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ME 3322A: Thermodynamics: Fall 2014
Homework Set # 10
Due Date: November 11, 2014

	Problem # in Textbook		Answer
	7th Ed.	**8th Ed.**	
1	7.21	7.21	22.3 m
2	7.33	7.33	W_{12}=-713.3 kJ; E_d=463.5 kJ
3	7.55	7.55	Exergy destruction rate=96.0 kW; annual cost=$ 68,544
4	7.104	7.104	a) 26.7%; b) 11%
5	7.115	7.115	88.9%

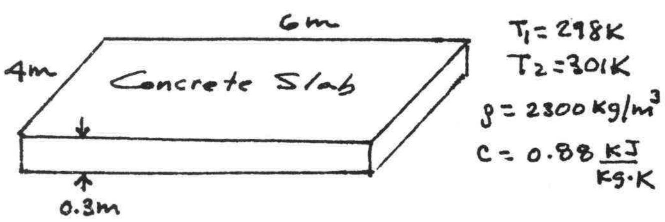
PROBLEM 7.21

7.21 A concrete slab measuring 0.3 m × 4 m × 6 m, initially at 298 K, is exposed to the sun for several hours, after which its temperature is 301 K. The density of the concrete is 2300 kg/m^3 and its specific heat is c = 0.88 kJ/kg · K. (a) Determine the increase in exergy of the slab, in kJ. (b) To what elevation, in m, would a 1000-kg mass have to be raised from zero elevation relative to the reference environment for its exergy to equal the exergy increase of the slab? Let T_0 = 298 K, p_0 = 1 atm, g = 9.81 m/s^2.

KNOWN: Data are provided for a concrete slab warmed by the sun.

FIND: Determine the exergy of the slab. Also, determine the elevation to which a 1000-kg mass would have to be raised for its exergy to equal that of the slab.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The slab is the closed system.
2. For the system, the effects of motion and gravity are not significant. $T_0 = 298K$, $p_0 = 1$ atm.
3. The slab is modeled as incompressible with known values of ρ and specific heat.

ANALYSIS: With assumption #2, Eq. 7.3 reduces to give

$$E_2 - E_1 = m\left[(u_2-u_1) + p_0(v_2 - v_1) - T_0(s_2-s_1)\right]$$

Since the slab is modeled as incompressible, $(u_2-u_1) = c(T_2-T_1)$, $(v_2-v_1) = 0$, and $(s_2-s_1) = c \ln \frac{T_2}{T_1}$.

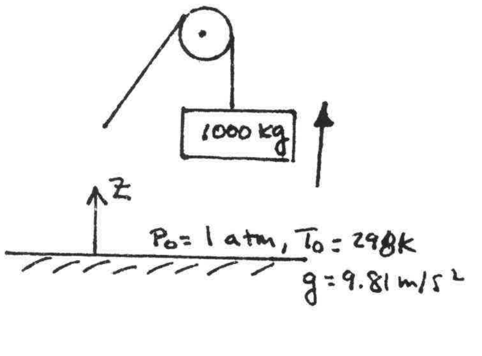
Collecting results,

$$E_2 - E_1 = m\,c\left[(T_2-T_1) - T_0 \ln\frac{T_2}{T_1}\right] \qquad (1)$$

$$m = \rho V = \left(2300\ \frac{kg}{m^3}\right)\left((6m)(4m)(0.3m)\right)$$

$$= 16{,}560\ kg$$

$$\Rightarrow \quad E_2 - E_1 = (16{,}560\ kg)\left(0.88\ \frac{kJ}{kg\cdot K}\right)\left[3\ K - 298K \ln\left(\frac{301}{298}\right)\right] = 218.6\ kJ$$



The elevation to which a 1000-kg mass would have to be raised for its exergy to equal 218.6 kJ is found as follows:

$$\Delta E = mgZ$$

$$\Rightarrow \quad Z = \frac{\Delta E}{mg}$$

$$= \frac{218.6\ kJ}{(1000\ kg)(9.81\ m/s^2)}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ kg\cdot m/s^2}{1\ N}\right|$$

$$= 22.3\ m$$

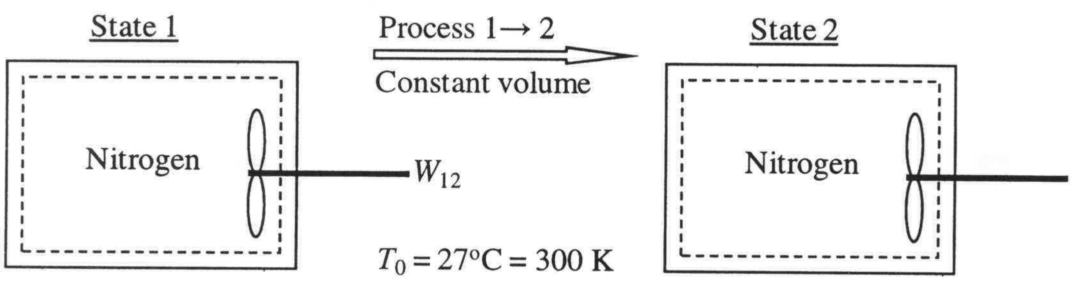
PROBLEM 7.33

7.33 Three kilograms of nitrogen initially at 47°C and 2 bar is contained within a rigid, insulated tank. The nitrogen is stirred by a paddle wheel until its pressure doubles. Employing the ideal gas model with constant specific heat evaluated at 300 K, determine the work and exergy destruction for the nitrogen, each in kJ. Ignore the effects of motion and gravity and let T_0= 300 K, p_0=1 bar.

KNOWN: Nitrogen in a closed, rigid, insulated tank is stirred by a paddle wheel.

FIND: Determine the work and exergy destruction for the nitrogen, each in kJ.

SCHEMATIC AND GIVEN DATA:



$T_0 = 27°C = 300$ K
$p_0 = 1$ bar

$m = 3$ kg
$T_1 = 47°C = 320$ K
$p_1 = 2$ bar

$p_2 = 2p_1 = 4$ bar

ENGINEERING MODEL:
(1) The closed system shown in the accompanying figure is 3 kg of N_2 with $Q_{12} = 0$.
(2) The process occurs at constant volume.
(3) N_2 is modeled as an ideal gas with constant specific heat evaluated at 300 K.
(4) T_0= 300 K, p_0=1 bar.
(5) Ignore the effects of motion and gravity.

ANALYSIS:

For the nitrogen as the system, evaluate the work using the energy balance simplified based on assumptions.

$$W_{12} = m(u_1 - u_2) = mc_v(T_1 - T_2) \qquad (1)$$

Using Table A-20 for N_2 at 300 K, $c_v = 0.743$ kJ/kg·K. Substituting into Eq. (1):

$$W_{12} = 3\,\text{kg}\left(0.743\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(320 - 640)\text{K} = -713.3\,\text{kJ} \quad \longleftarrow$$

Exergy destroyed can be determined from an exergy balance or from $\mathbf{E}_d = T_0\sigma$, where σ is the entropy produced as obtained from the entropy balance. Using the second of these approaches:

$$\Delta S = \int_1^2 \cancelto{0}{\left(\frac{\delta Q}{T}\right)_b} + \sigma$$

Rearranging and simplifying for an insulated system yields:

$$\sigma = \Delta S = m(s_2 - s_1)$$

Using Eq. 6.21 and noting $v_1 = v_2$:

$$\sigma = m(s_2 - s_1) = m\left(c_v \ln\left(\frac{T_2}{T_1}\right) + \frac{\bar{R}}{M}\ln\left(\frac{v_2}{v_1}\right)\right) = mc_v \ln\left(\frac{T_2}{T_1}\right) = 3\,\text{kg}\left(0.743\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{640}{320}\right) = 1.545\frac{\text{kJ}}{\text{K}}$$

Therefore,

$$\mathbf{E}_d = T_0\sigma = (300\,\text{K})1.545\frac{\text{kJ}}{\text{K}} = 463.5\,\text{kJ} \quad \longleftarrow$$

PROBLEM 7.55

7.55 Water vapor enters a valve with a mass flow rate of 2 kg/s at a temperature of 320°C and a pressure of 60 bar and undergoes a throttling process to 40 bar.

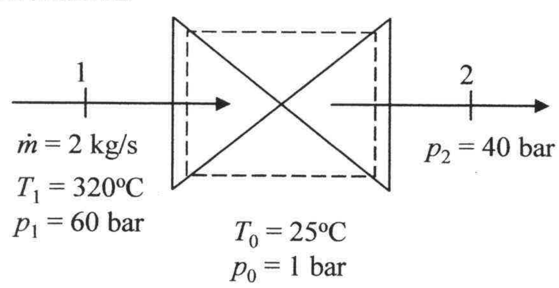
(a) Determine the flow exergy rates at the valve inlet and exit and the rate of exergy destruction, each in kW.
(b) Evaluating exergy at 8.5 cents per kW·h, determine the annual cost, in \$/year, associated with the exergy destruction, assuming 8400 hours of operation annually.

Let $T_0 = 25°C$, $p_0 = 1$ bar.

KNOWN: Water vapor at specified temperature and pressure undergoes a throttling process to a specified pressure.

FIND: Inlet and exit flow exergy rates, the rate of exergy destruction, and the annual cost associated with exergy destruction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram is at steady state.
2. For the throttling process, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and the effects of motion and gravity can be ignored.
3. Exergy is evaluated at 8.5 cents per kW·h.
4. $T_0 = 25°C$, $p_0 = 1$ bar.

ANALYSIS:
(a) At the inlet the water is superheated vapor. From Table A-4, $h_1 = 2952.6$ kJ/kg and $s_1 = 6.1846$ kJ/(kg·K).

Water at the reference state is compressed liquid. From Table A-2 at $T_0 = 25°C$, $h_0 \approx h_{f0} = 104.89$ kJ/kg and $s_0 \approx s_{f0} = 0.3674$ kJ/(kg·K).

Pressure is known at the exit so one additional property is required to fix State 2. The exit enthalpy can be determined from the steady-state, one-inlet, one-exit energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting heat transfer rate, power, and kinetic and potential energy effects, the energy balance simplifies to

$$0 = h_1 - h_2$$

Solving for the exit enthalpy gives

$$h_2 = h_1 = 2952.6 \text{ kJ/kg}$$

State 2 is superheated vapor. From Table A-4 (interpolated), $s_2 = 6.3456$ kJ/(kg·K).

The flow exergy rate at the valve inlet (neglecting kinetic and potential energy effects) is determined from

$$\dot{E}_{fi} = \dot{m}\,[h_1 - h_0 - T_0(s_1 - s_0)]$$

Substituting values gives

$$\dot{E}_{fi} = \left(2\frac{\text{kg}}{\text{s}}\right)\left[2952.6\frac{\text{kJ}}{\text{kg}} - 104.89\frac{\text{kJ}}{\text{kg}} - (298\text{ K})\left(6.1846\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.3674\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{E}_{fi} = \underline{\mathbf{2228.4\ kW}}$$

The flow exergy rate at the valve exit (neglecting kinetic and potential energy effects) is determined from

$$\dot{E}_{fe} = \dot{m}\,[h_2 - h_0 - T_0(s_2 - s_0)]$$

Substituting values gives

$$\dot{E}_{fe} = \left(2\frac{\text{kg}}{\text{s}}\right)\left[2952.6\frac{\text{kJ}}{\text{kg}} - 104.89\frac{\text{kJ}}{\text{kg}} - (298\text{ K})\left(6.3456\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.3674\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{E}_{fe} = \underline{\mathbf{2132.4\ kW}}$$

The rate of exergy destruction is determined from the steady-state control volume exergy rate balance for one inlet and one exit. Since there is no heat transfer and no work, the exergy rate balance Eq. 7.13b reduces to

$$0 = \dot{E}_{fi} - \dot{E}_{fe} - \dot{E}_d$$

Solving for rate of exergy destruction gives

1 $$\dot{E}_d = \dot{E}_{fi} - \dot{E}_{fe} = 2228.4\text{ kW} - 2132.4\text{ kW} = \underline{\mathbf{96.0\ kW}}$$

The annual economic cost of exergy destruction is

$$Cost = (96\text{ kW})\left(8.5\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(8400\frac{\text{h}}{\text{year}}\right)\left(\frac{\$}{100\text{ cents}}\right) = \underline{\mathbf{\$68{,}544/year}}$$

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1 Alternatively, the rate of exergy destruction can be evaluated from $\dot{E}_d = T_0\dot{\sigma}_{cv}$, which reduces in the present case to $\dot{E}_d = T_0\,\dot{m}\,(s_2 - s_1)$. This calculation is left as an exercise.

PROBLEM 7.104

7.104 Figure P7.104 provides two options for generating hot water at steady state. In (a), water heating is achieved by utilizing *industrial waste heat* supplied at a temperature of 500 K. In (b), water heating is achieved by an electrical resistor. For each case, devise and evaluate an exergetic efficiency. Compare the calculated efficiency values and comment. Stray heat transfer and the effects of motion and gravity are negligible. Let T_0 = 20°C, p_0 = 1 bar.

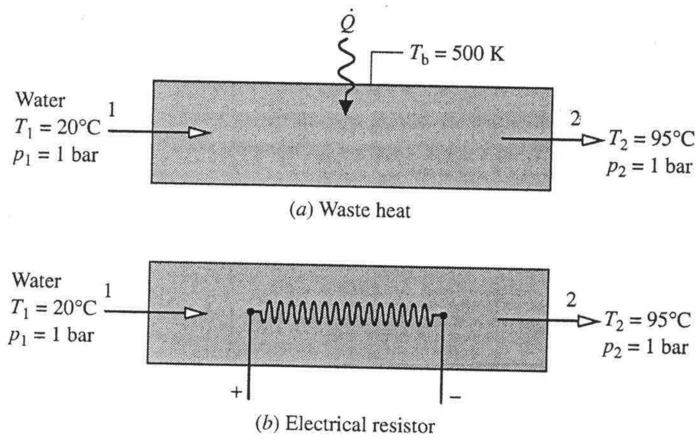


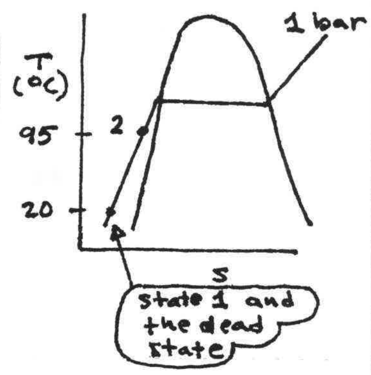
Fig. P7.104

KNOWN: Data are provided for two ways for generating hot water.

FIND: For each option, devise and evaluate an exergetic efficiency. Compare calculated values and comment.

ENGINEERING MODEL:

1. In each case, a control volume at steady state encloses the option: (a) and (b).
2. Stray heat transfer and the effects of motion and gravity are ignored. In (a), $\dot{W}=0$. In (b), $\dot{Q}_{cv}=0$.
3. $T_0 = 20°C$, $p_0 = 1 \text{bar}$



$h_1 \approx h_f(20°C) = 83.96 \text{ kJ/kg}$

$s_1 \approx s_f(20°C) = 0.2966 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$h_2 \approx h_f(95°C) = 397.96 \text{ kJ/kg}$

$s_2 \approx s_f(95°C) = 1.25 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

ANALYSIS:

CASE (a) An exergy rate balance reduces as follows:

$$0 = \left[1 - \frac{T_0}{T_b}\right]\dot{Q} - \cancel{\dot{W}}^{\,0} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

① $$\therefore \quad \left[1 - \frac{T_0}{T_b}\right]\dot{Q} = \dot{m}(e_{f2} - e_{f1}) + \dot{E}_d$$

$$\Rightarrow \quad \epsilon = \frac{\dot{m}(e_{f2} - e_{f1})}{\left[1 - \frac{T_0}{T_b}\right]\dot{Q}} = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{\left[1 - \frac{T_0}{T_b}\right](\dot{Q}/\dot{m})}$$

An energy rate balance reduces to give $(\dot{Q}/\dot{m}) = h_2 - h_1$. Collecting results

$$\epsilon = \frac{[(h_2-h_1) - T_0(s_2-s_1)]}{\left[1 - \frac{T_0}{T_b}\right](h_2 - h_1)} = \frac{\overbrace{(397.96 - 83.96)}^{314}\frac{\text{kJ}}{\text{kg}} - (293\text{K})(1.25 - 0.2966)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}}{\left[1 - \frac{293}{500}\right](397.96 - 83.96)\frac{\text{kJ}}{\text{kg}}}$$

$$= 0.267 \quad (26.7\%)$$

CASE (b) An exergy rate balance reduces as follows:

$$0 = \cancel{\sum\left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j}^{\,0} - \dot{W} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

② $$\therefore \quad (-\dot{W}) = \dot{m}(e_{f2} - e_{f1}) + \dot{E}_d$$

$$\Rightarrow \quad \epsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}/\dot{m})}$$

An energy rate balance reduces to give $(-\dot{W}) = \dot{m}(h_2 - h_1)$ or $(-\dot{W}/\dot{m}) = h_2 - h_1$.

PROBLEM 7.104 (Continued)

Collecting results,

$$\epsilon = \frac{(h_2 - h_1) - T_0 (s_2 - s_1)}{h_2 - h_1}$$

$$= \frac{(397.96 - 83.96)\frac{kJ}{kg} - 293K(1.25 - 0.2966)\frac{kJ}{kg \cdot K}}{(397.96 - 83.96)\frac{kJ}{kg}}$$

$$= 0.11 \quad (11\%)$$

Discussion: Based on the calculated exergetic efficiency values, the waste heat method (option(a)) is better matched to the water heating task than the resistor method (option (b)). Still, other factors should be taken into account: costs, maintenance, and safety, among others.

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1. In this option, $\left[1 - \frac{T_0}{T_b}\right]\dot{Q}$ is the exergy supplied. Using values from the subsequent solution,

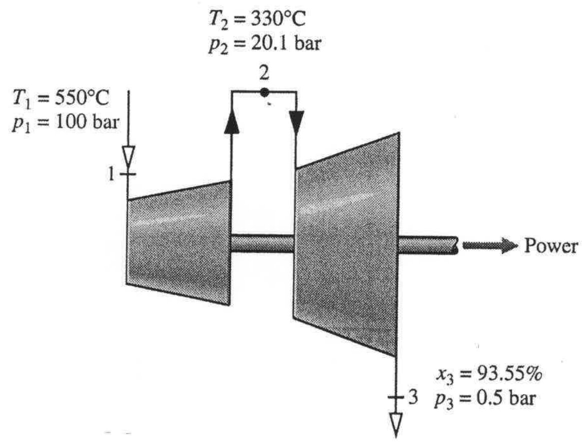
$$\left[1 - \frac{T_0}{T_b}\right]\dot{Q}/\dot{m} = \left(1 - \frac{293}{500}\right)\left(314 \frac{kJ}{kg}\right) = 130 \frac{kJ}{kg}$$

2. In this option $(-\dot{W})$ is the exergy supplied. Using values from the subsequent solution, $(-\dot{W}/\dot{m}) = 214$ kJ/kg.

PROBLEM 7.115

7.115 Figure P7.115 and the accompanying table provide steady-state operating data for a two-stage steam turbine. Stray heat transfer and the effects of motion and gravity are negligible. For each turbine stage, determine the work developed, in kJ per kg of steam flowing, and the exergetic turbine efficiency. For the overall two-stage turbine, devise and evaluate an exergetic efficiency. Let $T_0 = 298$ K, $p_0 = 1$ atm.

State	T(°C)	p(bar)	h(kJ/kg)	s(kJ/kg · K)
1	550	100	3500	6.755
2	330	20.1	3090	6.878
3	(x = 93.55%)	0.5	2497	7.174



KNOWN: Steady-state data are provided for a two-stage steam turbine.

FIND: For each turbine, evaloate $\dot{W}_t/\dot{m}$ and ϵ. Also, devise and evaluate an exergetic efficiency for the overall two-stage turbine.

ENGINEERING MODEL:

1. Control volumes at steady state enclose each turbine and the overall configuration.
2. Stray heat transfer and the effects of motion and gravity are ignored.
3. $T_0 = 298$K, $p_0 = 1$ atm.

ANALYSIS:

TURBINE 1 $\quad \frac{\dot{W}_t}{\dot{m}} = h_1 - h_2 = (3500 - 3090)\frac{kJ}{kg} = 410 \frac{kJ}{kg}$

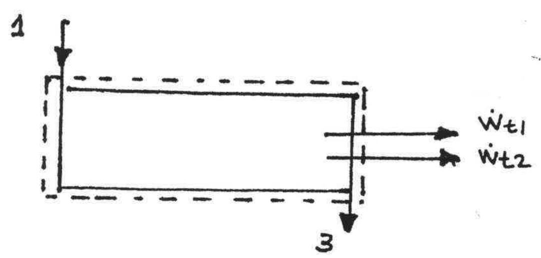
With Eq. Eq. 7.24,

$$\epsilon = \frac{\dot{W}_t/\dot{m}}{e_{f1} - e_{f2}} = \frac{\dot{W}_t/\dot{m}}{(h_1-h_2) - T_0(s_1 - s_2)} = \frac{410}{410 - 298(6.755 - 6.878)} = 0.91.8\ (91.8\%)$$

TURBINE 2 $\quad \frac{\dot{W}_t}{\dot{m}} = h_2 - h_3 = (3090 - 2497)\frac{kJ}{kg} = 593 \frac{kJ}{kg}$

$$\epsilon = \frac{\dot{W}_t/\dot{m}}{(h_2-h_3) - T_0(s_2 - s_3)} = \frac{593}{593 - 298(6.878 - 7.174)} = 0.871\ (87.1\%)$$

OVERALL



Exergy rate balance:

$$0 = \sum_j \left[1 - \frac{T_0}{T_j}\right]\cancel{\dot{Q}_j}^{\,0} - (\dot{W}_{t1} + \dot{W}_{t2}) + \dot{m}[e_{f1} - e_{f3}] - \dot{E}_d$$

$$\therefore \quad \dot{m}[e_{f1} - e_{f3}] = (\dot{W}_{t1} + \dot{W}_{t2}) + \dot{E}_d \quad \Rightarrow \quad \epsilon = \frac{\dot{W}_{t1} + \dot{W}_{t2}}{\dot{m}(e_{f1} - e_{f3})} \qquad (1)$$

Or,

$$\epsilon = \frac{(\dot{W}_{t1}/\dot{m}) + (\dot{W}_{t2}/\dot{m})}{(h_1 - h_3) - T_0(s_1 - s_3)} = \frac{(410 + 593)}{(3500 - 2497) - 298(6.755 - 7.174)} = 0.889\ (88.9\%)$$

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1. Using the exergy rate balance, the exergetic efficiency can be written as

$$\epsilon = \frac{(\dot{W}_{t1} + \dot{W}_{t2})}{(\dot{W}_{t1} + \dot{W}_{t2}) + \dot{E}_d}$$

, where $\dot{E}_d$ is the <u>total</u> rate of exergy destruction.