G.W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology

ME 3322A: Thermodynamics: Fall 2014
Homework Set # 8
Due Date: October 28, 2014

	Problem # in Textbook		Answer
	7th Ed.	**8th Ed.**	
1	5.59	5.58	b) 54.5 kW
2	5.69	5.60	a) 8.23, b) 28.8
3	6.21	6.21	Q=67.73 kJ
4	6.24	6.24	b) Q=W= -120 kJ
5	6.41	6.39	a) 596 K; b) 1.81 bar; c) 0.885 kJ/K
6	6.61	6.58	a) 533.7 R; b) 0.1789 Btu/R
7	6.69	6.68	

PROBLEM 5.59

At steady state, a refrigeration cycle operating between hot and cold reservoirs at 300 K and 275 K, respectively, removes energy by heat transfer from the cold reservoir at a rate of 600 kW.

(a) If the cycle's coefficient of performance is 4, determine the power input required, in kW.
(b) Determine the minimum theoretical power required, in kW, for *any* such cycle.

ANALYSIS:

(a) $\beta = \frac{\dot{Q}_c}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = \frac{\dot{Q}_c}{\beta} = \frac{600\,kW}{4} = 150\,kW$ ← (a)

(b) $\beta \le \beta_{MAX} \Rightarrow \frac{\dot{Q}_c}{\dot{W}_{cycle}} \le \frac{T_C}{T_H - T_C} \Rightarrow \dot{Q}_c\left[\frac{T_H - T_C}{T_C}\right] \le \dot{W}_{cycle}$

$600\,kW\left[\frac{25\,K}{275\,K}\right] \le \dot{W}_{cycle}$

$\Rightarrow \dot{W}_{cycle} \ge 54.5\,kW$ ← (b)

PROBLEM 5.60

An air conditioner operating at steady state maintains a dwelling at 20°C on a day when the outside temperature is 35°C. Energy is removed by heat transfer from the dwelling at a rate of 2800 J/s while the air conditioner's power input is 0.8 kW. Determine (a) the coefficient of performance of the air conditioner and (b) the power input required by a reversible refrigeration cycle providing the same cooling effect while operating between hot and cold reservoirs at 35°C and 20°C, respectively.

(a) $\beta = \frac{\dot{Q}_c}{\dot{W}_{cycle}} = \frac{(2800\,J/s)\left|\frac{1\,kJ}{10^3\,J}\right|\left|\frac{1\,kW}{1\,kJ/s}\right|}{0.8\,kW} = 3.5$ ←

(b) $\beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{293\,K}{15\,K} = 19.5$

For the same $\dot{Q}_c$: 2.8 kW, the reversible refrigeration cycle requires a power input of

$(\dot{W}_{cycle})_{min} = \frac{\dot{Q}_c}{\beta_{MAX}} = \frac{2.8\,kW}{19.5} = 0.14\,kW$ ←

PROBLEM 5.69

A heat pump is under consideration for heating a research station located on an Antarctica ice shelf. The interior of the station is to be kept at 15°C. Determine the maximum theoretical rate of heating provided by a heat pump, in kW per kW of power input, in each of two cases: The role of the cold reservoir is played by (a) the atmosphere at −20°C, (b) ocean water at 5°C.

For any heat pump on a time-rate basis, $\gamma = \frac{\dot{Q}_H}{\dot{W}_{in}}$.

Also, $\gamma_{MAX} = \frac{T_H}{T_H - T_C}$, where $T_H = 288K\ (15°C)$.

Collecting results

$$\left(\frac{\dot{Q}_H}{\dot{W}_{in}}\right)_{MAX} = \left[\frac{288\,K}{288K - T_C}\right] \quad (1)$$

(a) Atmosphere: $T_C = 253\,K\ (-20°C)$. Eq. (1) gives

$$\left(\frac{\dot{Q}_H}{\dot{W}_{in}}\right)_{MAX} = \frac{288\,K}{(288-253)K} = 8.23 \longleftarrow$$

(b) Ocean water: $T_C = 278K\ (5°C)$. Eq. (1) gives

1 $$\left(\frac{\dot{Q}_H}{\dot{W}_{in}}\right)_{MAX} = \frac{288K}{(288-278)K} = 28.8 \longleftarrow$$

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1. Use of ocean water as the cold reservoir appears to be advantageous thermodynamically. However, actual performance will depart significantly from these maximum values, and cost will be a factor in deciding on the preferred course of action.

PROBLEM 6.21

One kilogram of water in a piston–cylinder assembly undergoes the two internally reversible processes in series shown in Fig. P6.21. For each process, determine, in kJ, the heat transfer and the work.

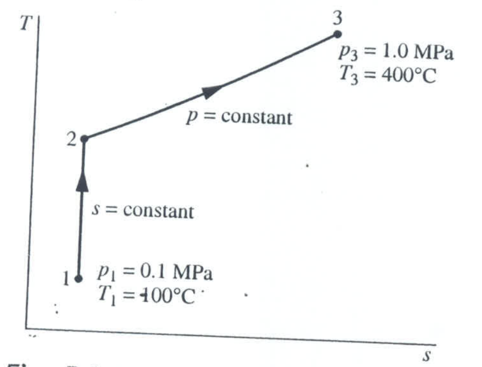
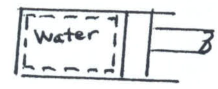


Fig. P6.21

SCHEMATIC & GIVEN DATA:



m = 1 kg

ENGR. MODEL:

1. The water is the closed system.
2. For the system, kinetic and potential energy changes can be ignored.
3. The processes are internally reversible.

ANALYSIS:

Process 1-2. With Eq. 6.23, $Q = 0$. An energy balance reduces to give $\Delta U = \overset{0}{\cancel{Q}} - W$

$\Rightarrow \quad W = -\Delta U = -m(u_2 - u_1)$

From Table A-4, $s_1 = 7.3614$ kJ/kg·K, $u_1 = 2506.7$ kJ/kg. Interpolating at 1.0 MPa with $s_2 = s_1$, $u_2 = 2904.97$ kJ/kg, $v_2 = 0.2912$ m³/kg

$$\therefore \; W = -(1\text{ kg})(2904.97 - 2506.7)\frac{\text{kJ}}{\text{kg}} = -398.27 \text{ kJ}$$

Process 2-3. $W = \int_2^3 p\,dV = mp(v_3 - v_2)$. From Table A-4, $v_3 = 0.3066$ m³/kg.

$$\therefore \; W = (1\text{ kg})\left(10^6\frac{\text{N}}{\text{m}^2}\right)(0.3066 - 0.2912)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 15.4 \text{ kJ}$$

An energy balance reduces to read, $\Delta U = Q - W \Rightarrow Q = \Delta U + W$

Thus, with $u_3 = 2957.3$ kJ/kg

$$Q = m(u_3 - u_2) + W$$
$$= (1\text{ kg})(2957.3 - 2904.97) + 15.4\text{ kJ}$$
$$= 67.73 \text{ kJ}$$

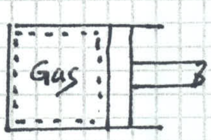
PROBLEM 6.24

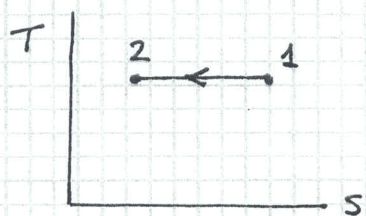
A gas within a piston–cylinder assembly undergoes an isothermal process at 400 K during which the change in entropy is −0.3 kJ/K. Assuming the ideal gas model for the gas and negligible kinetic and potential energy effects, evaluate the work, in kJ.

KNOWN: The change in entropy is known for an isothermal process of a gas within a piston-cylinder assembly.

FIND: Evaluate the work for the process of the gas, in kJ.

SCHEMATIC & GIVEN DATA:





$$(S_2 - S_1) = -0.3\ \frac{kJ}{K}$$

ENGINEERING MODEL:

1. The gas is the closed system, and the ideal gas model applies.
2. For the system, kinetic and potential energy effects can be ignored.
3. The process of the gas is isothermal and thus internally reversible.

ANALYSIS: An energy balance reduces as follows:

$$\underbrace{\Delta U}_{=0} + \underbrace{\cancel{\Delta KE}^{\,0} + \cancel{\Delta PE}^{\,0}}_{\text{see model \#2}} = Q - W$$

$= 0$ Since T = constant and the ideal gas model applies.

$$\Rightarrow \quad W = Q$$

With Eq. 6.23 we get $Q = \int_1^2 T\,dS$ (Model #3)

$$\therefore W = \int_1^2 T\,dS$$

$$= T\,\Delta S$$

$$= 400K\left(-0.3\ \frac{kJ}{kg}\right)$$

$$= -120\ kJ$$

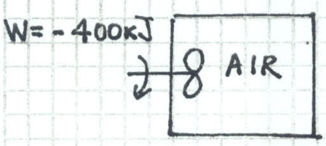
PROBLEM 6.41

Air contained in a rigid, insulated tank fitted with a paddle wheel, initially at 1 bar, 330 K and a volume of 1.93 m^3, receives an energy transfer by work from the paddle wheel in an amount of 400 kJ. Assuming the ideal gas model for the air, determine (a) the final temperature, in K, (b) the final pressure, in bar, and (c) the amount of entropy produced, in kJ/K. Ignore kinetic and potential energy.

KNOWN: Air in a rigid, insulated tank is stirred by a paddle wheel. State data and W are given.

FIND: Determine the final temperature and pressure and σ.

SCHEMATIC & GIVEN DATA:



$P_1 = 1$ bar, $T_1 = 330$K
$V = 1.93\ m^3$

ENGINEERING MODEL:

1. The air in the tank is the closed system.
2. For the system, $Q = 0$ and there are no effects of kinetic and potential energy.
3. The air is modeled as an ideal gas.

ANALYSIS:

(a) Applying an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$

$\Rightarrow \Delta U = -W$. Or, $m(u_2 - u_1) = -W \Rightarrow u_2 = u_1 - W/m$ (1)

We get u_1 from Table A-22: $u_1 = 235.61$ kJ/kg. To obtain m, apply the ideal gas model equation of state:

$$m = \frac{P_1 V}{R T_1} = \frac{(10^5 N/m^2)(1.93 m^3)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(330K)} = 2.04\ kg$$

Then Eq. (1) gives

$$u_2 = \left[235.61 - \frac{(-400)}{2.04}\right]\frac{kJ}{kg} = 431.69\ \frac{kJ}{kg}$$

Interpolating in Table A-22, $T_2 = 596$K

(b) Applying the ideal gas model equation of state,

$$\left.\begin{array}{l} P_1 V = m R T_1 \\ P_2 V = m R T_2 \end{array}\right\rangle \frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = P_1\left[\frac{T_2}{T_1}\right] = 1\text{ bar}\left[\frac{596K}{330K}\right] = 1.81\text{ bar}$$

(c) Applying an entropy balance, $\Delta S = \cancel{\int \frac{\delta Q}{T}}^{\,0} + \sigma$

$$\Rightarrow \sigma = m[S_2 - S_1] = m\left[s_2^\circ - s_1^\circ - R \ln\frac{P_2}{P_1}\right]$$

With s° data from Table A-22,

$$\sigma = 2.04 kg\left[2.40188 - 1.79783 - \frac{8.314}{28.97}\ln(1.81)\right]\frac{kJ}{kg\cdot K}$$

$$= 0.885\ \frac{kJ}{K}$$

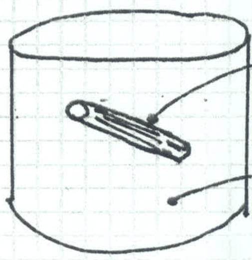
PROBLEM 6.61

A 33.8-lb aluminum bar, initially at 200°F, is placed in a tank together with 249 lb of liquid water, initially at 70°F, and allowed to achieve thermal equilibrium. The aluminum bar and water can be modeled as incompressible with specific heats 0.216 Btu/lb · °R and 0.998 Btu/lb · °R, respectively. For the aluminum bar and water as the system, determine (a) the final temperature, in °F, and (b) the amount of entropy produced within the tank, in Btu/°R. Ignore heat transfer between the system and its surroundings.

KNOWN: An aluminum bar is quenched in a tank of water.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the water plus bar. The total volume remains constant.
2. For the system, $Q = 0$, $W = 0$. Kinetic and potential energy play no role.
2. The water and bar are each modeled as incompressible with specific heats $c_a = 0.216$ Btu/lb·°R, $c_w = 0.998$ Btu/lb·°R.

ANALYSIS: (a) The energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$.

Thus, $\Delta U]_{aluminum} + \Delta U]_{water} = 0$. Using Eq. 3.20a, we get

$$m_a c_a [T_f - T_a] + m_w c_w [T_f - T_w] = 0$$

Solving for T_f, the final temperature,

$$T_f = \frac{m_a c_a T_a + m_w c_w T_w}{m_a c_a + m_w c_w}$$

$$= \frac{(33.8\,\text{lb})(0.216\,\text{Btu/lb·°R})(660°R) + (249\,\text{lb})(0.998\,\text{Btu/lb·°R})(530°R)}{((33.8)(0.216) + (249)(0.998))\,\text{Btu/°R}}$$

$$= 533.7\,°R \quad (74°F)$$

(b) Applying an entropy balance, $\Delta S = \cancel{\int_1^2 \delta Q/T}^{\,0} + \sigma$. Or $\sigma = (\Delta S)_a + (\Delta S)_w$. With Eq. 6.13

$$\sigma = m_a c_a \ln\frac{T_f}{T_a} + m_w c_w \ln\frac{T_f}{T_w}$$

$$= (33.8\,\text{lb})\left(0.216\,\frac{\text{Btu}}{\text{lb·°R}}\right)\ln\left[\frac{533.7°R}{660°R}\right] + (249)(0.998)\ln\left[\frac{533.7}{530}\right]$$

$$= 0.1789\,\frac{\text{Btu}}{°R}$$

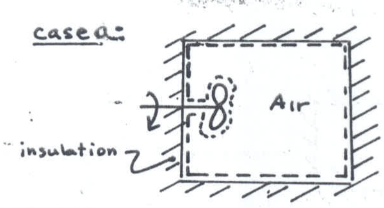
PROBLEM 6.69

KNOWN: A system consisting of air at a specified state undergoes constant volume processes in which the temperature increases in each of two ways.

FIND: (a) When the air is stirred adiabatically, determine the entropy produced. (b) When the air is heated by a reservoir at temperature T, plot the entropy produced versus T. Compare and discuss.

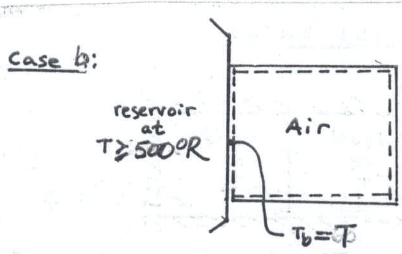
SCHEMATIC & GIVEN DATA:

case a:



$T_1 = 300\,K$
$P_1 = 1\ bar$
$T_2 = 500K$

Case b:



ENGR. MODEL: (1) The system consists of the air. (2) Air is modeled as an ideal gas. (3) In case (a), $Q = 0$, $W \neq 0$. In case (b), $Q \neq 0$, $W = 0$.

ANALYSIS: The change in entropy of the air is required in the evaluation of σ in each case. Thus, with data from Table A-22 and the ideal gas equation which gives $P_2/P_1 = T_2/T_1$

$$s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} = 2.21952 - 1.70203 - \frac{8.314}{28.97} \ln \frac{500}{300} = 0.3709\ kJ/kg\cdot K$$

CASE a: An entropy balance reduces to give

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b^{\,0} + \sigma_a \Rightarrow \frac{\sigma_a}{m} = s_2 - s_1 = 0.3709\ kJ/kg\cdot K \quad \text{(a)}$$

CASE b: An entropy balance reduces to give

$$\Delta S = \frac{Q}{T_b} + \sigma_b \Rightarrow \frac{\sigma_b}{m} = (s_2 - s_1) - \frac{Q/m}{T_b}$$

To find Q, write an energy balance: $\Delta U = Q - \cancel{W}^{\,0}$. Thus, with data from Table A-22

$$\frac{Q}{m} = u(T_2) - u(T_1) = 359.49 - 214.07 = 145.42\ kJ/kg$$

Thus

$$\frac{\sigma_b}{m} = 0.3709 - \frac{145.42}{T} \quad \text{(b)}$$

Sample calculation: When $T = 500K$, $\sigma_b/m = 0.08\ kJ/kg\cdot K$.

Equation (b) is plotted below. Comparing the two cases, we get

$$(\sigma_b/m) = (\sigma_a/m) - (145.42/T).$$

Thus, the entropy produced by heating is always less than the entropy produced by stirring. The entropy produced by heating approaches the entropy produced stirring at higher reservoir temperatures (mathematically, as $T \to \infty$). See the accompanying plot.