Process Automation Laboratory - PID control

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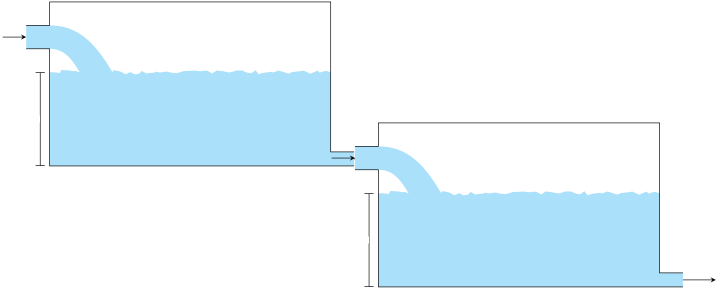
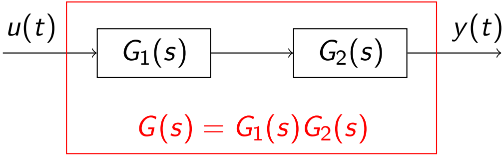
2020-08-31

Second-order models

Two first-order models in series



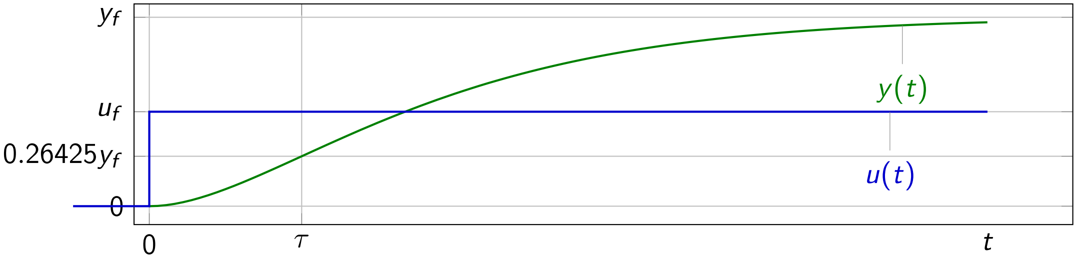


$$G(s) = G_1(s)G_2(s)$$

Fitting second-order critically-damped model

Model with two identical time-constants. Assuming model

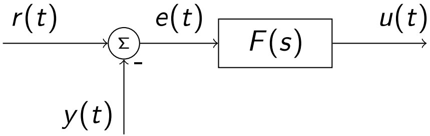
$$Y(s) = \frac{K}{(s\tau + 1)^2} U(s) \quad \stackrel{U(s)=\frac{u_f}{s}}{\Longrightarrow} \quad y(t) = u_f K \left(1 - (1 + \frac{t}{\tau})\mathrm{e}^{-\frac{t}{\tau}}\right) u_H(t)$$



$$y_f = \lim_{t\to\infty} y(t) = u_f K \quad \Rightarrow \quad K = \frac{y_f}{u_f}.$$

Feedback control

The PID controller



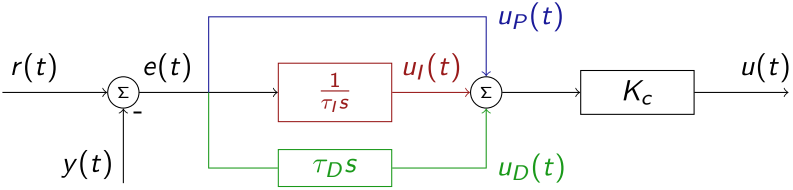
Parallel form (ISA)

$$F(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Series form

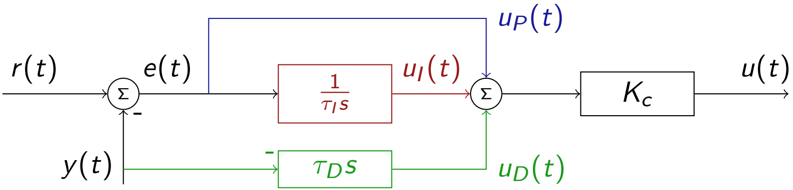
$$F(s) = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)(\tau_D s + 1)$$

The PID - Parallel form



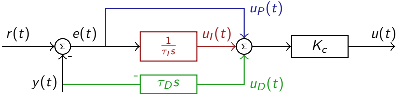
$$u(t) = K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(\xi)d\xi + \tau_D\frac{d}{dt}e(t)\right)$$

The PID - Parallel form, modified D-part

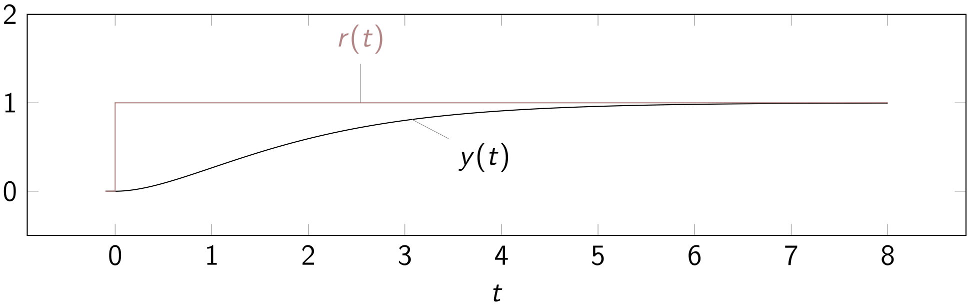


$$u(t) = K_c\left(e(t) + \overbrace{\frac{1}{\tau_I}\int_0^t e(\xi)d\xi}^{u_I(t)} + \underbrace{\tau_D \frac{d}{dt}\big(-y(t)\big)}_{u_D(t)}\right)$$

The PID - Parallel form



$$u(t) = K_c\left(e(t) + \overbrace{\frac{1}{\tau_I}\int_0^t e(\xi)d\xi}^{u_I(t)} + \underbrace{\tau_D\frac{d}{dt}(-y(t))}_{u_D(t)}\right)$$



Activity Sketch the error signal $e(t)$, the derivative signal $u_D(t)$ and the integral signal $u_I(t)$ (use $\tau_I = \tau_D = 1$)

The PID - Parallel form, solution

$$u(t) = K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(\xi)d\xi - \tau_D\frac{d}{dt}y(t)\right)$$