Computerized control - homework 2

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Due 2018-08-24 at midnight

Exercises

1 Sample the continuous-time harmonic oscillator

Consider the harmonic oscillator with transfer function

$$G(s) = \frac{\omega_0^2 s}{s^2 + \omega_0^2}. \tag{1}$$

1. Determine the poles of $G(s)$.
2. Compute the pulse-transfer function by zero-order-hold sampling (step-invariant sampling) of the transfer function $G(s)$.
3. Determine the poles of the pulse-transfer function. What is the relationship between these poles and the corresponding poles of $G(s)$?
4. Let $\omega_0 = 1$. Describe in words how the poles depend on the sampling period h. In particular, discuss what happens when $\omega_0 h > \pi$.

2 Simulate the continuous- and the discrete-time harmonic oscillator

Use matlab's control toolbox to simulate the system and verify your calculations.

Define systems

First, define the continuous-time system in (1)

```
omega0 = ?; % Use the average of the last digit of your two phone numbers.
sys_c = tf(?,?)
```

Sample the system using the function `c2d`

```
h = 0.2/omega0; % This gives about 30 samples per period
sys_c2d = c2d(?,?)
```

Define the discrete-time system you calculated in the first part of the homework

```
den = [1 a1 a2];
num = [b1 b2];
sys_d = tf(num, den, h)
```

Verify that the two discrete-time systems `sys_c2d` and `sys_d` are identical.

Simulate step responses

Simulate for 4 complete periods

```
Tc = linspace(0, 4*(2*pi/omega0), 800);
[yc,tc] = step(sys_c, Tc);

Td = h*(0:120);
[yd,td] = step(sys_d, Td);

figure()
clf
plot(tc,yc)
hold on
plot(td,yd, 'r*')
```

Verify that the step response of the discrete-time system is exactly equal to that of the continuous-time system at the sampling instants. Explain why this is so!

Compute the discrete step response yourself

Write some lines of code that solves the difference equation

$$y(k+2) = -a_1 y(k+1) - a_2 y(k) + b_1 u(k+1) + b_2 u(k)$$

for the harmonic oscillator. Use the initial state $y(-1) = y(0) = 0$ and compute the response to a step sequence

$$u(k) = \begin{cases} 1, & k \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

Verify that your solution is the same as when using the `step` function in the previous exercise in this homework.