5.36

5.36 A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in **Video V5.6** and Fig. P5.36. Determine the minimum volume flowrate needed to tip the block.

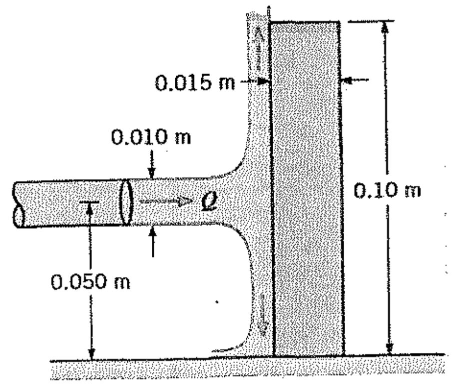


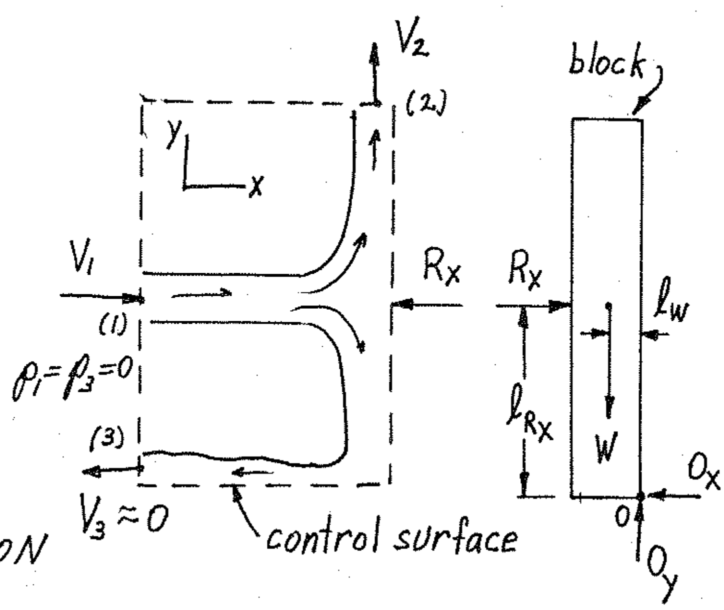
FIGURE P5.36

From the free body diagram of the block when it is ready to tip $\sum M_0 = 0$, or

$R_x \ell_{R_x} = W \ell_w$ where R_x is the fore that the water puts on the block.

Thus,

$$R_x = \frac{W \ell_w}{\ell_{R_x}} = \frac{6\,N\left(\frac{0.015}{2}\,m\right)}{0.050\,m} = 0.90\,N$$



For the control volume shown the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \sum F_x$$

becomes

$$V_1 \rho (-V_1) A_1 = -R_x \quad \text{or} \quad V_1 = \sqrt{\frac{R_x}{\rho A_1}}$$

Thus,

$$V_1 = \sqrt{\frac{0.9\,N}{\left(999\,\frac{kg}{m^3}\right)\frac{\pi}{4}(0.01\,m)^2}} = 3.39\,\frac{m}{s}$$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.01\,m)^2\left(3.39\,\frac{m}{s}\right) = \underline{\underline{2.66 \times 10^{-4}\,\frac{m^3}{s}}}$$

5.42

5.42 Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.42 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.

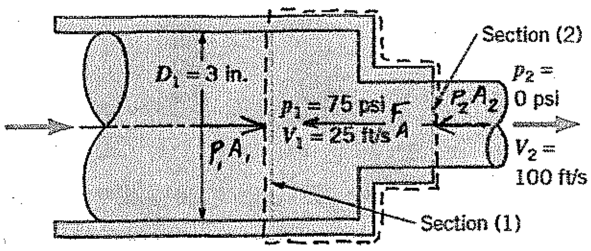


FIGURE P5.42

For this problem we include in the control volume the nozzle as well as the water at an instant between sections (1) and (2) as indicated in the sketch above. The horizontal forces acting on the contents of the control volume are shown in the sketch. Note that the atmospheric forces cancel out and are not shown. Application of the horizontal or x-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = p_1 A_1 - F_A - p_2 A_2 \qquad (1)$$

From the conservation of mass equation (Eq. 5.12) we obtain

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus Eq. (1) may be expressed as

$$\dot{m}(u_2 - u_1) = p_1 A_1 - F_A - p_2 A_2$$

or

$$F_A = p_1 A_1 - p_2 A_2 + \dot{m}(u_2 - u_1) = p_1 \frac{\pi D_1^2}{4} - p_2 \frac{\pi D_2^2}{4} - \rho u_1 \frac{\pi D_1^2}{4}(u_2 - u_1)$$

and

$$F_A = \left(75 \frac{lb}{in^2}\right)\frac{\pi (3 in.)^2}{4} - 0\ lb - \left(1.94 \frac{slugs}{ft^3}\right)\left(25 \frac{ft}{s}\right)\frac{\pi (3 in.)^2}{4} \frac{\left(100 \frac{ft}{s} - 25 \frac{ft}{s}\right)}{\left(144 \frac{in.^2}{ft^2}\right)}\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)$$

$$F_A = \underline{\underline{352}}\ lb$$

5.65

5.65 A horizontal circular jet of air strikes a stationary flat plate as indicated in Fig. 5.65. The jet velocity is 40 m/s and the jet diameter is 30 mm. If the air velocity magnitude remains constant as the air flows over the plate surface in the directions shown, determine: (a) the magnitude of F_A, the anchoring force required to hold the plate stationary; (b) the fraction of mass flow along the plate surface in each of the two directions shown; (c) the magnitude of F_A, the anchoring force required to allow the plate to move to the right at a constant speed of 10 m/s.

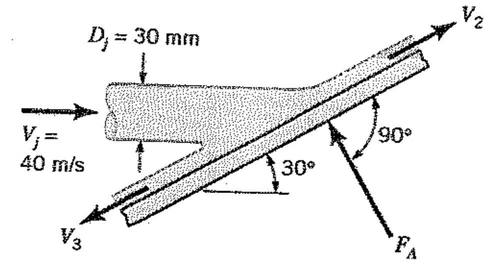
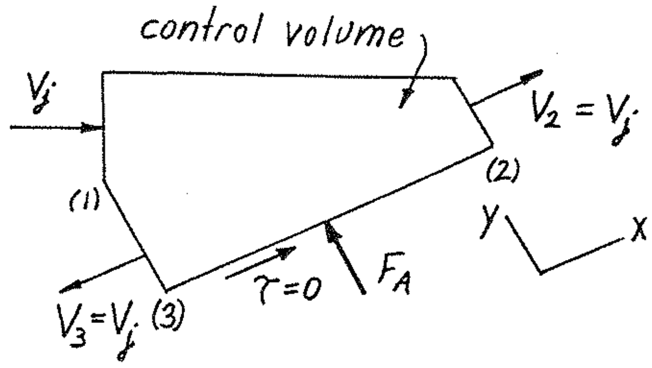


FIGURE P5.65



The non-deforming control volume shown in the sketch above is used.

(a) To determine the magnitude of F_A we apply the component of the linear momentum equation (Eq. 5.22) along the direction of F_A. Thus, $\int_{cs} v \, \rho \vec{V}\cdot\hat{n}\, dA = \Sigma F_y$, or

$$F_A = \dot{m} V_j \sin 30° = \rho A_j V_j V_j \sin 30° = \frac{\rho \pi D_j^2}{4} V_j^2 \sin 30°$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030 m)^2}{(4)} \left(40 \frac{m}{s}\right)^2 (\sin 30°) \left(\frac{1 \, N}{kg \cdot \frac{m}{s^2}}\right) = \underline{\underline{0.696 \, N}}$$

(b) To determine the fraction of mass flow along the plate surface in each of the 2 directions shown in the sketch above, we apply the component of the linear momentum equation parallel to the surface of the plate, $\int_{cs} u \rho \vec{V}\cdot\hat{n}\, dA = \Sigma F_x$, to obtain

$$R_{\substack{\text{along plate} \\ \text{surface}}} = \dot{m}_2 V_2 - \dot{m}_3 V_3 - \dot{m}_j V_j \cos 30° \qquad (1)$$

(con't)

5.65 (con't)

Since the air velocity magnitude remains constant, the value of $R_{\text{along plate surface}}$ is zero.* Thus from Eq. 1 we obtain

$$\dot{m}_3 V_3 = \dot{m}_2 V_2 - \dot{m}_j V_j \cos 30° \qquad (2)$$

Since $V_3 = V_2 = V_j$, Eq. 2 becomes

$$\dot{m}_3 = \dot{m}_2 - \dot{m}_j \cos 30° \qquad (3)$$

From conservation of mass we conclude that

$$\dot{m}_j = \dot{m}_2 + \dot{m}_3 \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$\dot{m}_3 = \dot{m}_j - \dot{m}_3 - \dot{m}_j \cos 30°$$

or

$$\dot{m}_3 = \dot{m}_j \frac{(1-\cos 30°)}{2} = \dot{m}_j (0.0670)$$

and

$$\dot{m}_2 = \dot{m}_j (1 - 0.067) = \dot{m}_j (0.933)$$

Thus, $\underline{\underline{\dot{m}_2 \text{ involves } 93.3\% \text{ of } \dot{m}_j}}$ and $\underline{\underline{\dot{m}_3 \text{ involves } 6.7\% \text{ of } \dot{m}_j}}$.

(c) To determine the magnitude of F_A required to allow the plate to move to the right at a constant speed of $10 \frac{m}{s}$, we use a non-deforming control volume like the one in the sketch above that moves to the right with a speed of $10 \frac{m}{s}$. The translating control volume linear momentum equation (Eq. 5.29) leads to

$$F_A = \frac{\rho \pi D_j^2}{4} \left(V_j - 10 \frac{m}{s}\right)^2 \sin 30°$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030\,m)^2}{4} \left(40 \frac{m}{s} - 10 \frac{m}{s}\right)^2 (\sin 30°) \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

and

$$\underline{\underline{F_A = 0.391\ N}}$$

* Since $V_1 = V_2 = V_3$ and $p_1 = p_2 = p_3$ and $z_1 = z_2 = z_3$ it follows that the Bernoulli equation is valid from $1 \to 2$ and $1 \to 3$.
Thus, there are no viscous effects (Bernoulli equation is valid only for inviscid flow) so that $\tau = 0$. Hence, $R_{\text{along plate}} = 0$.

5.76

5.76 Thrust vector control is a new technique that can be used to greatly improve the maneuverability of military fighter aircraft. It consists of using a set of vanes in the exit of a jet engine to deflect the exhaust gases as shown in Fig. P5.76. **(a)** Determine the pitching moment (the moment tending to rotate the nose of the aircraft up) about the aircraft's mass center (cg) for the conditions indicated in the figure. **(b)** By how much is the thrust (force along the centerline of the aircraft) reduced for the case indicated compared to normal flight when the exhaust is parallel to the centerline?

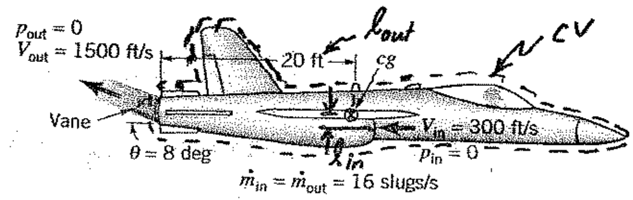


FIGURE P5.76

For part (a) we apply the component of the moment-of-momentum equation that is perpendicular to the plane of the sketch of the aircraft to the contents of the control volume shown to get

$$l_{out} V_{out} \sin\theta \, \dot{m}_{out} - l_{in} V_{in} \dot{m}_{in} = \text{pitching moment}$$

$$\frac{(20 \text{ ft})(1500 \frac{ft}{s}) \sin 8^\circ (16 \frac{slugs}{s})}{1 \frac{slug \cdot ft}{lb \cdot s^2}} - \frac{l_{in} (300 \frac{ft}{s})(16 \frac{slugs}{s})}{1 \frac{slug \cdot ft}{lb \cdot s^2}} = \text{pitching moment}$$

$$66{,}800 - 4800\, l_{in} \quad ft \cdot lb = \text{pitching moment}$$

For part (b) we apply the horizontal component of the linear momentum equation to the contents of the control volume to get

$$V_{out} \cos\theta \, \dot{m}_{out} - V_{in} \dot{m}_{in} = \text{thrust}$$

So

$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = V_{out} (\cos 0^\circ - \cos 8^\circ) \dot{m}_{out}$$

or

$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = \frac{(1500 \frac{ft}{s})(\cos 0^\circ - \cos 8^\circ)(16 \frac{slugs}{s})}{1 \frac{slug \cdot ft}{lb \cdot s^2}}$$

and

$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = \underline{\underline{234}} \text{ lb}$$