Boolean algebra, logic diagrams and truth tables

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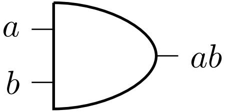
AND and OR

$a, b \in \{0, 1\}$

AND

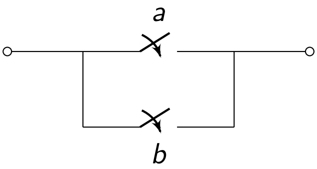
a	b	a AND b, ab
0	0	0
0	1	0
1	0	0
1	1	1

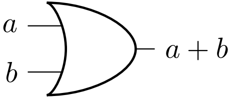
Closed circuit $\Leftrightarrow$ 1
Open circuit $\Leftrightarrow$ 0



OR

a	b	a OR b, $a + b$
0	0	0
0	1	1
1	0	1
1	1	1



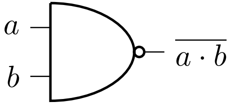


NAND and NOR

$a, b \in \{0, 1\}$

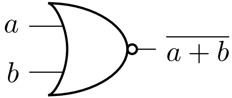
NAND

a	b	a NAND b, $\overline{a \cdot b}$
0	0	1
0	1	1
1	0	1
1	1	0



NOR

a	b	a NOR b, $\overline{a + b}$
0	0	
0	1	
1	0	
1	1	



Boolean algebra, contd

$x, y, z \in \{0, 1\}$

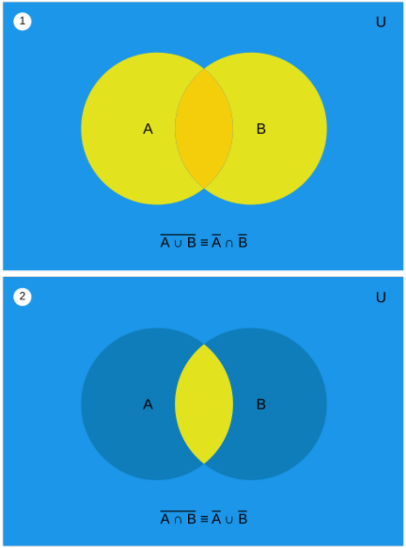
	Property	Dual
Properties of 0 and 1	$x + 0 = x$	$x \cdot 0 = 0$
	$x + 1 = 1$	$x \cdot 1 = x$
Idempotency	$x + x = x$	$x \cdot x = x$
Complementarity	$x + \overline{x} = 1$	$x \cdot \overline{x} = 0$
Involution	$\overline{\overline{x}} = x$	
Commutative	$x + y = y + x$	$x \cdot y = y \cdot x$
Associative	$(x + y) + z = x + (y + z)$	$(xy)z = z(yz)$
Distributive	$x \cdot (y + z) = xy + xz$	$x + yz = (x + y)(x + z)$

Boolean algebra, contd

$x, y \in \{0, 1\}$

	Theorem	Dual
Absorption	$x + xy = x(1 + y) = x$	$x(x + y) = x$
Logic adjacency	$xy + x\overline{y} = x(y + \overline{y}) = x$	$(x + y)(x + \overline{y}) = x$
De Morgan's	$\overline{x + y} = \overline{x} \cdot \overline{y}$	$\overline{xy} = \overline{x} + \overline{y}$

DeMorgan's theorem



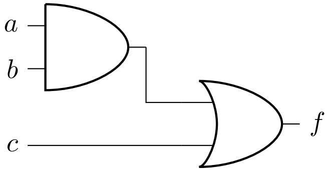
From wikipedia

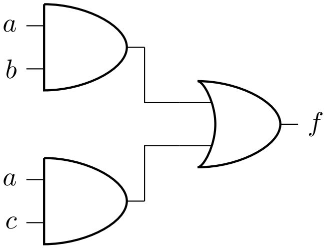
Simplify functions

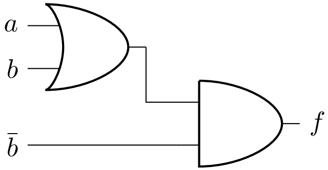
1. $f = (a + b)(a + c)$
2. $f = a + \overline{a}b$

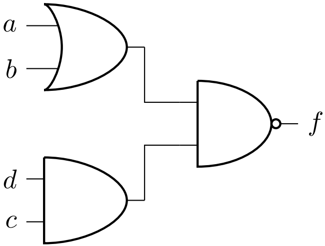
Logic diagram $\to$ function

Determine the function represented by the logic diagrams









Function $\rightarrow$ logic diagram

Draw the diagram corresponding to the boolean function

1. $f = (a + b)(a + c)$
2. $f = a + \overline{a}b$

Group exercise

1. Enter breakout room
2. One of you downloads and shares this presentation
3. Work together on the problems in the previous three slides
   3.1 Simplify functions
   3.2 Determine function from logic diagram
   3.3 Draw logic diagram from function