ME 4189 Spring 2016
Homework assignment #1- Solutions
Due on Thursday 01/21/2015

- 1.31

1 Problem 1.31

Moment equilibrium around point O:

$$0 = M_O = Fl + F_1 l_1 + F_2 l_2 + F_3 l$$
$$F = -\frac{F_1 l_1 + F_2 l_2 + F_3 l}{l} \qquad (1)$$

where

$$F_1 = -k_1 x_1 = -k_1 \frac{l_1}{l} x \qquad (2)$$
$$F_2 = -k_2 x_2 = -k_2 \frac{l_2}{l} x \qquad (3)$$
$$F_3 = -k_3 x \qquad (4)$$

such that :

$$F = k_1 (\frac{l_1}{l})^2 x + k_2 (\frac{l_2}{l})^2 x + k_3 x \qquad (5)$$

Using the values of l_1, l_2, k_1, k_2 and k_3, we obtain:

$$F = k\Big[(\frac{1}{3})^2 + (\frac{2}{3})^2 + 3\Big] x$$
$$= 4kx \qquad (6)$$
$$= k_{eff} x$$

where

$$k_{eff} = 4k \qquad (7)$$

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- 1.76

1.76 $\vec{x}_1 = 1 + 2i = a_1 + a_2 i$, $\vec{x}_2 = 3 - 4i = b_1 + b_2 i$

$\vec{x} = \vec{x}_1 + \vec{x}_2 = (a_1 + b_1) + i(a_2 + b_2) = 4 - 2i$

$= A e^{i\theta} = A \cos\theta + i A \sin\theta$

$A = \sqrt{4^2 + (-2)^2} = 4.4721$

$\theta = \tan^{-1}\left(\frac{-2}{4}\right) = -26.5651^\circ$

In radians, theta=-0.4636 rad

- 1.82

1.82 (a) $x(t) = \frac{A}{1000}\cos(50t + \alpha)$ m where A is in mm ---- (E_1)

$$x(0) = \frac{A}{1000}\cos\alpha = 0{\cdot}003 \;, \qquad A\cos\alpha = 3 \quad \text{---- } (E_2)$$

$$\dot{x}(0) = -\frac{50A}{1000}\sin\alpha = 1 \;, \qquad A\sin\alpha = -20 \quad \text{---- } (E_3)$$

$$A = \left\{(A\cos\alpha)^2 + (A\sin\alpha)^2\right\}^{1/2} = 20{\cdot}2237 \text{ mm}$$

$$\alpha = \tan^{-1}\left(\frac{A\sin\alpha}{A\cos\alpha}\right) = \tan^{-1}(-6{\cdot}6667) = -81{\cdot}4692^\circ = -1{\cdot}4219 \text{ rad}$$

$$x(t) = 20{\cdot}2237\cos(50t - 1{\cdot}4219) \text{ mm}$$

(b) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

Eq. (E_1) can be expressed as $x(t) = A\cos 50t\cdot\cos\alpha - A\sin 50t\cdot\sin\alpha$

$$= A_1\cos\omega t + A_2\sin\omega t$$

where $\omega = 50$, $A_1 = A\cos\alpha$, $A_2 = -A\sin\alpha$

$$\therefore\; x(t) = (3\cos 50t + 20\sin 50t) \text{ mm}$$

- 1.97

2 Problem 1.97

$$x(t) = -3\sin 5t - 2\cos 5t \tag{8}$$

We want to express in the form:

$$\begin{aligned} x(t) &= A\cos(5t + \phi) \\ &= A\cos 5t\cos\phi - A\sin 5t\sin\phi \end{aligned} \tag{9}$$

Therefore,

$$A\cos\phi = -2 \tag{10}$$

$$A\sin\phi = 3 \tag{11}$$

which gives

$$\tan\phi = -\frac{3}{2} \tag{12}$$

$$A^2 = 2^2 + 3^2 = 13 \tag{13}$$

Therefore

$$\phi = \tan^{-1}(-\frac{3}{2}) = -0.9828 \tag{14}$$

$$A = \pm\sqrt{13} = \pm 3.6065 \tag{15}$$

Should we choose the positive or negative root for A?

At t=0, x(0)=-2.0. If we choose the positive root, we get:

$$x(0) = \sqrt{13}\cos(-0.9828) = 2 \tag{16}$$

hence the positive root is not the solution. Hence $A = -3.6065$, and $x(t)$ can be written:

$$x(t) = -3.6065\cos(5t - 0.9828) \tag{17}$$

- 2.7

(2.7) For small angular rotation of bar PQ about P,

$$\frac{1}{2}(k_{12})_{eq}(\theta l_3)^2 = \frac{1}{2}k_1(\theta l_1)^2 + \frac{1}{2}k_2(\theta l_2)^2$$

i.e., $$(k_{12})_{eq} = \left(k_1 l_1^2 + k_2 l_2^2\right) / l_3^2$$

Let k_{eq} = overall spring constant at Q.

$$\frac{1}{k_{eq}} = \frac{1}{(k_{12})_{eq}} + \frac{1}{k_3}$$

$$k_{eq} = \frac{(k_{12})_{eq}\, k_3}{(k_{12})_{eq} + k_3} = \frac{\left\{k_1\left(\frac{l_1}{l_3}\right)^2 + k_2\left(\frac{l_2}{l_3}\right)^2\right\} k_3}{k_1\left(\frac{l_1}{l_3}\right)^2 + k_2\left(\frac{l_2}{l_3}\right)^2 + k_3}$$

Natural frequency:

$$\begin{aligned}
\omega_n &= \sqrt{\frac{K_{eq}}{m}} \\
&= \sqrt{\frac{k_3 k_1(\frac{l_1}{l_3})^2 + k_3 k_2(\frac{l_2}{l_3})^2}{m\left[k_1(\frac{l_1}{l_3})^2 + k_2(\frac{l_2}{l_3})^2 + k_3\right]}} \\
&= \sqrt{\frac{k_3 k_1 l_1^2 + k_3 k_2 l_2^2}{m\left[k_1 l_1^2 + k_2 l_2^2 + k_3 l_3^2\right]}}
\end{aligned}$$