5.100

5.100 A 100-ft-wide river with a flowrate of 2400 ft^3/s flows over a rock pile as shown in Fig. P5.100. Determine the direction of flow and the head loss associated with the flow across the rock pile.

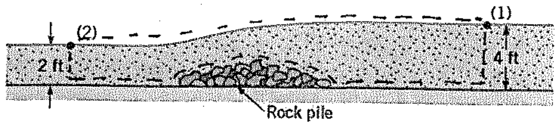


FIGURE P5.100

To determine the direction of flow we will assume a direction, use the energy equation (Eq. 5.84) and calculate the head loss. If the head loss is positive, our assumed direction of flow is correct. If the head loss is negative which is not physically possible, our assumed direction of flow is wrong.

So, assuming the flow is from right to left or from point (1) to point (2) in the sketch above, we get using Eq. 5.84

$$\overbrace{\cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\rho}}}^{\text{same pressure}} + \frac{V_1^2}{2g} + z_1 + \underbrace{\cancel{h_s}}_{0,\ \text{no shaft work}} - h_L$$

Now

$$V_1 = \frac{Q}{A_1} = \frac{\left(2400\ \frac{ft^3}{s}\right)}{(4\ ft)(100\ ft)} = 6\ \frac{ft}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{\left(2400\ \frac{ft^3}{s}\right)}{(2\ ft)(100\ ft)} = 12\ \frac{ft}{s}$$

So

$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + z_1 - z_2 = \frac{\left(6\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} - \frac{\left(12\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} + 4\ ft - 2\ ft$$

$h_L = \underline{\underline{0.32\ ft}}$ and since h_L is positive, our assumed <u>right to left</u> flow is correct

5.108

5.108 For the 180° elbow and nozzle flow shown in Fig. P5.108 determine the loss in available energy from section (1) to section (2). How much additional available energy is lost from section (2) to where the water comes to rest?

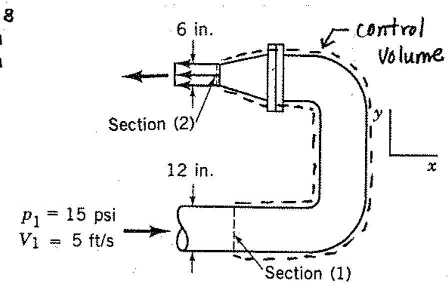


FIGURE P5.108

For solving the first part of this problem, the control volume shown in the sketch above is used. To determine the loss accompanying flow from section 1 to section 2 Eq. 5.79 can be used as follows.

$$_1loss_2 = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

Since x-y coordinates are specified we assume that the flow is horizontal and $z_1 - z_2 = 0$. Also, $p_2 = p_{atm} = 0$ psi.

From the conservation of mass principle we conclude that

$$V_2 = \frac{V_1 A_1}{A_2} = V_1\left(\frac{D_1^2}{D_2^2}\right)$$

Thus

$$_1loss_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1^2}{D_2^2}\right)^2\right] = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$$

or

$$_1loss_2 = \frac{\left(15\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)} + \frac{\left(5\ \frac{ft}{s}\right)^2}{2}\left[1 - \left(\frac{12\ in.}{6\ in.}\right)^4\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$_1loss_2 = \underline{\underline{926}}\ \frac{ft \cdot lb}{slug}$$

For the second part of this problem we consider the flow of a fluid particle from section 2 to a state of rest, a. Eq. 5.79 leads to

$$_2loss_a = \frac{V_2^2}{2}$$

Note that we have assumed that $p_2 = p_a = p_{atm}$ and $z_2 = z_a$.

Thus

$$_2loss_A = \frac{V_2^2}{2} = \frac{V_1^2}{2}\left(\frac{D_1^2}{D_2^2}\right)^2 = \frac{V_1^2}{2}\left(\frac{D_1}{D_2}\right)^4 = \frac{\left(5\ \frac{ft}{s}\right)^2\left(\frac{12\ in.}{6\ in.}\right)^4\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)}{2}$$

$$_2loss_A = \underline{\underline{200}}\ \frac{ft \cdot lb}{slug}$$

5.117

5.117 Water is supplied at 150 ft³/s and 60 psi to a hydraulic turbine through a 3-ft inside diameter inlet pipe as indicated in Fig. P5.117. The turbine discharge pipe has a 4-ft inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10-in. Hg vacuum. If the turbine develops 2500 hp, determine the power lost between sections (1) and (2).

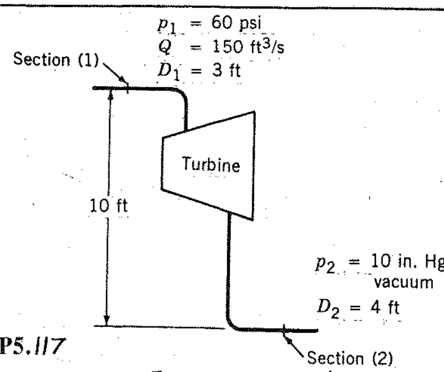


FIGURE P5.117

For flow between sections (1) and (2), Eq. 5.82 leads to

$$\text{power loss} = \rho Q\left[\left(\frac{P_1-P_2}{\rho}\right) + g(z_1-z_2) + \frac{(V_1^2-V_2^2)}{2}\right] - \dot{W}_{\substack{shaft\\ net\ out}} \qquad (1)$$

From given data

$$P_2 = \frac{(-10\ in.\ Hg)(13.6)\left(1.94\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{ft}{s^2}\right)}{\left(12\ \frac{in.}{ft}\right)}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) = -708\ \frac{lb}{ft^2}$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{(4)\left(150\ \frac{ft^3}{s}\right)}{\pi\,(3ft)^2} = 21.22\ \frac{ft}{s}$$

From conservation of mass (Eq. 5.13)

$$V_2 = V_1\frac{A_1}{A_2} = V_1\frac{D_1^2}{D_2^2} = \left(21.22\ \frac{ft}{s}\right)\frac{(3ft)^2}{(4ft)^2} = 11.94\ \frac{ft}{s}$$

From Eq. 1

$$\text{power loss} = \frac{\left(1.94\ \frac{slugs}{ft^3}\right)\left(150\ \frac{ft^3}{s}\right)}{\left(550\ \frac{ft.lb}{s.hp}\right)}\left\{\frac{\left(60\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)+\left(708\ \frac{lb}{ft^2}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)}\right.$$

$$\left. + \left(32.2\ \frac{ft}{s^2}\right)(10\ ft)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) + \left[\frac{\left(21.22\ \frac{ft}{s}\right)^2-\left(11.94\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right\}$$

$$-2500hp$$

or

$$\text{power loss} = \underline{\underline{301}}\ hp$$

5.130

5.130 Near the downstream end of a river spillway, a hydraulic jump often forms, as illustrated in Fig. P5.130. The velocity of the channel flow is reduced abruptly across the jump. Using the conservation of mass and linear momentum principles, derive the expression for h_2,

$$h_2 = -\frac{h_1}{2} + \sqrt{\left(\frac{h_1}{2}\right)^2 + \frac{2V_1^2 h_1}{g}}$$

The loss of available energy across the jump can also be determined if energy conservation is considered. Derive the loss expression

$$\text{jump loss} = \frac{g(h_2 - h_1)^3}{4h_1h_2}$$

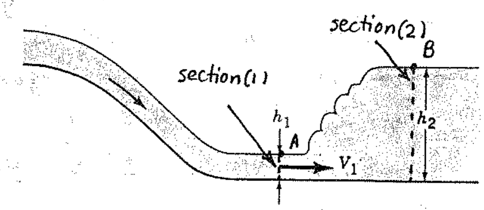


FIGURE P5.130

Application of the horizontal component of the linear momentum equation (Eq. 5.22) to the water in the control volume from section (1) to section (2) leads to, for unit width of flow,

$$-R_x + \frac{\gamma h_1^2}{2} - \frac{\gamma h_2^2}{2} = -V_1 \rho h_1 V_1 + V_2 \rho h_2 V_2 \qquad (1)$$

Since the jump occurs over a short distance we drop R_x from Eq. 1. Also from conservation of mass (Eq. 5.13) we obtain

$$V_2 = V_1 \frac{h_1}{h_2} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$1 - \left(\frac{h_2}{h_1}\right)^2 = \frac{2V_1^2}{gh_1}\left[\frac{1}{\left(\frac{h_2}{h_1}\right)} - 1\right]$$

or

$$\left(1 - \frac{h_2}{h_1}\right)\left(1 + \frac{h_2}{h_1}\right) = \frac{2V_1^2}{gh_1} \frac{\left(1 - \frac{h_2}{h_1}\right)}{\left(\frac{h_2}{h_1}\right)}$$

and

$$\left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right) - \frac{2V_1^2}{gh_1} = 0 \qquad (3)$$

From Eq. 3 we obtain

$$\frac{h_2}{h_1} = \frac{-1 \pm \sqrt{1 + \frac{8V_1^2}{gh_1}}}{2}$$

or

(con't)

5.130 (con't)

$$h_2 = -\frac{h_1}{2} + \sqrt{\left(\frac{h_1}{2}\right)^2 + \frac{2V_1^2}{g}h_1}$$

The other quadratic root is not meaningful.

Application of the energy equation (Eq. 5.82) to the flow from point A to point B shown on the sketch above leads to

$$\text{jump loss} = \frac{V_A^2 - V_B^2}{2} + g(z_A - z_B) = \frac{V_1^2 - V_2^2}{2} + g(h_1 - h_2) \quad (4)$$

Combining Eqs. 2, 3 and 4 we obtain

$$\text{jump loss} = \frac{gh_1}{4}\left[\left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1}\right]\left[1 - \left(\frac{h_1}{h_2}\right)^2\right] + g(h_1 - h_2)$$

or

$$\text{jump loss} = \frac{g}{4h_2h_1}(h_2 - h_1)^3$$

5.133

5.133 When fluid flows through an abrupt expansion as indicated in Fig. P5.133, the loss in available energy across the expansion, $loss_{ex}$, is often expressed as

$$loss_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.

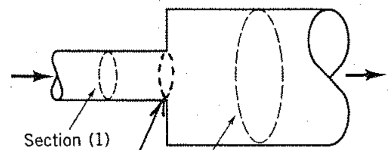


FIGURE P5.133

Applying the energy equation (Eq. 5.82) to the flow from section (1) to section (2) we obtain

$$loss_{ex} = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

Applying the axial direction component of the linear momentum equation (Eq. 5.22) to the fluid contained in the control volume from section (1) to section (2) we obtain

$$R_x + P_1 A_1 - P_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (2)$$

Now, if we consider section (1) as occurring at the end of the smaller diameter pipe (the beginning of the larger diameter pipe) as indicated in the sketch above, Eq. 1 still yields the expansion loss and Eq. 2 becomes

$$R_x + P_1 A_2 - P_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (3)$$

Note that with section (1) positioned at the end of the smaller diameter pipe, P_1 acts over area A_2. Also, because of the jet flow from the smaller diameter pipe into the larger diameter pipe, the value of R_x will be small enough compared to the other terms in Eq. 3 that we can drop R_x. From Eq. 3

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1^2 \frac{A_1}{A_2} \qquad (4)$$

Combining Eqs. 1 and 4 we obtain

$$loss_{ex} = V_2^2 - V_1^2 \frac{A_1}{A_2} + \frac{V_1^2 - V_2^2}{2}$$

(con't)

5.133 (con't)

From conservation of mass (Eq. 5.13) we have

$$V_2 = V_1 \frac{A_1}{A_2} \qquad (6)$$

Combining Eqs. 5 and 6 we get

$$loss_{ex} = V_1^2\left(\frac{A_1}{A_2}\right)^2 - V_1^2\left(\frac{A_1}{A_2}\right) + \frac{V_1^2 - V_1^2\left(\frac{A_1}{A_2}\right)^2}{2}$$

or

$$loss_{ex} = \frac{V_1^2}{2}\left[2\left(\frac{A_1}{A_2}\right)^2 - 2\frac{A_1}{A_2} + 1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

and

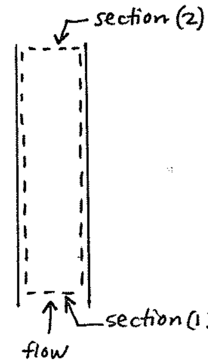
$$loss_{ex} = \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2$$

5.135

5.135 Water flows vertically upward in a circular cross section pipe. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left(\frac{R - r}{R}\right)^{1/7} \hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe inside radius, and, r = radius from pipe axis. Develop an expression for the loss in available energy between sections (1) and (2).



For determining loss we use the energy equation for non-uniform flows, Eq. 5.87. Thus,

$$\text{loss} = \frac{p_1 - p_2}{\rho} + \frac{\alpha_1 \overline{V}_1^2 - \alpha_2 \overline{V}_2^2}{2} + g(z_1 - z_2) \qquad (1)$$

From conservation of mass (Eq. 5.13) we have

$$\overline{V}_1 = \overline{V}_2$$

Also, with Eq. 5.86 for the kinetic energy coefficient, α, we have

$$\alpha_1 = 1.0$$

since the velocity profile at section (1) is uniform. At section (2) we solve Eq. 5.86 (see solution for problem 5.125(c)) and obtain

$$\alpha_2 = 1.06$$

Thus, Eq. 1 yields

$$\text{loss} = \frac{p_1 - p_2}{\rho} - 0.06 \frac{\overline{V}_1^2}{2} + g(z_1 - z_2)$$