Master 2 Financial Economics – Université Paris 1 Panthéon-Sorbonne

Empirical Application 1

Asset pricing

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I. EMPIRICAL APPLICATION 1

Question 1: Consider the series of the quotations for a stock index over two different periods and justify the choice of the periods and of the frequency

Of all the possible options, we have selected the S&P500 Stock Index (SP500) to conduct our empirical analysis, due to its representativeness of the state of the world's financial markets.

As we still are experiencing the fallout of the COVID-19 pandemic, we wanted to compare such times of turmoil with another meaningful event of the 21st century: the Global Financial Crisis of 2007-2008.

Therefore, we have selected the following two periods:

- 01/01/2005 to 03/01/2012 for the Global Financial Crisis
- 30/01/2019 to 10/09/2021 for the COVID-19 Pandemic

We overextended both periods, before and after the main event, in order to study the series on a bigger time scale and to capture the evolution of the SP500 during its recovery. Finally, we opted for a daily frequency to seize the high volatility present on the markets during these two events.

Question 2: Take the logarithm of the selected time series

A. Examine whether the series are martingales or not

From the first Chapter of the Asset Pricing lecture, we know that a random walk without drift is a martingale (although the reverse is not true).

Therefore, examining whether the series are martingales implies verifying that the series are random walks without drift, i.e. that they verify the following conditions:

i. The random variable X at time $t + 1$ corresponds to the value of the X variable at time t plus a "surprise" factor εt :

$$X_{t+1} = X_t + \varepsilon_t$$

ii. ε_t is a weak white noise process, where:

$$E(\varepsilon_t) = 0$$
$$Var(\varepsilon_t) = \sigma^2$$
$$Cov(\varepsilon_t, \varepsilon_t') = 0$$

a. Testing condition (i)

The below results of the ADF test (specified as "none" in R code, so as to regress X_t only on its [-1] lagged values) suggesting the presence of a unit root for both of our time series.

S&P500 during the subprime crisis: with a value of 0.089, the t-statistic for "Z.lag.1" (which is the coefficient $\gamma = a - 1$ in front of the lag variable X_{t-1}) does not exceed the critical value at the 5% level (of -1.95), in absolute value. Likewise, its p-value does not meet the 5% significance level.

This therefore means that the coefficient $\gamma = a - 1$ is not statistically different from 0, which implies that we cannot reject that the value of $a = 1$.

We therefore cannot reject the null hypothesis H_0 of the existence of a unit root.

R-Code 1

ADF test on the Subprime crisis sample

```
> logSUBts_ADF <- ur.df(logSUBts, type="none", selectlags="AIC")
> summary(logSUBts_ADF)

###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.095384 -0.005281  0.000966  0.006083  0.108083 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
z.lag.1     4.377e-06  4.869e-05   0.090    0.928    
z.diff.lag -1.234e-01  2.366e-02  -5.215 2.05e-07 ***
---
Signif. codes:  
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01451 on 1760 degrees of freedom
Multiple R-squared:  0.01522,	Adjusted R-squared:  0.0141 
F-statistic:  13.6 on 2 and 1760 DF,  p-value: 1.374e-06


Value of test-statistic is: 0.0899 

Critical values for test statistics: 
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

S&P500 during the COVID-19 outbreak: with a value of 1.8289, the t-statistic for “Z.lag.1” (which is the coefficient $\gamma = a - 1$ in front of the lag variable X_{t-1}) does not exceed the critical value at the 5% level (of -1.95) in absolute value. Likewise, its p-value does not meet the 5% significance level.

This therefore means that the coefficient γ = a – 1 is not statistically different from 0, which implies that we cannot reject that the value of a = 1.

We therefore cannot reject the null hypothesis H_0 of the existence of a unit root at the 5% significance level.

R-Code 2

ADF test on the COVID-19 crisis sample

```
> logCOVIDts_ADF <- ur.df(logCOVIDts, type="none", selectlags="AIC")
> summary(logCOVIDts_ADF)

###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
      Min        1Q    Median        3Q       Max
-0.116085 -0.004236  0.000765  0.005903  0.079730

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
z.lag.1     1.244e-04  6.805e-05   1.829   0.0679 .
z.diff.lag -3.025e-01  3.724e-02  -8.122 2.28e-15 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01412 on 655 degrees of freedom
Multiple R-squared:  0.09391,	Adjusted R-squared:  0.09115
F-statistic: 33.94 on 2 and 655 DF,  p-value: 9.398e-15


Value of test-statistic is: 1.8289

Critical values for test statistics:
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

Therefore, both ADF tests suggest that our two samples are likely affected by a unit root as we cannot reject the null hypothesis ($H_0 : presence\ of\ a\ unit\ root$). Therefore, the values of X_t are pertinent for explaining the values of X_{t+1}. The series of the S&P500 during the Subprime crisis and during the COVID-19 outbreak seem to be random walks without drift and could therefore be considered as following a martingale process.

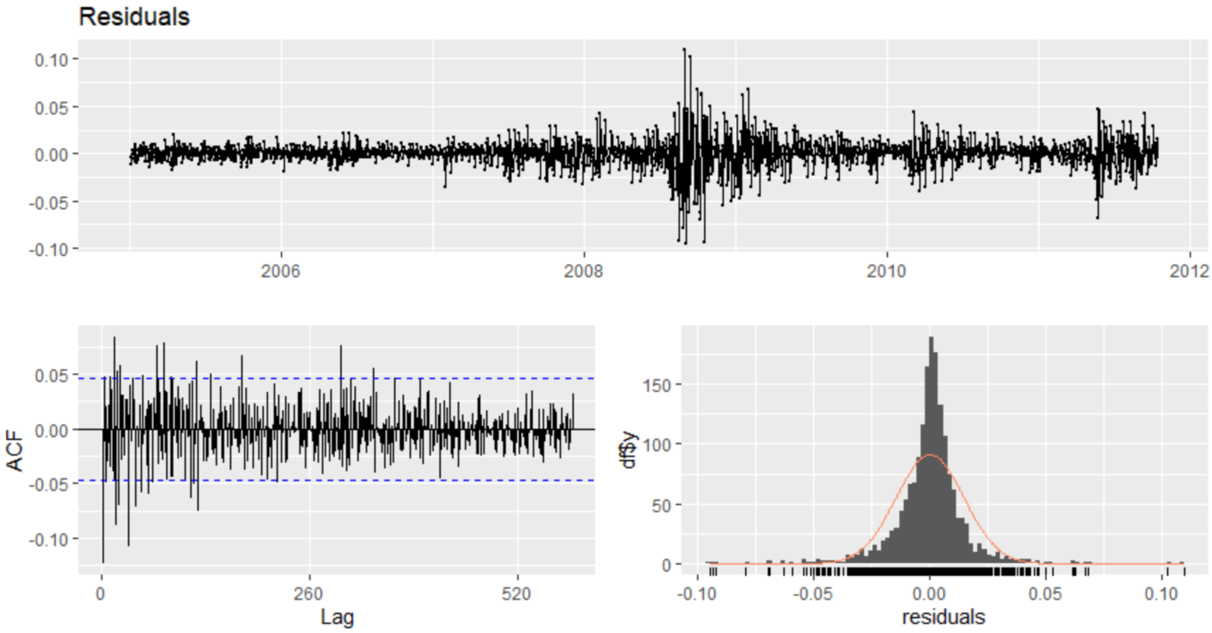
b. Testing condition (ii)

We plot the residuals for both series' RW(1) without drift. The below charts and tests suggest that residuals are not weak white noise. Therefore, as opposed to what we demonstrate in the above section, residuals suggest that both series cannot be described as random walk without drift and thereby do not follow a martingale process.

S&P500 during the subprime crisis:

Figure 1

Residuals analysis of the Subprime crisis sample



- The residuals are centered around 0, which verifies the property that E(ε_t) = 0. However, variance of residuals is clearly not constant, as shown by the spikes in variance of residuals in H2 2018 and in mid-2011. We can therefore assume that the assumption Var(ε_t) = σ^2 is violated.
- To analyze whether Cov($\varepsilon_{t,}$ ε_{t}') = 0:
  - The ACF of the RW(1)'s residuals shows that some of the errors' lags are significantly greater than 0 (outside the bounds): this means that some lags of ε_t have an explanatory power for ε_t. This shows that ε_t is not a white noise process.
  - We confirm our conclusion that ε_t are not white noise by applying a Ljung-Box test to the residuals, to check whether the first ε_t autocorrelation values behave like a white noise series:

R-Code 3

Box-Ljung test on the Subprime crisis sample

```
> Box.test(Residuals_RW1_logSUBts,lag=20,type="Lj")

        Box-Ljung test

data:  Residuals_RW1_logSUBts
X-squared = 95.429, df = 20, p-value = 8.21e-12
```

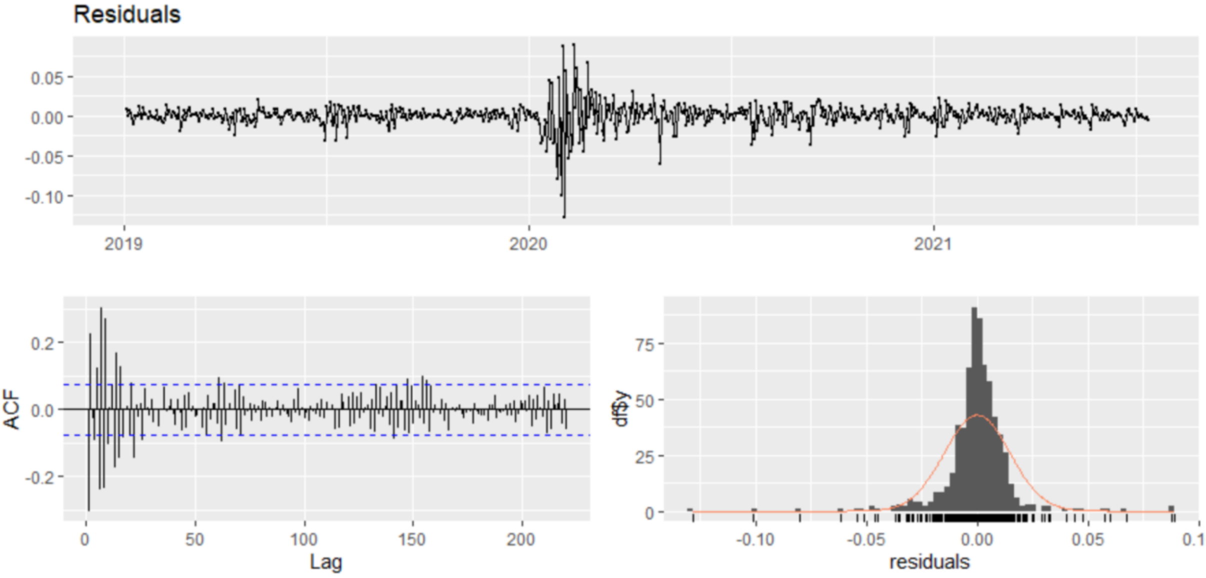
The Box-Ljung test tests whether the errors are white noise (H0) or whether the autocorrelations in residuals are non-zero. We can see that the p-value is extremely small, compared to the 0.05 significance level. We therefore reject the hypothesis that the residuals are white noise.

Therefore, the S&P500 was not a random walk (1) without drift during the subprimes crisis, implying that it was not following a martingale process.

<u>S&P500 during the COVID-19 outbreak:</u>

Figure 2

Residuals analysis of the COVID-19 crisis sample



- The residuals are centered around 0, which verifies the property that $E(\varepsilon_t) = 0$.
- However, variance of residuals is clearly not constant, as shown by the spike in variance of residuals in H1 2020.
- To test whether $Cov(\varepsilon_t, \varepsilon_{t'}) = 0$:
  - The ACF of the RW(1)'s residuals shows that some of the errors' lags are significantly greater than 0 (outside the bounds): this means that some lags of ε_t have an explanatory power for ε_t. This shows that ε_t is not a white noise process.
  - We confirm our conclusion that ε_t are not white noise by applying a Ljung-Box test to the residuals, to check whether the first ε_t autocorrelation values behave like a white noise series:

R-Code 4

Box-Ljung test on the COVID-19 crisis sample

```
> Box.test(Residuals_RW1_logCOVIDts,lag=20,type="Lj")

	Box-Ljung test

data:  Residuals_RW1_logCOVIDts
X-squared = 381.63, df = 20, p-value < 2.2e-16
```

The Box-Ljung test tests whether the errors are white noise (H0) or whether the autocorrelations in residuals are non-zero (H_A). We can see that the p-value is extremely small, compared to the 0.05 significance level. We therefore reject the hypothesis that the residuals are white noise.

Therefore, the S&P500 was not a random walk (1) without drift during the COVID-19 crisis, implying that it was not following a martingale process.

B. If not, try to specify a strong random walk with drift for each series, with the relevant test(s).

<u>S&P500 during the subprimes crisis:</u>

We firstly estimate an AR(1) model with an intercept.

R-Code 5

Estimation of an AR(1) model on the Subprime crisis sample

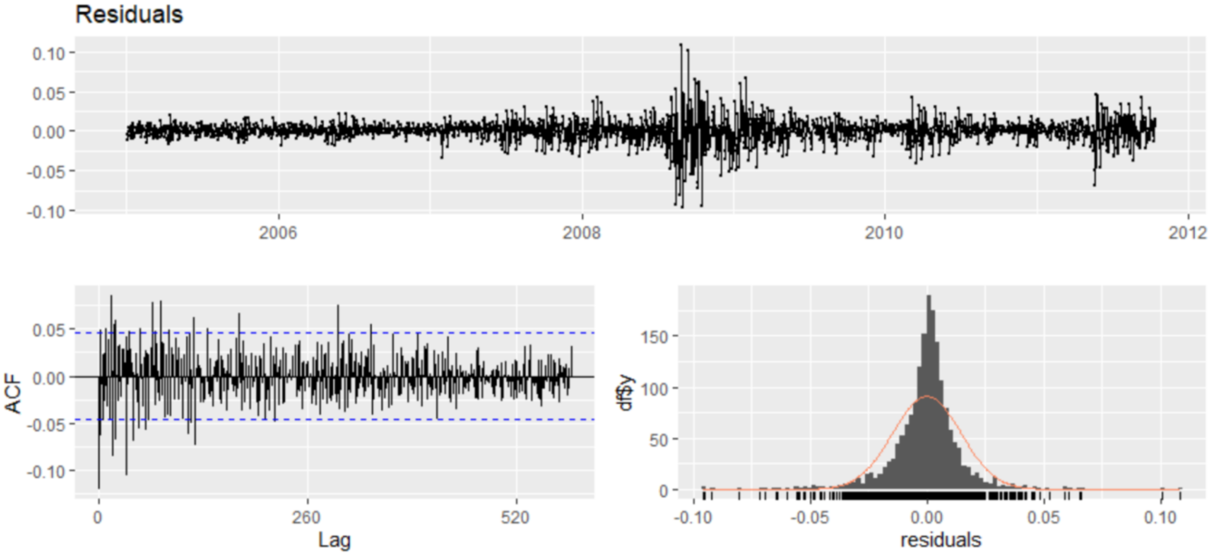
```
Call:
arma(x = logSUBts, order = c(1, 0), include.intercept = TRUE)

Model:
ARMA(1,0)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0955786 -0.0054342  0.0009397  0.0060443  0.1082297 

Coefficient(s):
           Estimate  Std. Error  t value Pr(>|t|)    
ar1        0.995598    0.002242  444.120   <2e-16 ***
intercept  0.031285    0.015917    1.966   0.0494 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Fit:
sigma^2 estimated as 0.0002132,  Conditional Sum-of-Squares = 0.38,  AIC = -9901.55
```

The above regression results seem to confirm that both the lag term and the constant are statistically significantly different from 0, confirming that the series could follow a strong random walk with drift.

We then check the regression’s residuals:

Figure 3

Residuals analysis of the Subprime crisis sample



From the above results, we can conclude that the regression's residuals still do not follow a white noise process:

- The residuals are centered around 0, which verifies the property that $E(\varepsilon_t) = 0$.
- However, variance of residuals is clearly not constant, as shown by the spike in variance of residuals in H2 2009 and H1 2012.
- To test whether $Cov(\varepsilon_t, \varepsilon_t') = 0$:
  - The ACF of the RW(1)'s residuals shows that some of the errors' lags are significantly greater than 0 (outside the bounds): this means that some lags of ε_t have an explanatory power for ε_t. This shows that ε_t is not a white noise process.
  - We confirm our conclusion that ε_t are not white noise by applying a Ljung-Box test to the residuals, to check whether the first ε_t autocorrelation values behave like a white noise series:

R-Code 6

Box-Ljung test on the Subprime crisis sample

```
> Box.test(Residuals_RW2_logSUBts,lag=20,type="Lj")

        Box-Ljung test

data:  Residuals_RW2_logSUBts
X-squared = 94.02, df = 20, p-value = 1.457e-11
```

The Box-Ljung test tests whether the errors are white noise (H0) or whether the autocorrelations in residuals are non-zero. We can see that the p-value is extremely small, compared to the 0.05 significance level. We therefore reject the hypothesis that the residuals are white noise.

Therefore, the S&P500 was not a strong random walk with drift during the subprimes crisis.

S&P500 during the COVID-19 outbreak:

We firstly estimate an AR(1) model with an intercept.

R-Code 7

Estimatation of an AR(1) model on the COVID-19 crisis sample

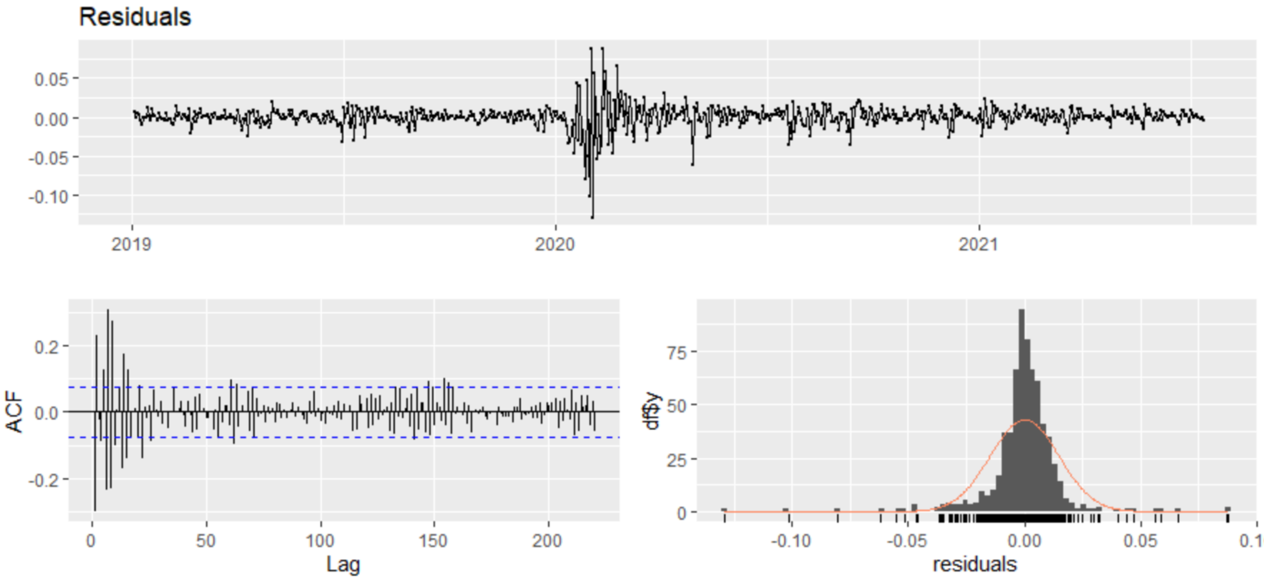
```
Call:
arma(x = logCOVIDts, order = c(1, 0), include.intercept = TRUE)

Model:
ARMA(1,0)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.1290674 -0.0044156  0.0007405  0.0061742  0.0876696 

Coefficient(s):
           Estimate  Std. Error  t value Pr(>|t|)    
ar1        0.996883    0.003721  267.893   <2e-16 ***
intercept  0.026051    0.030175    0.863    0.388    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Fit:
sigma^2 estimated as 0.0002187,  Conditional Sum-of-Squares = 0.14,  AIC = -3679.71
```

The above regression results seem to confirm that the lag term is statistically significantly different from 0 (given its very small p-value). However, its intercept term appears not to be significant. This result does not confirm that the series follows a strong random walk with drift.

We then check the regression's residuals:

Figure 4

Residuals analysis of the COVID-19 crisis sample



From the above results, we can conclude that the regression's residuals still do not follow a white noise process:

- The residuals are centered around 0, which verifies the property that $E(\varepsilon_t) = 0$.
- However, variance of residuals is clearly not constant, as shown by the spike in variance of residuals in H1 2020, during the outbreak of the COVID-19 crisis.
- To test whether $Cov(\varepsilon_t, \varepsilon_t') = 0$:
  - The ACF of the RW(1)'s residuals shows that some of the errors' lags are significantly greater than 0 (outside the bounds): this means that some lags of ε_t have an explanatory power for ε_t. This shows that ε_t is not a white noise process.
  - We confirm our conclusion that ε_t are not white noise by applying a Ljung-Box test to the residuals, to check whether the first ε_t autocorrelation values behave like a white noise series:

R-Code 8

Box-Ljung test on the COVID-19 crisis sample

```
> Box.test(Residuals_RW2_logCOVIDts,lag=20,type="Lj")

	Box-Ljung test

data:  Residuals_RW2_logCOVIDts
X-squared = 379.63, df = 20, p-value < 2.2e-16
```

The Box-Ljung test tests whether the errors are white noise (H0) or whether the autocorrelations in residuals are non-zero. We can see that the p-value is extremely small, compared to the 0.05 significance level. We therefore reject the hypothesis that the residuals are white noise.

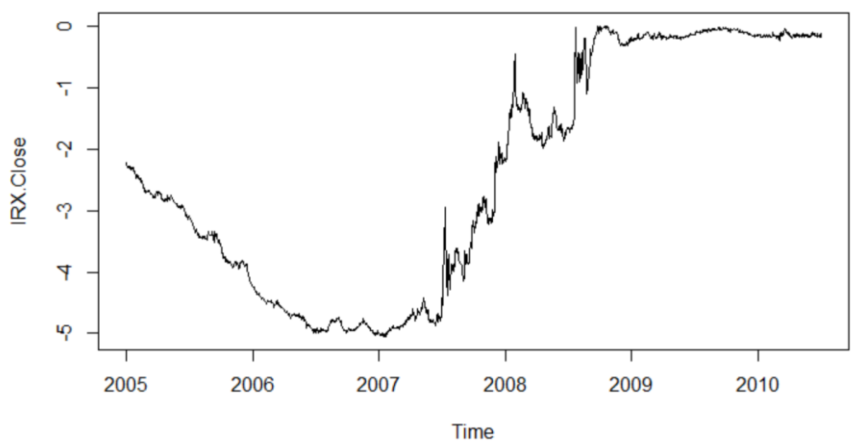
Therefore, the S&P500 was not a strong random walk with drift during the COVID-19 crisis.

C. Then turn to the series of excess returns and repeat the analysis. Comment your results.

<u>S&P 500 excess returns – Subprime crisis period:</u>

Figure 5

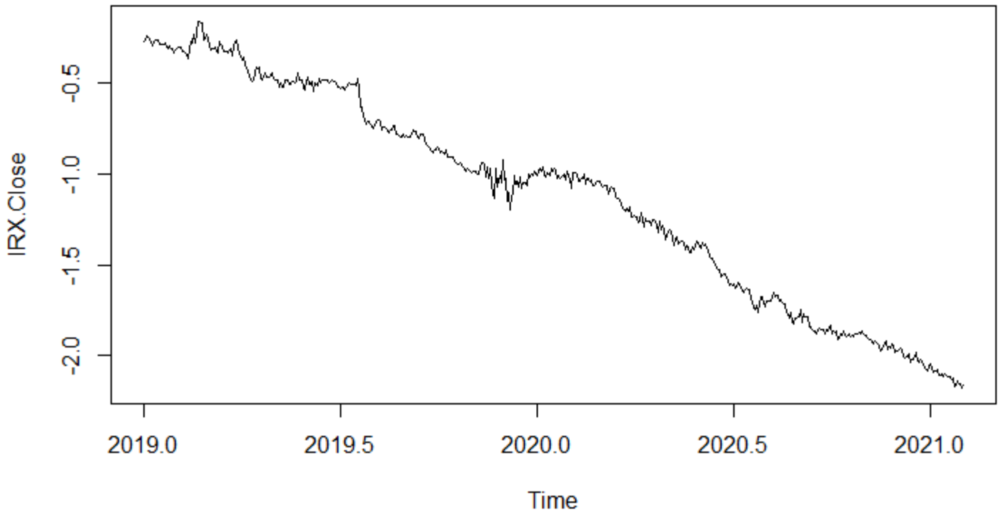
S&P500 excess returns on the Subprime crisis sample



<u>S&P 500 excess returns – Covid-19 crisis period:</u>

Figure 6

S&P500 excess returns on the COVID-19 crisis sample



Checking if excess returns of the S&P500 follow a martingale during the subprimes crisis.

R-Code 9

ADF test on the Subprime crisis sample

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max
-0.85456 -0.02201 -0.00217  0.01891  0.81987

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
z.lag.1    -0.0005223  0.0006740  -0.775    0.438
z.diff.lag  0.0212220  0.0264547   0.802    0.423

Residual standard error: 0.07816 on 1427 degrees of freedom
Multiple R-squared:  0.0008663, Adjusted R-squared:  -0.000534
F-statistic: 0.6186 on 2 and 1427 DF,  p-value: 0.5388


Value of test-statistic is: -0.775

Critical values for test statistics:
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

With a value of -0.775, the t-statistic for "Z.lag.1" (which is the coefficient $\gamma = a - 1$ in front of the lag variable X_{t-1}) is not sufficiently negative to exceed the critical value at the 10% level (of -1.62). Likewise, its p-value does not meet the 10% significance level.

This therefore means that the coefficient $\gamma = a - 1$ is not statistically different from 0, which implies that we cannot reject that the value of $a = 1$.

We therefore cannot reject the null hypothesis H0 of the existence of a unit root.

Checking if excess returns of the S&P500 follow a martingale during the covid-19 crisis.

R-Code 10

ADF test on the COVID-19 crisis sample

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.118798 -0.014978  0.000012  0.012970  0.154179 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
z.lag.1     0.0033637  0.0009251   3.636 0.000304 ***
z.diff.lag -0.3170958  0.0410021  -7.734 5.17e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02689 on 539 degrees of freedom
Multiple R-squared:  0.1102,	Adjusted R-squared:  0.1069 
F-statistic: 33.39 on 2 and 539 DF,  p-value: 2.138e-14


Value of test-statistic is: 3.6359 

Critical values for test statistics: 
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

With a value of 3.6359, the t-statistic for "Z.lag.1" (which is the coefficient $\gamma = a - 1$ in front of the lag variable X_{t-1}) is not sufficiently negative to exceed the critical value at the 10% level (of -1.62). Likewise, its p-value does not meet the 10% significance level.

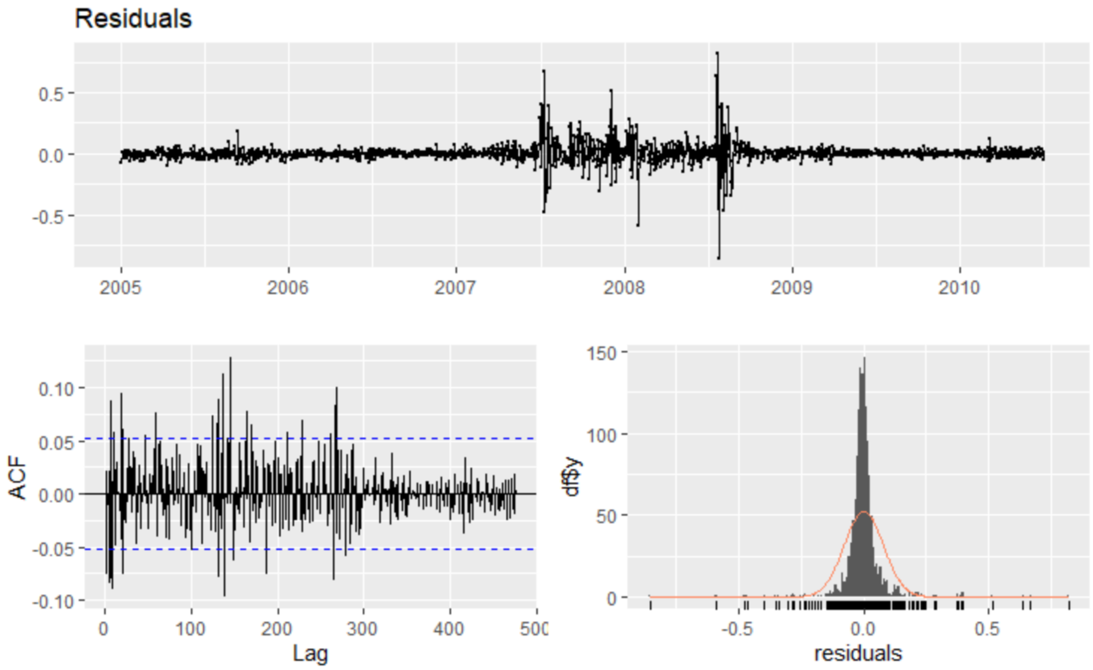
This therefore means that the coefficient $\gamma = a - 1$ is not statistically different from 0, which implies that we cannot reject that the value of a = 1.

We therefore cannot reject the null hypothesis H0 of the existence of a unit root.

<u>Residuals of RW without drift for S&P500 during subprime crisis</u>

Figure 7

Residuals analysis of the Subprime crisis sample



From the above results, we can conclude that the regression's residuals still do not follow a white noise process:

- The residuals are centered around 0, which verifies the property that $E(\varepsilon_t) = 0$.
- However, variance of residuals is clearly not constant, as shown by the spike in variance of residuals in H1 2020, during the outbreak of the COVID-19 crisis.
- To test whether $Cov(\varepsilon_t, \varepsilon_t') = 0$:
  - The ACF of the RW(1)'s residuals shows that some of the errors' lags are significantly greater than 0 (outside the bounds): this means that some lags of ε_t have an explanatory power for ε_t. This shows that ε_t is not a white noise process.
  - We confirm our conclusion that ε_t are not white noise by applying a Ljung-Box test to the residuals, to check whether the first ε_t autocorrelation values behave like a white noise series:

R-Code 11

Box-Ljung test on the Subprime crisis sample

```
> Box.test(Residuals_SUBER1,lag=20,type="Lj")

        Box-Ljung test

data:  Residuals_SUBER1
X-squared = 83.854, df = 20, p-value = 8.622e-10
```

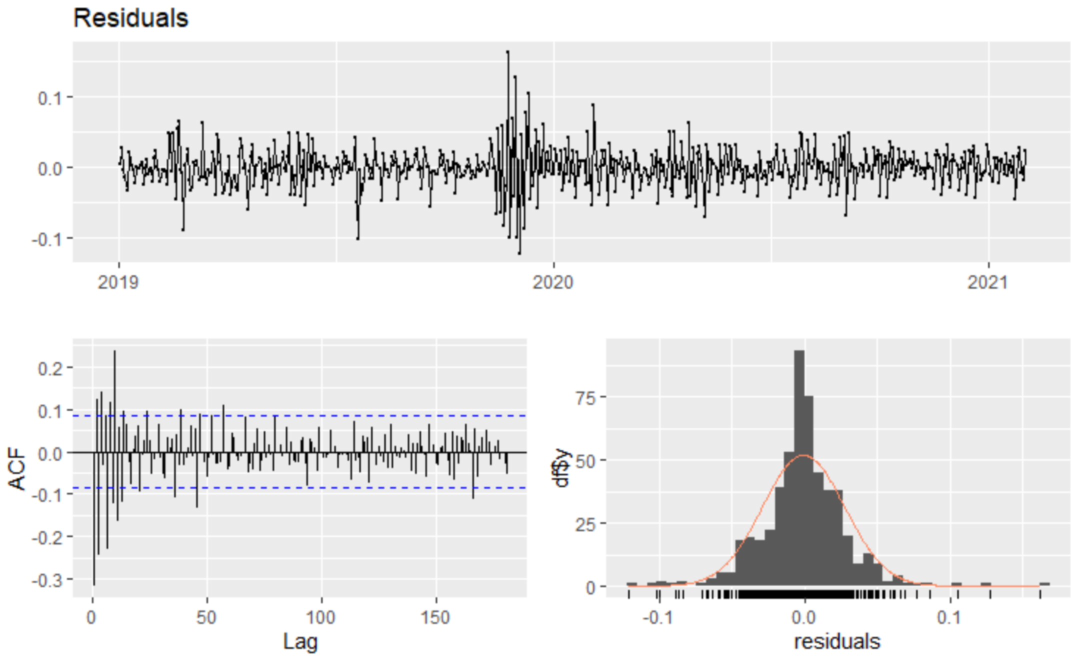
The Box-Ljung test tests whether the errors are white noise (H0) or whether the autocorrelations in residuals are non-zero. We can see that the p-value is extremely small, compared to the 0.05 significance level. We therefore reject the hypothesis that the residuals are white noise.

Therefore, the S&P500's excess returns was not a martingale during the subprime crisis.

Residuals of RW without drift for S&P500 during covid crisis

Figure 8

Residuals analysis of the Subprime crisis sample



From the above results, we can conclude that the regression's residuals still do not follow a white noise process:

- The residuals are centered around 0, which verifies the property that $E(\varepsilon_t) = 0$.
- However, variance of residuals is clearly not constant, as shown by the spike in variance of residuals in H1 2020, during the outbreak of the COVID-19 crisis.
- To test whether $Cov(\varepsilon_t, \varepsilon_t') = 0$:
  - The ACF of the RW(1)'s residuals shows that some of the errors' lags are significantly greater than 0 (outside the bounds): this means that some lags of ε_t have an explanatory power for ε_t. This shows that ε_t is not a white noise process.
  - We confirm our conclusion that ε_t are not white noise by applying a Ljung-Box test to the residuals, to check whether the first ε_t autocorrelation values behave like a white noise series:

R-Code 12

Box-Ljung test on the COVID-19 crisis sample

```
> Box.test(Residuals_COVER1,lag=20,type="Lj")

	Box-Ljung test

data:  Residuals_COVER1
X-squared = 225.99, df = 20, p-value < 2.2e-16
```

The Box-Ljung test tests whether the errors are white noise (H0) or whether the autocorrelations in residuals are non-zero. We can see that the p-value is extremely small, compared to the 0.05 significance level. We therefore reject the hypothesis that the residuals are white noise.

Therefore, the S&P500's excess returns was not a martingale during the covid-19 crisis.

D. Same analysis with a strong random walk with drift

<u>S&P 500 during subprime crisis</u>

R-Code 13

```
> summary(RW2_SUBER)

Call:
arma(x = ExcessReturns_SUBTS, order = c(1, 0), include.intercept
 = TRUE)

Model:
ARMA(1,0)

Residuals:
      Min        1Q    Median        3Q       Max
-0.856175 -0.022083 -0.002307  0.018493  0.818886
```

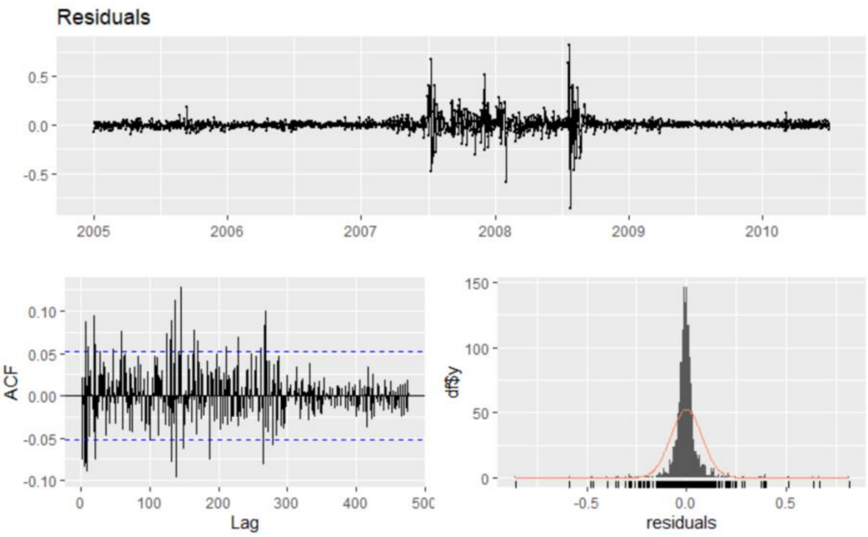
```
Coefficient(s):
           Estimate  Std. Error  t value Pr(>|t|)
ar1       0.9996368   0.0010768   928.33   <2e-16 ***
intercept 0.0005624   0.0033030     0.17    0.865
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Fit:
sigma^2 estimated as 0.006108,  Conditional Sum-of-Squares = 8.7
3,  AIC = -3230.54
```

The above regression results seem to confirm that both the lag term is statistically significantly different from 0, but not the constant, suggesting that the series does not follow a strong random walk with drift.

We check regression residuals

Figure 9

Residuals analysis of the Subprime crisis sample



From the above results, we can conclude that the regression's residuals do not follow a white noise process:

- The residuals are centered around 0, which verifies the property that $E(\varepsilon_t) = 0$.
- However, variance of residuals is clearly not constant, as shown by the spike in variance of residuals in H2 2007, upon occurence of the subprimes crisis.
- To test whether $Cov(\varepsilon_t, \varepsilon_t') = 0$:
  - The ACF of the RW(1)'s residuals shows that some of the errors' lags are significantly greater than 0 (outside the bounds): this means that some lags of ε_t have an explanatory power for ε_t. This shows that ε_t is not a white noise process.
  - We confirm our conclusion that ε_t are not white noise by applying a Ljung-Box test to the residuals, to check whether the first ε_t autocorrelation values behave like a white noise series:

R-Code 14

Box-Ljung test on the Subprime crisis sample

```
> Box.test(Residuals_SUBER2,lag=20,type="Lj")

        Box-Ljung test

data:  Residuals_SUBER2
X-squared = 83.896, df = 20, p-value = 8.481e-10
```

The Box-Ljung test tests whether the errors are white noise (H0) or whether the autocorrelations in residuals are non-zero. We can see that the p-value is extremely small, compared to the 0.05 significance level. We therefore reject the hypothesis that the residuals are white noise.

Therefore, the S&P500's excess returns was not a strong random during the subprime crisis.

<u>S&P 500 during covid-19 crisis</u>

R-Code 15

```
Call:
arma(x = ExcessReturns_COVTS, order = c(1, 0), include.intercept = TRUE)

Model:
ARMA(1,0)

Residuals:
       Min         1Q     Median         3Q        Max 
-1.206e-01 -1.332e-02  1.837e-05  1.399e-02  1.633e-01 
```

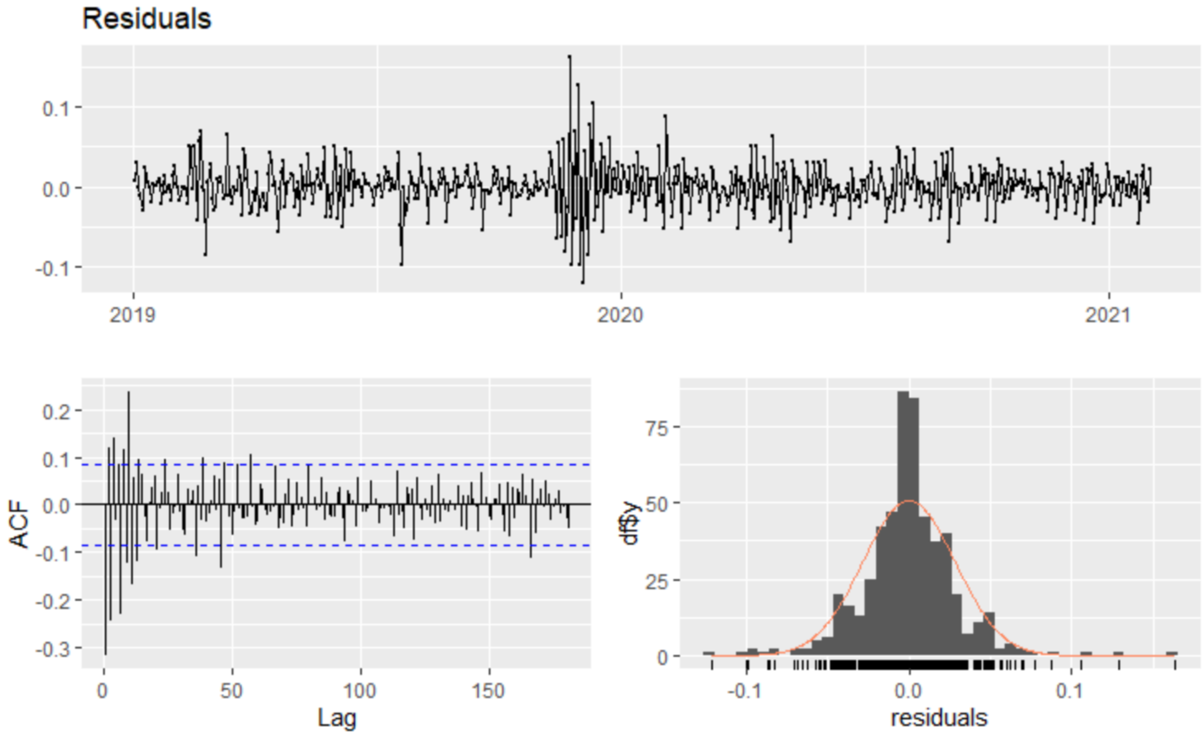
```
Coefficient(s):
           Estimate  Std. Error  t value Pr(>|t|)    
ar1        0.999813    0.002088  478.764   <2e-16 ***
intercept -0.003697    0.002630   -1.406     0.16    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Fit:
sigma^2 estimated as 0.0007975,  Conditional Sum-of-Squares = 0.43,  AIC = -2328.8
```

The above regression results seem to confirm that both the lag term is statistically significantly different from 0, but not the constant, suggesting that the series does not follow a strong random walk with drift.

We check the regression's residuals:

Figure 10

Residuals analysis of the COVID-19 crisis sample



From the above results, we can conclude that the regression's residuals do not follow a white noise process:

- The residuals are centered around 0, which verifies the property that $E(\varepsilon_t) = 0$.
- However, variance of residuals is clearly not constant, as shown by the spike in variance of residuals in H1 2020, upon occurence of the covid-19 crisis.
- To test whether $Cov(\varepsilon_t, \varepsilon_{t'}) = 0$:
  - The ACF of the RW(1)'s residuals shows that some of the errors' lags are significantly greater than 0 (outside the bounds): this means that some lags of ε_t have an explanatory power for ε_t. This shows that ε_t is not a white noise process.
  - We confirm our conclusion that ε_t are not white noise by applying a Ljung-Box test to the residuals, to check whether the first ε_t autocorrelation values behave like a white noise series:

R-Code 16

Box-Ljung test on the COVID-19 crisis sample

```
> Box.test(Residuals_COVER2,lag=20,type="Lj")

	Box-Ljung test

data:  Residuals_COVER2
X-squared = 226.22, df = 20, p-value < 2.2e-16
```

The Box-Ljung test tests whether the errors are white noise (H0) or whether the autocorrelations in residuals are non-zero. We can see that the p-value is extremely small, compared to the 0.05 significance level. We therefore reject the hypothesis that the residuals are white noise.

Therefore, the S&P500's excess returns was not a strong random during the subprime crisis.