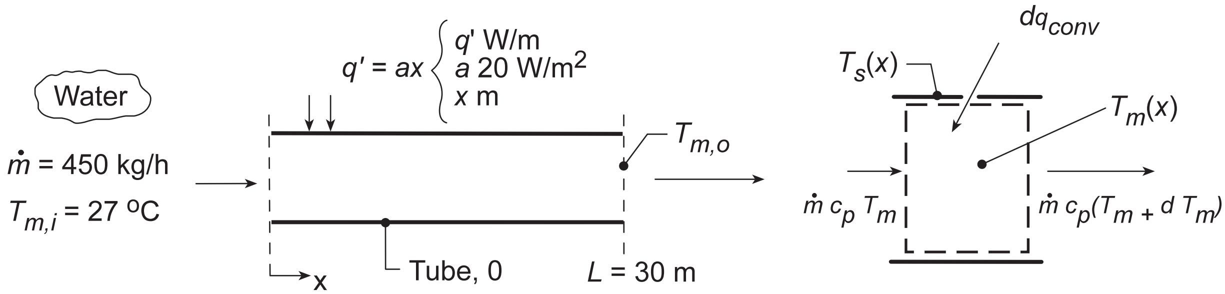
# PROBLEM 8.12

**KNOWN:** Internal flow with prescribed wall heat flux as a function of distance.

**FIND:** (a) Beginning with a properly defined differential control volume, the temperature distribution, $T_m(x)$, (b) Outlet temperature, $T_{m,o}$, (c) Sketch $T_m(x)$, and $T_s(x)$ for fully developed *and* developing flow conditions, and (d) Value of uniform wall flux $q_s''$ (instead of $q_s' = ax$) providing same outlet temperature as found in part (a); sketch $T_m(x)$ and $T_s(x)$ for this heating condition.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant properties, (3) Incompressible liquid with negligible viscous dissipation.

**PROPERTIES:** *Table A.6*, Water (300 K): $c_p$ = 4.179 kJ/kg·K.

**ANALYSIS:** (a) Applying energy conservation to the control volume above,

$$dq_{conv} = \dot{m}c_p dT_m \tag{1}$$

where $T_m(x)$ is the mean temperature at any cross-section and $dq_{conv} = q' \cdot dx$. Hence,

$$ax = \dot{m}c_p \frac{dT_m}{dx}. \tag{2}$$

Separating and integrating with proper limits gives

$$a\int_{x=0}^{x} x dx = \dot{m}c_p \int_{T_{m,i}}^{T_m(x)} dT_m \qquad\qquad T_m(x) = T_{m,i} + \frac{ax^2}{2\dot{m}c_p} \tag{3,4}$$ <

(b) To find the outlet temperature, let x = L, then

$$T_m(L) = T_{m,o} = T_{m,i} + aL^2\big/2\dot{m}c_p\,. \tag{5}$$

Solving for $T_{m,o}$, we find

$$T_{m,o} = 27^\circ C + \frac{20\,W/m^2\left(30\,m^2\right)}{2\left(450\,kg/h/\left(3600\,s/h\right)\right)\times 4179\,J/kg\cdot K} = 27^\circ C + 17.2^\circ C = 44.2^\circ C\,.$$ <

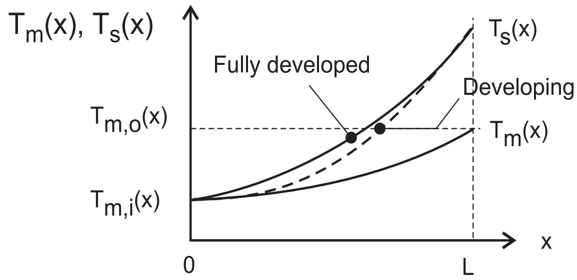
(c) For *linear wall heating*, $q_s' = ax$, the fluid temperature distribution along the length of the tube is quadratic as prescribed by Eq. (4). From the convection rate equation,

$$q_s' = h(x)\cdot \pi D\left(T_s(x) - T_m(x)\right) \tag{6}$$

For fully developed flow conditions, h(x) = h is a constant; hence, $T_s(x)$ - $T_m(x)$ increases linearly with x. For developing conditions, h(x) will decrease with increasing distance along the tube eventually achieving the fully developed value.

Continued...

## PROBLEM 8.12 (Cont.)



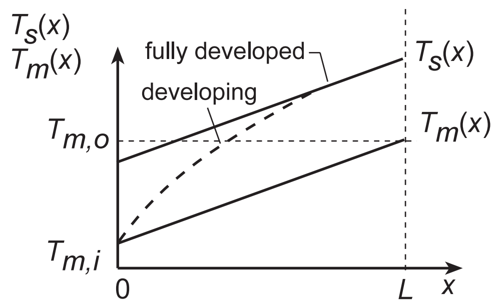

(d) For *uniform wall heat flux heating*, the overall energy balance on the tube yields

$$q = q_s'' \pi DL = \dot{m} c_p \left( T_{m,o} - T_{m,i} \right)$$

Requiring that $T_{m,o}$ = 44.2°C from part (a), find

$$q_s'' = \frac{(450/3600)\,\text{kg/s} \times 4179\,\text{J/kg}\cdot\text{K}\,(44.2 - 27)\,\text{K}}{\pi D \times 30\,\text{m}} = 95.3/D\ \text{W/m}^2 \qquad <$$

where D is the diameter (m) of the tube which, when specified, would permit determining the required heat flux, $q_s''$. For uniform heating, Section 8.3.2, we know that $T_m(x)$ will be linear with distance. $T_s(x)$ will also be linear for fully developed conditions and appear as shown below when the flow is developing.




**COMMENTS:** (1) Note that $c_p$ should be evaluated at $T_m$ = (27 + 44)°C/2 = 309 K.

(2) Why did we show $T_s(0) = T_m(0)$ for both types of history when the flow was developing?

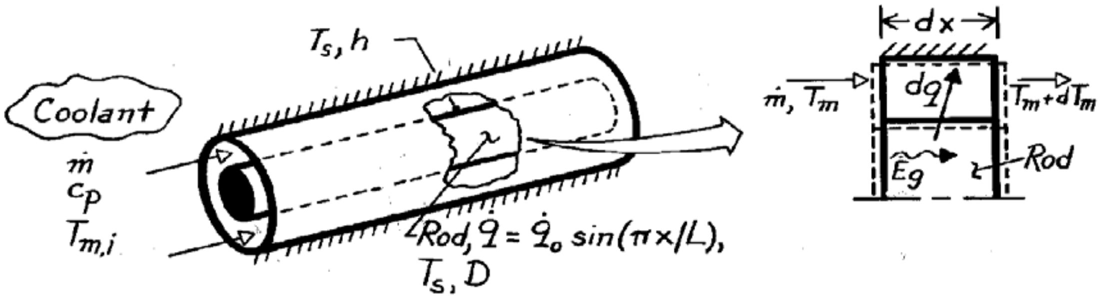
(3) Why must $T_m(x)$ be linear with distance in the case of uniform wall flux heating?

# PROBLEM 8.14

**KNOWN:** Geometry and coolant flow conditions associated with a nuclear fuel rod. Axial variation of heat generation within the rod.

**FIND:** (a) Axial variation of local heat flux and total heat transfer rate, (b) Axial variation of mean coolant temperature, (c) Axial variation of rod surface temperature and location of maximum temperature.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant fluid properties, (3) Uniform surface convection coefficient, (4) Negligible axial conduction in rod and fluid, (5) Incompressible liquid with negligible viscous dissipation, (6) Outer surface is adiabatic.

**ANALYSIS:** (a) Performing an energy balance for a control volume about the rod,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad -dq + \dot{E}_g = 0$$

or

$$-q''(\pi D\,dx) + \dot{q}_o \sin(\pi x/L)\left(\pi D^2/4\right) dx = 0 \qquad q'' = \dot{q}_o (D/4) \sin(\pi x/L). \qquad <$$

The total heat transfer rate is then

$$q = \int_0^L q''\, \pi D\, dx = \left(\pi D^2/4\right)\dot{q}_o \int_0^L \sin(\pi x/L)\, dx$$

$$q = \frac{\pi D^2}{4}\dot{q}_o \left(-\frac{L}{\pi}\cos\frac{\pi x}{L}\right)\Big|_0^L = \frac{D^2 \dot{q}_o L}{4}(1+1)$$

$$q = \frac{D^2 L}{2}\dot{q}_o. \qquad (1) \quad <$$

(b) Performing an energy balance for a control volume about the coolant,

$$\dot{m}\, c_p\, T_m + dq = \dot{m}\, c_p \left(T_m + dT_m\right) = 0.$$

Hence

$$\dot{m}\, c_p\, d\,T_m = dq = (\pi D\, dx)\, q''$$

$$\frac{d\,T_m}{dx} = \frac{\pi D}{\dot{m}\, c_p}\frac{\dot{q}_o D}{4}\sin\left(\frac{\pi x}{L}\right).$$

Continued …

## PROBLEM 8.14 (Cont.)

Integrating,

$$T_m(x) - T_{m,i} = \frac{\pi D^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \int_0^x \sin \frac{\pi x}{L} dx$$

$$T_m(x) = T_{m,i} + \frac{L D^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \left[1 - \cos \frac{\pi x}{L}\right] \qquad (2)$$ <

(c) From Newton's law of cooling,

$$q'' = h\left(T_s - T_m\right).$$

Hence

$$T_s = \frac{q''}{h} + T_m$$

$$T_s = \frac{\dot{q}_o D}{4h} \sin \frac{\pi x}{L} + T_{m,i} + \frac{LD^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \left[1 - \cos \frac{\pi x}{L}\right].$$ <

To determine the location of the maximum surface temperature, evaluate

$$\frac{d T_s}{dx} = 0 = \frac{\dot{q}_o D\pi}{4hL} \cos \frac{\pi x}{L} + \frac{LD^2}{4} \frac{\dot{q}_o}{\dot{m} c_p} \frac{\pi}{L} \sin \frac{\pi x}{L}$$

or

$$\frac{1}{hL} \cos \frac{\pi x}{L} + \frac{D}{\dot{m} c_p} \sin \frac{\pi x}{L} = 0.$$

Hence

$$\tan \frac{\pi x}{L} = -\frac{\dot{m} c_p}{D h L}$$

$$x = \frac{L}{\pi} \tan^{-1}\left(-\frac{\dot{m} c_p}{D h L}\right) = x_{max}.$$ <

**COMMENTS:** Note from Eq. (2) that

$$T_{m,o} = T_m(L) = T_{m,i} + \frac{L D^2 \dot{q}_o}{2 \dot{m} c_p}$$

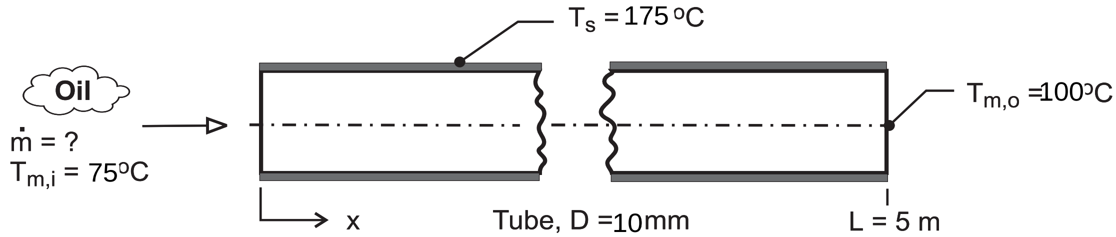
which is equivalent to the result obtained by combining Eq. (1) and Eq. 8.34.

## PROBLEM 8.28

**KNOWN:** Oil at 75°C enters a single-tube preheater of 10-mm diameter and 5-m length; tube surface maintained at 175°C by swirling combustion gases.

**FIND:** Determine the flow rate and heat transfer rate when the outlet temperature is 100°C.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Laminar flow, (2) Tube wall is isothermal, (3) Incompressible liquid with negligible viscous dissipation, (4) Constant properties.

**PROPERTIES:** *Table A-5*, Engine oil, new ($T_m = (T_{m,i} + T_{m,o})/2$ = 361 K): $\rho$ = 847.5 kg/m$^3$, $c_p$ = 2163 J/kg·K, $\nu = 2.931 \times 10^{-5}$ m$^2$/s, k = 0.1379 W/m·K, Pr = 390.2, $\mu$ = 0.0245.

**ANALYSIS:** The overall energy balance, Eq. 8.34, and rate equation, Eq. 8.41b, are

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) \tag{1}$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m} c_p}\right) \tag{2}$$

Not knowing the flow rate $\dot{m}$, the Reynolds number cannot be calculated. Assume that the flow is laminar. Since Pr > 5, the average convection coefficient can be estimated using the Hausen correlation, Eq. 8.57, with Eq. 8.56 for the Graetz number:

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)Re_D Pr}{1 + 0.04\left[(D/L)Re_D Pr\right]^{2/3}} \tag{3}$$

where all properties are evaluated at $T_m = (T_{m,i} + T_{m,o})/2$. The Reynolds number follows from Eq. 8.6,

$$Re_D = 4\dot{m}/\pi D\mu \tag{4}$$

A tedious trial-and-error solution is avoided by using *IHT* to solve the system of equations with the following result:

| $Re_D$ | $\overline{Nu}_D$ | $\bar{h}_D$ (W/m$^2$·K) | q(W) | $\dot{m}$(kg/h) | |
|---|---|---|---|---|---|
| 130 | 7.25 | 100 | 1360 | 90 | < |

Note that the flow is laminar, and evaluating $x_{fd,t}$ using Eq. 8.23, find $x_{fd,t}$ = 25 m, so the flow is not thermally fully developed.

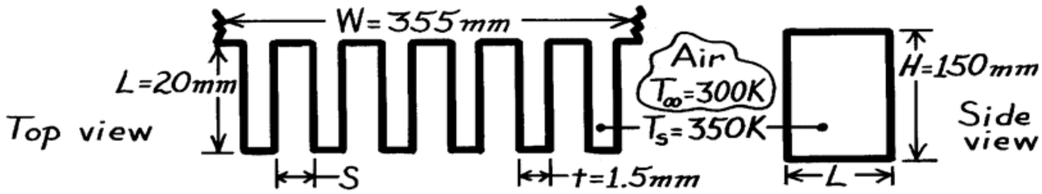
**COMMENT:** Use of the Baehr and Stephan correlation for the combined entry problem yields the identical values. Hence it may also be used.

# PROBLEM 9.9

**KNOWN:** Dimensions of vertical rectangular fins. Temperature of fins and quiescent air.

**FIND:** (a) Optimum fin spacing, (b) Rate of heat transfer from an array of fins at the optimal spacing.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Fins are isothermal, (2) Radiation effects are negligible, (3) Air is quiescent.

**PROPERTIES:** *Table A-4*, Air ($T_f$ = 325K, 1 atm): $\nu = 18.41 \times 10^{-6}\ m^2/s$, k = 0.0282 W/m·K, Pr = 0.703.

**ANALYSIS:** (a) If fins are too close, boundary layers on adjoining surfaces will coalesce and heat transfer will decrease. If fins are too far apart, the surface area becomes too small and heat transfer decreases. $S_{op} \approx \delta_{x=H}$. From Fig. 9.4, the edge of boundary layer corresponds to

$$\eta = (\delta / H)\ (Gr_H / 4)^{1/4} \approx 5.$$

Hence, $$Gr_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{9.8\ m/s^2 (1/325K)\ 50K\ (0.15m)^3}{\left(18.41\times10^{-6} m^2/s\right)^2} = 1.5\times10^7$$

$$\delta(H) = 5(0.15m)/\left(1.5\times10^7/4\right)^{1/4} = 0.017m = 17mm \qquad S_{op} \approx 34mm.$$ <

(b) The number of fins N can be found as

$$N = W/\left(S_{op} + t\right) = 355/35.5 = 10$$

and the rate is $q = 2\ N\ \bar{h}(H \cdot L)\ (T_s - T_\infty)$.

For laminar flow conditions

$$\overline{Nu}_H = 0.68 + 0.67\ Ra_L^{1/4} / \left[1 + (0.492/Pr)^{9/16}\right]^{4/9}$$

$$\overline{Nu}_H = 0.68 + 0.67\left(1.5\times10^7 \times 0.703\right)^{1/4} / \left[1 + (0.492/0.703)^{9/16}\right]^{4/9} = 30$$

$$\bar{h} = k\ Nu_H / H = 0.0282\ W/m\cdot K(30)/0.15\ m = 5.6\ W/m^2\cdot K$$

$$q = 2(10)5.6\ W/m^2\cdot K(0.15m \times 0.02m)\ (350 - 300)K = 16.8\ W.$$ <

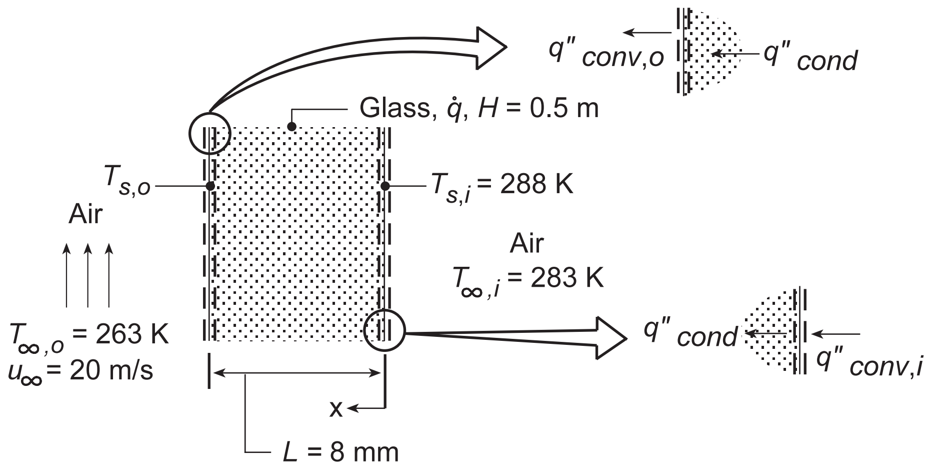
**COMMENTS:** Part (a) result is a conservative estimate of the optimum spacing. The increase in area resulting from a further reduction in S would more than compensate for the effect of fluid entrapment due to boundary layer merger. From a more rigorous treatment (see Section 9.7.1), $S_{op} \approx 10$ mm is obtained for the prescribed conditions.

# PROBLEM 9.27

**KNOWN:** Boundary conditions associated with a rear window experiencing uniform volumetric heating.

**FIND:** (a) Volumetric heating rate $\dot{q}$ needed to maintain inner surface temperature at $T_{s,i}$ = 15°C, (b) Effects of $T_{\infty,o}$, $u_\infty$, and $T_{\infty,i}$ on $\dot{q}$ and $T_{s,o}$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, one-dimensional conditions, (2) Constant properties, (3) Uniform volumetric heating in window, (4) Convection heat transfer from interior surface of window to interior air may be approximated as free convection from a vertical plate, (5) Heat transfer from outer surface is due to forced convection over a flat plate in parallel flow.

**PROPERTIES:** *Table A.3*, Glass (300 K): k = 1.4 W/m·K: *Table A.4*, Air ($T_{f,i}$ = 12.5°C, 1 atm): $\nu$ = $14.6 \times 10^{-6}$ m$^2$/s, k = 0.0251 W/m·K, $\alpha$ = $20.59 \times 10^{-6}$ m$^2$/s, $\beta$ = (1/285.5) = $3.503 \times 10^{-3}$ K$^{-1}$, Pr = 0.711; ($T_{f,o} \approx$ 0°C): $\nu$ = $13.49 \times 10^{-6}$ m$^2$/s, k = 0.0241 W/m·K, Pr = 0.714.

**ANALYSIS:** (a) The temperature distribution in the glass is governed by the appropriate form of the heat equation, Eq. 3.44, whose general solution is given by Eq. 3.45.

$$T(x) = -(\dot{q}/2k)x^2 + C_1 x + C_2 .$$

The constants of integration may be evaluated by applying appropriate boundary conditions at x = 0. In particular, with T(0) = $T_{s,i}$, $C_2 = T_{s,i}$. Applying an energy balance to the inner surface, $q''_{cond} = q''_{conv,i}$

$$-k\frac{dT}{dx}\bigg|_{x=0} = \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) \qquad -k\left(-\frac{\dot{q}}{k}x + C_1\right)\bigg|_{x=0} = \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)$$

$$C_1 = -\left(\bar{h}_i/k\right)\left(T_{\infty,i} - T_{s,i}\right)$$

$$T(x) = -(\dot{q}/2k)x^2 - \frac{\bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)}{k}x + T_{s,i} \tag{1}$$

The required generation may then be obtained by formulating an energy balance at the outer surface, where $q''_{cond} = q''_{conv,o}$. Using Eq. (1),

$$-k\frac{dT}{dx}\bigg|_{x=L} = \bar{h}_o\left(T_{s,o} - T_{\infty,o}\right) \tag{2}$$

Continued...

$$-k\left.\frac{dT}{dx}\right|_{x=L} = -k\left(-\frac{\dot{q}L}{k}\right) + \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) = \dot{q}L + \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) \tag{3}$$

Substituting Eq. (3) into Eq. (2), the energy balance becomes

$$\dot{q}L = \bar{h}_o\left(T_{s,o} - T_{\infty,o}\right) + \bar{h}_i\left(T_{s,i} - T_{\infty,i}\right) \tag{4}$$

where $T_{s,o}$ may be evaluated by applying Eq. (1) at x = L.

$$T_{s,o} = -\frac{\dot{q}L^2}{2k} - \frac{\bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)}{k}L + T_{s,i}\,. \tag{5}$$

The *inside* convection coefficient may be obtained from Eq. 9.26. With

$$Ra_H = \frac{g\beta\left(T_{s,i} - T_{\infty,i}\right)H^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\left(3.503\times10^{-3}\,\text{K}^{-1}\right)(15-10)\,\text{K}\,(0.5\,\text{m})^3}{14.60\times10^{-6}\,\text{m}^2/\text{s}\times20.59\times10^{-6}\,\text{m}^2/\text{s}} = 7.137\times10^7\,,$$

$$\overline{Nu}_H = \left[0.825 + \frac{0.387Ra_H^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right]^2 = \left[0.825 + \frac{0.387\left(7.137\times10^7\right)^{1/6}}{\left[1+(0.492/0.711)^{9/16}\right]^{8/27}}\right]^2 = 55.2$$

$$\bar{h}_i = \overline{Nu}_H\frac{k}{H} = \frac{55.2\times0.0251\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}} = 2.77\,\text{W/m}^2\cdot\text{K}$$

The *outside* convection coefficient may be obtained by first evaluating the Reynolds number. With

$$Re_H = \frac{u_\infty H}{\nu} = \frac{20\,\text{m/s}\times0.5\,\text{m}}{13.49\times10^{-6}\,\text{m}^2/\text{s}} = 7.413\times10^5$$

and with $Re_{x,c} = 5\times10^5$, mixed boundary layer conditions exist. Hence,

$$\overline{Nu}_H = \left(0.037\,Re_H^{4/5} - 871\right)Pr^{1/3} = \left[0.037\left(7.413\times10^5\right)^{4/5} - 871\right](0.714)^{1/3} = 864$$

$$\bar{h}_o = \overline{Nu}_H\,(k/H) = \left(864\times0.0241\,\text{W/m}\cdot\text{K}\right)/0.5\,\text{m} = 41.6\,\text{W/m}^2\cdot\text{K}\,.$$

Eq. (5) may now be expressed as

$$T_{s,o} = -\frac{\dot{q}\,(0.008\,\text{m})^2}{2\left(1.4\,\text{W/m}\cdot\text{K}\right)} - \frac{2.77\,\text{W/m}^2\cdot\text{K}\,(10-15)\,\text{K}}{1.4\,\text{W/m}\cdot\text{K}}\times0.008\,\text{m} + 288\,\text{K} = -2.286\times10^{-5}\,\dot{q} + 288.1\,\text{K}$$

or, solving for $\dot{q}$,

$$\dot{q} = -43{,}745\left(T_{s,o} - 288.1\right) \tag{6}$$

and substituting into Eq. (4),

$$-43{,}745\left(T_{s,o} - 288.1\right)(0.008\,\text{m}) = 41.6\,\text{W/m}^2\cdot\text{K}\left(T_{s,o} - 263\,\text{K}\right) + 2.77\,\text{W/m}^2\cdot\text{K}\left(288\,\text{K} - 283\,\text{K}\right).$$

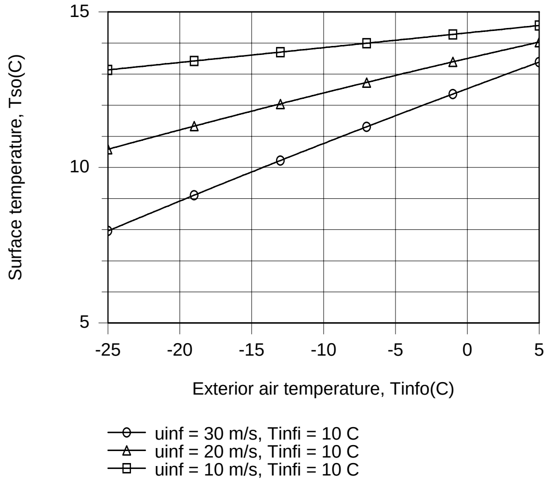
It follows that $T_{s,o}$ = 285.4 K in which case, from Eq. (6)

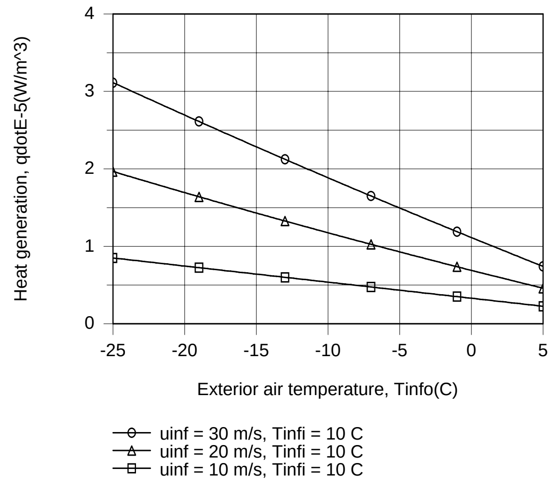
$$\dot{q} = 118\,\text{kW/m}^3\,.$$ <

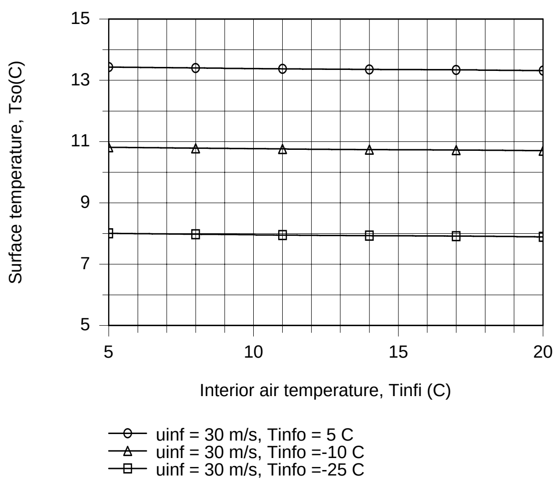
(b) The parametric calculations were performed using the *One-Dimensional*, *Steady-state Conduction* Model of IHT with the appropriate *Correlations* and *Properties* Tool Pads, and the results are as follows.

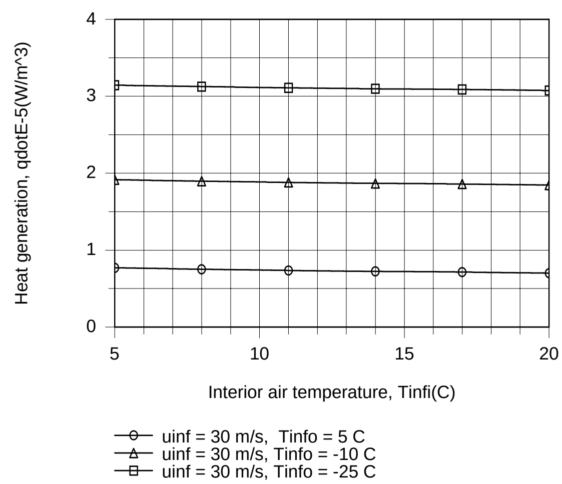
Continued...

## PROBLEM 9.27 (Cont.)




For fixed $T_{s,i}$ and $T_{\infty,i}$, $T_{s,o}$ and $\dot{q}$ are strongly influenced by $T_{\infty,o}$ and $u_\infty$, increasing and decreasing, respectively, with increasing $T_{\infty,o}$ and decreasing and increasing, respectively with increasing $u_\infty$. For fixed $T_{s,i}$ and $u_\infty$, $T_{s,o}$ and $\dot{q}$ are independent of $T_{\infty,i}$, but increase and decrease, respectively, with increasing $T_{\infty,o}$.

**COMMENTS:** In lieu of performing a surface energy balance at x = L, Eq. (4) may also be obtained by applying an energy balance to a control volume about the entire window.