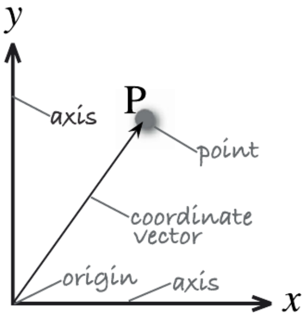
Pose en 2D

Kjartan Halvorsen

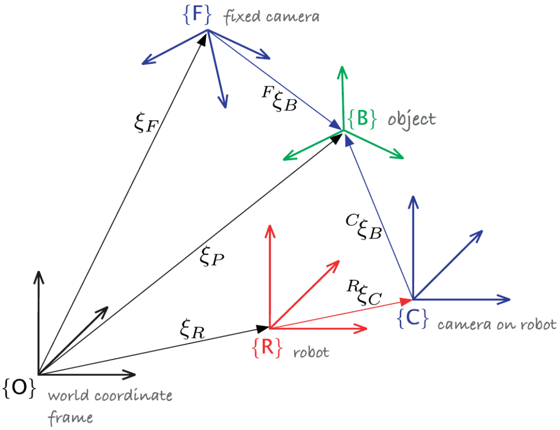
February 3, 2023

Definiciones



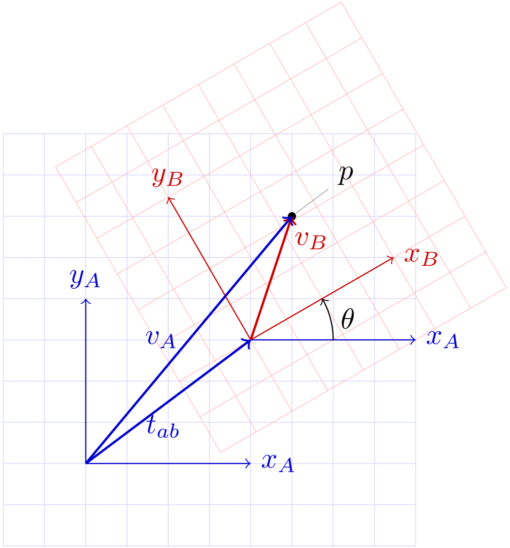
Peter Corke *Robotics, vision and control*

Uso de sistemas de referencia en robótica

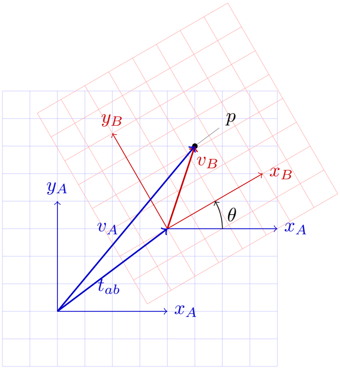


Peter Corke *Robotics, vision and control*

Pose en 2D



Pose en 2D

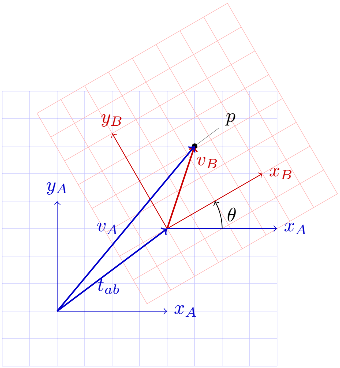


El vector $\vec{x_B}$ en sistema $\{A\}$:

$$\vec{x_B} = \cos\theta\vec{x_A} + \sin\theta\vec{y_A} = \begin{bmatrix}\vec{x_A} & \vec{y_A}\end{bmatrix}\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}$$

$${}^{A}\vec{x_B} = \cos\theta\,{}^{A}\vec{x_A} + \sin\theta\,{}^{A}\vec{y_A} = \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}$$

Pose en 2D



El vector $\vec{x_B}$ en sistema $\{A\}$:

$$\vec{x_B} = \cos\theta\vec{x_A} + \sin\theta\vec{y_A} = \begin{bmatrix}\vec{x_A} & \vec{y_A}\end{bmatrix}\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}$$

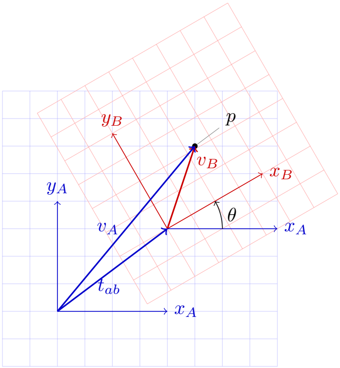
$$^A\vec{x_B} = \cos\theta\,^A\vec{x_A} + \sin\theta\,^A\vec{y_A} = \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}$$

El vector $\vec{y_B}$ en sistema $\{A\}$:

$$\vec{y_B} = -\sin\theta\vec{x_A} + \cos\theta\vec{y_A} = \begin{bmatrix}\vec{x_A} & \vec{y_A}\end{bmatrix}\begin{bmatrix}-\sin\theta \\ \cos\theta\end{bmatrix}$$

$$^A\vec{y_B} = -\sin\theta\,^A\vec{x_A} + \cos\theta\,^A\vec{y_A} = \begin{bmatrix}-\sin\theta \\ \cos\theta\end{bmatrix}$$

Pose en 2D



El vector $\vec{x_B}$ en sistema $\{A\}$:

$$\vec{x_B} = \cos\theta \vec{x_A} + \sin\theta \vec{y_A} = \begin{bmatrix} \vec{x_A} & \vec{y_A} \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$^A\vec{x_B} = \cos\theta \, ^A\vec{x_A} + \sin\theta \, ^A\vec{y_A} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

El vector $\vec{y_B}$ en sistema $\{A\}$:

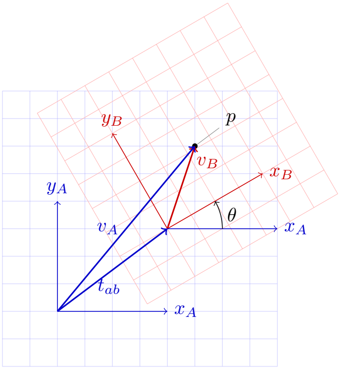
$$\vec{y_B} = -\sin\theta \vec{x_A} + \cos\theta \vec{y_A} = \begin{bmatrix} \vec{x_A} & \vec{y_A} \end{bmatrix} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$^A\vec{y_B} = -\sin\theta \, ^A\vec{x_A} + \cos\theta \, ^A\vec{y_A} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

El punto P en sistema $\{A\}$: $p_A = O_A + v_A$, donde:

$$\vec{v_A} = \vec{t_{ab}} + \vec{v_B} = \vec{t_{ab}} +^B p_x \vec{x_B} +^B p_y \vec{y_B}$$

Pose en 2D



El vector $\vec{x_B}$ en sistema $\{A\}$:

$$\vec{x_B} = \cos\theta \vec{x_A} + \sin\theta \vec{y_A} = \begin{bmatrix} \vec{x_A} & \vec{y_A} \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$^A\vec{x_B} = \cos\theta^A\vec{x_A} + \sin\theta^A\vec{y_A} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

El vector $\vec{y_B}$ en sistema $\{A\}$:

$$\vec{y_B} = -\sin\theta \vec{x_A} + \cos\theta \vec{y_A} = \begin{bmatrix} \vec{x_A} & \vec{y_A} \end{bmatrix} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

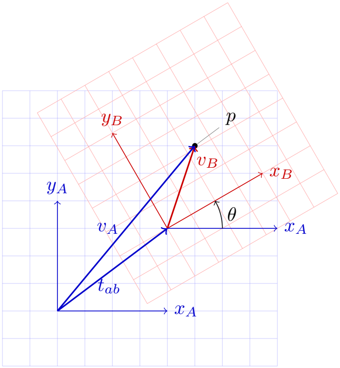
$$^A\vec{y_B} = -\sin\theta^A\vec{x_A} + \cos\theta^A\vec{y_A} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

El punto P en sistema $\{A\}$: $p_A = O_A + v_A$, donde:

$$\vec{v_A} = \vec{t_{ab}} + \vec{v_B} = \vec{t_{ab}} +^B p_x\vec{x_B} +^B p_y\vec{y_B}$$

$$^A\vec{v_A} =^A \vec{t_{ab}} +^B p_x^A\vec{x_B} +^B p_y^A\vec{y_B} =^A \vec{t_{ab}} + \begin{bmatrix} ^A\vec{x_B} & ^A\vec{y_B} \end{bmatrix} \begin{bmatrix} ^Bp_x \\ ^Bp_y \end{bmatrix}$$

Pose en 2D

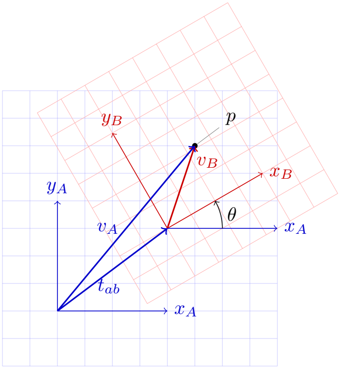


El punto P en sistema $\{A\}$: $p_A = O_A + v_A$, donde:

$$\vec{v_A} = \vec{t_{ab}} + \vec{v_B} = \vec{t_{ab}} +^B p_x \vec{x_B} +^B p_y \vec{y_B}$$

$$^A\vec{v_A} =^A \vec{t_{ab}} +^B p_x^A \vec{x_B} +^B p_y^A \vec{y_B} =^A \vec{t_{ab}} + \begin{bmatrix} ^A\vec{x_B} & ^A\vec{y_B} \end{bmatrix} \begin{bmatrix} ^B p_x \\ ^B p_y \end{bmatrix}$$

Pose en 2D



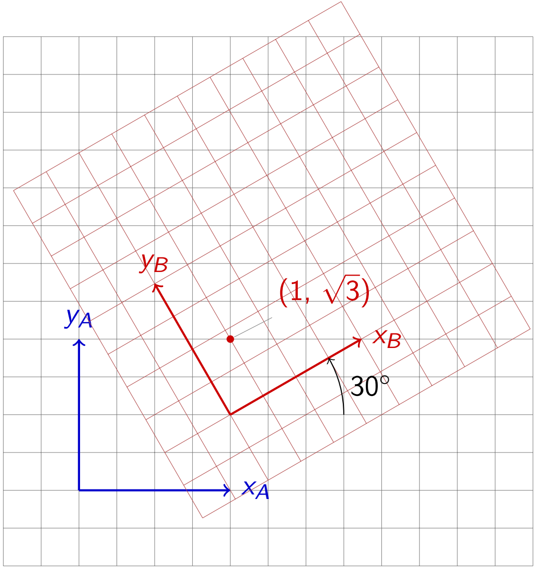
El punto P en sistema $\{A\}$: $p_A = O_A + v_A$, donde:

$$\vec{v_A} = \vec{t_{ab}} + \vec{v_B} = \vec{t_{ab}} + {}^B p_x \vec{x_B} + {}^B p_y \vec{y_B}$$

$${}^A\vec{v_A} = {}^A\vec{t_{ab}} + {}^B p_x^A \vec{x_B} + {}^B p_y^A \vec{y_B} = {}^A\vec{t_{ab}} + \begin{bmatrix} {}^A\vec{x_B} & {}^A\vec{y_B} \end{bmatrix} \begin{bmatrix} {}^B p_x \\ {}^B p_y \end{bmatrix}$$

$$= {}^A\vec{t_{ab}} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} {}^B p_x \\ {}^B p_y \end{bmatrix}$$

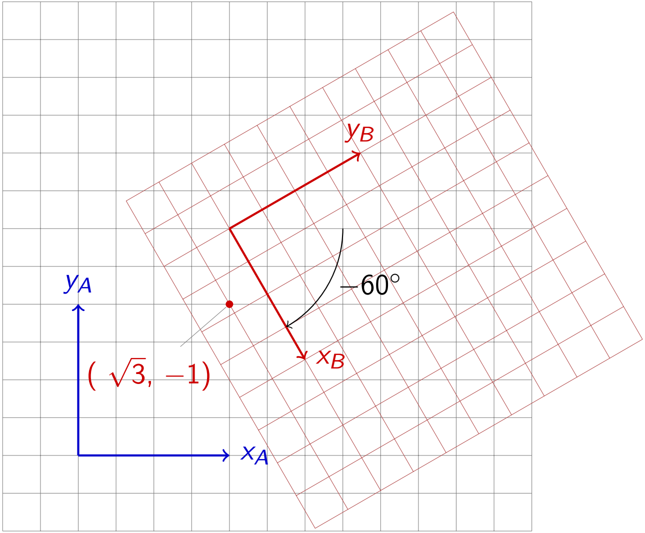
Ejemplo



Determine la matriz de rotación R_{ab} y la translación t_{ab} que definen la transformación entre $\{B\}$ y $\{A\}$. Verifique que

$$
\begin{aligned}
{}^{A}p = \begin{bmatrix} 4 \\ 4 \end{bmatrix} &= R_{ab}^{B}p + t_{ab} \\
&= R_{ab} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} + t_{ab}
\end{aligned}
$$

Ejercicio



Determine la matriz de rotación R_{ab} y la translación t_{ab} que definen la transformación entre $\{B\}$ y $\{A\}$. Verifique que

$$
\begin{aligned}
{}^{A}p = \begin{bmatrix} 4 \\ 4 \end{bmatrix} &= R_{ab}^{B} p + t_{ab} \\
&= R_{ab} \begin{bmatrix} \sqrt{3} \\ -1 \end{bmatrix} + t_{ab}
\end{aligned}
$$

Programar