Process Automation Laboratory - Modeling first-order systems

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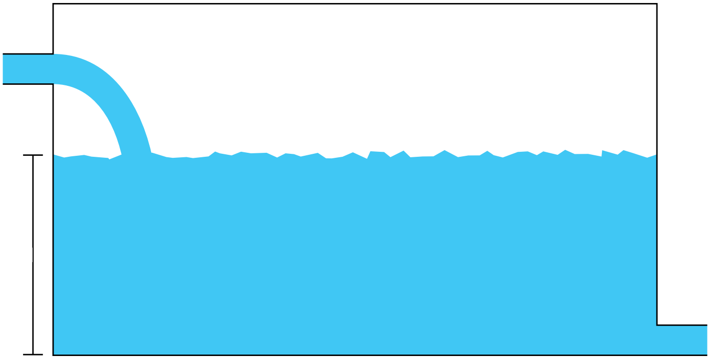
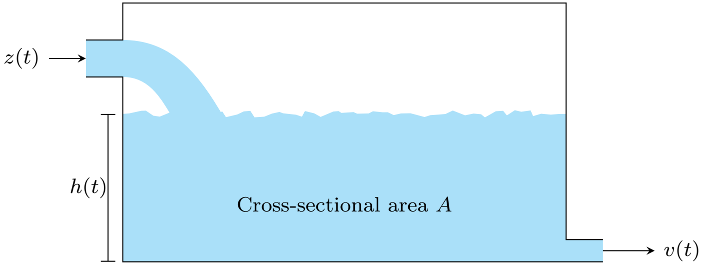
First-order system example: A tank



1. What is the state of the system?
2. What is the input signal and output signal?

First-order system example: A tank

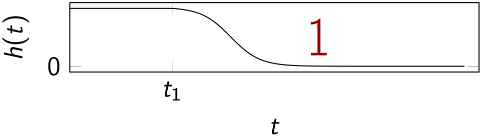


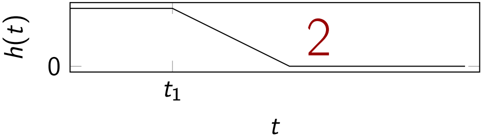
$$\frac{d}{dt}(Ah) = z(t) - x(t) = z(t) - a\sqrt{2gh} \quad \Rightarrow$$

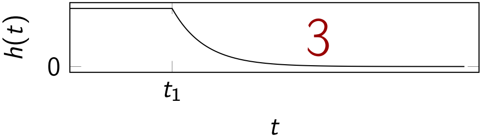
$$\frac{d}{dt}h(t) = -\frac{a\sqrt{2g}}{A}\sqrt{h(t)} + \frac{1}{A}z(t)$$

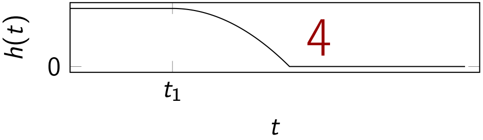
Intuition

Individual activity A constant inflow has been present since forever, but at time t_1 the flow in is suddenly shut off. Which of the responses of the water level $h(t)$ below is correct?

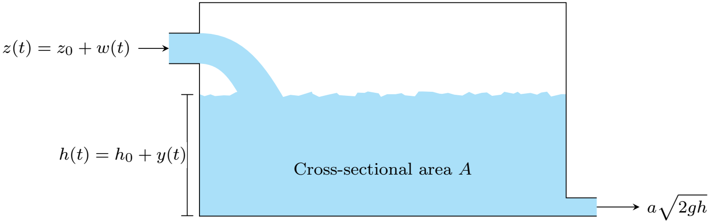








Deviation variables



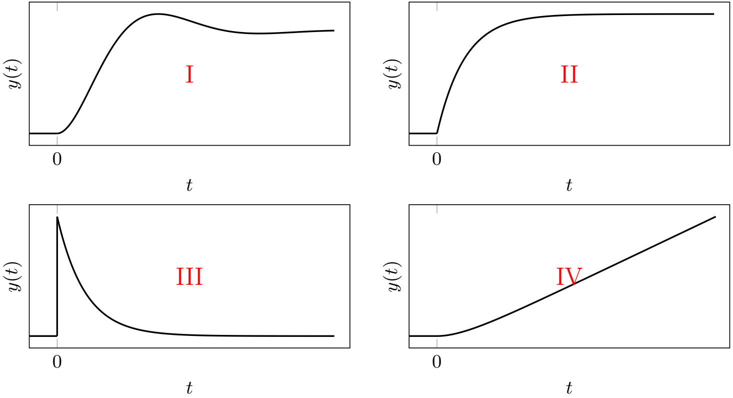
Flow in: $z(t) = z_0 + w(t)$. Level of water: $h(t) = h_0 + y(t)$. The constants h_0 and z_0 define an *operating point*.

$$\frac{d}{dt}h(t) = -\frac{a\sqrt{2g}}{A}\sqrt{h(t)} + \frac{1}{A}z(t)$$

Individual activity Given h_0 determine the operating point for the inflow, z_0, such that the system is in equilibrium at the operating point.

Intuition

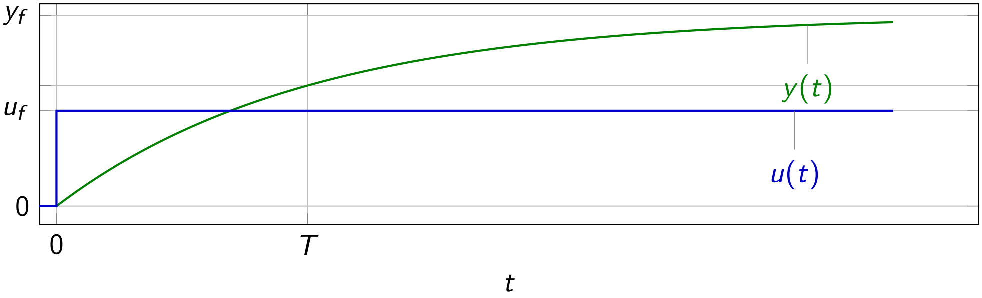
Which change $y(t)$ in the water level corresponds to a step change $w(t)$ in the inflow?



Fitting a first-order model

Assuming a plant model of first-order with time-constant T

$$Y(s) = \frac{K}{sT+1}U(s) \quad \overset{U(s)=\frac{u_f}{s}}{\Longrightarrow} \quad y(t) = u_f K\left(1-\mathrm{e}^{-\frac{t}{T}}\right)u_H(t)$$

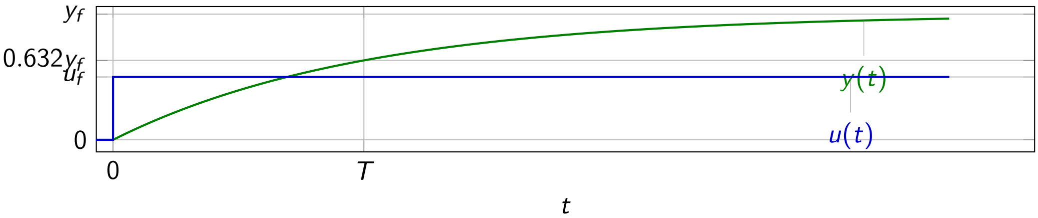


Individual activity Evaluate the response $y(t)$ at the time instant $t = T$ and for $t \to \infty$!

Fitting a first-order model

Assuming a plant model of first-order with time-constant T

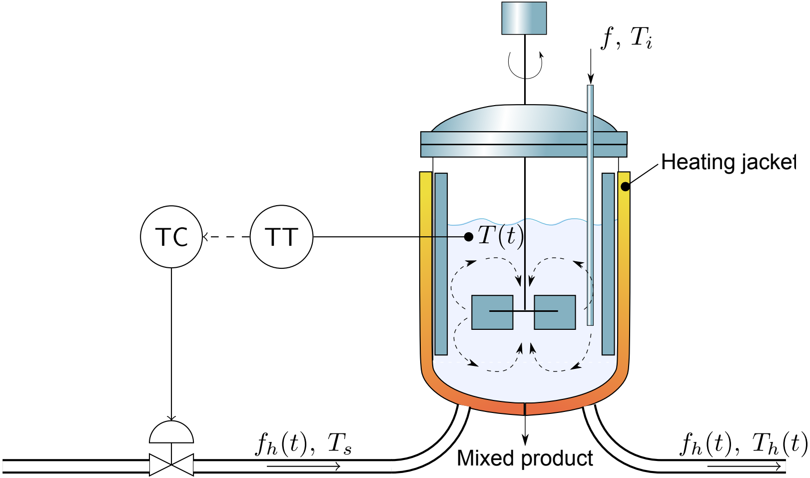
$$Y(s) = \frac{K}{sT+1} U(s) \quad \overset{U(s)=\frac{u_f}{s}}{\Longrightarrow} \quad y(t) = u_f K \left(1 - \mathrm{e}^{-\frac{t}{T}}\right) u_H(t)$$



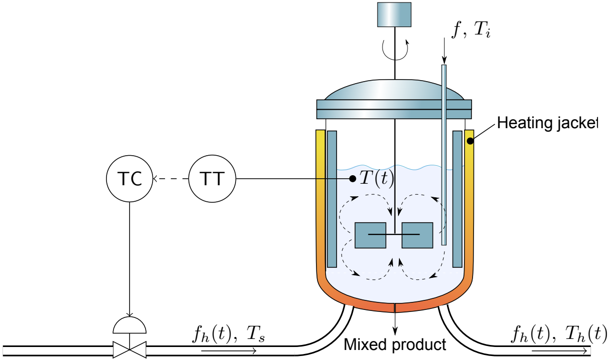
Time-constant: Find the time $t = T$ at which the response has reached 63.2% of its final value

Gain: $y_f = \lim_{t\to\infty} y(t) = K u_f \quad \Rightarrow \quad K = \frac{y_f}{u_f}$

A Continuous Stirred Tank Reactor



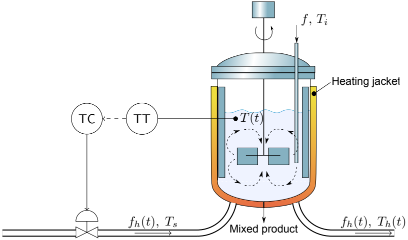
A Continuous Stirred Tank Reactor



Assume:

1. Constant flow f through the tank reactor
2. Constant temperatures T_i and T_s
3. Perfect mixing in the tank reactor
4. Perfect mixing in the heating jacket
5. Isothermic reaction

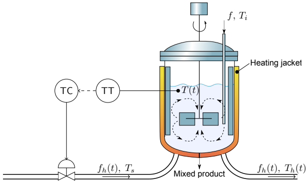
A Continuous Stirred Tank Reactor



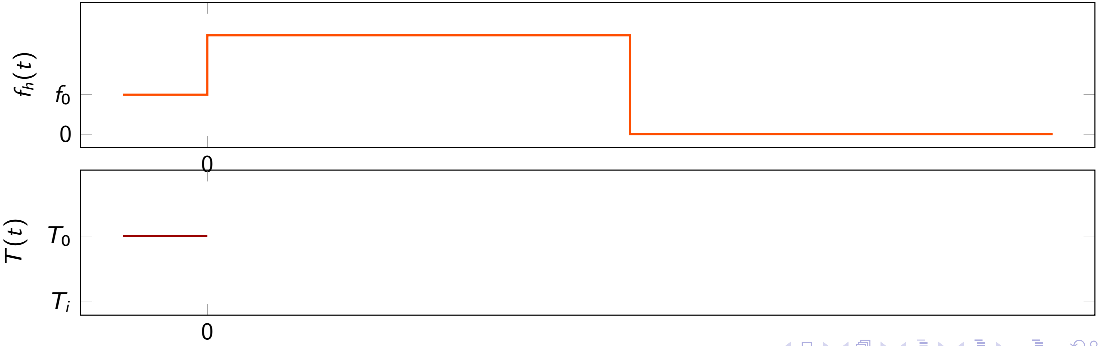
Energy balance:

$$\frac{dT(t)}{dt} = k_1\big(T_i - T(t)\big) + k_2\big(T_h(t) - T(t)\big)$$
$$\frac{dT_h(t)}{dt} = k_3 f_h(t)\big(T_s - T_h(t)\big) - k_4\big(T_h(t) - T(t)\big)$$

Intuition



f, T_i
Heating jacket
TC
TT
$T(t)$
$f_h(t)$, T_s
Mixed product
$f_h(t)$, $T_h(t)$

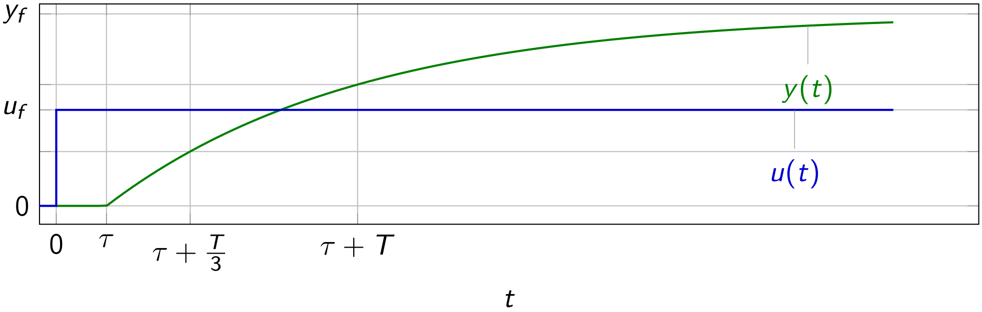


$f_h(t)$
f_0
0
0
$T(t)$
T_0
T_i
0

Fitting first-order model with delay

Assuming a plant model of first-order with time constant T and delay τ

$$Y(s) = \frac{K e^{-s\tau}}{sT+1} U(s) \quad \xRightarrow{U(s)=\frac{u_f}{s}} \quad y(t) = u_f K \left(1 - e^{-\frac{t-\tau}{T}}\right) u_H(t-\tau)$$

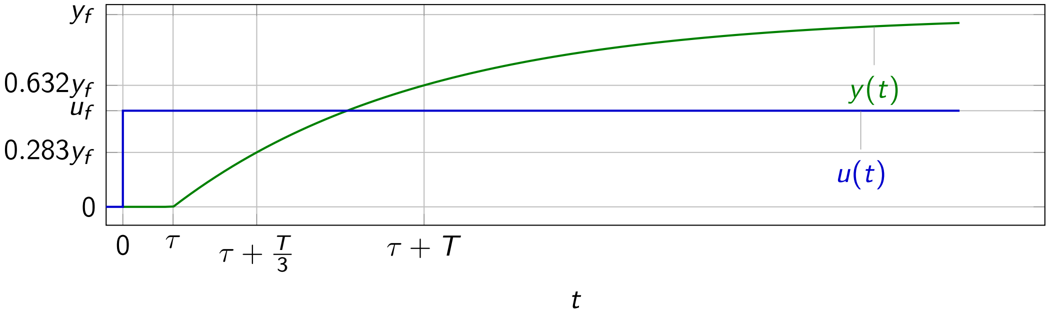


Individual activity Evaluate the response $y(t)$ at the two time instants $t = \tau + \frac{T}{3}$ and $t = \tau + T$!

Fitting first-order model with delay

Assuming a plant model of first-order with time constant T and delay τ

$$Y(s) = \frac{K\mathrm{e}^{-s\tau}}{sT+1}U(s) \quad \overset{U(s)=\frac{u_f}{s}}{\Longrightarrow} \quad y(t) = u_f K\left(1-\mathrm{e}^{-\frac{t-\tau}{T}}\right)u_H(t-\tau)$$



$$y_f = \lim_{t\to\infty} y(t) = u_f K \quad \Rightarrow \quad K = \frac{y_f}{u_f}.$$

First-order model with delay - example

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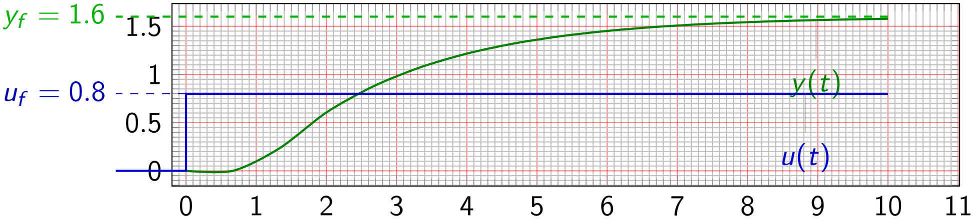

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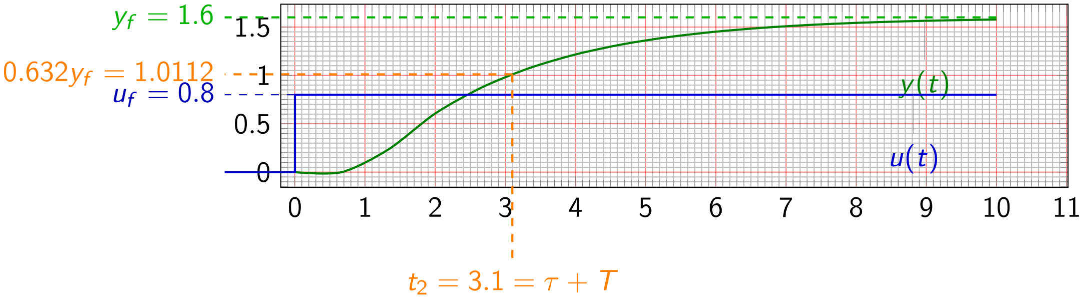

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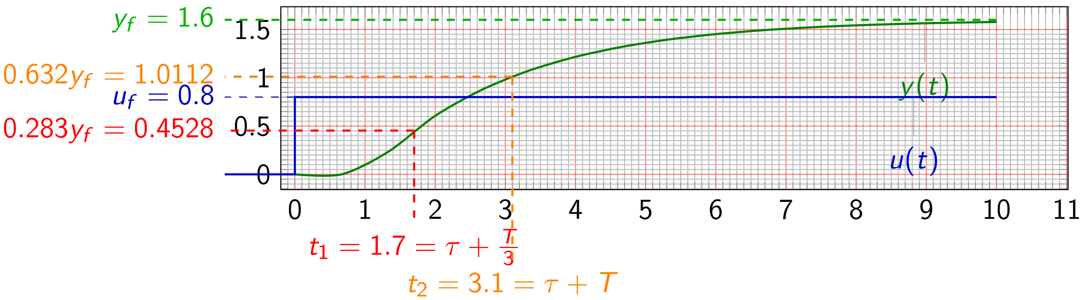

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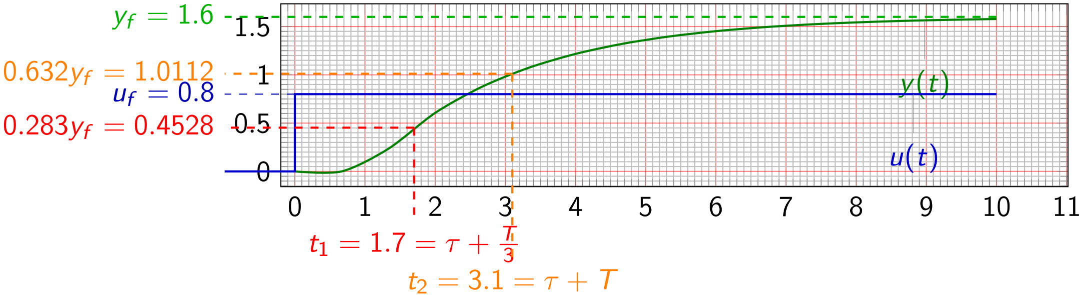

$$\begin{cases} 1.7 = \tau + \frac{T}{3} \\ 3.1 = \tau + T \end{cases} \Rightarrow \begin{cases} \tau = 1 \\ T = 2.1 \end{cases}, \qquad K = \frac{y_f}{u_f} = \frac{1.6}{0.8} = 2$$

First-order model with delay - example

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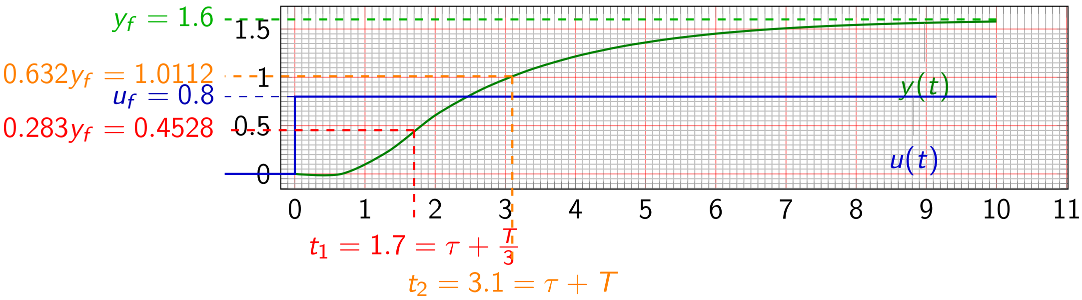

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