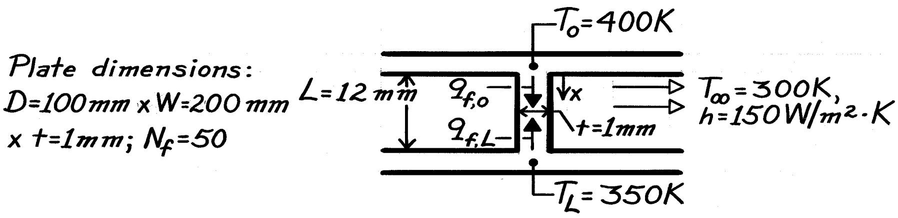
PROBLEM 3.142

KNOWN: Arrangement of fins between parallel plates. Temperature and convection coefficient of air flow in finned passages. Maximum allowable plate temperatures.

FIND: (a) Expressions relating fin heat transfer rates to end temperatures, (b) Maximum power dissipation for each plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Uniform h, (6) Negligible variation in T_∞, (7) Negligible contact resistance.

PROPERTIES: *Table A.1*, Aluminum (pure), 375 K: k = 240 W/m·K.

ANALYSIS: (a) The general solution for the temperature distribution in a fin is

$$\theta(x) \equiv T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary conditions: $\theta(0) = \theta_o = T_o - T_\infty$, $\quad \theta(L) = \theta_L = T_L - T_\infty$.

Hence $\quad \theta_o = C_1 + C_2 \qquad \theta_L = C_1 e^{mL} + C_2 e^{-mL}$

$$\theta_L = C_1 e^{mL} + \left(\theta_o - C_1\right) e^{-mL}$$

$$C_1 = \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} \qquad C_2 = \theta_o - \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} = \frac{\theta_o e^{mL} - \theta_L}{e^{mL} - e^{-mL}}.$$

Hence
$$\theta(x) = \frac{\theta_L e^{mx} - \theta_o e^{m(x-L)} + \theta_o e^{m(L-x)} - \theta_L e^{-mx}}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o\left[e^{m(L-x)} - e^{-m(L-x)}\right] + \theta_L\left(e^{mx} - e^{-mx}\right)}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \sinh m(L-x) + \theta_L \sinh mx}{\sinh mL}.$$

The fin heat transfer rate is then

$$q_f = -kA_c \frac{dT}{dx} = -kDt\left[-\frac{\theta_o m}{\sinh mL}\cosh m(L-x) + \frac{\theta_L m}{\sinh mL}\cosh mx\right].$$

Hence
$$q_{f,o} = kDt\left(\frac{\theta_o m}{\tanh mL} - \frac{\theta_L m}{\sinh mL}\right)$$ <

$$q_{f,L} = kDt\left(\frac{\theta_o m}{\sinh mL} - \frac{\theta_L m}{\tanh mL}\right).$$ <

Continued …

(b)
$$m=\left(\frac{hP}{kA_c}\right)^{1/2}=\left(\frac{150\ \text{W/m}^2\cdot\text{K}\left(2\times0.1\ \text{m}+2\times0.001\ \text{m}\right)}{240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}}\right)^{1/2}=35.5\ \text{m}^{-1}$$

$$mL=35.5\ \text{m}^{-1}\times0.012\ \text{m}=0.43$$

$$\sinh\ mL=0.439 \qquad \tanh\ mL=0.401 \qquad \theta_o=100\ \text{K} \qquad \theta_L=50\ \text{K}$$

$$q_{f,o}=240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}\right)$$

$q_{f,o}=115.4\ \text{W}$ (*from* the top plate)

$$q_{f,L}=240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}\right)$$

$q_{f,L}=87.8\ \text{W}.$ (*into* the bottom plate)

Maximum power dissipations are therefore

$$q_{o,max}=N_f q_{f,o}+\left(W-N_f t\right)Dh\theta_o$$

$$q_{o,max}=50\times115.4\ \text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times100\ \text{K}$$

$$q_{o,max}=5770\ \text{W}+225\ \text{W}=5995\ \text{W} \qquad <$$

$$q_{L,max}=-N_f q_{f,L}+\left(W-N_f t\right)Dh\theta_o$$

$$q_{L,max}=-50\times87.8\text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times50\ \text{K}$$

$$q_{L,max}=-4390\ \text{W}+112\text{W}=-4278\ \text{W}. \qquad <$$

COMMENTS: (1) It is of interest to determine the air velocity needed to prevent excessive heating of the air as it passes between the plates. If the air temperature change is restricted to $\Delta T_\infty = 5$ K, its flowrate must be

$$\dot{m}_{air}=\frac{q_{tot}}{c_p\Delta T_\infty}=\frac{1717\ \text{W}}{1007\ \text{J/kg}\cdot\text{K}\times5\ \text{K}}=0.34\ \text{kg/s}.$$

Its mean velocity is then

$$V_{air}=\frac{\dot{m}_{air}}{\rho_{air}A_c}=\frac{0.34\ \text{kg/s}}{1.16\ \text{kg/m}^3\times0.012\ \text{m}\left(0.2-50\times0.001\right)\text{m}}=163\ \text{m/s}.$$

Such a velocity would be impossible to maintain. To reduce it to a reasonable value, e.g. 10 m/s, A_c would have to be increased substantially by increasing W (and hence the space between fins) and by increasing L. The present configuration is impractical from the standpoint that 1717 W could not be transferred to air in such a small volume.

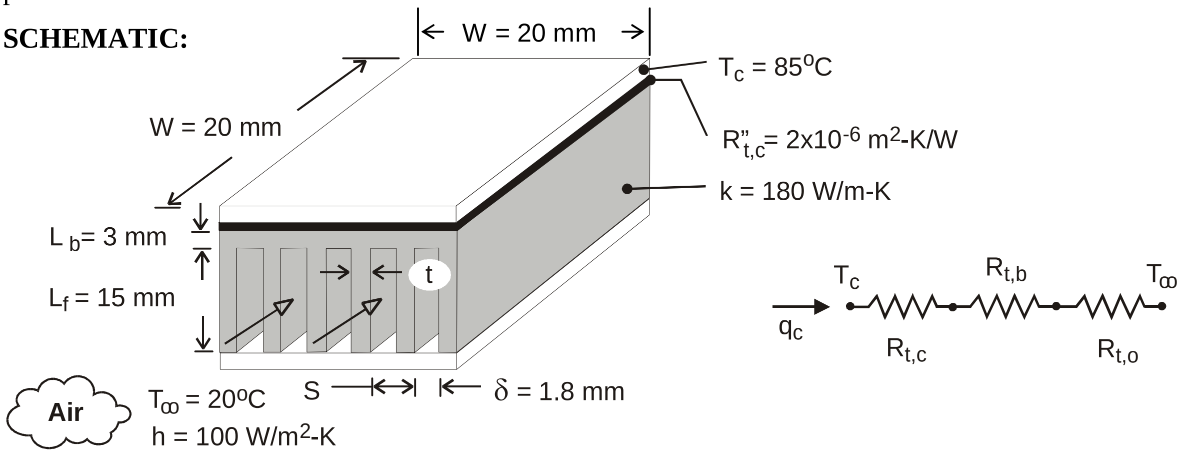
(2) A negative value of $q_{L,max}$ implies that the bottom plate must be cooled externally to maintain the plate at 350 K.

PROBLEM 3.144

KNOWN: Dimensions and maximum allowable temperature of an electronic chip. Thermal contact resistance between chip and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through the heat sink.

FIND: (a) Maximum allowable chip power for heat sink with prescribed number of fins, fin thickness, and fin pitch, and (b) Effect of fin thickness/number and convection coefficient on performance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surfaces of heat sink, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{tot}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

where $R_{t,c} = R''_{t,c} / W^2 = 2\times10^{-6}\,m^2\cdot K/W / (0.02m)^2 = 0.005\ K/W$ and $R_{t,b} = L_b / k\left(W^2\right)$

$= 0.003m / 180\ W/m\cdot K\,(0.02m)^2 = 0.042\ K/W$. From Eqs. (3.108), (3.107), and (3.104)

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \qquad \eta_o = 1 - \frac{N A_f}{A_t}(1-\eta_f), \qquad A_t = N A_f + A_b$$

where $A_f = 2WL_f = 2 \times 0.02m \times 0.015m = 6 \times 10^{-4}\ m^2$ and $A_b = W^2 - N(tW) = (0.02m)^2 - 11(0.182 \times 10^{-3}\ m \times 0.02m) = 3.6 \times 10^{-4}\ m^2$. With $mL_f = (2h/kt)^{1/2} L_f = (200\ W/m^2\cdot K/180\ W/m\cdot K \times 0.182 \times 10^{-3} m)^{1/2}\ (0.015m) = 1.17$, $\tanh mL_f = 0.824$ and Eq. (3.92) yields

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

It follows that $A_t = 6.96 \times 10^{-3}\ m^2$, $\eta_o = 0.719$, $R_{t,o} = 2.00$ K/W, and

$$q_c = \frac{(85-20)°C}{(0.005 + 0.042 + 2.00)K/W} = 31.8\ W \qquad <$$

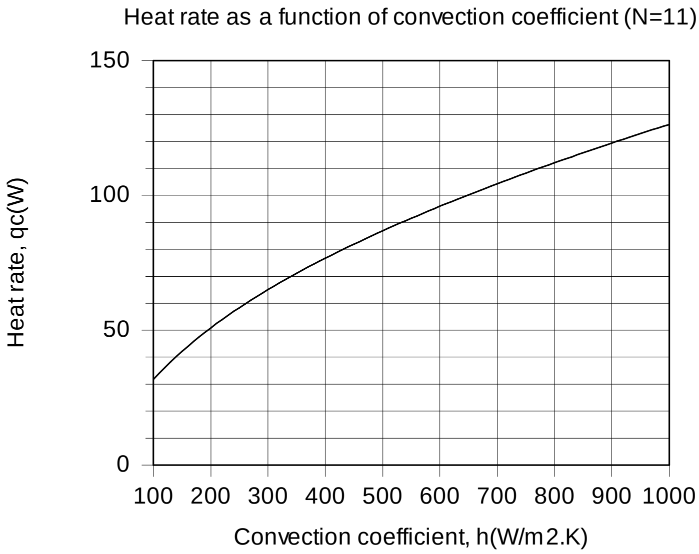
(b) The following results are obtained from parametric calculations performed to explore the effect of decreasing the number of fins and increasing the fin thickness.

Continued …

PROBLEM 3.144 (Cont.)

N	t(mm)	η_f	$R_{t,o}$ (K/W)	q_c (W)	A_t (m^2)
6	1.833	0.957	2.76	23.2	0.00378
7	1.314	0.941	2.40	26.6	0.00442
8	0.925	0.919	2.15	29.7	0.00505
9	0.622	0.885	1.97	32.2	0.00569
10	0.380	0.826	1.89	33.5	0.00632
11	0.182	0.704	2.00	31.8	0.00696

Although η_f (and η_o) increases with decreasing N (increasing t), there is a reduction in A_t which yields a minimum in $R_{t,o}$, and hence a maximum value of q_c, for N = 10. For N = 11, the effect of h on the performance of the heat sink is shown below.



With increasing h from 100 to 1000 W/m^2·K, $R_{t,o}$ decreases from 2.00 to 0.47 K/W, despite a decrease in η_f (and η_o) from 0.704 (0.719) to 0.269 (0.309). The corresponding increase in q_c is significant.

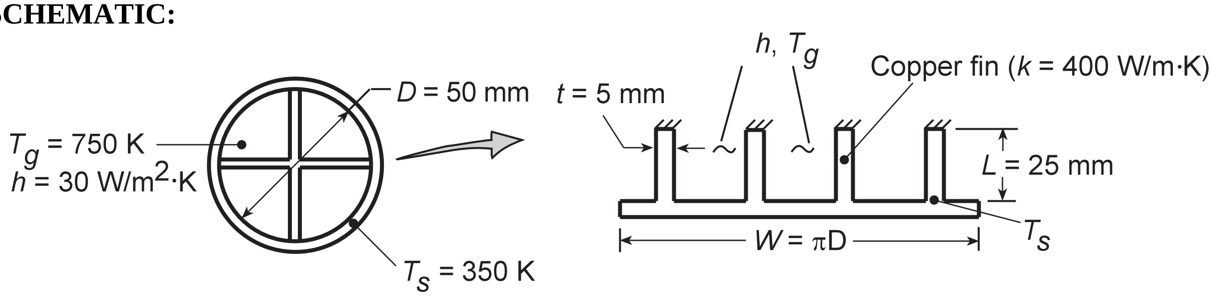
COMMENTS: (1) The heat sink significantly increases the allowable heat dissipation. If it were not used and heat was simply transferred by convection from the surface of the chip with h = 100 W/m^2·K, R_{tot} = 2.05 K/W from Part (a) would be replaced by $R_{cnv} = 1/hW^2$ = 25 K/W, yielding q_c = 2.60 W. (2) The air temperature will increase as it flows through the heat sink. Therefore the required air velocity will be greater than determined here. See Problem 11.89.

PROBLEM 3.149

KNOWN: Diameter and internal fin configuration of copper tubes submerged in water. Tube wall temperature and temperature and convection coefficient of gas flow through the tube.

FIND: Rate of heat transfer per tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional fin conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Tube wall may be unfolded and represented as a plane wall with four straight, rectangular fins, each with an adiabatic tip (since, by symmetry, there can be no heat flow along the fins where they cross).

ANALYSIS: The rate of heat transfer per unit tube length is:

$$q'_t = \eta_o h A'_t \left(T_g - T_s\right)$$

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1 - \eta_f\right)$$

$$NA'_f = 4 \times 2L = 8\left(0.025\text{m}\right) = 0.20\text{m}$$

$$A'_t = NA'_f + A'_b = 0.20\text{m} + \left(\pi D - 4t\right) = 0.20\text{m} + \left(\pi \times 0.05\text{m} - 4 \times 0.005\text{m}\right) = 0.337\text{m}$$

For an adiabatic fin tip,

$$\eta_f = \frac{q_f}{q_{max}} = \frac{M \tanh mL}{h\left(2L \cdot 1\right)\left(T_g - T_s\right)}$$

$$M = \left[h2\left(1\text{m} + t\right)k\left(1\text{m} \times t\right)\right]^{1/2}\left(T_g - T_s\right) \approx \left[30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)400\,\text{W/m} \cdot \text{K}\left(0.005\text{m}^2\right)\right]^{1/2}\left(400\text{K}\right) = 4382\text{W}$$

$$mL = \left\{\left[h2\left(1\text{m} + t\right)\right]/\left[k\left(1\text{m} \times t\right)\right]\right\}^{1/2} L \approx \left[\frac{30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)}{400\,\text{W/m} \cdot \text{K}\left(0.005\text{m}^2\right)}\right]^{1/2} 0.025\text{m} = 0.137$$

Hence, tanh mL = 0.136, and

$$\eta_f = \frac{4382\text{W}\left(0.136\right)}{30\,\text{W/m}^2 \cdot \text{K}\left(0.05\text{m}^2\right)\left(400\text{K}\right)} = \frac{595\text{W}}{600\text{W}} = 0.992$$

$$\eta_o = 1 - \frac{0.20}{0.337}\left(1 - 0.992\right) = 0.995$$

$$q'_t = 0.995\left(30\,\text{W/m}^2 \cdot \text{K}\right)0.337\text{m}\left(400\text{K}\right) = 4025\,\text{W/m}$$

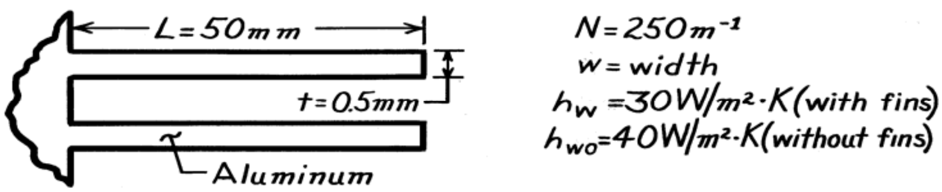
COMMENTS: Alternatively, $q'_t = 4q'_f + h\left(A'_t - A'_f\right)\left(T_g - T_s\right)$. Hence, q' = 4(595 W/m) + 30 W/m²·K (0.137 m)(400 K) = (2380 + 1644) W/m = 4024 W/m.

PROBLEM 3.155

KNOWN: Dimensions and number of rectangular aluminum fins. Convection coefficient with and without fins.

FIND: Percentage increase in heat transfer resulting from use of fins.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Negligible fin contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1,* Aluminum, pure: $k \approx 240$ W/m·K.

ANALYSIS: Evaluate the fin parameters

$$L_c = L + t/2 = 0.05025\text{m}$$

$$A_p = L_c t = 0.05025\text{m} \times 0.5 \times 10^{-3}\text{m} = 25.13 \times 10^{-6}\ \text{m}^2$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = \left(0.05025\text{m}\right)^{3/2} \left[\frac{30\ \text{W/m}^2 \cdot \text{K}}{240\ \text{W/m} \cdot \text{K} \times 25.13 \times 10^{-6}\text{m}^2}\right]^{1/2}$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = 0.794$$

It follows from Fig. 3.19 that $\eta_f \approx 0.72$. Hence,

$$q_f = \eta_f q_{max} = 0.72\ h_w 2wL\ \theta_b$$

$$q_f = 0.72 \times 30\ \text{W/m}^2 \cdot \text{K} \times 2 \times 0.05\text{m} \times \left(w\ \theta_b\right) = 2.16\ \text{W/m} \cdot \text{K}\left(w\ \theta_b\right)$$

With the fins, the heat transfer from the walls is

$$q_w = N\ q_f + \left(1 - Nt\right) w\ h_w\ \theta_b$$

$$q_w = 250 \times 2.16 \frac{\text{W}}{\text{m} \cdot \text{K}}\left(w\ \theta_b\right) + \left(1\text{m} - 250 \times 5 \times 10^{-4}\text{m}\right) \times 30\ \text{W/m}^2 \cdot \text{K}\ \left(w\ \theta_b\right)$$

$$q_w = \left(540 + 26.3\right)\frac{\text{W}}{\text{m} \cdot \text{K}}\left(w\ \theta_b\right) = 566\ w\ \theta_b.$$

Without the fins, $q_{wo} = h_{wo}\ 1\text{m} \times w\ \theta_b = 40\ w\ \theta_b$. Hence the percentage increase in heat transfer is

$$\frac{q_w - q_{wo}}{q_{wo}} = \frac{\left(566 - 40\right) w\ \theta_b}{40\ w\ \theta_b} = 13.15 = 1315\%$$ <

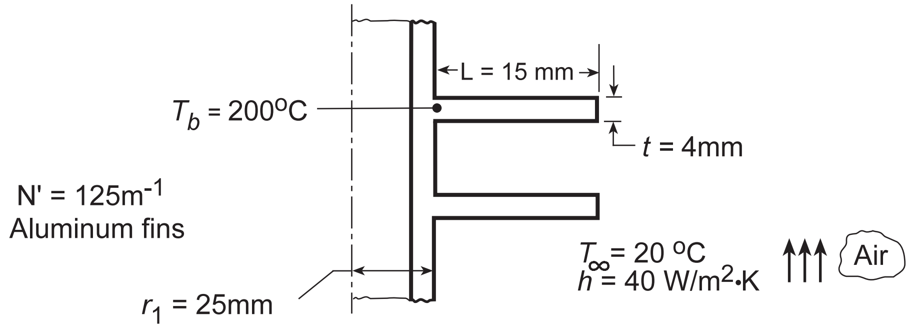
COMMENTS: If the infinite fin approximation is made, it follows that $q_f = (hPkA_c)^{1/2}\ \theta_b = [h_w 2wkwt]^{1/2}\ \theta_b = (30 \times 2 \times 240 \times 5{\times}10^{-4})^{1/2}\ w\ \theta_b = 2.68\ w\ \theta_b$. Hence, q_f is overestimated.

PROBLEM 3.159

KNOWN: Dimensions and base temperature of aluminum fins of rectangular profile. Ambient air conditions.

FIND: (a) Fin efficiency and effectiveness, (b) Rate of heat transfer per unit length of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Negligible base contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.20 are

$$r_{2c} = r_2 + t/2 = 40\,\text{mm} + 2\,\text{mm} = 0.042\,\text{m} \qquad L_c = L + t/2 = 15\,\text{mm} + 2\,\text{mm} = 0.017\,\text{m}$$

$$r_{2c}/r_1 = 0.042\,\text{m}/0.025\,\text{m} = 1.68 \qquad A_p = L_c t = 0.017\,\text{m} \times 0.004\,\text{m} = 6.8\times10^{-5}\,\text{m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = \left(0.017\,\text{m}\right)^{3/2}\left[40\,\text{W/m}^2\cdot\text{K}\big/240\,\text{W/m}\cdot\text{K}\times 6.8\times10^{-5}\,\text{m}^2\right]^{1/2} = 0.11$$

The fin efficiency is $\eta_f \approx 0.97$. From Eq. 3.91,

$$q_f = \eta_f q_{max} = \eta_f h A_{f\,(ann)}\theta_b = 2\pi\eta_f h\left[r_{2c}^2 - r_1^2\right]\theta_b$$

$$q_f = 2\pi \times 0.97 \times 40\,\text{W/m}^2\cdot\text{K}\left[\left(0.042\right)^2 - \left(0.025\right)^2\right]\text{m}^2 \times 180^\circ\text{C} = 50\,\text{W}$$

<

From Eq. 3.86, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{50\,\text{W}}{40\,\text{W/m}^2\cdot\text{K}\;\;2\pi\left(0.025\,\text{m}\right)\left(0.004\,\text{m}\right)180^\circ\text{C}} = 11.05$$

<

(b) The rate of heat transfer per unit length is

$$q' = N'q_f + h\left(1 - N't\right)\left(2\pi r_1\right)\theta_b$$

$$q' = 125 \times 50\,\text{W/m} + 40\,\text{W/m}^2\cdot\text{K}\left(1 - 125 \times 0.004\right)\left(2\pi \times 0.025\,\text{m}\right) \times 180^\circ\text{C}$$

$$q' = \left(6250 + 565\right)\text{W/m} = 6.82\,\text{kW/m}$$

<

COMMENTS: Note the dominant contribution made by the fins to the total heat transfer.