Block-diagram algebra

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The model of the Hummer EV

ODE

$$
\begin{aligned}
m\dot{y} &= -2kv_0 y + u, \\
\dot{y} + \frac{2 \cdot 1.44 \cdot 22}{5000} y &= \frac{1}{5000} u, \\
\dot{y} + 0.013y &= 0.0002u, \\
78.9\dot{y} + y &= 0.016u.
\end{aligned}
$$

Transfer function

$$Y(s) = \underbrace{\frac{\overbrace{0.016}^{K}}{\underbrace{78.9}_{\tau}\, s + 1}}_{G(s)} U(s)$$

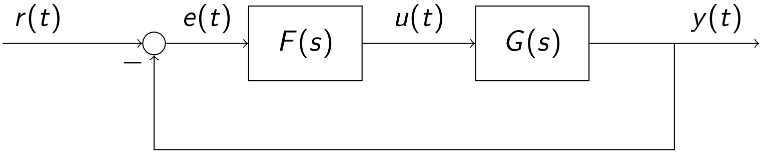
Laplace transform

$$(78.9s + 1)Y(s) = 0.016U(s)$$

Block diagram

$$u(t) \longrightarrow \boxed{\frac{K}{\tau s + 1}} \longrightarrow y(t)$$

Feedback control



Block-diagram algebra

Transfer function from $r(t)$ to $y(t)$:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

Block-diagram algebra

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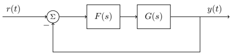
Mason's gain formula for simple systems with one loop only:

$$G_c(s) = \frac{\text{Forward path gain}}{1 + \text{Loop gain}}$$

Block diagram algebra

Activity Pair the block-diagram with the correct closed-loop transfer function!

A



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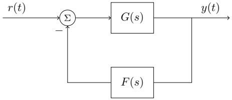

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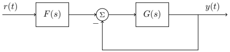

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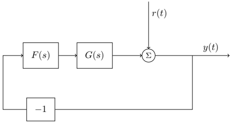

I

$$\frac{Y(s)}{R(s)} = \frac{G(s)F(s)}{1+G(s)}$$

II

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)F(s)}$$

III

$$\frac{Y(s)}{R(s)} = \frac{1}{1+G(s)F(s)}$$

IV

$$\frac{Y(s)}{R(s)} = \frac{G(s)F(s)}{1+G(s)F(s)}$$