Master 2 Financial Economics – Université Paris 1 Panthéon-Sorbonne

Empirical Application 4

Asset pricing

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September 2021

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Topic 1 – Interpolation of the ECB yield curve

The Yield Curve is a graphical representation of observed yields on debts instruments, such as government bonds. As those yields may vary depending on the maturity, the main goal of the Yield Curve is to capture those variations across the duration of bonds. Therefore, the x-axis is a time scale whereas the y-axis depicts the yield to maturity.

For our analysis, we chose to study government nominal bond, all triple A issuer companies. The maturities are 3 and 6 months and from 1 year to 30 years with business day frequency, provided by European Central Bank. The range date is from 2006-12-28 to 2009-07-23. We imported our data thanks to the "YieldCurve" package in Rstudio.

Chart 1

Spot ECB yield curve (observed at 28/12/2006)

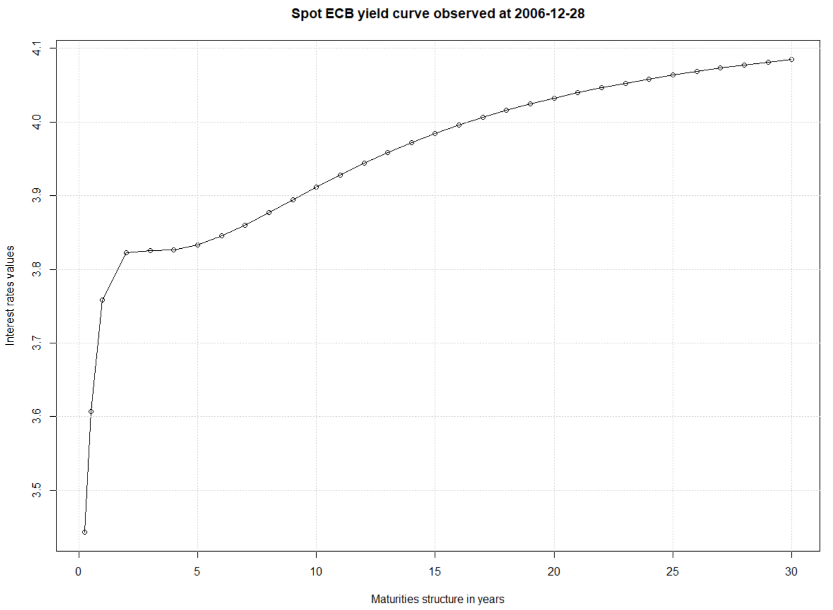
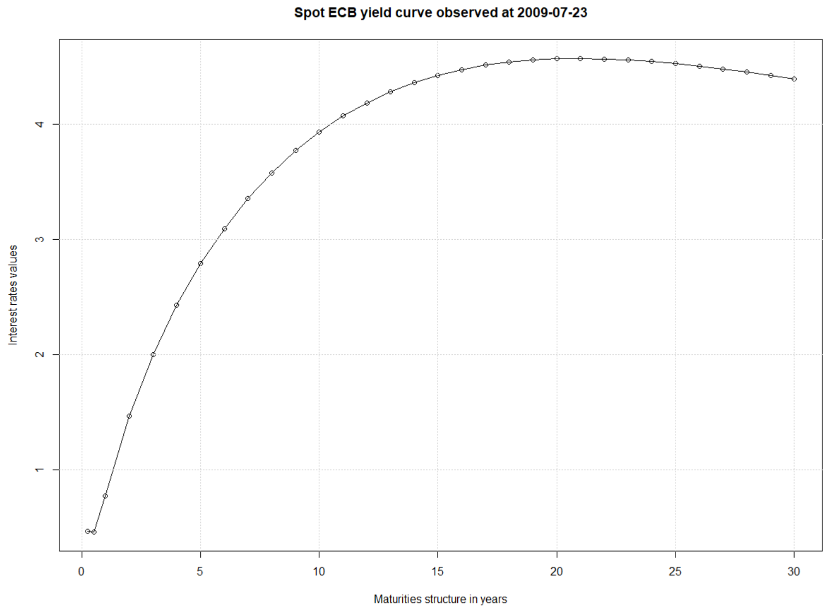


Chart 2

Spot ECB yield curve (observed at 23/07/2009)



Those two graphs above represent different observed yield curve at different periods. We decided to study the variation in our sample by focusing on the first period observation versus the last period of observation. We notice different shapes of the curve depending on the period of observation.

The overall shape of a Yield Curve is a great indicator of market confidence. As interest rates vary depending on the maturity, one can easily observe preference for the present, with the short-term interest rates, versus preference for the future. Therefore, one can also anticipate future short-term interest rates thanks to the slope; an upward slope indicates an anticipation for higher interest rates whereas a downward slope is synonymous with lower rates.

In the first graph, the leap observed in the early stage of maturities can be interpreted as an expectation for higher short-term rates. It seems that the trend slows down for medium-term rates and accelerates once again in the long run. In our second graph, we observe a parabola with a turning point for long-term rates, meaning that investors anticipated a rise in interest rates for both short and medium-term maturities while they were slightly more optimistic about the long run as they accepted lower interest rates.

The Nelson Siegel model characterizes the yield curve with four distinct parameters: the level parameter β_{0t} which captures the long-term behavior of the yield curve, the slope parameter β_{1t} also known as the short-term factor, the curvature parameter β_{2t} that seize medium-term effects and an additional fixed parameter λ.

Equation 1

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\frac{1-\exp(-\lambda\tau)}{\lambda\tau} + \beta_{2t}\left(\frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau)\right)$$

From our data, we estimated those parameters for our two periods of interest:

Table 1

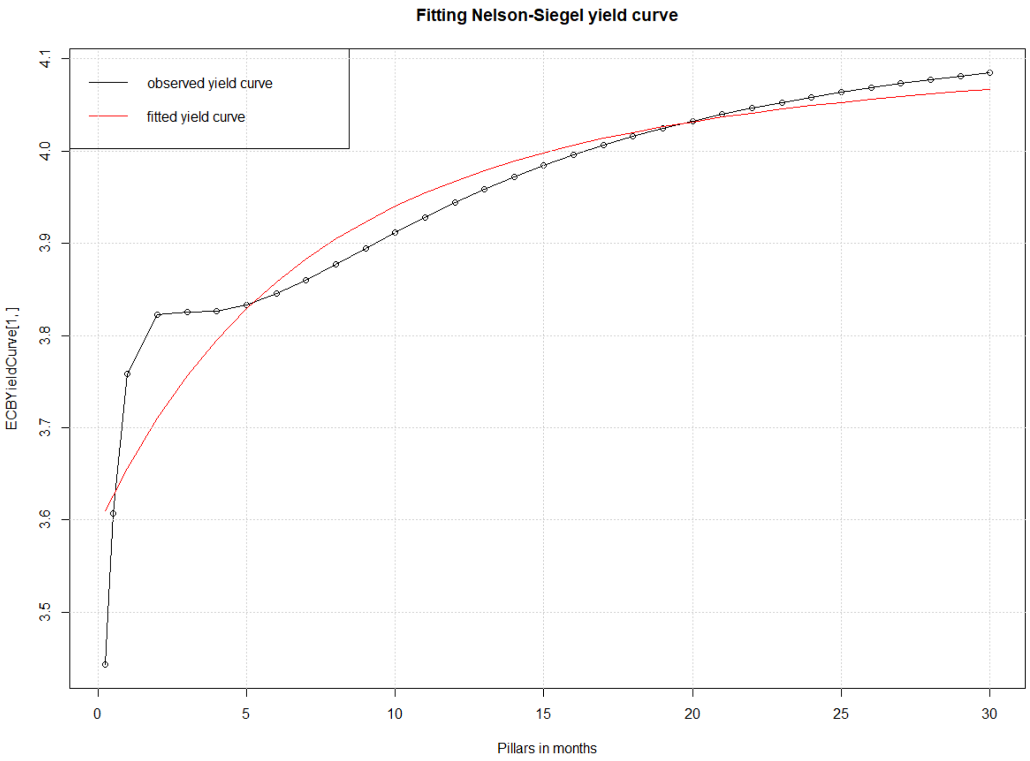
Nelson-Siegel parameters of the ECB yield curve

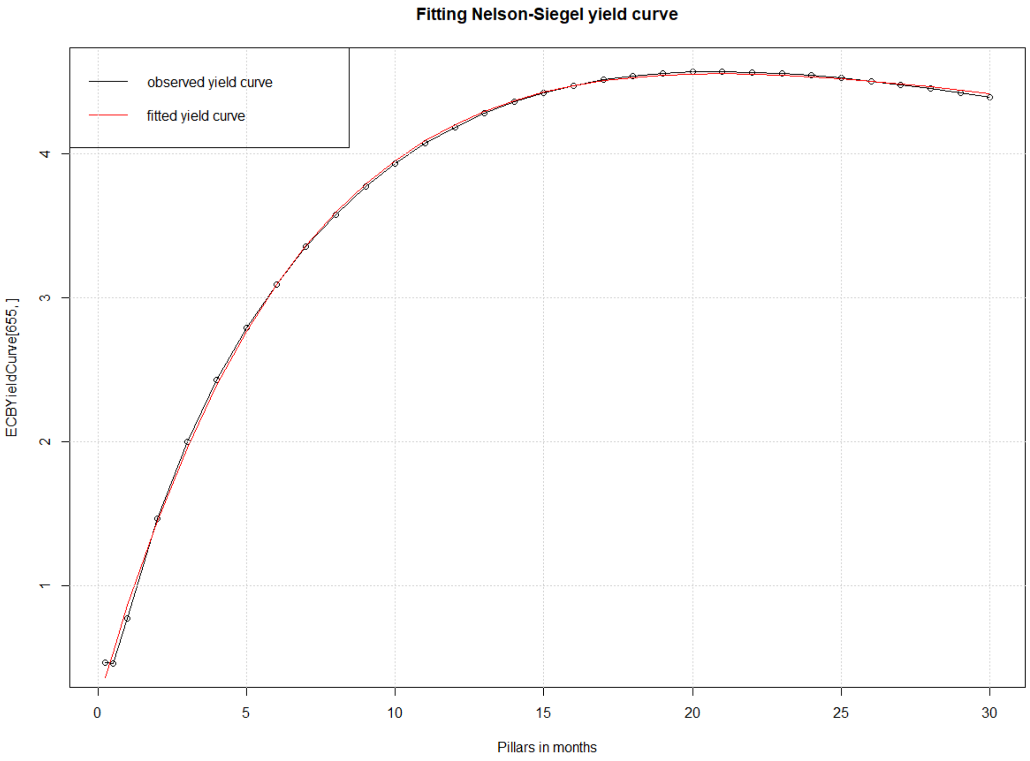
	beta_0	beta_1	beta_2	lambda
2006-12-28	4.139318	-0.5465237	1.104602e-02	0.24733284
2007-01-01	4.139314	-0.5419122	-5.825462e-03	0.21738091
2007-01-02	4.116384	-0.5267525	3.921797e-03	0.23140025
2007-01-03	4.133333	-0.5380867	1.977204e-03	0.23140025
2007-01-04	4.169871	-0.5609113	-3.677112e-04	0.24733284
2007-01-07	4.184103	-0.5714263	1.211699e-02	0.21738091
2009-07-16	2.210098479	-2.06827399	10.834020	0.11385807
2009-07-19	2.586431875	-2.41895815	10.131993	0.11761146
2009-07-20	2.665455770	-2.50572503	9.950763	0.11761146
2009-07-21	2.754276554	-2.59258668	9.688432	0.11761146
2009-07-22	2.792420594	-2.62968783	9.607885	0.11761146
2009-07-23	2.906929032	-2.72891769	9.308067	0.12159026

Thanks to this estimation of the parameters, we can confirm and explain the difference in the observed behaviors of the Yield Curve over our two periods. There has been a significant drop for the β_{0t} parameter which translates in a decreasing long-term interest rate for the last period. Due to the difference in scale between our two graphs, one might have thought that the yield curve's sensitivity for short-term interest rate was higher for our first period. However, it is quite the opposite, as shown by the difference for the β_{1t} parameters: short-term interest rates climb faster in our last periods of observation. Finally, the biggest gap observed between the estimated parameters is for the β_{2t} which is represented by the yield range's contrast. For our first period of observation, regardless of the maturity, the yield is around 3.4% to 4.1% which represents only a variation of 0.7 percentage point whereas this variation is around 4 percentage point four the last period of observation.

Chart 3

ECB yield curve – observed (in black) VS fitted (in red)





Finally, we decided to fit our observed yield curve with the parameters obtained thanks to the Nelson Siegel model. As the second curve is closer to a perfect parabola, the model fits perfectly with the observed data whereas it is less accurate for the non-parabolic series, but it stills capture the general observed trend.

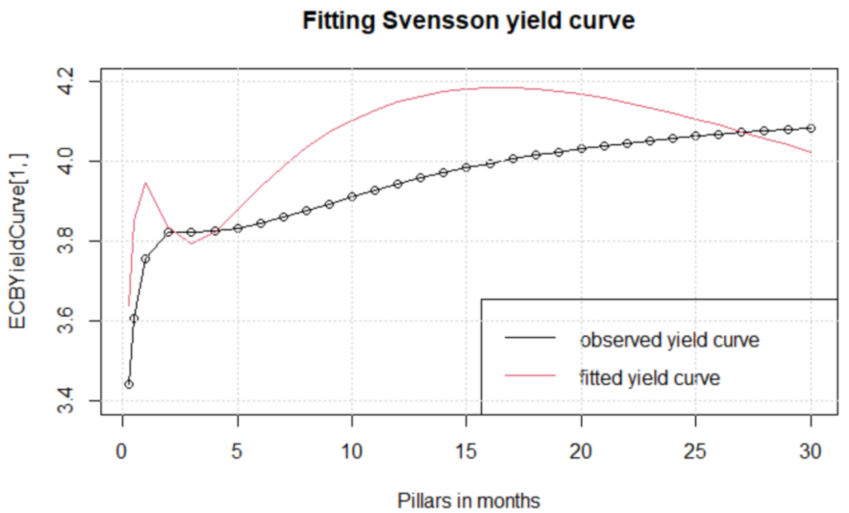
The “YieldCurve” package also offers the possibility to estimate parameters for the Svensson model that also considers the forward rate. In the words of its creator: “The forward rates are interpreted as indicating market expectations of the time-path of future interest rates, future inflation rates, and future currency depreciation rates. They separate market expectations for the short, medium and long term more easily than the standard yield curve”. The parameters β_{3t} allows for more than one local extremum along the maturity profile which can improve the fit of yield curves.

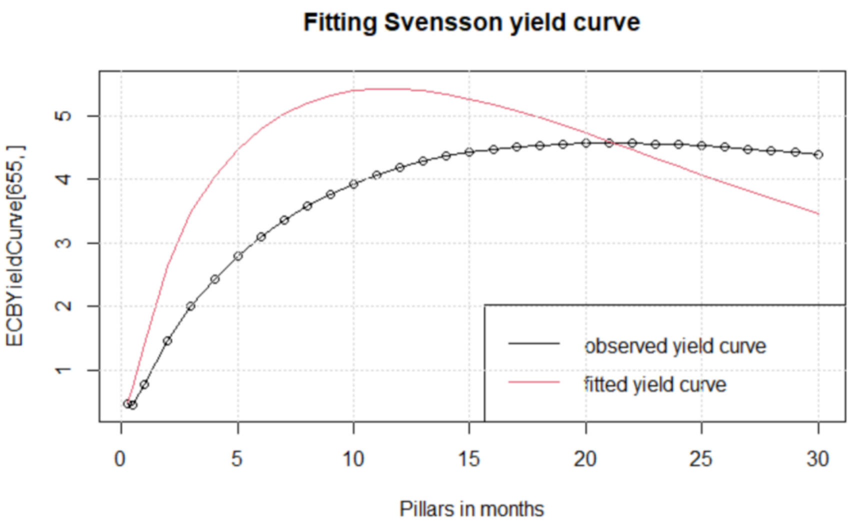
Equation 2

$$y_t(\tau) = \beta_0 + \beta_1 \frac{1-\exp(-\frac{\tau}{\lambda_1})}{\frac{\tau}{\lambda_1}} + \beta_2 \left[\frac{1-\exp(-\frac{\tau}{\lambda_1})}{\frac{\tau}{\lambda_1}} - \exp(-\frac{\tau}{\lambda_1})\right] + \beta_3 \left[\frac{1-\exp(-\frac{\tau}{\lambda_2})}{\frac{\tau}{\lambda_2}} - \exp(-\frac{\tau}{\lambda_2})\right]$$

Chart 4

ECB yield curve – observed (in black) VS fitted (in red)



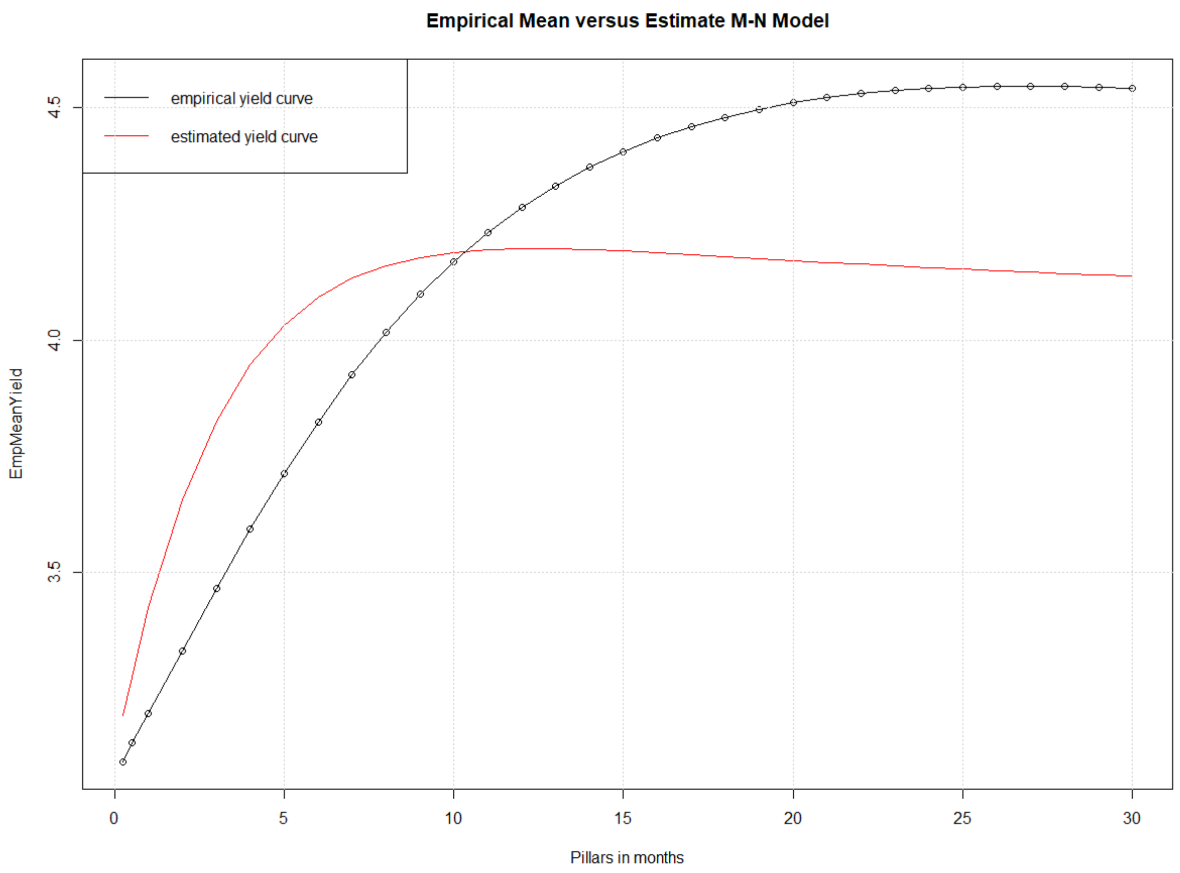


Thanks to its fourth parameter, the Svensson model captures the observed trend in a more efficient way than the Nelson Siegel model for our first period of observation. However, as our second Yield Curve is almost a perfect parabola, it fails to fit it.

To conclude our study, we produced a graphical representation of the empirical yield mean for each maturity and the mean for each Nelson Siegel parameters estimated over the period of observation. Despite a slight gap between the two curves, we could state that the Nelson Siegel model still provides quite a reliable representation of an observed yield curve over those 655 periods.

Chart 5

ECB yield curve – empirical mean (in black) VS mean for estimated Nelson-Siegel parameters (in red)



Topic 2 – Factorial analysis of the ECB yield curve

In this section, we aim at summarizing the main possible future changes in the shape of the yield curve by using a small number of factors.

Our starting point for this analysis is the Nelson-Siegel yield curve decomposition model. This model expresses the yield of a bond with maturity τ as a function of three unobserved factors $\boldsymbol{\beta_{0t}}$, $\boldsymbol{\beta_{1t}}$, $\boldsymbol{\beta_{2t}}$:

Equation 1

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\frac{1-\exp(-\lambda\tau)}{\lambda\tau} + \beta_{2t}\left(\frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau)\right)$$

Where:

$\boldsymbol{Y_t(\tau)} =$ the Yield To Maturity (YTM) for a bond with maturity τ at date t, namely the total return anticipated on a bond if the bond is held until it matures.

$\boldsymbol{\beta_{0t}} =$ the level of the yield curve. This component accounts for the impact of the overall level of the yield curve on the interest rates.

$\boldsymbol{\beta_{1t}} =$ the spread between short term and long-term rate. A positive change in this component will imply a rise in short term interest rates and a decrease in long-term rates – thus resulting in a decrease in the maturity spread and therefore in the yield curve slope. Conversely, a negative change in this component imply an increase in the maturity spread and therefore in a stronger yield curve slope.

$\boldsymbol{\beta_{2t}} =$ the term structure curvature. This component will predominantly affect medium-term interest rates as a consequence.

$\boldsymbol{\lambda_\tau} =$ the rate of decay of the exponential function. As highlighted by Diebold and Li (2005) "small values of λt produce slow decay and can better fit the curve at long maturities, while large values of λt produce fast decay and can better fit the curve at short maturities"[1].

We can now confront this model to the ECB yield curve data, to examine whether those three factors – (i) change in the yield curve level, (ii) maturity spread (or the curve's 'twist') and (iii) its curvature – are useful for explaining yield curve changes. As a reminder, we use the spot rates of Euro Area AAA-rated government bonds across 30 maturities.

By differencing the ECB yield curve data, we now obtain the changes in the yield curve, to which our factorial analysis through Principal Component Analysis (PCA) will be applied.

[1] Francis X. Diebold, Canlin Li, Forecasting the term structure of government bond yields, May 2005, Journal of Econometrics

The below table confirms that the yield curve change can be explained by three principal components, as those explain 94.5% of cumulative variance in yield curve change data, with component 1 in particular explaining nearly 75% of the data's variations.

Table 1

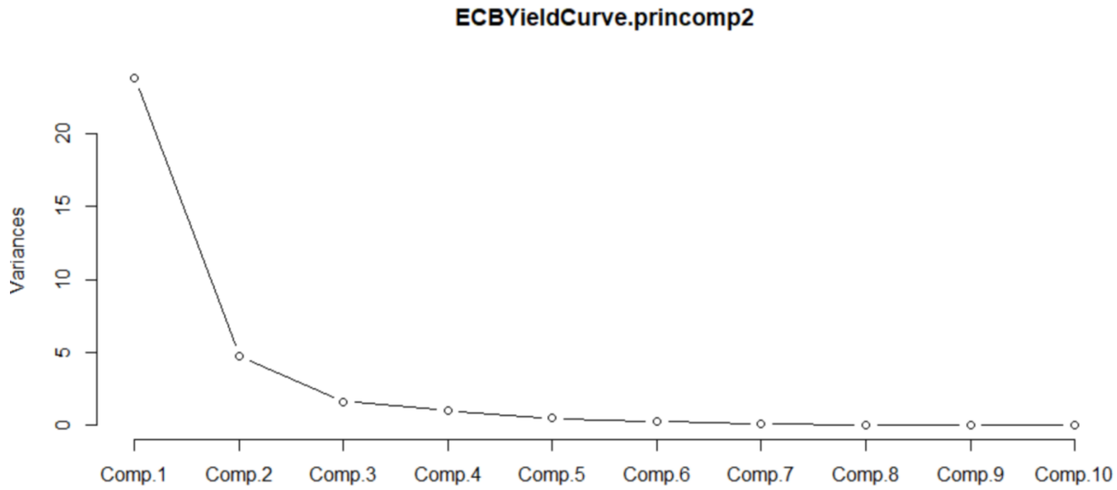
ECB yield curve principal component analysis summary output

```
> summary(ECBYieldCurve.princomp)
Importance of components:
                          Comp.1    Comp.2
Standard deviation     0.2312708 0.1073934
Proportion of Variance 0.7384159 0.1592263
Cumulative Proportion  0.7384159 0.8976422
                           Comp.3
Standard deviation     0.05851445
Proportion of Variance 0.04727000
Cumulative Proportion  0.94491220
                           Comp.4
Standard deviation     0.04994953
Proportion of Variance 0.03444468
Cumulative Proportion  0.97935688
                           Comp.5
Standard deviation     0.02927783
Proportion of Variance 0.01183415
Cumulative Proportion  0.99119103
                            Comp.6
Standard deviation     0.022143889
Proportion of Variance 0.006769664
Cumulative Proportion  0.997960693
                            Comp.7
Standard deviation     0.011188234
Proportion of Variance 0.001728154
Cumulative Proportion  0.999688848
                             Comp.8
Standard deviation     0.0044025094
Proportion of Variance 0.0002675839
Cumulative Proportion  0.9999564314
```

The below plot allows us a clearer visual representation of how much each component explains the variations in the data. Following the "Elbow method" (i.e. choosing the number of components up to the point where additionally explained marginal variation is insignificant), we can see that no significant additional variability is explained after component 3. This chart confirms that we can drop all the remaining components and keep the 3 first components.

Chart 1

ECB yield curve PCA: importance of components



We know that the three first components explain most of the variations in the yield curve. We can therefore analyze the PCA's loadings. As a PCA is about formulating a linear combination of factors (or components) that define the variations of a variable, the loadings express the correlation between the variable and the principal component.

The below table of loadings expresses how individual dimensions (maturities) contribute to each component.

Table 2

ECB yield curve Principal Component Analysis – loadings

```
> factor.loadings
         Comp.1        Comp.2       Comp.3
X3M  0.03658103  0.0019875001  0.554725627
X6M  0.05068604 -0.2146966718  0.563817364
X1Y  0.11257858 -0.2764579748  0.342846113
X2Y  0.13870533 -0.2824558816  0.150385556
X3Y  0.14185225 -0.2966386911  0.050934031
X4Y  0.14761789 -0.2931180650 -0.006039029
X5Y  0.15660723 -0.2772659960 -0.045102357
X6Y  0.16640418 -0.2525841902 -0.075824508
X7Y  0.17526163 -0.2217268365 -0.100015367
X8Y  0.18241108 -0.1875079829 -0.117552228
X9Y  0.18781266 -0.1523241464 -0.128241748
X10Y 0.19174570 -0.1178917519 -0.132834514
X11Y 0.19455960 -0.0852230058 -0.131991254
X12Y 0.19657463 -0.0546890579 -0.126666945
```

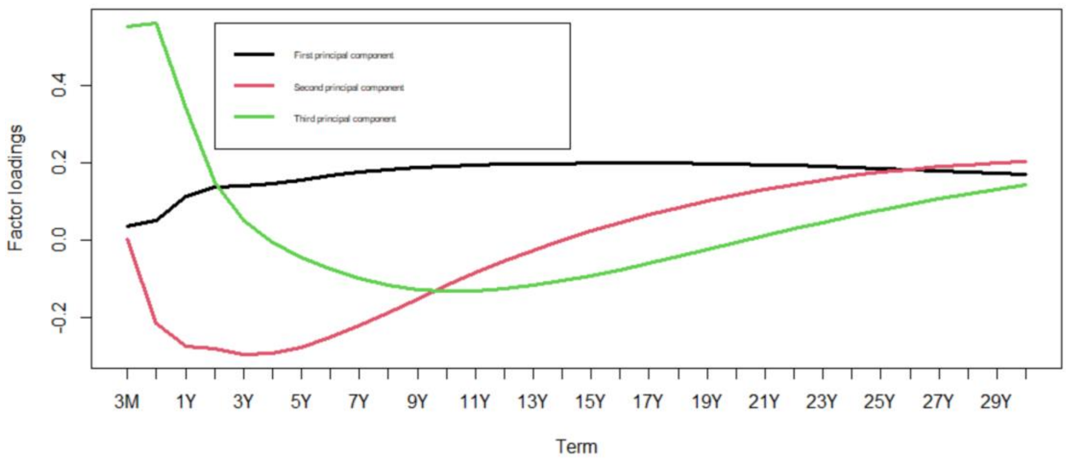
X13Y	0.19799150	-0.0264447462	-0.117831011
X14Y	0.19895750	-0.0003907268	-0.106173355
X15Y	0.19953849	0.0236561850	-0.092368482
X16Y	0.19977551	0.0458110876	-0.076903817
X17Y	0.19966873	0.0661797707	-0.060278548
X18Y	0.19922241	0.0848953470	-0.042859650
X19Y	0.19841979	0.1021013720	-0.024978937
X20Y	0.19727162	0.1177530332	-0.006804662
X21Y	0.19577091	0.1320019885	0.011202674
X22Y	0.19393515	0.1448656623	0.028921103
X23Y	0.19179320	0.1563489752	0.046157067
X24Y	0.18936890	0.1665697615	0.062724497
X25Y	0.18670073	0.1755787029	0.078601707
X26Y	0.18382633	0.1834452707	0.093625296
X27Y	0.18079606	0.1902278633	0.107703036
X28Y	0.17763757	0.1960611040	0.120832569
X29Y	0.17440635	0.2009898414	0.133008973
X30Y	0.17111151	0.2051494682	0.144328807

From this table, positive loading values suggest a variable and a principal component (PC) that are positively correlated. Conversely, negative loadings indicate a negative correlation. Large loading values indicate that a variable has a strong effect on the given principal component. Therefore, our aim is now to identify the maturities at which loading values are more pronounced, to visualize at which maturities the correlation between the principal component and changes in the yield curve is the strongest. We plot the PC loadings for better visual representation of their behavior across maturities.

Plotting those three factors' loadings across all maturities, we notice how loading values evolve for each maturity, which "gives an initial intuition about the vector's economic meaning"[2].

Chart 2

ECB yield curve PCA: loadings by principal components



[2] https://towardsdatascience.com/decomposing-predicting-the-euro-yield-curve-b3ad1670fdbb

In particular, the stability and persistence across maturities of Principal Component 1 (PC1) loadings (the black curve) suggests that PC1 captures the impact of the level of the yield curve. Indeed, as the PC1 loadings are nearly the same across all maturities, an increase in the first factor would affect yields of all maturities identically, which amounts to a change in level of the yield curve.

Regarding the second component, we notice its loadings (the red curve) are higher in absolute value for short maturities (from 3 months to 3 years). This is consistent with the theory implied by the Nelson-Siegel model, whereby increases in $\boldsymbol{\beta_{1t}}$ affect short-term yields more strongly than long-term yields, which has therefore an impact on the yield curve slope. Therefore, we can safely conclude that PC2 factor captures the effect of $\boldsymbol{\beta_{1t}}$, namely the "yield curve slope" factor, i.e. the spread between short-term and long-term rates.

Finally, we notice that the third component loadings (the green curve) are higher in absolute value for medium-term maturities and are significantly smaller for longer-term maturities. The third factor therefore captures the change in the curvature ($\boldsymbol{\beta_{2t}}$), which is mostly reflected in the medium-term maturities. An increase in $\boldsymbol{\beta_{2t}}$ will predominantly influence medium-term maturities, thereby increasing the curve's curvature.

In conclusion, in the light of the Nelson-Siegel model of the yield curve, we have therefore (i) successfully identified the three factors that underpin changes in the yield curve and (ii) decomposed the ECB yield curve changes according to those three components.