$\boxed{6.95}$ (1)

$$[k] = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \qquad [m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$= \begin{bmatrix} 100 & -100 \\ -100 & 150 \end{bmatrix} \times 10^3 \qquad = \begin{bmatrix} 80 & 0 \\ 0 & 40 \end{bmatrix}$$

<u>Natural frequencies</u>

$$\det\left([k] - \omega^2 [m] \right) = 0$$

$$\det \left(\begin{bmatrix} 100 \cdot 10^3 - 80\omega^2 & -100 \times 10^3 \\ -100 \times 10^3 & 150 \times 10^3 - 40\omega^2 \end{bmatrix} \right) = 0$$

$$(100 \cdot 10^3 - 80\omega^2)(150 \cdot 10^3 - 40\omega^2) - (100 \cdot 10^3)^2 = 0$$

$$\cdot 10^3 \cdot \left(150\,000 - 4000\omega^2 - 12000\omega^2 + 3200\omega^4 - 10000 \right) = 0$$

$$= 150\cancel{000} - 16\cancel{000}\,\omega^2 + 3.2\cancel{00}\,\omega^4 = 0$$

$$\Rightarrow \omega_2 = 68.3015 \text{ rad/s}$$

$$\omega_1 = 18.3013 \text{ rad/s}$$

$$(100 \cdot 10^3 - 80\omega_1^2)\, X_1^{(1)} - 100 \times 10^3\, X_2^{(1)} = 0$$

$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{100 \times 10^3 - 80\omega_1^2}{100 \times 10^3}$$

$\lambda_1 = 0.752$

$\lambda_2 = -2.732$

$\{\vec{X}^{(1)}\} = \begin{pmatrix} 1 \\ 0.752 \end{pmatrix} X_1^{(1)}$

$\{\vec{X}^{(2)}\} = \begin{pmatrix} 1 \\ -2.732 \end{pmatrix} X_1^{(2)}$

<u>Mass normalizing the modal vectors</u>

$\langle \vec{X}^{(1)} \rangle [m] \{\vec{X}^{(1)}\} = 1$

$\Rightarrow (1 \quad 0.752) \begin{bmatrix} 80 & 0 \\ 0 & 40 \end{bmatrix} \begin{pmatrix} 1 \\ 0.752 \end{pmatrix} (X_1^{(1)})^2 = 1$

$X_1^{(1)} = 0.0993$

$\{\vec{X}^{(1)}\} = \begin{pmatrix} 1 \\ 0.752 \end{pmatrix} 0.0993 = \begin{Bmatrix} 0.09929 \\ 0.07268 \end{Bmatrix}$

$[1 \quad -2.732] \begin{bmatrix} 80 & 0 \\ 0 & 40 \end{bmatrix} \begin{Bmatrix} 1 \\ -2.732 \end{Bmatrix} X_1^{(2)\,2} = 1$

$\Rightarrow X_1^{(2)} = 0.0514$

$\{\vec{X}^{(2)}\} = 0.0514 \begin{Bmatrix} 1 \\ -2.732 \end{Bmatrix}$

$= \begin{Bmatrix} 0.0514 \\ -0.14042 \end{Bmatrix}$

$$\{\vec{q}(t)\} = \begin{Bmatrix} \dfrac{1}{\omega_1} \displaystyle\int_0^t Q_1(\tau) \sin(\omega_1(t-\tau))\, d\tau \\ \dfrac{1}{\omega_2} \displaystyle\int_0^t Q_2(\tau) \sin(\omega_2(t-\tau))\, d\tau \end{Bmatrix}$$

$$= \cancel{\frac{1}{18.03\,\omega_1} \int_0^t 181.7 \sin \omega_1(t-\tau)}$$

$$= \begin{Bmatrix} \dfrac{-1}{\omega_1^2} Q_1 \cos(\omega_1(t-\tau))\Big|_0^t \\ -\dfrac{1}{\omega_2} Q_2 \cos(\omega_2(t-\tau))\Big|_0^t \end{Bmatrix}$$

$$= \begin{Bmatrix} +\dfrac{Q_1}{\omega_1^2} (1 - \cos \omega_1 t) \\ +\dfrac{Q_2}{\omega_2^2} (1 - \cos \omega_2 t) \end{Bmatrix}$$

$$= \begin{Bmatrix} +\dfrac{181.5}{18.03^2} (1 - \cos \omega_1 t) \\ -\dfrac{351.05}{68.3015^2} (1 - \cos \omega_2 t) \end{Bmatrix}$$

$$= \begin{Bmatrix} +0.5583 (1 - \cos \omega_1 t) \\ -0.0753 (1 - \cos \omega_2 t) \end{Bmatrix}$$

Mass - normalized mass matrix:

$$[X] = \begin{bmatrix} 0.09929 & 0.0514 \\ 0.07268 & -0.14092 \end{bmatrix}$$

Force vector:

$$\{\vec{f}\} = \begin{Bmatrix} 0 \\ k_2 \Delta x \end{Bmatrix} \quad \text{where } \Delta x = 0.05\,m$$

$$= \begin{Bmatrix} 0 \\ 2500 \end{Bmatrix} \quad \text{for } t > 0$$

prior to the step, $\{\vec{x}(0)\} = \{\vec{0}\}$

$$\{\dot{\vec{x}}(0)\} = \{\vec{0}\}$$

$$\Rightarrow \quad \{\vec{q}(0)\} = \{\vec{0}\}$$

$$\{\dot{\vec{q}}(0)\} = \{\vec{0}\}$$

Modal force vector:

$$\{\vec{Q}\} = [X]^T \{\vec{f}\}$$

$$= \begin{bmatrix} 0.09929 & 0.07268 \\ 0.0514 & -0.14092 \end{bmatrix} \begin{Bmatrix} 0 \\ 2500 \end{Bmatrix}$$

$$= \begin{Bmatrix} 181.7 \\ -351.05 \end{Bmatrix}$$

$$\{\vec{x}(t)\} = [X]\,\{\vec{a}(t)\}$$

$$= \begin{bmatrix} 0.09929 & 0.0514 \\ 0.07268 & -0.14042 \end{bmatrix} \begin{Bmatrix} 0.5583\,(1-\cos\omega_1 t) \\ -0.0753\,(1-\cos\omega_2 t) \end{Bmatrix}$$

$$= \begin{Bmatrix} 0.0554\,(1-\cos\omega_1 t) - 0.0039\,(1-\cos\omega_2 t) \\ 0.0406\,(1-\cos\omega_1 t) + 0.0106(1-\cos\omega_2 t) \end{Bmatrix}$$