L^2 \quad \rho \doteq FL^{-4}T^2 \quad V_1 \doteq LT^{-1}$

From the pi theorem, 5 - 3 = 2 pi terms required.

By inspection for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p}{\rho V_1^2} \doteq \frac{FL^{-2}}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p}{\rho V_1^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing A_1 and A_2):

$$\Pi_2 = \frac{A_1}{A_2}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p}{\rho V_1^2} = \phi\left(\frac{A_1}{A_2}\right)$$

Using the data given, it follows that:

$\Delta p/\rho V_1^2$	0.067	0.162	0.212	0.239	0.254
A_1/A_2	0.080	0.200	0.296	0.416	0.488

A plot of these data is shown on the next page.

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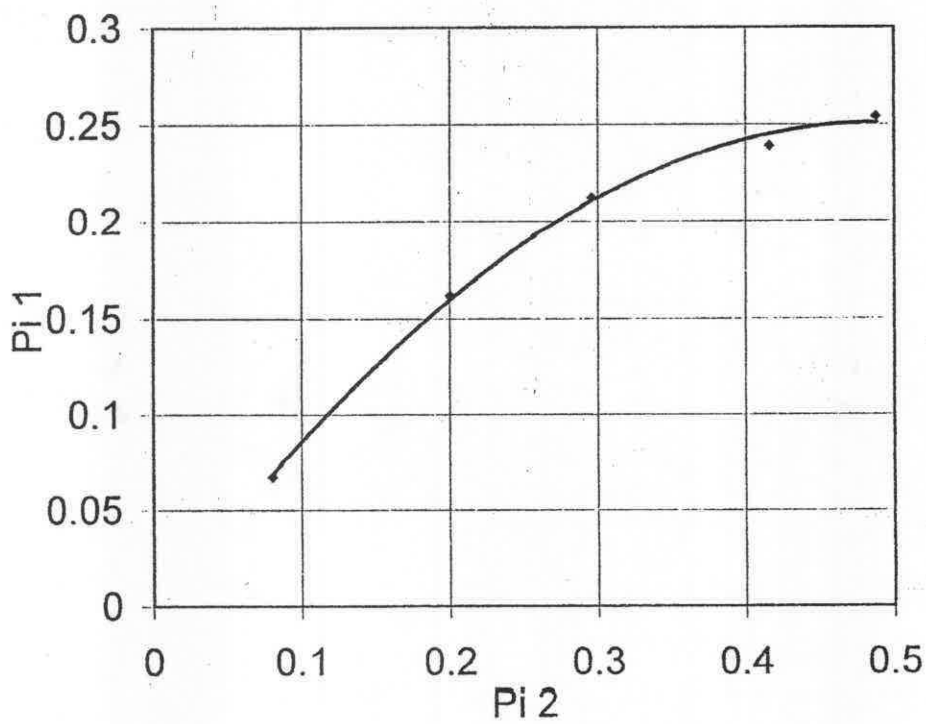
7.32 (con't)



The curve drawn on the graph is a 2nd order polynomial giving the equation

$$\frac{\Delta p}{\rho V_1^2} = -1.10\left(\frac{A_1}{A_2}\right)^2 + 1.07\left(\frac{A_1}{A_2}\right) - 0.0103$$

Thus, for $A_1/A_2 = 0.35$ and $V_1 = 3.75\ ft/s$ with water ($\rho = 1.94\ slugs/ft^3$)

$$\Delta p = \left(1.94\ \frac{slugs}{ft^3}\right)\left(3.75\ \frac{ft}{s}\right)^2\left[-1.10\,(0.35)^2 + 1.07\,(0.35) - 0.0103\right]$$

$$= 6.26\ \frac{lb}{ft^2}$$

7.35

7.35 The pressure drop per unit length, Δp_ℓ, for the flow of blood through a horizontal small diameter tube is a function of the volume rate of flow, Q, the diameter, D, and the blood viscosity, μ. For a series of tests in which $d = 2$ mm, and $\mu = 0.004\ \text{N·s/m}^2$, the following data were obtained, where the Δp listed was measured over the length, $\ell = 300$ mm.

Q (m³/s)	Δp (N/m²)
3.6×10^{-6}	1.1×10^4
4.9×10^{-6}	1.5×10^4
6.3×10^{-6}	1.9×10^4
7.9×10^{-6}	2.4×10^4
9.8×10^{-6}	3.0×10^4

Perform a dimensional analysis for this problem, and make use of the data given to determine a general relationship between Δp_ℓ and Q (one that is valid for other values of D, ℓ, and μ).

$$\Delta p_\ell = f(Q, D, \mu)$$

$$\Delta p_\ell \doteq FL^{-3} \qquad Q \doteq L^3T^{-1} \qquad D \doteq L \qquad \mu \doteq FL^{-2}T$$

From the pi theorem, $4-3=1$ pi term required.

By inspection:

$$\Pi_1 = \frac{\Delta p_\ell D^4}{\mu Q} \doteq \frac{(FL^{-3})(L)^4}{(FL^{-2}T)(L^3T^{-1})} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p_\ell D^4}{\mu Q} \doteq \frac{(ML^{-2}T^{-2})(L)^4}{(ML^{-1}T^{-1})(L^3T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

Since there is only 1 pi term, it follows that

$$\frac{\Delta p_\ell D^4}{\mu Q} = C$$

where C is a constant. For the data given

$$\frac{\Delta p_\ell D^4}{\mu Q} = \left(\frac{\Delta p}{0.3\,m}\right)\frac{(0.002\,m)^4}{(0.004\ N\cdot s/m^2)\,Q} = 1.33\times10^{-8}\,\frac{\Delta p}{Q}$$

and therefore using the data in the table

$\frac{\Delta p_\ell D^4}{\mu Q}$	40.6	40.7	40.1	40.4	40.7

Thus, the average value for $C = 40.5$ and

$$\underline{\underline{\Delta p_\ell = 40.5\,\frac{\mu Q}{D^4}}}$$

7.48

7.48 When a fluid flows slowly past a vertical plate of height h and width b (see Fig. P7.48), pressure develops on the face of the plate. Assume that the pressure, p, at the midpoint of the plate is a function of plate height and width, the approach velocity, V, and the fluid viscosity, μ. Make use of dimensional analysis to determine how the pressure, p, will change when the fluid velocity, V, is doubled.

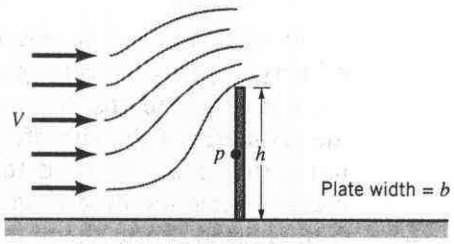


FIGURE P7.48

$$p = f(h, b, V, \mu)$$

$p \doteq FL^{-2}$ $\quad h \doteq L$ $\quad b \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem $5-3=2$ pi terms required.

By inspection, for Π_1 (containing p):

$$\Pi_1 = \frac{ph}{V\mu} \doteq \frac{(FL^{-2})(L)}{(LT^{-1})(FL^{-2}T)} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{ph}{V\mu} = \frac{(ML^{-1}T^{-2})(L)}{(LT^{-1})(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore OK$$

For Π_2 (containing b):

$$\Pi_2 = \frac{b}{h}$$

Which is obviously dimensionless. Thus,

$$\frac{ph}{V\mu} = \phi\left(\frac{b}{h}\right)$$

so that

$$p = \frac{V\mu}{h}\,\phi\left(\frac{b}{h}\right) \qquad (1)$$

From Eq. (1) it follows that for a given geometry and viscosity, if the velocity, V, is doubled the pressure, p, will be doubled.

7.67

7.67 Air bubbles discharge from the end of a submerged tube as shown in Fig. P7.67. The bubble diameter, D, is assumed to be a function of the air flowrate, Q, the tube diameter, d, the acceleration of gravity, g, the density of the liquid, ρ, and the surface tension of the liquid, σ. **(a)** Determine a suitable set of dimensionless variables for this problem. **(b)** Model tests are to be run on the earth for a prototype that is to be operated on a planet where the acceleration of gravity is 10 times greater than that on earth. The model and prototype are to use the same fluid, and the prototype tube diameter is 0.25 in. Determine the tube diameter for the model and the required model flowrate if the prototype flowrate is to be 0.001 ft³/s.

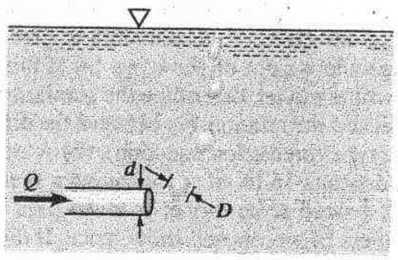


FIGURE P7.67

(a) $$D = f(Q, d, g, \rho, \sigma)$$

$D \doteq L \quad Q \doteq L^3T^{-1} \quad d \doteq L \quad g \doteq LT^{-2} \quad \rho \doteq FL^{-4}T^2 \quad \sigma \doteq FL^{-1}$

From the pi theorem, $6-3 = 3$ pi terms required, and a dimensional analysis yields

$$\frac{D}{d} = \phi\left(\frac{\sigma}{\rho g d^2}, \frac{Q}{\sqrt{g}\, d^{5/2}}\right)$$

(b) For similarity

$$\frac{\sigma_m}{\rho_m g_m d_m^2} = \frac{\sigma}{\rho g d^2}$$

and with $\sigma_m = \sigma$, $\rho_m = \rho$

$$d_m = \sqrt{\frac{g}{g_m}}\, d = \sqrt{\frac{10 g_m}{g_m}}\,(0.25\text{ in.}) = \underline{\underline{0.791\text{ in.}}}$$

Also,

$$\frac{Q_m}{\sqrt{g_m}\, d_m^{5/2}} = \frac{Q}{\sqrt{g}\, d^{5/2}}$$

so that

$$Q_m = \sqrt{\frac{g_m}{g}}\left(\frac{d_m}{d}\right)^{5/2} Q = \sqrt{\frac{1}{10}}\left(\frac{0.791\text{ in.}}{0.25\text{ in.}}\right)^{5/2}\left(0.001\frac{\text{ft}^3}{\text{s}}\right)$$

$$= \underline{\underline{5.63\times 10^{-3}\ \frac{\text{ft}^3}{\text{s}}}}$$

With the above similarity requirements met

$$\frac{D_m}{d_m} = \frac{D}{d} \quad \text{or} \quad \frac{D_m}{D} = \frac{d_m}{d} = \underline{\underline{\sqrt{10}}}$$

7.93

7.93 A liquid is contained in a pipe that is closed at one end as shown in Fig. P7.93. Initially the liquid is at rest, but if the end is suddenly opened the liquid starts to move. Assume the pressure p_1 remains constant. The differential equation that describes the resulting motion of the liquid is

$$\rho \frac{\partial v_z}{\partial t} = \frac{p_1}{\ell} + \mu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right)$$

where v_z is the velocity at any radial location, r, and t is time. Rewrite this equation in dimensionless form using the liquid density, ρ, the viscosity, μ, and the pipe radius, R, as reference parameters.

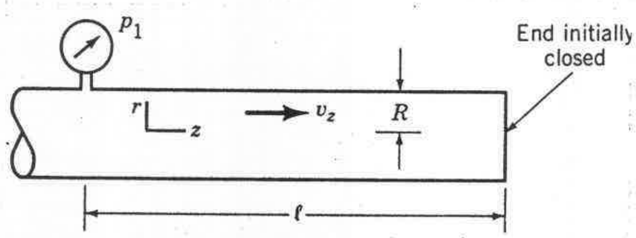


FIGURE P7.93

Let $r^* = \frac{r}{R}$, $t^* = \frac{t}{\tau}$, and $v_z^* = \frac{v_z}{V}$ where τ is some combinations of the parameters ρ, μ, and R having the dimensions of time, and V is some combination of the same parameters having the dimensions of a velocity. Let

$$\tau = \frac{\rho R^2}{\mu} \doteq \frac{(FL^{-4}T^2)(L)^2}{FL^{-2}T} \doteq T$$

and

$$V = \frac{\mu}{\rho R} \doteq \frac{FL^{-2}T}{(FL^{-4}T^2)(L)} \doteq LT^{-1}$$

With these dimensionless variables:

$$\frac{\partial v_z}{\partial t} = \frac{\partial (V v_z^*)}{\partial t^*} \frac{\partial t^*}{\partial t} = V \frac{\partial v_z^*}{\partial t^*}\left(\frac{1}{\tau}\right) = \left(\frac{\mu}{\rho R}\right)\left(\frac{\mu}{\rho R^2}\right)\frac{\partial v_z^*}{\partial t^*} = \left(\frac{\mu}{\rho}\right)^2 \frac{1}{R^3} \frac{\partial v_z^*}{\partial t^*}$$

$$\frac{\partial v_z}{\partial r} = \frac{\partial (V v_z^*)}{\partial r^*} \frac{\partial r^*}{\partial r} = V \frac{\partial v_z^*}{\partial r^*}\left(\frac{1}{R}\right) = \left(\frac{\mu}{\rho R}\right)\left(\frac{1}{R}\right)\frac{\partial v_z^*}{\partial r^*} = \frac{\mu}{\rho R^2} \frac{\partial v_z^*}{\partial r^*}$$

$$\frac{\partial^2 v_z}{\partial r^2} = \frac{\mu}{\rho R^2} \frac{\partial}{\partial r^*}\left(\frac{\partial v_z^*}{\partial r^*}\right)\frac{\partial r^*}{\partial r} = \frac{\mu}{\rho R^2} \frac{\partial^2 v_z^*}{\partial r^{*2}}\left(\frac{1}{R}\right) = \frac{\mu}{\rho R^3} \frac{\partial^2 v_z^*}{\partial r^{*2}}$$

The original differential equation can now be expressed as

$$\left[\rho \left(\frac{\mu}{\rho}\right)^2 \frac{1}{R^3}\right] \frac{\partial v_z^*}{\partial t^*} = \frac{p_1}{\ell} + \left[\mu \left(\frac{\mu}{\rho R^3}\right)\right]\left(\frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial v_z^*}{\partial r^*}\right)$$

or

$$\underline{\underline{\frac{\partial v_z^*}{\partial t^*} = \frac{p_1 \rho R^3}{\ell \mu^2} + \frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v_z^*}{\partial r^*}}}$$

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