

Computerized Control - Sampling and antialiasing

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Repetition

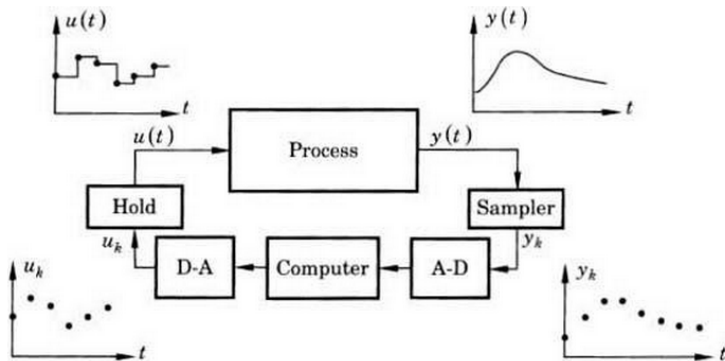
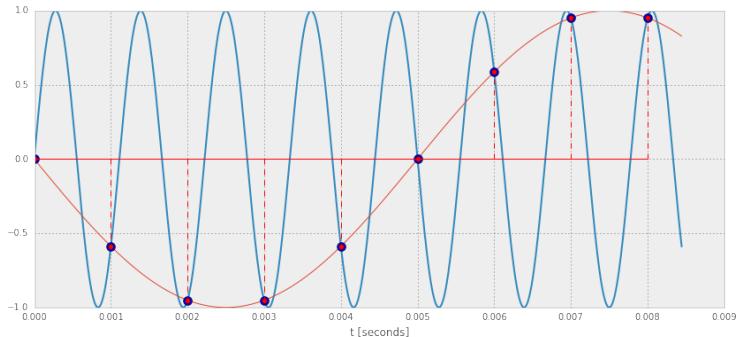


Figure 7.2 Relationships among the measured signal, control signal, and their representations in the computer.

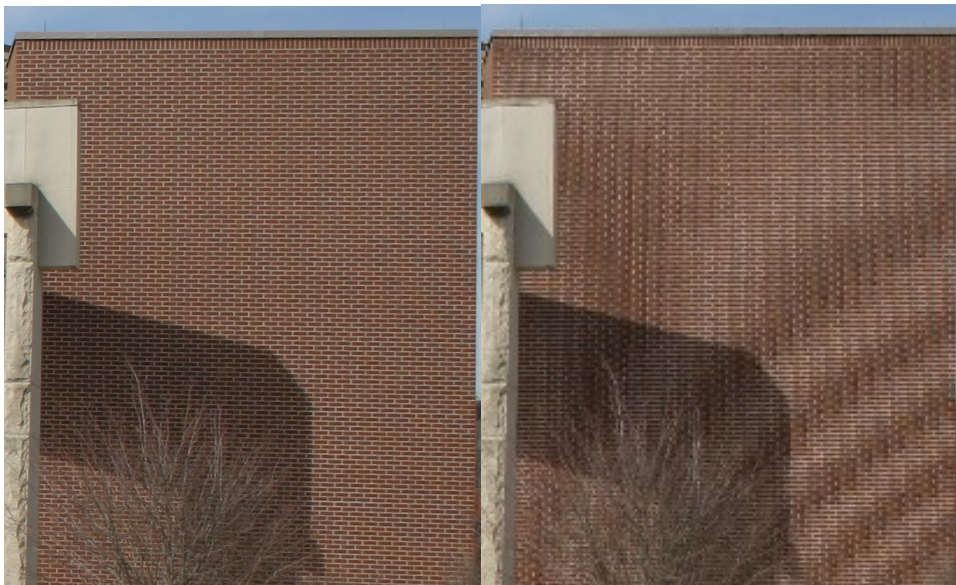
The problem

If a continuous-time signal with frequency content (bandwidth) ω_B is sampled at too low sampling rate ($\omega_s < 2\omega_B$), then the energy at higher frequencies is folded onto lower frequencies.



A high-frequency sinusoid ($\omega_1 = 1800\pi$ rad/s) masquerading as a lower frequency sinusoid (200π rad/s) due to aliasing.

Spatial aliasing



The sampling theorem

Shannon and Nyquist:

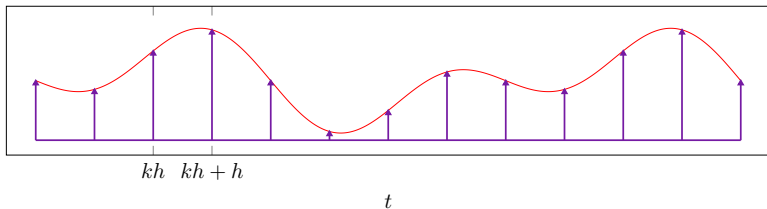
A continuous-time signal with Fourier transform that is zero outside the interval $(-\omega_0, \omega_0)$ can be completely reconstructed from equidistant samples of the signal, as long as the sampling frequency is at least $2\omega_0$.

The impulse modulation model

The **impulse train**, a.k.a the **Dirac comb**:

$$m(t) = \sum_{k=-\infty}^{\infty} \delta(t - kh)$$

$$f_s(t) = f(t)m(t) = f(t) \sum_{k=-\infty}^{\infty} \delta(t - kh) = \sum_{k=-\infty}^{\infty} f(t)\delta(t - kh) = \sum_{k=-\infty}^{\infty} f(kh)\delta(t - kh)$$



Fourier transform (spectrum) of the sampled signal

The Fourier transform of f_s and the Fourier transform of f are related as

$$F_s(\omega) = \frac{1}{h} \sum_n F(\omega + n\omega_s).$$

Because the Fourier transform of the sampled signal equals the Fourier transform of the continuous-time signal repeated at every multiple of the sampling frequency and added, we get *frequency-folding* or *aliasing*.

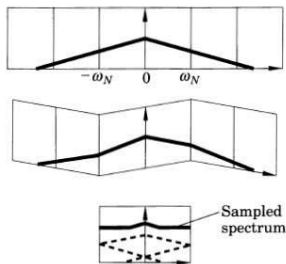


Figure 7.11 Frequency folding.

Alias frequency

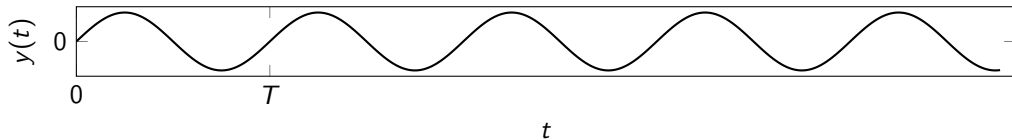
To find the low frequency alias $\omega_a < \omega_N$ of a high frequency sinusoid ω_1 , The expression

$$\omega_a = |((\omega_1 + \omega_N) \bmod \omega_s) - \omega_N|$$

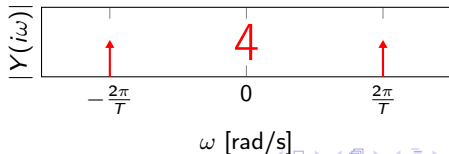
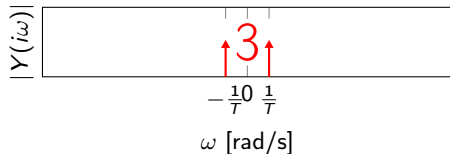
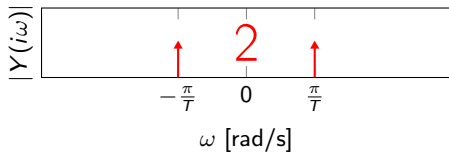
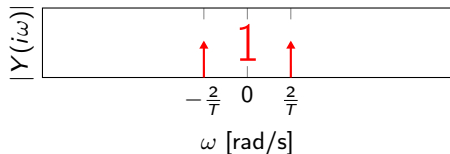
can be used (equation 7.10 in the textbook).

Exercise 1: Spectrum of a sinusoid

Consider the signal

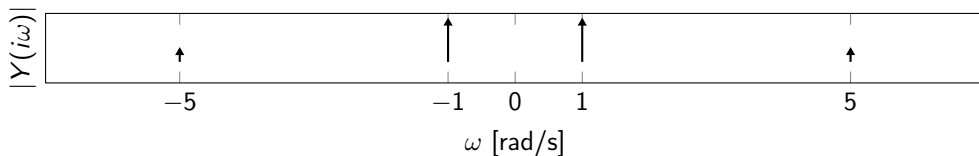


Which of the below is the correct Fourier transform (magnitude plot shown)?

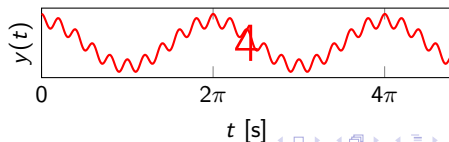
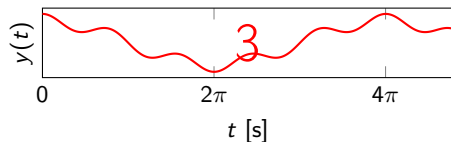
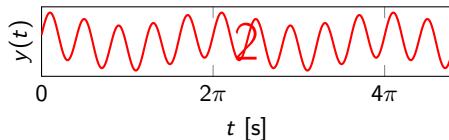
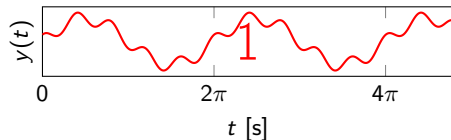


Exercise 2: Two sinusoids

Consider a signal with Fourier transform

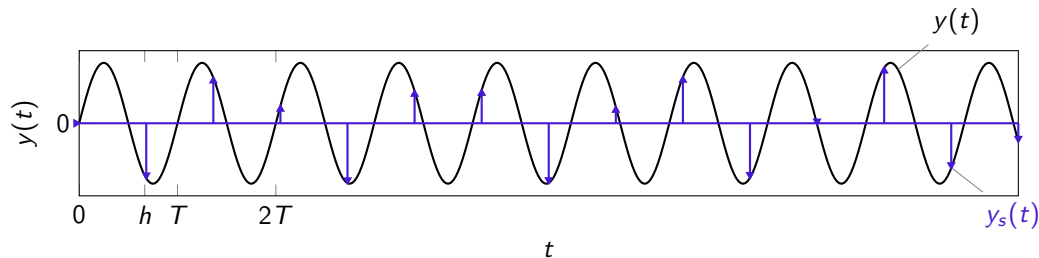


Which of the below time series could this Fourier transform correspond to?



Exercise 3: Spectrum of a sampled sinusoid

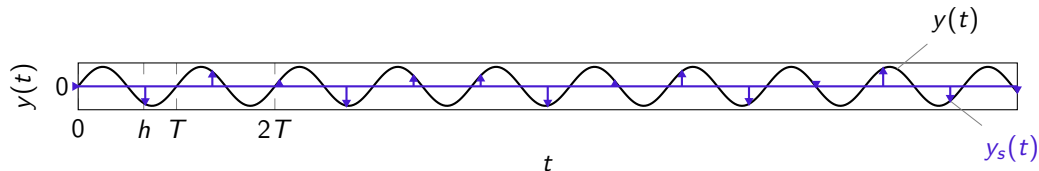
Consider the continuous and sampled signals with sampling period $h = \frac{2}{3}T$



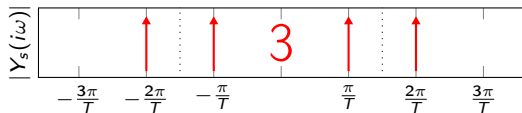
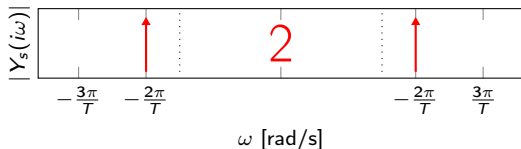
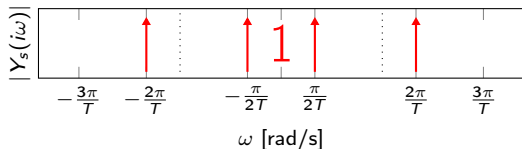
What is the frequency of the sinusoid? What is the sampling frequency ω_s and the Nyquist frequency ω_N ?

Exercise 3: Spectrum of a sampled sinusoid

Consider the continuous and sampled signals with sampling period $h = \frac{2}{3}T$

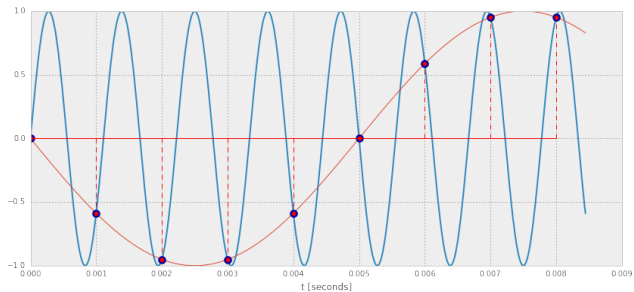


Which of the below corresponds to the Fourier transform of the **sampled signal**?



Aliasing exercise

If a continuous-time signal with frequency content (bandwidth) ω_B is sampled at too low sampling rate ($\omega_s < 2\omega_B$), then the energy at higher frequencies is folded onto lower frequencies.



A high-frequency sinusoid ($\omega_1 = 1800\pi$ rad/s) masquerading as a lower frequency sinusoid (200π rad/s) due to aliasing when sampled with $h = 10^{-3}$ s.

Draw the spectrum (lines) of the two sinusoids. Mark the Nyquist frequency and verify that the alias frequency is obtained by folding about the Nyquist frequency.

Noisy measurements

Antialiasing filter

The **Bessel filter** is often used. From wikipedia:

In electronics and signal processing, a Bessel filter is a type of analog linear filter with a maximally flat group/phase delay (maximally linear phase response), which preserves the wave shape of filtered signals in the passband. Bessel filters are often used in audio systems.

Why use a Bessel filter as antialiasing filter?

Antialiasing filter

The **Bessel filter** is often used. From wikipedia:

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Why use as antialiasing filter?

- ▶ Preserves wave shapes \Rightarrow very little distortion of signals in the passband
- ▶ Maximally linear phase response $\Rightarrow \arg H \approx -T\omega$, Can be modelled as a pure delay