Harddisk drive control design exercise (Å&W ex 1.2)

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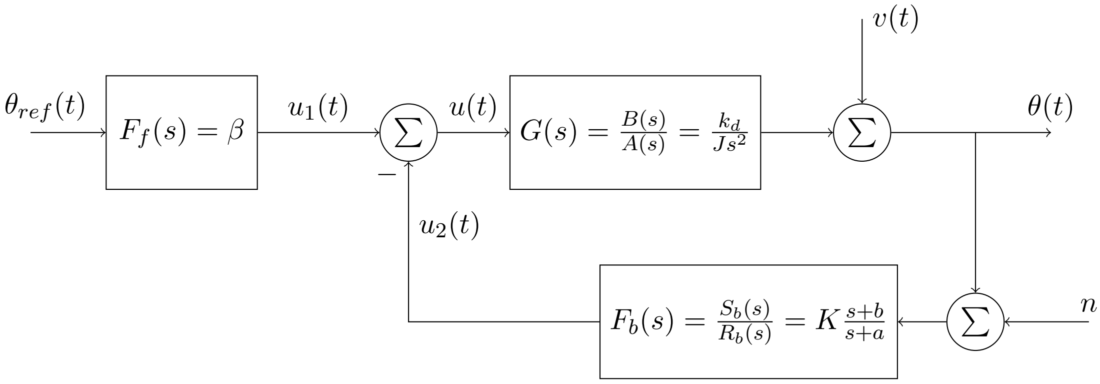
The closed-loop transfer functions



1. **Show that the closed-loop system from the three input signals $\theta_{ref}(t)$, $v(t)$ and $n(t)$ is given by**

$$\Theta(s) = \frac{G(s)F_f(s)}{1+G(s)F_b(s)}\Theta_{ref}(s) + \underbrace{\frac{1}{1+G(s)F_b(s)}}_{S(s)}V(s) - \underbrace{\frac{G(s)F_b(s)}{1+G(s)F_b(s)}}_{T(s)}N(s)$$

2. **Show that $S(s) + T(s) = 1$**

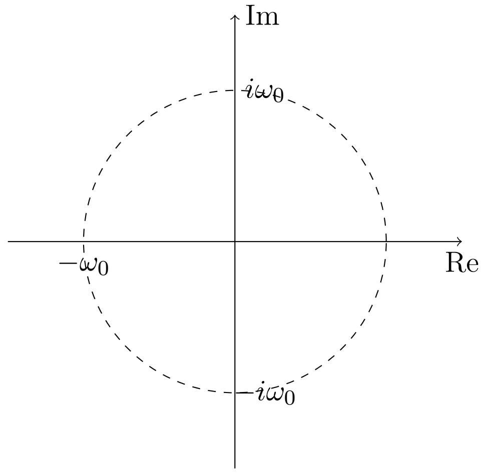
The characteristic equation

Show that the characteristic equation for the closed-loop system is

$$A(s)R_b(s) + B(s)S_b(s) = s^3 + as^2 + \frac{Kk_d}{J}s + \frac{Kk_d}{J}b = 0. \tag{1}$$

Desired closed-loop poles

The closed-loop system is of order three, so there are three poles to specify. Assume that the specification on the speed of the response of the closed-loop system requires the poles to be at a distance of ω_0 from the origin. **Sketch suitable positions of the three poles in the s-plane below.**



Determine the controller parameters

A suitable set of three closed-loop poles is obtained as the roots of the characteristic polynomial

$$s^3 + 2\omega_0 s^2 + 2\omega_0^2 s + \omega_0^3. \tag{2}$$

By comparing the characteristic polynomial in (1) to the desired polynomial in (2), we can determine the controller parameters. **Find the parameters of the feedback controller.**

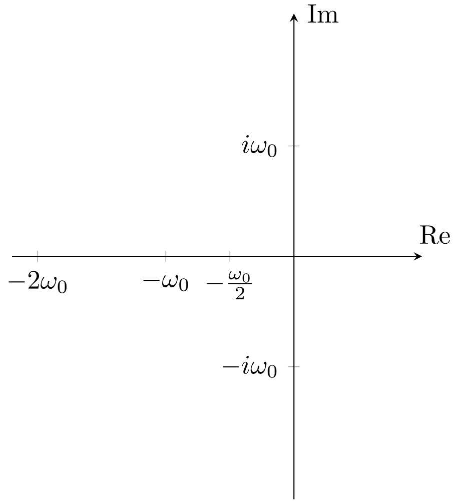
$a =$

$b =$

$K =$

Root locus

Consider the closed-loop characteristic equation $1 + K\frac{s+b}{s^2(s+a)} = 0$ using the values for a and b found above (but leave $K > 0$ undetermined). **Draw the root locus w.r.t the controller gain K.**



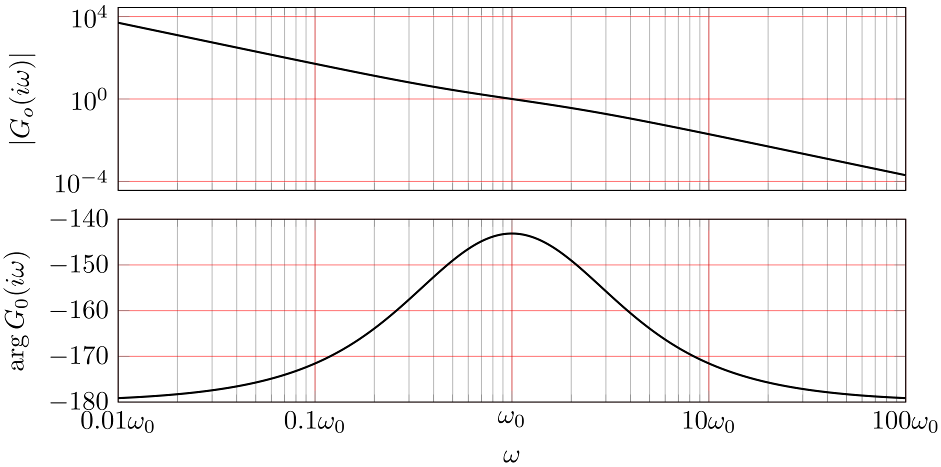
Is $F_b(s) = K\frac{s+b}{s+a}$ a *lead-compensator* or a *lag-compensator*?

The loop gain

Show that the loop gain of the closed-loop system is

$$G_o(s) = G(s)F_b(s) = \frac{\omega_0^2(2s+\omega_0)}{s^2(s+2\omega_0)}.$$

Determine and mark the (amplitude) crossover frequency ω_c, the phase-crossover frequency ω_p, the phase margin φ_m and the gain margin A_m in the bodeplot of the loop gain below.



The set-point weighting

The closed-loop transfer function from reference signal to output is

$$G_c(s) = \frac{G(s)F_f(s)}{1 + G(s)F_b(s)} = \frac{\beta \frac{k_d/J}{s^2}}{1 + \frac{k_d/J}{s^2} K \frac{s+b}{s+a}} = \frac{\beta \frac{k_d}{J}(s+a)}{s^2(s+a) + K\frac{k_d}{J}(s+b)}.$$

Determine β such that the closed-loop transfer function has unit static gain ($G_c(0) = 1$).

Discretizing the controller

The control law is

$$\begin{aligned} U(s) &= F_f(s)\Theta_{ref}(s) - F_b(s)\Theta(s) \\ &= \underbrace{\beta\Theta_{ref}(s)}_{U_1(s)} - \underbrace{K\frac{s+b}{s+a}\Theta(s)}_{U_2(s)} \end{aligned}$$

which means that it an be written in the time-domain as $u(t) = u_1(t) - u_2(t)$, where

$$u_1(t) = \beta\theta_{ref}(t), \tag{3}$$

and the output feedback $u_2(t)$ is

$$U_2(s) = K \cdot \frac{s+b}{s+a}\Theta(s)$$

which be written as the ODE

$$\dot{u}_2 + au_2 = K(\dot{\theta} + b\theta).$$

The term $u_1(t)$ is straightforward to discretize as

$$u_1(t) = \beta\theta_{ref}(t) \quad \Rightarrow \quad u_1(kh) = \beta\theta_{ref}(kh).$$

The second term $u_2(t)$ requires a discrete approximation of the derivative. There are a number of different alternatives. The simplest is the Euler forward approximation

$$\dot{x}(t) = \frac{dx}{dt} \approx \frac{x(t+h) - x(t)}{h},$$

where h is the sampling interval. **Show that the Euler forward approximation gives**

$$u_2(kh+h) = (1-ah)u_2(kh) + K\Big(\theta(kh+h) + (1-bh)\theta(kh)\Big).$$

For which values of h is the discretized feedback controller a stable system?