ME 4189 Spring 2016
Homework assignment #2 - Solutions

1 Problem 2.97

Equation of motion for damped pendulum:

$$ml^2\ddot{\theta} + c\theta + mgl\sin\theta = 0 \tag{1}$$

If angle is small,

$$ml^2\ddot{\theta} + c\theta + mgl\theta = 0 \tag{2}$$

Which can rewritten as:

$$\ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2\theta = 0 \tag{3}$$

where ω_n is the (undamped) natural frequency and is given by:

$$\omega_n = \sqrt{\frac{g}{l}} \tag{4}$$

and ξ is the damped ratio:

$$\xi = \frac{c}{2ml^2\omega_n} \tag{5}$$

For this problem, we know that the undamped natural frequency is equal to 0.5Hz. Hence

$$l = \frac{g}{\omega_n^2} = \frac{9.81}{\pi^2} = 0.9940\text{m} \tag{6}$$

The damped natural frequency is equal to 0.45 Hz. The relationship between the damped natural frequency and the undamped natural frequency is:

$$\omega_d = \omega_n\sqrt{1-\xi^2} \tag{7}$$

Hence,

$$\xi = \sqrt{\frac{\omega_n^2 - \omega_d^2}{\omega_n^2}} = \sqrt{(2\pi)^2\frac{0.5^2 - 0.45^2}{(2\pi 0.5)^2}} = 0.43 \tag{8}$$

The damping constant is given by:

$$c = 2\xi ml^2\omega_n = 2 \times 0.43 \times (2\pi) \times 0.994^2 = 2.67Nms/rad \tag{9}$$

2.102 (i) (a) Viscous damping, (b) Coulomb damping.

(iii) (a) $\tau_d = 0.2$ sec, $f_d = 5$ Hz, $\omega_d = 31.416$ rad/sec.
(b) $\tau_n = 0.2$ sec, $f_n = 5$ Hz, $\omega_n = 31.416$ rad/sec.

(ii) (a) $\dfrac{x_i}{x_{i+1}} = e^{\varsigma\, \omega_n\, \tau_d}$

$$\ln\left(\frac{x_i}{x_{i+1}}\right) = \ln 2 = 0.6931 = \frac{2\,\pi\,\varsigma}{\sqrt{1-\varsigma^2}}$$

or $39.9590\ \varsigma^2 = 0.4804$ or $\varsigma = 0.1096$

Since $\omega_d = \omega_n\ \sqrt{1-\varsigma^2}$, we find

$$\omega_n = \frac{\omega_d}{\sqrt{1-\varsigma^2}} = \frac{31.416}{\sqrt{0.98798}} = 31.6065 \text{ rad/sec}$$

$$k = m\ \omega_n^2 = \left(\frac{500}{9.81}\right)(31.6065)^2 = 5.0916\ (10^4)\ \text{N/m}$$

$$\varsigma = \frac{c}{c_c} = \frac{c}{2\ m\ \omega_n}$$

Hence $c = 2\ m\ \omega_n\ \varsigma = 2\ (\frac{500}{9.81})\ (31.6065)\ (0.1096) = 353.1164$ N–s/m

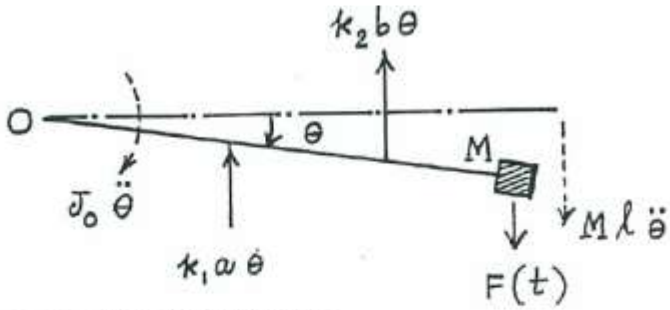
(b) From Eq. (2.135):

$$k = m\ \omega_n^2 = \frac{500}{9.81}\ (31.416)^2 = 5.0304\ (10^4)\ \text{N/m}$$

Using N = W = 500 N,

$$\mu = \frac{0.002\ k}{4\ W} = \frac{(0.002)\ (5.0304\ (10^4))}{4\ (500)} = 0.0503$$

3.24



Equation of motion for rotational motion about the hinge O:

$$(J_0 + M\,\ell^2)\,\ddot{\theta} + (k_1\,a^2 + k_2\,b^2)\,\theta = F(t)\,\ell = F_0\,\ell\,\sin\omega t \tag{1}$$

Steady state response (using Eqs. (3.3) and (3.6)):

$$\theta_p(t) = \Theta \sin \omega t \tag{2}$$

$$\text{where} \quad \Theta = \frac{F_0\,\ell}{(k_1\,a^2 + k_2\,b^2) - (J_0 + M\,\ell^2)\,\omega^2} \tag{3}$$

$$\text{and} \quad J_0 = \frac{m\,\ell^2}{12} + m\,(\frac{\ell}{2})^2 = \frac{1}{3}\,m\,\ell^2 \tag{4}$$

For given data, $J_0 = \frac{1}{3}\,(10)\,(1^2) = 3.3333\ \text{kg--m}^2$, $\omega = \frac{1000\,(2\,\pi)}{60} = 104.72$ rad/sec, and

$$\Theta = \frac{500\,(1)}{5000\,(0.25^2 + 0.5^2) - (3.3333 + 50\,(1^2))\,(104.72^2)} = -\,8.5718\,(10^{-4})\ \text{rad}$$

3.27.

$k = 4000$ N/m, $m = 10$ kg, $c = 40$ N-s/m, $F(t) = 200 \cos 10t$,
$F_0 = 200$ N, $\omega = 10$ rad/s, $x_0 = 0.1$ m, $\dot{x}_0 = 0$

$\omega_n = \sqrt{\frac{k}{m}} = 20$ rad/s, $\delta_{st} = \frac{F_0}{k} = \frac{200}{4000} = 0.05$ m

$\zeta = c/c_c = \left(c/2\sqrt{km}\right) = \left(40/2\sqrt{4000(10)}\right) = 0.1$

$\omega_d = \sqrt{1-\zeta^2}\,\omega_n = \sqrt{1-(0.1)^2}\,(20) = 19.899749$ rad/s

$r = \frac{\omega}{\omega_n} = \frac{10}{20} = 0.5$

$$X = \delta_{st}\Big/\sqrt{(1-r^2)^2 + (2\zeta r)^2} = \frac{0.05}{\left\{(1-0.5^2)^2 + (2(0.1)(0.5))^2\right\}^{\frac{1}{2}}}$$

$= 0.066082$ m

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = \tan^{-1}\left(\frac{2 * 0.1 * 0.5}{1-0.5^2}\right) = 0.132552 \text{ rad}$$

steady state response, Eq. (3.25):

$$x_p(t) = X \cos(\omega t - \phi)$$
$$= 0.066082 \cos(10t - 0.132552) \text{ m}$$

$$x(t) = X_0 e^{-\zeta\omega_n t} \cos(\omega_d t - \phi_0) + X \cos(\omega t - \phi) \quad (E.1)$$

Using the initial conditions x_0 and $\dot{x}_0$, Eq. (E.1) gives

$$x_0 = X_0 \cos\phi_0 + X \cos\phi \quad (E.2)$$

or $$X_0 \cos\phi_0 = x_0 - X \cos\phi \quad (E.3)$$

$$\dot{x}_0 = -\zeta\omega_n X_0 \cos\phi_0 + \omega_d X_0 \sin\phi_0 + \omega X \sin\phi \quad (E.4)$$

or $$X_0 \sin\phi_0 = \frac{1}{\omega_1}\left\{\dot{x}_0 + \zeta\omega_n X_0 \cos\phi_0 - \omega X \sin\phi\right\} \quad (E.5)$$

$$X_0 \cos\phi_0 = x_0 - X\cos\phi = -0.065502$$

$$X_0 \sin\phi_0 = \frac{1}{\omega_d}\left\{\dot{x}_0 + \zeta\omega_n X_0\cos\phi_0 - \omega X\sin\phi\right\} = 0.491547$$

$$X_0 = \left\{(X_0\cos\phi_0)^2 + (X_0\sin\phi_0)^2\right\}^{\frac{1}{2}} = 0.495892$$

$$\phi_0 = \tan^{-1}\left(\frac{X_0\sin\phi_0}{X_0\cos\phi_0}\right) = -1.438320$$

Thus the total response, Eq. (3.35), is given by

$$x(t) = 0.495892\ e^{-2t}\cos(19.899749\,t + 1.438320)$$
$$+\ 0.066082\cos(10\,t - 0.132552) \quad m$$