Master 2 Financial Economics – Université Paris 1 Panthéon-Sorbonne

Empirical Application 3

Financial Econometrics

Roland BOUILLOT
(Roland.Bouillot@etu.univ-paris1.fr)

Khalil JANBEK
(Khalil.Janbek@etu.univ-paris1.fr)

Mehdi LOUAFI
(Mehdi.Louafi@etu.univ-paris1.fr)

September 2021

EMPIRICAL APPLICATION 3

From the empirical application 1 and 2, we have chosen three financial series: the 5-year breakeven inflation expectations (*Breakeven5Y*), the S&P500 stock index (*SP500*) and the WTI oil prices (*WTI*). However, for this third application, we decided to change some of our variables to address some econometric issues we have found running the multiple tests of this third empirical application. We still focus our study over the November 2011 to August 2021 period (totaling 118 monthly observations) and keep the S&P500 stock index (*SP500*) and the WTI oil prices (*WTI*) variables. The novelty in this third application is the introduction of the Industrial Production (*INDPRO*) and the Consumer Price Index (*CPI*) variables. The summary of the aforementioned variables can be found in Table 2.

In this third empirical application, we will use this new set of variables to find evidence of a long-run cointegration relationship, estimate a VECM model, conduct some robustness checks and forecast each of the series at horizon 10.

I. Argument your choice of series by referring to a financial issue you are interested in

As we introduced some new variables, it was important for us to describe them more in detail and explain our new identification framework. This study aims at empirically assessing the extent to which selected fundamental macroeconomic variables (*CPI, WTI, IPROD*) are linked in the long run with the S&P500 U.S. stock index (*SP500*).

The sources of the data mostly come from governmental authorities. All the presented variables are retrieved from the Federal Reserve Bank of St. Louis' Economic Database (FRED). The S&P500 stock index comes from the S&P Dow Jones Indices division of S&P Global. The CPI, WTI and Industrial Production variables have been publicly published by the U.S. Bureau of Labor Statistics, the U.S. Energy Information Administration and the Board of Governors of the Federal Reserve System respectively.

The choice of these three macroeconomic fundamentals is driven by their use in the economic literature and are likely to have an impact on business conditions, firms and thus affect the equity market.

- *Consumer Price Index (CPI)*

Past literature finds evidence of a negative relationship between (expected and unexpected) inflation rate and the stock market (Fama, 1981, 1990). In this study, the consumer price index is used as a proxy for inflation. The intuition behind a negative relationship regarding expected inflation is that inflation is often linked to nominal interest rates and if nominal interest rates increase, the discount rates also increase. This implication has a negative influence over cash flows and then stock prices. The relation regarding unexpected inflation comes from the fact that an unanticipated increase in prices would shift resources from investment to consumption. This would induce a reduced amount of shares traded and thus a decrease in stock prices.

- *Industrial Production (IPROD)*

Industrial production is used in this study to approximate the level of economic activity. Other studies choose to exploit gross domestic product or gross national product data. As increasing industrial production is associated to an increase in economic activity, we expect this variable to have a positive relation with the stock market. Indeed, improving business conditions has a positive impact on firm's earnings, higher dividends, and thus an increase in stock prices.

- *WTI oil price (WTI)*

Oil prices can be seen as a proxy to assess the level of international economic activity. In a globalized world, many S&P500 firms have ties with foreign suppliers or customers. Controlling for extra-domestic activity can explain part of the stock market price evolution, both in a positive and negative direction.

Note that before proceeding to the estimation of the model, all variables are transformed in their natural logarithms. The advantage of the logarithmic transformation is that it reduces extremes values in the data and smoothes the impact of outliers.

Our study follows the contemporary identification outline by testing our data against non-stationarity through ADF and PP tests. Once the data is proven to be integrated of order one I(1), we perform a Johansen cointegration test. Finally, we use a cointegrated vector error-correction model to find evidence of the long-run dynamics of the time series.

The model can be expressed as:

$$\mathrm{SP500}_t = \beta_0 + \beta_1 * CPI_t + \beta_2 * WTI_t + \beta_3 * IPROD_t + \varepsilon_t \quad \textbf{(1)}$$

Some descriptive statistics can be found in Table 3 and 4. The plotted untransformed and log-transformed time series are displayed in Figure 3.1.

II. Fitting the data

In order to use our data to obtain evidence of the long-run relationship and be able to do some forecasting, we need to be sure it complies with the usual statistical quality to fit our model. In empirical applications 1 & 2 we have already done some tests, which are re-ran with this new set of variables. We present the main results of those fitting tests and some complementary figures can be found in the dedicated annex (testing for a trend, seasonality, additional plotted variables).

In a VAR model, when the variables are integrated of order one (or more), unrestricted estimation can be subject to the hazards of regressions involving non-sationary variables. However, the presence of non-stationary variables can lead to the possibility of cointegrated relationships. The first step to estimate such model is to determine the cointegrating rank, that is the number of cointegrating relations in our data sample.

In order to do so, we have to prove that our selected variables are integrated of order 1. We perform the ADF and PP stationarity tests on our variables. The results can be found in the following figures.

Figure 26

ADF stationarity tests

```
> adf.test(lSP500, k=1)

	Augmented Dickey-Fuller Test

data:  lSP500
Dickey-Fuller = -2.5693, Lag order = 1, p-value = 0.3402
alternative hypothesis: stationary

> adf.test(lWTI, k=1)

	Augmented Dickey-Fuller Test

data:  lWTI
Dickey-Fuller = -2.8885, Lag order = 1, p-value = 0.2077
alternative hypothesis: stationary

> adf.test(lCPI, k=1)

	Augmented Dickey-Fuller Test

data:  lCPI
Dickey-Fuller = -2.8981, Lag order = 1, p-value = 0.2037
alternative hypothesis: stationary

> adf.test(lIPROD, k=1)

	Augmented Dickey-Fuller Test

data:  lIPROD
Dickey-Fuller = -3.7921, Lag order = 1, p-value = 0.02176
alternative hypothesis: stationary
```

Figure 27

PP stationarity tests

```
> pp.test(lSP500)

	Phillips-Perron Unit Root Test

data:  lSP500
Dickey-Fuller Z(alpha) = -14.787, Truncation lag parameter = 4, p-value = 0.2604
alternative hypothesis: stationary

> pp.test(lWTI)

	Phillips-Perron Unit Root Test

data:  lWTI
Dickey-Fuller Z(alpha) = -11.778, Truncation lag parameter = 4, p-value = 0.4346
alternative hypothesis: stationary

> pp.test(lCPI)

	Phillips-Perron Unit Root Test

data:  lCPI
Dickey-Fuller Z(alpha) = -2.5547, Truncation lag parameter = 4, p-value = 0.9519
alternative hypothesis: stationary

> pp.test(lIPROD)

	Phillips-Perron Unit Root Test

data:  lIPROD
Dickey-Fuller Z(alpha) = -17.757, Truncation lag parameter = 4, p-value = 0.09687
alternative hypothesis: stationary
```

As displayed in Figures 26 and 27, we fail to reject the null hypothesis of non-stationarity for each of our variables, either testing with the ADF or PP method. Those results mean that our variables have a unit root and thus are non-stationary. More detailed results can be found in Figure 3.7.

To find the order of integration of our variables, we need to first-differentiate them. We re-run the ADF and PP tests on the newly generated first-differenced log-transformed variables. Results are displayed in the following figures.

Figure 28

ADF stationarity tests (first-differenced variables)

```
> adf.test(lSP500_DIFF_ts, k=1)

        Augmented Dickey-Fuller Test

data:  lSP500_DIFF_ts
Dickey-Fuller = -9.3055, Lag order = 1, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In adf.test(lSP500_DIFF_ts, k = 1) : p-value smaller than printed p-value
> adf.test(lWTI_DIFF_ts, k=1)

        Augmented Dickey-Fuller Test

data:  lWTI_DIFF_ts
Dickey-Fuller = -8.2841, Lag order = 1, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In adf.test(lWTI_DIFF_ts, k = 1) : p-value smaller than printed p-value
> adf.test(lCPI_DIFF_ts, k=1)

        Augmented Dickey-Fuller Test

data:  lCPI_DIFF_ts
Dickey-Fuller = -6.1063, Lag order = 1, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In adf.test(lCPI_DIFF_ts, k = 1) : p-value smaller than printed p-value
> adf.test(lIPROD_DIFF_ts, k=1)

        Augmented Dickey-Fuller Test

data:  lIPROD_DIFF_ts
Dickey-Fuller = -8.9879, Lag order = 1, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In adf.test(lIPROD_DIFF_ts, k = 1) : p-value smaller than printed p-value
```

Figure 29

PP stationarity tests (first-differenced variables)

```
> pp.test(lSP500_DIFF_ts)

	Phillips-Perron Unit Root Test

data:  lSP500_DIFF_ts
Dickey-Fuller Z(alpha) = -107.73, Truncation lag parameter = 4, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In pp.test(lSP500_DIFF_ts) : p-value smaller than printed p-value
> pp.test(lWTI_DIFF_ts)

	Phillips-Perron Unit Root Test

data:  lWTI_DIFF_ts
Dickey-Fuller Z(alpha) = -78.9, Truncation lag parameter = 4, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In pp.test(lWTI_DIFF_ts) : p-value smaller than printed p-value
> pp.test(lCPI_DIFF_ts)

	Phillips-Perron Unit Root Test

data:  lCPI_DIFF_ts
Dickey-Fuller Z(alpha) = -52.4, Truncation lag parameter = 4, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In pp.test(lCPI_DIFF_ts) : p-value smaller than printed p-value
> pp.test(lIPROD_DIFF_ts)

	Phillips-Perron Unit Root Test

data:  lIPROD_DIFF_ts
Dickey-Fuller Z(alpha) = -75.219, Truncation lag parameter = 4, p-value = 0.01
alternative hypothesis: stationary

Warning message:
In pp.test(lIPROD_DIFF_ts) : p-value smaller than printed p-value
```

As displayed in Figures 28 and 29, this time we reject the null hypothesis of non-stationarity for each of our variables, either testing with the ADF or PP method. First-differencing our variables removed the unit roots previously identified in our sample, meaning that each of our variables are integrated of order 1 (as none of variables needed a second-difference transformation).

i. Testing for Cointegration Rank

Now that we have proven that our variables are integrated of order 1 (I(1)), it is now possible to proceed to the cointegration rank tests. Although two tests exist to identify cointegration ranks -the Engle-Granger and the Johansen cointegration tests- our study focuses on the Johansen test since it allows predicting more accurately the number of cointegration vectors in a model with more than two variables.

In the Johansen test framework, there are two statistics for testing the hypothesis that the cointegrating rank is at most r ($< k$). In one case, the alternative hypothesis is that the rank is k and the t-stat is known as the **Trace statistic**. In the second case, the alternative hypothesis is that the rank is $r + 1$, and is known as the **Max eigenvalue statistic**.

We perform an optimal lag selection to find the number of lags to use as an input in our Trace and Eigen tests.

Figure 30

Optimal number of lags tests

```
> optimal_lag$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     2      2      2      2
```

The optimal number of lags identified by the AIC, HQ, SC and FPE tests is unanimous as it advises using two lags.

- *Trace test results*

Figure 31

Trace test results

```
> Trace <- ca.jo(model1, type = "trace", ecdet = "const", K = 2, spec="longrun")
> summary(Trace)

######################
# Johansen-Procedure #
######################

Test type: trace statistic , without linear trend and constant in cointegration

Eigenvalues (lambda):
[1] 3.112172e-01 1.074983e-01 8.247602e-02 6.270423e-02 3.328231e-16

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 3 |  7.51  7.52  9.24 12.97
r <= 2 | 17.50 17.85 19.96 24.60
r <= 1 | 30.69 32.00 34.91 41.07
r = 0  | 73.94 49.65 53.12 60.16

Eigenvectors, normalised to first column:
(These are the cointegration relations)

            lSP500.l2      lWTI.l2     lCPI.l2 lIPROD.l2    constant
lSP500.l2   1.0000000  1.000000000  1.00000000  1.000000    1.000000
lWTI.l2     0.1969781  0.002090487 -0.05720252 -1.165099    3.936334
lCPI.l2    -3.8305109 -5.493746812 -5.36662285 -7.342773   43.952488
lIPROD.l2  -2.7281056 -1.783429474  2.87213519  1.020151  -16.165705
constant   22.4719606 26.332403001  4.40972841 26.840532 -155.343873

Weights W:
(This is the loading matrix)

            lSP500.l2      lWTI.l2       lCPI.l2     lIPROD.l2      constant
lSP500.d  0.041515358 -0.068818098 -0.0799628628  0.0017723117  2.168547e-15
lWTI.d   -0.008764795  0.160547366 -0.2118590521  0.0584097197 -5.018639e-14
lCPI.d    0.003207975  0.001525725  0.0002889592  0.0004287667 -6.115515e-16
lIPROD.d  0.009659717  0.059947723 -0.0044409115 -0.0029497249 -1.253953e-14
```

The results from the Trace test suggest that our model rejects the $r = 0$ hypothesis, meaning that there is at least 1 cointegrated relation. However, we fail to reject the $r \leq 1$ hypothesis and all the higher rank hypotheses.

- *Max eigenvalues test results*

Figure 32

Trace test results

```
> Eigen <- ca.jo(model1, type = "eigen", ecdet = "const", K = 2, spec="longrun")
> summary(Eigen)

######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , without linear trend and constant in cointegration

Eigenvalues (lambda):
[1] 3.112172e-01 1.074983e-01 8.247602e-02 6.270423e-02 3.328231e-16

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 3 |  7.51  7.52  9.24 12.97
r <= 2 |  9.98 13.75 15.67 20.20
r <= 1 | 13.19 19.77 22.00 26.81
r = 0  | 43.25 25.56 28.14 33.24

Eigenvectors, normalised to first column:
(These are the cointegration relations)

            lSP500.l2      lWTI.l2     lCPI.l2 lIPROD.l2     constant
lSP500.l2   1.0000000  1.000000000  1.00000000  1.000000     1.000000
lWTI.l2     0.1969781  0.002090487 -0.05720252 -1.165099     3.936334
lCPI.l2    -3.8305109 -5.493746812 -5.36662285 -7.342773    43.952488
lIPROD.l2  -2.7281056 -1.783429474  2.87213519  1.020151   -16.165705
constant   22.4719606 26.332403001  4.40972841 26.840532 -155.343873

Weights W:
(This is the loading matrix)

             lSP500.l2      lWTI.l2       lCPI.l2     lIPROD.l2      constant
lSP500.d   0.041515358 -0.068818098 -0.0799628628  0.0017723117  2.168547e-15
lWTI.d    -0.008764795  0.160547366 -0.2118590521  0.0584097197 -5.018639e-14
lCPI.d     0.003207975  0.001525725  0.0002889592  0.0004287667 -6.115515e-16
lIPROD.d   0.009659717  0.059947723 -0.0044409115 -0.0029497249 -1.253953e-14
```

The results from the Max Eigenvalues test suggest that our model rejects again the $r = 0$ hypothesis, meaning that there is at least 1 cointegrated relation. However, we fail to reject the $r \leq 1$ hypothesis and all the higher rank hypotheses.

This is an encouraging result since it can be interpreted as the existence of a long-run relationship between the stock market under study and at least one of the selected economic variables.

ii. <u>Estimating the VEC Model</u>

Now that we have the proof that a cointegration relation exists in our model, we build a VEC Model in order to find the beta estimates of our model and do some forecasts. Our model is calibrated with the following parameters :

- 2 lags
- 1 cointegration rank
- A constant
- A 2OLS estimation (since we only have 1 cointegration relation, otherwise we would have used the Maximum Likelihood method)

Our estimated VECM can be found in the following figure.

Figure 33

VEC Model estimation

```
> VEC1 <- VECM(model1, 2, r=1, include =("const"), estim=("2OLS"))
> summary(VEC1)
#############
###Model VECM
#############
Full sample size: 118   End sample size: 115
Number of variables: 4  Number of estimated slope parameters 40
AIC -3823.528   BIC -3705.496   SSR 1.686311
Cointegrating vector (estimated by 2OLS):
   lSP500      lWTI      lCPI  lIPROD
r1      1 0.0149889 -4.236656 2.60687


                ECT                Intercept            lSP500 -1          lWTI -1             lCPI -1
Equation lSP500 -0.0559(0.0286).   0.0171(0.0068)*      -0.1613(0.1184)    0.0255(0.0378)      0.1162(3.1625)
Equation lWTI   -0.1728(0.0932).   -0.0079(0.0223)      0.5913(0.3866)     0.1263(0.1234)      8.0147(10.3251)
Equation lCPI   0.0009(0.0009)     0.0009(0.0002)***    0.0021(0.0039)     0.0005(0.0012)      0.5392(0.1039)***
Equation lIPROD -0.0028(0.0097)    -4.5e-05(0.0023)     0.1004(0.0404)*    0.0605(0.0129)***   0.0776(1.0799)
                lIPROD -1          lSP500 -2           lWTI -2            lCPI -2             lIPROD -2
Equation lSP500 -0.0833(0.3055)    -0.2612(0.1180)*    0.0614(0.0387)     -0.9199(3.0111)     0.0741(0.2573)
Equation lWTI   -2.6234(0.9973)**  -0.0504(0.3854)     0.0807(0.1264)     -7.5097(9.8306)     -0.0087(0.8400)
Equation lCPI   0.0022(0.0100)     0.0039(0.0039)      0.0015(0.0013)     -0.0867(0.0989)     -0.0154(0.0085).
Equation lIPROD 0.0423(0.1043)     0.0697(0.0403).     0.0032(0.0132)     -0.7567(1.0281)     -0.0606(0.0879)
```

We are interested in the cointegrated vector estimated by 2OLS, showing a slightly positive relation between the S&P index and WTI oil prices, a negative relation between the S&P index and the CPI index and finally a positive relation between the S&P index and the Industrial Production index.

The model can be then expressed as:

$$\mathrm{SP500}_t = \beta_0 + \ -4.24 * CPI_t + 0.01 * WTI_t + 2.61 * IPROD_t + \ \varepsilon_t \quad \textbf{(1)}$$

Our findings are in line with the literature's results and confirm the hypotheses we made in the data description section.

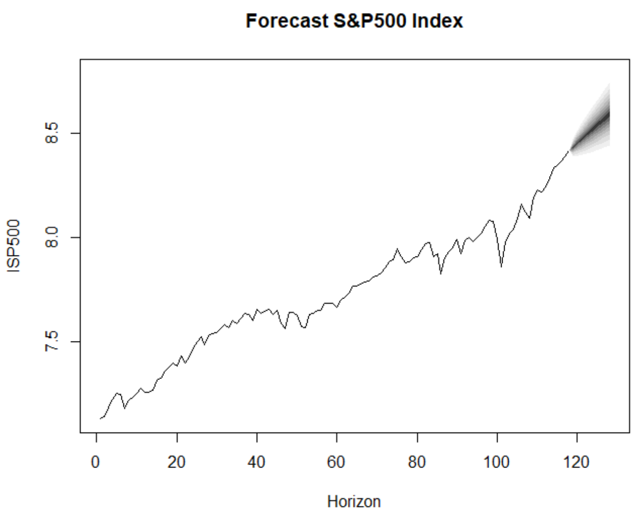
Some robustness checks (serial correlation, ARCH effects and normality, variance decomposition) and the impulse reaction functions can be found in the annex.

iii. Forecasting our model

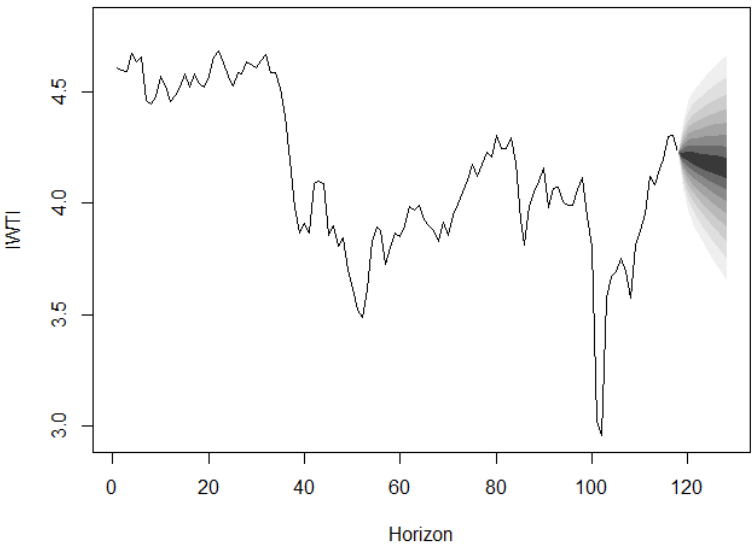
The following figures display the forecasts of our model at a 10 periods horizon.

Figure 34

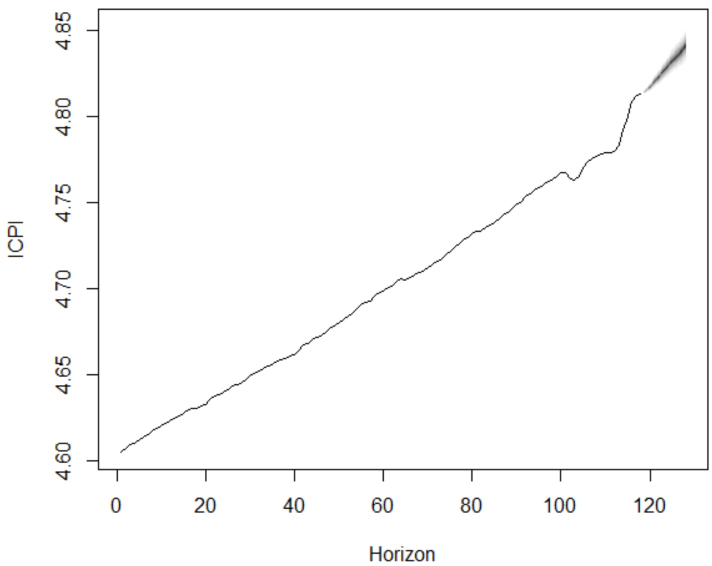
Forecasts estimations



Forecast WTI Oil prices



Forecast CPI



Forecast Industrial Production Index

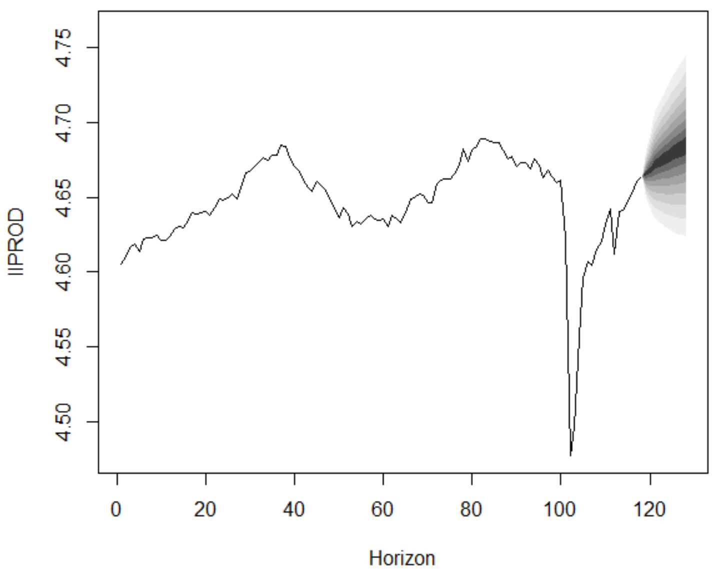


Table 2
Summary of variables – second set

Variable	Description	Units	Data source	Period
SP500	The S&P 500 is regarded as a gauge of the large cap U.S. equities market. The index includes 500 leading companies in leading industries of the U.S. economy, which are publicly held on either the NYSE or NASDAQ	Monthly, End of period Index, Not Seasonally Adjusted	S&P Dow Jones Indices LLC, S&P 500	November 2011 – August 2021
CPI	The Consumer Price Index for all urban consumers: all items less food and energy	Monthly Index Nov 2011 = 100 Seasonally adjusted	U.S. Bureau of Labor Statistics	November 2011 – August 2021
WTI	A crude stream produced in Texas and southern Oklahoma which serves as a reference for pricing a number of other crude streams and which is traded in the domestic spot market at Cushing, Oklahoma.	Monthly, End of period, Dollars per Barrel, Not Seasonally Adjusted	U.S. Energy Information Administration	November 2011 – August 2021
IPROD	The Industrial Production Index measures real output for all U.S. facilities manufacturing, mining, and electric, and gas utilities	Monthly, Index Nov 2011 = 100 Seasonally adjusted	Board of Governors of the Federal Reserve System (U.S.)	November 2011 – August 2021

Notes: All variables are gapless over the November 2011 – August 2021 period. All variables are retrieved from the Federal Reserve Bank of St. Louis' Economic Database. Frequency of the time series is monthly.

Table 3

Summary statistics – Untransformed variables

	N	Mean	S.D.	Min	Max
S&P500 Stock Index	118	2386.4	753.1	1247.0	4522.7
CPI Index	118	109.9	6.2	100.0	123.1
WTI oil price	118	66.0	22.8	19.2	108.0
Industrial Production Index	118	104.3	3.2	88.0	108.8

Notes: The sample includes 118 observations between November 2011 and August 2021. All variables are untransformed.

Table 4

Summary statistics – Log-transformed variables

	N	Mean	S.D.	Min	Max
S&P500 Stock Index	118	7.73	0.31	7.13	8.42
CPI Index	118	4.70	0.06	4.61	4.81
WTI oil price	118	4.13	0.36	2.96	4.68
Industrial Production Index	118	4.65	0.03	4.48	4.69

Notes: The sample includes 118 observations between November 2011 and August 2021. All variables are log-transformed.

ANNEXES – EMPIRICAL APPLICATION 3

Figure 3.1

Untransformed and Log-transformed variables

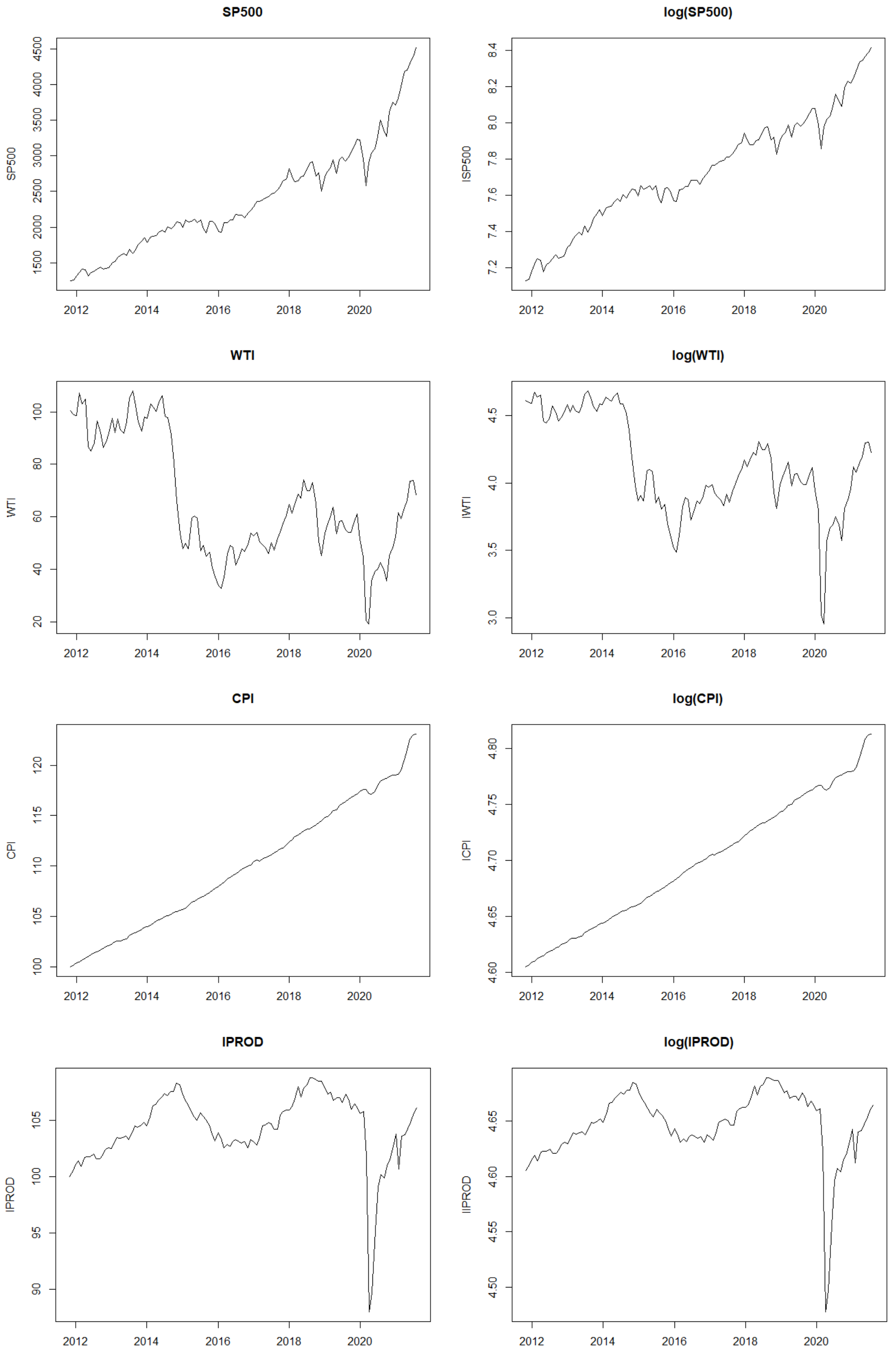
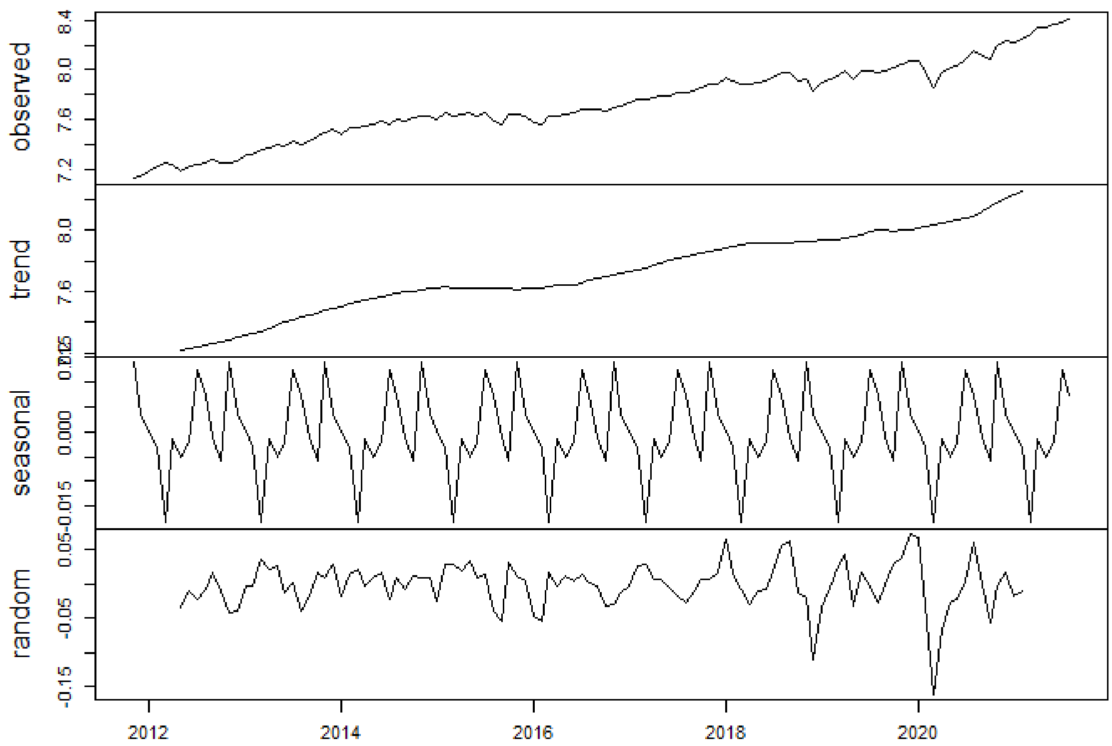


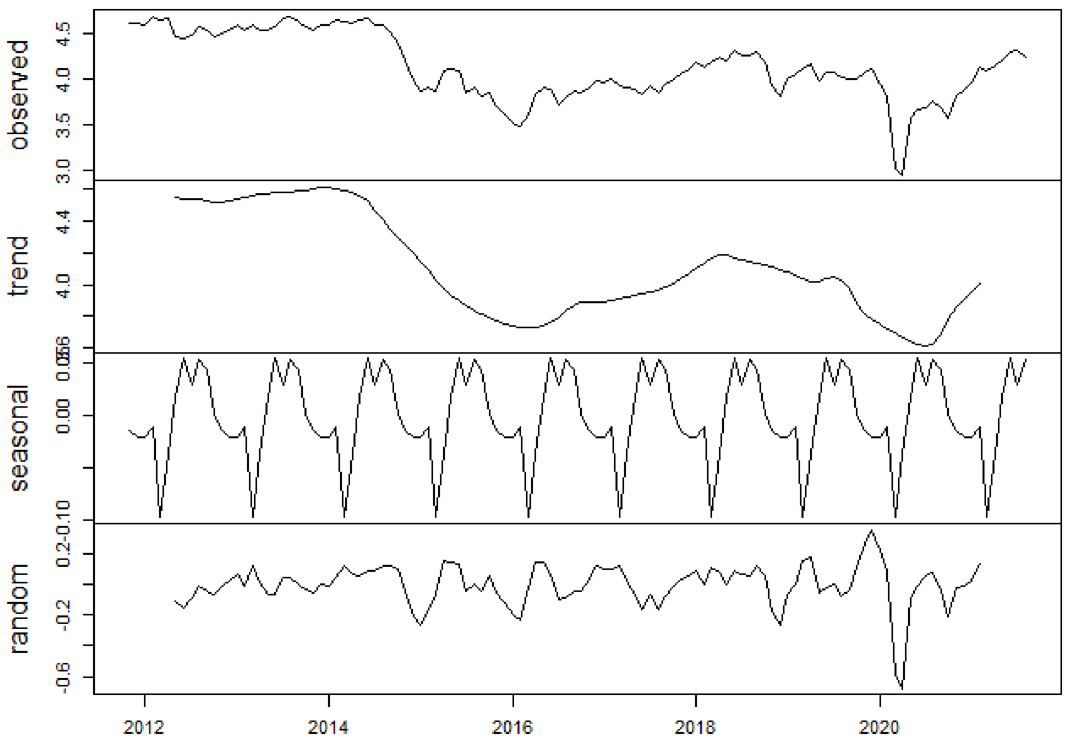
Figure 3.2

Decomposition of the time series

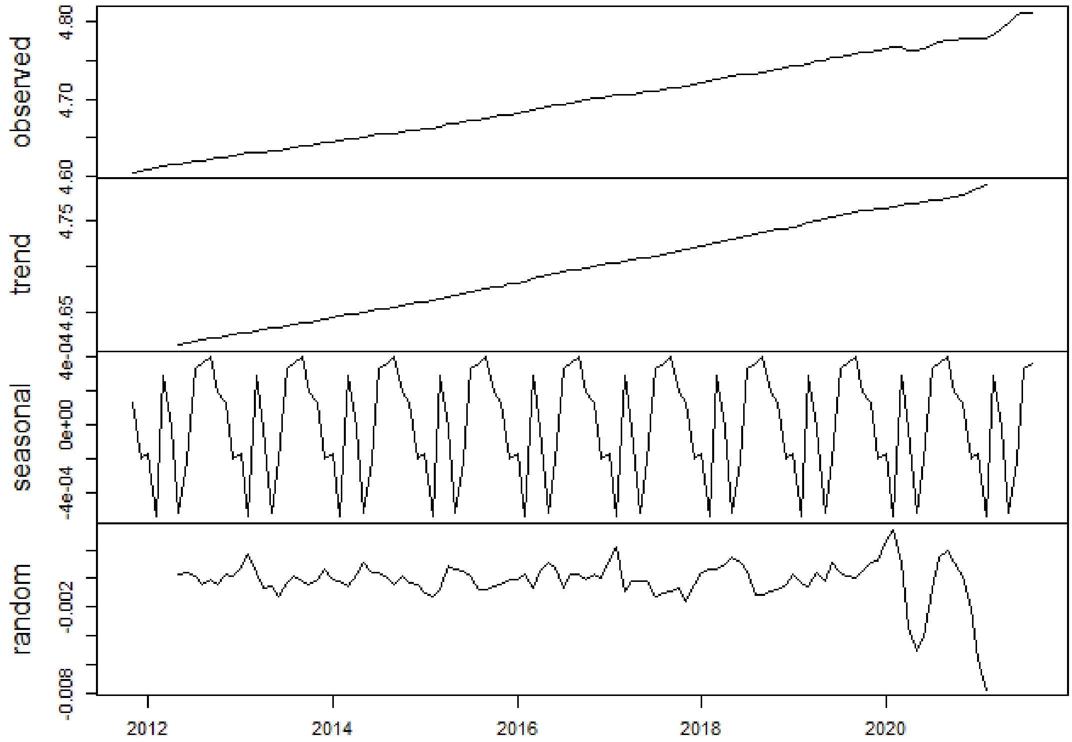
S&P500 Index



WTI Oil prices



CPI Index



Industrial Production Index

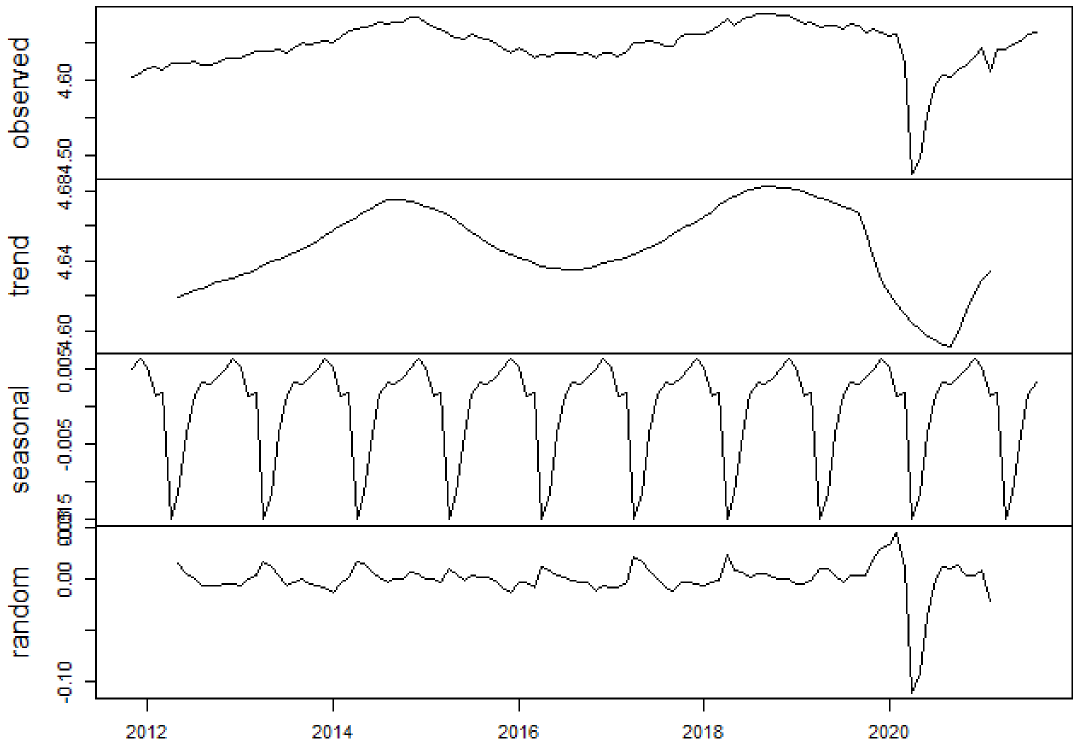


Figure 3.3

Trend tests

S&P500 Index

```
> summary(trend_lSP500)

Call:
tslm(formula = lSP500 ~ trend)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.240111 -0.042249 -0.007034  0.043920  0.168829 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 7.2027017  0.0121884  590.95   <2e-16 ***
trend       0.0088587  0.0001778   49.83   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.06578 on 116 degrees of freedom
Multiple R-squared:  0.9554,	Adjusted R-squared:  0.955 
F-statistic:  2483 on 1 and 116 DF,  p-value: < 2.2e-16
```

WTI Oil prices

```
> summary(trend_lWTI)

Call:
tslm(formula = lWTI ~ trend)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.89518 -0.16905  0.07977  0.19873  0.54895 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.5144991  0.0524335  86.100  < 2e-16 ***
trend       -0.0064985  0.0007648  -8.497 7.61e-14 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.283 on 116 degrees of freedom
Multiple R-squared:  0.3836,	Adjusted R-squared:  0.3783 
F-statistic:  72.2 on 1 and 116 DF,  p-value: 7.608e-14
```

CPI Index

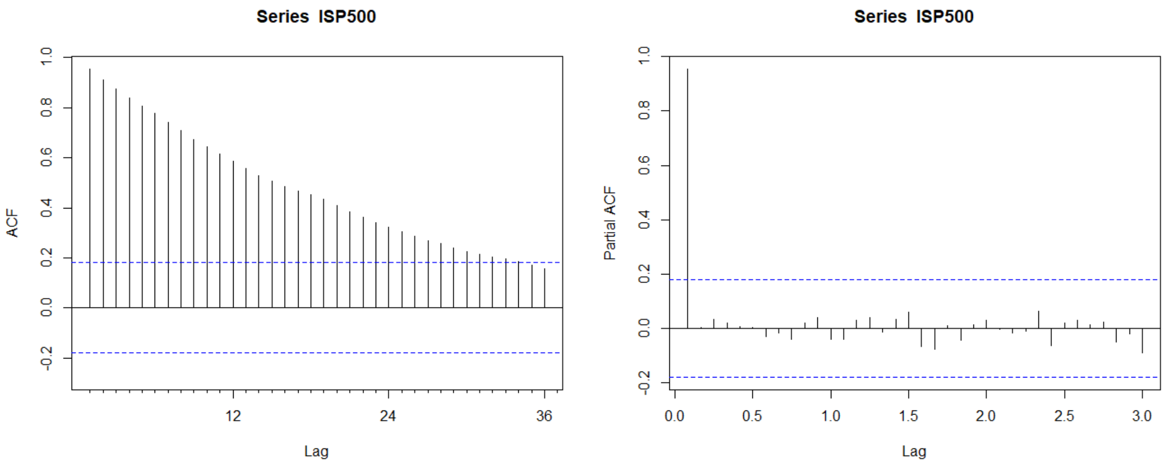
```
> summary(trend_lCPI)

Call:
tslm(formula = lCPI ~ trend)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0069991 -0.0023477 -0.0005271  0.0015819  0.0190797 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 4.600e+00  7.325e-04  6280.6   <2e-16 ***
trend       1.648e-03  1.068e-05   154.3   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.003953 on 116 degrees of freedom
Multiple R-squared:  0.9952,	Adjusted R-squared:  0.9951 
F-statistic: 2.381e+04 on 1 and 116 DF,  p-value: < 2.2e-16
```

Industrial Production Index

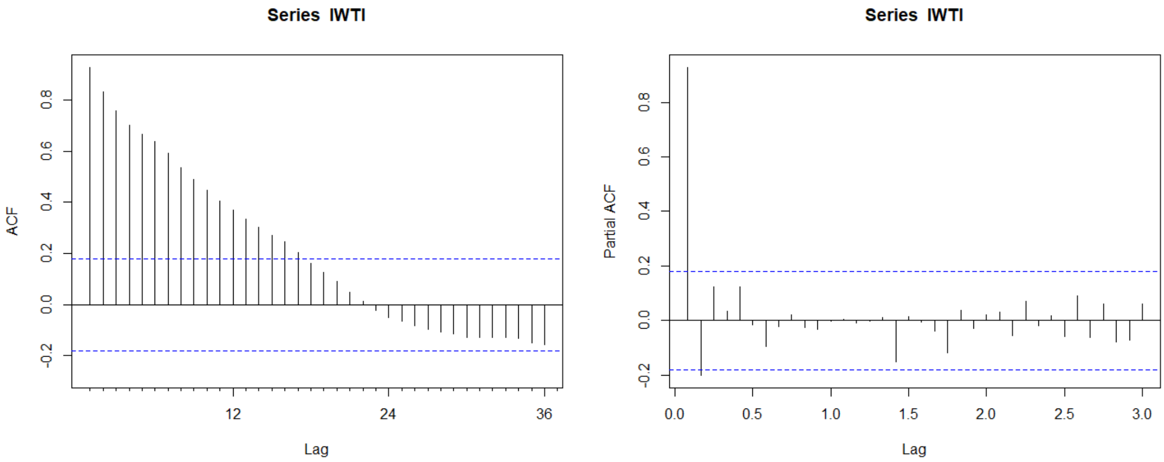
```
> summary(trend_lIPROD)

Call:
tslm(formula = lIPROD ~ trend)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.170254 -0.013499  0.003519  0.021433  0.042446 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 4.645e+00  5.903e-03 786.808   <2e-16 ***
trend       2.625e-05  8.611e-05   0.305    0.761    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.03186 on 116 degrees of freedom
Multiple R-squared:  0.0008005,	Adjusted R-squared:  -0.007813 
F-statistic: 0.09294 on 1 and 116 DF,  p-value: 0.761
```

Figure 3.4
Seasonality identification

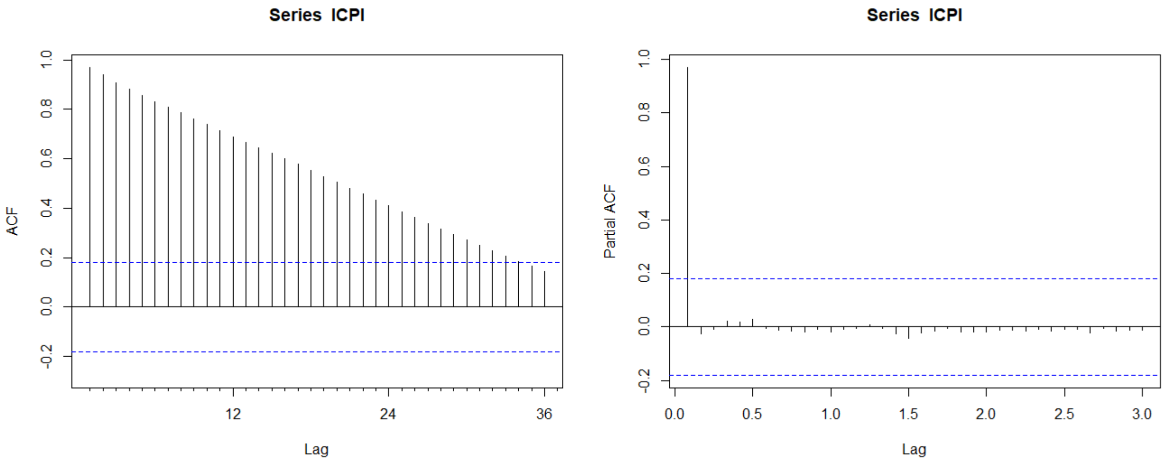
<u>S&P500 Index</u>



<u>WTI Oil prices</u>



<u>CPI Index</u>



Industrial Production Index

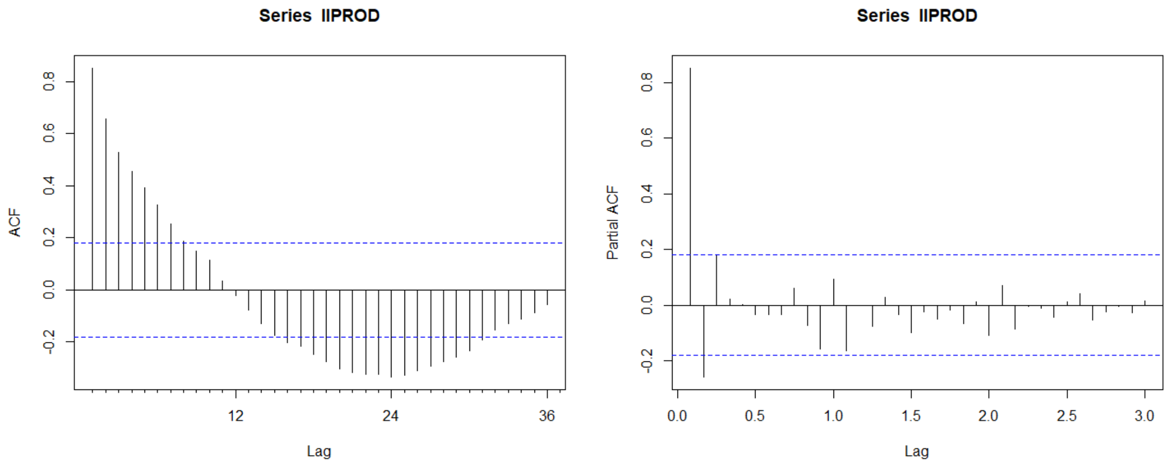


Figure 3.5
Seasonality tests

S&P500 Index

```
> isSeasonal(lSP500)
[1] FALSE
> summary(wo(lSP500))
Test used:  WO

Test statistic:  0
P-value:  1 1 0.8296793

The WO - test does not identify  seasonality
```

WTI Oil prices

```
> isSeasonal(lWTI)
[1] FALSE
> summary(wo(lWTI))
Test used:  WO

Test statistic:  0
P-value:  1 1 0.4818983

The WO - test does not identify  seasonality
```

CPI Index

```
> isSeasonal(lCPI)
[1] FALSE
> summary(wo(lCPI))
Test used:  WO

Test statistic:  0
P-value:  1 1 0.1388282

The WO - test does not identify  seasonality
```

Industrial Production Index

```
> isSeasonal(lIPROD)
[1] FALSE
> summary(wo(lIPROD))
Test used:  WO

Test statistic:  0
P-value:  1 1 0.956444

The WO - test does not identify  seasonality
```

Figure 3.6

Plotted first-differenced log variables

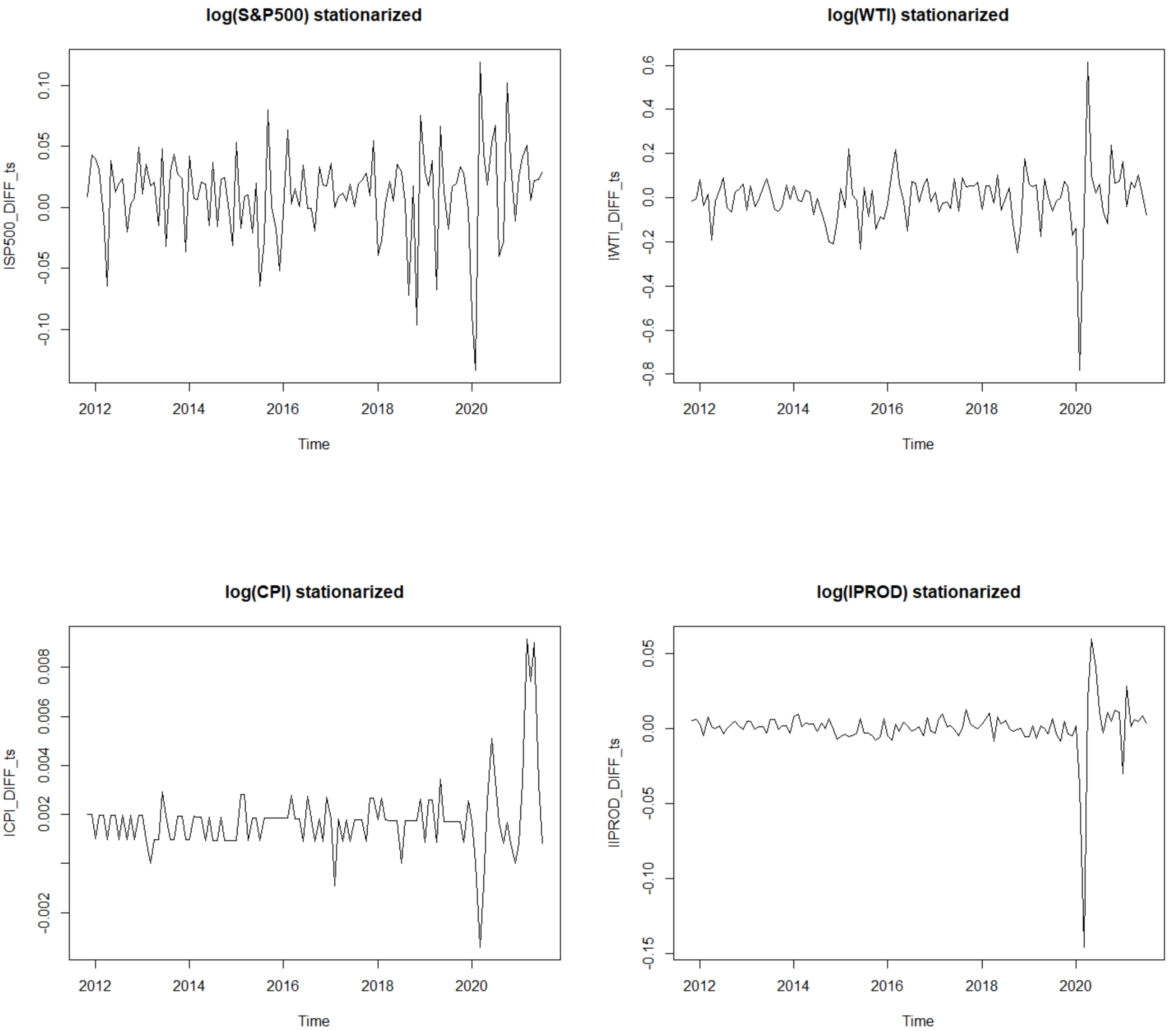


Figure 3.7

Detailed ADF unit root tests

S&P500 Index

```
> ur_lSP500_trend <- ur.df(lSP500, type =  "trend", selectlags =  "AIC")
> summary(ur_lSP500_trend)

###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
      Min        1Q    Median        3Q       Max
-0.161427 -0.013818  0.004853  0.021554  0.079717

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.0635929  0.4102567   2.593   0.0108 *
z.lag.1     -0.1463027  0.0569424  -2.569   0.0115 *
tt           0.0013178  0.0005073   2.597   0.0107 *
z.diff.lag  -0.0169997  0.0967274  -0.176   0.8608
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0367 on 112 degrees of freedom
Multiple R-squared:  0.06578,   Adjusted R-squared:  0.04076
F-statistic: 2.629 on 3 and 112 DF,  p-value: 0.05372


Value of test-statistic is: -2.5693 6.1188 3.3809

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

WTI Oil prices

```
> ur_lWTI_trend <- ur.df(lWTI, type =  "trend", selectlags =  "AIC")
> summary(ur_lWTI_trend)

###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max
-0.75874 -0.05364  0.01499  0.06243  0.51585

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.5477326  0.1962413   2.791  0.00618 **
z.lag.1     -0.1241793  0.0429912  -2.888  0.00465 **
tt          -0.0006434  0.0004544  -1.416  0.15951
z.diff.lag   0.2451067  0.0934233   2.624  0.00991 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1245 on 112 degrees of freedom
Multiple R-squared:  0.101,     Adjusted R-squared:  0.07687
F-statistic: 4.192 on 3 and 112 DF,  p-value: 0.007481


Value of test-statistic is: -2.8885 2.9174 4.3487

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

CPI Index

```
> ur_lCPI_trend <- ur.df(lCPI, type =  "trend", selectlags =  "AIC")
> summary(ur_lCPI_trend)

###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
       Min         1Q     Median         3Q        Max
-0.0040451 -0.0006692 -0.0001565  0.0006266  0.0059071

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.030e-01  1.734e-01   2.901  0.00447 **
z.lag.1     -1.093e-01  3.770e-02  -2.898  0.00452 **
tt           1.826e-04  6.147e-05   2.970  0.00364 **
z.diff.lag   6.495e-01  9.366e-02   6.935 2.76e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.001216 on 112 degrees of freedom
Multiple R-squared:  0.3395,    Adjusted R-squared:  0.3218
F-statistic: 19.19 on 3 and 112 DF,  p-value: 4.11e-10


Value of test-statistic is: -2.8981 10.6851 5.1269

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

Industrial Production Index

```
> ur_lIPROD_trend <- ur.df(lIPROD, type =  "trend", selectlags =  "AIC")
> summary(ur_lIPROD_trend)

###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
      Min        1Q    Median        3Q       Max
-0.139698 -0.002966  0.001406  0.005547  0.030223

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  8.473e-01  2.233e-01   3.795 0.000240 ***
z.lag.1     -1.822e-01  4.805e-02  -3.792 0.000242 ***
tt          -5.151e-06  4.387e-05  -0.117 0.906741
z.diff.lag   2.702e-01  9.041e-02   2.988 0.003448 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01582 on 112 degrees of freedom
Multiple R-squared:  0.1438,    Adjusted R-squared:  0.1208
F-statistic: 6.269 on 3 and 112 DF,  p-value: 0.000568


Value of test-statistic is: -3.7921 4.8227 7.2006

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

Figure 3.8

Serial correlation test

```
> VAR1 <- vec2var(Trace, r=1)
>
>   # Serial Correlation
> SerCorr <- serial.test(VAR1, lags.pt = 2, type = "PT.asymptotic")
> SerCorr

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object VAR1
Chi-squared = 22.668, df = 4, p-value = 0.0001475
```

Figure 3.9

ARCH effects test

```
>   #ARCH effects
> Arch <- arch.test(VAR1, lags.multi = 15, multivariate.only = TRUE)
> Arch

        ARCH (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 1010, df = 1500, p-value = 1
```

Figure 3.10

Normality, multivariate skewness and kurtosis tests

```
>   #normality
> Normal <- normality.test(VAR1, multivariate.only = TRUE)
> Normal
$JB

        JB-Test (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 507.59, df = 8, p-value < 2.2e-16


$Skewness

        Skewness only (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 98.935, df = 4, p-value < 2.2e-16


$Kurtosis

        Kurtosis only (multivariate)

data:  Residuals of VAR object VAR1
Chi-squared = 408.66, df = 4, p-value < 2.2e-16
```

Figure 3.11

Impulse Reaction function of the CPI Index to the S&P500 Index

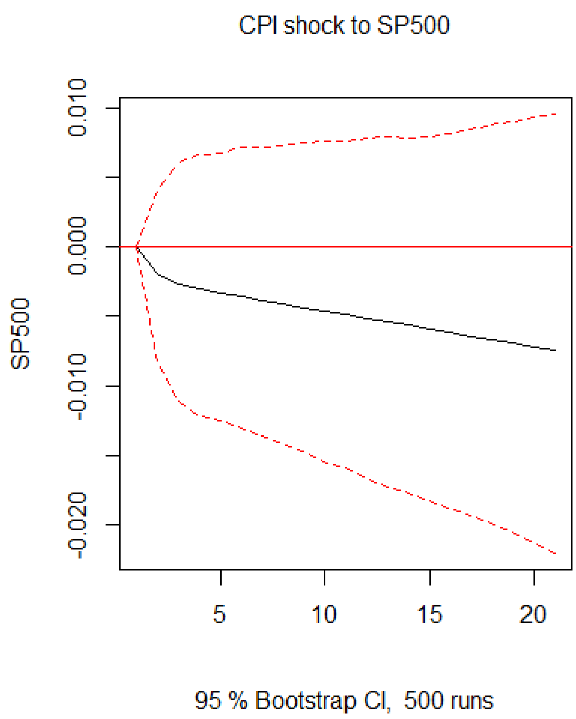


Figure 3.12

Impulse Reaction function of the WTI Oil prices to the S&P500 Index

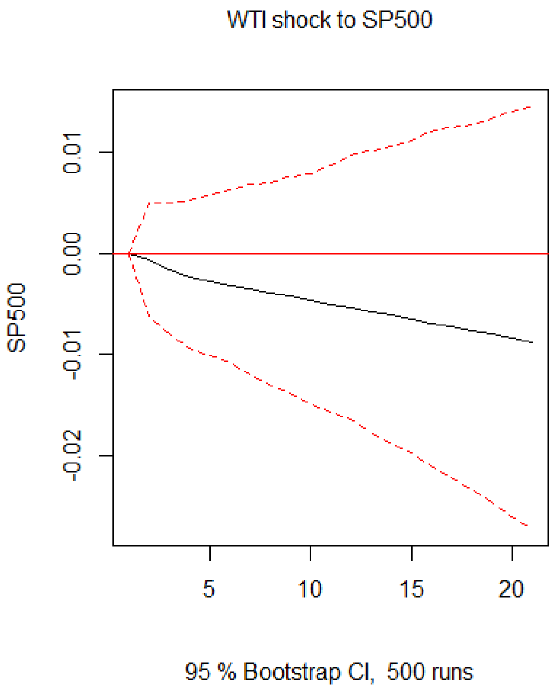


Figure 3.13

Impulse Reaction function of the Industrial Production Index to the S&P500 Index

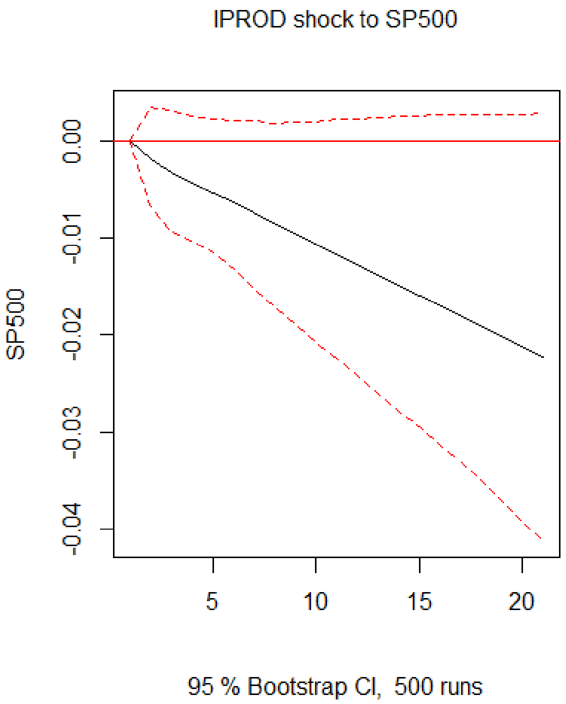
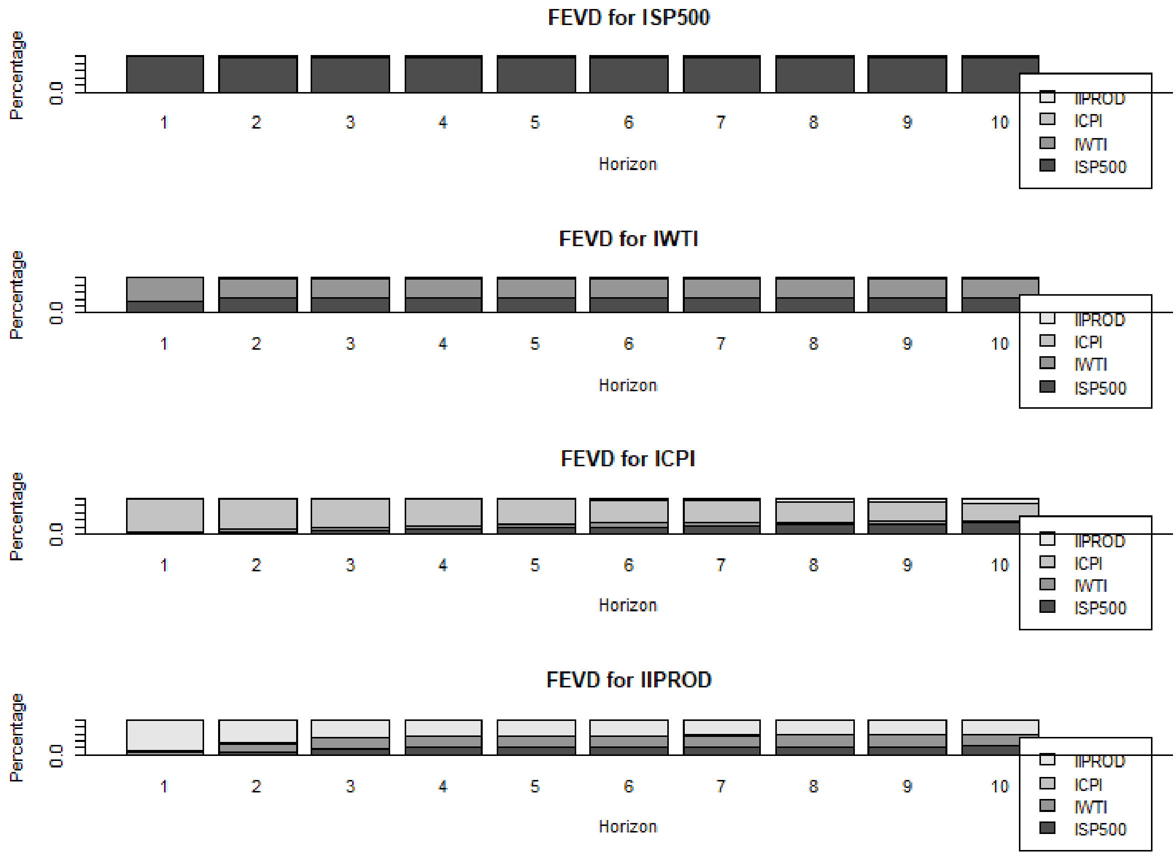


Figure 3.14

Variance decomposition


FEVD for ISP500
FEVD for IWTI
FEVD for ICPI
FEVD for IIPROD
Percentage
0.0
Horizon
1
2
3
4
5
6
7
8
9
10
IIPROD
ICPI
IWTI
ISP500