Computerized Control - Discretizing continuous-time controllers

Kjartan Halvorsen

2019-11-21

Goal of today's lecture

- Understand discrete approximations of continuous-time controllers

Result from quizz

- Frequency warping and prewarping
- How to choose approximation method

Discretizing a controller

- Have a controller $F(s)$ obtained from a design in continuous time.
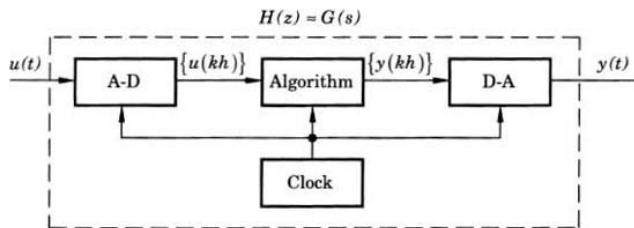- Discretize in order to implement on a computer

Figure 8.1 Approximating a continuous-time transfer function, $G(s)$, using a computer.

Discretization methods

1. Forward difference. Substitute

$$s = \frac{z-1}{h}$$

in $F(s)$ to get

$$F_d(z) = F(s')|_{s' = \frac{z-1}{h}}.$$

2. Backward difference. Substitute

$$s = \frac{z-1}{zh}$$

in $F(s)$ to get

$$F_d(z) = F(s')|_{s' = \frac{z-1}{zh}}.$$

Discretization methods, contd.

3. Tustin's method (also known as the bilinear transform). Substitute

$$s = \frac{2}{h}\frac{z-1}{z+1}$$

in $F(s)$ to get

$$F_d(z) = F(s')|_{s' = \frac{2}{h} \cdot \frac{z-1}{z+1}}.$$

4. Ramp invariance. This is similar to ZoH, which is step-invariant approximation. Since a unit ramp has z-transform $\frac{zh}{(z-1)^2}$ and Laplace-transform $1/s^2$, the discretization becomes

$$F_d(z) = \frac{(z-1)^2}{zh}\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{F(s)}{s^2}\right\}\right\}.$$

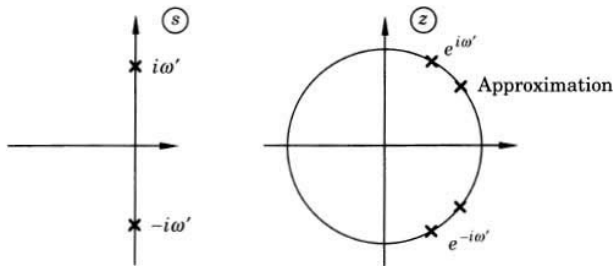# Frequency warping using Tustin's

Figure 8.3 Frequency distortion (warping) obtained with approximation.

The infinite positive imaginary axis in the s-plane is mapped to the finite-length upper half of the unit circle in the z-plane.

Exercise

Find the discrete approximation of the lead-compensator $F(s) = \frac{s+b}{s+a}$, and determine the pole for

1. Forward difference. Substitute

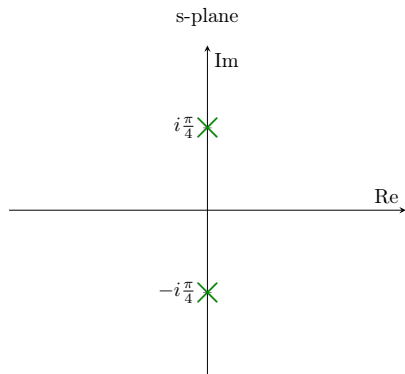$$F_d(z) = F(s')|_{s' = \frac{z-1}{h}}.$$

2. Backward difference. Substitute
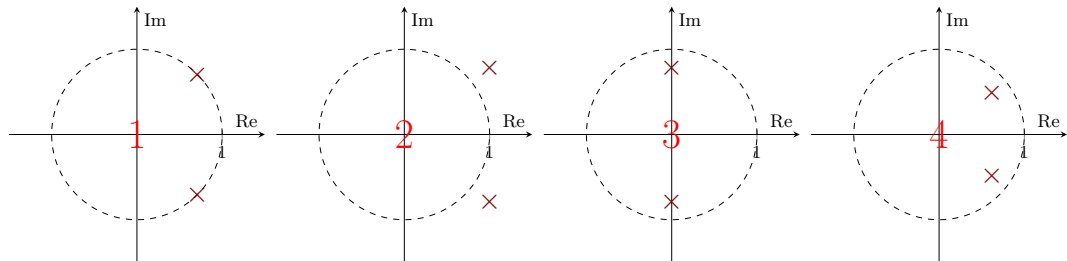
$$F_d(z) = F(s')|_{s' = \frac{z-1}{zh}}.$$

3. Tustin's approximation

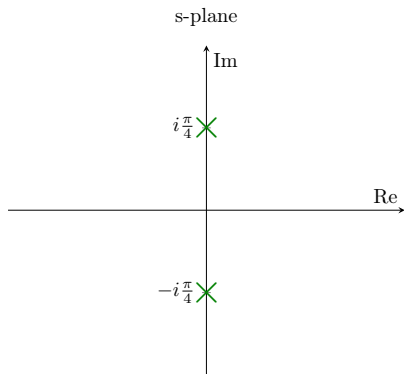$$F_d(z) = F(s')|_{s' = \frac{2}{h} \cdot \frac{z-1}{z+1}}.$$

Forward difference exercise

s-plane

Which of the below figures shows the correct mapping of the continuous-time poles using the **forward difference** $z = 1 + sh$ with $h = 1$?

Backward difference exercise

s-plane

Which of the below figures shows the correct mapping of the continuous-time poles using the **backward difference** $z = \frac{1}{1-sh}$ with $h = 1$?