Actuators

Kjartan Halvorsen

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Mechanical requirements

Mechanical energy

From Encyclopaedia Britannica

Mechanical energy *The sum of kinetic energy (the energy in movement) and the potential energy (energy stored in a system due to the position of its parts).*

$$E_M = \underbrace{K}_{\text{Kinetic energy}} + \underbrace{U}_{\text{Potential energy}}$$

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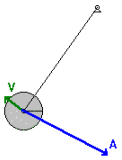
A point mass m with velocity v at a height h above the reference level, has mechanical energy $E_M = \frac{1}{2}mv^2 + mgh$.



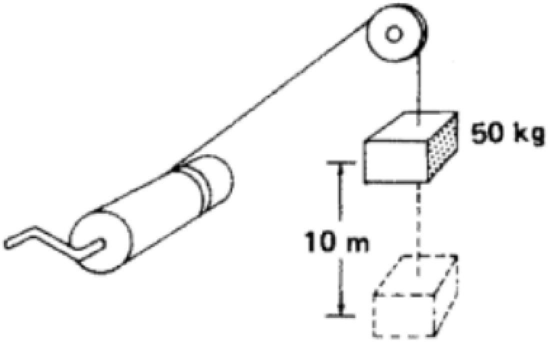
Source: Hubert Christiaen, wikipedia

Work

From Encyclopaedia Britannica

> *Work In physics, the measure of transfer of energy when an object is diplaced a certain distance by an external force which has a component in the direction of the displacement..*

Work

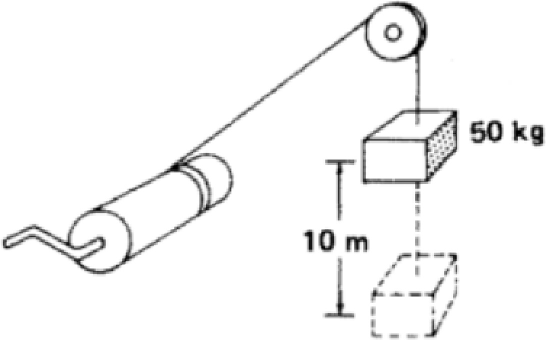


Activity A mass of $50\ \mathrm{kg}$ is lifted a distance of $10\ \mathrm{m}$. What is the work done?

Power

Definition The time-derivative of work.

Power



Activity A mass of 50 kg is lifted a distance of 10 m in 5 seocnds. What is the average power required?

Power and acceleration

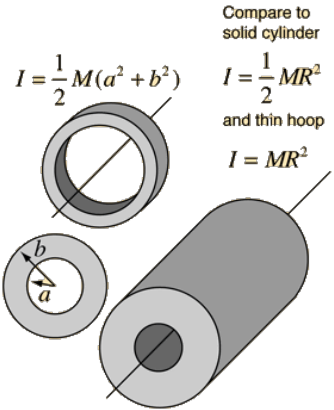
Activity The new Hummer EV has a mass of $m = 5000 \text{ kg}$, and can accelerate from $0 - 100 \text{ km/h}$ in three seconds. What is the average power required to achieve this (ignoring wind- and rolling resistance)?

Power in rotating systems

Torque multiplied with angular velocity

$$P = T\omega$$

Moment of inertia



Source: Georgia State University

Moment of inertia is a parameter in Newton's law that determines

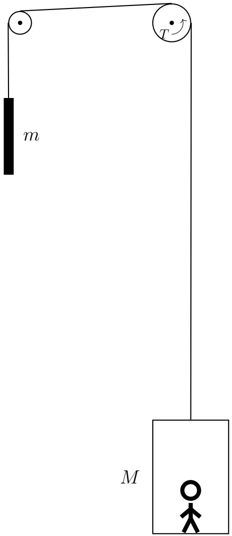
- the tendency of a body to resist angular acceleration:

$$\textcolor{red}{J}\dot{\omega} = \sum T_i$$

- the kinetic energy of a body rotating at a certain angular velocity:

$$K = \frac{1}{2}\textcolor{red}{J}\omega^2.$$

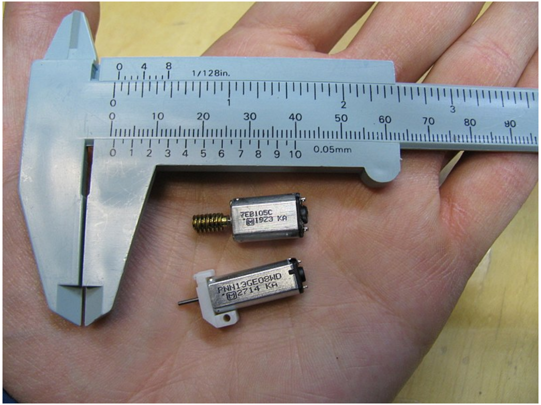
Power and torque requirements for an elevator



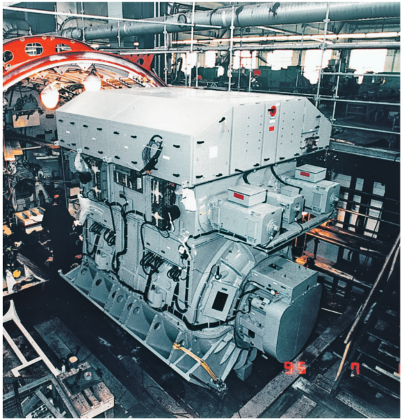
An elevator with mass $M = 1000$ kg, and a counterweight with mass $m = 800$ kg are connected by a wire which runs over a pulley with radius of $r = 0.4$ m. An electric motor is connected to the pulley via a transmission with gear ratio of 12:1 (12 revolutions of the motor for each revolution of the pulley). The motor has a moment of inertia of $J_m = 0.3 \text{ kgm}^2$. The inertia of the pulley can be ignored.

Activity (a) At what angular velocity is the motor rotating when the elevator is ascending at 4 m/s? (b) Determine the power and the motor torque necessary to lift the elevator at 4 m/s (assuming no friction). (c) The elevator takes two seconds to reach the velocity 4 m/s from zero. During this time, the elevator has moved up 4 m. Determine the average power and torque during the start.

The DC motor



Source: Wikipedia



DC-Prop drive

Source: Siemens AG

Force on an electrical conductor in a magnetic field

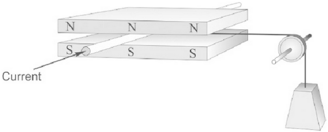


FIG. 1.14 Primitive linear d.c. motor.

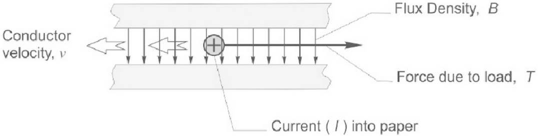
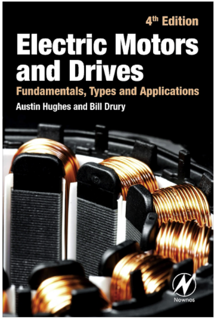
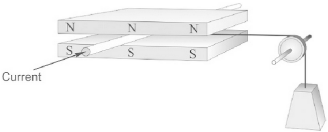


FIG. 1.15 Diagrammatic sketch of primitive linear d.c. motor.

Source



Force on an electrical conductor in a magnetic field



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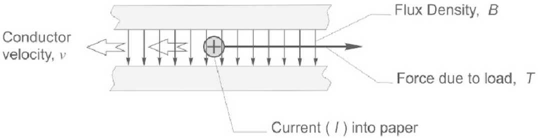


FIG. 1.15 Diagrammatic sketch of primitive linear d.c. motor.

The electromagnetic force in a conductor is proportional to the current and the strength of the magnetic field:

$$F = k_m I = (B l_m) I,$$

where B is the magnetic flux density in the gap, I is the current, an l_m is the length of the conductor.

Activity In a large motor of $4\ \mathrm{MW}$ with an axial length of $l_m = 2\ \mathrm{m}$, the magnetic flux density is $B = 0.8\ \mathrm{T}$ and the current is $I = 3\ \mathrm{kA}$. How many parallel conductors are needed to achieve a force of $F = 259.2\ \mathrm{kN}$?

The two equations of the DC motor

The force on the electrical conductor in the magnetic field

$$F(t) = k_m i(t) \quad \Leftrightarrow \quad T(t) = k_m r i(t),$$

where r is the radius of the motor.

Voltage generated in a conductor that moves in a magnetic field

$$e(t) = k_v v(t) \quad \Leftrightarrow \quad e(t) = k_v r \omega(t)$$

$e(t)$ is called *Back electro-motive force (Back e.m.f.)*.

Electrical and mechanical power

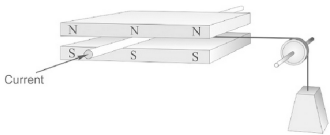


FIG. 1.14 Primitive linear d.c. motor.

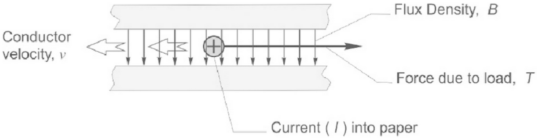


FIG. 1.15 Diagrammatic sketch of primitive linear d.c. motor.

With constant velocity v and ignoring friction and electrical resistance:

$$\text{Electromagnetic force} = \text{Mechanical force} \quad \Leftrightarrow \quad F = k_m I = Bl_m I = mg$$

$$\text{Electric power} = \text{Mechanical power} \quad \Leftrightarrow \quad \underbrace{V_1 I}_{P_e} = \underbrace{Fv = Bl_m Iv}_{P_m}$$

It is necessary to apply a voltaje V_1 across the cable to maintain the current I. This voltage is equal to the back e.m.f.

$$V_1 I = Bl_m Iv \quad \Rightarrow \quad V_1 = (Bl_m)v = k_v v = E$$

Back e.m.f.

Actividad What is the relationship between the two constants k_v and k_m?

Electrical and mechanical power

In practice some energy is lost due to the resistance in the electrical circuit.

$$\text{Electrical power drawn} = \text{Heat production} + \text{Mechanical power}$$
$$V_2 I = RI^2 + EI$$

Where $V_2 > V_1 = (Bl_m)v = E$.

The efficiency of the motor

$$\text{efficiency} = \frac{\text{Mechanical power}}{\text{Electrical power drawn}} = \frac{EI}{V_2 I} = \frac{E}{RI + E}$$

Activity An electri motor has a motor constant $k = 0.05\,\mathrm{kN/A}$ and an armature resistance of $R = 2\,\mathrm{m}\Omega$. It is producing a mechanical power of $4\,\mathrm{MW}$ at a velocity of $v = 10\,\mathrm{m/s}$. Calculate the back e.m.f E, the current I, then voltage V_2 and the efficiency.

Rotation

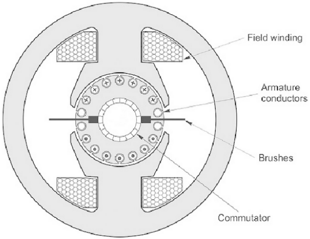


FIG. 3.1 Conventional (brushed) d.c. motor.

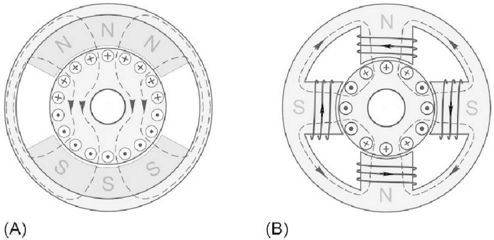
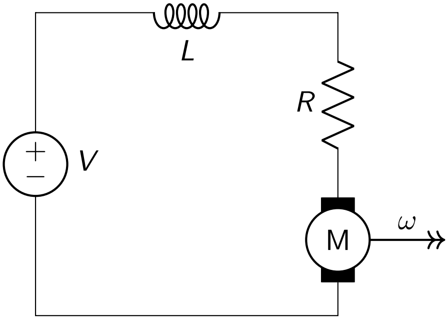
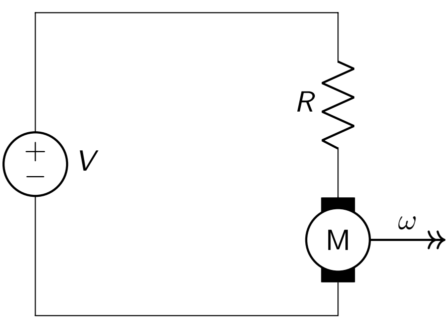


FIG. 3.2 Excitation (field) systems for d.c. motors (A) two-pole permanent magnet; (B) four-pole wound field.

Source: Hughes and Drury

Equivalent circuit





$$L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$$

$$Ri(t) + k\omega(t) = V$$

Newton: $J\frac{d}{dt}\omega(t) = ki(t) - T_l(t)$

Velocity with constant load

$$L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V(t) \tag{1}$$
$$J\frac{d}{dt}\omega(t) = ki(t) - T_l(t) \tag{2}$$

In steady-state: $i(t) = I$, $\omega(t) = \omega$.

$$RI + k\omega = V \tag{3}$$
$$0 = kI - T_l \tag{4}$$

Activity Write the angular velocity as function of the load T_l and the volage V

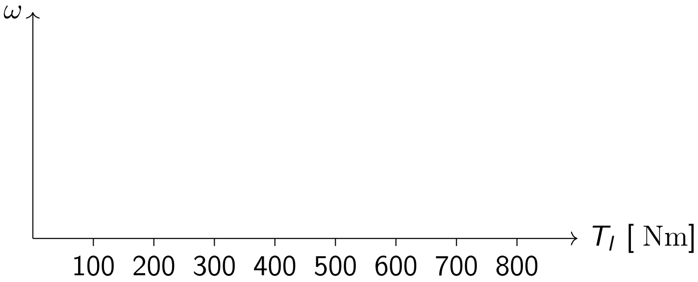
$$\omega = f(V, T_l) = \frac{V}{k} - \frac{RT_l}{k^2}$$

Velocity with constant load

$$\omega = f(V, T_l) = \frac{V}{k} - \frac{RT_l}{k^2}$$

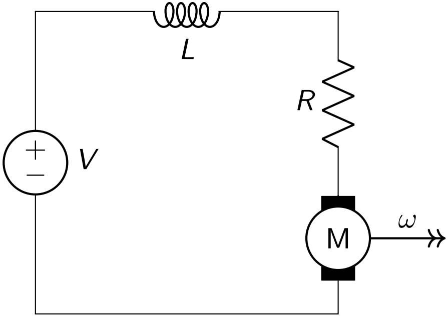
A specific motor has a motor constant $k = 4\ \mathrm{Nm/A}$ and armature resistence $R = 1\,\Omega$. A voltage of $V = 100\ \mathrm{V}$ is applied over the armature circuit.

Activity Sketch how the steady-state velocity depends on the load T_l. What is the stall torque (the torque that will cause the motor to stand still)?



Start-up

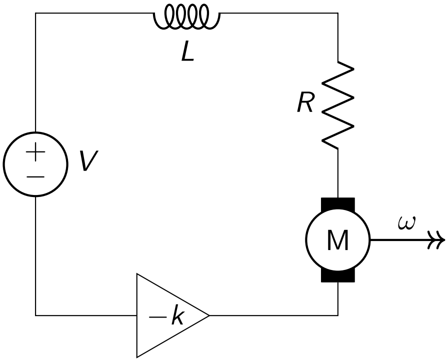
For a motor that is not rotating, the back e.m.f is zero, and only the resistance and the inductance of the armature limits the current.



$$L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$$

It is necessary to be careful with applying too much voltage at start-up to avoid excessive current in the motor.

Block-diagram of the equivalent circuit



$$L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$$

$$\frac{d}{dt}i(t) = \frac{1}{L}\Big(- Ri(t) - k\omega(t) + V\Big)$$