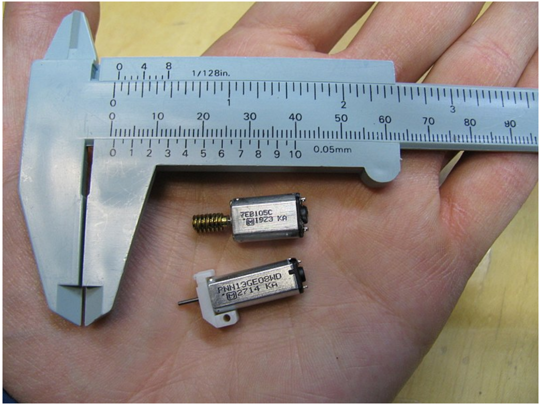
Espacio de estados

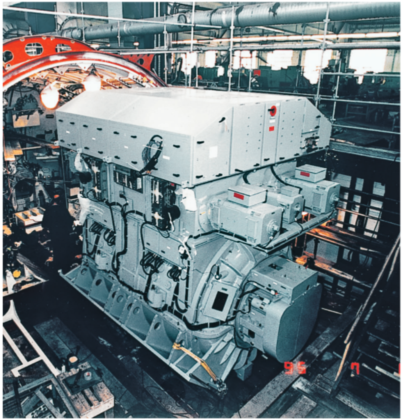
Kjartan Halvorsen

2021-05-24

El motor eléctrico de corriente directa



Fuente: Wikipedia



DC-Prop drive

Fuente: Siemens AG

El motor eléctrico de corriente directa

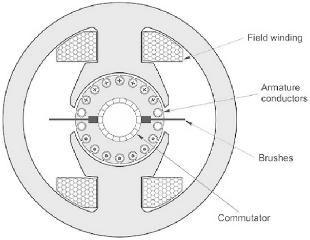


FIG. 3.1 Conventional (brushed) d.c. motor.

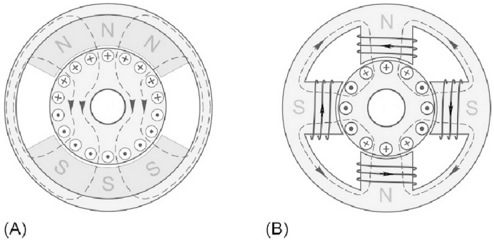


FIG. 3.2 Excitation (field) systems for d.c. motors (A) two-pole permanent magnet; (B) four-pole wound field.

Funte: Hughes and Drury

Las dos ecuaciónes del motor eléctrica CD

Torque generado por la corriente del rotor en el campo magnético

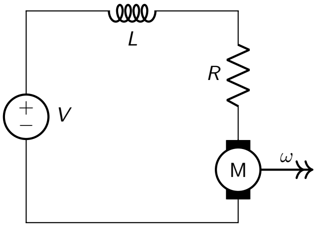
$$T_m(t) = ki(t)$$

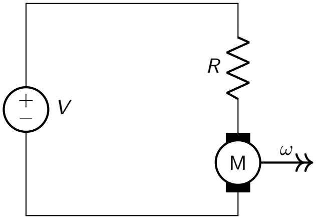
Voltaje generado por el movimiento del rotor en el campo magnético

$$e(t) = k\omega(t)$$

$e(t)$ se llama *Fuerza contraelectromotriz* o *Back electro-motive force (Back e.m.f.)* en inglés.

Circuito equivalente



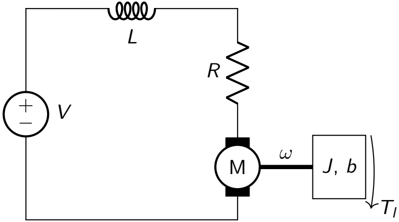


$L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$

$Ri(t) + k\omega(t) = V$

Newton: $J\frac{d}{dt}\omega(t) = ki(t) - b\omega(t) + T_l(t)$

El modelo en espacio de estados



Kirchoff: $L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$

Newton: $J\frac{d}{dt}\omega(t) = ki(t) - b\omega(t) + T_l(t)$

Vector de estados $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Li \\ J\omega \end{bmatrix}$

Vector de entradas $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ T_l \end{bmatrix}$

Vector de salidas $y = \begin{bmatrix} \omega \\ i \end{bmatrix}$

$$\begin{aligned}
\dot{x}_1 &= (\dot{Li}) = -Ri - k\omega + V = -\frac{R}{L}(Li) - \frac{k}{J}(J\omega) + V \\
&= -\frac{R}{L}x_1 - \frac{k}{J}x_2 + u_1 \\
\dot{x}_2 &= (\dot{J\omega}) = ki - b\omega + T_l = \frac{k}{L}(Li) - \frac{b}{J}(J\omega) + T_l \\
&= \frac{k}{L}x_1 - \frac{b}{J}x_2 + u_2
\end{aligned}$$

El modelo en espacio de estados

$$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{k}{J}x_2 + u_1$$

$$\dot{x}_2 = \frac{k}{L}x_1 - \frac{b}{J}x_2 + u_2$$

Vector de estados

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Li \\ J\omega \end{bmatrix}$$

Vector de entradas

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ T_l \end{bmatrix}$$

Vector de salidas $y = \begin{bmatrix} \omega \\ i \end{bmatrix}$

$$\dot{x} = \overbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}^{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

El modelo en espacio de estados

$$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{k}{J}x_2 + u_1$$
$$\dot{x}_2 = \frac{k}{L}x_1 - \frac{b}{J}x_2 + u_2$$

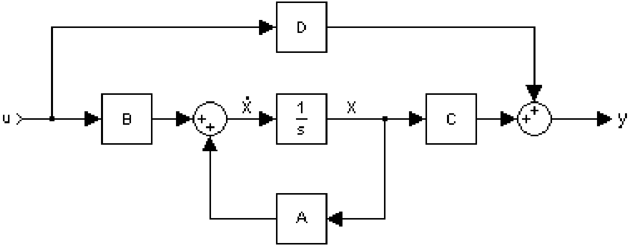
Vector de estados
$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Li \\ J\omega \end{bmatrix}$$

Vector de entradas
$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ T_l \end{bmatrix}$$

Vector de salidas $y = \begin{bmatrix} \omega \\ i \end{bmatrix}$

$$\dot{x} = \overbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{k}{J} \\ \frac{k}{L} & -\frac{b}{J} \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}^{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{J} \\ \frac{1}{L} & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Espacio de estados



{Fuente: Wikipedia }