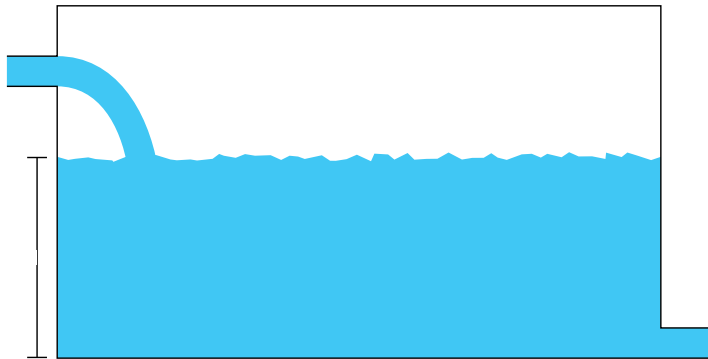


Process Automation Laboratory - Modeling first-order systems

Kjartan Halvorsen

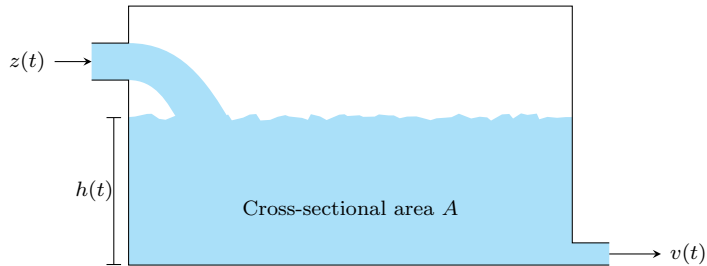
August 19, 2021

First-order system example: A tank



1. What is the **state** of the system?
2. What is the **input signal** and **output signal**?

First-order system example: A tank

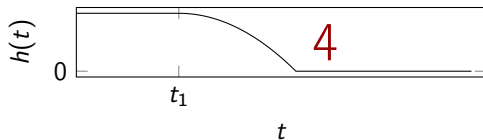
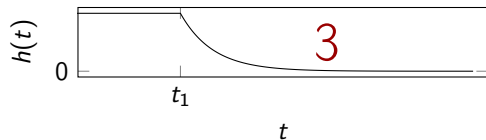
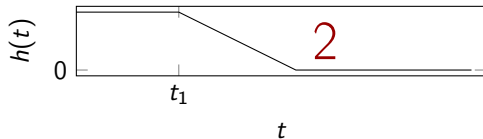
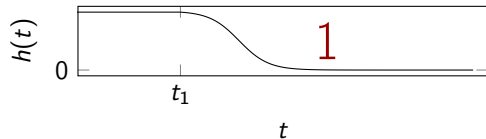


$$\begin{aligned}\frac{d}{dt}(Ah) &= z(t) - x(t) = z(t) - a\sqrt{2gh} \Rightarrow \\ \frac{d}{dt}h(t) &= -\frac{a\sqrt{2g}}{A}\sqrt{h(t)} + \frac{1}{A}z(t)\end{aligned}$$

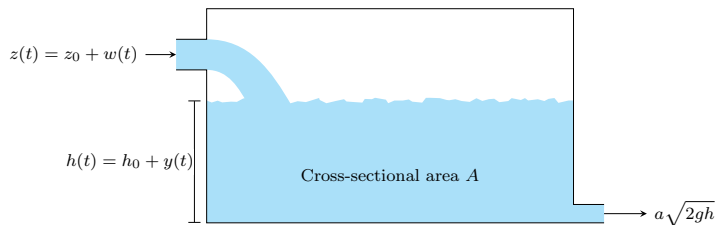
Intuition



Individual activity A constant inflow has been present since forever, but at time t_1 the flow in is suddenly shut off. Which of the responses of the water level $h(t)$ below is correct?



Deviation variables



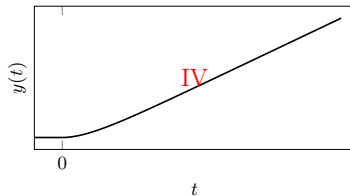
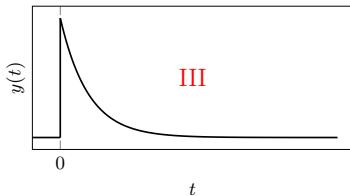
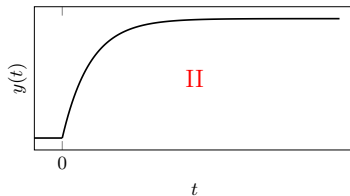
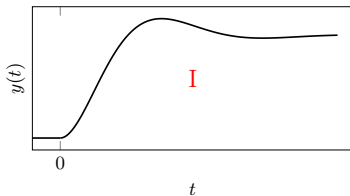
Flow in: $z(t) = z_0 + w(t)$. Level of water: $h(t) = h_0 + y(t)$. The constants h_0 and z_0 define an *operating point*.

$$\frac{d}{dt}h(t) = -\frac{a\sqrt{2g}}{A}\sqrt{h(t)} + \frac{1}{A}z(t)$$

Individual activity Given h_0 determine the operating point for the inflow, z_0 , such that the system is in equilibrium at the operating point.

Intuition

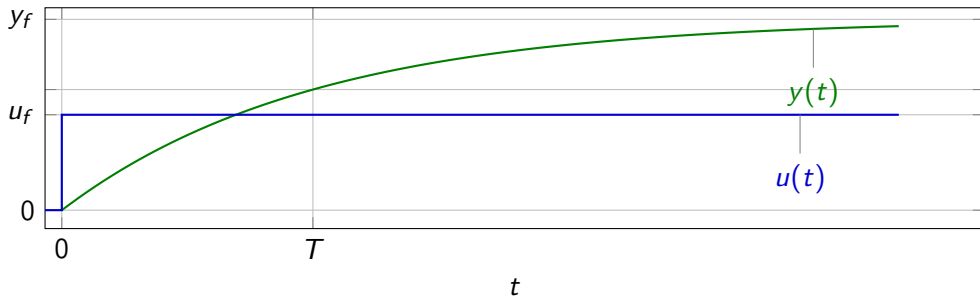
Which change $y(t)$ in the water level corresponds to a step change $w(t)$ in the inflow?



Fitting a first-order model

Assuming a plant model of first-order with time-constant T

$$Y(s) = \frac{K}{sT + 1} U(s) \quad U(s) = \frac{u_f}{s} \quad \Longrightarrow \quad y(t) = u_f K (1 - e^{-\frac{t}{T}}) u_H(t)$$

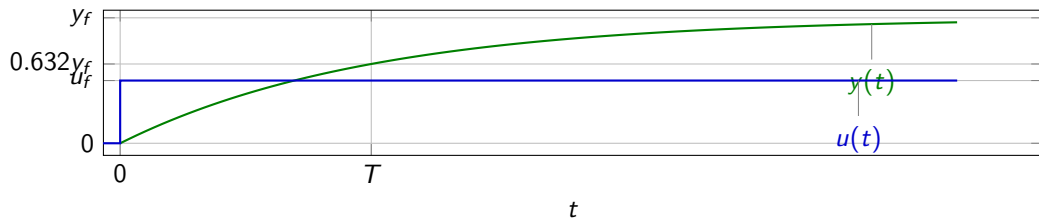


Individual activity Evaluate the response $y(t)$ at the time instant $t = T$ and for $t \rightarrow \infty$!

Fitting a first-order model

Assuming a plant model of first-order with time-constant T

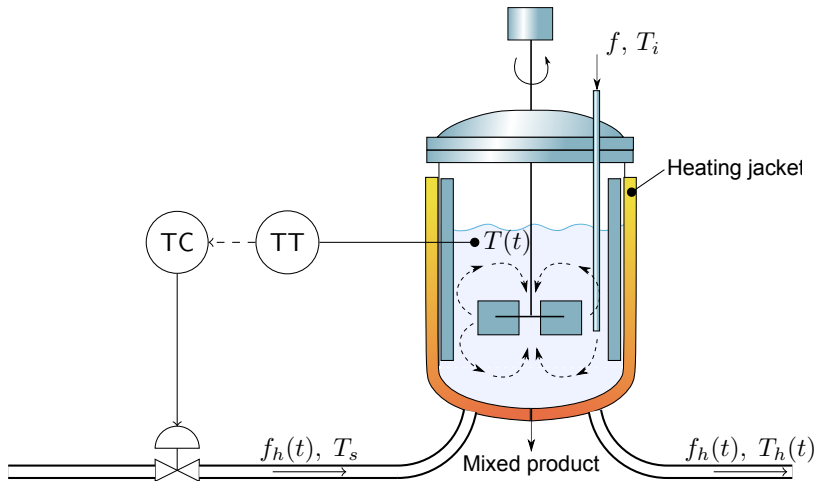
$$Y(s) = \frac{K}{sT + 1} U(s) \quad U(s) = \frac{u_f}{s} \quad \Rightarrow \quad y(t) = u_f K (1 - e^{-\frac{t}{T}}) u_H(t)$$



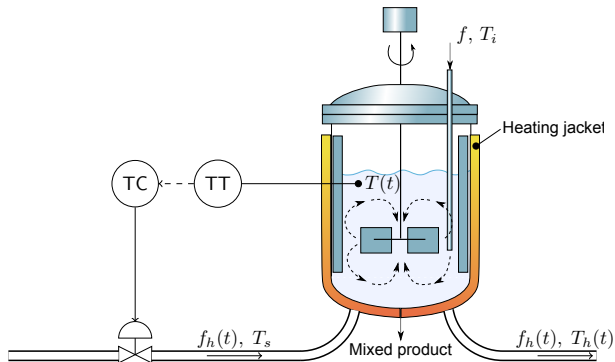
Time-constant: Find the time $t = T$ at which the response has reached 63.2% of its final value

Gain: $y_f = \lim_{t \rightarrow \infty} y(t) = K u_f \quad \Rightarrow \quad K = \frac{y_f}{u_f}$

A Continuous Stirred Tank Reactor



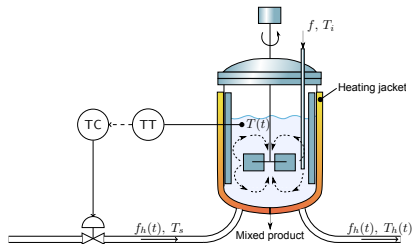
A Continuous Stirred Tank Reactor



Assume:

1. Constant flow f through the tank reactor
2. Constant temperatures T_i and T_s
3. Perfect mixing in the tank reactor
4. Perfect mixing in the heating jacket
5. Isothermic reaction

A Continuous Stirred Tank Reactor

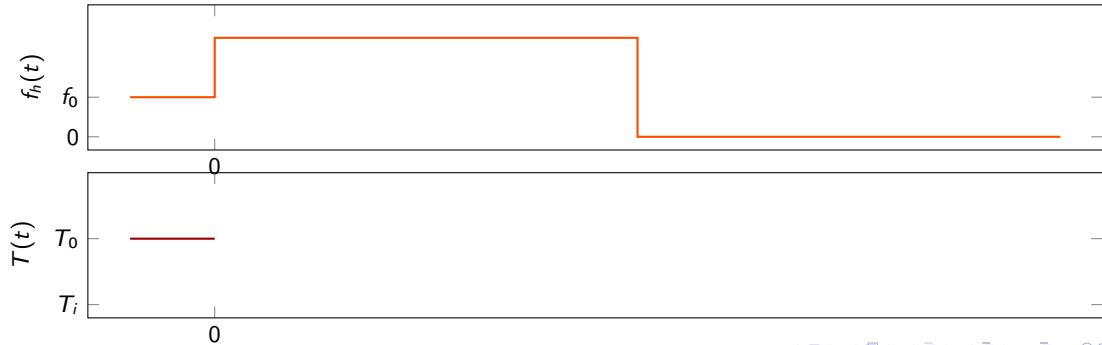
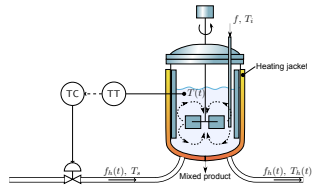


Energy balance:

$$\frac{dT(t)}{dt} = k_1(T_i - T(t)) + k_2(T_h(t) - T(t))$$

$$\frac{dT_h(t)}{dt} = k_3 f_h(t)(T_s - T_h(t)) - k_4(T_h(t) - T(t))$$

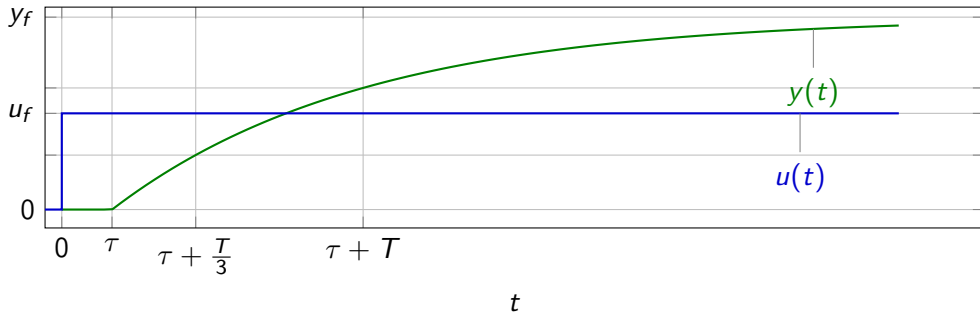
Intuition



Fitting first-order model with delay

Assuming a plant model of first-order with time constant T and delay τ

$$Y(s) = \frac{Ke^{-s\tau}}{sT + 1} U(s) \quad U(s) = \frac{u_f}{s} \quad \Rightarrow \quad y(t) = u_f K (1 - e^{-\frac{t-\tau}{T}}) u_H(t - \tau)$$

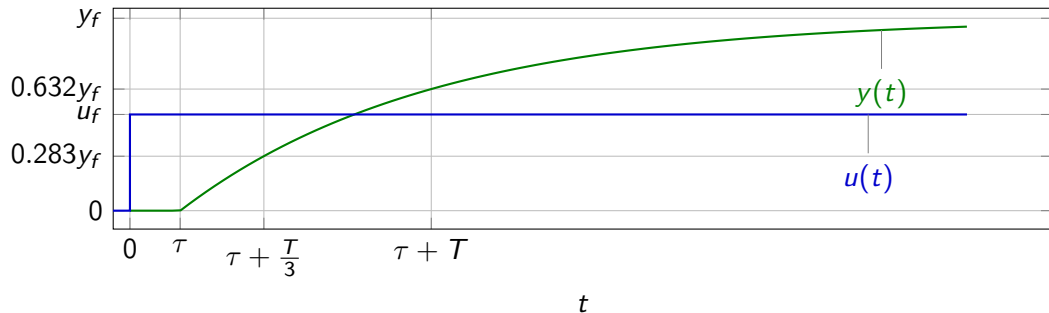


Individual activity Evaluate the response $y(t)$ at the two time instants $t = \tau + \frac{T}{3}$ and $t = \tau + T$!

Fitting first-order model with delay

Assuming a plant model of first-order with time constant T and delay τ

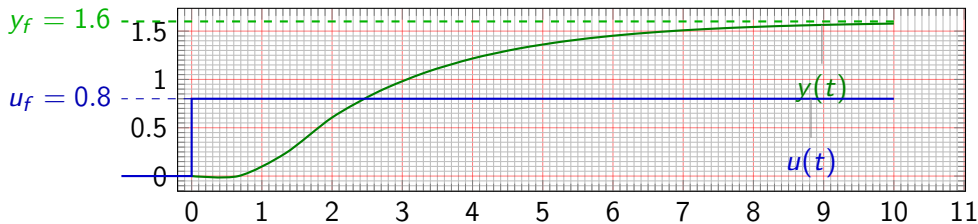
$$Y(s) = \frac{Ke^{-s\tau}}{sT + 1} U(s) \quad U(s) = \frac{u_f}{s} \quad \Rightarrow \quad y(t) = u_f K (1 - e^{-\frac{t-\tau}{T}}) u_H(t - \tau)$$



$$y_f = \lim_{t \rightarrow \infty} y(t) = u_f K \quad \Rightarrow \quad K = \frac{y_f}{u_f}$$

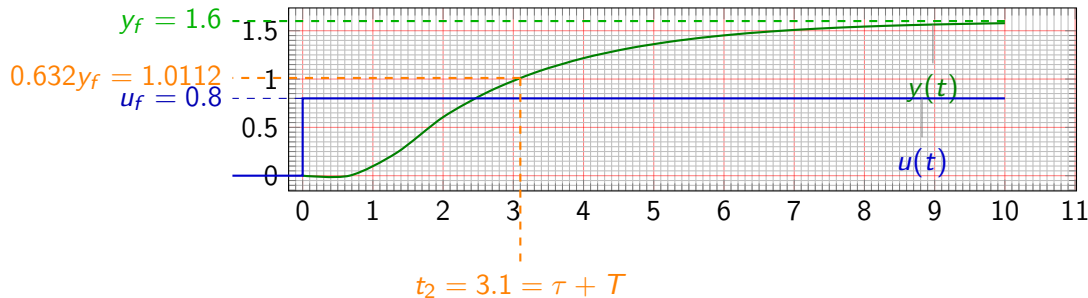
First-order model with delay - example

$$Y(s) = \frac{Ke^{-s\tau}}{sT + 1} U(s) \quad U(s) = \frac{u_f}{s} \quad \Rightarrow \quad y(t) = u_f K \left(1 - e^{-\frac{t-\tau}{T}}\right) u_s(t - \tau)$$



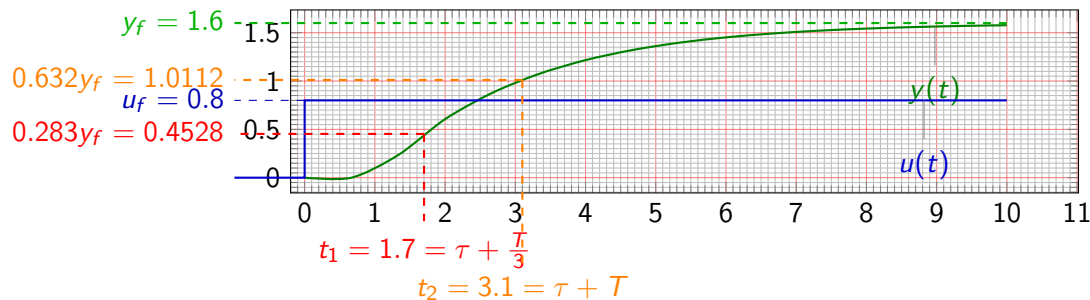
First-order model with delay - example

$$Y(s) = \frac{Ke^{-s\tau}}{sT + 1} U(s) \quad U(s) = \frac{u_f}{s} \quad \Longrightarrow \quad y(t) = u_f K \left(1 - e^{-\frac{t-\tau}{T}}\right) u_s(t - \tau)$$



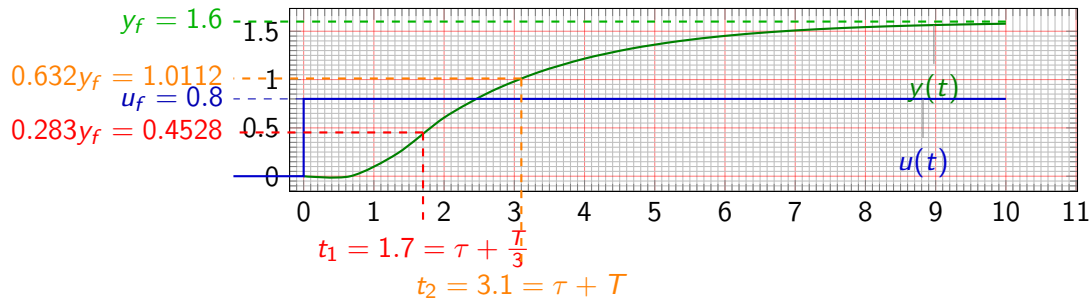
First-order model with delay - example

$$Y(s) = \frac{Ke^{-s\tau}}{sT + 1} U(s) \quad U(s) = \frac{u_f}{s} \quad \Rightarrow \quad y(t) = u_f K \left(1 - e^{-\frac{t-\tau}{T}}\right) u_s(t - \tau)$$



First-order model with delay - example

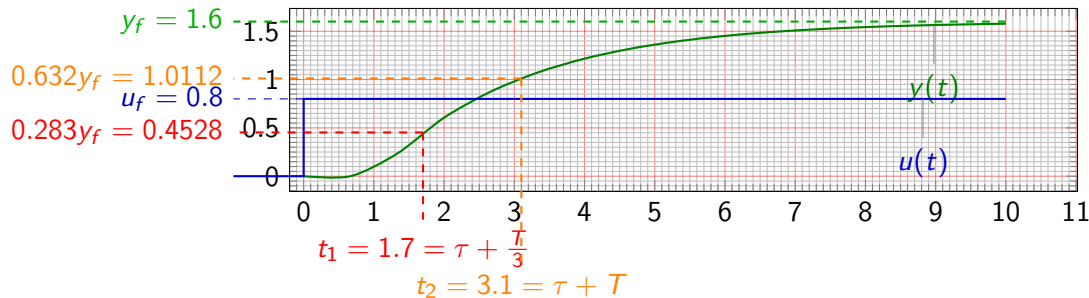
$$Y(s) = \frac{K e^{-s\tau}}{sT + 1} U(s) \quad \xrightarrow{U(s) = \frac{u_f}{s}} \quad y(t) = u_f K \left(1 - e^{-\frac{t-\tau}{T}}\right) u_s(t - \tau)$$



$$\begin{cases} 1.7 = \tau + \frac{T}{3} \\ 3.1 = \tau + T \end{cases} \Rightarrow \begin{cases} \tau = 1 \\ T = 2.1 \end{cases}, \quad K = \frac{y_f}{u_f} = \frac{1.6}{0.8} = 2$$

First-order model with delay - example

$$Y(s) = \frac{Ke^{-s\tau}}{sT + 1} U(s) \quad \xrightarrow{U(s) = \frac{u_f}{s}} \quad y(t) = u_f K \left(1 - e^{-\frac{t-\tau}{T}}\right) u_s(t - \tau)$$



$$\begin{cases} 1.7 = \tau + \frac{T}{3} \\ 3.1 = \tau + T \end{cases} \Rightarrow \begin{cases} \tau = 1 \\ T = 2.1 \end{cases}, \quad K = \frac{y_f}{u_f} = \frac{1.6}{0.8} = 2$$