State-space models

Kjartan Halvorsen

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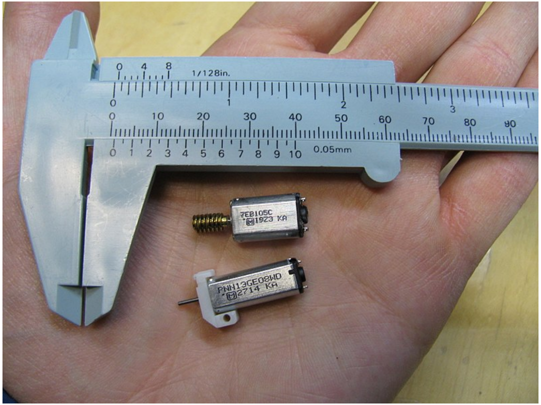
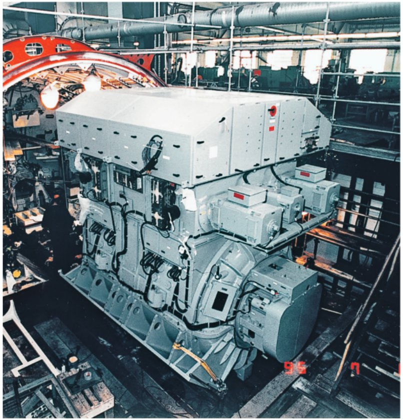
The DC motor comes in many sizes



Source: Wikipedia



DC-Prop drive

Source: Siemens AG

The DC motor - working principle

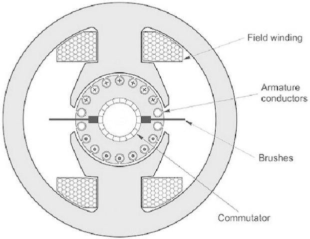


FIG. 3.1 Conventional (brushed) d.c. motor.

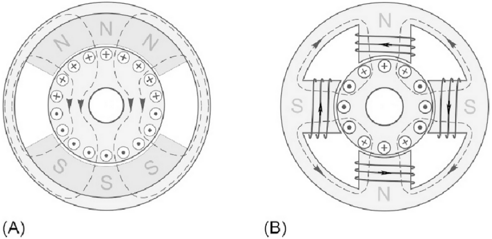


FIG. 3.2 Excitation (field) systems for d.c. motors (A) two-pole permanent magnet; (B) four-pole wound field.

Source: Hughes and Drury

The two equations of the DC motor

Torque generated by the current of the rotor in the magnetic field

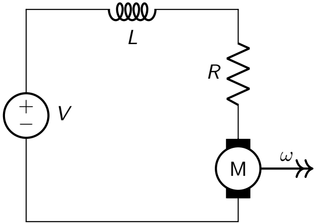
$$T_m(t) = ki(t)$$

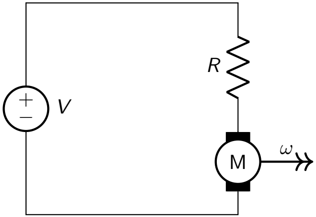
Voltage generated by the movement of conductors of the rotor in the magnetic field

$$e(t) = k\omega(t)$$

$e(t)$ is called *back electro-motive force* (back e.m.f.).

Equivalent circuit



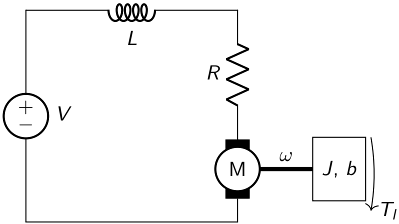


$L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$

$Ri(t) + k\omega(t) = V$

Newton: $J\frac{d}{dt}\omega(t) = ki(t) - b\omega(t) + T_l(t)$

The model on state-space form



Kirchoff: $L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$

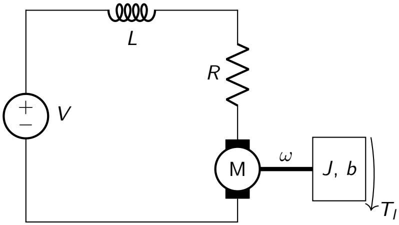
Newton: $J\frac{d}{dt}\omega(t) = ki(t) - b\omega(t) + T_l(t)$

State vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Li \\ J\omega \end{bmatrix}$

Input signals $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ T_l \end{bmatrix}$

Output signals $y = \begin{bmatrix} \omega \\ i \end{bmatrix}$

The model on state-space form



Kirchoff: $L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$

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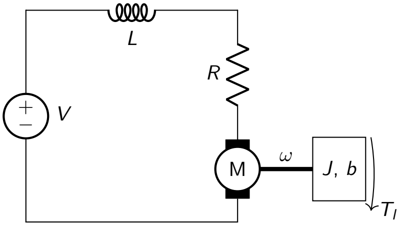
State vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Li \\ J\omega \end{bmatrix}$

Input signals $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ T_l \end{bmatrix}$

Output signals $y = \begin{bmatrix} \omega \\ i \end{bmatrix}$

$$\dot{x}_1 = (\dot{Li}) = -Ri - k\omega + V = -\frac{R}{L}(Li) - \frac{k}{J}(J\omega) + V$$
$$= -\frac{R}{L}x_1 - \frac{k}{J}x_2 + u_1$$

The model on state-space form



Kirchoff: $L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = V$

Newton: $J\frac{d}{dt}\omega(t) = ki(t) - b\omega(t) + T_l(t)$

State vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Li \\ J\omega \end{bmatrix}$

Input signals $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ T_l \end{bmatrix}$

Output signals $y = \begin{bmatrix} \omega \\ i \end{bmatrix}$

$$\begin{aligned}\dot{x}_1 &= (\dot{Li}) = -Ri - k\omega + V = -\frac{R}{L}(Li) - \frac{k}{J}(J\omega) + V \\ &= -\frac{R}{L}x_1 - \frac{k}{J}x_2 + u_1\end{aligned}$$

$$\begin{aligned}\dot{x}_2 &= (\dot{J\omega}) = ki - b\omega + T_l = \frac{k}{L}(Li) - \frac{b}{J}(J\omega) + T_l \\ &= \frac{k}{L}x_1 - \frac{b}{J}x_2 + u_2\end{aligned}$$

The model on state-space form

$$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{k}{J}x_2 + u_1$$
$$\dot{x}_2 = \frac{k}{L}x_1 - \frac{b}{J}x_2 + u_2$$

State vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Li \\ J\omega \end{bmatrix}$

Input signals $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ T_l \end{bmatrix}$

Output signals $y = \begin{bmatrix} \omega \\ i \end{bmatrix}$

$$\dot{x} = \overbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}^{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The model on state-space form

$$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{k}{J}x_2 + u_1$$
$$\dot{x}_2 = \frac{k}{L}x_1 - \frac{b}{J}x_2 + u_2$$

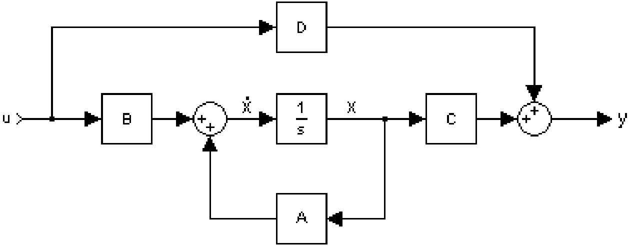
State vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Li \\ J\omega \end{bmatrix}$

Input signals $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ T_l \end{bmatrix}$

Output signals $y = \begin{bmatrix} \omega \\ i \end{bmatrix}$

$$\dot{x} = \overbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{k}{J} \\ \frac{k}{L} & -\frac{b}{J} \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}^{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{J} \\ \frac{1}{L} & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

State-space model



Source: Wikipedia