Homework Set 2

Due date: February 5, 2016 after class

Problem 1: Consider a plane wall with a thickness of l_w = 0.5m and <u>a heat sink</u> that is a function of temperature given as: $\dot{q}$ = -0.8T W/m^3, where T is in K. The temperature and heat flux at the inlet to the wall (x = 0) are 500 K and 0.4 kW/m^2, respectively, and the thermal conductivity is k_w = 20 W/m·K. At the exit of the wall, convective heat exchange occurs with the surrounding that are maintained at T_∞ = 250 K. Assuming steady, one-dimensional conduction in the wall, constant properties, and negligible radiative heat transfer to the environment, determine the following:

a) The temperature at the exit of the wall ($x = L$) in °C.

$$k\frac{d^2T}{dx} + \dot{q}_{\text{gen}} = 0$$

$$\frac{d^2T}{dx^2} - \frac{0.8\ \text{W}\cdot\text{m}^{-3}}{20\ \text{W/m}\cdot\text{K}}T = 0 = \frac{d^2T}{dx^2} - \frac{0.8\ \text{W}\cdot\text{m}^{-3}}{20\ \text{W/m}\cdot\text{K}}T = 0$$

$$\frac{d^2T}{dx^2} - 0.04T = 0$$

$$T = C_1\exp(-0.2x) + C_2\exp(0.2x)$$

Boundary conditions:

$$\left.\frac{dT}{dx}\right|_{x=0} = -\frac{q_{\text{w}}^{"}}{k} = -20\ \text{K/m and T(0)=500 K}$$

$$500 = C_1 + C_2$$

$$100 = C_1 - C_2$$

$$C_1 = 300\text{K and } C_2 = 200\text{K}$$

$$T = 300\exp(-0.2x) + 200\exp(0.2x)K$$

$$T(0.5) = 300\exp(-0.2\cdot 0.5) + 200\exp(0.2\cdot 0.5)K = 492.5\text{K or } 219.4°\text{C}$$

OR due to repeating roots

$$T = C_1^{'}\sinh(0.2x) + C_2^{'}\cosh(0.2x)$$

$$T(x=0) = C_1^{'}\sinh(0) + C_2^{'}\cosh(0) = 500\ \text{K} \Rightarrow C_2^{'} = 500\ \text{K}$$

$$\left.\frac{dT}{dx}\right|_{x=0} = -\frac{400\ \text{W}\cdot\text{m}^{-2}}{20\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}} = -20\ \text{K/m=0.2C}_1^{'}\text{cosh}(0) + 0.2\sinh(0) \Rightarrow \text{C}_1^{'} = -100$$

$$T(x) = -100\sinh(0.2x) + 500\cosh(0.2x)$$

$$T(x) = -100\sinh(0.2\times 0.5) + 500\cosh(0.2\times 0.5) = 492.5\ \text{K or } 219.4°\text{C}$$

b) The convective heat transfer coefficient at the exit to the surroundings in W/m²·K.

$$q''\big|_{x=0.5\text{m}} = -k\frac{dT}{dx}\bigg|_{x=0.5\text{m}} = -20\frac{\text{W}}{\text{m}\cdot\text{K}}\left[-0.2\cdot 300\exp(-0.2\cdot 0.5)+0.2\cdot 200\exp(0.2\cdot 0.5)\right]$$

$$q''\big|_{x=0.5\text{m}} = 201.7\ \text{W}\cdot\text{m}^{-2}$$

$$h = \frac{q_w''}{T(0.5)-T_\infty} = 0.8316\frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

Problem 2: Consider a cylindrical pipe with steam running through it at a temperature of T_s = 250 °C and a convective heat transfer coefficient of h = 100 W/m²·K. The inner and outer diameters of the pipe are 0.1m and 0.11 m, respectively. The pipe is covered with 0.05 m thick fiberglass pipe insulation to prevent heat loss, and there is a contact resistance between the insulation and the pipe of R'' = 0.002 m²·K/W. The temperature of the air on the outside of the insulation is T_∞ = 30 °C and the combined heat transfer coefficient is h = 25 W/m²·K. The thermal conductivities of the pipe and insulation are k_{pipe} =60.5 W/m·K and k_{ins} = 0.038 W/m·K, respectively. Assuming steady, radial conduction, determine the following:

(a) The temperature drop between the pipe and insulation due to contact resistance.

$$\frac{UA}{L} = \left[(2\pi h_i r_i)^{-1} + \left(\frac{\ln(r_o/r_i)}{2\pi k_{\text{pipe}}}\right) + \frac{R_c''}{2\pi r_o} + \left(\frac{\ln((r_o+d)/r_o)}{2\pi k_{ins}}\right) + \left(2\pi h_o(r_o+d)\right)^{-1}\right]^{-1} = 0.360\ \text{W/m}\cdot\text{K}$$

$$q' = \frac{UA}{L}\left[T_{i,\infty} - T_{o,\infty}\right] = 79.19\ \text{W/m}$$

$$\Delta T = \frac{R_c'' q'}{2\pi r_o} = 0.4583\ \text{K}$$

(b) The surface temperature on the inner surface of the pipe in °C.

$$q' = 2\pi r_i h_i\left[T_{i,\infty} - T_1\right] \Rightarrow T_1 = T_{i,\infty} - \frac{q'}{2\pi r_i h_i} = 247.5°\text{C}$$