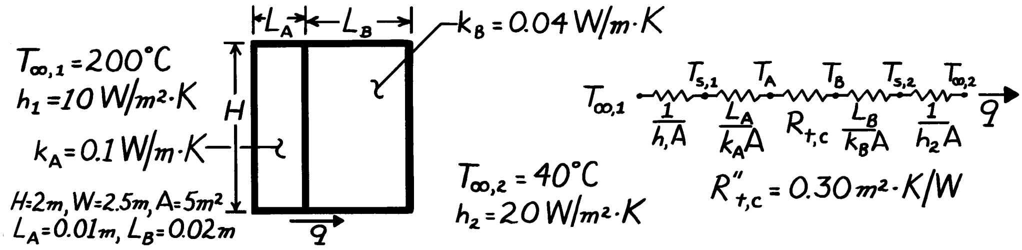
PROBLEM 3.29

KNOWN: Temperatures and convection coefficients associated with fluids at inner and outer surfaces of a composite wall. Contact resistance, dimensions, and thermal conductivities associated with wall materials.

FIND: (a) Rate of heat transfer through the wall, (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible radiation, (4) Constant properties.

ANALYSIS: (a) Calculate the total resistance to find the heat rate,

$$R_{tot} = \frac{1}{h_1A} + \frac{L_A}{k_AA} + R_{t,c} + \frac{L_B}{k_BA} + \frac{1}{h_2A}$$

$$R_{tot} = \left[\frac{1}{10\times 5} + \frac{0.01}{0.1\times 5} + \frac{0.3}{5} + \frac{0.02}{0.04\times 5} + \frac{1}{20\times 5}\right]\frac{K}{W}$$

$$R_{tot} = \left[0.02 + 0.02 + 0.06 + 0.10 + 0.01\right]\frac{K}{W} = 0.21\frac{K}{W}$$

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} = \frac{(200-40)^{\circ}C}{0.21\ K/W} = 762\ W.$$ <

(b) It follows that

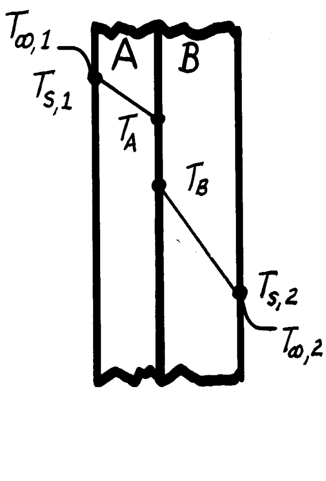
$$T_{s,1} = T_{\infty,1} - \frac{q}{h_1A} = 200^{\circ}C - \frac{762\ W}{50\ W/K} = 184.8^{\circ}C$$

$$T_A = T_{s,1} - \frac{qL_A}{k_AA} = 184.8^{\circ}C - \frac{762W \times 0.01m}{0.1\frac{W}{m\cdot K}\times 5m^2} = 169.6^{\circ}C$$

$$T_B = T_A - qR_{t,c} = 169.6^{\circ}C - 762W \times 0.06\frac{K}{W} = 123.8^{\circ}C$$

$$T_{s,2} = T_B - \frac{qL_B}{k_BA} = 123.8^{\circ}C - \frac{762W \times 0.02m}{0.04\frac{W}{m\cdot K}\times 5m^2} = 47.6^{\circ}C$$

$$T_{\infty,2} = T_{s,2} - \frac{q}{h_2A} = 47.6^{\circ}C - \frac{762W}{100W/K} = 40^{\circ}C$$

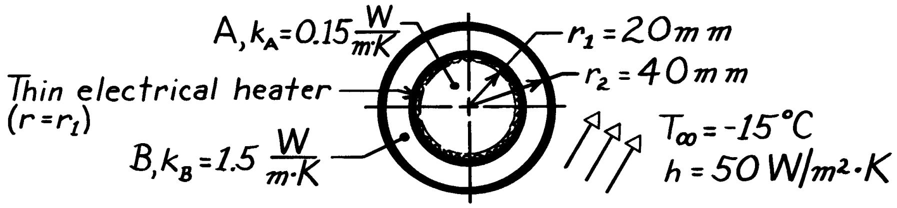


PROBLEM 3.52

KNOWN: Thin electrical heater fitted between two concentric cylinders, the outer surface of which experiences convection.

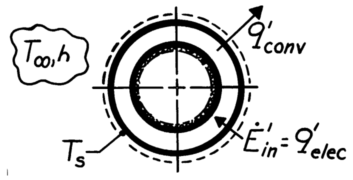
FIND: (a) Electrical power required to maintain outer surface at a specified temperature, (b) Temperature at the center.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Heater element has negligible thickness, (4) Negligible contact resistance between cylinders and heater, (5) Constant properties, (6) No generation.

ANALYSIS: (a) Perform an energy balance on the composite system to determine the power required to maintain $T(r_2) = T_s = 5°C$.



$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$+q'_{elec} - q'_{conv} = 0.$$

Using Newton's law of cooling,

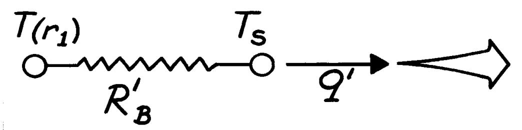
$$q'_{elec} = q'_{conv} = h \cdot 2\pi\ r_2 \left(T_s - T_\infty\right)$$

$$q'_{elec} = 50\frac{W}{m^2 \cdot K} \times 2\pi\left(0.040\text{m}\right)\ \left[5-\left(-15\right)\right]^\circ C = 251\ W/m.$$ <

(b) From a control volume about Cylinder A, we recognize that the cylinder must be isothermal, that is,

$$T(0) = T(r_1).$$

Represent Cylinder B by a thermal circuit:



$$q' = \frac{T\left(r_1\right) - T_s}{R'_B}$$

For the cylinder, from Eq. 3.28,

$$R'_B = \ln r_2 / r_1 / 2\pi\ k_B$$

giving

$$T\left(r_1\right) = T_s + q'R'_B = 5^\circ C + 251\frac{W}{m}\ \frac{\ln 40/20}{2\pi \times 1.5\ W/m \cdot K} = 23.5^\circ C$$

Hence, $T(0) = T(r_1) = 23.5°C$. <

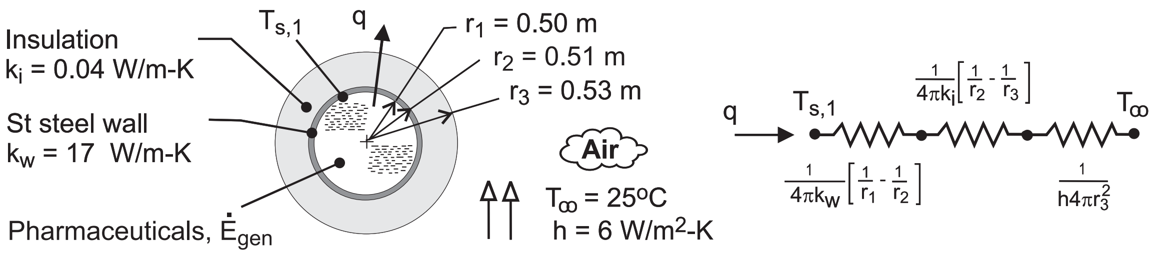
Note that k_A has no influence on the temperature T(0).

PROBLEM 3.70

KNOWN: Inner diameter, wall thickness and thermal conductivity of spherical vessel containing heat generating medium. Inner surface temperature without insulation. Thickness and thermal conductivity of insulation. Ambient air temperature and convection coefficient.

FIND: (a) Thermal energy generated within vessel, (b) Inner surface temperature of vessel with insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional, radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Neglect radiation due to relatively low emissivity of stainless steel in part (a). In part (b), insulation resistance dominates.

ANALYSIS: (a) From an energy balance performed at an instant for a control surface about the pharmaceuticals, $\dot{E}_g = q$, in which case, without the insulation

$$\dot{E}_g = q = \frac{T_{s,1} - T_\infty}{\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi r_2^2 h}} = \frac{(50-25)°C}{\frac{1}{4\pi(17\,W/m\cdot K)}\left(\frac{1}{0.50m} - \frac{1}{0.51m}\right) + \frac{1}{4\pi(0.51m)^2\,6\,W/m^2\cdot K}}$$

$$\dot{E}_g = q = \frac{25°C}{\left(1.84\times10^{-4} + 5.10\times10^{-2}\right)K/W} = 489\,W$$ <

(b) With the insulation,

$$T_{s,1} = T_\infty + q\left[\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{1}{4\pi r_3^2 h}\right]$$

$$T_{s,1} = 25°C + 489\,W\left[1.84\times10^{-4} + \frac{1}{4\pi(0.04)}\left(\frac{1}{0.51} - \frac{1}{0.53}\right) + \frac{1}{4\pi(0.53)^2\,6}\right]\frac{K}{W}$$

$$T_{s,1} = 25°C + 489\,W\left[1.84\times10^{-4} + 0.147 + 0.047\right]\frac{K}{W} = 120°C$$ <

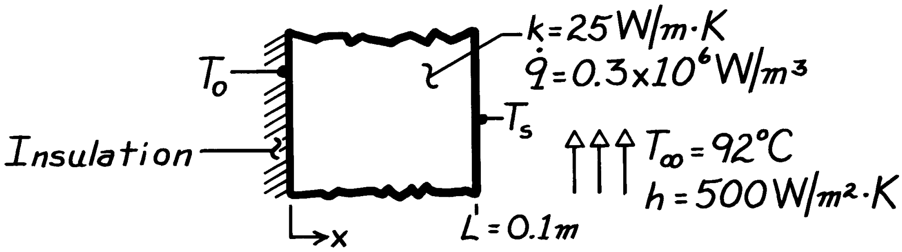
COMMENTS: The thermal resistance associated with the vessel wall is negligible, and without the insulation the dominant resistance is due to convection. The thermal resistance of the insulation is approximately three times that due to convection. Radiation may not be negligible, and would have the effect of increasing the heat loss rate (for fixed inner surface temperature) or decreasing the inner surface temperature (for fixed heat loss rate).

PROBLEM 3.81

KNOWN: Plane wall with internal heat generation which is insulated at the inner surface and subjected to a convection process at the outer surface.

FIND: Maximum temperature in the wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform volumetric heat generation, (3) Inner surface is adiabatic.

ANALYSIS: The temperature at the inner surface is given by Eq. 3.48 and is the maximum temperature within the wall,

$$T_o = \dot{q}L^2/2k + T_s.$$

The outer surface temperature follows from Eq. 3.51,

$$T_s = T_\infty + \dot{q}L/h$$

$$T_s = 92^\circ C + 0.3\times10^6 \frac{W}{m^3} \times 0.1m/500W/m^2\cdot K = 92^\circ C + 60^\circ C = 152^\circ C.$$

It follows that

$$T_o = 0.3\times10^6 W/m^3 \times (0.1m)^2 / 2\times25W/m\cdot K + 152^\circ C$$

$$T_o = 60^\circ C + 152^\circ C = 212^\circ C. \qquad <$$

COMMENTS: The heat flux leaving the wall can be determined from knowledge of h, T_s and T_∞ using Newton's law of cooling.

$$q''_{conv} = h(T_s - T_\infty) = 500W/m^2\cdot K(152-92)^\circ C = 30kW/m^2.$$

This same result can be determined from an energy balance on the entire wall, which has the form

$$\dot{E}_g - \dot{E}_{out} = 0$$

where

$$\dot{E}_g = \dot{q}AL \qquad \text{and} \qquad \dot{E}_{out} = q''_{conv}\cdot A.$$

Hence,

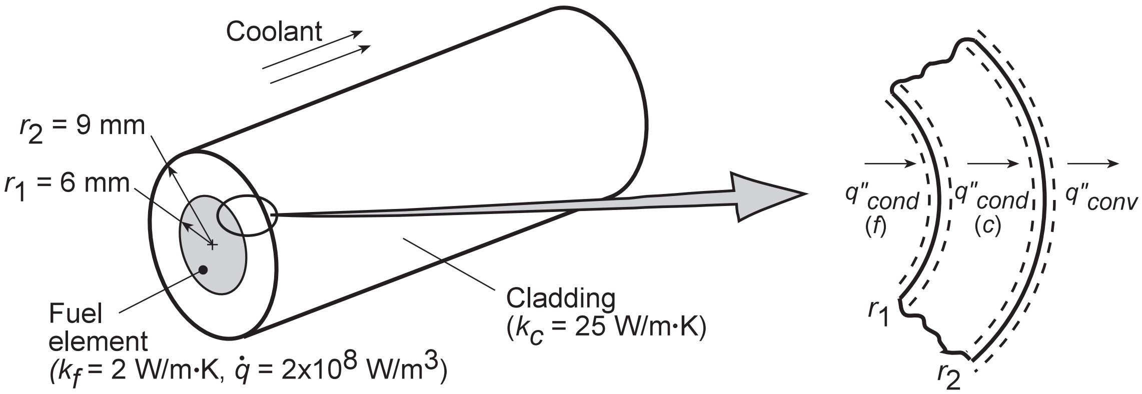
$$q''_{conv} = \dot{q}L = 0.3\times10^6 W/m^3 \times 0.1m = 30kW/m^2.$$

PROBLEM 3.98

KNOWN: Radii and thermal conductivities of reactor fuel element and cladding. Fuel heat generation rate. Temperature and convection coefficient of coolant.

FIND: (a) Expressions for temperature distributions in fuel and cladding, (b) Maximum fuel element temperature for prescribed conditions, (c) Effect of h on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible contact resistance, (4) Constant properties.

ANALYSIS: (a) From Eqs. 3.54 and 3.28, the heat equations for the fuel (f) and cladding (c) are

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_f}{dr}\right) = -\frac{\dot{q}}{k_f} \qquad (0 \le r \le r_1) \qquad\qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right) = 0 \qquad (r_1 \le r \le r_2)$$

Hence, integrating both equations twice,

$$\frac{dT_f}{dr} = -\frac{\dot{q}r}{2k_f} + \frac{C_1}{k_f r} \qquad T_f = -\frac{\dot{q}r^2}{4k_f} + \frac{C_1}{k_f}\ln r + C_2 \tag{1,2}$$

$$\frac{dT_c}{dr} = \frac{C_3}{k_c r} \qquad T_c = \frac{C_3}{k_c}\ln r + C_4 \tag{3,4}$$

The corresponding boundary conditions are:

$$dT_f/dr)_{r=0} = 0 \qquad\qquad T_f(r_1) = T_c(r_1) \tag{5,6}$$

$$-k_f\left.\frac{dT_f}{dr}\right)_{r=r_1} = -k_c\left.\frac{dT_c}{dr}\right)_{r=r_1} \qquad -k_c\left.\frac{dT_c}{dr}\right)_{r=r_2} = h\left[T_c(r_2) - T_\infty\right] \tag{7,8}$$

Note that Eqs. (7) and (8) are obtained from surface energy balances at r_1 and r_2, respectively. Applying Eq. (5) to Eq. (1), it follows that $C_1 = 0$. Hence,

$$T_f = -\frac{\dot{q}r^2}{4k_f} + C_2 \tag{9}$$

From Eq. (6), it follows that

$$-\frac{\dot{q}r_1^2}{4k_f} + C_2 = \frac{C_3 \ln r_1}{k_c} + C_4 \tag{10}$$

Continued...

PROBLEM 3.98 (Cont.)

Also, from Eq. (7),

$$\frac{\dot{q}r_1}{2} = -\frac{C_3}{r_1} \qquad \text{or} \qquad C_3 = -\frac{\dot{q}r_1^2}{2} \tag{11}$$

Finally, from Eq. (8), $-\frac{C_3}{r_2} = h\left[\frac{C_3}{k_c}\ln r_2 + C_4 - T_\infty\right]$ or, substituting for C_3 and solving for C_4

$$C_4 = \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty \tag{12}$$

Substituting Eqs. (11) and (12) into (10), it follows that

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} - \frac{\dot{q}r_1^2 \ln r_1}{2k_c} + \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty$$

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \tag{13}$$

Substituting Eq. (13) into (9),

$$T_f = \frac{\dot{q}}{4k_f}\left(r_1^2 - r^2\right) + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \tag{14}$$ <

Substituting Eqs. (11) and (12) into (4),

$$T_c = \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty\,. \tag{15}$$ <

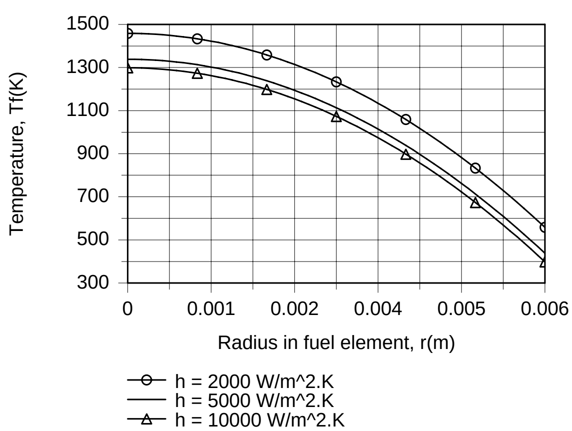
(b) Applying Eq. (14) at r = 0, the maximum fuel temperature for h = 2000 $W/m^2\cdot K$ is

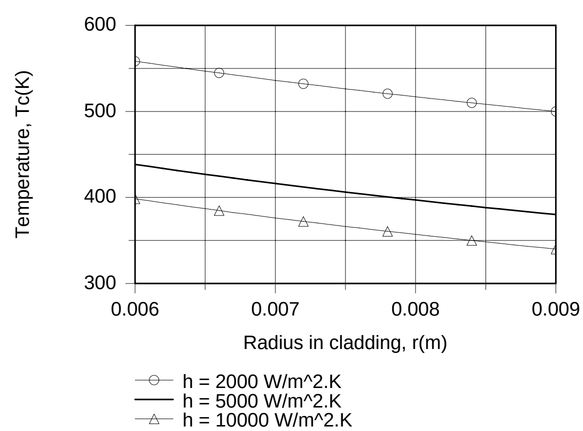
$$T_f(0) = \frac{2\times10^8\ W/m^3 \times (0.006\,m)^2}{4\times 2\ W/m\cdot K} + \frac{2\times10^8\ W/m^3 \times (0.006\,m)^2}{2\times 25\ W/m\cdot K}\ln\frac{0.009\,m}{0.006\,m}$$

$$+\frac{2\times10^8\ W/m^3\,(0.006\,m)^2}{2\times(0.009\,m)\,2000\ W/m^2\cdot K} + 300\,K$$

$$T_f(0) = (900 + 58.4 + 200 + 300)\,K = 1458\,K\,.$$ <

(c) Temperature distributions for the prescribed values of h are as follows:





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PROBLEM 3.98 (Cont.)

Clearly, the ability to control the maximum fuel temperature by increasing h is limited, and even for $h \rightarrow \infty$, $T_f(0)$ exceeds 1000 K. The overall temperature drop, $T_f(0) - T_\infty$, is influenced principally by the low thermal conductivity of the fuel material.

COMMENTS: For the prescribed conditions, Eq. (14) yields, $T_f(0) - T_f(r_1) = \dot{q}r_1^2 / 4k_f = (2\times10^8 \text{ W/m}^3)(0.006 \text{ m})^3/8 \text{ W/m}\cdot\text{K} = 900$ K, in which case, with no cladding and $h \rightarrow \infty$, $T_f(0) = 1200$ K. To reduce $T_f(0)$ below 1000 K for the prescribed material, it is necessary to reduce $\dot{q}$.