PID exercises

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1 A PID controller with anti-windup

The figure below taken from Åström & Wittenmark shows a block-diagram of a PID controller with anti-windup.

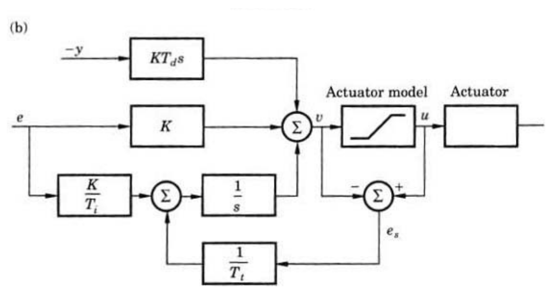


Figure 8.10 Controller with antiwindup. A system in which the actuator output is measured is shown in (a) and a system in which the actuator output is estimated from a mathematical model is shown in (b).

Consider the PID controller as having the output signal $v(t)$ and three input signals: The control error $e(t) = y_{ref}(t) - y(t)$, the output feedback signal $y(t)$ and the output signal of the actuator model $u(t)$.

1. Determine the transfer functions in
$$V(s) = F_y(s)Y(s) + F_e(s)E(s) + F_u(s)U(s).$$
2. Determine the pole(s) of the controller.
3. Determine the steady-state output of the controller for step input signals (all at once) $Y(s) = \frac{\alpha}{s}$, $E(s) = \frac{\beta}{s}$ and $U(s) = \frac{\gamma}{s}$.
4. How should we interpret the steady-state value if the actuator is not saturating, so that $u(t) = v(t)$?

1.1 Solution

1. Transfer functions

$$V = -KT_d sY + KE + 1/s\big(K/T_i E + 1/T_t(U - V)\big)$$

$$V + 1/(T_t s)V = -KT_d sY + (K + K/(T_i s))E + 1/(T_t s)U$$

$$V = \frac{-KT_d s}{1 + 1/(T_t s)}Y + \frac{K + K/(T_i s)}{1 + 1/(T_t s)}E + \frac{1/(T_t s)}{1 + 1/(T_t s)}$$

$$V = \frac{-KT_d s^2}{s + 1/T_t}Y + \frac{K(s + 1/T_i)}{s + 1/T_t}E + \frac{1/T_t}{s + 1/T_t}U$$

2. Final value for step-response from the three insignals:

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} s\frac{-KT_d s^2}{s + 1/T_t}\alpha/s + \lim_{s\to 0} s\frac{K(s + 1/T_i)}{s + 1/T_t}\beta/s + \lim_{s\to 0} s\frac{1/T_t}{s + 1/T_t}\gamma/s$$

$$\lim_{t\to\infty} v(t) = 0\cdot\alpha + \frac{KT_t}{T_i}\beta + \gamma$$

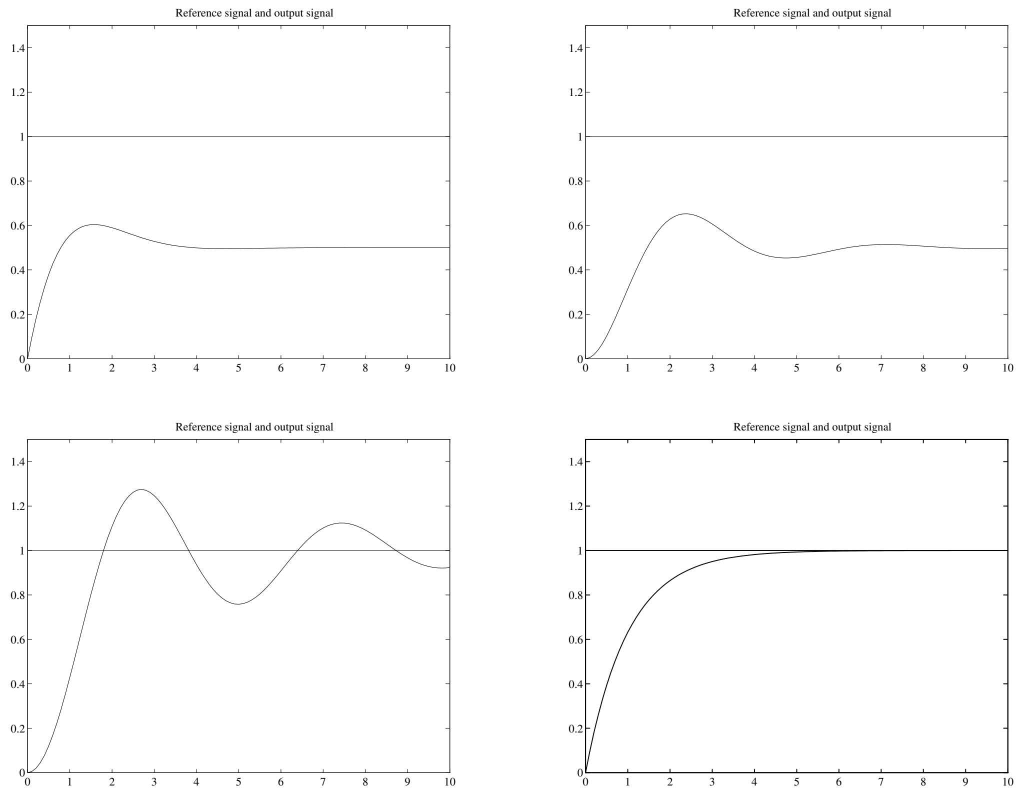
3. Interpretation: Output of controller does not grow, even with constant error signal $e(t)$. If $u = v$, then the final value must be $v(\infty) = \gamma$. This can only happen if $\beta = 0$, i.e. the control error is zero. So, as long as the actuator does not saturate, the controller will give zero steady-state error. If the actuator does saturate, then \$v(∞) $>$ γ \$ and so $\beta > 0$, i.e. there will be a steady-state error

2 The effect of PID controller parameters

Assume a certain system being controlled by a PID regulator, $U(s) = (K_P + K_I\frac{1}{s} + K_D s)E(s)$. In the figure below shows four step responses for the parameter triples

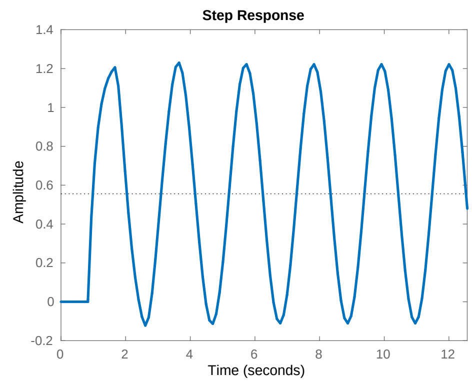
$$\begin{array}{llll} i) & K_P = 1 & K_I = 0 & K_D = 0 \\ ii) & K_P = 1 & K_I = 1 & K_D = 0 \\ iii) & K_P = 1 & K_I = 0 & K_D = 1 \\ iv) & K_P = 1 & K_I = 1 & K_D = 1 \end{array}$$

Match each one of the parameter triples to one of the step responses. Justify your answer!



3 Ziegler-Nichols ultimate gain tuning

Ziegler and Nichols developed their tuning rules for PID regulators given that the system can be approximated as a first order system with time delay: $G(s) = \frac{e^{-sT}}{s+a}$. We perform a resonance test according to Ziegler-Nichols' ultimate gain method and with the proportional gain $K = 5 = K_u$ we get the closed-loop step response seen below



1. What is the period T_u and angular frequency ω_u of the oscillations?
2. Determine the parameters T and a in the transfer function $G(s)$.
3. Tune a PI regulator
$$u(t) = K_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau \right)$$
according to Ziegler-Nichols' recipe.

Controller type	K_p	T_i	T_d
P	$0.5K_u$		
PI	$0.45K_u$	$T_u/1.2$	
PD	$0.8K_u$		$T_u/8$
PID	$0.6K_u$	$T_u/2$	$T_u/8$

4. The Nyquist plot of the resulting loop gain $G_o(s) = F(s)G(s)$ is shown below. What are the stability margins?

5. The cross-over frequency is $\omega_c = 0.39$ rad/s with the tuned PI controller. Determine a sampling period h for a discrete-time implementation of the PI controller, such that the phase margin is reduced by about 10°.

3.1 Solution

1. Period $T_u = (12 - 1.5)/5 = 2.1$ sec. Corresponding to $\omega_u = \frac{2\pi}{T_u} = 3.0$ rad/s.
2. The loop gain is $G_o(s) = 5\frac{e^{-sT}}{s+a}$. Since we have undamped oscillations of frequency $\omega_u = 3$ rad/s, then the closed-loop system must have poles on the imaginary axis at $\pm i\omega_u$. That is have $1 + G_o(i\omega_u) = 0$ or $G_o(i\omega_u) = -1$. The Nyquist curve goes through the critical point -1. We get the equations
$$|G_o(i\omega_u)| = 1$$

$$\arg G_o(i\omega_u) = \pi$$

3.
$$F(s) = 0.45K_u(1 + \frac{1.2}{T_u s}) = 2.25(1 + \frac{1.2}{2.1s}).$$

4. Phase margin about 100 degrees, amplitude margin about 2.

5. The discretization gives a delay of about h/2, which contributes with negative phase to the loop gain. We get $G_{d,o}(i\omega) \approx \mathrm{e}^{-i\omega h/2} G_o(i\omega)$. At the cross-over frequency, the negative phase contribution from delay is $\arg \mathrm{e}^{-i\omega_c h/2} = -\omega_c h/2 = -10 * pi/180$ which gives $\omega_c h = 2 * 20 * pi/180 = 0.35$ Note: rule-of-thumb says $\omega_c h \approx 0.15 - 0.5$. Here: h $= 0.35/0.39 \approx 0.90$ s.