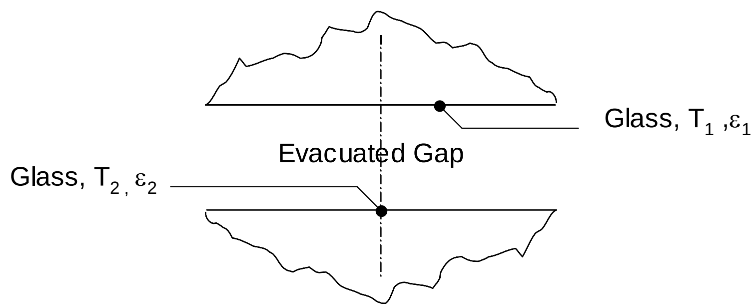
## PROBLEM 13.44

**KNOWN:** Temperatures and emissivity of glass surfaces.

**FIND:** Heat flux through the window for case 1: $\varepsilon_1 = \varepsilon_2 = 0.95$, case 2: $\varepsilon_1 = \varepsilon_2 = 0.05$, and case 3: $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.95$.

**SCHEMATIC**:




**ASSUMPTIONS**: (1) Diffuse-gray surfaces with uniform radiosity and irradiation distributions, (2) Infinite parallel glass surfaces.

**ANALYSIS:** For case 1, the net radiation heat flux between the glass sheets is

$$q''_{rad} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(293\text{K}^4 - 263\text{K}^4\right)}{\frac{1}{0.95} + \frac{1}{0.95} - 1} = 133\ \text{W/m}^2 \qquad <$$

Likewise, for case 2: $\varepsilon_1 = \varepsilon_2 = 0.05$, $q''_{rad} = 3.76\ \text{W/m}^2$, <

and for case 3: $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.95$, $q''_{rad} = 7.31\ \text{W/m}^2$. <

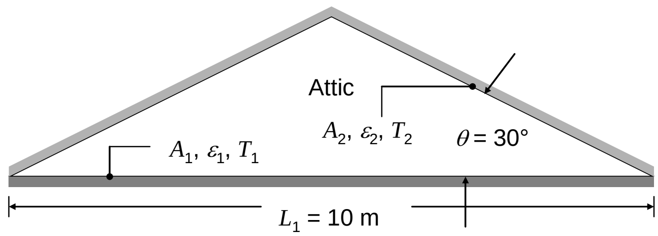
**COMMENTS:** The reduction associated with case 2 is [(133 – 3.76)/133] × 100 = 97 % while the reduction associated with case 3 is 94.5%. Both cases 2 and 3 provide a significant reduction in the heat flux relative to the uncoated glass of case 1. The decision to specify single- or double-surface coating depends on the cost of applying the low-emissivity coating.

## PROBLEM 13.48

**KNOWN:** Dimensions of attic. Emissivity of aluminum foil and of surfaces prior to application of the foil .

**FIND:** (a) Reduction of radiation heat transfer from the hot roof to the attic floor if foil is installed on the bottom of the roof, (b) Reduction in radiation heat load if foil is installed on the top of the attic floor, (c) Reduction if foiled is installed on both the attic floor and bottom of roof.

**SCHEMATIC**:




**ASSUMPTIONS**: (1) Diffuse-gray surfaces of uniform radiosity and irradiation, (2) Temperatures of surfaces are unaffected by surface treatment, (3) Two-dimensional configuration.

**ANALYSIS:** From Eqn. 13.23 we know that the ratio of the radiation heat load after installation of the foil to the radiation heat load prior to the installation of the foil is

$$R = \left[\frac{1-\varepsilon_o}{\varepsilon_o A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_o}{\varepsilon_o A_2}\right] \Bigg/ \left[\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right] \tag{1}$$

On a per-unit depth basis, $A_1$ = 10 m$^2$ and $A_2$ = 10 m$^2$/cos(30°) = 11.55 m$^2$, while from inspection, $F_{12}$ = 1.

(a) $\underline{\varepsilon_1 = 0.85,\ \varepsilon_2 = 0.07}$. Evaluation of Eqn. (1) yields $R$ = 0.105. <

(b) $\underline{\varepsilon_1 = 0.07,\ \varepsilon_2 = 0.85}$. Evaluation of Eqn. (1) yields $R$ = 0.092. <

(c) $\underline{\varepsilon_1 = \varepsilon_2 = 0.07}$. Evaluation of Eqn. (1) yields $R$ = 0.052. <

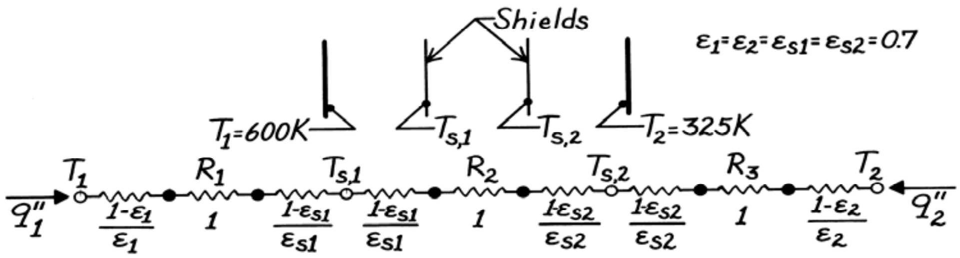
**COMMENTS:** (1) The reduction in the radiation heat load is least when the foil is installed on the bottom of the attic roof only. The surface formed by the attic roof is relatively large compared to Surface 1. As the roof becomes more steeply pitched, $A_2$ will increase and will eventually form a large surroundings. Installing foil on the roof, as its area becomes much larger, will become more ineffective. (2) The reduction is most significant when both surfaces are covered with foil, as expected. (3) Over time, the foil installed on the floor may become covered with a layer of dust, reducing the effectiveness of the foil installation on that surface.

## PROBLEM 13.56

**KNOWN:** Two radiation shields positioned in the evacuated space between two infinite, parallel planes.

**FIND:** Steady-state temperature of the shields.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) All surfaces are diffuse-gray and (2) All surfaces are parallel and of infinite extent.

**ANALYSIS:** The planes and shields can be represented by a thermal circuit from which it follows that

$$q_1'' = -q_2'' = \frac{\sigma\left(T_1^4 - T_2^4\right)}{R_1'' + R_2'' + R_3''} = \frac{\sigma\left(T_1^4 - T_{s1}^4\right)}{R_1''} = \frac{\sigma\left(T_{s1}^4 - T_{s2}^4\right)}{R_2''} = \frac{\sigma\left(T_{s2}^4 - T_2^4\right)}{R_3''}.$$

Since all the emissivities involved are equal, $R_1'' = \dfrac{A_1}{A_1F_{12}} = 1 = R_2'' = R_3''$, so that

$$T_{s1}^4 = T_1^4 - \frac{R_1''}{R_1'' + R_2'' + R_3''}\left(T_1^4 - T_2^4\right) = T_1^4 - (1/3)\left(T_1^4 - T_2^4\right)$$

$$T_{s1}^4 = (600\text{ K})^4 - (1/3)\left(600^4 - 325^4\right)\text{K}^4 \qquad T_{1s} = 548\text{ K}$$ <

$$T_{s2}^4 = T_2^4 + \frac{R_3''}{R_1'' + R_2'' + R_3''}\left(T_1^4 - T_2^4\right) = T_2^4 + (1/3)\left(T_1^4 - T_2^4\right)$$

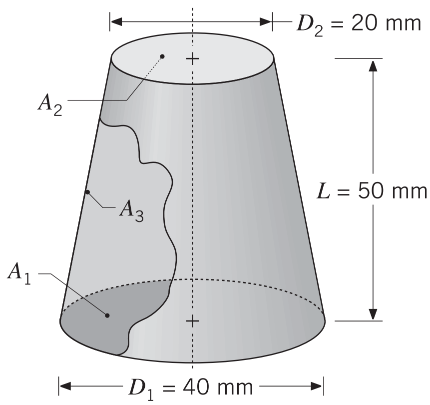
$$T_{s2}^4 = (325\text{ K})^4 + (1/3)\left(600^4 - 325^4\right)\text{K}^4 \qquad T_{s2} = 474\text{ K}.$$ <

# PROBLEM 13.66

**KNOWN:** Furnace in the form of a truncated conical section, floor (1) maintained at $T_1$ = 1000 K by providing a heat flux $q''_{1,in} = 2200\ \text{W/m}^2$; lateral wall (3) perfectly insulated; radiative properties of all surfaces specified.

**FIND:** (a) Temperature of the upper surface, $T_2$, and of the lateral wall $T_3$, and (b) $T_2$ and $T_3$ if all the furnace surfaces are black instead of diffuse-gray, with all other conditions remain unchanged. Explain effect of $\varepsilon_2$ on your results.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Furnace is a three-surface, diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Lateral surface is adiabatic, and (4) Negligible convection effects.

**ANALYSIS:** For the three-surface enclosure, write the radiation surface energy balances, Eq. 13.21, to find the radiosities of the three surfaces.

$$\frac{E_{b,1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - J_3}{1/A_1 F_{13}} \tag{1}$$

$$\frac{E_{b,2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - J_3}{1/A_2 F_{23}} \tag{2}$$

$$\frac{E_{b,3} - J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} = \frac{J_3 - J_1}{1/A_3 F_{31}} + \frac{J_3 - J_2}{1/A_3 F_{32}} \tag{3}$$

where the blackbody emissive powers are of the form $E_b = \sigma T^4$ with $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$. From Eq. 13.19, the net radiation leaving $A_1$ is

$$q_1 = \frac{E_{b,1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \tag{4}$$

$$q_1 = q''_{1,in} \cdot A_1 = 2200\ \text{W/m}^2 \times \pi (0.040\ \text{m})^2 / 4 = 2.76\ \text{W}$$

Continued …

## PROBLEM 13.66 (Cont.)

Since the lateral surface is adiabatic,

$$q_3 = \frac{E_{b,3} - J_3}{(1-\varepsilon_3)/\varepsilon_3 A_3} = 0 \qquad (5)$$

from which we recognize $E_{b,3} = J_3$, but will find that as an outcome of the analysis. For the enclosure, N = 3, there are $N^2 = 9$ view factors, for which N (N - 1)/2 = 3 must be directly determined. Calculations for the $F_{ij}$ are summarized in Comments.

With the foregoing five relations, we can determine the five unknowns: $J_1$, $J_2$, $J_3$, $E_{b,2}$, and $E_{b,3}$. The temperatures $T_2$ and $T_3$ will be evaluated from the relation $E_b = \sigma T^4$. Using this analysis approach with the relations in the *IHT* workspace, the results for (a) the diffuse-gray surfaces and (b) black surfaces are tabulated below. <

| | $J_1$ (kW/m$^2$) | $J_2$ (kW/m$^2$) | $J_3$ (kW/m$^2$) | $T_2$ (K) | $T_3$ (K) |
|---|---|---|---|---|---|
| (a) Diffuse-gray | 55.76 | 45.30 | 53.48 | 896 | 986 |
| (b) Black | 56.70 | 46.24 | 54.42 | 950 | 990 |

**COMMENTS:** (1) From the tabulated results, it follows that the temperatures of the lateral and top surfaces will be higher when the surfaces are black, rather than diffuse-gray as specified.

(2) From Eq. (5) for the net heat radiation leaving the lateral surface, $A_3$, the rate is zero since the wall is adiabatic. The consequences are that the blackbody emissive power and the radiosity are equal, and that the emissivity of the surface has no effect in the analysis. That is, this surface emits and absorbs at the same rate; the net is zero.

(3) For the enclosure, N = 3, there are $N^2 = 9$ view factors, for which

$$N\,(N-1)/2 = 3 \times 2/2 = 3$$

must be directly determined. We used the *IHT Tools | Radiation | View Factors Relations* model that sets up the summation rules and reciprocity relations for the N surfaces. The user is required to specify the 3 $F_{ij}$ that must be determined directly; by inspection, $F_{11} = F_{22} = 0$; and $F_{12}$ can be evaluated using the parallel coaxial disk relation, Table 13.2 (Fig. 13.5). This model is also provided in *IHT* to simplify the calculation task. The results of the view factor analysis are:

$$F_{12} = 0.03348 \qquad F_{13} = 0.9665$$

$$F_{21} = 0.1339 \qquad F_{23} = 0.8661$$

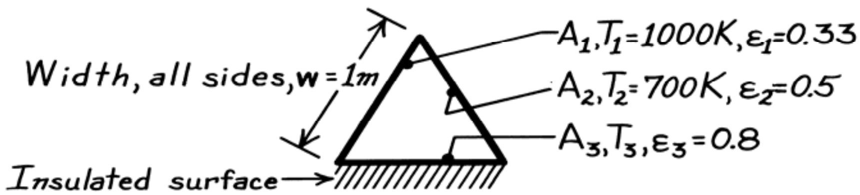
(4) An alternative method of solution for part (a) is to treat the enclosure of part (a) as described in Section 13.3.5. For part (b), the black enclosure analysis is described in Section 13.2. We chose to use the direct approach, Section 13.3.2, to develop a general 3-surface enclosure code in *IHT* that can also handle black surfaces (caution: use ε = 0.999, not 1.000).

## PROBLEM 13.75

**KNOWN:** Very long, triangular duct with walls that are diffuse-gray.

**FIND:** (a) Net radiation transfer from surface $A_1$ per unit length of duct, (b) The temperature of the insulated surface, (c) Influence of $\varepsilon_3$ on the results; comment on exactness of results.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Surfaces are diffuse-gray, (2) Duct is very long; end effects negligible.

**ANALYSIS:** (a) The duct approximates a three-surface enclosure for which the third surface ($A_3$) is re-radiating. Using Eq. 13.30 with $A_3 = A_R$, the net exchange is

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\dfrac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left(1/A_1F_{1R} + 1/A_2F_{2R}\right)^{-1}} + \dfrac{(1-\varepsilon_2)}{\varepsilon_2 A_2}} \qquad (1)$$

From symmetry, $F_{12} = F_{1R} = F_{2R} = 0.5$. With $A_1 = A_2 = w \cdot \ell$, where $\ell$ is the length normal to the page and w = 1 m,

$$q_1' = q_1/\ell = \left(q_1/A_1\right)w$$

$$q_1' = \frac{(56,700 - 13,614)\,\text{W/m}^2 \times 1\,\text{m}}{\dfrac{(1-0.33)}{0.33} + \dfrac{1}{0.5 + (1/0.5 + 1/0.5)^{-1}} + \dfrac{(1-0.5)}{0.5}} = 9874\ \text{W/m}. \qquad <$$

(b) From a radiation balance on $A_R$,

$$q_R = q_3 = 0 = \frac{E_{b3} - J_1}{(A_3F_{31})^{-1}} + \frac{E_{b3} - J_2}{(A_3F_{32})^{-1}} \qquad \text{or} \qquad E_{b3} = \frac{J_1 + J_2}{2}. \qquad (2)$$

To evaluate $J_1$ and $J_2$, use Eq. 13.19,

$$J_i = E_{b,i} - \frac{q_i}{A_i}\frac{(1-\varepsilon_i)}{\varepsilon_i} \quad \begin{cases} J_1 = 56,700 - (9874)\dfrac{1-0.33}{0.33} = 36,653\ \text{W/m}^2 \\[2ex] J_2 = 13,614 - (-9874)\dfrac{1-0.5}{0.5} = 23,488\ \text{W/m}^2 \end{cases}$$

From Eq. (2), now find

$$T_3 = \left(E_{b3}/\sigma\right)^{1/4} = \left([J_1 + J_2]/2\sigma\right)^{1/4} = \left(\frac{(36,653 + 23,488)\,\text{W/m}^2}{2\left(5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\right)}\right)^{1/4} = 853\ \text{K}. \qquad <$$

(c) Since $A_3$ is adiabatic or re-radiating, $J_3 = Eb_3$. Therefore, the value of $\varepsilon_3$ is of no influence on the radiation exchange or on $T_3$. In using Eq. (1), we require uniform radiosity over the surfaces. This requirement is not met near the corners. For best results we should subdivide the areas such that they represent regions of uniform radiosity. Of course, the analysis then becomes much more complicated.