Computerized Control - discrete-time systems, z-transform

Kjartan Halvorsen

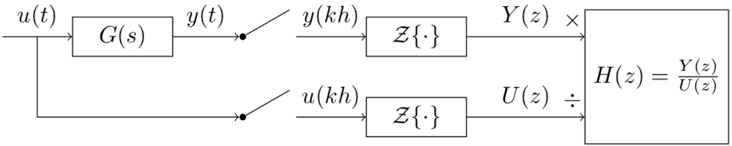
2018-08-29

Result from quizz

- Compute the z-transform
- Pulse transfer function/operator
- Modified z-transform

Impulse- step- and ramp-invariant sampling

The idea is to sample the continuous-time system's response to a step input, in order to obtain a discrete approximation which is exact (at the sampling instants) for such an input signal.



- Impulse-invariant sampling: $u(t) = \delta(t)$
- Step-invariant sampling (zero order hold): $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$
- Ramp-invariant sampling: $u(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Step-invariant sampling, or zero-order-hold sampling

Let the input to the continuous-time system be a step $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$, which has Laplace transform $U(s) = \frac{1}{s}$. In the Laplace-domain we get

$$Y(s) = G(s)\frac{1}{s}$$

1. Obtain the time-response by inverse Laplace: $y(t) = \mathcal{L}^{-1}\{Y(s)\}$
2. Sample the time-response to obtain the sequence $y(kh)$ and apply the z-transform to obtain $Y(z) = \mathcal{Z}\{y(kh)\}$
3. Calculate the pulse-transfer function by dividing with the z-transform of the input signal $U(z) = \frac{z}{z-1}$.

$$H(z) = \frac{Y(z)}{U(z)} = \frac{z-1}{z}Y(z)$$

Example: First-order system

Let's apply step-invariant sampling to the system

$$G(s) = \frac{1}{s - \lambda}.$$

Do on your own: The double integrator

$$G(s) = \frac{1}{s^2}$$