Computerized control - homework 4

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1 PID tuning

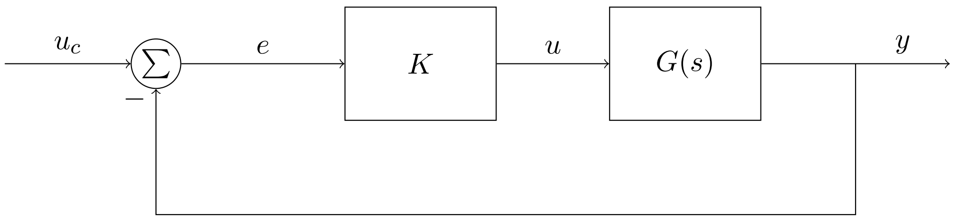
Your task in this homework is to find parameters for the PID controller

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} \int^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right).$$

Consider the continuous-time system with transfer function

$$G(s) = \frac{\mathrm{e}^{-0.2s}}{s+2}$$

Use Ziegler-Nichols tuning (ultimate-sensitivity method). In this method, one determines the critical gain of pure proportional control of the system. That means, to determine the value of K for which the closed loop system below gives self-sustained oscillations.



1.1 Analytical solution

Determine the ultimate gain by calculations.

1. Determine the frequency ω_p for which the nyquist curve for $G(s)$ crosses the negative real axis. That is, solve for ω in
$$\arg G(i\omega) = -\pi.$$
Actually, this leads to a transcendental equation. You can solve this numerically using, for example, the function `fsolve` in matlab.
2. Determine the gain $K = K_u$ such that the nyquist curve for the open loop system $KG(s)$ passes through the point $(-1, 0)$.

This will give you the ultimate gain K_u and period of the oscillations T_u needed to apply the tuning rules.

1.2 Determine by simulation

1. Use matlab (or simulink) to implement the system. To define the transfer function of the plant you can do

```
s = tf('s');
G = exp(-0.2*s)/(s+2)
```

2. Plot a Bode diagram using `margin(G)`. Verify that the phase curve crosses $-180°$ at the frequency ω_p that you determined in the previous exercise. Include the bode-diagram in your report.
3. Define the closed loop system with proportional feedback, K.
4. Simulate step-responses for various values of the gain K. Increase K until the system shows sustained (neither increasing nor decaying) oscillations. Note the corresponding value, K_u and the period of the oscillations, T_u. Verify that they are the same as (or close to) the values you determined analytically. Include a figure of the step-response for critical gain K_u to your report (plot showing 10-20 periods of oscillations).

1.3 Implement a PID-controller

Use Table 8.3 in Å&W to determine the parameters K_p, T_i and T_d based on K_u and ω_p.

1. Implement the (continuous-time) controller and the closed loop system.
2. Plot the Bode diagram for the closed-loop system and include in your report. What is the bandwidth of the closed-loop system?
3. Plot the Bode diagram and Nyquist diagram for the open-loop system $F(s)G(s)$. What is the phase marginal? Include a figure (Bode or Nyquist) in your report, where you indicate the phase marginal.
4. Simulate a step response for the closed-loop system and include in your report. You will probably need to use a Padé-approximation of the delay. To get an LTI-system with approximated delays use `Gcp = pade(Gc, 4)` for a 4th order Padé approximation. What is the overshoot of the step response in percent of the final value?

2 Solution

2.1 Analytical solution

1. The equation to solve to find the phase-crossover frequency is

$$\arg G(i\omega) = -\pi.$$

Write this as

$$\arg \mathrm{e}^{-0.2i\omega} - \arg(i\omega + 2) + \pi = 0,$$

or

$$-0.2\omega - \frac{\omega}{2} + \pi = 0.$$

This can be solved in matlab with the line `fsolve(inline('-atan2(x,2) - 0.2*x + pi'), 0)` and gives the answer

$$\omega_p = 8.953.$$

2. The phase-crossover frequency is the frequency for which the Nyquist curve crosses the negative real axis. Multiplying $G(i\omega)$ with a real, positive gain K does not change the phase and so the phase-crossover frequency remains the same. $KG(i\omega)$ will cross the negative real axis at ω_p. To find the ultimate gain K_u that will cause the Nyquist curve to pass throgh the point (-1,0), solve

$$|K_u G(i\omega_p)| = 1,$$

which lead to

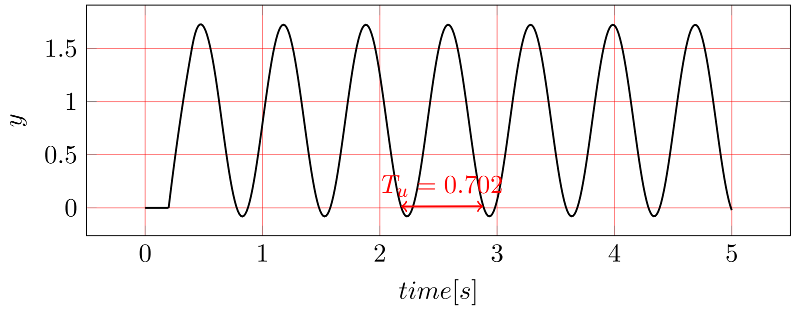
$$K_u = \frac{1}{|G(i\omega_p)|} = \frac{|i\omega_p + 2|}{|\mathrm{e}^{-0.2i\omega_p}|} = \sqrt{\omega_p^2 + 4} \approx 9.174.$$

2.2 Simulation

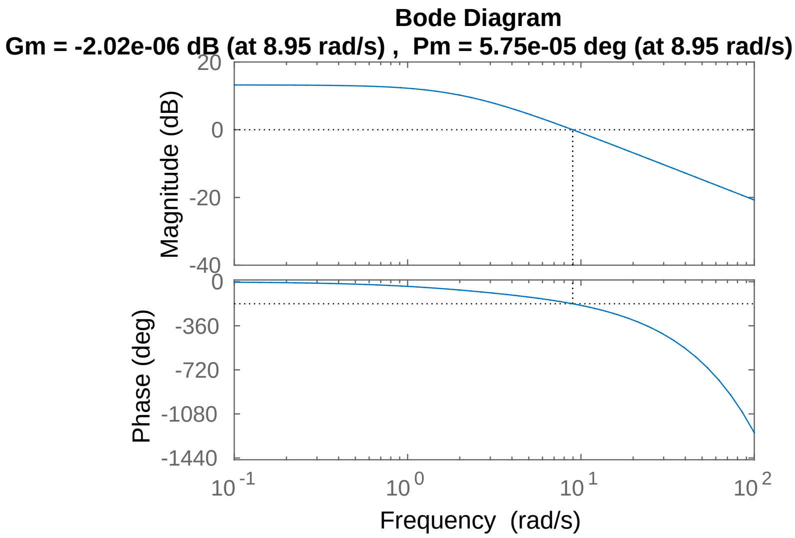
Here is example matlab-code for generating the model and simulating a step:

```
T = 0.2;
s = tf('s');
G = exp(-s*T) / (s+2)
Ku = 1/abs(evalfr(G, i*wp))
Gc = feedback(Ku*G, 1);
step(Gc)
```

The figure below shows the first five seconds of the step response. Clearly, the period of the oscillations is $T_u = 0.702$.



Bode-diagram of open-loop transfer function with ultimate gain (using `margin`)

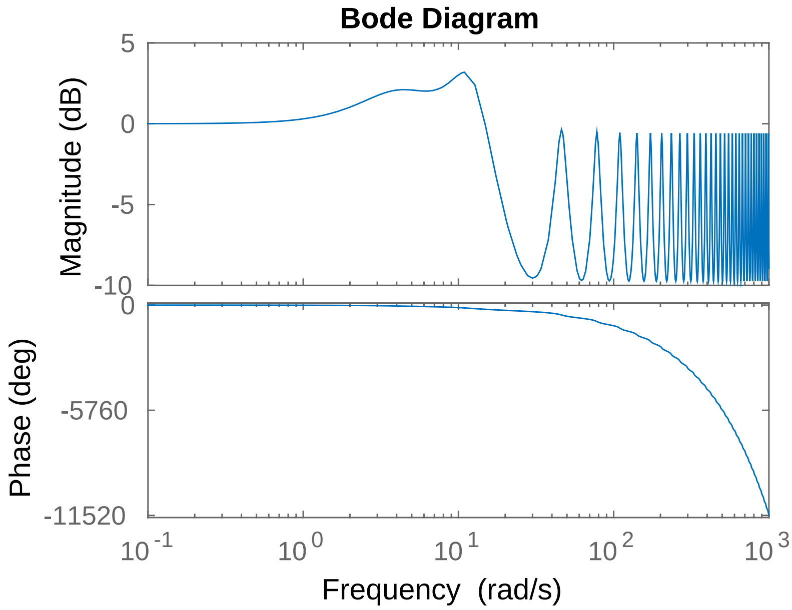


2.3 Implement PID

Using table 8.3 from Å&W we obtain the PID controller with parameters

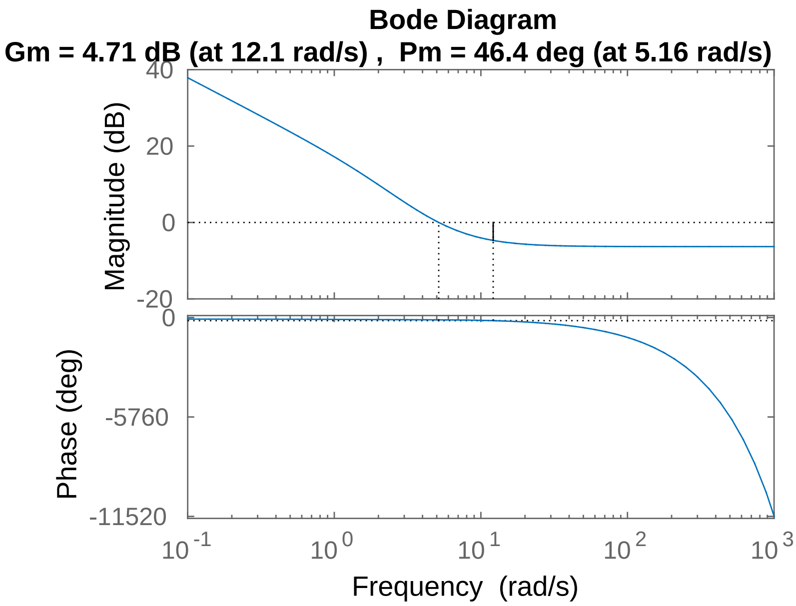
$$K_p = 0.6K_u = 5.5$$
$$T_i = 0.5T_u = 0.35$$
$$T_d = T_u/8 = 0.088$$

The Bode diagram of the closed loop system with the PID controller is given below.

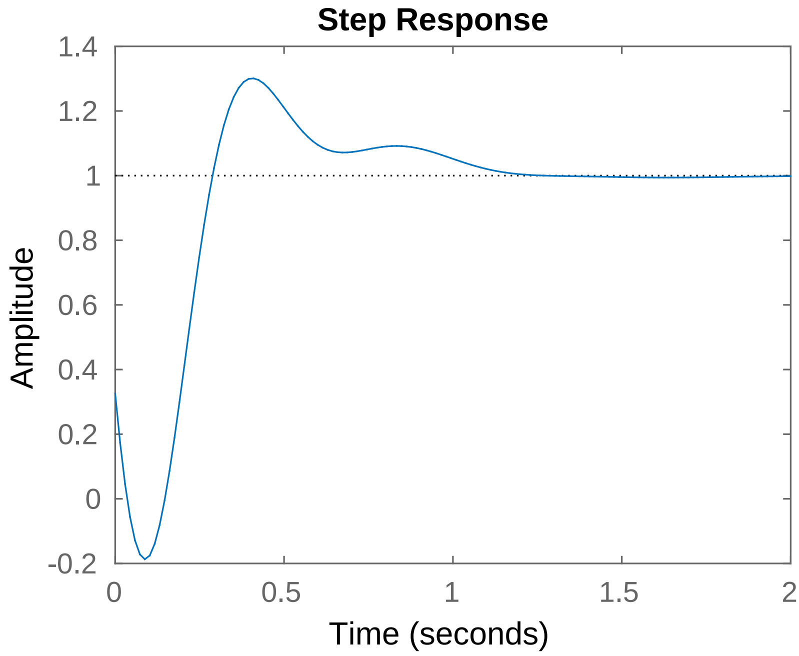


The closed-loop system has bandwidth 17.2 rad/s.

The Bode diagram of the open-loop system shows a phase margin of 46.4 degrees:



With a padé approximation of the delay, the step response of the closed loop system is given below



The initial response of the close-loop system before the delay of the open-loop system is due to the approximation. It should be exactly zero. The overshoot is about 30%.