

## Modeling and Control for a Semi-active Suspension with a Magnetorheological Damper including the actuator dynamics

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### Abstract

*Reported researches on semi-active suspension neglect the actuator dynamics, missing important information that can deteriorate the suspension's performance. The present research proposes a new semi-active suspension model with a magnetorheological damper, including its dynamics. Three control approaches are applied to this suspension model to improve passenger comfort and vehicle stability. The well known control strategies: skyhook, groundhook and hybrid, are adequated for the model and simulated using Matlab. Exhaustive simulation results compare comfort and stability between an average city car passive suspension and the proposed semi-active suspension. The suspension with hybrid control shows the best performance complying, at the same time, with vehicle stability and passenger comfort.*

### 1. Introduction

Two critical areas in developing automotive technology are: passenger comfort and vehicle stability. Contemporaneous suspension research is focused on finding means to improve both issues at the same time, but problems arise because comfort and stability are opposite targets, thus a fixed type of suspension has to be selected according to a particular application.

During the middle of the 20th century, scientists started to work with fluids capable of experiment physical changes in a small period of time, for instance, rheologic fluids become semi-solids when are exposed to magnetic fields or electric currents. Their viscosity change in less than ten milliseconds, [1]. Four decades later, magnetorheologic fluids were introduced to automotive applications. The efforts were focused on

intelligent suspensions from the first intelligent approaches as discussed in [2] to the newest control proposals as reported in [3].

Experiments with semi-active suspensions have confirmed better results than their passive counterparts but with a higher degree of complexity in their model and control because of the nonlinear phenomena inherent in their composition. Modeling and control strategies for semi-active suspensions are two principal areas where automotive research has been concentrated.

The objective is to find a suspension capable of improving the performance requirements related to comfort and stability. Therefore, it is required to have a suspension model that can change its damping characteristics in real-time. This investigation takes a Two Degrees Of Freedom (2-DOF), semi-active suspension model based in the modified Bouc-Wen model, [1] reported in [4] and applies three control strategies looking for an improvement of passenger comfort and vehicle stability.

The article is organized as follows: Section 2 includes the proposed model and control laws. Section 3 introduces the performance criteria referenced in this paper work. Section 4 presents the simulation work and results. The ending sections conclude the investigation and propose further research directions.

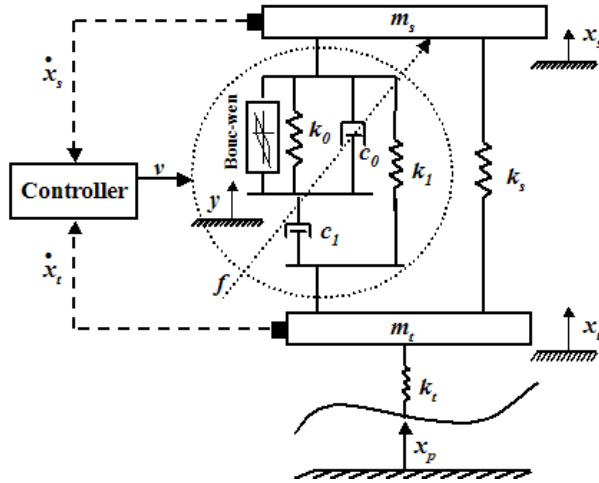
### 2. Proposed Model and Control

#### 2.1 Vehicle suspension model

The proposed model in [4] refers to the well known 2-DOF one quarter car vehicle with a MR (magnetorheological) damper. The dynamics of the damper are modeled with the modified Bouc-Wen model, [1], a highly accurate physical based model. As the result, a semi-active suspension model with real-time control is obtained.

The Bouc-Wen model contains: the nonlinear behavior of a viscous fluid going through an orifice, the hysteresis due to the damper and the saturation because of physical constraints, [5]. The new suspension model includes all these characteristics and adds the two masses of the quarter vehicle, generating a semi-active gray box design. Figure 1 shows the semi-active suspension model with a magnetorheological damper and the controller.

In Figure 1,  $m_s$  represents one quarter of the suspended mass;  $m_t$  represents the non-suspended mass (tire, damper, spring, etc.),  $x_s$  and  $x_t$  are the masses displacements and  $x_p$  represents the road disturbances.  $k_t$  is the tire stiffness, while  $k_s$  is the spring between the tire and the chassis. The variables related to the modified Bouc-Wen model are:  $k_0$  a large velocities stiffness,  $c_0$  a viscous damping,  $k_1$  is the MR damper accumulator stiffness,  $c_1$  models the roll-off at low velocities, and  $y$  is an internal variable included in the modified Bouc-Wen model, [1].



**“Figure 1. One quarter vehicle suspension model with a magnetorheological damper”**

Regarding the controller,  $\dot{x}_s$  and  $\dot{x}_t$  represents the sprung mass absolute velocity and unsprung mass absolute velocity respectively. From the control laws, explained in subsection 2.2, the controller generates a voltage  $v$  that applied to the magnetorheological damper, modifies the force  $f$  of the semi-active suspension.

Based on the system shown in Figure 1 and performing Newton's second law in each mass and in the internal variable  $y$ , the following equations are obtained:

$$m_s \ddot{x}_s = c_0(\dot{x}_s - \dot{y}) - k_0(x_s - y) - (k_1 + k_s)(x_s - x_t) - \alpha z \quad (1)$$

$$m_t \ddot{x}_t = c_1(\dot{x}_t - \dot{y}) - k_t(x_t - x_p) - (k_1 + k_s)(x_t - x_s) \quad (2)$$

$$\dot{y} = \left( \frac{1}{c_0 + c_1} \right) \left[ c_0 \dot{x}_s + c_1 \dot{x}_t - k_0(y - x_s) + \alpha z \right] \quad (3)$$

The hysteresis dynamic, added as the state variable  $z$  in equations (1) and (3), and reported in [1], is function of the displacement's time history and it is defined as:

$$\dot{z} = -\gamma \left| \dot{x}_s - \dot{y} \right| |z|^{n-1} - \beta (\dot{x}_s - \dot{y}) |z|^n + A (\dot{x}_s - \dot{y}) \quad (4)$$

In equation 4,  $n = 2$  as explained in [6]. In addition and regarding the suspension model in Figure 1, the controller response is a voltage  $v$  that will change a force  $f$  generated with the semi-active suspension represented by all the elements inside the dash circle. The voltage affects the internal characteristics of the magnetorheological damper, [1] according to equations (5), (6) and (7).

$$\alpha = \alpha_a + \alpha_b u \quad (5)$$

$$c_0 = c_{0a} + c_{0b} u \quad (6)$$

$$c_1 = c_{1a} + c_{1b} u \quad (7)$$

## 2.2 Control Strategies

Different control approaches have been applied to vehicle suspensions. This paper considers three control laws: skyhook, groundhook and hybrid strategies.

The suspension performance criteria for this research are based in passenger comfort and vehicle stability. Skyhook control is designed to maximize comfort, because its control algorithm acts on the sprung mass. The equivalency for the skyhook strategy is an hypothetical damper  $c_{sky}$  that connects the sprung mass with a fixed fictitious frame in the sky as shown in the sprung mass  $m_s$  in Figure 2. Assume that  $c_{gnd}$  does not exist.

As proposed in [7], the control laws are presented in the form of IF-THEN rules as follows:

$$\text{IF } V_s V_{st} \geq 0 \text{ THEN } f = c_{sky} |V_s| \quad (8)$$

$$\text{IF } V_s V_{st} < 0 \text{ THEN } f = 0 \quad (9)$$

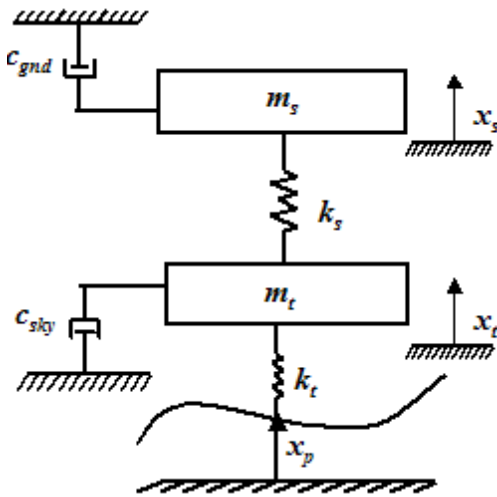
In equations (8) and (9),  $V_{st}$  ( $\dot{x}_s - \dot{x}_t$ ) is the velocity of the sprung mass  $m_s$  relative to the unsprung mass  $m_t$ .  $V_s$  is the absolute velocity of  $m_s$ .  $f$  is the force applied to the magnetorheological damper and it is proportional to a  $c_{sky}$  heuristic gain and to the magnitude of  $V_s$ .

Groundhook control intends to improve vehicle stability. It adopts a hypothetical damper  $c_{gnd}$  connected between the unsprung mass and a fixed fictitious frame in the ground. Refer to mass  $m_t$  in Figure 2 and assume that  $c_{sky}$  does not exist. The groundhook control laws, reported in [7], are the equations (10) and (11).

$$\text{IF } -V_t V_{st} \geq 0 \text{ THEN } f = c_{gnd} |V_t| \quad (10)$$

$$\text{IF } -V_t V_{st} < 0 \text{ THEN } f = 0 \quad (11)$$

In equations (10) and (11),  $V_t$  is the absolute velocity of  $m_t$ .  $f$  is the force applied to the damper, proportional to a  $c_{gnd}$  empirical gain and to the magnitude of  $V_t$ .



**“Figure 2. Hybrid control strategy schematic. Analyzing each mass separately, it is possible to identify skyhook and groundhook components”**

The problem with these approaches is the tradeoff handling among the criteria to maximize. For example, skyhook leads to a poor performance in stability while groundhook induce low comfort indicators. If the objective is to have a good performance for both stability and comfort, another strategy is required.

Hybrid control is a combination of skyhook and groundhook. It contains the two approaches and adds a variable factor  $\alpha$  to favor one over the other according with performance requirements. Figure 2 presents a schematic of hybrid control; consider both  $c_{gnd}$  and  $c_{sky}$  elements.

In Figure 2,  $V_s$  and  $V_t$  are the absolute velocities of the sprung and unsprung masses respectively. Equations (12) to (15) govern the hybrid approach as developed in [7].

$$\text{IF } V_s V_{st} \geq 0 \text{ THEN } \sigma_{sky} = c_{sky} |V_s| \quad (12)$$

$$\text{IF } V_s V_{st} < 0 \text{ THEN } \sigma_{sky} = 0 \quad (13)$$

$$\text{IF } -V_t V_{st} \geq 0 \text{ THEN } \sigma_{gnd} = c_{gnd} |V_t| \quad (14)$$

$$\text{IF } -V_t V_{st} < 0 \text{ THEN } \sigma_{gnd} = 0 \quad (15)$$

In equations (12) to (15),  $V_{st}$  represents the velocity of the sprung mass relative to the unsprung mass.  $c_{sky}$  and  $c_{gnd}$  are constant values as used in skyhook and groundhook, while  $\sigma_{sky}$  and  $\sigma_{gnd}$  are the independent contributions of each strategy. The actions coming from each approach are combined as in equation (16) to generate the final control action that is going to be applied to the damper as a voltage. Refer to Figure 1.

$$f = g_{hib} [\alpha \sigma_{sky} + (1 - \alpha) \sigma_{gnd}] \quad (16)$$

In equation (16),  $g_{hib}$  is an heuristic gain that affects the intensity of the final control action.  $\alpha$  can take values from 0 to 1. If  $\alpha$  is 0.5, skyhook and groundhook strategies apport equally to the resulting control action. Lower values of  $\alpha$  favor groundhook control while values of  $\alpha$  bigger than 0.5 favor skyhook control.

### 3. Performance Criteria

Vehicle stability is a function of the response to changes in direction (driver's command), and inputs generated by the environment, [8]. Stability is focused on the vehicle's ability to keep stability when it is exposed to external disturbances. In the present study, disturbances will be limited to those coming from the road surface, thus the stability in this investigation is specifically the road holding.

Passenger comfort is a subjective index depending on human perception. Comfort is related to temperature, car dimensions, etc. This research manages the comfort related to mechanical vibration. This work takes the road profile as the generator of mechanical vibrations acting on passengers' body.

Comfort and stability will be limited to vertical motion because the model is limited to a quarter of vehicle, where the sprung mass is related to comfort and the unsprung mass in associated to vehicle stability.

A variety of measuring criteria for automotive suspension performance have been reported as in [9]. Due to the characteristics of the two degrees of freedom model, in this research, the performance criteria is based in the work reported in [10] and listed as follows:

Passenger comfort in low frequencies (0-4Hz). Limit the chassis vertical displacement ( $x_s$ ) to be the

same of the road ( $x_p$ ). Reduce in 6dB, the relation dB ( $x_s/x_p$ ) obtained with an average passive suspension.

Road holding between 8 and 15Hz. Limit the tire vertical displacement ( $x_t$ ) to be the same of the road ( $x_p$ ). Reduce in 6dB, the relation dB ( $x_t/x_p$ ) performed by a regular passive suspension.

The purpose of the performance indicators proposed in [10] is to analyze the response of both masses around their resonant frequencies (particular frequency of vibration where the amplitude input-output is maximum). It is well known, [11], for an average two degree of freedom city vehicle model, that the sprung mass has its resonant frequency near 1.2Hz, while for the unsprung mass, the resonant frequency is between 10 and 11 Hz.

This investigation compares the performance of three semi-active suspension model against a passive average suspension. The passive suspension utilized herein is 1,000 Ns/m.

#### 4. Simulation Work and Results

The semi-active suspension model shown in Figure 1 was simulated in Matlab-Simulink. The equations that represent the system are fully explained in [4]. The non-linear dynamics of the magnetorheological damper are included in these equations as well as the variables that relate the controller's output and the actuator.

The values for the one quarter vehicle represent an average vehicle, with a quarter of the sprung mass around 350 kg, an unsprung mass of about 31 kg; a tire stiffness of 108,000 N/m, and a suspension spring of 20,900 N/m. The values corresponding to the semi-active damper and its dynamics were taken from the magnethoreological damper characterized in [1].

According to the performance criteria in [10], the results have to be reviewed in the frequency domain. The first thought is a bode diagram; however, the reader is missing the non-linear nature of the system, thus the bode diagram concept cannot be applied. This circumstance requires the computation of a describing function, [12].

The describing function, also referred as pseudobode, is an output/input relation in steady state for a sine input with fixed amplitude and that sweeps a frequency range. Although a pseudobode is an approximation, it is a very helpful tool to obtain the frequency response in non-linear systems.

To calculate the pseudobode, the system was excited with a 0.01m fixed amplitude sine wave. The frequency range was divided in 351 segments of 0.1Hz each. The initial frequency was 0.05Hz and then

increased in steps of 0.1Hz until reaching the 35.05Hz. For each frequency, a lecture of the system outputs ( $x_s/x_p$ ) and ( $x_t/x_p$ ) was gathered. The comfort performance was computed with the relation ( $x_s/x_p$ ) in steady-state for the sprung mass; and stability was calculated with ( $x_t/x_p$ ) in steady-state for the unsprung mass. Each pseudobode was finally obtained with 351 samples saved in defined arrays.

The model in Simulink is called by a .m program responsible of collecting and drawing the pseudobode diagrams. In addition, the .m program controls the frequency of the sine wave, which represents the road disturbances  $x_p$  entering into the system.

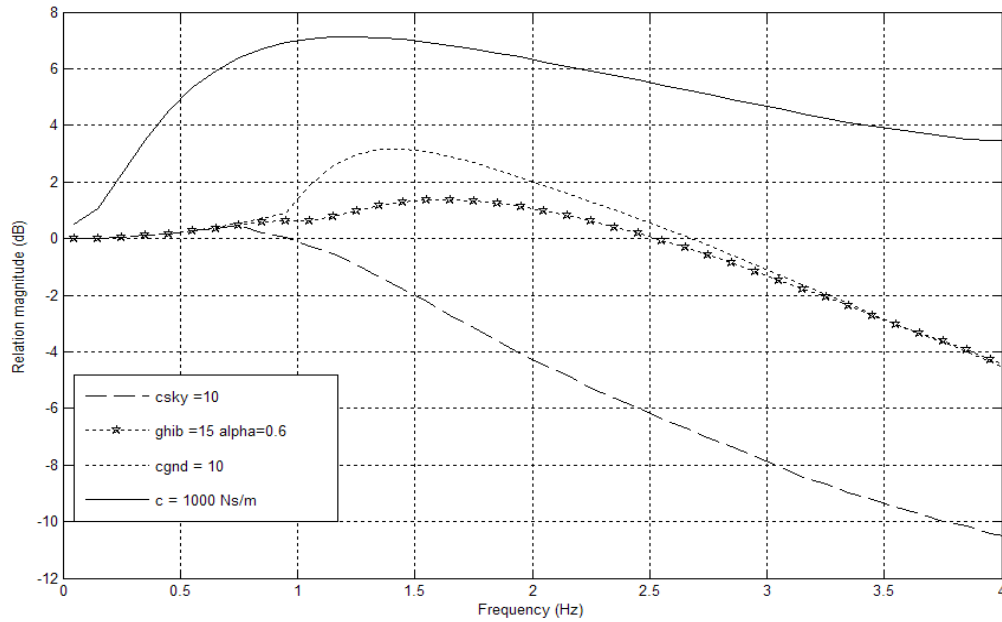
Figures 3 and 4 show the suspension responses for a passive suspension of 1,000 Ns/m, and a semi-active suspension running three control approaches (skyhook, groundhook and hybrid). Since the most important results to analyze are the amplitudes around the resonant frequencies, thus a close-up to each interest frequency range is done.

Because the best values for  $c_{sky}$ ,  $c_{gnd}$ ,  $g_{hib}$  and  $\alpha$  were obtained empirically, a lot of trials were run varying the gains. The best gain value of each nonlinear strategy was used to calculate the pseudobodes.  $c_{sky}=10$  generates the best comfort results,  $c_{gnd}=10$  provides the best vehicle stability, and  $g_{hib}=17$  with  $\alpha=0.6$  provides a semi-active suspension with the best trade-off that accomplishes the two suspension performance criteria proposed in [10].

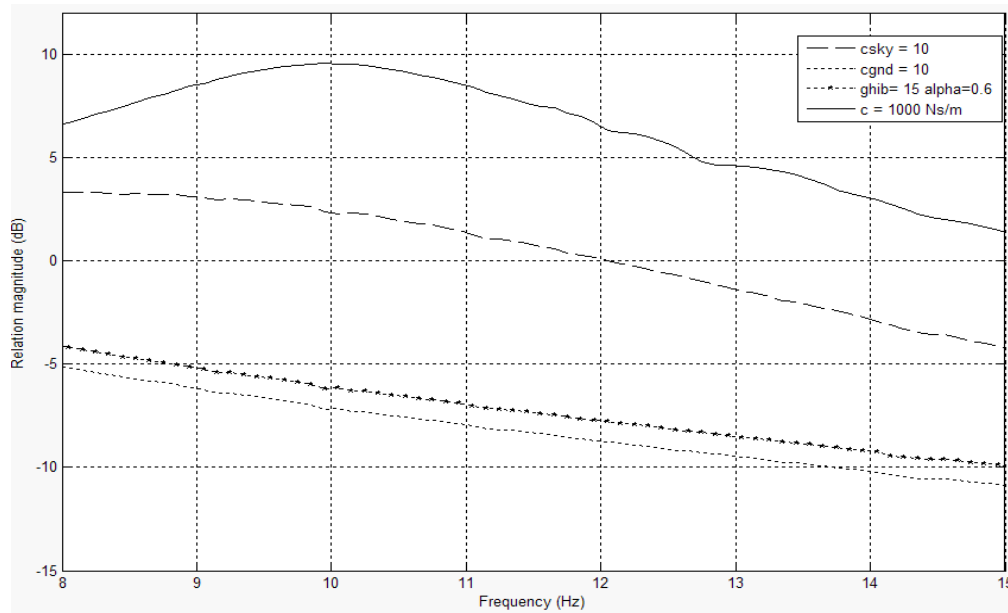
In [7], skyhook, groundhook and hybrid are applied keeping the gains between 0 and 1. The present study applies gains bigger than 1 and obtains better results, however, a saturation appears with high gains. For example,  $c_{sky}=10$  gives the best results for comfort, but as  $c_{sky}$  is moved up, the results turns worse. The same phenomenon occurs with  $c_{gnd}$  and  $g_{hib}$ . In the case of the hybrid approach, the  $\alpha$  was manually chosen to get the best response.

Another important issue is the physical meaning of the control gains. In equations (8), (10) and (16), if the gains ( $c_{sky}$ ,  $c_{gnd}$  or  $g_{hib}$ ) are increased, the control action will also be amplified. Physically the saturation value according to the damper characteristics used in [1], manages a range between 0 and 3 volts, thus the manipulation has to be limited to work inside these bounds. The output of the control strategies is saturated by software (simulink saturation block). The saturation block limits the controller to send manipulations between 0 and 3 volts.

Figure 3 presents the results related to passenger comfort. To comply to the performance criteria in [10]; for the sprung mass, the relation ( $x_s/x_p$ ) obtained with



**“Figure 3 Passenger comfort results. Relation ( $x_s/x_p$ ) between the chassis vertical displacement and the road profile magnitude. Notice the close-up at the frequencies of interest (0-4 Hz)”**



**“Figure 4 Vehicle stability results. Relation ( $x_t/x_p$ ) between the tire vertical displacement and the road profile magnitude. Notice the close-up at the frequencies of interest (8-15Hz)”**

the passive suspension has to be reduced in 6dB around the resonant frequency. Groundhook strategy does not achieve this objective, while skyhook generates the best performance. The hybrid controller is in the acceptable limits, but is not as good as skyhook.

In Figure 4, the pseudobode diagrams ( $x_t/x_p$ ) for the unsprung mass are presented. The performance of the passive suspension has to be improved by reducing the relation ( $x_t/x_p$ ) in 6dB. The best stability performance is provided by the groundhook approach, while skyhook sometimes complies with the requirement and sometimes fails or barely complies. In the range

8–15 Hz, the hybrid control scheme always approves the specification, performing almost as good as the groundhook strategy.

## 5. Conclusions

Different well studied control strategies were applied to a new semi-active suspension model based on a quarter vehicle model and including the actuator dynamics. The proposed model can be used with skyhook, groundhook and hybrid approaches to generate good performance results.

This research shows that skyhook is the best approach for passenger comfort while groundhook is the best strategy for vehicle stability. However, they fail to comply with both performances at the same time, forcing to seek an hybrid approach that deals with more than one criterion.

The semi-active suspension model with hybrid control obtains the best results and complies with the performance criteria. This suspension model maintains passenger comfort and vehicle stability, improving, by far, the performance obtained with an average vehicle passive suspension.

## 6. Future Work

The hybrid gain  $g_{hib}$  and  $\alpha$  factor were tuned in an heuristic form. Another option for future work is to implement an algorithm to calculate the values for  $g_{hib}$  and  $\alpha$  factors. It is also convenient to explore another control approaches.

The analysis of this research is limited to vertical motion, thus some variables are not taking in consideration. It could be probable to have a more accurate analysis including more vehicle dynamics.

The next step is the identification of our own magnetorheological damper to obtain the Bouc-wen parameters. Later, a control on a real vehicle will be performed. Some research works have reported the vertical acceleration of the chassis as a control variable, therefore including this analysis could provide more information about the suspension performance.

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