Computerized control - homework 1

Kjartan Halvorsen

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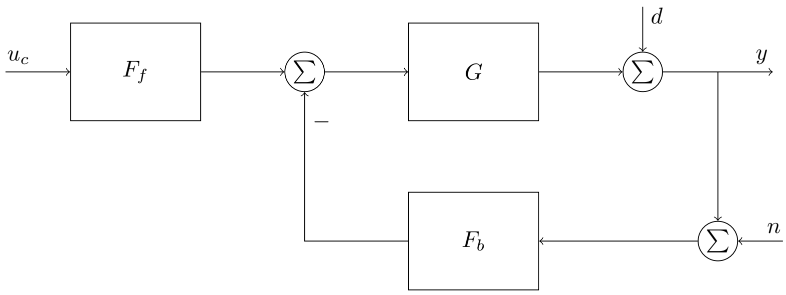
1 Exercise

1.1 Block-diagram calculation

The block-diagram below shows a so-called *two-degrees-of-freedom* feedback control system. Calculate the transfer function from each of the signals u_c (command signal), d (disturbance signal) and n (measurement noise) to the system output y.



1.2 Solution to a first-order system

Consider the first-order system

$$\dot{x}(t) = -x(t) + u(t)$$

1. Write the solution to the system for the input signal

$$u(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

   The initial value is $x(t) = 0$.

2. Sketch the solution $x(t)$ for $0 \leq t \leq 4$, (or generate the plot on the computer).
3. On Blackboard you can find a simulink-file with an implementation of the system. Use this to verify your solution. Include a screen-dump of the simulation output in your report.

2 Solution

2.1 Block-diagram

One alternative is to use Mason's rule. Note that the denominator of the closed-loop transfer functions will be

$$1 + GF_b.$$

The numerators are given by the direkt path from the respective input to the output, so we get

$$\frac{Y}{U_c} = \frac{GF_f}{1+GF_b}$$
$$\frac{Y}{D} = \frac{1}{1+GF_b}$$
$$\frac{Y}{N} = -\frac{GF_b}{1+GF_b}$$

The other alternative is to compute directly in the given block-diagram. We get

$$Y = D + G\big(F_f U_c - F_b(N+Y)\big).$$

Moving terms with Y on the left hand side we get

$$Y + GF_b Y = D + GF_f U_c - GF_b N.$$

Deviding by $1 + GF_b$ on both sides gives

$$Y = \frac{1}{1+GF_b} D + \frac{GF_f}{1+GF_b} U_c - \frac{GF_b}{1+GF_b} N.$$

2.2 First-order system

a) The system can be solved by direct integration of the differential equation, or using the Laplace transform.

Using Laplace, we write the input as the sum of two step signals, of which the second is delayed. The solution is then given by summing the solution to each of the two step signals. Write the input as

$$u(t) = u_1(t) + u_2(t),$$

with

$$u_1(t) = \begin{cases} 1, t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$
$$U_1(s) = \frac{1}{s}.$$
$$u_2(t) = -u_1(t-1) = \begin{cases} -1, t \geq 1 \\ 0, & \text{otherwise} \end{cases}$$
$$U_2(s) = -\mathrm{e}^{-s} U_1(s) = \frac{\mathrm{e}^{-s}}{s}$$

Solving using the Laplace transform we get

$$sX = -X + U_1 + U_2$$

which gives

We can also solve the differential equation by integration using the solution

$$x(t) = x(0)\mathrm{e}^{-t} + \int_0^t \mathrm{e}^{-(t-\tau)} u(\tau) d\tau$$

Since $u(t)$ is equal to 1 for $0 \leq t < 1$ and zero elsewhere, the integration is quite simple. For $t < 1$ we get (since $x(0) = 0$)

$$x(t) = \int_0^t \mathrm{e}^{-t+\tau} d\tau = \int_{-t}^0 \mathrm{e}^{\tau'} d\tau' = \left[\mathrm{e}^{\tau'}\right]_{-t}^0 = 1 - \mathrm{e}^{-t}.$$

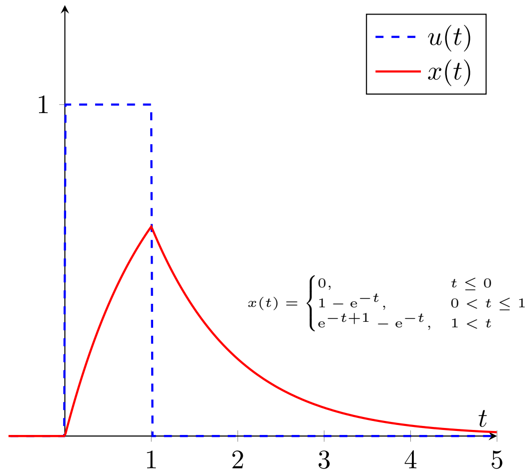
And for $t > 1$ we get

$$x(t) = \int_0^1 \mathrm{e}^{-t+\tau} d\tau = \int_{-t}^{-t+1} \mathrm{e}^{\tau'} d\tau' = \left[\mathrm{e}^{\tau'}\right]_{-t}^{-t+1} = \mathrm{e}^{-t+1} - \mathrm{e}^{-t}.$$

So,

$$x(t) = \begin{cases} 0, & t \leq 0 \\ 1 - \mathrm{e}^{-t}, & 0 < t \leq 1 \\ \mathrm{e}^{-t+1} - \mathrm{e}^{-t}, & t > 1 \end{cases}$$

b) Plot of the solution



c) Simulink-model with simulation