Homework Set 5

Due date: March 28, 2016 at the beginning of class

Problem 1: The hemispherical spectral emissivity, ε_λ of a selective diffuse surface of TiO_2 shown schematically in Figure 1 is $\varepsilon_\lambda = 0.90$ for $\lambda \leq 0.6$ μm, and $\varepsilon_\lambda = 0.25$ for $\lambda > 0.6$ μm. Calculate the hemispherical, total absorptivity, α, for incident radiation from a black source at 2000 K and for incident solar radiation assuming a blackbody at 5780 K. Calculate the equilibrium temperature when the normal incident solar flux is 800 W/m^2 assuming a perfectly insulated (i.e. adiabatic) backside of the TiO_2 and no convective heat transfer at the surface. *Hint:* iteration may be required to determine T_{eq}.

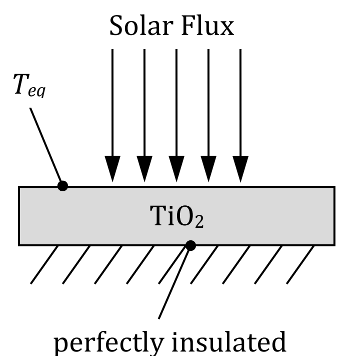

Figure 1.

Apply Kirchhoff's law:

$$\varepsilon_\lambda(\lambda,\mathrm{T}) = \alpha_\lambda(\lambda,\mathrm{T})$$

Integration:

$$\alpha(\mathrm{T}) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,\mathrm{T})\mathrm{E}_{\lambda b}(\lambda,\mathrm{T})d\lambda}{\sigma \mathrm{T}^4} = \frac{0.9\int_0^{0.6\,\mu m} \mathrm{E}_{\lambda b}(\lambda,\mathrm{T})d\lambda + 0.25\int_{0.6\,\mu m}^{\infty} \mathrm{E}_{\lambda b}(\lambda,\mathrm{T})d\lambda}{\sigma \mathrm{T}^4}$$

$$\mathrm{F}_{0-0.6\,\mu m\,\mathrm{T}} = \frac{\int_0^{0.6\,\mu m} \mathrm{E}_{\lambda b}(\lambda,\mathrm{T})d\lambda}{\sigma \mathrm{T}^4}$$

$$\mathrm{F}_{0.6\,\mu m\,\mathrm{T}-\infty} = 1 - \mathrm{F}_{0-0.6\,\mu m\,\mathrm{T}}$$

$$\alpha(\mathrm{T}) = 0.9\mathrm{F}_{0-0.6\,\mu m\,\mathrm{T}} + 0.25\left(1 - \mathrm{F}_{0-0.6\,\mu m\,\mathrm{T}}\right)$$

For T=2000 K

$$\mathrm{F}_{0-0.6\,\mu m\,2000\,\mathrm{K}} = 0.00213$$

$$\alpha(2000\ \mathrm{K}) = 0.9 \times 0.00213 + 0.25 \times (1 - 0.00213)$$

$$\Rightarrow \alpha(2000\ \mathrm{K}) = 0.2514$$

For T=5780 K

$$\mathrm{F}_{0-0.6\,\mu m\,5780\,\mathrm{K}} \cong 0.37766$$

$$\alpha(5780\ \mathrm{K}) = 0.9 \times 0.37766 + 0.25 \times (1 - 0.37766)$$

$$\Rightarrow \underline{\underline{\alpha(5780\ \mathrm{K}) = 0.4955}}$$

Initial guess: T_{eq} =1500 K

$$\varepsilon\left(T_{eq}\right) = 0.9F_{0-0.6\ \mu m\ T_{eq}} + 0.25\left(1 - F_{0-0.6\ \mu m\ T_{eq}}\right)$$

$$F_{0-0.6\ \mu m \times 1500\ K} = 8.70 \times 10^{-5}$$

$$\varepsilon\left(1500\ K\right) = 0.9 \times 8.70 \times 10^{-5} + 0.25 \times \left(1 - 8.70 \times 10^{-5}\right) = 0.2501$$

From an energy balance:

$$\frac{Q_i}{A} = \alpha\left(T_{sun}\right) I_{Solar\ Flux} \cos\theta_S = \frac{Q_e}{A} = \varepsilon\left(T_{eq}\right)\sigma T_{eq}^4$$

$$T_{eq} = \sqrt[4]{\frac{\alpha\left(T_{sun}\right) I_{Solar\ Flux} \cos\theta_S}{\varepsilon\left(T_{eq}\right)\sigma}}$$

$$T_{eq} = \sqrt[4]{\frac{0.4955 \times 800 \frac{W}{m^2} \cos 0}{0.2501 \times 5.61051 \times 10^{-8} \frac{W}{m^2 K^4}}}$$

$$T_{eq} = 410.0\ K$$

Recalculating the $F_{0-\lambda T} = F_{0-0.6 \times 410\ K} = 0.000$ ε=0.25

$$\Rightarrow T_{eq} = \sqrt[4]{\frac{0.4955 \times 800 \frac{W}{m^2} \cos 0}{0.25 \times 5.61051 \times 10^{-8} \frac{W}{m^2 K^4}}}$$

$$\Rightarrow \underline{\underline{T_{eq} = 410.0\ K}}$$

Problem 2: Determine all view factors between diffuse surfaces (A_1, A_2, and A_3) shown schematically with dimensions in Figure 2. *Hint:* A_2, can be calculated as: $A_2 = \pi\left(r_1 + r_3\right)\sqrt{\left(r_3 - r_1\right)^2 + h^2}$.

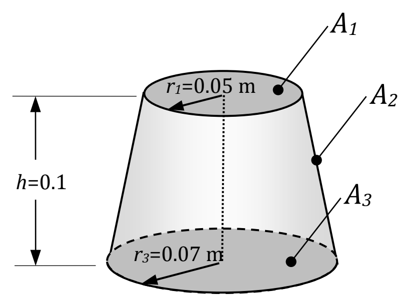

Figure 2.

Surface 1

$$R_1 = \frac{r_3}{h} = \frac{0.05\ m}{0.1\ m} = 0.5$$

$$R_2 = \frac{r_1}{h} = \frac{0.07\ m}{0.1\ m} = 0.7$$

$$X = 1 + \frac{1 + R_2^2}{R_1^2} = 1 + \frac{1 + 0.7^2}{0.5^2} = 6.96$$

$$\Rightarrow F_{1-1} = 0$$

$$F_{1-3} = \frac{1}{2}\left(X - \sqrt{X^2 - 4\left(\frac{R_2}{R_1}\right)^2}\right) = \frac{1}{2}\left(6.96 - \sqrt{6.96^2 - 4\left(\frac{0.7}{0.5}\right)^2}\right) = 0.294$$

$$F_{1-2} = 1 - F_{1-1} - F_{1-2} = 1 - 0.294 = 0.706$$

Surface 3

$$\Rightarrow F_{3-1} = F_{1-3}\frac{A_1}{A_3} = F_{1-3}\left(\frac{r_1}{r_3}\right)^2 = 0.294\left(\frac{0.05\text{ m}}{0.07\text{ m}}\right)^2 = 0.150$$

$$\Rightarrow F_{3-3} = 0$$

$$\Rightarrow F_{3-2} = 1 - F_{3-1} - F_{3-3} = 1 - 0.15 = 0.85$$

Surface 2

$$A_2 = \pi\left(r_1 + r_3\right)\sqrt{\left(r_3 - r_1\right)^2 + h^2} = \pi\left(0.05\text{ m} + 0.07\text{ m}\right)\sqrt{\left(0.02\text{ m}\right)^2 + 0.1\text{ m}^2} = 0.0384\text{ m}^2$$

$$\Rightarrow F_{2-1} = F_{1-2}\frac{A_1}{A_2} = F_{1-2}\frac{\pi r_1^2}{A_2} = 0.706\times\frac{\pi\times 0.05^2}{0.0384} = 0.1442$$

$$\Rightarrow F_{2-3} = F_{3-2}\frac{A_3}{A_2} = F_{1-2}\frac{\pi r_1^2}{A_2} = 0.85\times\frac{\pi\times 0.07^2}{0.0384} = 0.3407$$

$$\Rightarrow F_{2-2} = 1 - F_{2-3} - F_{2-1} = 1 - 0.1442 - 0.3407 = 0.5151$$