5.2

5.2 An incompressible fluid flows horizontally in the x-y plane with a velocity given by

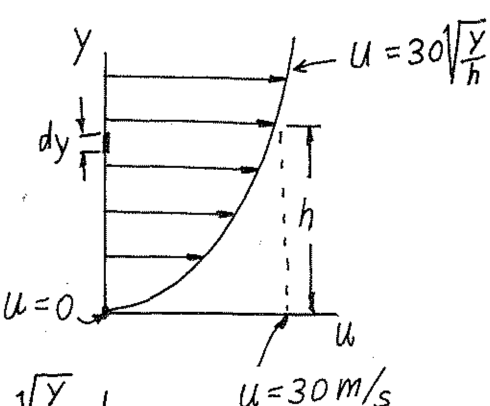
$$u = 30\,(y/h)^{1/2} \text{ m/s}, \; v = 0$$

where y and h are in meters and h is a constant. Determine the average velocity for the portion of the flow between $y = 0$ and $y = h$.

From Eq. 5.7

$$\bar{V} = \frac{\int_A \rho \vec{V}\cdot\hat{n}\,dA}{\rho A} \quad \text{or with } \rho = \text{constant},$$

$$\bar{V} = \frac{\int_A \vec{V}\cdot\hat{n}\,dA}{A}$$



Consider a unit depth normal to the x-y plane so that

$A = 1 \times h = h$ and $dA = 1 \times dy = dy$

Thus,

$$\bar{V} = \frac{\int u\,dA}{A} = \frac{\int_0^h u\,dy}{h} = \frac{\int_0^h 30\sqrt{\frac{y}{h}}\,dy}{h}$$

$$= \frac{30}{h^{3/2}} \int_0^h y^{1/2}\,dy = \frac{30}{h^{3/2}}\, y^{3/2}\left(\frac{2}{3}\right)\Bigg|_{y=0}^{y=h} = \frac{2}{3}(30) \text{ m/s}$$

or

$$\bar{V} = \underline{\underline{20 \text{ m/s}}}$$

5.3

5.3 Water flows steadily through the horizontal piping system shown in Fig. P5.3. The velocity is uniform at section (1), the mass flowrate is 10 slugs/s at section (2), and the velocity is nonuniform at section (3). **(a)** Determine the value of the quantity $\frac{D}{Dt}\int_{sys} \rho \, d\forall$, where the system is the water contained in the pipe bounded by sections (1), (2), and (3). **(b)** Determine the mean velocity at section (2). **(c)** Determine, if possible, the value of the integral $\int_{(3)} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$ over section (3). If it is not possible, explain what additional information is needed to do so.

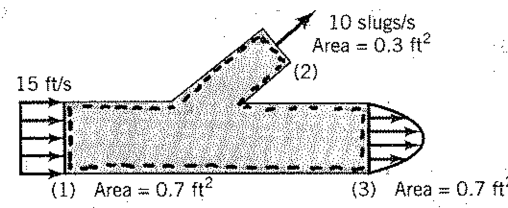


FIGURE P5.3

Use the control volume shown with the dashed lines in the figure above.

(a) From the conservation of mass principle we get

$$\frac{D}{Dt}\int_{sys} \rho \, d\forall = 0$$ since $\int_{sys} \rho \, d\forall$ is the unchanging mass of the system.

(b) $\dot{m}_2 = \rho A_2 \overline{V}_2$ thus

$$\overline{V}_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{10 \frac{slugs}{s}}{\left(1.94 \frac{slugs}{ft^3}\right)(0.3\ ft^2)} = \underline{\underline{17.2}}\ \frac{ft}{s}$$

(c) $\dot{m}_3 = \int_{A_3} \rho \vec{V} \cdot \hat{n} \, dA$ and from the conservation of mass principle we get

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

Thus

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = \rho A_1 \overline{V}_1 - \dot{m}_2 = \left(1.94 \frac{slugs}{ft^3}\right)(0.7\,ft^2)\left(15 \frac{ft}{s}\right) - 10 \frac{slugs}{s}$$

$$\dot{m}_3 = \underline{\underline{10.4}}\ \frac{slugs}{s} = \int_{A_3} \rho \vec{V} \cdot \hat{n} \, dA$$

5.25

5.25 Flow of a viscous fluid over a flat plate surface results in the development of a region of reduced velocity adjacent to the wetted surface as depicted in Fig. P5.25. This region of reduced flow is called a boundary layer. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value U. All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate surface is also U. If the x direction velocity profile at section (2) is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

develop an expression for the volume flowrate through the edge of the boundary layer from the leading edge to a location downstream at x where the boundary layer thickness is δ.

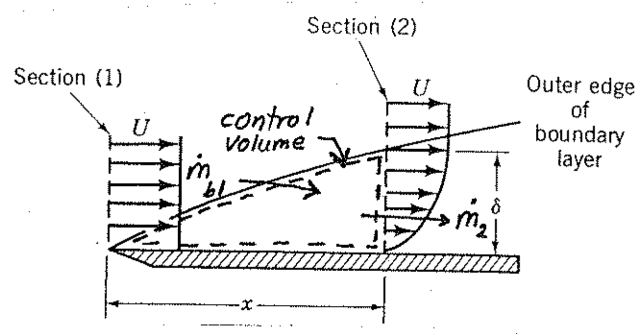


FIGURE P5.25

From the conservation of mass principle applied to the flow through the control volume shown in the figure we have

$$\dot{m}_{bl} = \dot{m}_2 = \int_{A_2} \rho \vec{V} \cdot \hat{n} \, dA$$

For incompressible flow

$$\rho Q_{bl} = \rho U \ell \delta \int_0^1 \left(\frac{y}{\delta}\right)^{\frac{1}{7}} d\left(\frac{y}{\delta}\right)$$

where

ℓ = width of the plate

and thus

$$\underline{\underline{Q_{bl} = \frac{7}{8} U \ell \delta}}$$

5.30

5.30 A hypodermic syringe (see Fig. P5.30) is used to apply a vaccine. If the plunger is moved forward at the steady rate of 20 mm/s and if vaccine leaks pass the plunger at 0.1 of the volume flowrate out the needle opening, calculate the average velocity of the needle exit flow. The inside diameters of the syringe and the needle are 20 mm and 0.7 mm.

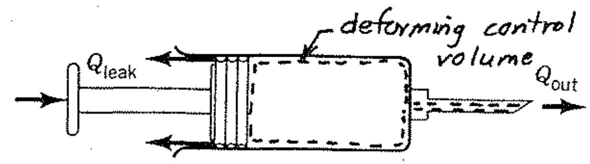


FIGURE P5.30

Using a deforming control volume and the conservation of mass principle (Eq. 5.17) as outlined in Example 5.8, we obtain (see Eq. 8 of Example 5.8)

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{leak} = 0 \qquad (1)$$

Since ρ = constant, $Q_{leak} = 0.1\, Q_2$ and $Q_2 = A_2 V_2$ we obtain from Eq. 1

$$1.1\, A_2 V_2 = A_1 V_p$$

or

$$V_2 = \left(\frac{A_1}{A_2}\right)\frac{V_p}{1.1} = \left(\frac{d_1^2}{d_2^2}\right)\frac{V_p}{1.1} = \frac{(20\,mm)^2(20\,mm/s)}{(0.7\,mm)^2(1.1)\left(1000\,\frac{mm}{m}\right)}$$

and

$$V_2 = \underline{\underline{14.8}}\ \frac{m}{s}$$