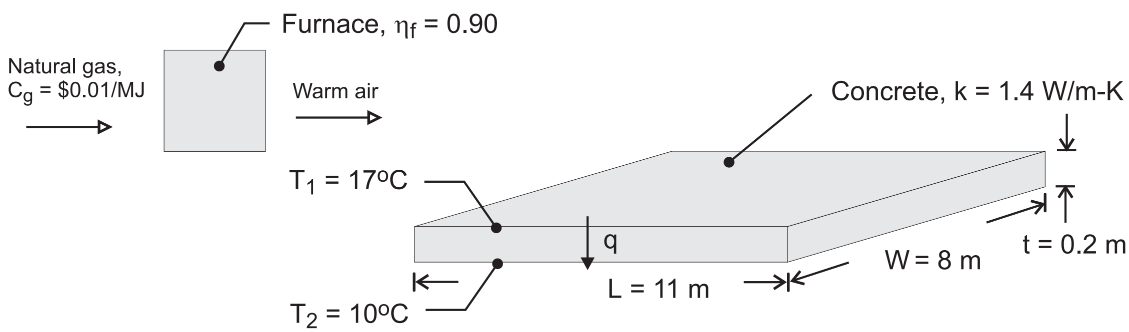
PROBLEM 1.4

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,W/m\cdot K\,(11\,m \times 8\,m)\frac{7°C}{0.20\,m} = 4312\ W$$ <

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,W \times \$0.02/MJ}{0.9 \times 10^6 J/MJ}(24\,h/d \times 3600\,s/h) = \$8.28/d$$ <

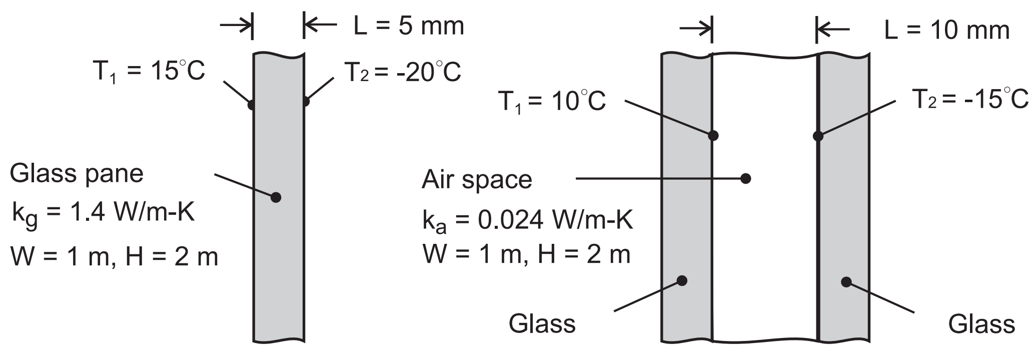
COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 1.9

KNOWN: Width, height, thickness and thermal conductivity of a single pane window and the air space of a double pane window. Representative winter surface temperatures of single pane and air space.

FIND: Heat loss through single and double pane windows.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through glass or air, (2) Steady-state conditions, (3) Enclosed air of double pane window is stagnant (negligible buoyancy induced motion).

ANALYSIS: From Fourier's law, the heat losses are

Single Pane: $$q_g = k_g A \frac{T_1 - T_2}{L} = 1.4 \text{ W/m}\cdot\text{K}\left(2\text{m}^2\right)\frac{35\ ^\circ\text{C}}{0.005\text{m}} = 19,600 \text{ W} \qquad <$$

Double Pane: $$q_a = k_a A \frac{T_1 - T_2}{L} = 0.024\left(2\text{m}^2\right)\frac{25\ ^\circ\text{C}}{0.010 \text{ m}} = 120 \text{ W} \qquad <$$

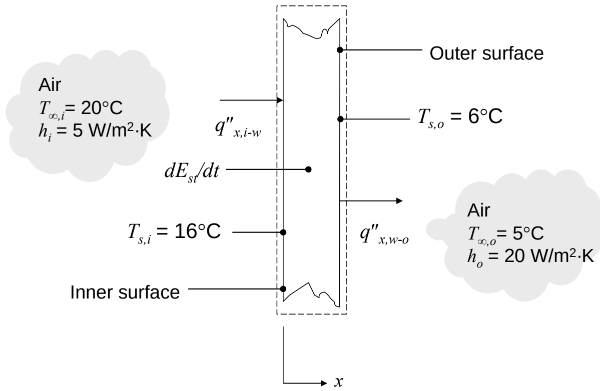
COMMENTS: Losses associated with a single pane are unacceptable and would remain excessive, even if the thickness of the glass were doubled to match that of the air space. The principal advantage of the double pane construction resides with the low thermal conductivity of air (~ 60 times smaller than that of glass). For a fixed ambient outside air temperature, use of the double pane construction would also increase the surface temperature of the glass exposed to the room (inside) air.

PROBLEM 1.20

KNOWN: Inner and outer surface temperatures of a wall. Inner and outer air temperatures and convection heat transfer coefficients.

FIND: Heat flux from inner air to wall. Heat flux from wall to outer air. Heat flux from wall to inner air. Whether wall is under steady-state conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation, (2) No internal energy generation.

ANALYSIS: The heat fluxes can be calculated using Newton's law of cooling. Convection from the inner air to the wall occurs in the positive x-direction:

$$q''_{x,i-w} = h_i(T_{\infty,i} - T_{s,i}) = 5\ \text{W/m}^2 \cdot \text{K} \times (20°\text{C} - 16°\text{C}) = 20\ \text{W/m}^2 \qquad <$$

Convection from the wall to the outer air also occurs in the positive x-direction:

$$q''_{x,w-o} = h_o(T_{s,o} - T_{\infty,o}) = 20\ \text{W/m}^2 \cdot \text{K} \times (6°\text{C} - 5°\text{C}) = 20\ \text{W/m}^2 \qquad <$$

From the wall to the inner air:

$$q''_{w-i} = h_i(T_{s,i} - T_{\infty,i}) = 5\ \text{W/m}^2 \cdot \text{K} \times (16°\text{C} - 20°\text{C}) = -20\ \text{W/m}^2 \qquad <$$

An energy balance on the wall gives

$$\frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} = A(q''_{x,i-w} - q''_{x,w-o}) = 0$$

Since $dE_{st}/dt = 0$, the wall *could be* at steady-state and the *spatially-averaged* wall temperature is not changing. However, it is possible that stored energy is increasing in one part of the wall and decreasing in another, therefore we cannot tell if the wall is at steady-state or not. If we found $dE_{st}/dt \neq 0$, we would know the wall was not at steady-state. $<$

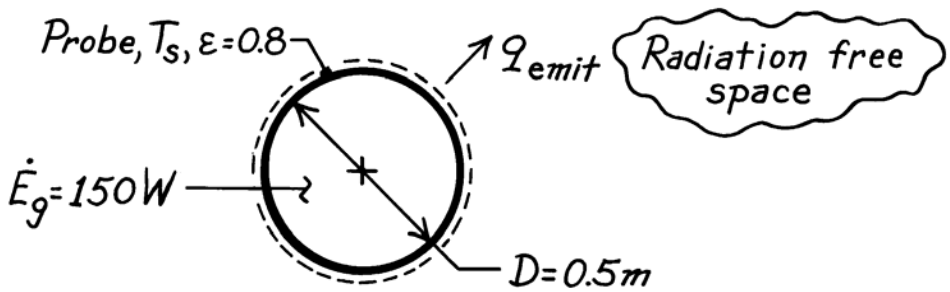
COMMENTS: The heat flux from the wall to the inner air is equal and opposite to the heat flux from the inner air to the wall.

PROBLEM 1.30

KNOWN: Diameter and emissivity of spherical interplanetary probe. Power dissipation within probe.

FIND: Probe surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation incident on the probe.

ANALYSIS: Conservation of energy dictates a balance between energy generation within the probe and radiation emission from the probe surface. Hence, at any instant

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$\varepsilon A_s \sigma T_s^4 = \dot{E}_g$$

$$T_s = \left(\frac{\dot{E}_g}{\varepsilon \pi D^2 \sigma}\right)^{1/4}$$

$$T_s = \left(\frac{150\text{W}}{0.8\pi(0.5\text{ m})^2\, 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\right)^{1/4}$$

$$T_s = 254.7\text{ K.}$$ <

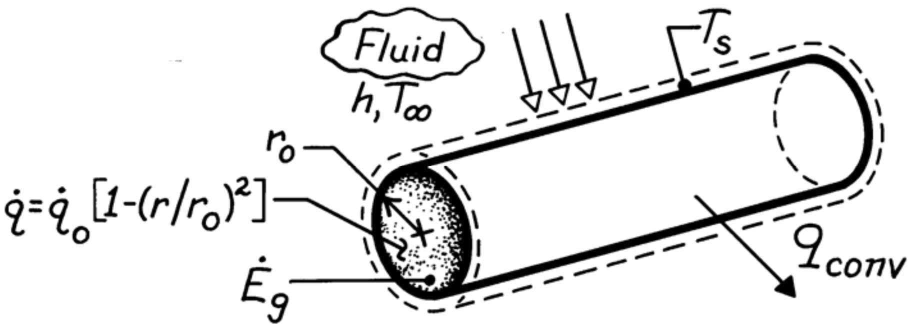
COMMENTS: Incident radiation, as, for example, from the sun, would increase the surface temperature.

PROBLEM 1.44

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Total energy generation rate and surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature drop across thin container wall.

ANALYSIS: The rate of energy generation is

$$\dot{E}_g = \int \dot{q} dV = \dot{q}_o \int_0^{r_o} \left[1 - (r/r_o)^2\right] 2\pi r L dr$$

$$\dot{E}_g = 2\pi L \dot{q}_o \left(r_o^2/2 - r_o^2/4\right)$$

or per unit length,

$$\dot{E}'_g = \frac{\pi \dot{q}_o r_o^2}{2}. \qquad \qquad <$$

Performing an energy balance for a control surface about the container yields, at an instant,

$$\dot{E}'_g - \dot{E}'_{out} = 0$$

and substituting for the convection heat rate per unit length,

$$\frac{\pi \dot{q}_o r_o^2}{2} = h\left(2\pi r_o\right)\left(T_s - T_\infty\right)$$

$$T_s = T_\infty + \frac{\dot{q}_o r_o}{4h}. \qquad \qquad <$$

COMMENTS: The temperature within the radioactive wastes increases with decreasing r from T_s at r_o to a maximum value at the centerline.