8.23

8.23 Oil flows through the horizontal pipe shown in Fig. P8.23 under laminar conditions. All sections are the same diameter except one. Which section of the pipe (A, B, C, D, or E) is slightly smaller diameter than the others? Explain.

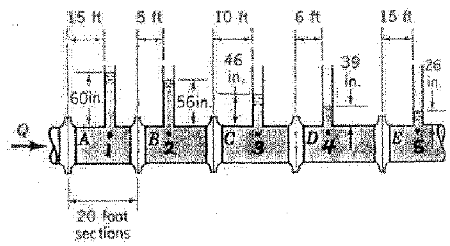
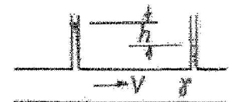


FIGURE P8.23

For laminar flow in a horizontal pipe $Q = \frac{\pi D^4}{128\mu}\frac{\Delta p}{\ell}$, where $Q_A = Q_B = Q_C = Q_D = Q_E$. Thus $\frac{\Delta p}{\ell} \sim \frac{1}{D^4}$. The smallest diameter pipe has the largest $\frac{\Delta p}{\ell}$, where $\Delta p = \gamma h$



Let $a = \frac{\Delta p}{\ell})_{pipe\ A}$, $b = \frac{\Delta p}{\ell})_{pipe\ B}$, etc.

Hence, from the data in the figure for the section between (1) and (2):

$5a + 5b = \gamma \frac{(60-56)}{12}$, where a and $b \sim \frac{lb}{ft^3}$ and $\gamma \sim \frac{lb}{ft^3}$. (1)

Similarly, from (2) to (3)

$15b + 10c = \gamma \frac{(56-46)}{12}$, (2)

from (3) to (4)

$10c + 6d = \gamma \frac{(46-39)}{12}$, (3)

and from (4) to (5)

$14d + 15e = \gamma \frac{(39-26)}{12}$ (4)

Eqs. (1) through (4) can be written as

(5) $a + b = 0.0667\gamma$
(6) $1.5b + c = 0.0833\gamma$
(7) $c + 0.6d = 0.0583\gamma$
(8) $d + 1.071e = 0.0774\gamma$

From the problem statement, 4 pipes are the same diameter, one is smaller diameter. Thus, 4 of the 5 variables (a, b, c, d, e) should be equal, one larger than the others.

Assume $a > b = c = d = e$. From Eq. (6), $1.5b + b = 0.0833\gamma$ or $b = 0.0333\gamma$ but from Eq. (7), $b + 0.6b = 0.0583\gamma$ or $b = 0.0364\gamma$ which is not the same as that from Eq. (6).

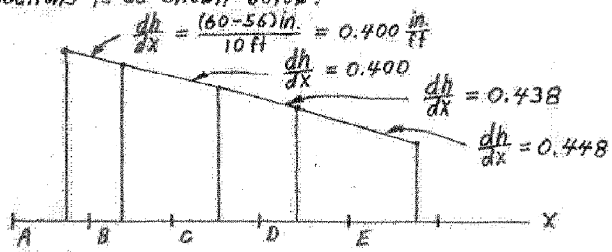
Assuming $b > a = c = d = e$, or $c > a = b = d = e$, or $e > a = b = c = d$ lead to similar inconsistencies. However, if we assume $d > a = b = c = e$ we obtain from Eq (5): $a = 0.0333\gamma$; from Eq. (6): the same value of a; from Eq. (7): $d = 0.0417\gamma$; the same value of d from Eq. (8).

(con't)

8.23

Thus, $a=b=c=e$ and $d>a$. <u>That is, the small pipe is pipe D.</u>

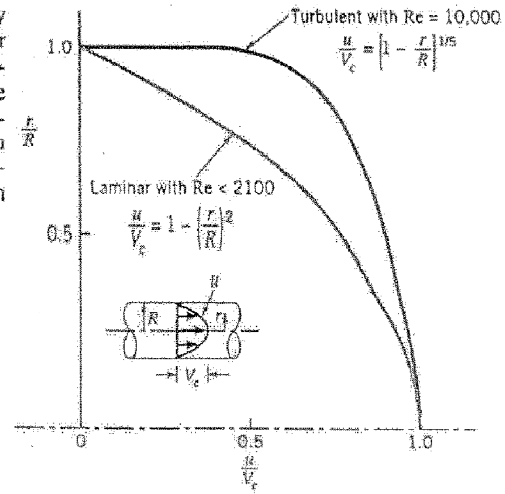
Note: This result can also be obtained as follows. From the given data the pressure gradient (average) between piezometer tube locations is as shown below.



Given that all section have the same diameter except for one, it follows (based on the different $\frac{dh}{dx}$ values) that the diameter of section D is less than that of the others.

8.30

8.30 As shown in Video V8.10 and Fig P8.30 the velocity profile for laminar flow in a pipe is quite different from that for turbulent flow. With laminar flow the velocity profile is parabolic; with turbulent flow at Re = 10,000 the velocity profile can be approximated by the power-law profile shown in the figure. (a) For laminar flow, determine at what radial loaction you would place a Pitot tube if it is to measure the average velocity in the pipe. (b) Repeat part (a) for turbulent flow with Re = 10,000.



■ FIGURE P8.30

For laminar or turbulent flow,

$$Q = AV = \pi R^2 V = \int u\,dA = \int u(2\pi r\,dr) = 2\pi \int_{r=0}^{R} u r\,dr$$

a) Laminar flow:

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \left(\frac{r}{R}\right)^2\right] dr = 2\pi V_c \left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \pi \frac{R^2}{2} V_c$$

Thus, $V = \frac{1}{2} V_c$ For $u = V = \frac{V_c}{2}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{1}{2} = 1 - \left(\frac{r}{R}\right)^2, \text{ or } \left(\frac{r}{R}\right)^2 = \frac{1}{2} \quad \text{Thus, } r = \frac{1}{\sqrt{2}} R = \underline{\underline{0.707R}}$$

b) Turbulent flow

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \frac{r}{R}\right]^{1/5} dr = 2\pi R^2 V_c \int_0^1 \left(\frac{r}{R}\right)\left[1 - \left(\frac{r}{R}\right)\right]^{1/5} d\left(\frac{r}{R}\right)$$

Let $y = 1 - \left(\frac{r}{R}\right)$ so that $\left(\frac{r}{R}\right) = 1 - y$ and $d\left(\frac{r}{R}\right) = -dy$

Thus,

$$\pi R^2 V = 2\pi R^2 V_c \int_{y=1}^{y=0} (1-y)\,y^{1/5}(-dy) = 2\pi R^2 V_c \int_0^1 \left(y^{1/5} - y^{6/5}\right) dy$$

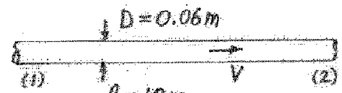
$$= 2\pi R^2 V_c \left[\frac{5}{6} - \frac{5}{11}\right] = 2\pi R^2 V_c \left(\frac{25}{66}\right)$$

or $V = \frac{50}{66} V_c$ For $u = V = \frac{50}{66}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{50}{66} = \left[1 - \frac{r}{R}\right]^{1/5} \quad \text{or} \quad \frac{r}{R} = 0.750 \text{ so that } r = \underline{\underline{0.750\,R}}$$

8.45

8.45 Water flows through a horizontal 60-mm-diameter galvanized iron pipe at a rate of 0.02 m^3/s. If the pressure drop is 135 kPa per 10 m of pipe, do you think this pipe is **(a)** a new pipe, **(b)** an old pipe with a somewhat increased roughness due to aging, or **(c)** a very old pipe that is partially clogged by deposits? Justify your answer.



For the horizontal pipe ($z_1 = z_2$) with $V_1 = V_2$ the energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \text{ reduces to } p_1 - p_2 = f\frac{\ell}{D}\frac{1}{2}\rho V^2$$

or $135 \times 10^3 \frac{N}{m^2} = f \frac{10m}{0.06m} \frac{1}{2}(999 \frac{kg}{m^3})(7.07\frac{m}{s})^2$, or $f = 0.0324$

where we have used $V = \frac{Q}{A} = \dfrac{0.02 \frac{m^3}{s}}{\frac{\pi}{4}(0.06m)^2} = 7.07 \frac{m}{s}$

With $Re = \frac{VD}{\nu} = \dfrac{(7.07\frac{m}{s})(0.06m)}{1.12 \times 10^{-6}\frac{m^2}{s}} = 3.79 \times 10^5$ and $\frac{\varepsilon}{D} = \frac{0.15 \text{ mm}}{60 \text{ mm}} = 2.5 \times 10^{-3}$

for a new galvanized iron pipe (see Table 8.1), the friction factor should be (see Fig. 8.20) $f = 0.0255$. Since this is less than the actual value $f = 0.0324$, the pipe is not a new pipe.

With $Re = 3.79 \times 10^5$ and $f = 0.0324$ we obtain from Fig. 8.20 a relative roughness of $\frac{\varepsilon}{D} = 0.006$. This is approximately twice the roughness of a new pipe — certainly quite possible. A very old partially clogged pipe would have considerably greater head loss. Thus, the pipe is an old pipe with somewhat increased roughness.

8.59

8.59 Water flows from the container shown in Fig. P8.59. Determine the loss coefficient needed in the valve if the water is to "bubble up" 3 in. above the outlet pipe.

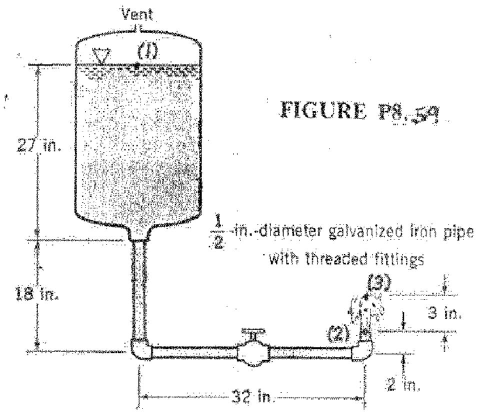


FIGURE P8.59

Determine the flowrate from Bernoulli equation:

$\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3$, where $z_3-z_2=3\text{ in.}$, $p_2=p_3=0$, and $V_3=0$

Thus, $V_2=\sqrt{2g(z_3-z_2)}=\sqrt{2(32.2\frac{ft}{s^2})(\frac{3}{12}ft)}=4.01\frac{ft}{s}$

Also,

$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3+(f\frac{\ell}{D}+\Sigma K_L)\frac{V^2}{2g}$, where $p_1=p_3=0$, $V_1=V_3=0$,

$z_1=\frac{(18+27)}{12}ft=3.75\,ft$, $z_3=\frac{(2+3)}{12}ft=0.417\,ft$, and $V=4.01\frac{ft}{s}$

Thus, $z_1=z_3+(f\frac{\ell}{D}+\Sigma K_L)\frac{V^2}{2g}$, where $\Sigma K_L=K_{L_{entrance}}+2K_{L_{elbow}}+K_{L_{valve}}$

or $\Sigma K_L=0.2+2(1.5)+K_{L_{valve}}=3.2+K_{L_{valve}}$

From Table 8.1 $\frac{\varepsilon}{D}=(0.0005\,ft/(0.5/12\,ft))=0.012$, and

$$Re=\frac{VD}{\nu}=\frac{(4.01\frac{ft}{s})(\frac{0.5}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}}=13{,}800$$

Thus, from Fig. 8.20,

$f=0.043$

Hence,

$$3.75\,ft=0.417\,ft+\left(0.043\left(\frac{18+32+2}{0.5}\right)+3.2+K_{L_{valve}}\right)\frac{(4.01\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$K_{L_{valve}}=\underline{\underline{5.68}}$

8.64

8.64 As shown in Fig. P8.64, water flows from one tank to another through a short pipe whose length is n times the pipe diameter. Head losses occur in the pipe and at the entrance and exit. (See Video V8.10) Determine the maximum value of n if the major loss is to be no more than 10% of the minor loss and the friction factor is 0.02.

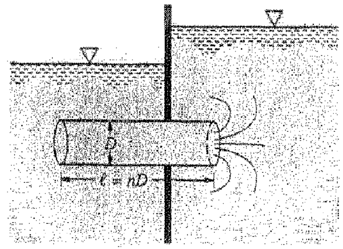


FIGURE P8.64

If $h_{L_{major}} = 10\% h_{L_{minor}}$, then

$$10\, f \frac{\ell}{D} \frac{V^2}{2g} = \sum K_L \frac{V^2}{2g} \quad \text{or} \quad \frac{\ell}{D} = \frac{\sum K_L}{10 f} \qquad (1)$$

where $\sum K_L = K_{L_{entrance}} + K_{L_{exit}} = 0.8 + 1 = 1.8$

Thus, with $f = 0.02$ and $\ell = nD$ Eq. (1) becomes

$$\frac{nD}{D} = \frac{1.8}{10(0.02)}$$

or

$$n = \underline{\underline{9}}$$

8.93

8.93 Water at 40 °F is pumped from a lake as shown in Fig. P8.93. What is the maximum flowrate possible without cavitation occurring?

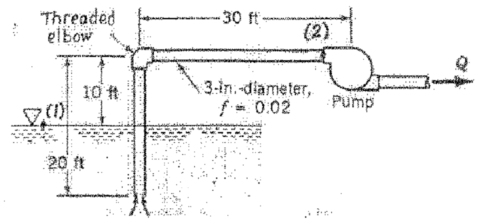


FIGURE P8.93

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}, \text{ where } z_1=0,\ z_2=10\,ft, \qquad (1)$$

$p_1=14.7\frac{lb}{in^2}(abs)$, $V_1=0$, $V_2=V$, and from Table B.1 $p_2=0.1217\frac{lb}{in^2}(abs)=17.52\frac{lb}{ft^2}$

Thus, with the given $f=0.02$ we obtain from Eq. (1)

$$\frac{(14.7\frac{lb}{in^2})(144\frac{in^2}{ft^2})-17.52\frac{lb}{ft^2}}{62.4\frac{lb}{ft^3}}=10ft+\left(0.02\left(\frac{50\,ft}{\frac{3}{12}ft}\right)+1+1.5+0.8\right)\frac{V^2}{2(32.2\frac{ft}{s^2})}$$

where we have used $K_L=0.8$ for the entrance, $K_L=1.5$ for the $90°$ elbow (see Fig. 8.22 and Table 8.2)

Thus, $V=14.46\frac{ft}{s}$ so that $Q=AV=\frac{\pi}{4}\left(\frac{3}{12}ft\right)^2\left(14.46\frac{ft}{s}\right)=\underline{\underline{0.710\ \frac{ft^3}{s}}}$