Modeling and automation - Evidence 1

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Time Thursday January 26 2023

Place 4102

Permitted aids The single page with your own notes.

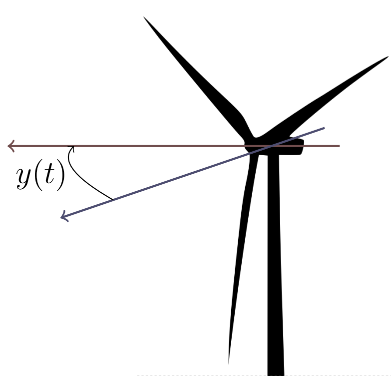
All answers should be readable and well motivated (if nothing else is written). Solutions/motivations should be written on the provided spaces in this exam. Use the last page if more space is needed.

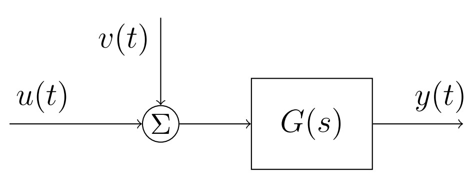
Good luck!

Matricula and name:

Yaw control of a wind turbine

Yaw is the horizontal angle between the direction the wind is blowing from and the direction the wind turbine is facing. Most wind turbines in commercial use have active control of the yaw angle in order to optimize energy production.





Assume that the friction force opposing the turning of the turbine is viscuous (proportional to the velocity), and that the wind force is causing a disturbance torque on the head. The dynamics of the system can then be described by the differential equation

$$J\ddot{y}(t) + f\dot{y}(t) = u(t) + v(t), \tag{1}$$

where $y(t)$ is the yaw angle, $u(t)$ is the torque produced by the motor turning the head, $v(t)$ is the disturbance torque, J is the moment of inertia and f is the friction coefficient.

Problems

Exercise 1

(a) Show that the plant model (1) can be expressed in the Laplace-domain as

$$Y(s) = G(s)\big(U(s) + V(s)\big) = \frac{b}{s(s+a)}\big(U(s) + V(s)\big),$$

with $b = \frac{1}{J}$ and $a = \frac{f}{J}$.

Calculations:

(b) Determine the poles of the system

Calculations:

(c) Choose a state vector x, and write the system (1) on state-space form.

$$\dot{x} = Ax + B \begin{bmatrix} u \\ v \end{bmatrix}$$
$$y = Cx$$

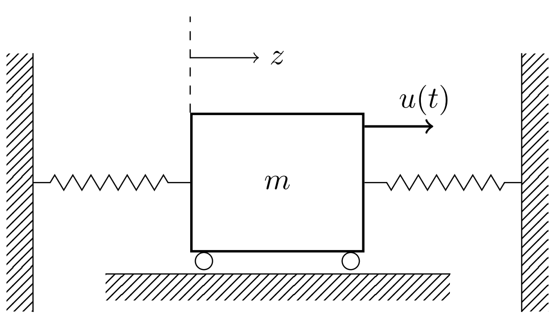
Note that B is a matrix with two columns, $B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$, so an equivalent way of writing the state-space model is

$$\dot{x} = Ax + B_1 u + B_2 v$$
$$y = Cx$$

Calculations:

A spring-mass system

The figure below shows a system consisting of one mass connected with two **identical** springs to two rigid walls. The mass can move without friction in the horizontal direction.



The displacement z is a deviation from the equilibrium position when the mass is in the middle position between the two walls. There is an external force $u(t)$ acting on the mass.

Exercise 2

(a) Draw a free-body diagram.

Drawing:

(b) The spring is nonlinear, and the magnitude of the force in the spring is given by

$$F_s = a_1 x + a_2 x^3,$$

where x is the deviation from its resting length. When the mass is in equilibrium, the two springs are stretched the same amount x_0 (since they are identical). The corresponding forces in the two springs is F_0 (the two forces are of the same magnitude but have opposite direction). **Determine the parameter k in a linear model of the spring**

$$F(t) = k\, z(t),$$

where $z(t)$ and $F(t)$ are deviation variables, i.e. $x(t) = x_0 + z(t)$ and $F_s(t) = F_0 + F(t)$.

Calculations:

(c) What are the dimensions in SI-units of the parameters k, a_1 and a_2 respectively?

$[k] =$

$[a_1] =$

$[a_2] =$

,

(d) Determine the differential equation that describes the system.

Calculations: