Homework Set 6

Due date: April 4, 2016 at the beginning of class

Problem 1: Two diffuse-gray concentric cylinders are separated by a thin diffuse-gray cylindrical radiation shield of radius r_{sh} = 1.5 m as shown in Figure 1. The internal cylinder A_1 of radius r_1 = 1.25 m is at temperature T_1 = 600 K. The external cylinder A_2 of radius r_2 = 1.75 m is at temperature T_2 = 300 K. Emissivities of the cylinders and the shield are $\varepsilon_1 = \varepsilon_{sh} = \varepsilon_2 = 0.1$.

a) Calculate the shield temperature.
b) Calculate the heat transfer rate from cylinder 1 to cylinder 2.
c) Calculate the ratio of the heat transfer rate from cylinder 1 to cylinder 2 with the shield, to that without the shield.

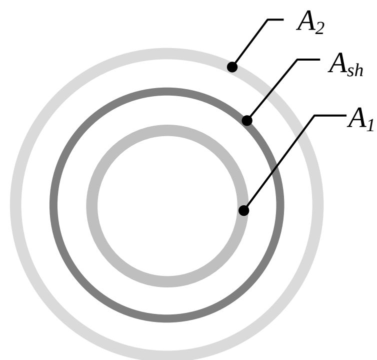

Figure 1.

a) Calculate the shield temperature

Energy is conserved, hence: $q_1 = q_{\text{shield}} = q_2$

$$q_1 = \frac{\sigma T_1^4 - \sigma T_{\text{shield}}^4}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1} + \frac{1-\varepsilon_{\text{shield}}}{A_{\text{shield}}\varepsilon_{\text{shield}}}} = q_{\text{shield}} = \frac{\sigma T_{\text{shield}}^4 - \sigma T_2^4}{\frac{1-\varepsilon_{\text{shield}}}{A_{\text{shield}}\varepsilon_{\text{shield}}} + \frac{1}{A_{\text{shield}}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$T_{\text{shield}} = 502.4 \text{ K}$$

b) Calculate the heat transfer rate from cylinder 1 to cylinder 2.

$$\frac{q_1}{l} = \frac{\sigma T_1^4 - \sigma T_{\text{shield}}^4}{\frac{1-\varepsilon_1}{2\pi r_1\varepsilon_1} + \frac{1}{2\pi r_1} + \frac{1-\varepsilon_{\text{shield}}}{2\pi r_{\text{shield}}\varepsilon_{\text{shield}}}} = 1677 \text{ W}\cdot\text{m}^{-1}$$

d) Calculate the ratio of the heat transfer rate from cylinder 1 to cylinder 2 with the shield, to that without the shield.

$$\frac{q_{1/\text{no shield}}}{l} = \frac{2\pi\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon}{r_1\varepsilon} + \frac{1}{r_1} + \frac{1-\varepsilon}{r_2\varepsilon}}$$

$$\frac{q_{1/\text{w shield}}}{l} = \frac{2\pi\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon}{r_1\varepsilon} + \frac{1}{r_1} + 2\left(\frac{1-\varepsilon}{r_{\text{shield}}\varepsilon}\right) + \frac{1}{r_{\text{shield}}} + \frac{1-\varepsilon}{r_2\varepsilon}}$$

$$\frac{q_{1/\text{no shield}}}{q_{1/\text{w shield}}} = \frac{\frac{1-\varepsilon}{r_1\varepsilon} + \frac{1}{r_1} + 2\left(\frac{1-\varepsilon}{r_{\text{shield}}\varepsilon}\right) + \frac{1}{r_{\text{shield}}} + \frac{1-\varepsilon}{r_2\varepsilon}}{\frac{1-\varepsilon}{r_1\varepsilon} + \frac{1}{r_1} + \frac{1-\varepsilon}{r_2\varepsilon}} = 0.5092$$

Problem 2: Consider a well-insulated cylindrical enclosure with a diameter and height of 0.7 m and 1 m, respectively. Heat is provided to the enclosure by a heating element through surface A_1. A 0.5 m hole is cut in the top of the cylinder shown schematically in Figure 2. The surfaces A_1, A_2, and A_3 are maintained at temperatures of T_1= 1500 K, T_2=1329.8 K, and T_3=1344.8 K, respectively, with emissivities of ε_1= 0.8 and ε_2=ε_3=0.6, respectively. Assume diffuse-gray surfaces, non-participating media, and uniform radiative power on the surfaces.

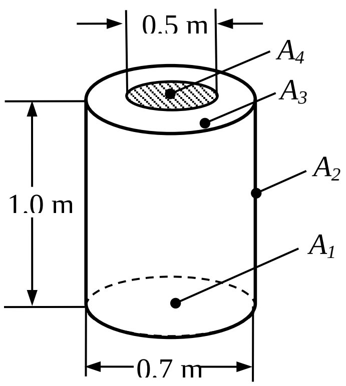

Figure 2.

a) Compute all view factors for surfaces A_1 ($F_{1\text{-}j}$) and A_4 ($F_{4\text{-}j}$)

For i=1, j=1:4

$$F_{1-1} = 0$$

$$R_1 = \frac{r_1}{h} = \frac{0.35\text{m}}{1.0\text{m}} \quad R_2 = \frac{0.25\text{m}}{1.0\text{m}} \quad \text{S}=1+\frac{1+R_2^2}{R_1^2}$$

$$F_{1-4} = 0.5\left[S - \sqrt{S^2 - 4\left(\frac{R_2}{R_1}\right)^2}\right] = 0.0530$$

$$F_{1-3} = F_{1-(3+4)} - F_{1-4}$$

$$R_1 = \frac{r_1}{h} = \frac{0.35\text{m}}{1.0\text{m}} \quad R_2 = \frac{0.35\text{m}}{1.0\text{m}} \quad X=1+\frac{1+R_2^2}{R_1^2}$$

$$\Rightarrow F_{1-(3+4)} = 0.0994$$

$$\Rightarrow F_{1-3} = 0.0994 - 0.0530 = 0.0464$$

$$\Rightarrow F_{1-2} = 1 - F_{1-(3+4)} = 0.9006$$

For i=4, j=1:4

$$\Rightarrow F_{4-1} = \frac{A_1}{A_4} F_{1-4} = \frac{(0.7\text{m})^2}{(0.5\text{m})^2} 0.0530 = 0.1039$$

$$\Rightarrow F_{4-4} = 0$$

$$\Rightarrow F_{4-3} = 0$$

$$\Rightarrow F_{4-2} = 1 - F_{4-1} - F_{4-3} - F_{4-4} = 1 - 0.1039 = 0.8961$$

b) Compute the net heat flux for the heated surface (q_1) and escaping through the opening (q_4).

$$q_2 = q_3 = 0$$

$$q_2 = 0 = \frac{E_{b,2} - J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}} \Rightarrow E_{b,2} = J_2$$

$$q_3 = 0 = \frac{E_{b,3} - J_3}{\frac{1-\varepsilon_3}{A_3\varepsilon_3}} \Rightarrow E_{b,3} = J_3$$

$$T_1 = 0 \text{ K}, \varepsilon_1 = 1 \Rightarrow J_1 = 0$$

$$\frac{q_1}{A_1} = \frac{E_{b,1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1}} = \left[J_1 - E_{b,2}\right] F_{12} + \left[J_1 - E_{b,3}\right] F_{13} + J_1 F_{14}$$

$$J_1 = 263{,}293 \text{ W}\cdot\text{m}^{-2}, q_1'' = 95004 \text{ W}\cdot\text{m}^{-2}$$

$$\frac{q_4}{A_4} = F_{41} J_1 + F_{42}\sigma T_2^4 + F_{43}\sigma T_3^4 = 186{,}242 \text{ W}\cdot\text{m}^{-2}$$