Computerized Control - Polynomial pole placement (RST)

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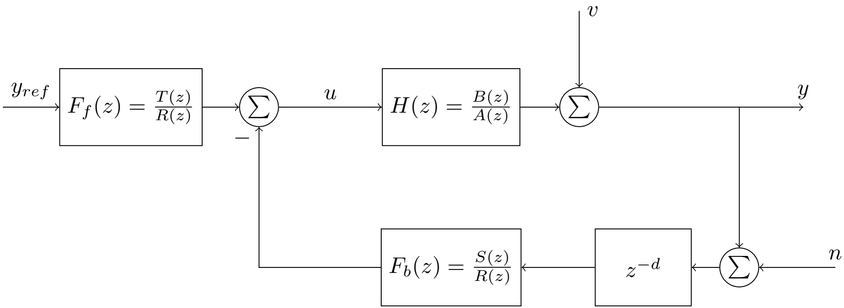
Goal of today's lecture

- Understand how to design an RST controller

Two-degree-of-freedom controller



Procedure

Given plant model $H(z) = \frac{B(z)}{A(z)}$ and specifications on the desired closed-loop poles $A_{cl}(z)$

1. Find polynomials $R(z)$ and $S(z)$ with $n_R \geq n_S$ such that

$$A(z)R(z)z^d + B(z)S(z) = A_{cl}(z)$$

2. Factor the closed-loop polynomials as $A_{cl}(z) = A_c(z)A_o(z)$, where $n_{A_o} \leq n_R$. Choose

$$T(z) = t_0 A_o(z),$$

where $t_0 = \frac{A_c(1)}{B(1)}$.

The control law is then

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k).$$

The closed-loop response to the command signal is given by

$$A_c(q)y(k) = t_0 B(q)u_c(k).$$

Determining the order of the controller

With Diophantine equation

$$A(z)R(z)z^d + B(z)S(z) = A_{cl}(z) \qquad (*)$$

and feedback controller

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0 z^n + s_1 z^{n-1} + \cdots + s_n}{z^n + r_1 z^{n-1} + \cdots + r_n}$$

How should we choose the order of the controller? Note:

- the controller has $n + n + 1 = 2\deg R + 1$ unknown parameters
- the LHS of $(*)$ has degree $\deg\left(A(z)R(z)z^d + B(z)S(z)\right) = \deg A + \deg R + d$
- The diophantine gives as many (nontrivial) equations as the degree of the polynomials on each side when we set the coefficients equal.

$\Rightarrow$ Choose $\deg R$ so that $2\deg R + 1 = \deg A + \deg R + d$

Determining the order of the controller - Exercise 1

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b}{z+a}$$

and $d = 0$ (no delay), what is the appropriate degree of the controller

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0 z^n + s_1 z^{n-1} + \cdots + s_n}{z^n + r_1 z^{n-1} + \cdots + r_n}$$

so that all parameters can be determined from the diophantine equation

$$A(z)R(z) + B(z)S(z) = A_c(z)A_o(z)?$$

1. $n = 0$
2. $n = 1$
3. $n = 2$
4. $n = 3$

Determining the order of the controller - Exercise 1

With the plant model

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so that all parameters can be determined from the diophantine equation

$$A(z)R(z) + B(z)S(z) = A_c(z)A_o(z)?$$

1. $n = 0$	2.
3.	4.

Determining the order of the controller - Exercise 2

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 z + b_1}{z^2 + a_1 z + a_2}$$

and $d = 2$, what is the appropriate degree of the controller

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0 z^n + s_1 z^{n-1} + \cdots + s_n}{z^n + r_1 z^{n-1} + \cdots + r_n}$$

so that all parameters can be determined from the diophantine equation

$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)?$$

1. $n = 1$
2. $n = 2$
3. $n = 3$
4. $n = 4$

Determining the order of the controller - Exercise 2

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 z + b_1}{z^2 + a_1 z + a_2}$$

and $d = 2$, what is the appropriate degree of the controller

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0 z^n + s_1 z^{n-1} + \cdots + s_n}{z^n + r_1 z^{n-1} + \cdots + r_n}$$

so that all parameters can be determined from the diophantine equation

$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)?$$

1.	2.
3. $n = 3$	4.

Determining the order of the controller - Exercise 3

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 z + b_1}{z^2 + a_1 z + a_2}$$

and $d = 2$ the appropriate degree of the controller is 3

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0 z^3 + s_1 z^2 + s_2 z + s_3}{z^3 + r_1 z^2 + r_2 z + r_3}.$$

What are the possible choices of the degree of the observer polynomial $A_o(z)$ in

$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)?$$

1. less than 2	2. less than 3
3. higher than 2	4. less than or equal to 3

Determining the order of the controller - Exercise 3

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 z + b_1}{z^2 + a_1 z + a_2}$$

and $d = 2$ the appropriate degree of the controller is 3

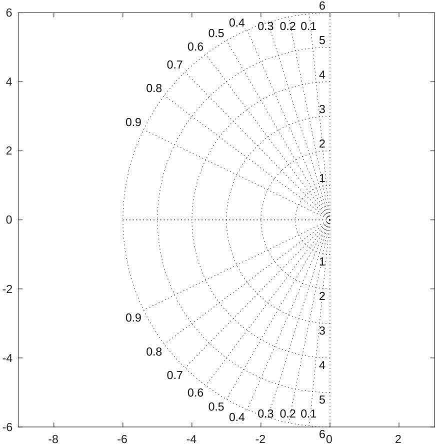
$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0 z^3 + s_1 z^2 + s_2 z + s_3}{z^3 + r_1 z^2 + r_2 z + r_3}.$$

What are the possible choices of the degree of the observer polynomial $A_o(z)$ in

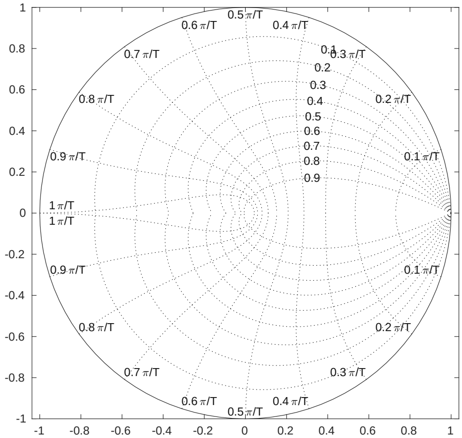
$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)?$$

1.
2.
3.
4. less than or equal to 3

Where to place the closed-loop poles?



s-plane



z-plane