4.4

The consequences are numerous, and the oxide can be beneficial as well as detrimental. In sliding contact, the oxide is a hard surface that, as a result, is wear resistant [see Eq. (4.6) on p. 145], and it can also protect the substrate from further chemical attack. However, if an oxide wear particle spalls from the surface, a detrimental three-body wear situation can result. Also, as discussed in Chapter 2, the hard surface layers may be detrimental from a fatigue standpoint if their ductility is compromised. Finally, if a material is plastically deformed, as in the processes described in Chapters 6 and 7, the oxide layer may crack or even break off, resulting in a surface finish that may be unacceptable for the particular application.

4.9

This situation indicates that profilometer traces are not exact duplicates of actual surfaces and that such readings can be misleading for precise study of surfaces. (Note, however, that the roughness in Fig. 4.4 on p. 137 is highly exaggerated because of the differences between the horizontal and vertical scales.) For example, surfaces with deep narrow valleys will be measured smoother than they really are. This can have significant effects on the estimating the fatigue life, corrosion, and proper assessment of the capabilities of various manufacturing processes.

4.57

This solution uses the continuous forms of roughness given by Eqs. (4.1) and (4.2) on p. 134.

(a) The equation of a sine wave with amplitude a is

$$y = a \sin \frac{2\pi x}{l}$$

Thus, Eq. (4.1) gives

$$R_a = \frac{1}{l}\int_0^l \left| a \sin \frac{2\pi x}{l} \right| dx = \frac{2a}{l}\int_0^{l/2} \sin \frac{2\pi x}{l} dx$$

Integrating,

$$\begin{aligned} R_a &= -\frac{2a}{l}\frac{l}{2\pi}\left(\cos \frac{2\pi x}{l}\right)_0^{l/2} \\ &= -\frac{a}{\pi}(\cos \pi - \cos 0) = -\frac{a}{\pi}(-1-1) \\ &= \frac{2a}{\pi} \end{aligned}$$

To evaluate R_q for a sine wave, recall that

$$\int \sin^2 u \, du = \frac{u}{2} - \frac{1}{4}\sin 2u$$

which can be obtained from any calculus book or table of integrals. Therefore, from Eq. (4.2),

$$R_q^2 = \frac{1}{l}\int_0^l y^2 \, dx = \frac{1}{l}\int_0^l a^2 \sin^2 \frac{2\pi x}{l} \, dx$$

Evaluating the integral,

$$\begin{aligned} R_q^2 &= \frac{a^2}{l}\frac{l}{2\pi}\left[\frac{2\pi x}{2l} - \frac{1}{4}\sin \frac{4\pi x}{l}\right]_0^l \\ &= \frac{a^2}{2\pi}\left[(\pi - 0) - (0-0)\right] = \frac{a^2}{2} \end{aligned}$$

So that $R_q = \frac{a}{\sqrt{2}}$, and

$$\frac{R_a}{R_q} = \frac{2a/\pi}{a/\sqrt{2}} = \frac{2\sqrt{2}}{\pi} \approx 0.90$$

(b) For a saw-tooth profile, we can use symmetry to evaluate R_a and R_q over one-fourth of the saw tooth. The equation for the curve over this range is:

$$y = \frac{4a}{l}x$$

so that, from Eq. (4.1),

$$R_a = \frac{4}{l}\int_0^{l/4} \frac{4ax}{l} dx$$

Evaluating the integral,

$$R_a = \frac{16a}{l^2}\left(\frac{1}{2}x^2\right)_0^{l/4} = \frac{16a}{l^2}\frac{1}{2}\left(\frac{l^2}{16} - 0\right) = \frac{a}{2}$$

From Eq. (4.2),

$$R_q^2 = \frac{4}{l}\int_0^{l/4} y^2 \, dx = \frac{4}{l}\int_0^{l/4} \frac{16a^2}{l^2}x^2 \, dx$$

Evaluating the integral,

$$R_q^2 = \frac{64a^2}{l^3}\left(\frac{1}{3}x^3\right)_0^{l/4} = \frac{64a^2}{l^3}\frac{1}{3}\frac{l^3}{64} = \frac{a^2}{3}$$

Therefore, $R_q = \frac{a}{\sqrt{3}}$ and

$$\frac{R_a}{R_q} = \frac{a/2}{a/\sqrt{3}} = \frac{\sqrt{3}}{2} \approx 0.866$$

(c) For a square wave with amplitude a,

$$R_a = \frac{1}{l}\int_0^l a\,dx = \frac{a}{l}(x)_0^l = a$$

and

$$R_q^2 = \frac{1}{l}\int_0^l a^2\,dx = \frac{a^2}{l}(x)_0^l = a^2$$

so that $R_q = a$. Therefore,

$$\frac{R_a}{R_q} = \frac{a}{a} = 1.0$$

4.68

In this case each row is a sample, and the sample size is 4. From Table 4.3, A2=0.729, D4=2.282, and $D_3 = 0$. We calculate averages and ranges and fill in the chart as follows:

x_1	x_2	x_3	x_4	$\bar{x}$	R
0.65	0.75	0.67	0.65	0.6800	0.10
0.69	0.73	0.70	0.68	0.7000	0.05
0.65	0.68	0.65	0.61	0.6475	0.07
0.64	0.65	0.60	0.60	0.6225	0.05
0.68	0.72	0.70	0.66	0.6900	0.06
0.70	0.74	0.65	0.71	0.7000	0.09

Therefore, the average of averages is $\bar{\bar{x}} = 0.6733$, and the average range is $\bar{R} = 0.07$. Eqs. (4.11) and (4.12) give the upper and lower control limits for the averages as

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = (0.6733) + (0.729)(0.07)$$

or $\text{UCL}_{\bar{x}}$=0.7243.

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = (0.6733) - (0.729)(0.07)$$

or $\text{LCL}_{\bar{x}}$=0.6223. For the ranges, Eqs. (4.13) and (4.14) yield

$$\text{UCL}_R = D_4\bar{R} = (2.282)(0.07) = 0.1600$$

$$\text{LCR}_R = D_3\bar{R} = (0)(0.07) = 0$$