5.20b

$$\{x(0)\} = \begin{Bmatrix} 0.2 \\ 0 \end{Bmatrix} \qquad \{\dot{x}(0)\} = \begin{Bmatrix} 0 \\ 5.0 \end{Bmatrix}$$

$r_1 = 1.1805$

$r_2 = -0.8471$

$\omega_1 = 30.2853$

$\omega_2 = 114.38$

$$\{x(t)\} = \begin{Bmatrix} 1 \\ r_1 \end{Bmatrix} X_1^{(1)} \cos(\omega_1 t + \phi_1) + \begin{Bmatrix} 1 \\ r_2 \end{Bmatrix} X_1^{(2)} \cos(\omega_2 t + \phi_2)$$

Displacement IC

$$\Rightarrow \begin{cases} 0.2 = X_1^{(1)} \cos\phi_1 + X_1^{(2)} \cos\phi_2 \\ 0 = r_1 X_1^{(1)} \cos\phi_2 + r_2 X_1^{(2)} \cos\phi_2 \end{cases}$$

$$\Rightarrow (r_2 - r_1) X_1^{(1)} \cos\phi_1 = 0.2 r_2$$

$$X_1^{(1)} \cos\phi_1 = \frac{0.2 r_2}{r_2 - r_1} = 0.0836$$

$$X_1^{(2)} \cos\phi_2 = \frac{0.2 r_1}{r_1 - r_2} = 0.1164$$

Velocity IC

$$\begin{cases} 0 = -\omega_1 X_1^{(1)} \sin\phi_1 - \omega_2 X_1^{(2)} \sin\phi_2 \\ 5 = -\omega_1 r_1 X_1^{(1)} \sin\phi_1 - \omega_2 r_2 X_1^{(2)} \sin\phi_2 \end{cases}$$

$$\Rightarrow X_1^{(1)} \sin\phi_1 = \frac{5}{\omega_1 (r_2 - r_1)} = -0.0814$$

$$X_1^{(2)} \sin\phi_2 = \frac{5}{\omega_2 (r_1 - r_2)} = 0.0216$$

$$\Rightarrow X_1^{(1)} = \sqrt{0.0836^2 + 0.0814^2} = 0.1167$$

$$X_1^{(2)} = \sqrt{0.1164^2 + 0.0716^2} = 0.1184$$

$$\phi_1 = \sin^{-1}\left[\frac{-0.0814}{X_1^{(1)}}\right] = -0.7719$$

$$\phi_2 = \sin^{-1}\left[\frac{0.0716}{X_1^{(2)}}\right] = 0.1835$$

$$\Rightarrow x_1[t] = 0.1167\cos(\omega_1 t - 0.7719) \quad (46^\circ)$$
$$+ 0.1184\cos(\omega_2 t + 0.1835) \quad (10.51^\circ)$$
$$= -0.1167\cos(\omega_1 t + 2.3697) \quad (135^\circ)$$
$$- 0.1184\cos(\omega_2 t + 2.95811) \quad (169.48^\circ)$$

$$x_2[t] = 0.1378\cos(\omega_1 t - 0.7719)$$
$$- 0.1003\cos(\omega_2 t + 0.1835)$$
$$= -0.1378\cos(\omega_2 t + 0.1835)$$
$$+ 0.1003\cos(\omega_2 t + 2.95811)$$