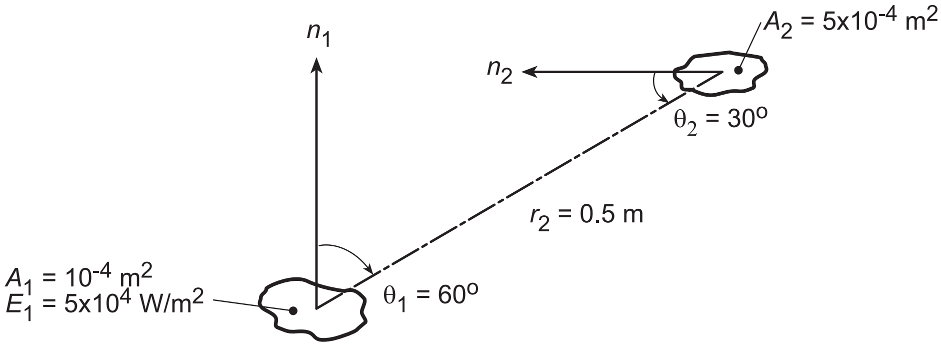
PROBLEM 12.6

KNOWN: A diffuse surface of area $A_1 = 10^{-4} m^2$ emits diffusely with total emissive power $E = 5 \times 10^4$ W/m^2 .

FIND: (a) Rate this emission is intercepted by small surface of area $A_2 = 5 \times 10^{-4}$ m^2 at a prescribed location and orientation, (b) Irradiation G_2 on A_2, and (c) Compute and plot G_2 as a function of the separation distance r_2 for the range $0.25 \le r_2 \le 1.0$ m for zenith angles $\theta_2 = 0$, 30 and 60°.

SCHEMATIC:



ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) A_1 may be approximated as a differential surface area and that $A_2 / r_2^2 << 1$.

ANALYSIS: (a) The rate at which emission from A_1 is intercepted by A_2 follows from Eq. 12.11 written on a total rather than spectral basis.

$$q_{1\to 2} = I_{e,1}(\theta,\phi) A_1 \cos\theta_1 d\omega_{2-1}. \tag{1}$$

Since the surface A_1 is diffuse, it follows from Eq. 12.16 that

$$I_{e,1}(\theta,\phi) = I_{e,1} = E_1/\pi \ . \tag{2}$$

The solid angle subtended by A_2 with respect to A_1 is

$$d\omega_{2-1} \approx A_2 \cdot \cos\theta_2 / r_2^2 \ . \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1) with numerical values gives

$$q_{1\to 2} = \frac{E_1}{\pi} \cdot A_1 \cos\theta_1 \cdot \frac{A_2 \cos\theta_2}{r_2^2} = \frac{5\times 10^4 \text{ W}/\text{m}^2}{\pi \text{ sr}} \times \left(10^{-4} \text{m}^2 \times \cos 60^\circ\right) \times \left[\frac{5\times 10^{-4} \text{m}^2 \times \cos 30^\circ}{(0.5\text{m})^2}\right] \text{sr} \tag{4}$$

$$q_{1\to 2} = 15,915 \text{ W}/\text{m}^2\text{sr} \times \left(5\times 10^{-5} \text{m}^2\right) \times 1.732\times 10^{-3} \text{sr} = 1.378\times 10^{-3} \text{W} . \quad <$$

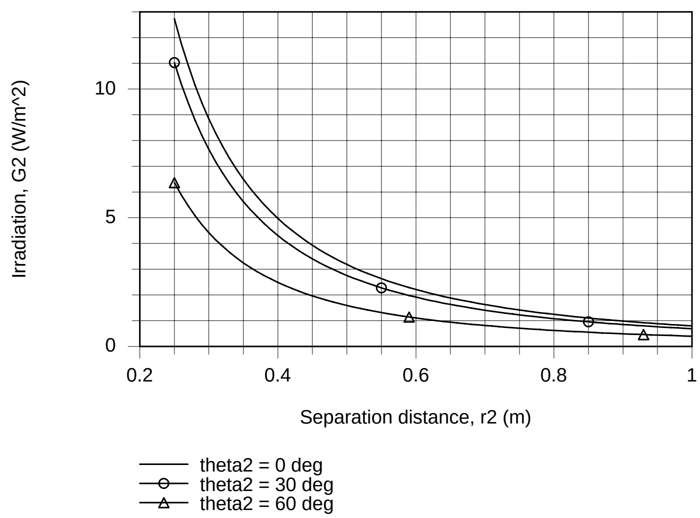
(b) From section 12.3.3, the irradiation is the rate at which radiation is incident upon the surface per unit surface area,

$$G_2 = \frac{q_{1\to 2}}{A_2} = \frac{1.378\times 10^{-3} \text{W}}{5\times 10^{-4} \text{m}^2} = 2.76 \text{ W}/\text{m}^2 \tag{5} \quad <$$

(c) Using the IHT workspace with the foregoing equations, the G_2 was computed as a function of the separation distance for selected zenith angles. The results are plotted below.

Continued...

PROBLEM 12.6 (Cont.)



For all zenith angles, G_2 decreases with increasing separation distance r_2 . From Eq. (3), note that $d\omega_{2-1}$ and, hence G_2, vary inversely as the square of the separation distance. For any fixed separation distance, G_2 is a maximum when $\theta_2 = 0°$ and decreases with increasing θ_2, proportional to cos θ_2.

COMMENTS: (1) For a diffuse surface, the intensity, I_e, is independent of direction and related to the emissive power as $I_e = E/\pi$. Note that π has the units of $[\text{sr}]$ in this relation.

(2) Note that Eq. 12.12 is an important relation for determining the radiant power leaving a surface in a prescribed manner. It has been used here on a total rather than spectral basis.

(3) Returning to part (b) and referring to Figure 12.10, the irradiation on A_2 may be expressed as

$$G_2 = I_{i,2} \cos\theta_2 \frac{A_1 \cos\theta_1}{r_2^2}$$

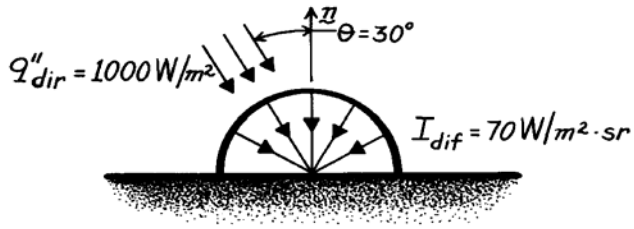
Show that the result is $G_2 = 2.76\ \text{W/m}^2$. Explain how this expression follows from Eq. 12.18.

PROBLEM 12.9

KNOWN: Flux and intensity of direct and diffuse components, respectively, of solar irradiation.

FIND: Total irradiation.

SCHEMATIC:



ANALYSIS: Since the irradiation is based on the actual surface area, the contribution due to the direct solar radiation is

$$G_{dir} = q''_{dir} \cdot \cos\theta.$$

From Eq. 12.17 the contribution due to the diffuse radiation is

$$G_{dif} = \pi I_{dif}.$$

Hence

$$G = G_{dir} + G_{dif} = q''_{dir} \cdot \cos\theta + \pi I_{dif}$$

or

$$G = 1000\,\mathrm{W/m^2} \times 0.866 + \pi\,\mathrm{sr} \times 70\,\mathrm{W/m^2 \cdot sr}$$

$$G = (866 + 220)\,\mathrm{W/m^2}$$

or

$$G = 1086\ \mathrm{W/m^2}. \qquad <$$

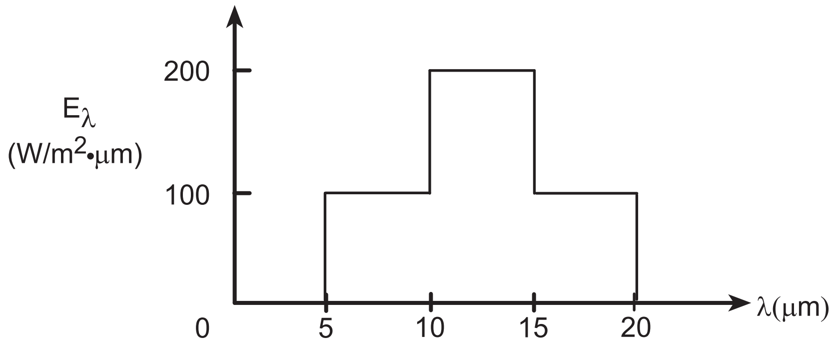
COMMENTS: Although a diffuse approximation is often made for the non-direct component of solar radiation, the actual directional distribution deviates from this condition, providing larger intensities at angles close to the direct beam.

PROBLEM 12.16

KNOWN: Spectral distribution of E_λ for a diffuse surface.

FIND: (a) Total emissive power E, (b) Total intensity associated with directions $\theta = 0^o$ and $\theta = 30^o$, and (c) Fraction of emissive power leaving the surface in directions $\pi/4 \le \theta \le \pi/2$.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse emission.

ANALYSIS: (a) From Eq. 12.14 it follows that

$$E = \int_0^\infty E_\lambda(\lambda)\,d\lambda = \int_0^5 (0)\,d\lambda + \int_5^{10}(100)\,d\lambda + \int_{10}^{15}(200)\,d\lambda + \int_{15}^{20}(100)\,d\lambda + \int_{20}^\infty (0)\,d\lambda$$

$$E = 100\ \text{W/m}^2\cdot\mu\text{m}\ (10 - 5)\ \mu\text{m} + 200\text{W/m}^2\cdot\mu\text{m}\ (15 - 10)\ \mu\text{m} + 100\ \text{W/m}^2\cdot\mu\text{m}\ (20–15)\ \mu\text{m}$$

$$E = 2000\ \text{W/m}^2$$ <

(b) For a diffuse emitter, I_e is *independent* of θ and Eq. 12.17 gives

$$I_e = \frac{E}{\pi} = \frac{2000\ \text{W}/\text{m}^2}{\pi\ \text{sr}}$$

$$I_e = 637\ \text{W}/\text{m}^2\cdot\text{sr}$$ <

(c) Since the surface is diffuse, use Eqs. 12.13 and 12.17,

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_0^{2\pi}\int_{\pi/4}^{\pi/2} I_e \cos\theta \sin\theta\, d\theta\, d\phi}{\pi I_e}$$

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_{\pi/4}^{\pi/2}\cos\theta\sin\theta\,d\theta\int_0^{2\pi} d\phi}{\pi} = \frac{1}{\pi}\left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} \phi\Big|_0^{2\pi}$$

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{1}{\pi}\left[\frac{1}{2}(1^2 - 0.707^2)(2\pi - 0)\right] = 0.50$$ <

COMMENTS: (1) Note how a spectral integration may be performed in parts.

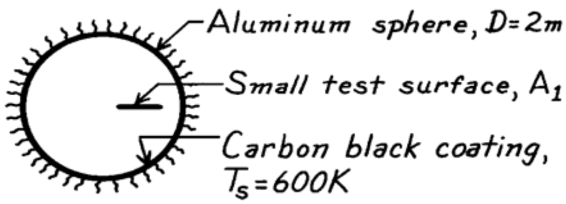
(2) In performing the integration of part (c), recognize the significance of the diffuse emission assumption for which the intensity is uniform in all directions.

PROBLEM 12.22

KNOWN: Evacuated, aluminum sphere (D = 2m) serving as a radiation test chamber.

FIND: Irradiation on a small test object when the inner surface is lined with carbon black and at 600K. What effect will surface coating have?

SCHEMATIC:



ASSUMPTIONS: (1) Sphere walls are isothermal, (2) Test surface area is small compared to the enclosure surface.

ANALYSIS: It follows from the discussion of Section 12.4 that this isothermal sphere is an enclosure behaving as a blackbody. For such a condition, see Fig. 12.11(c), the irradiation on a small surface within the enclosure is equal to the blackbody emissive power at the temperature of the enclosure. That is

$$G_1 = E_b(T_s) = \sigma T_s^4$$

$$G_1 = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(600\mathrm{K})^4 = 7348\,\mathrm{W/m^2}. \qquad <$$

The irradiation is independent of the nature of the enclosure surface coating properties.

COMMENTS: (1) The irradiation depends only upon the enclosure surface temperature and is independent of the enclosure surface properties. (2) Note that the test surface area must be small compared to the enclosure surface area. This allows for inter-reflections to occur such that the radiation field, within the enclosure will be uniform (diffuse) or isotropic. (3) The irradiation level would be the same if the enclosure were not evacuated since, in general, air would be a non-participating medium.

PROBLEM 12.28

KNOWN: Various surface temperatures.

FIND: (a) Wavelength corresponding to maximum emission for each surface, (b) Fraction of solar emission in UV, VIS and IR portions of the spectrum.

ASSUMPTIONS: (1) Spectral distribution of emission from each surface is approximately that of a blackbody, (2) The sun emits as a blackbody at 5800 K.

ANALYSIS: (a) From Wien’s displacement law, Eq. 12.31, the wavelength of maximum emission for blackbody radiation is

$$\lambda_{max} = \frac{C_3}{T} = \frac{2898\ \mu m \cdot K}{T}.$$

For the prescribed surfaces

Surface	Sun (5800K)	Tungsten (2500K)	Hot metal (1500K)	Skin (305K)	Cool metal (60K)	
$\lambda_{max}(\mu m)$	0.50	1.16	1.93	9.50	48.3	<

(b) From Fig. 12.3, the spectral regions associated with each portion of the spectrum are

Spectrum	Wavelength limits, μm
UV	0.01 – 0.4
VIS	0.4 – 0.7
IR	0.7 - 100

For T = 5800K and each of the wavelength limits, from Table 12.1 find:

$\lambda(\mu m)$	10^{-2}	0.4	0.7	10^{2}
$\lambda T(\mu m \cdot K)$	58	2320	4060	5.8×10^{5}
$F_{(0\to\lambda)}$	0	0.125	0.491	1

Hence, the fraction of the solar emission in each portion of the spectrum is:

$F_{UV} = 0.125 - 0 = 0.125$ <

$F_{VIS} = 0.491 - 0.125 = 0.366$ <

$F_{IR} = 1 - 0.491 = 0.509.$ <

COMMENTS: (1) Spectral concentration of surface radiation depends strongly on surface temperature. (2) Much of the UV solar radiation is absorbed in the earth’s atmosphere.