State feedback exercise

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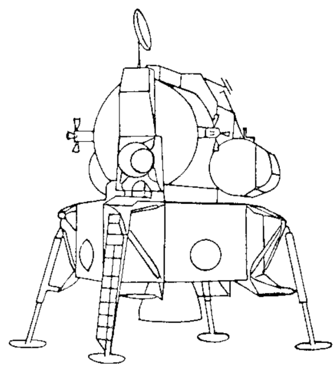
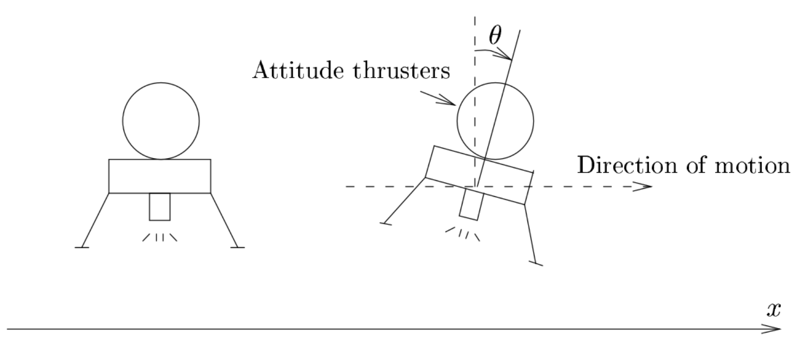
1 Velocity control for the Apollo Lunar Excursion Module

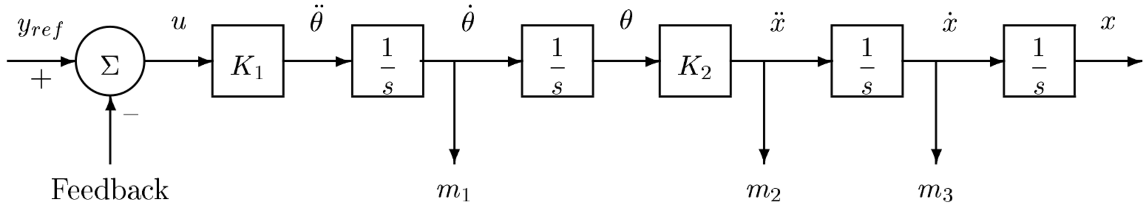


Consider the module hovering a short distance above the surface of the moon using its main engine. If the pitch angle of the module (angle between the vertical line and the direction of movement) differs from zero, a horizontal component of the force is obtained and the module is accelerating along the surface.



The block-diagram below shows the connection between the input signal $u(t)$ (the control signal to the attitude thrusters), the pitch angle $\theta(t)$ and the horizontal position coordinate

$x(t)$.



The module is obeying Newton's second law of motion in both the θ-direction and in the x-direction, and there are no damping (friction) present. The open-loop transfer function from the control signal of the astronaut (y_{ref}) to velocity $\dot{x}$ is

$$\frac{K_1 K_2}{s^3}$$

which is impossible to control by hand.

In order to make it possible for the astronaut to control the lunar module, we change the dynamics of the module by internal state feedback. The following signals are available:

m_1 the attitude angular velocity measured using rate gyro.

m_2 the acceleration in x-direction measured using accelerometers positioned on gyro-stabilized platforms.

m_3 the velocity in x-direction measured using doppler-radar.

Calculate a state-feedback using these signals such that the closed-loop system obtains its poles in $s = -\frac{1}{2}$ and the control signal of the astronaut becomes the reference signal of the velocity in x-direction.

1.1 Solution

Choose state variables according to the available signals.

$$\begin{aligned} z_1(t) &= m_1(t) = \dot{\theta}(t) \\ z_2(t) &= m_2(t) = \ddot{x}(t) \\ z_3(t) &= m_3(t) = \dot{x}(t) \end{aligned}$$

We have the following ODE's governing the dynamics

$$\begin{aligned} \dot{z}_1(t) &= \ddot{\theta}(t) = K_1 u(t) \\ \dot{z}_2(t) &= K_2 \dot{\theta} = K_2 z_1(t) \\ \dot{z}_3(t) &= \ddot{x}(t) = z_2(t) \end{aligned}$$

This gives the state space model

$$\dot{z} = \begin{bmatrix} 0 & 0 & 0 \\ K_2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} z + \begin{bmatrix} K_1 \\ 0 \\ 0 \end{bmatrix} u \tag{1}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} z \tag{2}$$

Introducing the linear state feedback

$$u = -Lz + l_0 r = -l_1 z_1 - l_2 z_2 - l_3 z_3 + l_0 r$$

we get the closed-loop system

$$\begin{aligned} \dot{z} &= (A - BK)z + l_0 B r \\ &= \left(\begin{bmatrix} 0 & 0 & 0 \\ K_2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} - K_1 \begin{bmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) z + l_0 B r \\ &= \begin{bmatrix} -K_1 l_1 & -K_1 l_2 & -K_1 l_3 \\ K_2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} z + l_0 B r \end{aligned}$$

with characteristic polynomial given by

$$\begin{aligned} \det(sI - (A - BK)) &= \det \begin{bmatrix} s + K_1 l_1 & K_1 l_2 & K_1 l_3 \\ -K_2 & s & 0 \\ 0 & -1 & s \end{bmatrix} \\ &= (s + K_1 l_1) \det \begin{bmatrix} s & 0 \\ -1 & s \end{bmatrix} - K_1 l_2 \det \begin{bmatrix} -K_2 & 0 \\ 0 & s \end{bmatrix} + K_1 l_3 \det \begin{bmatrix} -K_2 & s \\ 0 & -1 \end{bmatrix} \\ &= (s + K_1 l_1) s^2 + K_1 K_2 l_2 s + K_1 K_2 l_3 \\ &= s^3 + K_1 l_1 s^2 + K_1 K_2 l_2 s + K_1 K_2 l_3 \end{aligned}$$

The desired characteristic polynomial is (three poles in -0.5)

$$(s + 0.5)^3 = s^3 + \frac{3}{2}^2 + \frac{3}{4} s + \frac{1}{8}.$$

Setting the coefficients equal in the two characteristic polynomials gives the system of equations

$$K_1 l_1 = \frac{3}{2} \tag{3}$$

$$K_1 K_2 l_2 = \frac{3}{4} \tag{4}$$

$$K_1 K_2 l_3 = \frac{1}{8} \tag{5}$$

with straight-forward solution

$$l_1 = \frac{3}{2K_1} \tag{6}$$

$$l_2 = \frac{3}{4K_1K_2} \tag{7}$$

$$l_3 = \frac{1}{8K_1K_2} \tag{8}$$

It can be shown that the state feedback does not change the numerator in the transfer function. The open-loop system has transfer function

$$G(s) = \frac{B(s)}{A(s)} = \frac{K_1K_2}{s^3},$$

so the closed-loop system must have transfer function

$$G_c(s) = l_0 \frac{K_1K_2}{s^3 + \frac{3}{2}^2 + \frac{3}{4}s + \frac{1}{8}}.$$

We can now choose the gain l_0 so that the closed-loop system has unit static gain from the reference signal $r(t)$ to the output $y(t) = \dot{x}(t)$

$$G_c(0) = l_0 \frac{K_1K_2}{\frac{1}{8}} = 1$$

gives

$$l_0 = \frac{1}{8K_1K_2}.$$