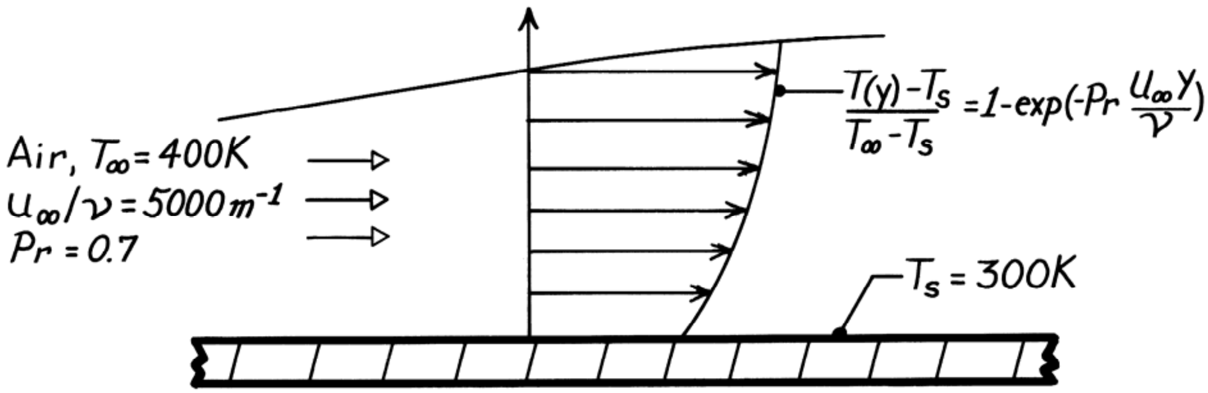
PROBLEM 6.3

KNOWN: Boundary layer temperature distribution.

FIND: Surface heat flux.

SCHEMATIC:



PROPERTIES: *Table A-4*, Air (T_s = 300K): k = 0.0263 W/m·K.

ANALYSIS: Applying Fourier's law at y = 0, the heat flux is

$$q''_s = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = -k\left(T_\infty - T_s\right)\left[Pr\frac{u_\infty}{\nu}\right]\exp\left[-Pr\frac{u_\infty y}{\nu}\right]\Bigg|_{y=0}$$

$$q''_s = -k\left(T_\infty - T_s\right)Pr\frac{u_\infty}{\nu}$$

$$q''_s = -0.0263\ \text{W/m}\cdot\text{K}\left(100\text{K}\right)0.7\times 5000\ \text{1/m}.$$

$$q''_s = -9205\ \text{W/m}^2.$$ <

COMMENTS: (1) Negative flux implies convection heat transfer to the surface.

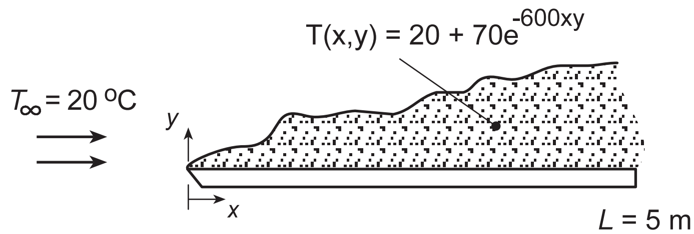
(2) Note use of k at T_s to evaluate q''_s from Fourier's law.

PROBLEM 6.12

KNOWN: Temperature distribution in boundary layer for air flow over a flat plate.

FIND: Variation of local convection coefficient along the plate and value of average coefficient.

SCHEMATIC:



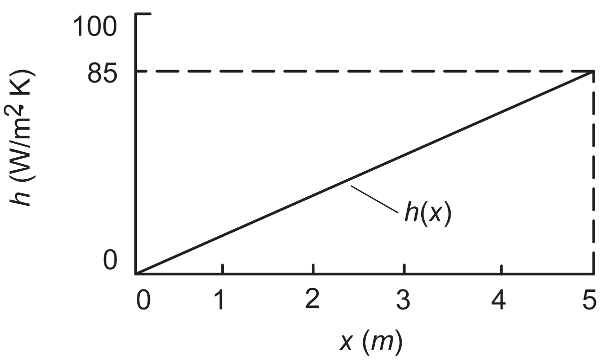
ANALYSIS: From Eq. 6.5,

$$h = -\frac{k\,\partial T/\partial y|_{y=0}}{(T_s - T_\infty)} = +\frac{k(70 \times 600x)}{(T_s - T_\infty)}$$

where $T_s = T(x,0) = 90°C$. Evaluating k at the arithmetic mean of the freestream and surface temperatures, $\overline{T}$ = (20 + 90)°C/2 = 55°C = 328 K, Table A.4 yields k = 0.0284 W/m·K. Hence, with $T_s - T_\infty$ = 70°C = 70 K,

$$h = \frac{0.0284\,\text{W/m}\cdot\text{K}\,(42{,}000x)\,\text{K/m}}{70\,\text{K}} = 17x\left(\text{W}/\text{m}^2\cdot\text{K}\right)$$ <

and the convection coefficient increases linearly with x.



The average coefficient over the range 0 ≤ x ≤ 5 m is

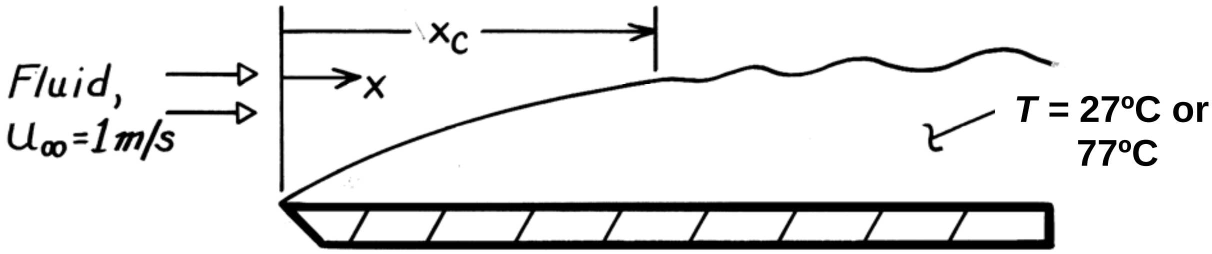
$$\overline{h} = \frac{1}{L}\int_0^L h\,dx = \frac{17}{5}\int_0^5 x\,dx = \frac{17}{5}\left.\frac{x^2}{2}\right|_0^5 = 42.5\,\text{W}/\text{m}^2\cdot\text{K}$$ <

PROBLEM 6.21

KNOWN: Transition Reynolds number. Velocity and temperature of atmospheric air, engine oil, and mercury flow over a flat plate.

FIND: Distance from leading edge at which transition occurs for each fluid.

SCHEMATIC:



ASSUMPTIONS: Transition Reynolds number is $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: For the fluids at T = 300 K and 350 K:

Fluid	*Table*	ν(m^2/s) T = 300 K	T = 350 K
Air (1 atm)	A-4	15.89×10^{-6}	20.92×10^{-6}
Engine Oil	A-5	550×10^{-6}	41.7×10^{-6}
Mercury	A-5	0.1125×10^{-6}	0.0976×10^{-6}

ANALYSIS: The point of transition is

$$x_c = Re_{x,c} \frac{\nu}{u_\infty} = \frac{5 \times 10^5}{1 \text{ m/s}} \nu.$$

Substituting appropriate viscosities, find

Fluid	x_c(m) T = 300 K	T = 350 K
Air	7.95	10.5
Oil	275	20.9
Mercury	0.056	0.049

<

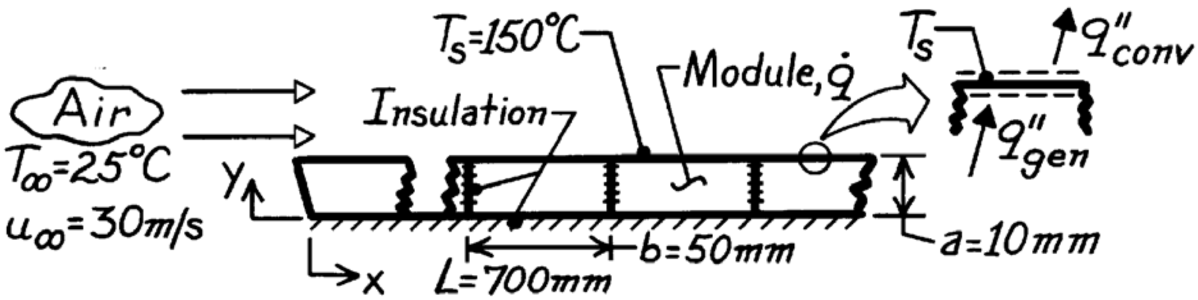
COMMENTS: (1) Note the great disparity in transition length for the different fluids. Due to the effect which viscous forces have on attenuating the instabilities which bring about transition, the distance required to achieve transition increases with increasing ν. (2) Note the temperature-dependence of the transition length, in particular for engine oil. (3) As shown in Example 6.4, the variation of the transition location can have a significant effect on the average heat transfer coefficient associated with convection to or from the plate.

PROBLEM 7.8

KNOWN: Flat plate comprised of rectangular modules of surface temperature T_s, thickness a and length b cooled by air at 25°C and a velocity of 30 m/s. Prescribed thermophysical properties of the module material.

FIND: (a) Required power generation for the module positioned 700 mm from the leading edge of the plate and (b) Maximum temperature in this module.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow at leading edge of plate, (2) Transition Reynolds number of 5×10^5, (3) Heat transfer is one-dimensional in y-direction within each module, (4) $\dot{q}$ is uniform within module, (5) Negligible radiation heat transfer.

PROPERTIES: Module material (given): k = 5.2 W/m·K, c_p = 320 J/kg·K, ρ = 2300 kg/m^3; *Table A-4*, Air $\left(\overline{T}_f = (T_s + T_\infty)/2 = 360 \text{ K, 1 atm}\right)$: k = 0.0308 W/m·K, ν = 22.02 × 10^{-6} m^2/s, Pr = 0.698.

ANALYSIS: (a) The module power generation follows from an energy balance on the module surface,

$$q''_{conv} = q''_{gen}$$

$$\overline{h}\left(T_s - T_\infty\right) = \dot{q} \cdot a \qquad \text{or} \qquad \dot{q} = \frac{\overline{h}\left(T_s - T_\infty\right)}{a}.$$

To select a convection correlation for estimating $\overline{h}$, first find the Reynolds numbers at x = L.

$$Re_L = \frac{u_\infty L}{\nu} = \frac{30 \text{ m/s} \times 0.70 \text{ m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.537 \times 10^5.$$

Since the flow is turbulent over the module, the approximation $\overline{h} \approx h_x \; (L + b/2)$ is appropriate, with

$$Re_{L+b/2} = \frac{30 \text{ m/s} \times \left(0.700 + 0.050/2\right)\text{m}}{22.02 \times 10^{-6}\text{m}^2/\text{s}} = 9.877 \times 10^5.$$

Using the turbulent flow correlation with x = L + b/2 = 0.725 m,

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$Nu_x = 0.0296\left(9.877 \times 10^5\right)^{4/5}\left(0.698\right)^{1/3} = 1640$$

$$\overline{h} \approx h_x = \frac{Nu_x k}{x} = \frac{1640 \times 0.0308 \text{ W/m} \cdot \text{K}}{0.725} = 69.7 \text{ W/m}^2 \cdot \text{K}.$$

Continued …

PROBLEM 7.8 (Cont.)

Hence,

$$\dot{q} = \frac{69.7\ \text{W/m}^2 \cdot \text{K}(150 - 25)\text{K}}{0.010\ \text{m}} = 8.713 \times 10^5\ \text{W/m}^3. \qquad <$$

(b) The maximum temperature within the module occurs at the surface next to the insulation (y = 0). For one-dimensional conduction with thermal energy generation, use Eq. 3.42 to obtain

$$T(0) = \frac{\dot{q}a^2}{2k} + T_s = \frac{8.713 \times 10^5\ \text{W/m}^3 \times (0.010\ \text{m})^2}{2 \times 5.2\ \text{W/m} \cdot \text{K}} + 150^\circ\text{C} = 158.4^\circ\text{C}. \qquad <$$

COMMENTS: An alternative approach for estimating the average heat transfer coefficient for the module follows from the relation

$$q_{\text{module}} = q_{0 \to L+b} - q_{0 \to L}$$

$$\bar{h} \cdot b = \bar{h}_{L+b} \cdot (L + b) - \bar{h}_L \cdot L \qquad \text{or} \qquad \bar{h} = \bar{h}_{L+b} \frac{L+b}{b} - \bar{h}_L \frac{L}{b}.$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation is

$$\overline{Nu}_x = \left(0.037 Re_x^{4/5} - 871\right) Pr^{1/3}$$

With x = L + b and x = L, find

$$\bar{h}_{L+b} = 54.79\ \text{W/m}^2 \cdot \text{K} \qquad \text{and} \qquad \bar{h}_L = 53.73\ \text{W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h} = \left[54.79 \frac{0.750}{0.050} - 53.73 \frac{0.700}{0.05}\right] \text{W/m}^2 \cdot \text{K} = 69.7\ \text{W/m}^2 \cdot \text{K}.$$

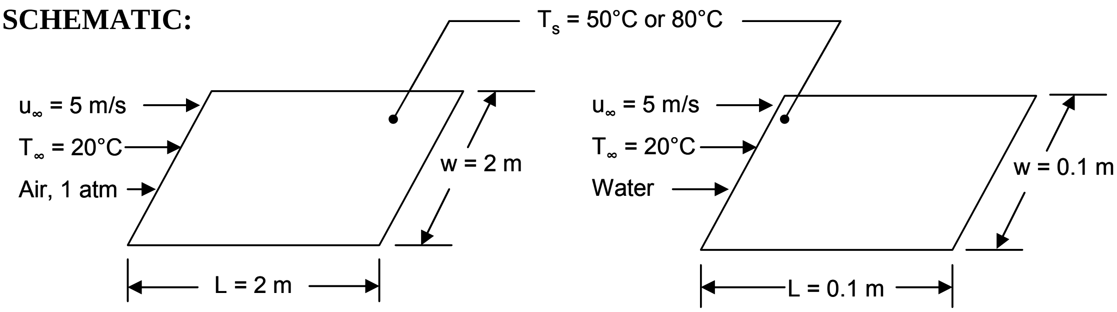
which is in excellent agreement with the approximate result employed in part (a).

PROBLEM 7.13

KNOWN: Dimensions and surface temperatures of a flat plate. Velocity and temperature of air and water flow parallel to the plate.

FIND: (a) Average convective heat transfer coefficient, convective heat transfer rate, and drag force when L = 2 m, w = 2 m. (b) Average convective heat transfer coefficient, convective heat transfer rate, and drag force when L = 0.1 m, w = 0.1 m.

SCHEMATIC:



(a) Air (b) Water

ASSUMPTIONS: (1) Steady-state conditions, (2) Boundary layer assumptions are valid, (3) Constant properties, (4) Transition Reynolds number is 5×10^5.

PROPERTIES: Using *IHT*, Air (p = 1 atm, T_f = 35°C = 308 K): Pr = 0.706, k = 26.9×10^{-3} W/m·K, $\nu = 1.669 \times 10^{-5}$ m^2/s, ρ = 1.135 kg/m^3. Air (p = 1 atm, T_f = 50°C = 323 K): Pr = 0.704, k = 28.0×10^{-3} W/m·K, $\nu = 1.82 \times 10^{-5}$ m^2/s, ρ = 1.085 kg/m^3. Water (T_f = 308 K): Pr = 4.85, k = 0.625 W/m·K, $\nu = 7.291 \times 10^{-7}$ m^2/s, ρ = 994 kg/m^3. Water (T_f = 323 K): Pr = 3.56, k = 0.643 W/m·K, $\nu = 5.543 \times 10^{-7}$ m^2/s, ρ = 988 kg/m^3.

ANALYSIS:

(a) We begin by calculating the Reynolds numbers for the two different surface temperatures:

$$Re_{L1} = \frac{u_\infty L}{\nu_1} = \frac{5\text{ m/s} \times 2\text{ m}}{1.669 \times 10^{-5}\text{ m}^2/\text{s}} = 5.99 \times 10^5$$

$$Re_{L2} = \frac{u_\infty L}{\nu_2} = \frac{5\text{ m/s} \times 2\text{ m}}{1.82 \times 10^{-5}\text{ m}^2/\text{s}} = 5.49 \times 10^5$$

Therefore, in both cases the flow is turbulent at the end of the plate and the conditions in the boundary layer are "mixed."

The average drag coefficient can be calculated from Equation 7.40. For the first case,

$$\overline{C}_{f,L1} = 0.074\, Re_{L1}^{-1/5} - 1742\, Re_{L1}^{-1}$$
$$= 0.074(5.99 \times 10^5)^{-1/5} - 1742(5.99 \times 10^5)^{-1} = 2.27 \times 10^{-3}$$

Then

$$F_{D1} = \overline{C}_{f,L1} \frac{1}{2} \rho u_\infty^2 A_s$$
$$= 2.27 \times 10^{-3} \times \frac{1}{2} \times 1.135\text{ kg/m}^3 \times (5\text{ m/s})^2 \times 8\text{ m}^2 = 0.257\text{ N}$$

$$= 0.257\text{ N}$$ <

Continued…

PROBLEM 7.13 (Cont.)

The average Nusselt number is calculated from Equation 7.38, with A = 871 for a transition Reynolds number of 5×10^5.

$$\overline{Nu}_{L1} = (0.037\, Re_L^{4/5} - 871)\, Pr^{1/3}$$

$$= \left[0.037(5.99 \times 10^5)^{4/5} - 871\right](0.706)^{1/3} = 604.$$

Then

$$\bar{h}_{L1} = \overline{Nu}_{L1} k/L = 604 \times 26.9 \times 10^{-3}\ \text{W/m} \cdot \text{K/2 m} = 8.13\ \text{W/m}^2 \cdot \text{K} \qquad <$$

and

$$q_1 = \bar{h}_{L1} A_s (T_s - T_\infty) = 8.13\ \text{W/m}^2 \cdot \text{K} \times 8\ \text{m}^2 \times (50°\text{C} - 20°\text{C}) = 1950\ \text{W} \qquad <$$

Similarly for $T_s = 80°C$ we find

$$F_{D2} = 0.227\ \text{N},\ \bar{h}_{L2} = 7.16\ \text{W/m}^2 \cdot \text{K},\ q_2 = 3440\ \text{W} \qquad <$$

(b) Repeating the calculations for water

$$Re_{L1} = \frac{u_\infty L}{\nu} = \frac{5\ \text{m/s} \times 0.1\ \text{m}}{7.291 \times 10^{-7}\ \text{m}^2/\text{s}} = 6.86 \times 10^5$$

$$Re_{L2} = 9.02 \times 10^5$$

The flow is turbulent at the end of the plate in both cases.

$$\overline{C}_{f,L1} = 0.074(6.86 \times 10^5)^{-1/5} - 1742(6.86 \times 10^5)^{-1} = 2.49 \times 10^{-3}$$

$$F_{D1} = 2.49 \times 10^{-3} \times 1/2 \times 994\ \text{kg/m}^3 \times (5\ \text{m/s})^2 \times 0.02\ \text{m}^2 = 0.620\ \text{N} \qquad <$$

$$\overline{Nu}_L = \left[0.037(6.86 \times 10^5)^{4/5} - 871\right](4.85)^{1/3} = 1450$$

$$\bar{h}_{L1} = 1450 \times 0.625\ \text{W/m} \cdot \text{K/0.1 m} = 9050\ \text{W/m}^2 \cdot \text{K} \qquad <$$

$$q_1 = 9050\ \text{W/m}^2 \cdot \text{K} \times 0.02\ \text{m}^2 \times (50°\text{C} - 20°\text{C}) = 5430\ \text{W} \qquad <$$

For the higher surface temperature,

$$F_{D2} = 0.700\ \text{N},\ \bar{h}_{L2} = 12{,}600\ \text{W/m}^2 \cdot \text{K},\ q_2 = 15{,}100\ \text{W} \qquad <$$

COMMENTS: (1) For air, kinematic viscosity increases with increasing temperature. This decreases the Reynolds number which causes the transition to turbulence to move downstream, thereby decreasing the drag force and average heat transfer coefficient. The heat transfer rate increases for the higher surface temperature, however, because of the greater temperature difference between the surface and air. (2) For water, kinematic viscosity decreases with increasing temperature, causing the opposite trends as for air. The heat transfer rate increases dramatically for the higher surface temperature because of the increases in both the heat transfer coefficient and temperature difference. (3) Even though the water flows over a plate that is 400 times smaller, the drag force and heat transfer rate are larger than for air because of the smaller viscosity and greater density, thermal conductivity, and Prandtl number. The discrepancy is particularly great for the hear transfer rate. (4) The problem highlights the importance of carefully accounting for the temperature dependence of thermal properties.