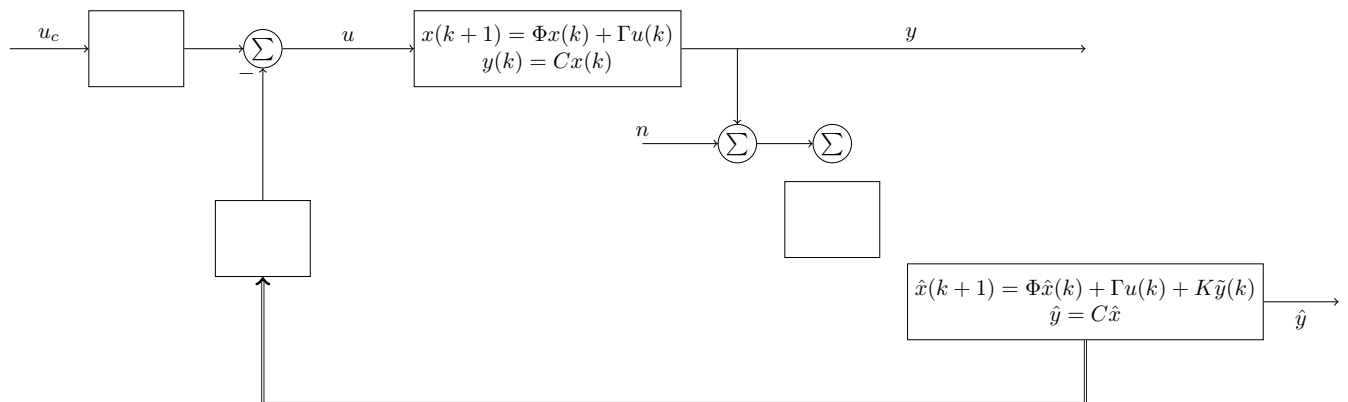


State feedback with observer exercise

Kjartan Halvorsen

March 16, 2018

Complete the block diagram



The control law is $u(k) = l_0 u_c(k) - L \hat{x}(k)$. The feedback to the observer is $K \tilde{y}(k) = K(y_m(k) - \hat{y}(k)) = K(y(k) + n(k) - C \hat{x}(k))$

Complete the augmented state-space model

$$\begin{bmatrix} x(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix} + \begin{bmatrix} \\ \end{bmatrix} u_c(k) + \begin{bmatrix} \\ \end{bmatrix} n(k)$$

$$y(k) = \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix}$$

Control of a tank model

Consider the system of two tanks in the figure below. The input signal is the flow of water into the upper tank, and the output signal is the level of the second tank.

In continuous-time the system is described by the state space system

$$\begin{aligned}\frac{dx}{dt} &= \begin{bmatrix} -0.0197 & 0 \\ 0.0178 & -0.0129 \end{bmatrix} x + \begin{bmatrix} 0.0263 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x.\end{aligned}$$

With sampling period $h = 12$ we obtain the discrete-time system

$$\begin{aligned}x(kh + h) &= \begin{bmatrix} 0.790 & 0 \\ 0.176 & 0.857 \end{bmatrix} x(kh) + \begin{bmatrix} 0.281 \\ 0.0296 \end{bmatrix} u(kh) \\ y(kh) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(kh).\end{aligned}$$

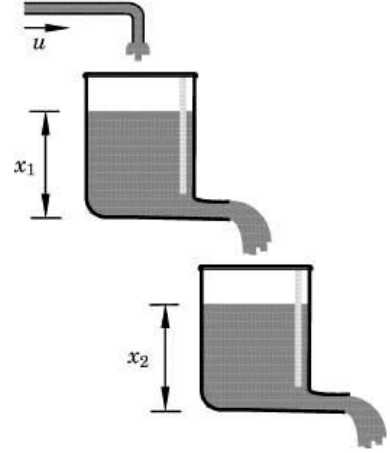
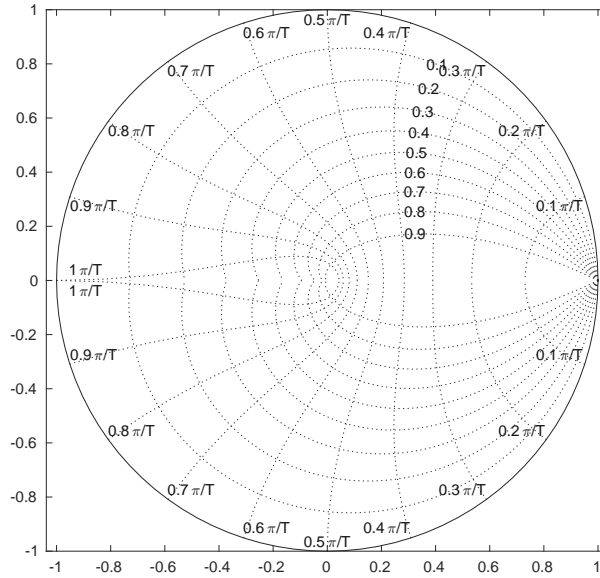


Figure 2.12 The two-tank process.

1. Determine an observer with poles that are twice as fast as the fastest mode of the plant.
2. Determine a state feedback gain L such that the closed-loop system has poles in $0.8 \pm 0.1j$.



Implementing the controller

Complete the pseudo-code (matlab) below for the implementation of the controller

```
% The plant model
n = 2;
h = 12;
Phi = [0.79 0; 0.176 0.857];
Gamma = [0.281; 0.0296];
C = [0 1];

% The controller and observer gains

L =

K =

% Initialize variables
u = 0; xhat = [0; 0];
% Run the controller
while run_controller() % The function run_controller will return 0 if the controller should stop
    y = get_output(); % Returns the sampled output signal from the plant

    % Update the observer

    xhat =

    % Compute the control signal

    u =

    % Send the control signal to the plant
    write_control_signal(u);
    % Wait until next sampling instant
    sleep(h);
end
```