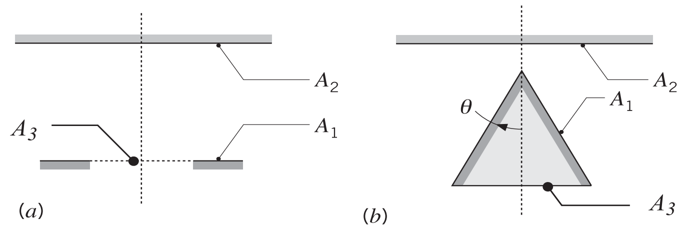
PROBLEM 13.3

KNOWN: Two arrangements (a) circular disk and coaxial, ring shaped disk, and (b) circular disk and coaxial, right-circular cone.

FIND: Derive expressions for the view factor F_{12} for the arrangements (a) and (b) in terms of the areas A_1 and A_2, and any appropriate hypothetical surface area, as well as the view factor for coaxial parallel disks (Table 13.2, Figure 13.5). For the disk-cone arrangement, sketch the variation of F_{12} with θ for $0 \le \theta \le \pi/2$, and explain the key features.

SCHEMATIC:



ASSUMPTIONS: Diffuse surfaces with uniform radiosities.

ANALYSIS: (a) Define the hypothetical surface A_3, a co-planar disk inside the ring of A_1. Using the additive view factor relation, Eq. 13.5,

$$A_{(1,3)}\,F_{(1,3)} = A_1\,F_{12} + A_3\,F_{32}$$

$$F_{12} = \frac{1}{A_1}\left[A_{(1,3)}\,F_{(1,3)} - A_3\,F_{32}\right] \qquad <$$

where the parenthesis denote a composite surface. All the F_{ij} on the right-hand side can be evaluated using Fig. 13.5.

(b) Define the hypothetical surface A_3, the disk at the bottom of the cone. The radiant power leaving A_2 that is intercepted by A_1 can be expressed as

$$F_{21} = F_{23} \qquad (1)$$

That is, the same power also intercepts the disk at the bottom of the cone, A_3. From reciprocity,

$$A_1\,F_{12} = A_2\,F_{21} \qquad (2)$$

and using Eq. (1),

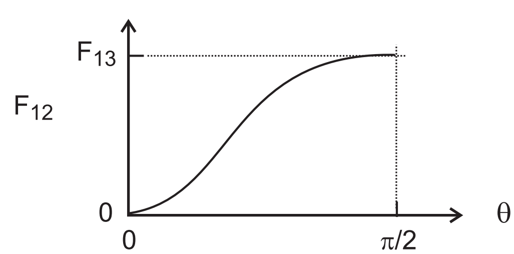
$$F_{12} = \frac{A_2}{A_1}\,F_{23} \qquad <$$

The variation of F_{12} as a function of θ is shown below for the disk-cone arrangement. In the limit when $\theta \to \pi/2$, the cone approaches a disk of area A_3. That is,

$$F_{12}\,(\theta \to \pi/2) = F_{13}$$

When $\theta \to 0$, the cone area A_2 diminishes so that

$$F_{12}\,(\theta \to 0) = 0$$

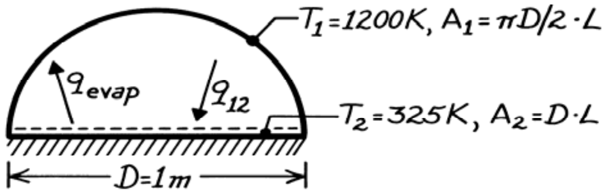


PROBLEM 13.18

KNOWN: Surface temperature of a semi-circular drying oven.

FIND: Drying rate per unit length of oven.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for furnace wall and water, (2) Convection effects are negligible and bottom is insulated, (3) Uniform surface irradiation and radiosity.

PROPERTIES: *Table A-6*, Water (325 K): $h_{fg} = 2.378 \times 10^6\ \mathrm{J/kg}$.

ANALYSIS: Applying a surface energy balance,

$$q_{12} = q_{evap} = \dot{m}\, h_{fg}$$

where it is assumed that the net radiation heat transfer to the water is balanced by the evaporative heat loss. From Eq. 13.13,

$$q_{12} = A_1\ F_{12}\ \sigma\left(T_1^4 - T_2^4\right).$$

From inspection and the reciprocity relation,

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{D \cdot L}{(\pi D/2) \cdot L} \times 1 = 0.637.$$

Hence

$$\dot{m}' = \frac{\dot{m}}{L} = \frac{\pi D}{2} F_{12}\, \sigma \frac{\left(T_1^4 - T_2^4\right)}{h_{fg}}$$

$$\dot{m}' = \frac{\pi (1\ \mathrm{m})}{2} \times 0.637 \times 5.67 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m^2 \cdot K^4}} \frac{(1200\ \mathrm{K})^4 - (325\ \mathrm{K})^4}{2.378 \times 10^6\ \mathrm{J/kg}}$$

or

$$\dot{m}' = 0.0492\ \mathrm{kg/s \cdot m}.$$ <

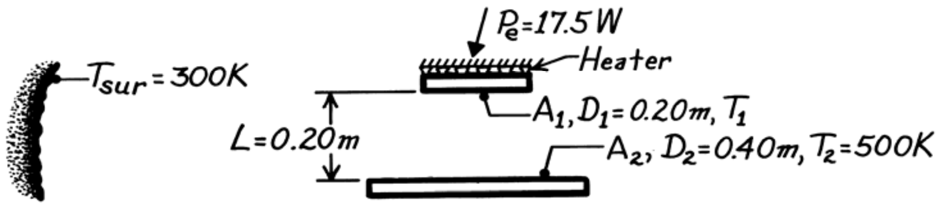
COMMENTS: (1) Air flow through the oven is needed to remove the water vapor. The water surface temperature, T_2, is determined by a balance between radiation heat transfer to the water and the convection of latent and sensible energy from the water. (2) Because the surfaces are black and isothermal, they have uniform radiosity.

PROBLEM 13.21

KNOWN: Coaxial, parallel black plates with surroundings. Lower plate (A_2) maintained at prescribed temperature T_2 while electrical power is supplied to upper plate (A_1).

FIND: Temperature of the upper plate T_1.

SCHEMATIC:



ASSUMPTIONS: (1) Plates are black surfaces of uniform temperature. (2) Backside of heater on A_1 insulated. (3) Uniform surface irradiation and radiosity.

ANALYSIS: The net radiation heat rate leaving A_i is

$$P_e = \sum_{j=1}^{N} q_{ij} = A_1 F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1 F_{13}\sigma\left(T_1^4 - T_3^4\right)$$

$$P_e = A_1\sigma\left[F_{12}\left(T_1^4 - T_2^4\right) + F_{13}\left(T_1^4 - T_{sur}^4\right)\right] \qquad (1)$$

From Fig. 13.5 for coaxial disks (see Table 13.2),

$$R_1 = r_1/L = 0.10\ \text{m}/0.20\ \text{m} = 0.5 \qquad R_2 = r_2/L = 0.20\ \text{m}/0.20\ \text{m} = 1.0$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+1^2}{(0.5)^2} = 9.0$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(r_2/r_1\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{9 - \left[9^2 - 4\left(0.2/0.1\right)^2\right]^{1/2}\right\} = 0.469.$$

From the summation rule for the enclosure A_1, A_2 and A_3 where the last area represents the surroundings with $T_3 = T_{sur}$, $F_{12} + F_{13} = 1$ and $F_{13} = 1 - F_{12} = 1 - 0.469 = 0.531$.

Substituting numerical values into Eq. (1), with $A_1 = \pi D_1^2/4 = 3.142\times10^{-2}\text{m}^2$,

$$17.5\ \text{W} = 3.142\times10^{-2}\text{m}^2 \times 5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left[0.469\left(T_1^4 - 500^4\right)\text{K}^4 + 0.531\left(T_1^4 - 300^4\right)\text{K}^4\right]$$

$$9.823\times10^9 = 0.469\left(T_1^4 - 500^4\right) + 0.531\left(T_1^4 - 300^4\right)$$

we find that $T_1 = 456\ \text{K}$. <

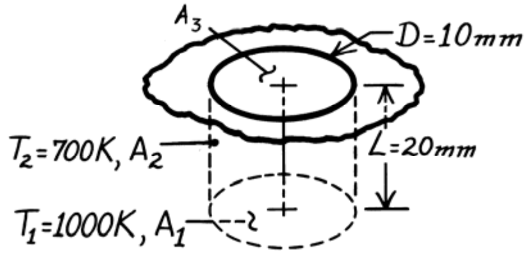
COMMENTS: (1) Note that if the upper plate were adiabatic, T_1 = 427 K. (2) Would you expect the surfaces to experience uniform irradiation? If the heater were constructed of a low thermal conductivity material, temperature gradients would likely develop in the radial direction. If this were the case, the heater surface would no longer be isothermal, and would no longer have a uniform radiosity. A more detailed analysis involving more radiation surface might be warranted in practice.

PROBLEM 13.29

KNOWN: Dimensions and temperatures of side and bottom walls in a cylindrical cavity.

FIND: Emissive power of the cavity.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for surfaces 1 and 2. (2) Uniform surface radiosity and irradiation distributions.

ANALYSIS: The emissive power is defined as

$$E = q_3 / A_3$$

where

$$q_3 = A_1\ F_{13}\ E_{b1} + A_2\ F_{23}\ E_{b2}.$$

From symmetry, $F_{23} = F_{21}$, and from reciprocity, $F_{21} = (A_1/A_2)\ F_{12}$. With $F_{12} = 1 - F_{13}$, it follows that

$$q_3 = A_1 F_{13} E_{b1} + A_1 (1 - F_{13}) E_{b2} = A_1 E_{b2} + A_1 F_{13} (E_{b1} - E_{b2}).$$

Hence, with $A_1 = A_3$,

$$E = \frac{q_3}{A_3} = E_{b2} + F_{13}(E_{b1} - E_{b2}) = \sigma T_2^4 + F_{13}\,\sigma \left(T_1^4 - T_2^4\right).$$

From Fig. 13.5, with $(L/r_i) = 4$ and $(r_j/L) = 0.25$, $F_{13} \approx 0.05$. Hence

$$E = 5.67\times10^{-8}\,W/m^2\cdot K^4 (700^4) K^4 + 0.05\times5.67\times10^{-8}\,W/m^2\cdot K^4 \left(1000^4 - 700^4\right) K^4$$

$$E = 1.36\times10^4\,W/m^2 + 0.22\times10^4\,W/m^2$$

$$E = 1.58\times10^4\ W/m^2.$$ <

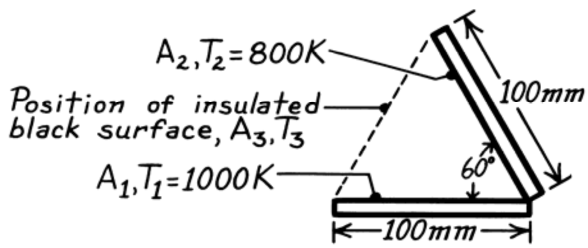
COMMENT: (1) The surfaces will not experience uniform irradiation. Will this affect the answer?

PROBLEM 13.40

KNOWN: Long, inclined black surfaces maintained at prescribed temperatures.

FIND: (a) Net radiation exchange between the two surfaces per unit length, (b) Net radiation transfer to surface A_2 with black, insulated surface positioned as shown below; determine temperature of this surface.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces behave as blackbodies, (2) Surfaces are very long in direction normal to page.

ANALYSIS: (a) The net radiation exchange between two black surfaces is

$$q_{12} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right)$$

Noting that A_1 = width×length (ℓ) and that from symmetry, $F_{12} = 0.5$, find

$$q'_{12} = \frac{q_{12}}{\ell} = 0.1\ \text{m} \times 0.5 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(1000^4 - 800^4\right) \text{K}^4 = 1680\ \text{W/m}.$$ <

(b) From Eq. 13.14,

$$q'_3 = \frac{q_3}{\ell} = \frac{A_3}{\ell} F_{31} \sigma \left(T_3^4 - T_1^4\right) + \frac{A_3}{\ell} F_{32} \sigma \left(T_3^4 - T_2^4\right) = 0$$

Since $F_{31} = F_{32}$, $\quad T_3 = \left[\left(T_1^4 + T_2^4\right)/2\right]^{1/4} = \left[\left(1000^4 + 800^4\right)/2\right]^{1/4} \text{K} = 916\ \text{K}.$ <

Also from Eq. 13.14,

$$q'_2 = \frac{q_2}{\ell} = \frac{A_2}{\ell} F_{21} \sigma \left(T_2^4 - T_1^4\right) + \frac{A_2}{\ell} F_{23} \sigma \left(T_2^4 - T_3^4\right)$$ <

$$= 0.1 \times 0.5 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \times \left(2 \times 800^4 - 1000^4 - 916^4\right) \text{K}^4 = -2508\ \text{W/m}$$