Master 2 Financial Economics – Université Paris 1 Panthéon-Sorbonne

Empirical Application 2

Financial Econometrics

Roland BOUILLOT
(Roland.Bouillot@etu.univ-paris1.fr)

Khalil JANBEK
(Khalil.Janbek@etu.univ-paris1.fr)

Mehdi LOUAFI
(Mehdi.Louafi@etu.univ-paris1.fr)

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EMPIRICAL APPLICATION 2

From the empirical application 1, we have chosen three financial series: the 5-year breakeven inflation expectations (*Breakeven5Y*), the S&P500 stock index (*SP500*) and the WTI oil prices (*WTI*). The times series span over 119 monthly observations, ranging from November 2011 to August 2021. The summary of the aforementioned variables can be found in Table 1.

In this second empirical application, we will reuse the variables to estimate a VAR model, conduct some robustness checks on our model and predict each of the series at horizon 2.

Question 1: Estimate a VAR model

Following the work done in the first applications, we use the de-trended and transformed variables to avoid possible stochastic trend issues. We first determine the lag order by running an Akaike (AIC), a Hannan–Quinn (HQ), a Schwarz (SC) and a Final prediction error (FPE) lag selection tests.

R-Code 34

AIC, HQ, SC and FPE lag selection tests for our VAR model

```
> lagselect$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     2      2      1      2
```

As three out of four tests recommend a 2 lags order, we decide to calibrate our model with 2 lags. Now that we have the optimal lag order for our VAR model, we can estimate it. The results are shown in the following R-Code boxes. We choose to include a constant per equation, as the only type of exogenous variable.

R-Code 35

VAR estimation results

```
> summary(Model1)

VAR Estimation Results:
=========================
Endogenous variables: Breakeven5Y, SP500, WTI
Deterministic variables: const
Sample size: 116
Log Likelihood: 427.086
Roots of the characteristic polynomial:
0.5357 0.5357 0.5167 0.5022 0.5022 0.4184
Call:
VAR(y = var1, p = 2, type = "const", exogen = NULL)
```

Note that the roots of the characteristic polynomial are inside the unit circle, meaning that our model is quite stable. Therefore, tests conducted on our VAR model, including impulse-response standard errors, can reasonably be expected to be valid.

R-Code 36

VAR estimation results for the Inflation Expectations variable

```
Estimation results for equation Breakeven5Y:
============================================
Breakeven5Y = Breakeven5Y.l1 + SP500.l1 + WTI.l1 + Breakeven5Y.l2 + SP500.l2 + WTI.l2 + const

               Estimate Std. Error t value Pr(>|t|)
Breakeven5Y.l1  0.071933   0.126960   0.567    0.572
SP500.l1        0.159591   0.523230   0.305    0.761
WTI.l1          0.205795   0.169888   1.211    0.228
Breakeven5Y.l2  0.133426   0.133053   1.003    0.318
SP500.l2       -0.744423   0.527327  -1.412    0.161
WTI.l2         -0.184937   0.138634  -1.334    0.185
const           0.007555   0.014963   0.505    0.615


Residual standard error: 0.1606 on 109 degrees of freedom
Multiple R-Squared: 0.06642,    Adjusted R-squared: 0.01503
F-statistic: 1.293 on 6 and 109 DF,  p-value: 0.2667
```

Coefficients for the 1st lag of the 5-year breakeven inflation rate, the S&P 500 and the WTI appear to have the expected sign (we can reasonably assume that those three regressors have a positive relationship with expected inflation). However, the 2nd lag of the S&P500 and the WTI have an unexpectedly negative sign. Moreover, quite surprisingly, neither the WTI, the S&P500 and their lagged variables are statistically significant for explaining variations in the 5-year breakeven inflation rate, as all p-values exceed by far the 10% significance level. The same goes for the lagged values of the 5-year breakeven inflation rate. This explains the very low adjusted-R^2 for this equation – whose analysis is valid in the present case, as this VAR model is estimated through OLS.

Nonetheless, as highlighted by Stock and Watson (2001)[1], due to the "complicated dynamics in the VAR, [Granger-causality tests, impulse responses, and forecast error variance decompositions] are more informative than the estimated VAR regression coefficients or R^2s, which typically go unreported."

Therefore, although those results appear to give no meaningful statistical conclusion, we do not need to conclude that our model is wrongly specified. We rather need to turn our focus to the tests mentioned above.

[1] J. Stock and M. Watson, Vector Autoregressions, Journal of Economic Perspectives – Volume 15, p. 104 / 115

R-Code 37

VAR estimation results for the Stock Index variable

```
Estimation results for equation SP500:
======================================
SP500 = Breakeven5Y.l1 + SP500.l1 + WTI.l1 + Breakeven5Y.l2 + SP500.l2 + WTI.l2 + const

                Estimate Std. Error t value Pr(>|t|)
Breakeven5Y.l1  0.012050   0.029355   0.410   0.6823
SP500.l1       -0.207923   0.120978  -1.719   0.0885 .
WTI.l1          0.056589   0.039280   1.441   0.1525
Breakeven5Y.l2 -0.031217   0.030763  -1.015   0.3125
SP500.l2       -0.214691   0.121925  -1.761   0.0811 .
WTI.l2          0.020214   0.032054   0.631   0.5296
const           0.002681   0.003460   0.775   0.4401
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.03714 on 109 degrees of freedom
Multiple R-Squared: 0.06896,    Adjusted R-squared: 0.01771
F-statistic: 1.345 on 6 and 109 DF,  p-value: 0.2433
```

As expected, lagged values of the S&P500 are statistically significant at the 10% level for explaining the S&P500. Nonetheless, the negative sign on those coefficients begs further analysis of determinants of S&P500 variations. As we are here primarily interested in explaining variations in the 5-year breakeven inflation rate, we consider that conducting such an analysis would be off-topic. None of the other variables and their lags are statistically significant in this equation, with p-values exceeding the 10% significance level. This explains the very low adjusted-R^2 for this equation of the VAR.

Nonetheless, as highlighted by Stock and Watson (2001), due to the “complicated dynamics in the VAR, [Granger-causality tests, impulse responses, and forecast error variance decompositions] are more informative than the estimated VAR regression coefficients or R^2s, which typically go unreported.”

Therefore, although those results appear to give no meaningful statistical conclusion, we do not need to conclude that our model is wrongly specified. We rather need to turn our focus to the tests mentioned above.

R-Code 38

VAR estimation results for the WTI variable

```
Estimation results for equation WTI:
====================================
WTI = Breakeven5Y.l1 + SP500.l1 + WTI.l1 + Breakeven5Y.l2 + SP500.l2 + WTI.l2 + const

                Estimate Std. Error t value Pr(>|t|)    
Breakeven5Y.l1  0.296592   0.078661   3.770 0.000265 ***
SP500.l1        0.665133   0.324180   2.052 0.042592 *  
WTI.l1          0.105799   0.105258   1.005 0.317056    
Breakeven5Y.l2 -0.047346   0.082436  -0.574 0.566925    
SP500.l2        0.153389   0.326718   0.469 0.639662    
WTI.l2         -0.242539   0.085894  -2.824 0.005646 ** 
const           0.000649   0.009271   0.070 0.944315    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.09953 on 109 degrees of freedom
Multiple R-Squared: 0.3778,     Adjusted R-squared: 0.3436 
F-statistic: 11.03 on 6 and 109 DF,  p-value: 1.391e-09
```

Estimation results for this equation are again quite surprising, as it seems that the first lags of the 5-year breakeven inflation rate and of the S&P500 are statistically significant (at the 0% and 1% levels respectively), as well as the 2^{nd} lag of the WTI (significant at the 0.1% level). Coefficient signs are positive for the 5-year breakeven inflation rate 1^{st} lag and for the S&P500 1^{st} lag. However, we cannot comment on those signs, as there is no economic interpretation that can be formulated in this case. Interestingly however, the second lag of the WTI is statistically significant in this equation, with a negative sign. This equation displays the highest adjusted-R^2 for this equation of the VAR. However, while the model offers some statistical insights on the relationship among the three variables at hand, it offers no meaningful economic insight.

Nonetheless, as highlighted by Stock and Watson (2001), due to the "complicated dynamics in the VAR, [Granger-causality tests, impulse responses, and forecast error variance decompositions] are more informative than the estimated VAR regression coefficients or R^2s, which typically go unreported."

Therefore, although those results appear to give no meaningful statistical conclusion, we do not need to conclude that our model is wrongly specified. We rather need to turn our focus to the tests mentioned above.

R-Code 39

VAR covariance matrix of residuals results

```
Covariance matrix of residuals:
             Breakeven5Y    SP500      WTI
Breakeven5Y     0.025805 0.003823 0.007847
SP500           0.003823 0.001380 0.001576
WTI             0.007847 0.001576 0.009906

Correlation matrix of residuals:
             Breakeven5Y  SP500    WTI
Breakeven5Y       1.0000 0.6408 0.4908
SP500             0.6408 1.0000 0.4263
WTI               0.4908 0.4263 1.0000
```

The model has been estimated, yet we also want to check for its robustness. We conduct a set of tests to identify serial correlation, heteroscedasticity, structural breaks and the normal distribution of residuals.

i. Serial correlation

R-Code 40

Serial correlation test for our VAR model

```
> Serial1 <- serial.test(Model1, lags.pt = 12, type = "PT.asymptotic")
> Serial1

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object Model1
Chi-squared = 94.809, df = 90, p-value = 0.3439
```

Autocorrelation or serial correlation is an undesired trait in a VAR model since it biases the estimators. In the conducted portmanteau test, the null hypothesis is :

$$H_0 : serial\ correlation\ is\ not\ present$$

The obtained p-value for our model is greater than the significance level of 0.05, meaning the null hypothesis cannot be rejected. We are satisfied with this result and move on to the next robustness test.

ii. Heteroscedasticity

R-Code 41

Heteroscedasticity test for our VAR model

```
> arch1 <- arch.test(Model1, lags.multi = 12, multivariate.only = T)
> arch1

        ARCH (multivariate)

data:  Residuals of VAR object Model1
Chi-squared = 447.78, df = 432, p-value = 0.2901
```

Heteroscedasticity in a time series takes the form of ARCH effects. To identify them, we run an ARCH test and find that we cannot reject the null hypothesis:

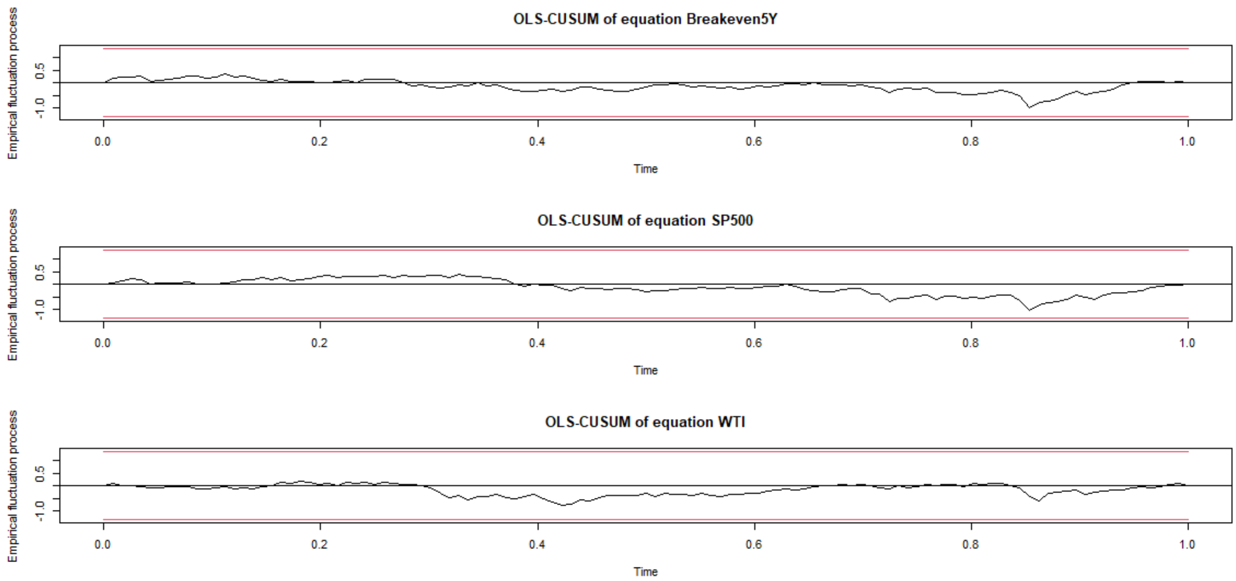
$$H_0 : heteroscedasticity\ is\ not\ present$$

since the p-value is greater than the significance level of 0.05, which is a good result for our model.

iii. Structural breaks

Figure 17

Structural breaks test for our VAR model



Testing for structural breaks means testing the stability of our model. As displayed in Figure 17, there are no fluctuations that go out-of-bounds, represented by the red lines, meaning no structural breaks are detected in our model.

iv. Normal distribution of residuals

R-Code 42

Normal distribution of residuals test for our VAR model

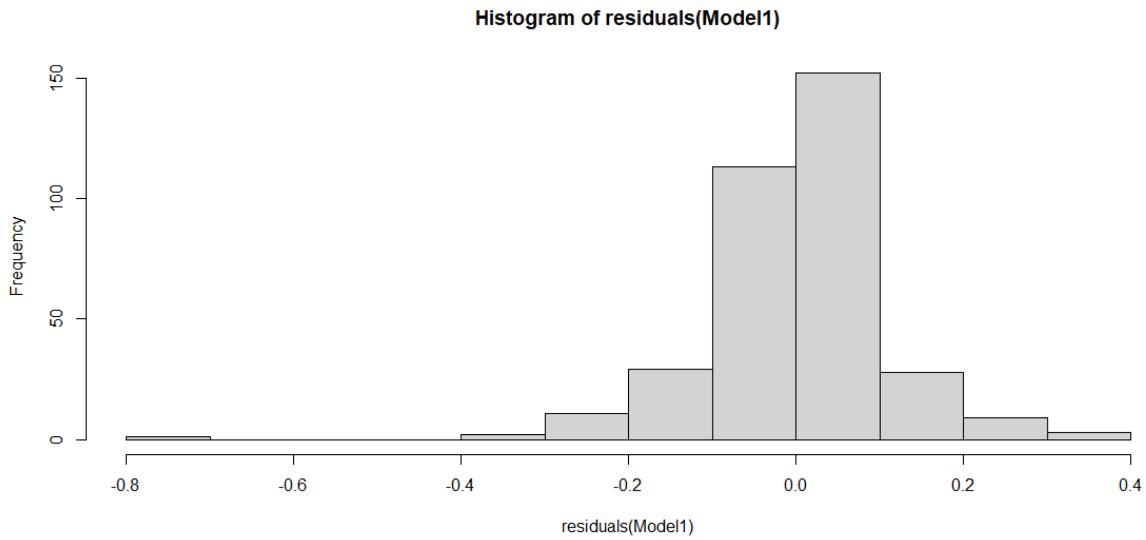
```
> shapiro.test(residuals(Model1))

        Shapiro-Wilk normality test

data:  residuals(Model1)
W = 0.9099, p-value = 1.474e-13
```

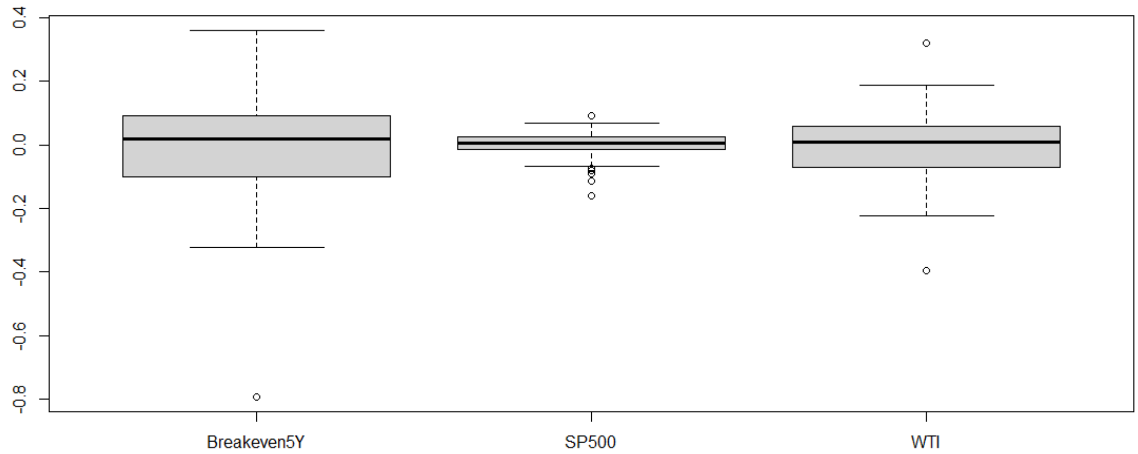
Finally, we check for the normal distribution of the residuals. We perform a Shapiro-Wilk normality test which seems to detect abnormally distributed residuals. However, relying on the Central Limit Theorem, we expect our sample to be large enough to have a normal distribution of its residuals and suspect that a large number of observations may distort the results of the Shapiro-Wilk test.

To support our hypothesis of the normality of our model's residuals, we check for their plotted distribution in Figure 18. As displayed, the vast majority of our residuals seem to be normally distributed. We conclude by assuming that our model has validated this test.

Figure 18

Distribution of residuals of our VAR model





Our model has been estimated and its robustness has been tested successfully. It is time to forecast our time series at horizon 2.

Question 2: Predict each of the series at horizon 2

Before we forecast each of our series at horizon 2 with our calibrated VAR model, we check for Granger causality between our variables. Then we plot the forecasts at horizon 2. Finally, we present the Impulse Reaction Function of each variable.

i. Granger causality tests

As our data is stationary and will be used for forecasting purposes, we want to identify Granger-causality between our model's variables. Testing for Granger causality examines whether lagged values of one variable helps to forecast the two other variables in the model. In this case, the null hypothesis is :

$$H_0 : X \ does \ not \ Granger \ cause \ Y$$

The results are displayed in the following R-Code boxes below.

R-Code 43

Granger causality test for the Inflation Expectations variable

```
> GrangerBRK <- causality(Model1, cause = 'Breakeven5Y')
> GrangerBRK
$Granger

        Granger causality H0: Breakeven5Y do not Granger-cause SP500 WTI

data:  VAR object Model1
F-Test = 4.3173, df1 = 4, df2 = 327, p-value = 0.00204
```

For the Inflation Expectations variable, the Granger causality test is conclusive as the p-value is lower than the 1% critical value threshold, meaning that inflation expectations are useful to predict both the stock index and the WTI variables in our VAR model.

R-Code 44

Granger causality test for the Stock Index variable

```
> GrangerSP500 <- causality(Model1, cause = 'SP500')
> GrangerSP500$Granger

        Granger causality H0: SP500 do not Granger-cause Breakeven5Y WTI

data:  VAR object Model1
F-Test = 1.9844, df1 = 4, df2 = 327, p-value = 0.09659
```

For the Stock Index variable, the Granger causality test is less conclusive as the p-value is just lower than the 10% critical value threshold, meaning that the S&P500 stock index is quite useful to predict both the inflation expectations and the WTI variables in our VAR model.

R-Code 45

Granger causality test for the WTI variable

```
> GrangerWTI <- causality(Model1, cause = 'WTI')
> GrangerWTI$Granger

        Granger causality H0: WTI do not Granger-cause Breakeven5Y SP500

data:  VAR object Model1
F-Test = 1.9352, df1 = 4, df2 = 327, p-value = 0.1043
```

For the WTI variable, the Granger causality test is inconclusive as the p-value is above the 10% critical value threshold, meaning that the WTI oil prices are not very useful to predict both the inflation expectations and the stock index variables in our VAR model.

ii. <u>Forecast at horizon 2 for each variable</u>

Using our calibrated VAR model, we can forecast each of our three financial variables at horizon 2. The plotted results are shown in the following figures.

Figure 19

Forecast of each variable at horizon 2

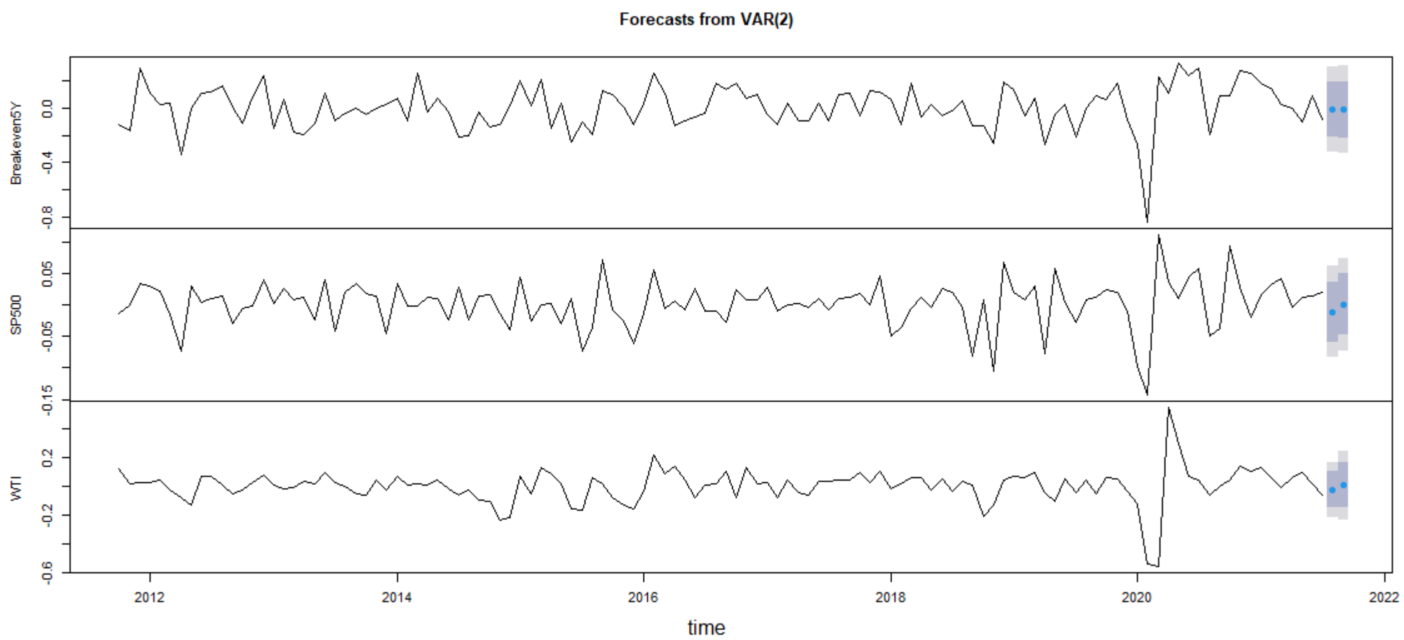


Figure 20

Forecast of the Inflation Expectations variable at horizon 2

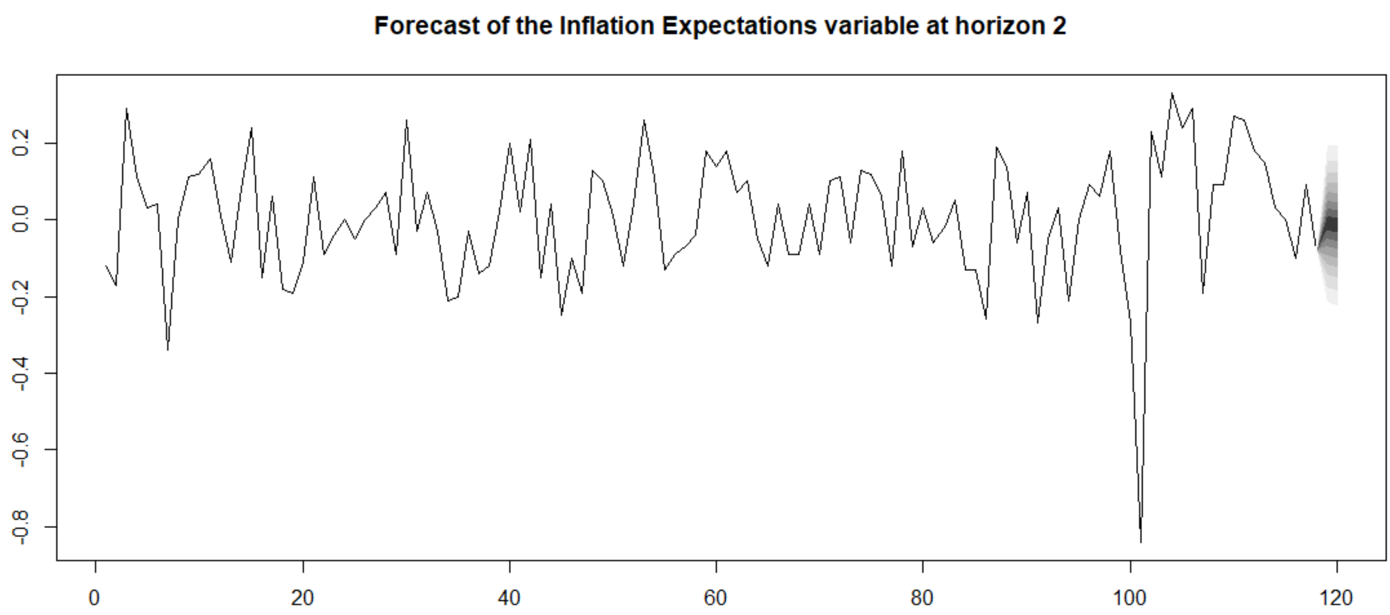


Figure 21

Forecast of the S&P500 Stock Index variable at horizon 2

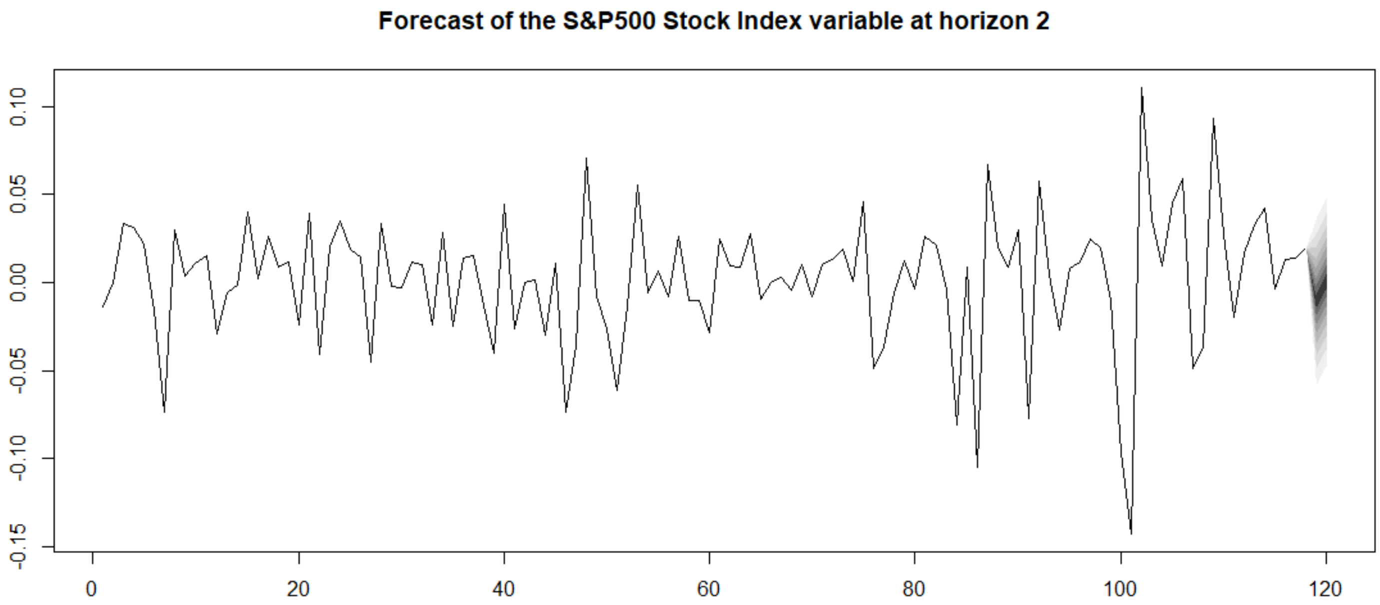
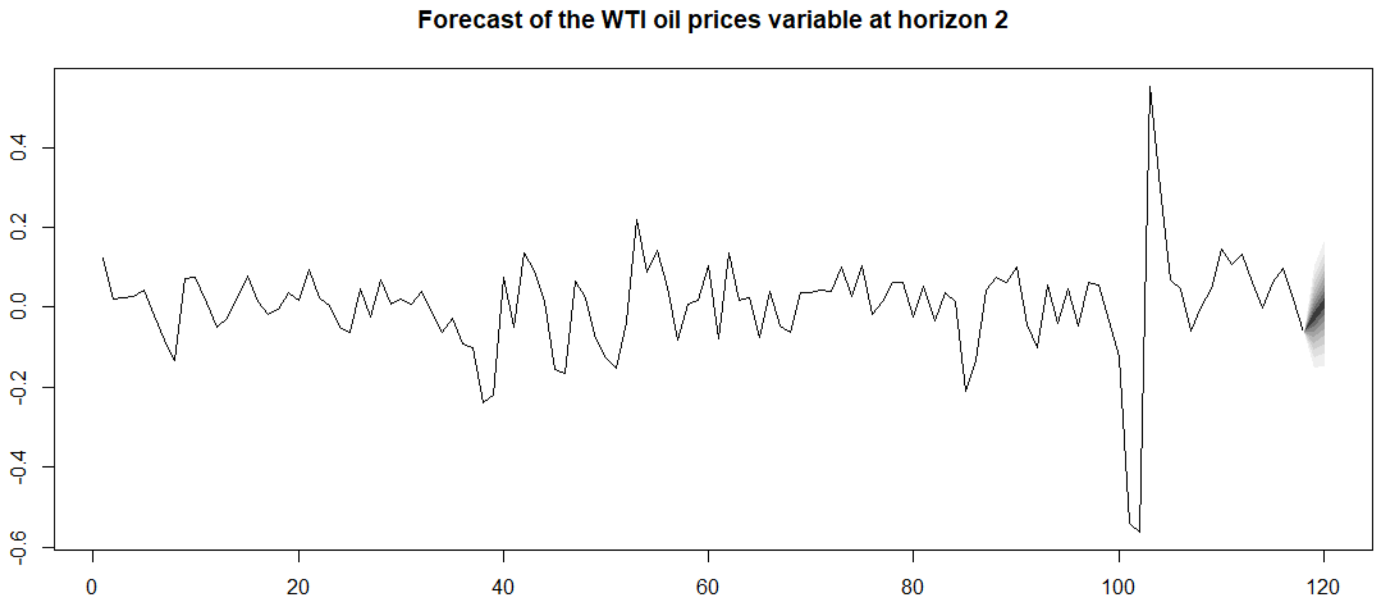


Figure 22

Forecast of the WTI oil prices variable at horizon 2



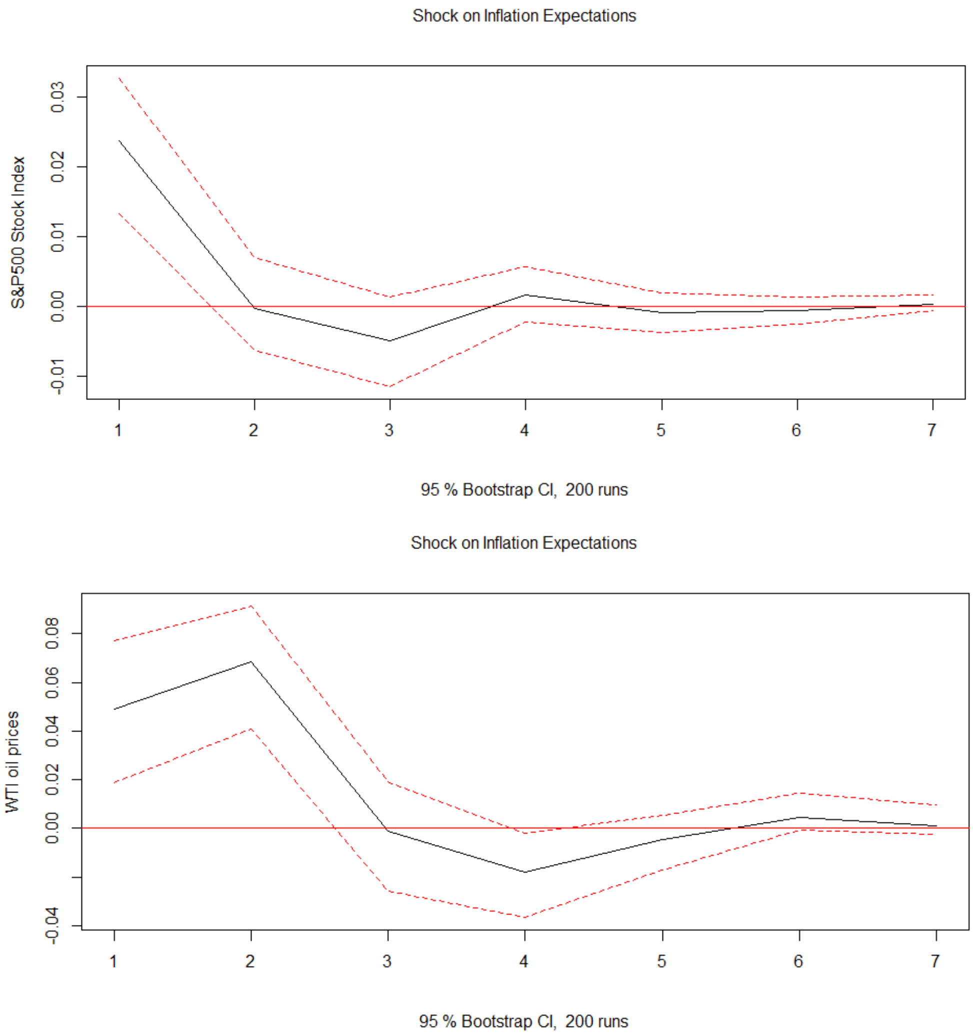
iii. <u>Impulse reaction functions of each variable</u>

As described by Stock and Watson (2001), impulse-responses "trace out the response of current and future values of each of the variables to a one-unit increase in the current value of one of the VAR errors, assuming that this error returns to zero in subsequent periods and that all other errors are equal to zero".

For each variable, we therefore predict the impulse reaction function (IRF) on the two remaining variables. We calibrate our IRFs for predicting the effects of the shock 6 months ahead, running 200 estimations, and delimiting the interval confidence at 95%. The results can be found in the following figures.

Figure 23

Impulse reaction function of a shock on Inflation Expectations on the Stock Index and the WTI variables

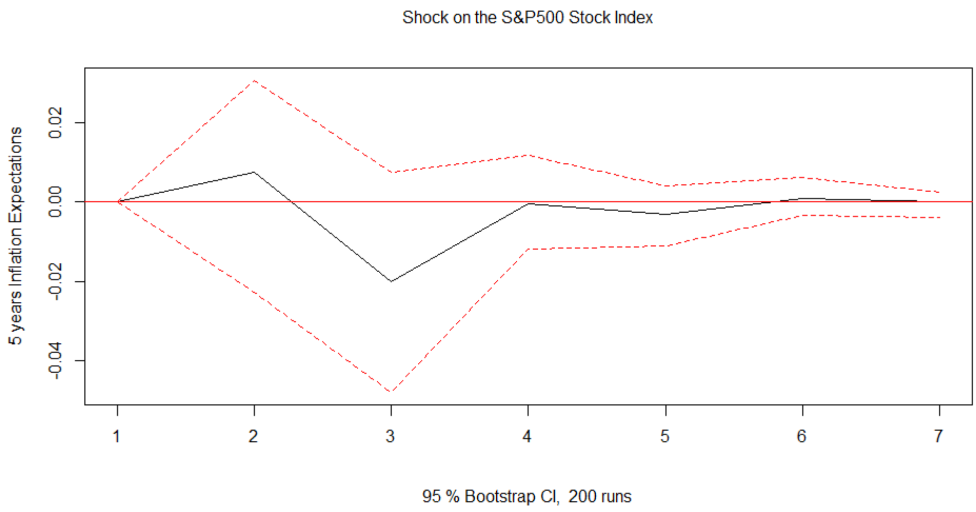


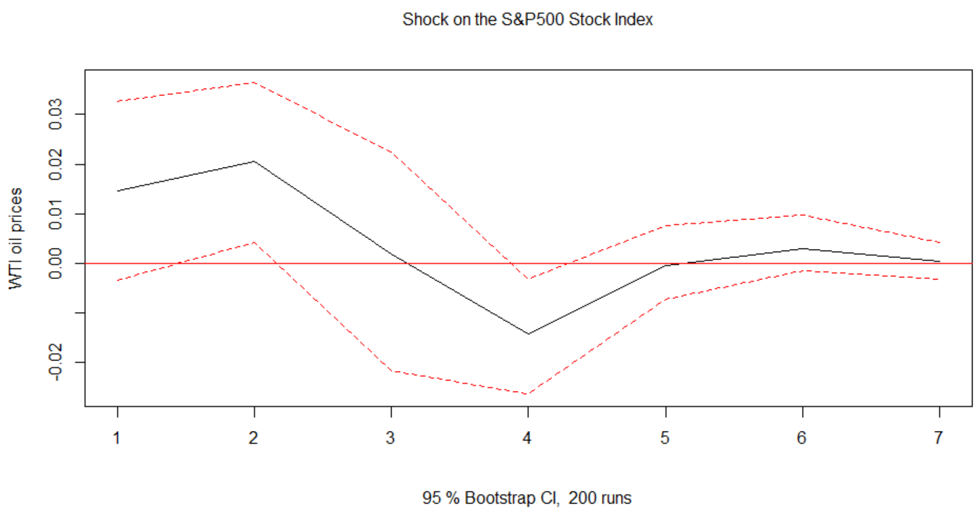
The impact of a contemporaneous shock on inflation expectations on S&P index and on WTI oil prices is positive in the very short run and gradually decreases (even reaching negative territory after 2 and 3 periods respectively) and finally disappears after 6 periods.

We do not predict shocks beyond 6 periods since we know that our VAR model is stationary and stationarity implies that the impact of a shock will quickly decay to 0. This is the case for this forecast, as shown by the curve which reverts to 0.

Figure 24

Impulse reaction function of a shock on the Stock Index on the Inflation expectations and the WTI variables



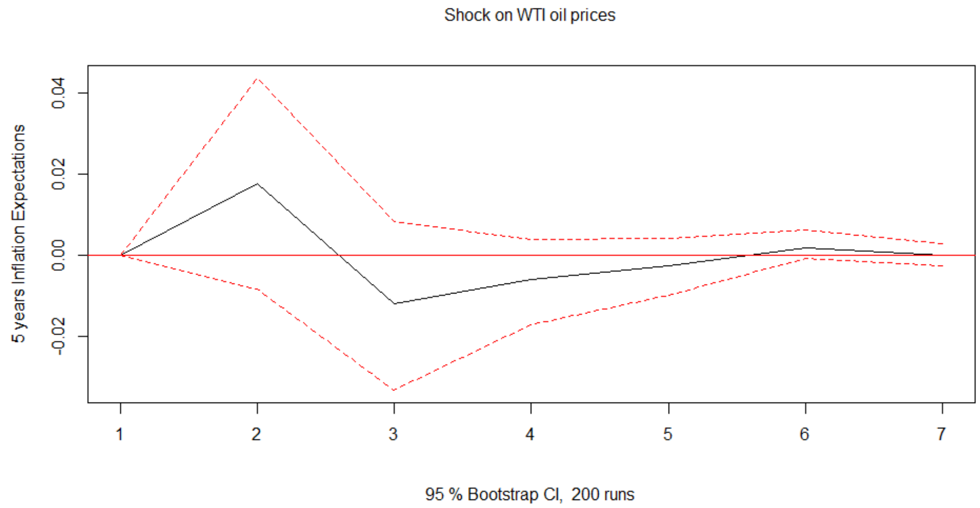


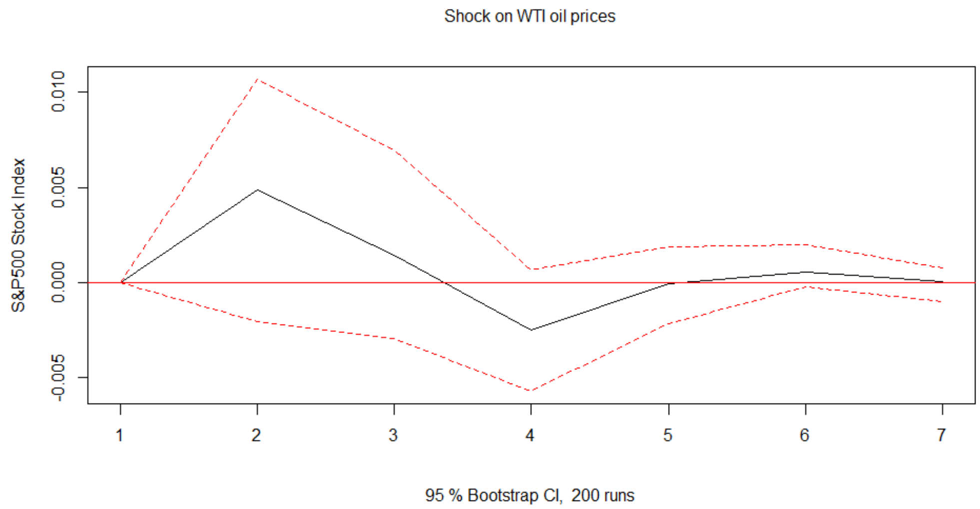
The impact of a contemporaneous shock on S&P index on 5-year inflation expectations has a mixed direction: it seems to be positive at period 1 but goes negative at period 2 and 3 to finally revert near to 0 in period 4 until 6. The impact on WTI oil prices is however more distinguishable since it is positive in the very short run, goes negative in periods 3 and 4 before reaching near 0 in the lasting periods.

We do not predict shocks beyond 6 periods since we know that our VAR model is stationary and stationarity implies that the impact of a shock will quickly decay to 0. This is the case for this forecast, as shown by the curve which reverts to 0.

Figure 25

Impulse reaction function of a shock on the WTI oil prices on the Inflation expectations and the Stock Index variables





The impact of a contemporaneous shock on WTI oil prices on inflation expectations is positive in period 1, then decreases to reach negative territory in period 2 until period 4 and finally reverts to 0 to the end of the forecast. The impact on the S&P index follows almost the same pattern: the impact is positive in periods 1 and 2, then goes negative in periods 3 and 4 and finally reverts to 0 in the lasting periods.

We do not predict shocks beyond 6 periods since we know that our VAR model is stationary and stationarity implies that the impact of a shock will quickly decay to 0. This is the case for this forecast, as shown by the curve which reverts to 0.

In conclusion to this second empirical application, we have estimated a VAR model using the previous empirical application's de-trended and transformed financial variables, we have conducted a set of robustness checks on our model which were successfully handled. We then tested for Granger-causality just before predicting and plotting each of our three time series at horizon 2. Finally, we predicted and plotted the impulse reaction functions of each of our three variables at a 6 month horizon.