State feedback exercise

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Input signal design

Consider the sampled double-integrator on state-space form

$$\begin{aligned}
x(kh+h) &= \underbrace{\begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}}_{\Phi(h)} x(kh) + \underbrace{\begin{bmatrix} \frac{h^2}{2} \\ h \end{bmatrix}}_{\Gamma(h)} u(kh) \\
y(kh) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(kh)
\end{aligned} \tag{1}$$

We want to find the input sequence $u(0)$, $u(h)$ that takes the system from the origin of the state space $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$ to the point $x(2h) = \begin{bmatrix} a & 0 \end{bmatrix}^{\mathrm{T}}$ in two time steps.

Iterate the state space model

… in order to find an expression for $x(2h)$ in terms of the unknown input signals

$$\begin{aligned}
x(h) &= \Phi x(0) + \Gamma u(0) \\
x(2h) &= \Phi x(h) + \Gamma u(h) = \Phi\left(\Phi x(0) + \Gamma u(0)\right) + \Gamma u(h) \\
&= \Phi^2 x(0) + \Phi\Gamma u(0) + \Gamma u(h) = \Phi^2 x(0) + \underbrace{\begin{bmatrix} \quad\quad\quad \end{bmatrix}}_{W_c} \begin{bmatrix} u(h) \\ u(0) \end{bmatrix}
\end{aligned}$$

Set up system of equations

With $u = \begin{bmatrix} u(h) & u(0) \end{bmatrix}^{\mathrm{T}}$ we get the linear system of equations $W_c u = x(2) - \Phi^2 x(0)$. In the particular case we are considering here this gives

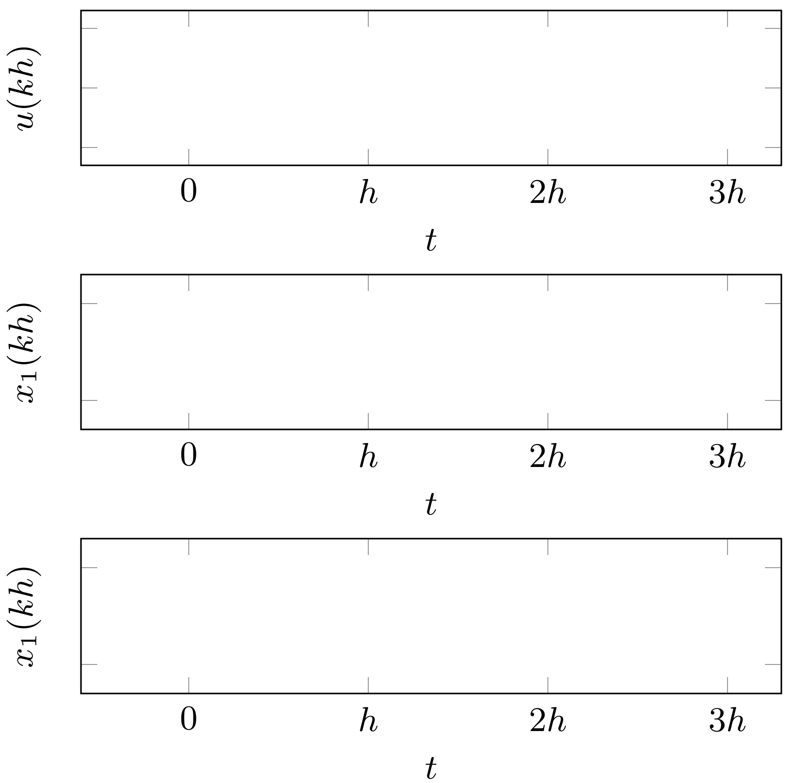
$$\begin{bmatrix} \quad\quad\quad & \\ & \quad\quad\quad \end{bmatrix} \begin{bmatrix} u(h) \\ u(0) \end{bmatrix} = \begin{bmatrix} \quad\quad \\ \quad\quad \end{bmatrix}.$$

Verify the result

Calculate the state sequence

$$x(h) =$$
$$x(2h) =$$

Plot the results



The deadbeat control law

We are looking for the linear control law $u(kh) = l_0 y_{ref}(kh) - Lx(kh) = l_0 y_{ref}(kh) - l_1 x_1(kh) - l_2 x_2(kh)$ that produced the result above.

Set up the equations

$k = 0$: $u(0) =$

$k = 1$: $u(h) =$

$k = 2$: $u(2h) =$

Solve the system of equations

Write the system of equations

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \begin{bmatrix} l_0 \\ l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}.$$

The poles of the closed-loop system

Inserting the control law $u(kh) = l_0 y_{ref}(kh) - Lx(kh)$ into the state-space system (1) gives the closed-loop system

$$
\begin{aligned}
x(kh+h) &= \Phi x(kh) + \Gamma u(kh) = \Phi x(kh) - \Gamma L x(kh) + \Gamma l_0 y_{ref}(kh) \\
&=
\end{aligned}
$$

Determine the poles of the closed-loop system.