Computerized control - homework 3

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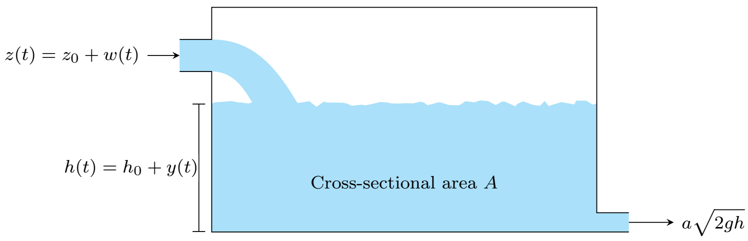
The system

Consider the linearized model of the tank that we looked at in class



Using the parameter values

$$A = 1, \qquad a = 0.1, \qquad g = 9.8,$$

and the operating point given by

$$h_0 = 1, \qquad z_0 = a\sqrt{2gh_0} \approx 0.44,$$

the linearized model of the tank is described by the first-order system

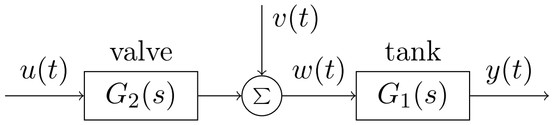
$$G_1(s) = \frac{1}{s + 0.44}$$

from the deviation in flow $w(t)$ to the deviation in level $y(t)$.

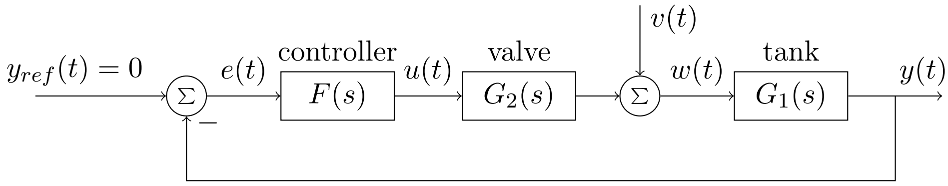
A valve is used to control the flow. The valve is a so-called control valve, which means it includes an inner controller that works as a position servo. That is, it will make sure the opening of the valve follows the input signal to the valve. This signal is named $u(t)$. The response of the opening of the valve $\theta(t)$ to the input signal $u(t)$ is well-described by a second-order, critically damped system

$$G_2(s) = \frac{1}{(0.5s + 1)(0.5s + 1)} = \frac{4}{(s + 2)(s + 2)}.$$

The flow through the valve depends also on the square root of the pressure difference across the valve. In a linearized model, a change in pressure enters as an additive disturbance to the system. The complete model of the process is given in the block-diagram below.



The level of the tank is measured, and is available for feedback control.



A simulation model (`simulink`) of the system is available on Blackboard under `Course Documents/Matlab and Simulink`

Exercises

Problem 1 - Tuning a PID

Perform a bumptest on the plant (valve+tank). This means to connect a step block (see `Sources` in the `Simulink Library Browser`) to the input of the valve. Determine the slope R, the apparent deadtime L, and the parameter $a = RL$ from the step response. See figure 8.13 in the text-book

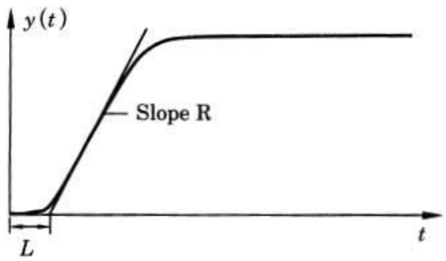


Figure 8.13 Determination of parameters $a = RL$ and L from the unit step response to be used in Ziegler-Nichols step-response method.

Include your simulated step-response in your report.

Determine a PID controller using table 8.2 in the book.

Table 8.2 PID parameters obtained from the Ziegler-Nichols step-response method.

Controller Type	K	T_i	T_d
P	$1/a$		
PI	$0.9/a$	$3L$	
PID	$1.2/a$	$2L$	$0.5L$

Problem 2 - Implement the PID in simulink

The controller is written

$$U(s) = K\left(U_c(s) - Y(s) + \frac{1}{sT_i}\big(U_c(s) - Y(s)\big) - \frac{sT_d}{1 + sT_d/N}Y(s)\right).$$

Set $N = 10$ and implement the controller in simulink using the values for K, T_i and T_d that you determined in Problem 1.

Simulate the closed-loop system's response to step changes in both the set point, $u_c(t)$ and the disturbance, $v(t)$. Include the step-responses in your report and comment on the results.

Problem 3 - Discrete PID

The discretized controller is written

$$R(\mathrm{q})u(kh) = T(\mathrm{q})u_c(kh) - S(\mathrm{q})y(kh),$$

where

$$\begin{aligned} R(\mathrm{q}) &= (\mathrm{q} - 1)(\mathrm{q} - a_d) \\ S(\mathrm{q}) &= s_0\,\mathrm{q}^2 + s_1\,\mathrm{q} + s_2 \\ T(\mathrm{q}) &= t_0\,\mathrm{q}^2 + t_1\,\mathrm{q} + t_2 \end{aligned}$$

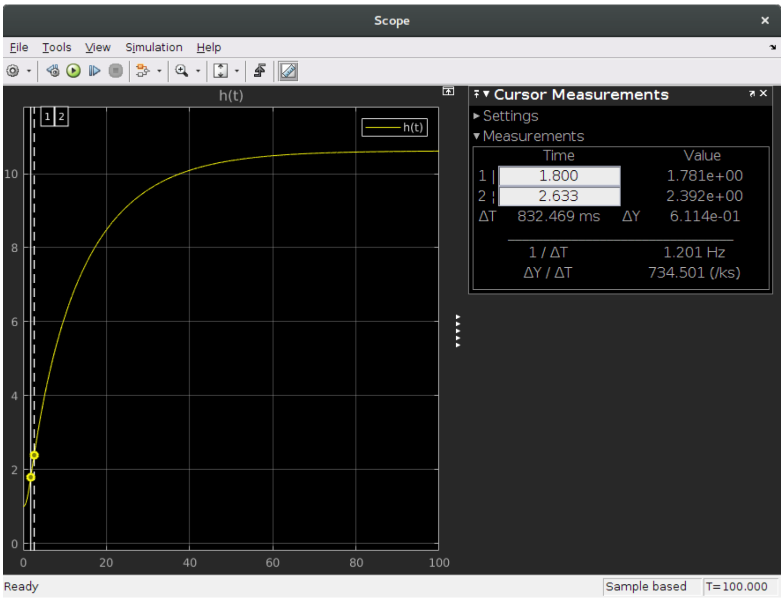
Determine the discrete PID controller parameters a_d, s_0, s_1, s_2, t_0, t_1 and t_2 using table 8.1 in the textbook (given below). You can use whichever of the three discretization methods provided.

	Special	Tustin	Ramp Equivalence
s_0	$K(1+b_d)$	$K(1+b_i+b_d)$	
s_1	$-K(1+a_d+2b_d-b_i)$	$-K\left(1+a_d+2b_d-b_i(1-a_d)\right)$	
s_2	$K(a_d+b_d-b_ia_d)$	$K(a_d+b_d-b_ia_d)$	
t_0	Kb	$K(b+b_i)$	
t_1	$-K(b(1+a_d)-b_i)$	$-K\left(b(1+a_d)-b_i(1-a_d)\right)$	
t_2	$Ka_d(b-b_i)$	$Ka_d(b-b_i)$	
a_d	$\dfrac{T_d}{Nh+T_d}$	$\dfrac{2T_d-Nh}{2T_d+Nh}$	$\exp\left(-\dfrac{Nh}{T_d}\right)$
b_d	Na_d	$\dfrac{2NT_d}{2T_d+Nh}$	$\dfrac{T_d}{h}(1-a_d)$
b_i	$\dfrac{h}{T_i}$	$\dfrac{h}{2T_i}$	$\dfrac{h}{2T_i}$

Solutions

Problem 1 - Tuning a PID

Below is the result from a bumptest on the plant (valve+tank).



The measurement points (t_1, y_1) and (t_2, y_2) were moved around in order to find two close points which gave the largest slope $R = \frac{y_2-y_1}{t_2-t_1}$. The apparent deadtime L is the intersection of the steepest tangent with the time axis. This can be found by noting that if we take the midpoint of y_1 and y_2 as the tangent point, then $R = \frac{y_2+y_1}{2L}$. We get

$$
\begin{aligned}
R &= \frac{y_2 - y_1}{t_2 - t_1} = 0.73 \\
L &= \frac{y_1 + y_2}{2R} = 2.85 \\
a &= RL = 2.09
\end{aligned}
$$

From table 8.2 we obtain the parameters

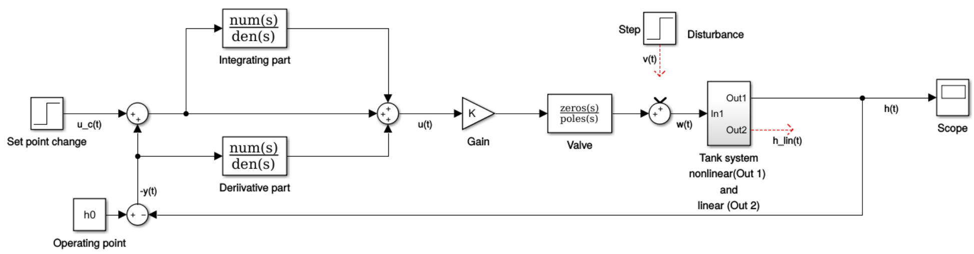
$$
\begin{aligned}
K &= 1.2/a = 0.575 \\
T_i &= 2L = 5.69 \\
T_d &= 0.5L = 1.42
\end{aligned}
$$

Problem 2 - Implement the PID in simulink

The controller is written

$$
U(s) = K\left(U_c(s) - Y(s) + \frac{1}{sT_i}\big(U_c(s) - Y(s)\big) - \frac{sT_d}{1 + sT_d/N}Y(s)\right).
$$

In `simulink` the model should look like the following

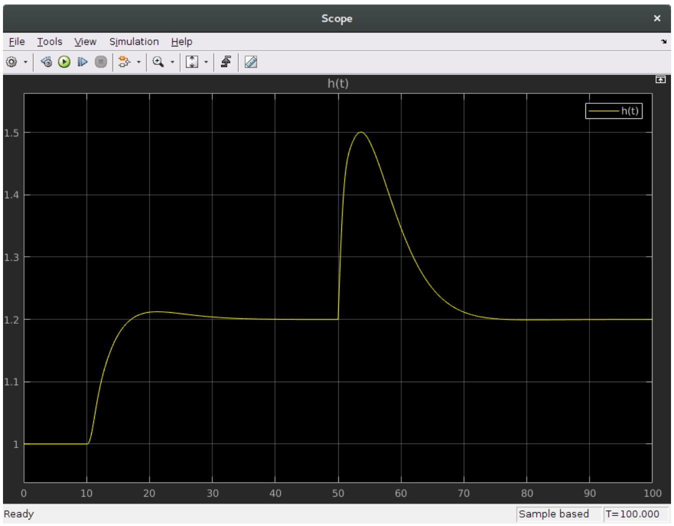


Note that the derivative part acts only on the feedback signal $-y(t)$. The blocks are

Derivative part num: [Td, 0], den: [Td/N, 1]

Integrating part num: [1], den: [Ti, 0]

A step response of the closed-loop system is given below. A step change in the set point occurs at time $t = 10$ and then a step in the disturbance occurs at time $t = 50$.



We can see that thanks to the integrating part of the controller, there is no steady-state error. It takes the system about 20 seconds to settle. There is some overshoot in the set-point response, but not too much. This should be acceptable.

Problem 3 - Discrete PID

The discretized controller is written

$$R(\mathrm{q})u(kh) = T(\mathrm{q})u_c(kh) - S(\mathrm{q})y(kh),$$

where

$$\begin{aligned} R(\mathrm{q}) &= (\mathrm{q}-1)(\mathrm{q}-a_d) \\ S(\mathrm{q}) &= s_0\,\mathrm{q}^2 + s_1\,\mathrm{q} + s_2 \\ T(\mathrm{q}) &= t_0\,\mathrm{q}^2 + t_1\,\mathrm{q} + t_2 \end{aligned}$$

Determine the discrete PID controller parameters a_d, s_0, s_1, s_2, t_0, t_1 and t_2 using table 8.1 in the textbook (given below).

Using the special discretization (and $b = 1$) we get

$$
\begin{aligned}
a_d &= \frac{T_d}{Nh + T_d} = \frac{1.42}{10h + 1.42} \\
s_0 &= K(1 + b_d) = K(1 + Na_d) = 0.575(1 + \frac{14.2}{10h + 1.42}) \\
s_1 &= -K(1 + a_d + 2b_d - b_i) = -0.575(1 + \frac{1.42 + 28.4}{10h + 1.42} - \frac{h}{5.69}) \\
s_2 &= K(a_d + b_d - b_i a_d) = 0.575\frac{1.42 + 14.2 - h/5.69}{10h + 1.42} \\
t_0 &= Kb = 0.575 \\
t_1 &= -K(b(1 + a_d) - b_i) = -0.575(\frac{10h + 1.42 + 1.42}{10h + 1.42} - \frac{h}{5.69}) \\
t_2 &= Ka_d(b - b_i) = 0.575\frac{1.42(1 - \frac{h}{5.69})}{10h + 1.42}
\end{aligned}
$$