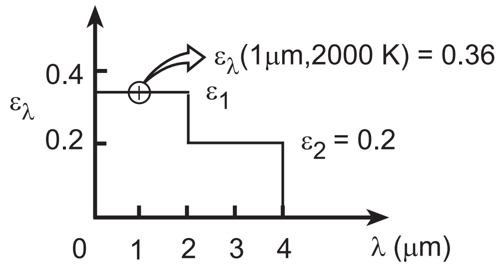
PROBLEM 12.37

KNOWN: Metallic surface with prescribed spectral, directional emissivity at 2000 K and 1 μm (see Example 12.7) and additional measurements of the spectral, hemispherical emissivity.

FIND: (a) Total hemispherical emissivity, ε, and the emissive power, E, at 2000 K, (b) Effect of temperature on the emissivity.

SCHEMATIC:



ANALYSIS: (a) The total, hemispherical emissivity, ε, may be determined from knowledge of the spectral, hemispherical emissivity, ε_λ, using Eq. 12.43.

$$\varepsilon(T) = \int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda,T)\,d\lambda \big/ E_b(T) = \varepsilon_1 \int_0^{2\mu m} \frac{E_{\lambda,b}(\lambda,T)d\lambda}{E_b(T)} + \varepsilon_2 \int_{2\mu m}^{4\mu m} \frac{E_{\lambda,b}(\lambda,T)d\lambda}{E_b(T)}$$

or from Eq. 12.34,

$$\varepsilon(T) = \varepsilon_1 F_{(0\to\lambda_1)} + \varepsilon_2 \left[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)} \right]$$

From Table 12.1,

$$\lambda_1 = 2\,\mu m, \quad T = 2000\text{ K}: \quad \lambda_1 T = 4000\,\mu m\cdot K, \quad F_{(0\to\lambda_1)} = 0.481$$

$$\lambda_2 = 4\,\mu m, \quad T = 2000\text{ K}: \quad \lambda_2 T = 8000\,\mu m\cdot K, \quad F_{(0\to\lambda_2)} = 0.856$$

Hence,

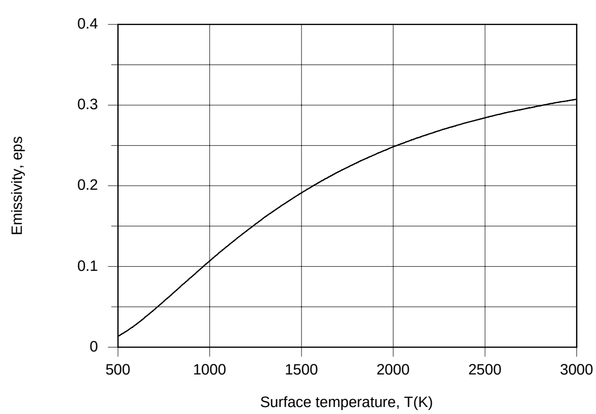
$$\varepsilon(T) = 0.36 \times 0.481 + 0.20(0.856 - 0.481) = 0.25$$ <

The total emissive power at 2000 K is

$$E(2000\text{ K}) = \varepsilon(2000\text{ K}) \cdot E_b(2000\text{ K})$$

$$E(2000\text{ K}) = 0.25 \times 5.67 \times 10^{-8}\,W/m^2\cdot K^4 \times (2000\text{ K})^4 = 2.27 \times 10^5\,W/m^2\,.$$ <

(b) Using the *Radiation* Toolpad of IHT, the following result was generated.



Continued...

PROBLEM 12.37 (Cont.)

At $T \approx 500$ K, most of the radiation is emitted in the far infrared region ($\lambda > 4$ μm), in which case $\varepsilon \approx 0$. With increasing T, emission is shifted to lower wavelengths, causing ε to increase. As $T \rightarrow \infty$, $\varepsilon \rightarrow 0.36$.

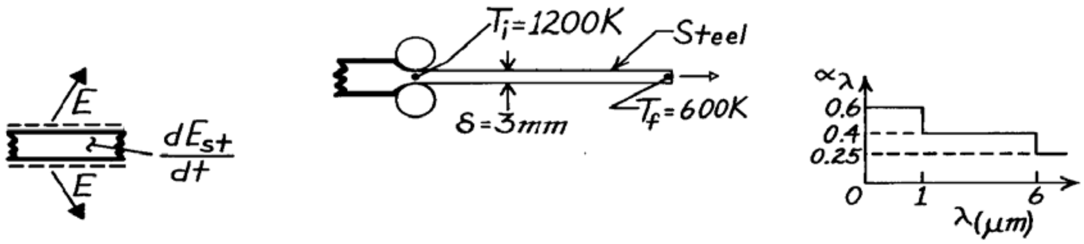
COMMENTS: Note that the value of ε_λ for $0 < \lambda \leq 2$ μm cannot be read directly from the ε_λ distribution provided in the problem statement. This value is calculated from knowledge of $\varepsilon_{\lambda,\theta}(\theta)$ in Example 12.7.

PROBLEM 12.47

KNOWN: Temperature, thickness and spectral emissivity of steel strip emerging from a hot roller. Temperature dependence of total, hemispherical emissivity.

FIND: (a) Initial total, hemispherical emissivity, (b) Initial cooling rate, (c) Time to cool to prescribed final temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible conduction (in longitudinal direction), convection and radiation from surroundings, (2) Negligible transverse temperature gradients.

PROPERTIES: Steel (given): $\rho = 7900\ \text{kg/m}^3$, $c = 640\ \text{J/kg}\cdot\text{K}$, $\varepsilon = 1200\varepsilon_i/T$ (K).

ANALYSIS: (a) The initial total hemispherical emissivity is

$$\varepsilon_i = \int_0^\infty \varepsilon_\lambda \left[E_{\lambda b}(1200)/E_b(1200)\right] d\lambda$$

and integrating by parts using values from Table 12.1, find

$$\lambda T = 1200\ \mu\text{m}\cdot\text{K} \to F_{(0-1\,\mu\text{m})} = 0.002;\ \lambda T = 7200\ \mu\text{m}\cdot\text{K} \to F_{(0-6\,\mu\text{m})} = 0.819$$

$$\varepsilon_i = 0.6\times 0.002 + 0.4(0.819-0.002) + 0.25(1-0.819) = 0.373.$$ <

(b) From an energy balance on a unit surface area of strip (top and bottom),

$$-\dot{E}_{out} = dE_{st}/dt \qquad -2\varepsilon\sigma T^4 = d(\rho\delta cT)/dt$$

$$\left.\frac{dT}{dt}\right)_i = -\frac{2\varepsilon_i\sigma T_i^4}{\rho\delta c} = \frac{-2(0.373)5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4(1200\ \text{K})^4}{7900\ \text{kg/m}^3(0.003\ \text{m})(640\ \text{J/kg}\cdot\text{K})} = -5.78\ \text{K/s}.$$ <

(c) From the energy balance,

$$\frac{dT}{dt} = -\frac{2\varepsilon_i(1200/T)\sigma T^4}{\rho\delta c},\ \int_{T_i}^{T_f}\frac{dT}{T^3} = -\frac{2400\varepsilon_i\sigma}{\rho\delta c}\int_0^t dt,\quad t = \frac{\rho\delta c}{4800\varepsilon_i\sigma}\left(\frac{1}{T_f^2}-\frac{1}{T_i^2}\right)$$

$$t = \frac{7900\ \text{kg/m}^3(0.003\,\text{m})640\ \text{J/kg}\cdot\text{K}}{4800\ \text{K}\times 0.373\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\left(\frac{1}{600^2}-\frac{1}{1200^2}\right)\text{K}^{-2} = 311\ \text{s}.$$ <

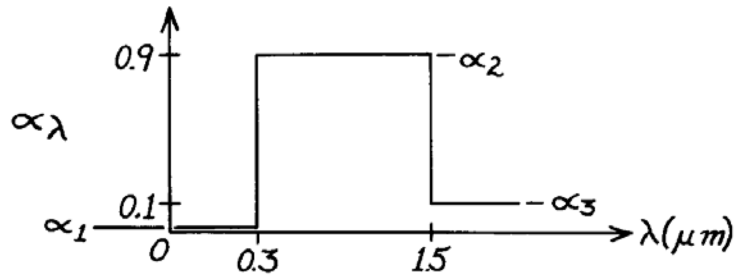
COMMENTS: Initially, from Eq. 1.9, $h_r \approx \varepsilon_i\sigma T_i^3 = 36.6\ \text{W/m}^2\cdot\text{K}$. Assuming a plate width of W = 1m, the Rayleigh number may be evaluated from $Ra_L = g\beta(T_i - T_\infty)(W/2)^3/\nu\alpha$. Assuming $T_\infty = 300$ K and evaluating properties at $T_f = 750$ K, $Ra_L = 1.8\times 10^8$. From Eq. 9.31, $Nu_L = 84$, giving $\overline{h} = 9.2\ \text{W/m}^2\cdot\text{K}$. Hence heat loss by radiation exceeds that associated with free convection. To check the validity of neglecting transverse temperature gradients, compute $Bi = h(\delta/2)/k$. With $h = 36.6\ \text{W/m}^2\cdot\text{K}$ and $k = 28\ \text{W/m}\cdot\text{K}$, $Bi = 0.002 \ll 1$. Hence the assumption is valid.

PROBLEM 12.56

KNOWN: Spectral, hemispherical absorptivity of an opaque surface.

FIND: (a) Solar absorptivity, (b) Total, hemispherical emissivity for T_s = 340K.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque, (2) $\varepsilon_\lambda = \alpha_\lambda$, (3) Solar spectrum has $G_\lambda = G_{\lambda,S}$ proportional to $E_{\lambda,b}$ (λ, 5800K).

ANALYSIS: (a) The solar absorptivity follows from Eq. 12.53.

$$\alpha_S = \int_0^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda, 5800\text{K})\, d\lambda \Big/ \int_0^\infty E_{\lambda,b}(\lambda, 5800\text{K})\, d\lambda.$$

The integral can be written in three parts using $F_{(0 \to \lambda)}$ terms.

$$\alpha_S = \alpha_1 F_{(0\to 0.3\mu m)} + \alpha_2 \left[F_{(0\to 1.5\mu m)} - F_{(0\to 0.3\mu m)}\right] + \alpha_3 \left[1 - F_{(0\to 1.5\mu m)}\right].$$

From Table 12.1,

$\lambda T = 0.3 \times 5800 = 1740\ \mu m \cdot K$	$F_{(0 \to 0.3\ \mu m)} = 0.0335$
$\lambda T = 1.5 \times 5800 = 8700\ \mu m \cdot K$	$F_{(0 \to 1.5\ \mu m)} = 0.8805.$

Hence,

$$\alpha_S = 0 \times 0.0355 + 0.9[0.8805 - 0.0335] + 0.1[1 - 0.8805] = 0.774.$$ <

(b) The total, hemispherical emissivity for the surface at 340K will be

$$\varepsilon = \int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, 340\text{K})\, d\lambda / E_b(340\text{K}).$$

If $\varepsilon_\lambda = \alpha_\lambda$, then using the α_λ distribution above, the integral can be written in terms of $F_{(0 \to \lambda)}$ values. It is readily recognized that since

$$F_{(0\to 1.5\,\mu m, 340K)} \approx 0.000 \qquad \text{at} \qquad \lambda T = 1.5 \times 340 = 510\ \mu m \cdot K$$

there is negligible spectral emissive power below 1.5 μm. It follows then that

$$\varepsilon = \varepsilon_\lambda = \alpha_\lambda = 0.1$$ <

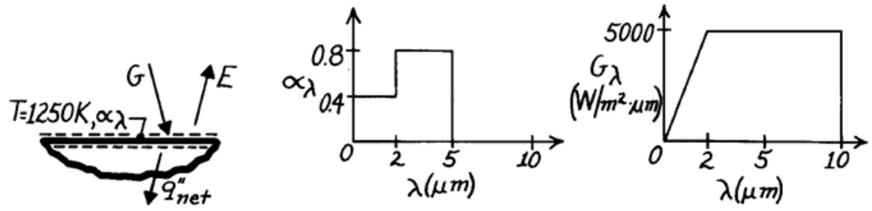
COMMENTS: The assumption $\varepsilon_\lambda = \alpha_\lambda$ can be satisfied if this surface were irradiated diffusely or if the surface itself were diffuse. Note that for this surface under the specified conditions of solar irradiation and surface temperature $\alpha_S \neq \varepsilon$. Such a surface is referred to as a spectrally selective surface.

PROBLEM 12.58

KNOWN: Spectral distribution of surface absorptivity and irradiation. Surface temperature.

FIND: (a) Total absorptivity, (b) Emissive power, (c) Nature of surface temperature change.

SCHEMATIC:



ASSUMPTIONS: (1) Opaque, diffuse surface behavior, (2) Convection effects are negligible.

ANALYSIS: (a) From Eqs. 12.51 and 12.52, the absorptivity is defined as

$$\alpha \equiv G_{abs} / G = \int_0^\infty \alpha_\lambda G_\lambda \, d\lambda / \int_0^\infty G_\lambda \, d\lambda.$$

The absorbed irradiation is,

$$G_{abs} = 0.4\left(5000\,W/m^2 \cdot \mu m \times 2\,\mu m\right)/2 + 0.8 \times 5000\,W/m^2 \cdot \mu m\left(5-2\right)\mu m + 0 = 14,000\ W/m^2.$$

The irradiation is,

$$G = \left(2\,\mu m \times 5000\ W/m^2 \cdot \mu m\right)/2 + \left(10-2\right)\mu m \times 5000\ W/m^2 \cdot \mu m = 45,000\ W/m^2.$$

Hence, $\alpha = 14,000\ W/m^2 / 45,000\ W/m^2 = 0.311.$ <

(b) From Eq. 12.43, the emissivity is

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b} \, d\lambda / E_b = 0.4 \int_0^2 E_{\lambda,b} \, d\lambda / E_b + 0.8 \int_2^5 E_{\lambda,b} \, d\lambda / E_b$$

From Table 12.1, $\lambda T = 2\ \mu m \times 1250K = 2500K$, $F_{(0-2)} = 0.162$

$\lambda T = 5\ \mu m \times 1250K = 6250K$, $F_{(0-5)} = 0.757.$

Hence, $\varepsilon = 0.4 \times 0.162 + 0.8\left(0.757 - 0.162\right) = 0.54.$

$$E = \varepsilon E_b = \varepsilon \sigma T^4 = 0.54 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \left(1250K\right)^4 = 74,751\ W/m^2.$$ <

(c) From an energy balance on the surface, the net heat flux *to* the surface is

$$q''_{net} = \alpha G - E = \left(14,000 - 74,751\right) W/m^2 = -60,751\,W/m^2.$$

Hence the temperature of the surface is *decreasing.* <

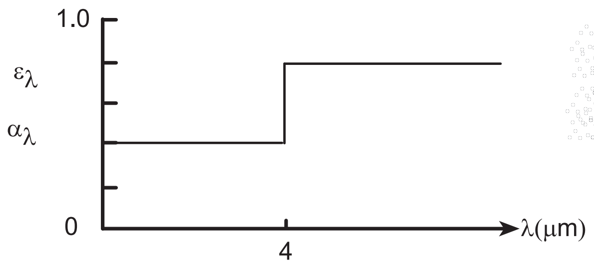
COMMENTS: Note that $\alpha \neq \varepsilon$. Hence the surface is not gray for the prescribed conditions.

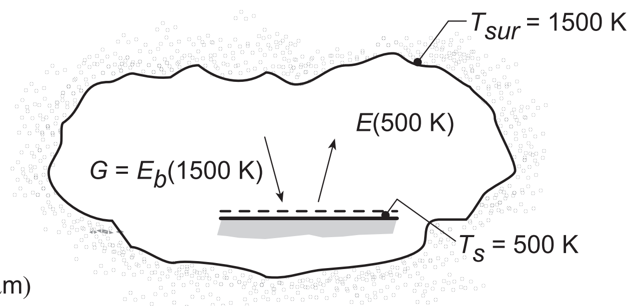
PROBLEM 12.71

KNOWN: Temperature and spectral characteristics of a diffuse surface at T_s = 500 K situated in a large enclosure with uniform temperature, T_{sur} = 1500 K.

FIND: (a) Sketch of spectral distribution of E_λ and $E_{\lambda,b}$ for the surface, (b) Net heat flux to the surface, $q''_{rad,in}$ (c) Compute and plot $q''_{rad,in}$ as a function of T_s for the range $500 \le T_s \le 1000$ K; also plot the heat flux for a diffuse, gray surface with total emissivities of 0.4 and 0.8; and (d) Compute and plot ε and α as a function of the surface temperature for the range $500 \le T_s \le 1000$ K.

SCHEMATIC:

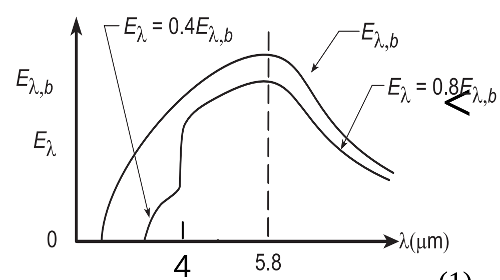




ASSUMPTIONS: (1) Surface is diffuse, (2) Convective effects are negligible, (3) Surface irradiation corresponds to blackbody emission at 1500 K.

ANALYSIS: (a) From Wien's displacement law, Eq. 12.31, $\lambda_{max} T$ = 2898 μm·K. Hence, for blackbody emission from the surface at T_s = 500 K,

$$\lambda_{max} = \frac{2898 \mu m \cdot K}{500\ K} = 5.80 \mu m\ .$$



<

(b) From an energy balance on the surface, the net heat flux to the surface is

$q''_{rad,in} = \alpha G - E = \alpha E_b\,(1500\ K) - \varepsilon E_b\,(500\ K)$. (1)

From Eq. 12.52,

$$\alpha = 0.4\int_0^4 \frac{E_{\lambda,b}(1500)}{E_b} d\lambda + 0.8\int_4^\infty \frac{E_{\lambda,b}(1500)}{E_b} d\lambda = 0.4F_{(0-4\mu m)} - 0.8[1 - F_{(0-4\mu m)}].$$

From Table 12.1 with λT = 4μm × 1500 K = 6000 μm·K, $F_{(0-4)}$ = 0.738, find

α = 0.4 × 0.738 + 0.8 (1 – 0.738) = 0.505.

From Eq. 12.43

$$\varepsilon = 0.4\int_0^4 \frac{E_{\lambda,b}(500)}{E_b} d\lambda + 0.8\int_4^\infty \frac{E_{\lambda,b}(500)}{E_b} d\lambda = 0.4F_{(0-4\mu m)} + 0.8[1 - F_{(0-4\mu m)}]\ .$$

From Table 12.1 with λT = 4μm × 500 K = 2000 μm·K, $F_{(0-4)}$ = 0.0667, find
ε = 0.4 × 0.0667 + 0.8 (1 – 0.0667) = 0.773.

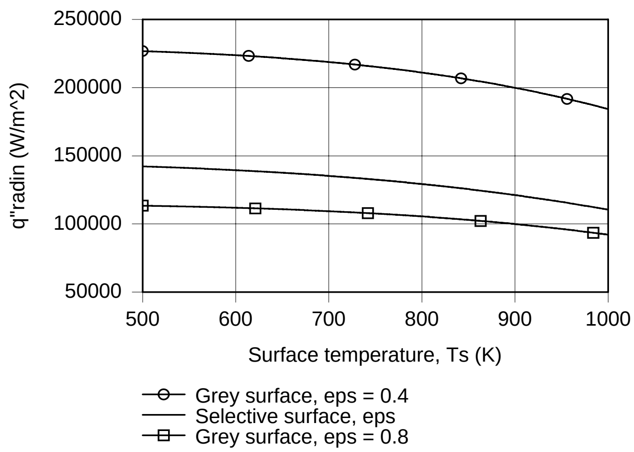
Hence, the net heat flux to the surface is

$$q''_{rad,in} = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 [0.505 \times (1500\ K)^4 - 0.773 \times (500\ K)^4] = 1.422 \times 10^5\ W/m^2\ .$$ <

Continued...

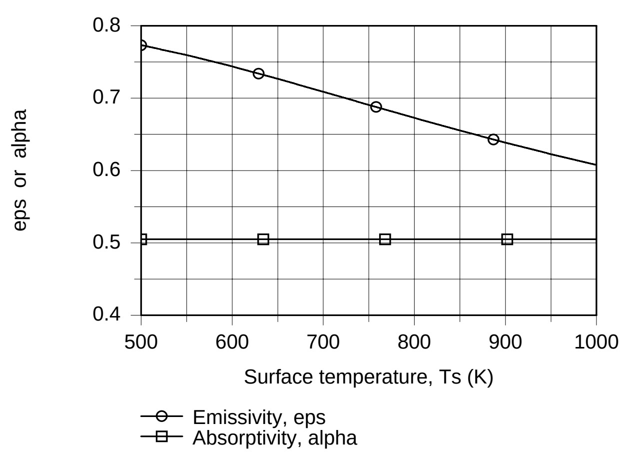
PROBLEM 12.71 (Cont.)

(c) Using the foregoing equations in the IHT workspace along with the *IHT Radiation Tool*, *Band Emission Factor*, $q''_{rad,in}$ was computed and plotted as a function of T_s.



The net radiation heat rate, $q''_{rad,in}$ decreases with increasing surface temperature since E increases with T_s and the absorbed irradiation remains constant according to Eq. (1). The heat flux is largest for the gray surface with $\varepsilon = 0.4$ and the smallest for the gray surface with $\varepsilon = 0.8$. As expected, the heat flux for the selective surface is between the limits of the two gray surfaces.

(d) Using the IHT model of part (c), the emissivity and absorptivity of the surface are computed and plotted below.



The absorptivity, $\alpha = \alpha(\alpha_\lambda, T_{sur})$, remains constant as T_s changes since it is a function of α_λ (or ε_λ) and T_{sur} only. The emissivity, $\varepsilon = \varepsilon(\varepsilon_\lambda, T_s)$ is a function of T_s and decreases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution? Under what condition would you expect $\alpha = \varepsilon$?