PID exercises

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1 A PID controller with anti-windup

The figure below taken from Åström & Wittenmark shows a block-diagram of a PID controller with anti-windup.

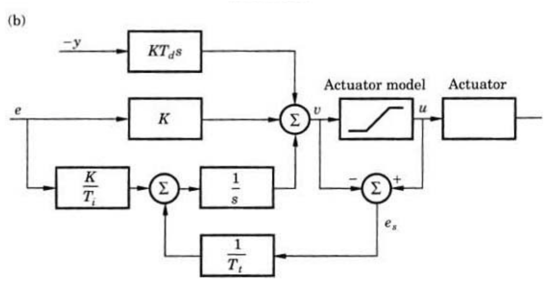


Figure 8.10 Controller with antiwindup. A system in which the actuator output is measured is shown in (a) and a system in which the actuator output is estimated from a mathematical model is shown in (b).

Consider the PID controller as having the output signal $v(t)$ and three input signals: The control error $e(t) = y_{ref}(t) - y(t)$, the output feedback signal $y(t)$ and the output signal of the actuator model $u(t)$.

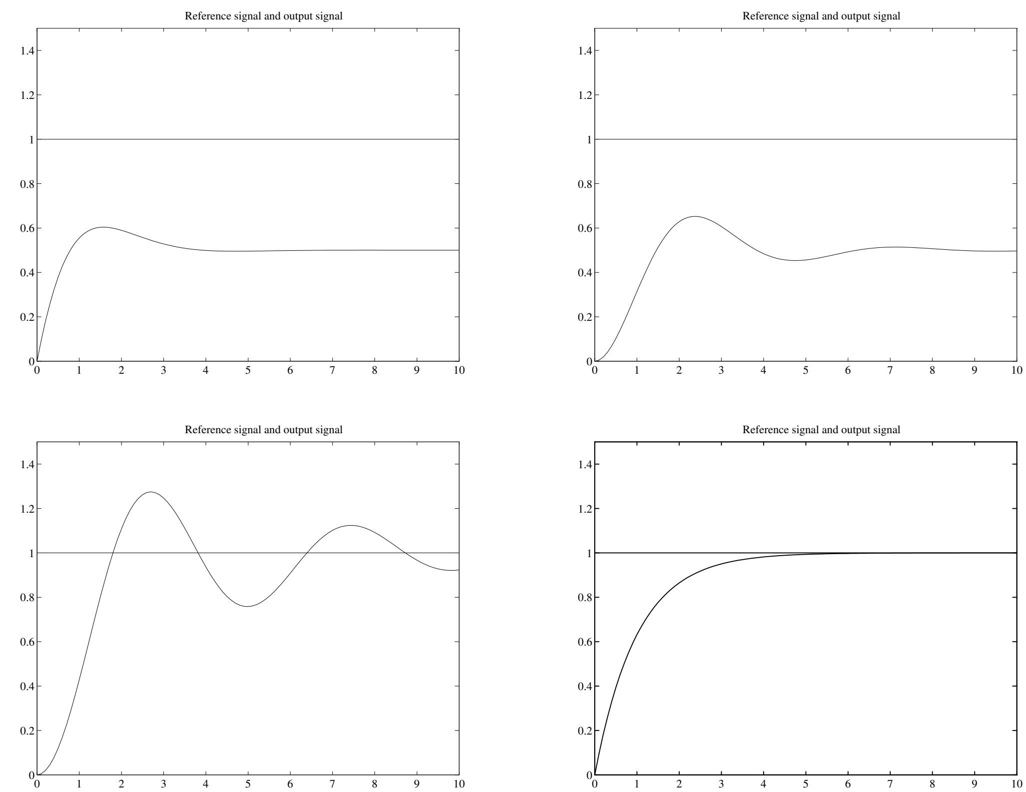
1. Determine the transfer functions in
$$V(s) = F_y(s)Y(s) + F_e(s)E(s) + F_u(s)U(s).$$
2. Determine the pole(s) of the controller.
3. Determine the steady-state output of the controller for step input signals (all at once) $Y(s) = \frac{\alpha}{s}$, $E(s) = \frac{\beta}{s}$ and $U(s) = \frac{\gamma}{s}$.
4. How should we interpret the steady-state value if the actuator is not saturating, so that $u(t) = v(t)$?

2 The effect of PID controller parameters

Assume a certain system being controlled by a PID regulator, $U(s) = (K_P + K_I\frac{1}{s} + K_D s)E(s)$. In the figure below shows four step responses for the parameter triples

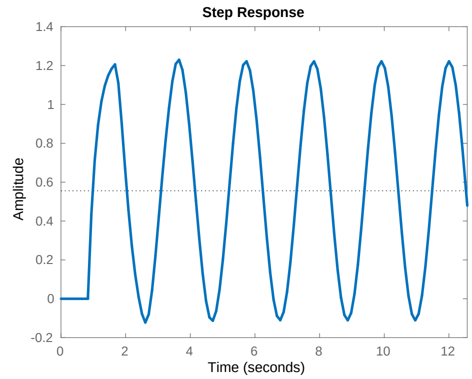
$$\begin{array}{rlll} i) & K_P = 1 & K_I = 0 & K_D = 0 \\ ii) & K_P = 1 & K_I = 1 & K_D = 0 \\ iii) & K_P = 1 & K_I = 0 & K_D = 1 \\ iv) & K_P = 1 & K_I = 1 & K_D = 1 \end{array}$$

Match each one of the parameter triples to one of the step responses. Justify your answer!



3 Ziegler-Nichols ultimate gain tuning

Ziegler and Nichols developed their tuning rules for PID regulators given that the system can be approximated as a first order system with time delay: $G(s) = \frac{e^{-sT}}{s+a}$. We perform a resonance test according to Ziegler-Nichols' ultimate gain method and with the proportional gain $K = 5 = K_u$ we get the closed-loop step response seen below



1. What is the period T_u and angular frequency ω_u of the oscillations?
2. Determine the parameters T and a in the transfer function $G(s)$.
3. Tune a PI regulator
$$u(t) = K_p\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right)$$
according to Ziegler-Nichols' recipe.

Controller type	K_p	T_i	T_d
P	$0.5K_u$		
PI	$0.45K_u$	$T_u/1.2$	
PD	$0.8K_u$		$T_u/8$
PID	$0.6K_u$	$T_u/2$	$T_u/8$

4. The Nyquist plot of the resulting loop gain $G_o(s) = F(s)G(s)$ is shown below. What are the stability margins?
5. The cross-over frequency is $\omega_c = 0.39$ rad/s with the tuned PI controller. Determine a sampling period h for a discrete-time implementation of the PI controller, such that the phase margin is reduced by about $10°$.