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ME 3322A: Thermodynamics: Fall 2014
Homework Set # 12
Due Date: December 2, 2014

	Problem # in Textbook		Answer
	7th Ed.	**8th Ed.**	
1	9.11	9.11	c) 53.3%; d) 142.2kPa
2	9.17	9.17	a) 0.1243 Btu; b) 16.2%
3	9.20	9.20	c) r=23.19 d) 975 kPa
4	9.43 Part a	9.43 Part a	a) 44.8%
5	9.54	9.54	a) 4.904×10^4 kW ; b) 1.0066×10^5 kW
6	9.59	9.59	a) 21.54 kg/s; b) 16,689 kW; c) 59.9%

9.11 Consider an air-standard Otto cycle. Operating data at principal states in the cycle are given in table below. The states are numbered as in Fig. 9.3. The mass of air is 0.002 kg. Determine

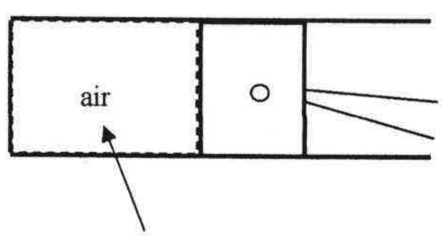
(a) the heat addition and heat rejection, each in kJ.
(b) the net work, in kJ.
(c) the thermal efficiency.
(d) the mean effective pressure, in kPa.

State	T (K)	p (kPa)	u (kJ/kg)
1	350	85	217.67
2	367.4	767.9	486.77
3	960	2005	725.20
4	458.7	127.8	329.01

KNOWN: An air-standard Otto cycle operates with property data given at principal states.

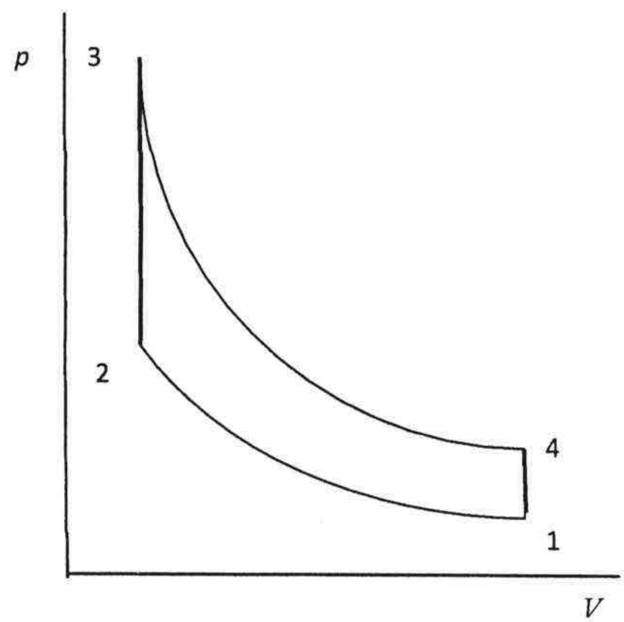
FIND: Determine (a) the heat addition and heat rejection, (b) the net work, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC AND GIVEN DATA:



$m = 0.002$ kg

See table for other data.



ENGINEERING MODEL:

1. Air, modeled as an ideal gas, is the system.
2. The compression and expansion processes are adiabatic.
3. Kinetic and potential energy effects are negligible

Problem 9.11 (Continued) – Page 2

ANALYSIS:

(a) The heat addition is determined is determined by using an energy balance for process 2-3.

$$Q_{\text{in}} = Q_{23} = m(u_3 - u_2) = (0.002\text{ kg})(725.02 - 486.77)\text{ kJ/kg} = \mathbf{0.4765\ kJ}$$

Similarly, the heat rejection is determined by using an energy balance for process 4-1.

$$Q_{\text{out}} = |Q_{41}| = m(u_4 - u_4) = (0.002)(329.01 - 217.67) = \mathbf{0.2227\ kJ}$$

(b) The net work is

$$W_{\text{cycle}} = Q_{\text{in}} - Q_{\text{out}} = 0.4765 - 0.2227 = \mathbf{0.2538\ kJ}$$

(c) The thermal efficiency

$$\eta = W_{\text{cycle}}/Q_{\text{in}} = \mathbf{0.533\ (53.3\%)}$$

(d) To determine the mean effective pressure, first find V_1 and V_2.

$$V_1 = \frac{mRT_1}{p_1} = \left[\frac{(0.002\text{ kg})\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}}\right)(305\text{ K})}{(85\text{ kPa})}\right]\left|\frac{1\text{ kPa}}{10^3\text{ N/m}^2}\right|\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right| = 2.06 \times 10^{-3}\ m^3$$

Similarly

$$V_2 = \frac{mRT_2}{p_2} = \left[\frac{(0.002)\left(\frac{8.314}{28.97}\right)(367.4)}{(767.9)}\right] = 2.75 \times 10^{-4}\text{ m}^3$$

Thus

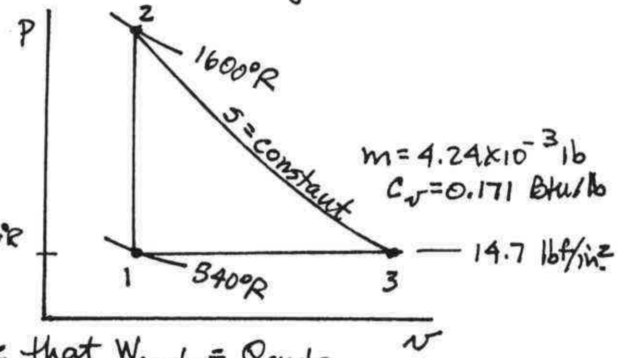
$$mep = \frac{W_{\text{cycle}}}{(V_1)(1-\frac{V_2}{V_1})} = \left[\frac{0.2538\text{ kJ}}{(2.06 \times 10^{-3} m^3)(1-\frac{0.000275}{0.00206})}\right]\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ kPa}}{10^3\text{ N/m}^2}\right| = \mathbf{142.2\ kPa}$$

PROBLEM 9.17

KNOWN: An air-standard Lenoir cycle has a known state at the beginning of the constant volume heat addition and a known maximum temperature. The mass is given.

FIND: Determine (a) the net work, and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The cycle is modeled as processes in a closed system. (2) The expansion process is isentropic. (3) The air is modeled as an ideal gas with constant specific heats; $c_v = 0.171$ Btu/lb·°R and $c_p = 0.24$ Btu/lb·°R. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: (a) To find the net work, note that $W_{cycle} = Q_{cycle}$ and

$$\dot{W}_{cycle} = Q_{12} + Q_{31}$$

For process 1-2: $m(u_2 - u_1) = Q_{12} - \cancel{W_{12}}^{\,0}$

$$Q_{12} = m(u_2 - u_1) = m c_v (T_2 - T_1)$$
$$= (4.24\times10^{-3}\ \text{lb})(0.171\ \tfrac{\text{Btu}}{\text{lb}\cdot°\text{R}})(1600 - 540)°\text{R}$$
$$= 0.7685\ \text{Btu}$$

For Process 2-3: $T_3 = T_2 (P_3/P_2)^{\frac{k-1}{k}}$. $P_2 = (T_2/T_1)p_1 = 43.56$ lbf/in.²
and $T_3 = (1600°\text{R})(14.7/43.56)^{0.2857} = 1173.1\ °\text{R}$

For Process 3-1: $m(u_1 - u_3) = Q_{31} - W_{31}$

$$W_{31} = m\int_3^1 p\,dv = mp(v_1 - v_3)$$

$$Q_{31} = m(u_1 - u_3) + mp(v_1 - v_3) = m c_p (T_1 - T_3)$$
$$= (4.24\times10^{-3})(0.24)(540 - 1173.1) = -0.6442\ \text{Btu}$$

Finally $W_{cycle} = Q_{12} + Q_{31} = 0.7685 + (-0.6442) = 0.1243$ Btu ← W_{cycle}

(b) The thermal efficiency is

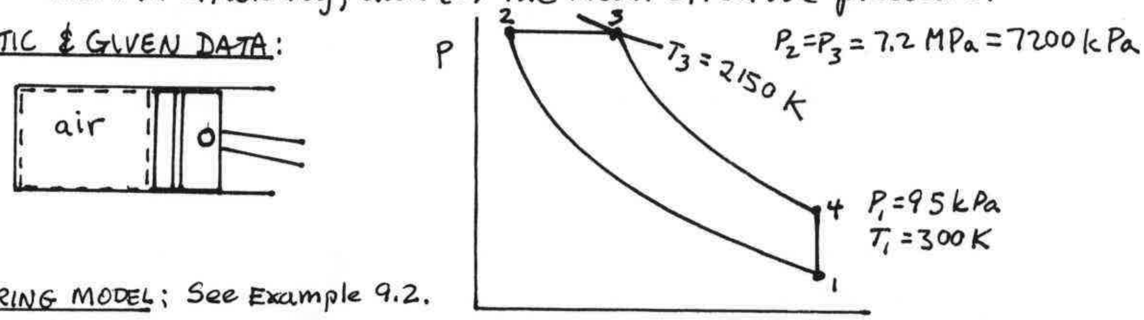
$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{W_{cycle}}{Q_{12}} = \frac{0.1243}{0.7685} = 0.162\ (16.2\%) \leftarrow \eta$$

PROBLEM 9.20

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

FIND: Determine (a) the compression ratio, (b) the cutoff ratio, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: Begin by fixing each principal state in the cycle (Table A-22).

State 1: $T_1 = 300$ K, $p_1 = 95$ kPa $\Rightarrow$ $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$, $p_{r1} = 1.3860$

State 2: For the isentropic compression

$$p_{r2} = p_{r1}\left(\frac{p_2}{p_1}\right) = (1.3860)\left(\frac{7200}{95}\right) = 105.04$$

Thus, $T_2 = 979.6$K, $v_{r2} = 26.793$, $h_2 = 1022.82$ kJ/kg

State 3: $T_3 = 2150$ K, $p_3 = 7200$ kPa $\Rightarrow$ $h_3 = 2440.3$ kJ/kg, $v_{r3} = 2.175$

State 4: For the isentropic expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2}\cdot\frac{v_2}{v_3} = \frac{v_1}{v_2}\cdot\frac{T_2}{T_3} = \frac{v_{r1}}{v_{r2}}\cdot\frac{T_2}{T_3} = \frac{621.2}{26.793}\cdot\frac{979.6}{2150} = 10.56$$

$$v_{r4} = \frac{v_4}{v_3}\cdot v_{r3} = 22.98 \Rightarrow T_4 = 1031 \text{ K},\ u_4 = 785.75 \text{ kJ/kg}$$

(a) The compression ratio is

$$r = \frac{v_1}{v_2} = \frac{v_{r1}}{v_{r2}} = \frac{621.2}{26.793} = 23.19 \longleftarrow r$$

(b) The cutoff ratio is

$$r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2150}{979.6} = 2.19 \longleftarrow r_c$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{(h_3 - h_2) - (u_4 - u_1)}{h_3 - h_2}$$

$$= \frac{(2440.3 - 1022.82) - (785.75 - 214.07)}{(2440.3 - 1022.82)}$$

$$= \frac{845.80}{1417.48} = 0.597\ (59.7\%) \longleftarrow \eta$$

PROBLEM 9.20 (Cont'd.) - Page 2

(d) The mean effective pressure is given as

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(300\ K)}{(95\ kPa)}\left|\frac{1\ kPa}{10^3\ N/m^2}\right|\left|\frac{10^3\ N \cdot m}{1\ kJ}\right|$$

$$= 0.9063\ m^3/kg$$

Thus

$$mep = \frac{(845.80\ kJ/kg)}{\left(0.9063\ \frac{m^3}{kg}\right)\left(1 - 1/23.19\right)}\left|\frac{10^3\ N \cdot m}{1\ kJ}\right|\left|\frac{1\ kPa}{10^3\ N/m^2}\right|$$

$$= 975\ kPa \longleftarrow \text{mep}$$

9.43 An ideal cold air standard Brayton cycle operates at steady state with compressor inlet conditions of 300 K and 100 kPa, fixed turbine inlet temperature of 1700 K, and $k = 1.4$. For the cycle,

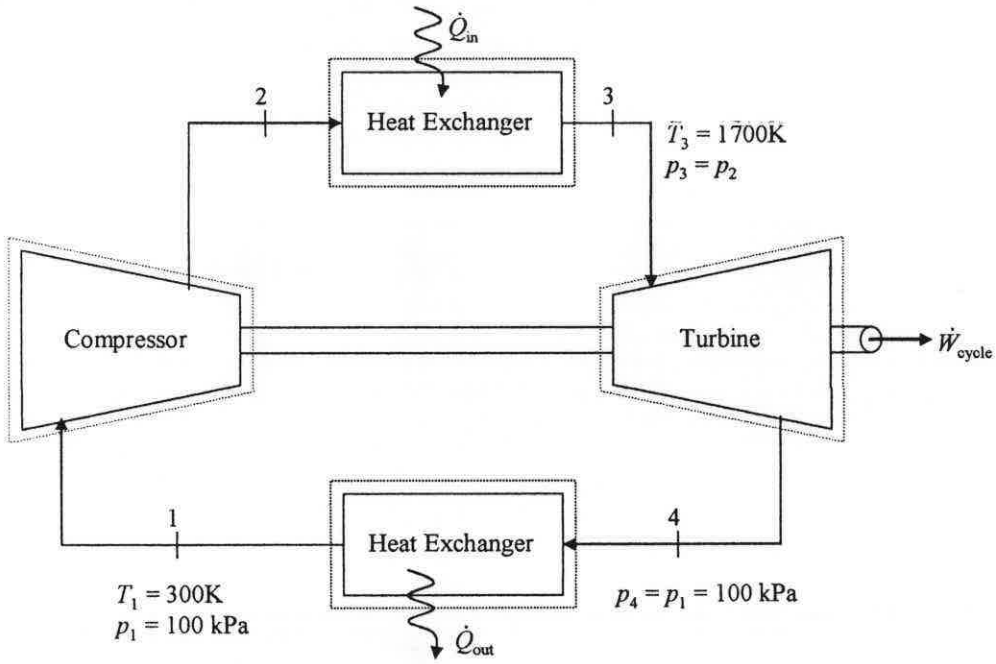
(a) determine the net power per unit mass flowing, in kJ/kg, and the thermal efficiency for a compressor pressure ratio of 8.

(b) plot the net power per unit mass flowing, in kJ/kg, and the thermal efficiency, each versus compressor pressure ratio ranging from 2 to 50.

KNOWN: An ideal cold air standard Brayton cycle operates with fixed compressor inlet conditions of 300 K and 100 kPa and fixed turbine inlet temperature of 1700.

FIND: (a) the net power per unit mass flowing and the thermal efficiency for a compressor pressure ratio of 8 and (b) plot the net power per unit mass flowing and the thermal efficiency, each versus compressor pressure ratio ranging from 2 to 50.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Air, modeled as an ideal gas, is the working fluid.
3. All processes of the working fluid are internally reversible.
4. The compressor and turbine operate adiabatically.
5. Kinetic and potential energy effects are negligible.
6. Specific heats of air are constant with $k = 1.4$.

Problem 9.43 (Continued) – Page 2

ANALYSIS: (a) The net work of the cycle per unit of mass flow using a cold air standard analysis

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = c_p[(T_3 - T_4) - (T_2 - T_1)]$$

<u>State 1</u>: $T_1 = 300$ K.

Process 1-2 is an isentropic process. Thus

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \Rightarrow T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = (300\text{ K})(8)^{(1.4-1)/1.4} = 543.4\text{ K}$$

<u>State 3</u>: $T_3 = 1700$ K.

Process 3-4 is an isentropic process. Thus

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{(k-1)/k} = \left(\frac{p_1}{p_2}\right)^{(k-1)/k} \Rightarrow T_4 = T_3\left(\frac{p_1}{p_2}\right)^{(k-1)/k} = (1700\text{ K})\left(\frac{1}{8}\right)^{(1.4-1)/1.4} = 938.5\text{ K}$$

From Table A-20 for air, $c_p = 1.005$ kJ/(kg·K). Solving for net work of the cycle per unit of mass flow

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \left(1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)[(1700\text{ K} - 938.5\text{ K}) - (543.4\text{ K} - 300\text{ K})] = \underline{\mathbf{520.7\ kJ/kg}}$$

Thermal efficiency is given by

$$\eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{c_p(T_3 - T_2)} = \frac{520.7\frac{\text{kJ}}{\text{kg}}}{\left(1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(1700\text{ K} - 543.4\text{ K})} = \underline{\mathbf{0.4480\ (44.80\%)}}$$

(b) **<u>IT code</u>**

```
/* ANALYSIS:  Cold Air Standard Analysis*/

cp = 1.005 // kJ/(kg-K)
k = 1.4
rp = 8
mdot = 1 // kg/s

/* State 1 */
p1 = 100 // kPa
T1 = 300 // K

/* State 2 */
```

Problem 9.43 (Continued) – Page 3

```
p2 = rp * p1 // kPa
T2 = T1*rp^((k-1)/k) // K

/* State 3 */
T3 = 1700 // K
p3 = p2 // kPa

/* State 4 */
p4 = p1 // kPa
T4 = T3*(1/rp)^((k-1)/k) // K

/* Energy Transfers and Cycle Performance */
Wdotcyclepermdot = cp*((T3 - T4) - (T2 - T1)) // kJ/kg
Qdotinpermdot = cp*(T3 - T2) // kJ/kg
eta = Wdotcyclepermdot / Qdotinpermdot
```

IT Results for pressure ratio of 8

eta	0.448
p2	800
p3	800
p4	100
Qdotinpermdot	1162
T2	543.4
T4	938.5
Wdotcyclepermdot	520.7
cp	1.005
k	1.4
mdot	1
p1	100
rp	8
T1	300
T3	1700

These values compare favorably with the values calculated above.

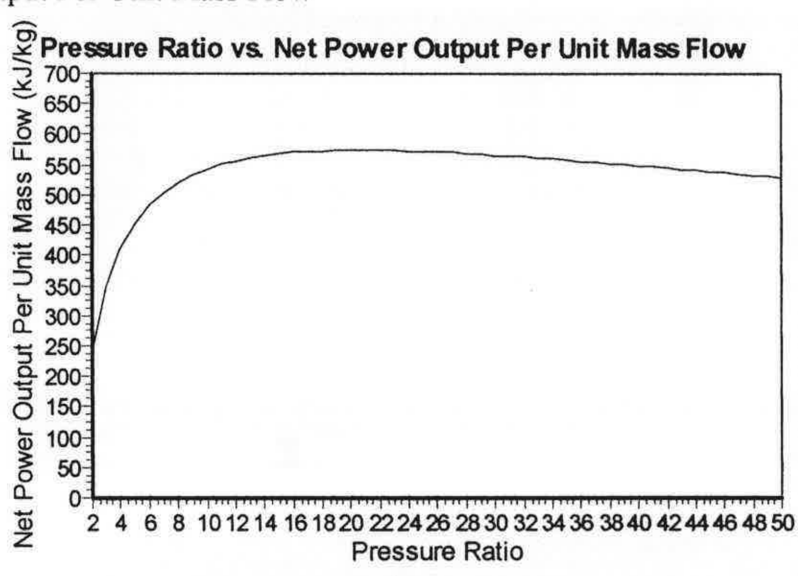
Problem 9.43 (Continued) – Page 4

IT Results for rp = 19 through rp = 23

eta	0.5688	0.5751	0.581	0.5865	0.5917
p2	1900	2000	2100	2200	2300
p3	1900	2000	2100	2200	2300
p4	100	100	100	100	100
Qdotinpermdot	1009	998.9	988.9	979.3	970
T2	695.8	706.1	716	725.6	734.8
T4	733	722.3	712.3	702.9	694
Wdotcyclepermdot	574.1	574.5	574.6	574.4	574
cp	1.005	1.005	1.005	1.005	1.005
k	1.4	1.4	1.4	1.4	1.4
mdot	1	1	1	1	1
p1	100	100	100	100	100
rp	19	20	21	22	23
T1	300	300	300	300	300
T3	1700	1700	1700	1700	1700

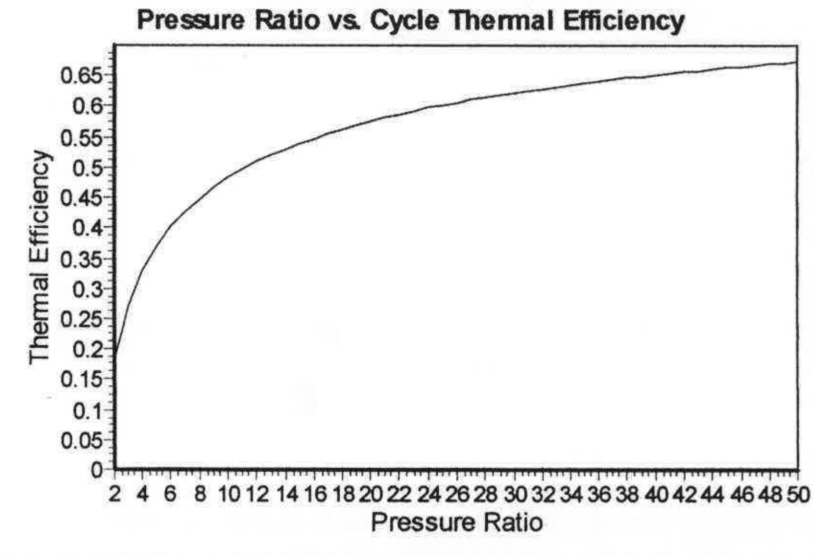
Plots

Net Power Output Per Unit Mass Flow



Problem 9.43 (Continued) – Page 5

Thermal Efficiency



Discussion

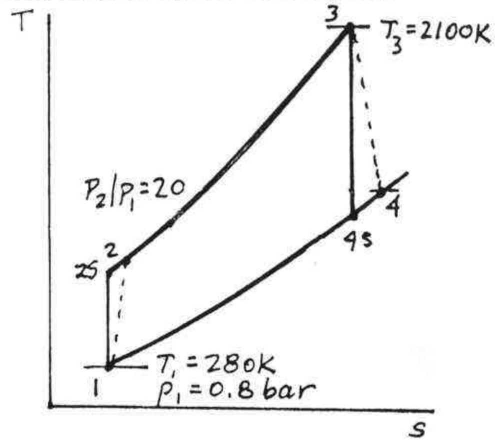
Maximum net work output per unit mass flow (574.6 kJ/kg) occurs at a pressure ratio of 21 using cold air standard analysis. Thermal efficiency continues to increase with increasing pressure ratio.

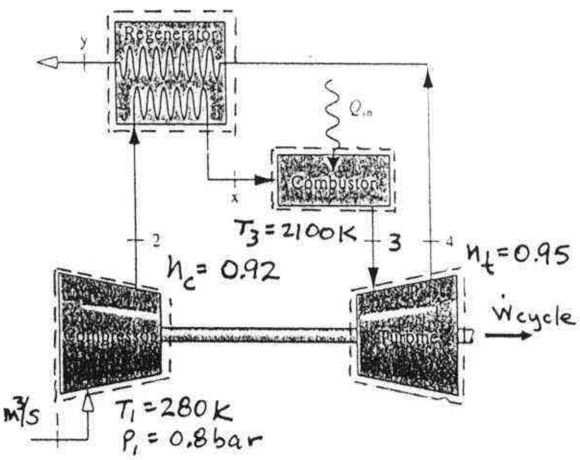
PROBLEM 9.54

KNOWN: Air enters the compressor of an air-standard Brayton cycle at a specified state and a given volumetric flow rate. The compressor pressure ratio and maximum cycle temperature are known. The compressor and turbine isentropic efficiencies are known.

FIND: Determine (a) the net power, (b) the rate of heat addition, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.7. Also, $\eta_c = 0.92$, $\eta_t = 0.95$

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = 280K \Rightarrow h_1 = 280.13\ kJ/kg$, $p_{r1} = 1.0889$

State 2: $p_{r2} = (P_2/P_1)\,p_{r1} = (20)(1.0889) = 21.778 \Rightarrow T_{2s} = 649.3K$, $h_{2s} = 659.13\ \frac{kJ}{kg}$

Using the isentropic compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 280.13 + (659.13 - 280.13)/(0.92) = 692.09\ kJ/kg$$

State 3: $T_3 = 2100K$; $h_3 = 2377.4\ kJ/kg$, $p_{r3} = 2559$

State 4: $p_{r4} = (P_4/P_3)\,p_{r3} = (\frac{1}{20})(2559) = 127.95 \Rightarrow T_{4s} = 1029.2K$, $h_{4s} = 1079.4\ \frac{kJ}{kg}$

Using the isentropic turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 2377.4 - (0.95)(2377.4 - 1079.4) = 1144.3\ kJ/kg$$

Now, determine the mass flow rate.

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1\,p_1}{R\,T_1} = \frac{(60\ m^3/s)(0.8\ bar)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(280K)}\left|\frac{10^5\ N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 59.73\ kg/s$$

(a) $\dot{W}_c = \dot{m}(h_2 - h_1) = (59.73\ \frac{kg}{s})(692.09 - 280.13)\ \frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right| = 2.461\times10^4\ kW$

$\dot{W}_t = \dot{m}(h_3 - h_4) = (59.73)(2377.4 - 1144.3) = 7.365\times10^4\ kW$

$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 4.904\times10^4\ kW$ ← $\dot{W}_{cycle}$

(b) $\dot{Q}_{in} = \dot{m}(h_3 - h_2) = (59.73)(2377.4 - 692.09) = 1.0066\times10^5\ kW$ ← $\dot{Q}_{in}$

(c) $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 4.904\times10^4 / 1.0066\times10^5 = 0.487\ (48.7\%)$ ← η

9.59 An ideal air-standard regenerative Brayton cycle produces 10 MW of power. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.14. Sketch the *T-s* diagram and determine

(a) the mass flow rate of air, in kg/s.

(b) the rate of heat transfer, in kW, to the working fluid passing through the combustor.

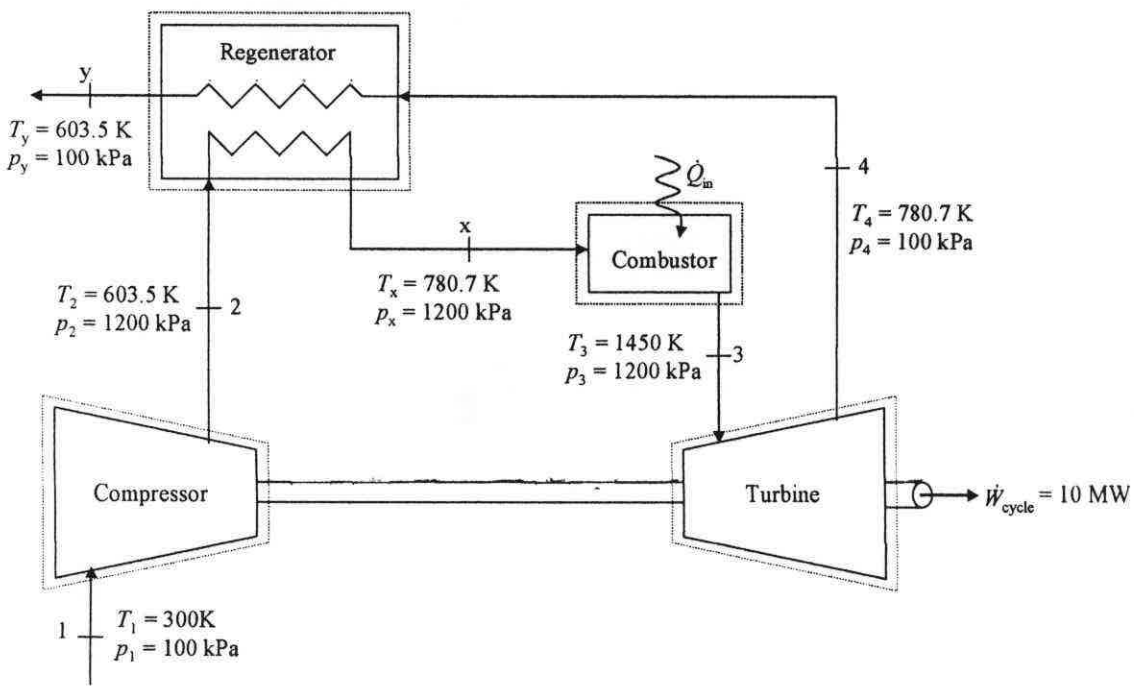
(c) the thermal efficiency.

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2	1200	603.5	610.65
x	1200	780.7	800.78
3	1200	1450	1575.57
4	100	780.7	800.78
y	100	603.5	610.65

KNOWN: An ideal air-standard regenerative Brayton cycle operates with property data given at principal states. The net power output of the cycle is given.

FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the combustor, and the thermal efficiency.

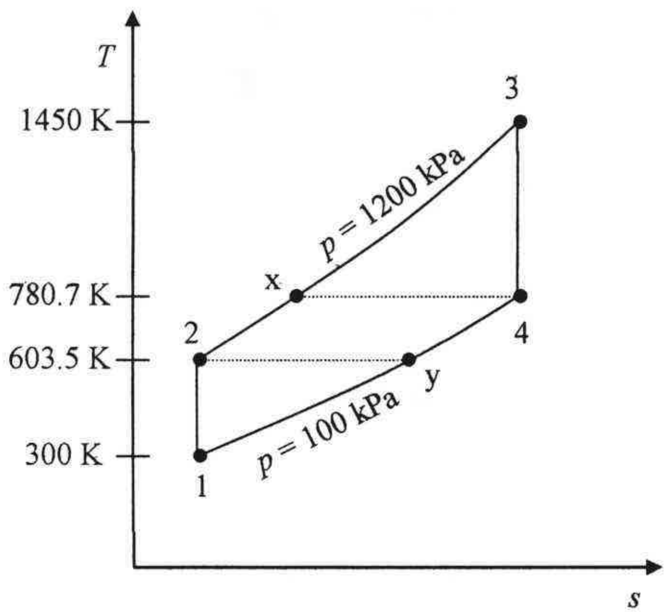
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and compressor operate adiabatically.
4. There are no pressure drops for flow through the regenerator and combustor.
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.

ANALYSIS: The *T-s* diagram for the cycle is shown below.



(a) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and compressor give

$$\dot{W}_{\mathrm{t}} = \dot{m}(h_3 - h_4) \quad \text{and} \quad \dot{W}_{\mathrm{c}} = \dot{m}(h_2 - h_1)$$

The net power of the cycle is

$$\dot{W}_{\mathrm{cycle}} = \dot{W}_{\mathrm{t}} - \dot{W}_{\mathrm{c}} = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{\mathrm{cycle}}}{[(h_3 - h_4) - (h_2 - h_1)]}$$

Inserting values

Problem 9.59 (continued) – Page 3

$$\dot{m} = \frac{10{,}000 \text{ kW}}{\left(1575.57\dfrac{\text{kJ}}{\text{kg}} - 800.78\dfrac{\text{kJ}}{\text{kg}}\right) - \left(610.65\dfrac{\text{kJ}}{\text{kg}} - 300.19\dfrac{\text{kJ}}{\text{kg}}\right)}\left|\frac{\text{kJ}/\text{s}}{\text{kW}}\right| = \underline{\mathbf{21.54\ kg/s}}$$

(b) The rate of heat transfer to the working fluid passing through the combustor can be determined by applying mass and energy balances to a control volume around the combustor to give

$$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_x) = \left(21.54\frac{\text{kg}}{\text{s}}\right)\left(1575.57\frac{\text{kJ}}{\text{kg}} - 800.78\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{kW}}{\text{kJ}/\text{s}}\right| = \underline{\mathbf{16{,}689\ kW}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (10{,}000 \text{ kW})/(16{,}689 \text{ kW}) = \underline{\mathbf{0.599\ (59.9\%)}}$$