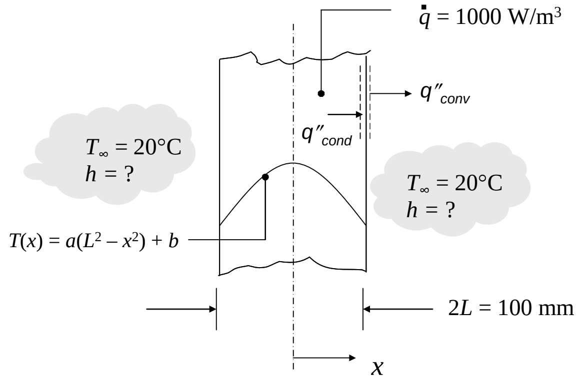
PROBLEM 2.10

KNOWN: Wall thickness. Thermal energy generation rate. Temperature distribution. Ambient fluid temperature.

FIND: Thermal conductivity. Convection heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: Under the specified conditions, the heat equation, Equation 2.21, reduces to

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

With the given temperature distribution, $d^2T/dx^2 = -2a$. Therefore, solving for k gives

$$k = \frac{\dot{q}}{2a} = \frac{1000\ \text{W/m}^3}{2 \times 10°\text{C/m}^2} = 50\ \text{W/m} \cdot \text{K}$$ <

The convection heat transfer coefficient can be found by applying the boundary condition at $x = L$ (or at $x = -L$),

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h\left[T(L) - T_\infty\right]$$

Therefore

$$h = \frac{-k\left.\dfrac{dT}{dx}\right|_{x=L}}{\left[T(L) - T_\infty\right]} = \frac{2kaL}{b - T_\infty} = \frac{2 \times 50\ \text{W/m} \cdot \text{K} \times 10°\text{C/m}^2 \times 0.05\ \text{m}}{30°\text{C} - 20°\text{C}} = 5\ \text{W/m}^2 \cdot \text{K}$$ <

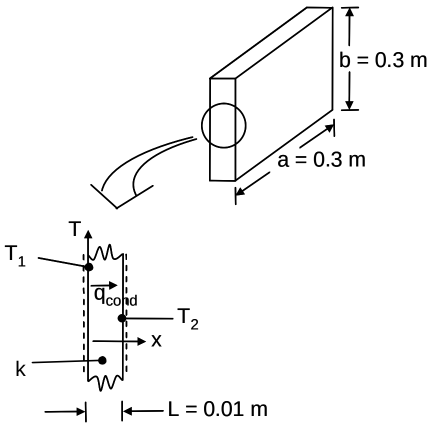
COMMENTS: (1) In Chapter 3, you will learn how to determine the temperature distribution. (2) The heat transfer coefficient could also have been found from an energy balance on the wall. With $\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = 0$, we find $-2hA[T(L) - T_\infty] + 2\dot{q}LA = 0$. This yields the same result for h.

PROBLEM 2.19

KNOWN: Dimensions of and temperature difference across an aircraft window. Window materials and cost of energy.

FIND: Heat loss through one window and cost of heating for 130 windows on 8-hour trip.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the x-direction, (3) Constant properties.

PROPERTIES: Table A.3, soda lime glass (300 K): k_{gl} = 1.4 W/m·K.

ANALYSIS: From Eq. 2.1,

$$q_x = -kA\frac{dT}{dx} = k\,a\,b\frac{(T_1 - T_2)}{L}$$

For glass,

$$q_{x,g} = 1.4\ \frac{W}{m \cdot K} \times 0.3\ m \times 0.3\ m \times \left[\frac{80°C}{0.01m}\right] = 1010\ W \qquad <$$

The cost associated with heat loss through N windows at a rate of R = \$1/kW·h over a t = 8 h flight time is

$$C_g = Nq_{x,g}Rt = 130 \times 1010\ W \times 1\ \frac{\$}{kW \cdot h} \times 8\ h \times \frac{1kW}{1000W} = \$1050 \qquad <$$

Repeating the calculation for the polycarbonate yields

$$q_{x,p} = 151\ W,\ C_p = \$157 \qquad <$$

while for aerogel,

$$q_{x,a} = 10.1\ W,\ C_a = \$10 \qquad <$$

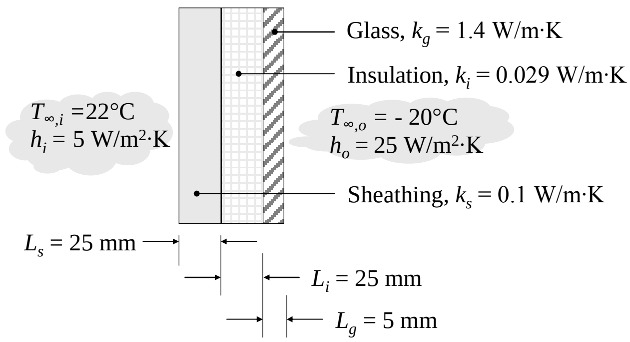
COMMENT: Polycarbonate provides significant savings relative to glass. It is also lighter (ρ_p = 1200 kg/m^3) relative to glass (ρ_g = 2500 kg/m^3). The aerogel offers the best thermal performance and is very light (ρ_a = 2 kg/m^3) but would be relatively expensive.

PROBLEM 3.5

KNOWN: Thermal conductivities and thicknesses of original wall, insulation layer, and glass layer. Interior and exterior air temperatures and convection heat transfer coefficients.

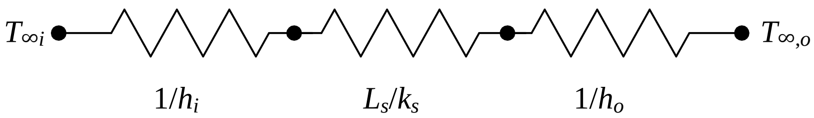
FIND: Heat flux through original and retrofitted walls.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible contact resistances.

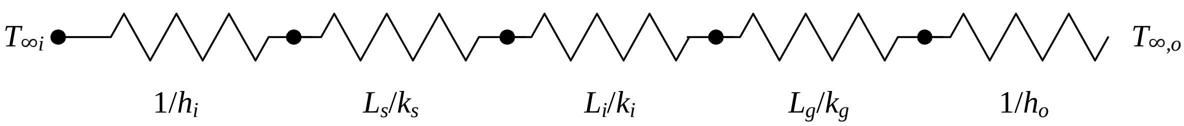
ANALYSIS: The original wall with convection inside and outside can be represented by the following thermal resistance network, where the resistances are each for a unit area:



Thus the heat flux can be expressed as

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_i} + \frac{L_s}{k_s} + \frac{1}{h_o}} = \frac{22°\text{C} - (-20°\text{C})}{\frac{1}{5\ \text{W/m}^2\cdot\text{K}} + \frac{0.025\ \text{m}}{0.1\ \text{W/m}\cdot\text{K}} + \frac{1}{25\ \text{W/m}^2\cdot\text{K}}} = 85.7\ \text{W/m}^2$$ <

The retrofitted wall has three layers. The thermal circuit can be represented as follows:



Thus the heat flux can be expressed as

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_i} + \frac{L_s}{k_s} + \frac{L_i}{k_i} + \frac{L_g}{k_g} + \frac{1}{h_o}}$$ <

$$= \frac{22°\text{C} - (-20°\text{C})}{\frac{1}{5\ \text{W/m}^2\cdot\text{K}} + \frac{0.025\ \text{m}}{0.1\ \text{W/m}\cdot\text{K}} + \frac{0.025\ \text{m}}{0.029\ \text{W/m}\cdot\text{K}} + \frac{0.005\ \text{m}}{1.4\ \text{W/m}\cdot\text{K}} + \frac{1}{25\ \text{W/m}^2\cdot\text{K}}} = 31.0\ \text{W/m}^2$$

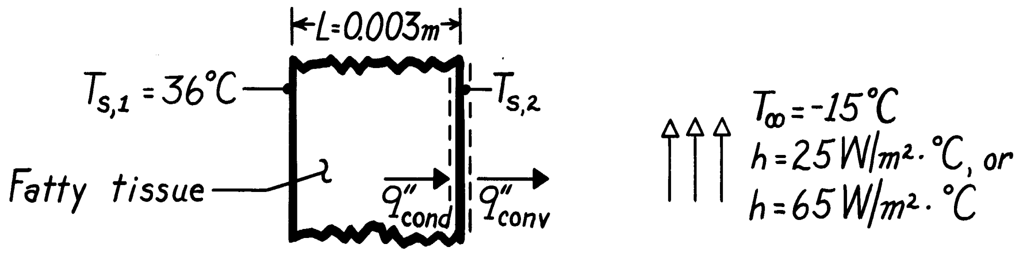
COMMENTS: The heat flux has been reduced to approximately one-third of the original value because of the increased resistance, which is mainly due to the insulation layer.

PROBLEM 3.10

KNOWN: A layer of fatty tissue with fixed inside temperature can experience different outside convection conditions.

FIND: (a) Ratio of heat loss for different convection conditions, (b) Outer surface temperature for different convection conditions, and (c) Temperature of still air which achieves same cooling as moving air (*wind chill* effect).

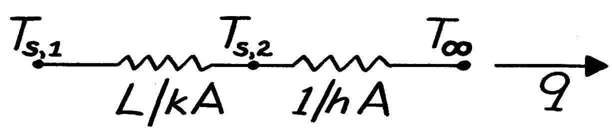
SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) Homogeneous medium with constant properties, (4) No internal heat generation (metabolic effects are negligible), (5) Negligible radiation effects.

PROPERTIES: *Table A-3*, Tissue, fat layer: k = 0.2 W/m·K.

ANALYSIS: The thermal circuit for this situation is



Hence, the heat rate is

$$q = \frac{T_{s,1} - T_\infty}{R_{tot}} = \frac{T_{s,1} - T_\infty}{L/kA + 1/hA}.$$

Therefore,

$$\frac{q''_{calm}}{q''_{windy}} = \frac{\left[\frac{L}{k} + \frac{1}{h}\right]_{windy}}{\left[\frac{L}{k} + \frac{1}{h}\right]_{calm}}.$$

Applying a surface energy balance to the outer surface, it also follows that

$$q''_{cond} = q''_{conv}.$$

Continued …

Hence,

$$\frac{k}{L}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_\infty\right)$$

$$T_{s,2}=\frac{T_\infty+\frac{k}{hL}T_{s,1}}{1+\frac{k}{hL}}.$$

To determine the wind chill effect, we must determine the heat loss for the windy day and use it to evaluate the hypothetical ambient air temperature, T'_∞, which would provide the same heat loss on a calm day, Hence,

$$q''=\frac{T_{s,1}-T_\infty}{\left[\frac{L}{k}+\frac{1}{h}\right]_{\text{windy}}}=\frac{T_{s,1}-T'_\infty}{\left[\frac{L}{k}+\frac{1}{h}\right]_{\text{calm}}}$$

From these relations, we can now find the results sought:

(a)
$$\frac{q''_{\text{calm}}}{q''_{\text{windy}}}=\frac{\frac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\frac{1}{65\text{ W/m}^2\cdot\text{K}}}{\frac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\frac{1}{25\text{ W/m}^2\cdot\text{K}}}=\frac{0.015+0.0154}{0.015+0.04}$$

$$\frac{q''_{\text{calm}}}{q''_{\text{windy}}}=0.553$$ <

(b)
$$\left.T_{s,2}\right]_{\text{calm}}=\frac{-15^\circ\text{C}+\frac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}36^\circ\text{C}}{1+\frac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}}=22.1^\circ\text{C}$$ <

$$\left.T_{s,2}\right]_{\text{windy}}=\frac{-15^\circ\text{C}+\frac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}36^\circ\text{C}}{1+\frac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}}=10.8^\circ\text{C}$$ <

(c)
$$T'_\infty=36^\circ\text{C}-\left(36+15\right)^\circ\text{C}\frac{\left(0.003/0.2+1/25\right)}{\left(0.003/0.2+1/65\right)}=-56.3^\circ\text{C}$$ <

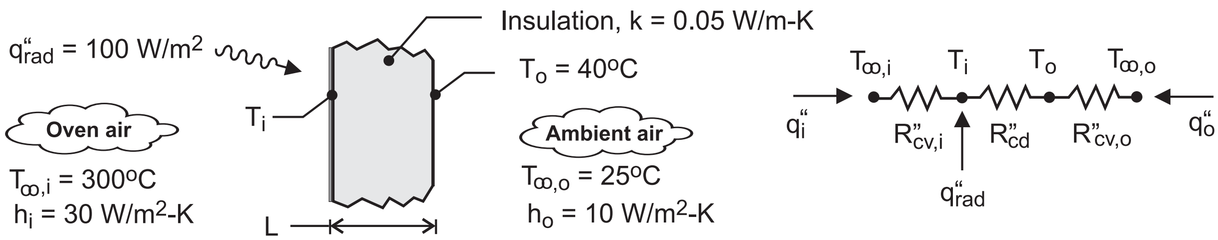
COMMENTS: The wind chill effect is equivalent to a decrease of $T_{s,2}$ by 11.3°C and increase in the heat loss by a factor of $(0.553)^{-1}$ = 1.81.

PROBLEM 3.19

KNOWN: Drying oven wall having material with known thermal conductivity sandwiched between thin metal sheets. Radiation and convection conditions prescribed on inner surface; convection conditions on outer surface.

FIND: (a) Thermal circuit representing wall and processes and (b) Insulation thickness required to maintain outer wall surface at $T_o = 40°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Thermal resistance of metal sheets negligible,(4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

(b) Perform energy balances on the i- and o- nodes finding

$$\frac{T_{\infty,i} - T_i}{R''_{cv,i}} + \frac{T_o - T_i}{R''_{cd}} + q''_{rad} = 0 \quad (1)$$

$$\frac{T_i - T_o}{R''_{cd}} + \frac{T_{\infty,o} - T_o}{R''_{cv,o}} = 0 \quad (2)$$

where the thermal resistances are

$$R''_{cv,i} = 1/h_i = 0.0333 \text{ m}^2 \cdot \text{K/W} \quad (3)$$

$$R''_{cd} = L/k = L/0.05 \text{ m}^2 \cdot \text{K/W} \quad (4)$$

$$R''_{cv,o} = 1/h_o = 0.100 \text{ m}^2 \cdot \text{K/W} \quad (5)$$

Substituting numerical values, and solving Eqs. (1) and (2) simultaneously, find

$$L = 86 \text{ mm}$$ <

COMMENTS: (1) The temperature at the inner surface can be found from an energy balance on the i-node using the value found for L.

$$\frac{T_{\infty,i} - T_i}{R''_{cv,o}} + \frac{T_{\infty,o} - T_i}{R''_{cd} + R''_{cv,i}} + q''_{rad} = 0 \qquad T_i = 298.3°C$$

It follows that T_i is close to $T_{\infty,i}$ since the wall represents the dominant resistance of the system.

(2) Verify that $q''_i = 50 \text{ W/m}^2$ and $q''_o = -150 \text{ W/m}^2$. Is the overall energy balance on the system satisfied?