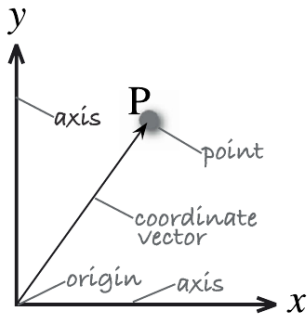


# Pose en 2D

Kjartan Halvorsen

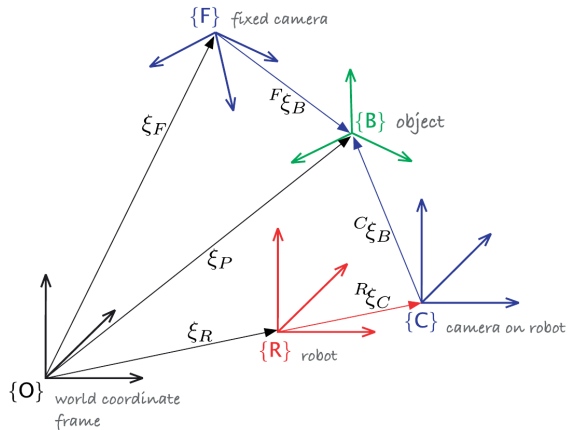
February 17, 2022

# Definiciones



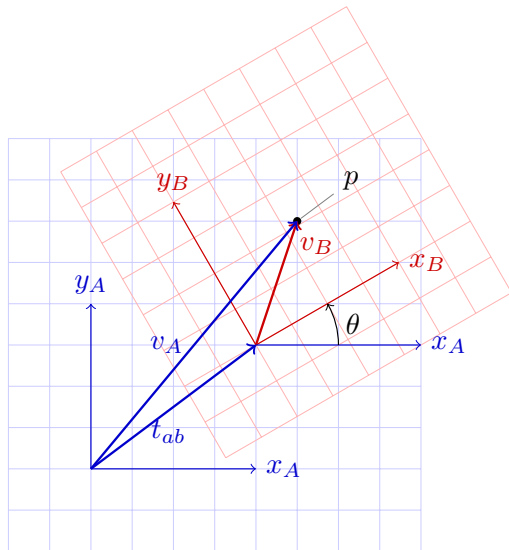
Peter Corke *Robotics, vision and control*

# Uso de sistemas de referencia en robótica



Peter Corke *Robotics, vision and control*

## Pose en 2D

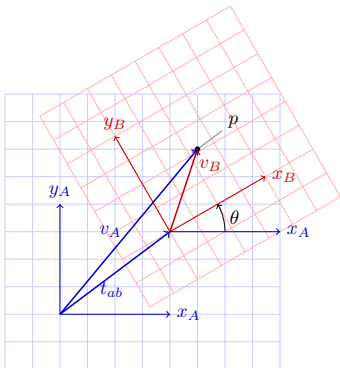


## Pose en 2D

El vector  $\vec{x}_B$  en sistema  $\{A\}$ :

$$\vec{x}_B = \cos \theta \vec{x}_A + \sin \theta \vec{y}_A = \begin{bmatrix} \vec{x}_A & \vec{y}_A \end{bmatrix} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

$${}^A \vec{x}_B = \cos \theta {}^A \vec{x}_A + \sin \theta {}^A \vec{y}_A = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$



## Pose en 2D

El vector  $\vec{x}_B$  en sistema  $\{A\}$ :

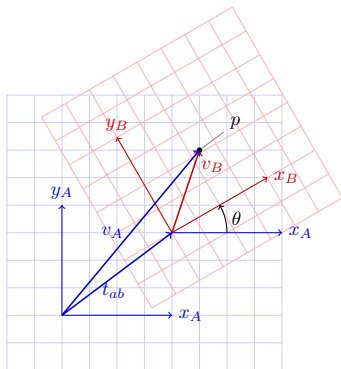
$$\vec{x}_B = \cos \theta \vec{x}_A + \sin \theta \vec{y}_A = \begin{bmatrix} \vec{x}_A & \vec{y}_A \end{bmatrix} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

$${}^A \vec{x}_B = \cos \theta {}^A \vec{x}_A + \sin \theta {}^A \vec{y}_A = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

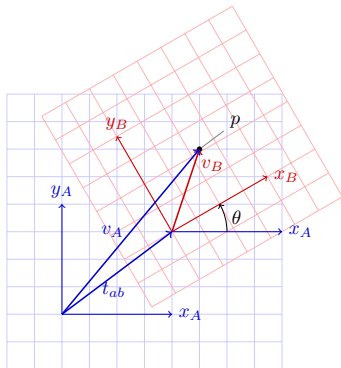
El vector  $\vec{y}_B$  en sistema  $\{A\}$ :

$$\vec{y}_B = -\sin \theta \vec{x}_A + \cos \theta \vec{y}_A = \begin{bmatrix} \vec{x}_A & \vec{y}_A \end{bmatrix} \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

$${}^A \vec{y}_B = -\sin \theta {}^A \vec{x}_A + \cos \theta {}^A \vec{y}_A = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$



## Pose en 2D



El vector  $\vec{x}_B$  en sistema  $\{A\}$ :

$$\vec{x}_B = \cos \theta \vec{x}_A + \sin \theta \vec{y}_A = \begin{bmatrix} \vec{x}_A & \vec{y}_A \end{bmatrix} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

$${}^A \vec{x}_B = \cos \theta {}^A \vec{x}_A + \sin \theta {}^A \vec{y}_A = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

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$${}^A \vec{y}_B = -\sin \theta {}^A \vec{x}_A + \cos \theta {}^A \vec{y}_A = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

El punto  $P$  en sistema  $\{A\}$ :  $p_A = O_A + v_A$ , donde:

$$\vec{v}_A = \vec{t}_{ab} + \vec{v}_B = \vec{t}_{ab} + {}^B p_x \vec{x}_B + {}^B p_y \vec{y}_B$$

## Pose en 2D

El vector  $\vec{x}_B$  en sistema  $\{A\}$ :

$$\vec{x}_B = \cos \theta \vec{x}_A + \sin \theta \vec{y}_A = \begin{bmatrix} \vec{x}_A & \vec{y}_A \end{bmatrix} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

$${}^A \vec{x}_B = \cos \theta {}^A \vec{x}_A + \sin \theta {}^A \vec{y}_A = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

El vector  $\vec{y}_B$  en sistema  $\{A\}$ :

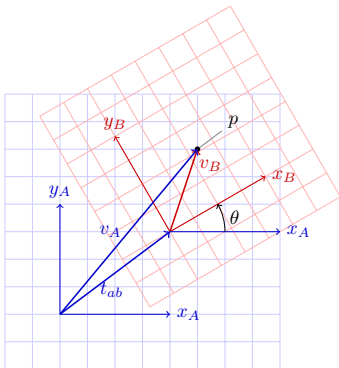
$$\vec{y}_B = -\sin \theta \vec{x}_A + \cos \theta \vec{y}_A = \begin{bmatrix} \vec{x}_A & \vec{y}_A \end{bmatrix} \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

$${}^A \vec{y}_B = -\sin \theta {}^A \vec{x}_A + \cos \theta {}^A \vec{y}_A = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

El punto  $P$  en sistema  $\{A\}$ :  $p_A = O_A + v_A$ , donde:

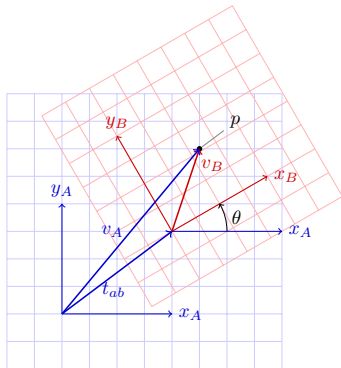
$$\vec{v}_A = \vec{t}_{ab} + \vec{v}_B = \vec{t}_{ab} + {}^B p_x \vec{x}_B + {}^B p_y \vec{y}_B$$

$${}^A \vec{v}_A = {}^A \vec{t}_{ab} + {}^B p_x {}^A \vec{x}_B + {}^B p_y {}^A \vec{y}_B = {}^A \vec{t}_{ab} + \begin{bmatrix} {}^A \vec{x}_B & {}^A \vec{y}_B \end{bmatrix} \begin{bmatrix} {}^B p_x \\ {}^B p_y \end{bmatrix}$$





## Pose en 2D

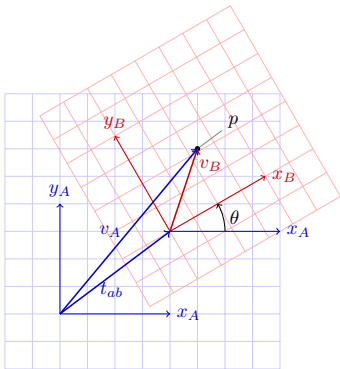


El punto  $P$  en sistema  $\{A\}$ :  $p_A = O_A + v_A$ , donde:

$$\vec{v}_A = \vec{t}_{ab} + \vec{v}_B = \vec{t}_{ab} + {}^B p_x \vec{x}_B + {}^B p_y \vec{y}_B$$

$${}^A \vec{v}_A = {}^A \vec{t}_{ab} + {}^B p_x^A \vec{x}_B + {}^B p_y^A \vec{y}_B = {}^A \vec{t}_{ab} + \begin{bmatrix} {}^A \vec{x}_B & {}^A \vec{y}_B \end{bmatrix} \begin{bmatrix} {}^B p_x \\ {}^B p_y \end{bmatrix}$$

## Pose en 2D



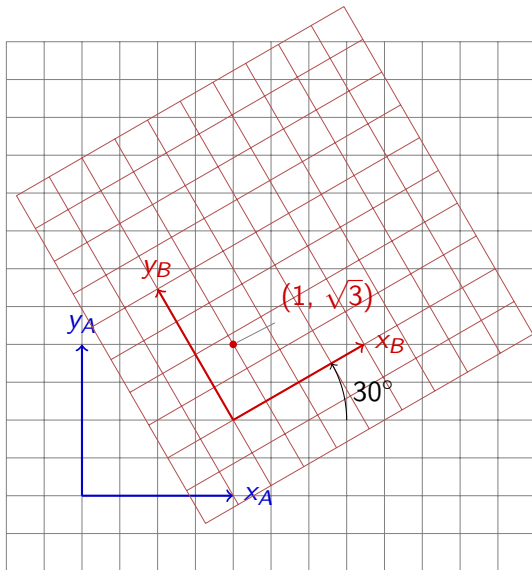
El punto  $P$  en sistema  $\{A\}$ :  $p_A = O_A + v_A$ , donde:

$$\vec{v}_A = \vec{t}_{ab} + \vec{v}_B = \vec{t}_{ab} + {}^B p_x \vec{x}_B + {}^B p_y \vec{y}_B$$

$${}^A \vec{v}_A = {}^A \vec{t}_{ab} + {}^B p_x {}^A \vec{x}_B + {}^B p_y {}^A \vec{y}_B = {}^A \vec{t}_{ab} + \begin{bmatrix} {}^A \vec{x}_B & {}^A \vec{y}_B \end{bmatrix} \begin{bmatrix} {}^B p_x \\ {}^B p_y \end{bmatrix}$$

$$= {}^A \vec{t}_{ab} + \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} {}^B p_x \\ {}^B p_y \end{bmatrix}$$

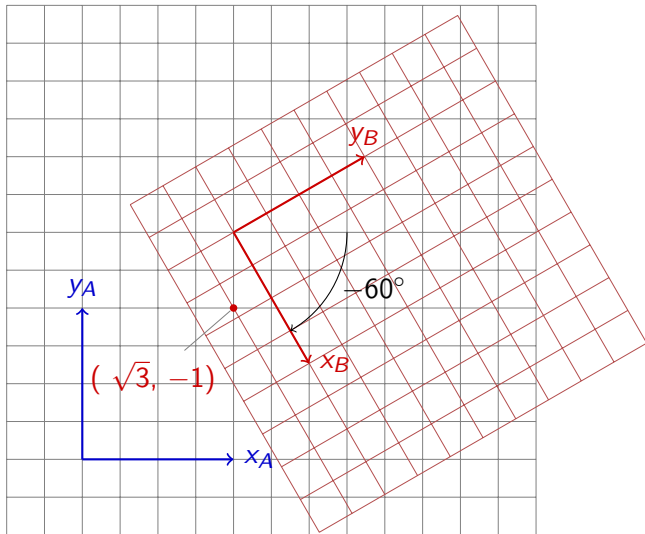
## Ejemplo



Determine la matriz de rotación  $R_{ab}$  y la translación  $t_{ab}$  que definen la transformación entre  $\{B\}$  y  $\{A\}$ . Verifique que

$$\begin{aligned} {}^A p &= \begin{bmatrix} 4 \\ 4 \end{bmatrix} = R_{ab}^B p + t_{ab} \\ &= R_{ab} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} + t_{ab} \end{aligned}$$

## Ejercicio



Determine la matriz de rotación  $R_{ab}$  y la translación  $t_{ab}$  que definen la transformación entre  $\{B\}$  y  $\{A\}$ . Verifique que

$$\begin{aligned} {}^A p &= \begin{bmatrix} 4 \\ 4 \end{bmatrix} = R_{ab}^B p + t_{ab} \\ &= R_{ab} \begin{bmatrix} \sqrt{3} \\ -1 \end{bmatrix} + t_{ab} \end{aligned}$$

# Programar