Computerized control - Homework 6

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1 Controller design by state feedback

Sampling the DC-motor with transfer function and state-space representation

$$G(s) = \frac{1}{s(s+1)}$$

$$\begin{aligned}\dot{x} &= \begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix} x + \begin{bmatrix}0\\1\end{bmatrix}\\ y &= \begin{bmatrix}1 & 0\end{bmatrix} x\end{aligned}$$

gives the discrete-time state-space model

$$\begin{aligned}x(k+1) = \Phi x(k) + \Gamma u(k) &= \begin{bmatrix}\mathrm{e}^{-h} & 1-\mathrm{e}^{-h}\\ 0 & 1\end{bmatrix} x(k) + \begin{bmatrix}\mathrm{e}^{-h}+h-1\\ h\end{bmatrix}\\ y(k) = Cx(k) &= \begin{bmatrix}1 & 0\end{bmatrix} x(k).\end{aligned}$$

1.1 Determine the pulse-transfer function

Show that the corresponding pulse-transfer operator is given by

$$H(q) = \frac{B(q)}{A(q)} = \frac{(q-1)(\mathrm{e}^{-h}+h-1)+h(1-\mathrm{e}^{-h})}{(q-1)(q-\mathrm{e}^{-h})}. \tag{1}$$

1.2 Reachability and observability

Show that the discrete-time system is both reachable and observable.

1.3 Design the feedback control

Choose a suitable sampling period h and determine L and m_0 in the state feedback

$$u(k) = -Lx(k) + m_0 u_c(k)$$

such that the closed loop system has poles in $0.5 \pm i0.5$, and so that the closed-loop system has static gain equal to 1.

1.4 Implement the model

Implement the model in Simulink or in Matlab. Simulate a step-response and attach to your report. Verify that the sampling period is reasonable based on the step-response: Determine the rising time and compare it to the sampling period you chose.