G.W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology

ME 3322A: Thermodynamics: Fall 2014
Homework Set # 14
(No due date, students are required to solve the problems and compare their solutions with the posted solutions.)

	Problem # in Textbook		Answer
	7^{th} Ed.	7^{th} Ed.	
1	9.61	9.61	b) 43.15 kg/s; c) 30,208 kW; d) 33.1%
2	9.63 Parts a-c	9.63 Parts a-c	a) 4.904×10^4 kW; c) 63.1%
3	9.67	9.68	a) 31%; b) 3.76 kg/s
4	9.80	9.80	a) 15.76 kg/s; b) 14058 kW

9.61 The cycle of Problem 9.60 is modified to include a regenerator with an effectiveness of 70%. Determine

(a) the specific enthalpy, in kJ.kg, and the temperature, in K, for each stream exiting the regenerator and sketch the *T-s* diagram.

(b) the mass flow rate of air, in kg/s.

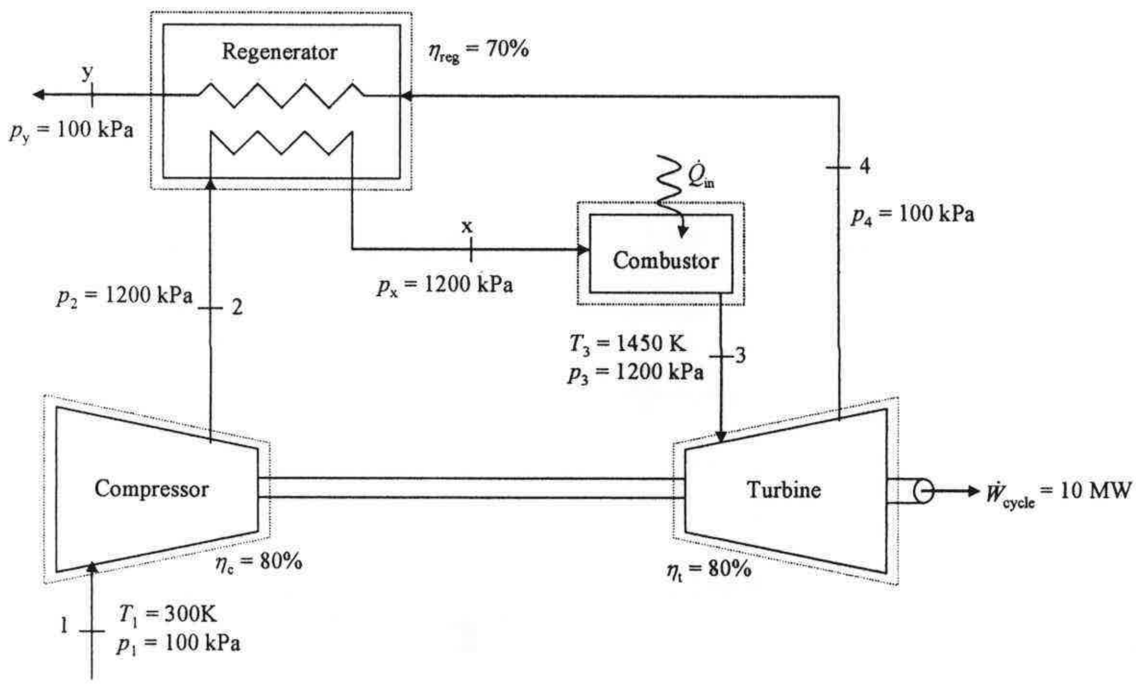
(c) the rate of heat transfer, in kW, to the working fluid passing through the combustor.

(d) the thermal efficiency.

KNOWN: An air-standard regenerative Brayton cycle operates with known states at the turbine and compressor inlets, known compressor and turbine isentropic efficiencies, and known regenerator effectiveness. The net power output of the cycle is given.

FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the combustor, and the thermal efficiency. Provide a summary table of pressure, temperature, and enthalpy for each state.

SCHEMATIC AND GIVEN DATA:

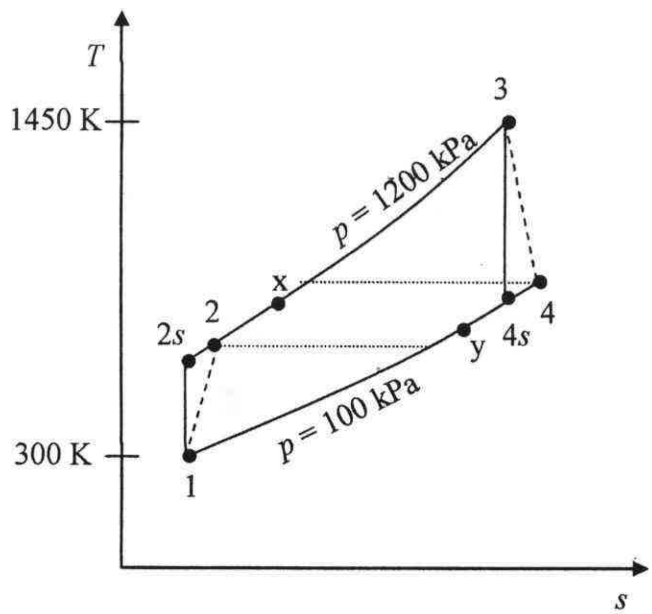


ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and compressor operate adiabatically.
3. There are no pressure drops for flow through the regenerator and combustor.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.

Problem 9.61 (Continued) – Page 2

ANALYSIS: (a) The *T*-*s* diagram for the cycle is shown below.



For the cycle with a regenerator effectiveness of less than 100%, all principal states are unchanged from Problem 9.60 except for states *x* and *y*.

State x can be determined using the regenerator effectiveness

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2}$$

Solving for h_x and inserting values

$$h_x = h_2 + \eta_{reg}(h_4 - h_2) = 688.27 \text{ kJ/kg} + (0.70)(955.74 \text{ kJ/kg} - 688.27 \text{ kJ/kg}) = 875.50 \text{ kJ/kg}$$

Interpolating in Table A-22, $T_x \approx 848.5$ K.

The specific enthalpy at state y can be determined from a mass and energy balances for a control volume enclosing the regenerator. With $\dot{Q} = 0$ and $\dot{W} = 0$

$$0 = \dot{m}\left(h_2 + h_4 - h_x - h_y\right)$$

Solving for h_y and inserting values

$$h_y = h_2 + h_4 - h_x = 688.27 \text{ kJ/kg} + 955.74 \text{ kJ/kg} - 875.50 \text{ kJ/kg} = 768.51 \text{ kJ/kg}$$

From Table A-22, $T_y \approx 751.1$ K.

Problem 9.61 (Continued) – Page 3

In summary

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2s	1200	603.5	610.65
2	1200	676.7	688.27
x	1200	848.5	875.50
3	1200	1450	1575.57
4s	100	780.7	800.78
4	100	920.3	955.74
y	100	751.1	768.51

(b) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and compressor give

$$\dot{W}_t = \dot{m}(h_3 - h_4) \quad \text{and} \quad \dot{W}_c = \dot{m}(h_2 - h_1)$$

The net power of the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{[(h_3 - h_4) - (h_2 - h_1)]}$$

Inserting values

$$\dot{m} = \frac{10{,}000\ \text{kW}}{\left(1575.57\dfrac{\text{kJ}}{\text{kg}} - 955.74\dfrac{\text{kJ}}{\text{kg}}\right) - \left(688.27\dfrac{\text{kJ}}{\text{kg}} - 300.19\dfrac{\text{kJ}}{\text{kg}}\right)}\left|\frac{\dfrac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = \underline{\mathbf{43.15\ kg/s}}$$

(c) The rate of heat transfer to the working fluid passing through the combustor can be determined by applying mass and energy balances to a control volume around the combustor to give

$$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_x)$$

$$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_x) = \left(43.15\frac{\text{kg}}{\text{s}}\right)\left(1575.57\frac{\text{kJ}}{\text{kg}} - 875.50\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{kW}}{\dfrac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{30{,}208\ kW}}$$

(d) The thermal efficiency is

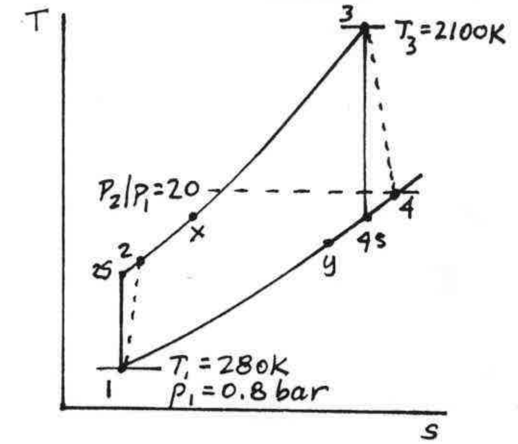
$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (10{,}000\ \text{kW})/(30{,}208\ \text{kW}) = \underline{\mathbf{0.331\ (33.1\%)}}$$

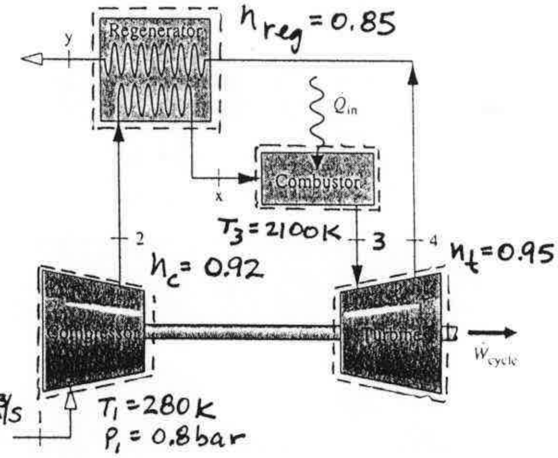
PROBLEM 9.63

KNOWN: Air enters the compressor of a regenerative air-standard Brayton cycle at a specified state and a given volumetric flow rate. The compressor pressure ratio and maximum cycle temperature are known. The compressor and turbine isentropic efficiencies and the regenerator effectiveness are known.

FIND: Determine (a) the net power, (b) the rate of heat addition, and (c) the thermal efficiency. Plot these quantities vs. regenerator effectiveness ranging from 0 to 100%. Discuss.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.7. Also, $\eta_c = 0.92$, $\eta_t = 0.95$, and $\eta_{reg} = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = 280K \Rightarrow h_1 = 280.13$ kJ/kg, $p_{r1} = 1.0889$

State 2: $p_{r2} = (p_2/p_1)\, p_{r1} = (20)(1.0889) = 21.778 \Rightarrow T_{2s} = 649.3K$, $h_{2s} = 659.13 \frac{kJ}{kg}$

Using the isentropic compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 280.13 + (659.13 - 280.13)/(0.92) = 692.09\ kJ/kg$$

State 3: $T_3 = 2100K$; $h_3 = 2377.4$ kJ/kg, $p_{r3} = 2559$

State 4: $p_{r4} = (p_4/p_3)\, p_{r3} = (\frac{1}{20})(2559) = 127.95 \Rightarrow T_{4s} = 1029.2K$, $h_{4s} = 1079.4 \frac{kJ}{kg}$

Using the isentropic turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 2377.4 - (0.95)(2377.4 - 1079.4) = 1144.3\ kJ/kg$$

State x: From the regenerator effectiveness; $\eta_{reg} = (h_x - h_2)/(h_4 - h_2)$

$$h_x = h_2 + \eta_{reg}(h_4 - h_2) = 692.09 + (0.85)(1144.3 - 692.09) = 1076.5\ \frac{kJ}{kg}$$

Now, determine the mass flow rate.

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1\, p_1}{R\,T_1} = \frac{(60\ m^3/s)(0.8\ bar)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(280K)} \left|\frac{10^5\ N/m^2}{1\ bar}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 59.73\ kg/s$$

(a) $\dot{W}_t = \dot{m}(h_3 - h_4) = (59.73\ \frac{kg}{s})(2377.4 - 1144.3)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right| = 7.365 \times 10^4\ kW$

$\dot{W}_c = \dot{m}(h_2 - h_1) = (59.73)(692.09 - 280.13) = 2.461 \times 10^4\ kW$

$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 4.904 \times 10^4\ kW$ ← $\dot{W}_{cycle}$

(b) $\dot{Q}_{in} = \dot{m}(h_3 - h_x) = (59.73)(2377.4 - 1076.5) = 7.770 \times 10^4\ kW$ ← $\dot{Q}_{in}$

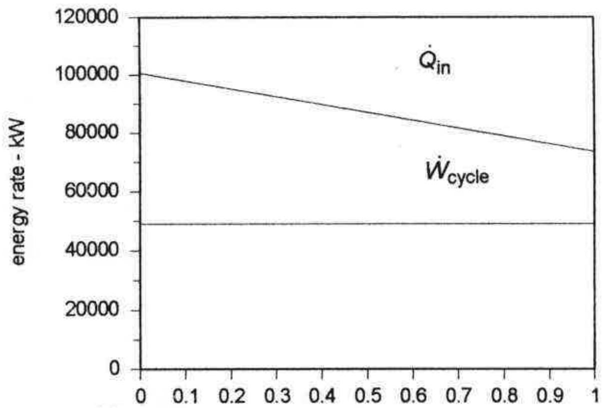
(c) The thermal efficiency is $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 0.631$ (63.1%) ← η

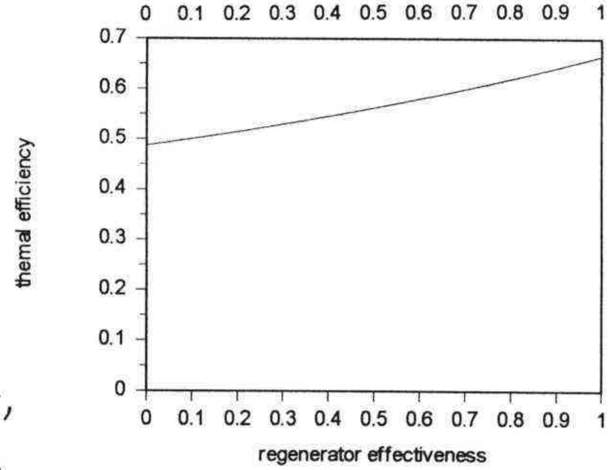
IT Code

```
T1 = 280  // K
p1 = 0.8  // bar
r = 20
p2 = r * p1
p3 = p2
T3 = 2100  // K
p4 = p3 / r
etac = 0.92
etat = 0.95
etareg = 0.85
AV1 = 60  // m³/s

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s1
s2s = s_hP("Air",h2s,p2)
h2 = h1 + (h2s - h1) / etac
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s3
s4s = s_hP("Air",h4s,p4)
h4 = h3 - etat * (h3 - h4s)
hx = h2 + etareg * (h4 - h2)

v1 = v_TP("Air",T1,p1)
mdot = AV1 / v1
Wdott = mdot * (h3 - h4)
Wdotc = mdot * (h2 - h1)
Wdotcycle = Wdott - Wdotc
Qdotin = mdot * (h3 - hx)
eta = Wdotcycle / Qdotin
```

IT Results for $\eta_{reg} = 0.85$, $\eta_c = 0.92$, and $\eta_t = 0.95$

$\dot{m}$ = 59.73 kg/s
$\dot{Q}_{in}$ = 7.768 x 10^4 kW
$\dot{W}_{cycle}$ = 4.903 x 10^4 kW
η = 0.6312
h_1 = 280 kJ/kg
h_2 = 691.9 kJ/kg
h_3 = 2376 kJ/kg
h_4 = 1143 kJ/kg
h_x = 1075 kJ/kg





Discussion

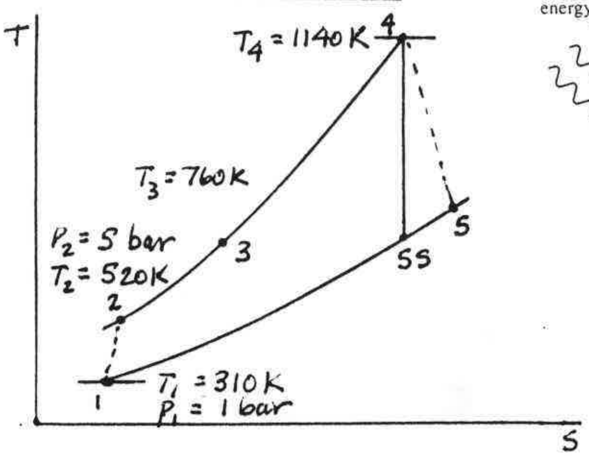
The states at the inlet and exit of the compressor and turbine, respectively, are not changed as the regenerator effectiveness changes. Hence, the power is constant. However, as the regenerator effectiveness increases, the specific enthalpy h_x increases and less heat addition is required in the combustor (external heat addition). The result is a substantial increase in the thermal efficiency.

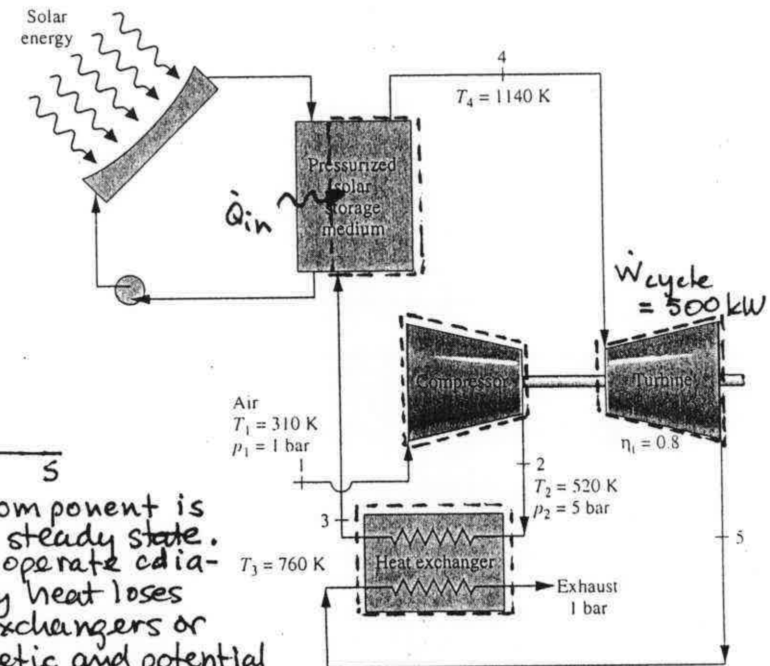
PROBLEM 9.67

KNOWN: Data are known for a regenerative gas turbine power plant using solar energy a the source of heat addition.

FIND: Determine (a) the thermal efficiency, and (b) the mass flow rate for a net power output of 500 kW.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is modeled as a control volume at steady state. (2) The compressor and turbine operate adiabatically. (3) There are no stray heat loses or pressure drops in the heat exchangers or interconnecting piping. (4) Kinetic and potential energy effects are negligible. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: For states 1-4, the temperatures are known. Thus

$$T_1 = 310\text{K} \Rightarrow h_1 = 310.24 \text{ kJ/kg}$$
$$T_2 = 520\text{K} \Rightarrow h_2 = 523.63 \text{ kJ/kg}$$
$$T_3 = 760\text{K} \Rightarrow h_3 = 778.18 \text{ kJ/kg}$$
$$T_4 = 1140\text{K} \Rightarrow h_4 = 1207.57 \text{ kJ/kg}$$

State 5: $p_{r5} = (p_5/p_4)\, p_{r4} = (1/5)(193.1) = 38.62 \Rightarrow h_{5s} = 774.49$ kJ/kg

Using $\eta_t = (h_4 - h_5)/(h_4 - h_{5s}) \Rightarrow h_5 = h_4 - \eta_t(h_4 - h_{5s})$
$= 861.11$ kJ/kg

(a) The thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_4-h_5)-(h_2-h_1)}{(h_4-h_3)} = \frac{(1207.57-861.11)-(523.63-310.24)}{(1207.57-778.18)}$$
$$= \frac{133.07}{429.39} = 0.31\ (31\%)$$

η

(b) For $\dot{W}_{cycle} = 500$ kW

$$\dot{m} = \frac{\dot{W}_{cycle}}{(h_4-h_5)-(h_2-h_1)} = \frac{500 \text{ kW}}{133.07 \text{ kJ/kg}}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right|$$
$$= 3.76 \text{ kg/s}$$

$\dot{m}$

9.80 An air-standard regenerative Brayton cycle operating at steady state with intercooling and reheat produces 10 MW of power. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.19. Sketch the *T-s* diagram for the cycle and determine

(a) the mass flow rate of air, in kg/s.

(b) the rate of heat transfer, in kW, to the working fluid passing through each combustor.

(c) the thermal efficiency.

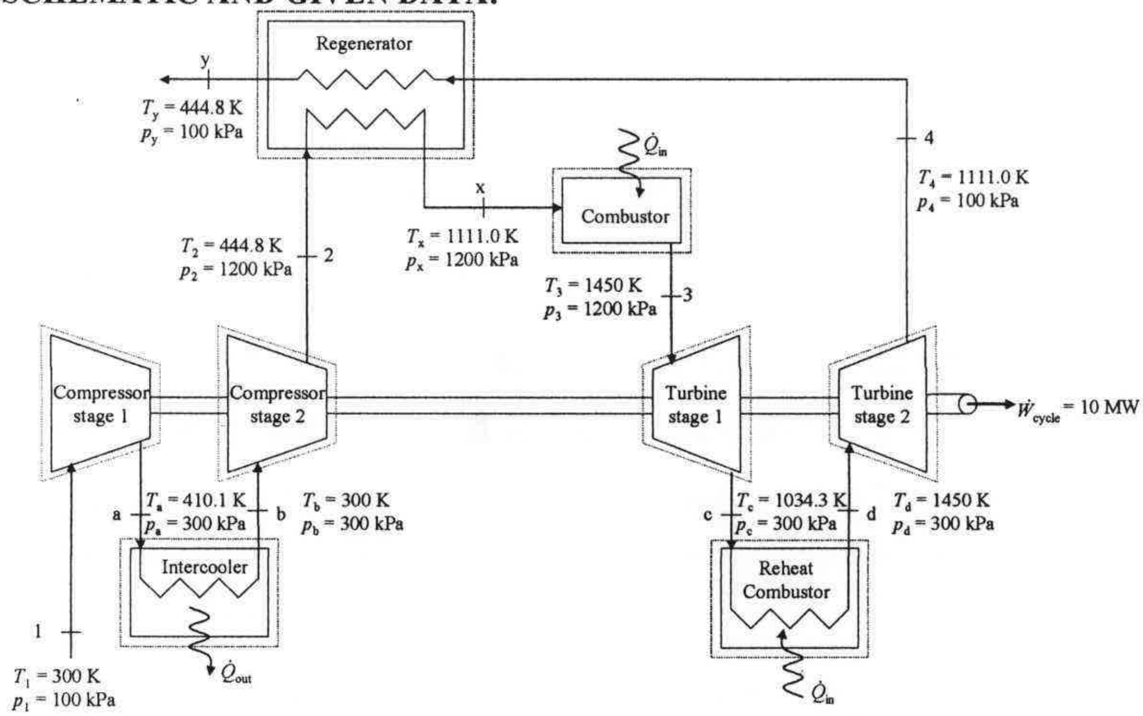
State	*p* (kPa)	*T* (K)	*h* (kJ/kg)
1	100	300	300.19
a	300	410.1	411.22
b	300	300	300.19
2	1200	444.8	446.50
x	1200	1111.0	1173.84
3	1200	1450	1575.57
c	300	1034.3	1085.31
d	300	1450	1575.57
4	100	1111.0	1173.84
y	100	444.8	446.50

KNOWN: An ideal air-standard regenerative Brayton cycle operates with property data given at principal states. The net power output of the cycle is given.

FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the combustor, and the thermal efficiency.

Problem 9.80 (Continued) – Page 2

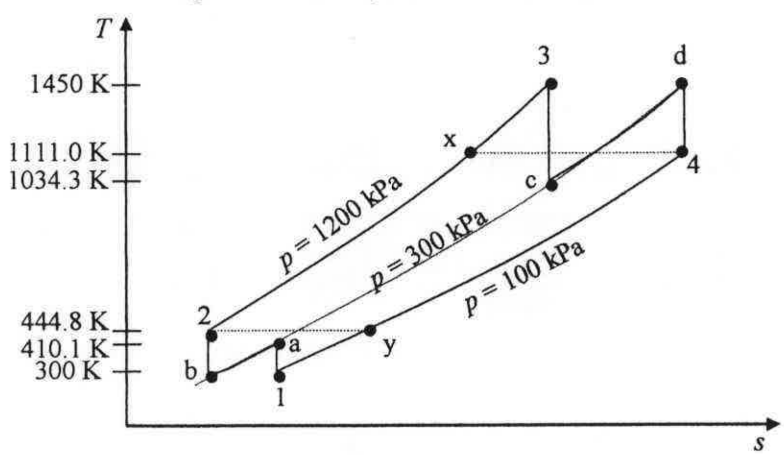
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine stages and compressor stages operate adiabatically.
4. There are no pressure drops for flow through the intercooler, regenerator, combustor, and reheat combustor.
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.

ANALYSIS: The *T-s* diagram for the cycle is shown below.



Problem 9.80 (Continued) – Page 3

(a) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine stages and compressor stages give

$$\dot{W}_{t1} = \dot{m}(h_3 - h_c)$$
$$\dot{W}_{t2} = \dot{m}(h_d - h_4)$$
$$\dot{W}_{c1} = \dot{m}(h_a - h_1)$$
$$\dot{W}_{c2} = \dot{m}(h_2 - h_b)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_{c1} - \dot{W}_{c2} = \dot{m}[(h_3 - h_c) + (h_d - h_4) - (h_a - h_1) - (h_2 - h_b)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{[(h_3 - h_c) + (h_d - h_4) - (h_a - h_1) - (h_2 - h_b)]}$$

Inserting values (Need to fix the conversion factors.)

$$\dot{m} = \frac{10{,}000\text{ kW}}{[(1575.57 - 1085.31) + (1575.57 - 1173.84) - (411.22 - 300.19) - (446.50 - 300.19)]\frac{\text{kJ}}{\text{kg}}}\left|\frac{\frac{1\text{ kJ}}{\text{s}}}{1\text{ kW}}\right|$$

$$\dot{m} = \underline{\mathbf{15.76\ kg/s}}$$

(b) The rate of heat transfer to the working fluid passing through the combustor and reheat combustor can be determined by applying mass and energy balances to control volumes around the combustor and reheat combustor to give

$$\dot{Q}_{in} = \dot{m}[(h_3 - h_x) + (h_d - h_c)]$$

$$\dot{Q}_{in} = \left(15.76\frac{\text{kg}}{\text{s}}\right)[(1575.57 - 1173.84) + (1575.57 - 1085.31)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{\frac{1\text{ kJ}}{\text{s}}}\right| = \underline{\mathbf{14{,}058\ kW}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = (10{,}000\text{ kW})/(14{,}058\text{ kW}) = \underline{\mathbf{0.711\ (71.1\%)}}$$