Sampling and anti-aliasing exercise

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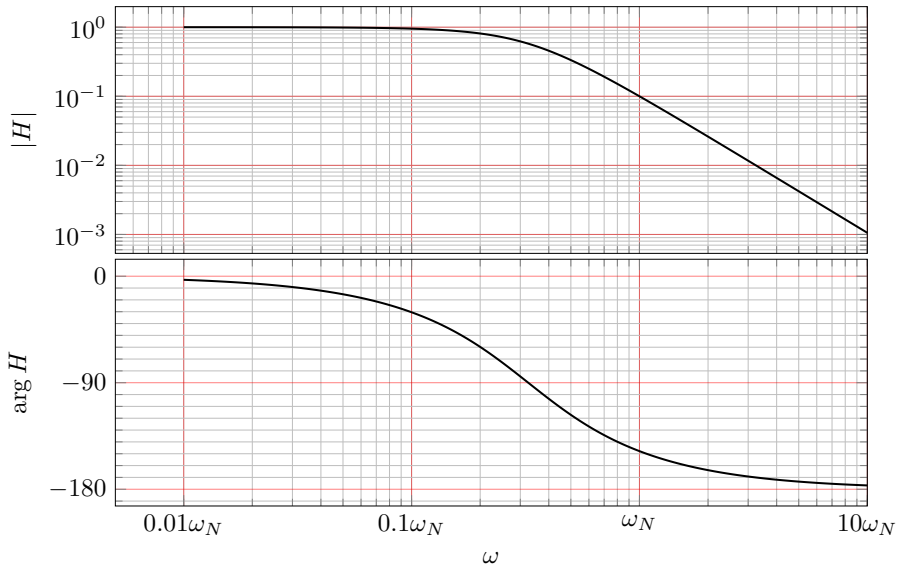
The Bessel filter

A second order Bessel filter

$$H(s) = \frac{3}{\left(s/\omega_0\right)^2 + 3\left(s/\omega_0\right) + 3},$$

has been designed to give attenuation of 0.1 at the Nyquist frequency, i.e. $|H(i\omega_N)| = 0.1$.



1. What is the bandwidth of the filter?
2. Determine the filter parameter ω_0 in terms of ω_N.
3. What is the phase shift at the Nyquist frequency?
4. Assume that the cross-over frequency of the loop gain is $\omega_c = 0.1\omega_N$. What is the phase shift at this frequency?

5. What delay T does this correspond to? Approximate the filter as a pure delay $H(i\omega) \approx e^{-i\omega T}$?

6. How large is the delay in terms of the sampling period $h = \frac{\pi}{\omega_N}$?

Choice of bandwidth of antialiasing filter

Table 7.3 The time delay T_d as a function of the desired attenuation at the Nyquist frequency for fourth- and sixth-order Bessel filters. The sampling period is denoted h.

	Fourth Order		Sixth Order	
β	ω_B/ω_N	T_d/h	ω_B/ω_N	T_d/h
0.001	0.1	5.6	0.2	4.8
0.01	0.2	3.2	0.3	3.1
0.05	0.3	2.1	0.4	2.3
0.1	0.4	1.7	0.4	2.0
0.2	0.5	1.4	0.5	1.7
0.5	0.7	0.9	0.7	1.2
0.7	1.0	0.7	1.0	0.9

The table can be used to find the delay incurred by an antialiasing filter. For instance, if we want to have an attenuation at the Nyquist frequency equal to $\beta = 0.01$, then a fourth-order Bessel low-pass filter will give a delay of about $T_d = 3.2h$, that is a delay of more than 3 sampling periods.

1. What is the advantage of using an antialiasing filter of higher order than 2?

2. If we can only accept a time delay of maximum two sampling periods, what is the attenutation we can obtain at the Nyquist frequency?

3. $\omega_B = \omega_N$ means that the antialiasing filter is chosen so that the Nyquist frequency is the same as the bandwidth of the filter. The attenuation at ω_N is then 0.7=-3dB. If the sampling period is 0.4 seconds. What is the time delay due to a sixth-order antialiasing filter?

4. What is the attenuation of a forth-order filter at $5\omega_B$

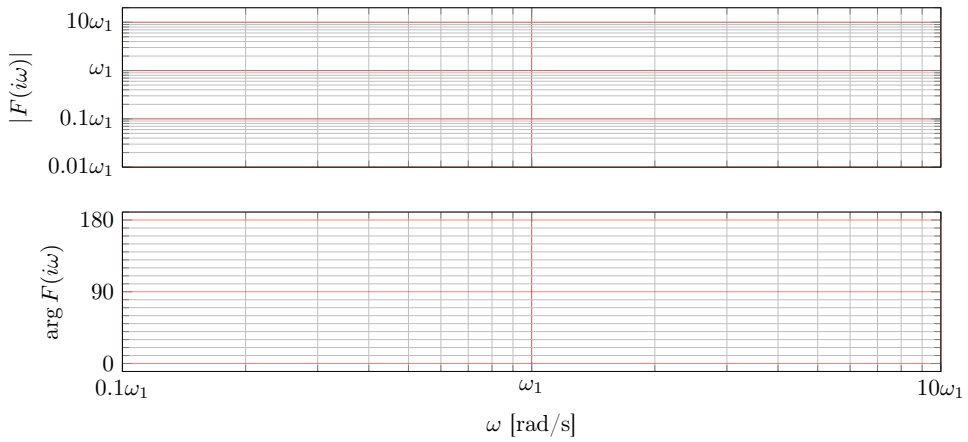
Notch filter, preliminary exercise

Sketch the Bode diagram of the (unrealizable) filter

$$F(s) = s^2 + \omega_1^2 = (s + i\omega_1)(s - i\omega_1)$$

$$|F(i\omega)| = |(i\omega + i\omega_1)||(i\omega - i\omega_1)| = |\omega - (-\omega_1)||\omega - \omega_1|$$

$$\arg F(i\omega) = \arg i(\omega + \omega_1) + \arg i(\omega - \omega_1) = 2\arg i + 0 + \arg(\omega - \omega_1)$$



Notch filter

Sketch the Bode diagram ($Q = 10$)of the second order continous-time notch filter

$$F(s) = \frac{s^2 + \omega_1^2}{s^2 + \frac{\omega_1}{Q}s + \omega_1^2}$$