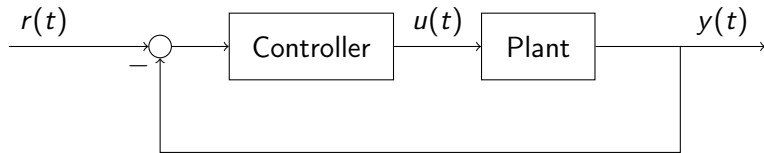


Design of control systems

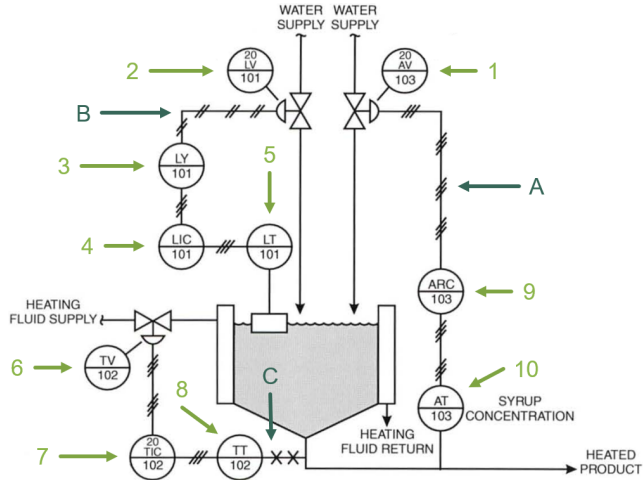
Kjartan Halvorsen

September 23, 2022

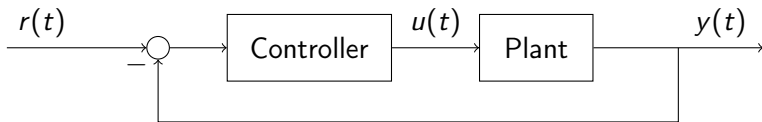
Feedback control



Feedback control systems are ubiquitous

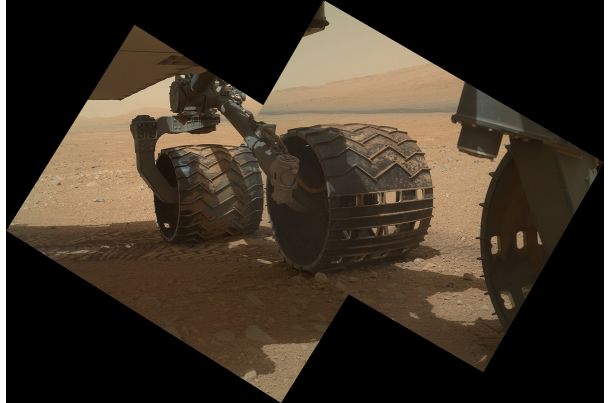


Feedback control systems



Controller design: Determine a feedback controller such that the controlled system performs according to given **performance specifications**.

The problem situation

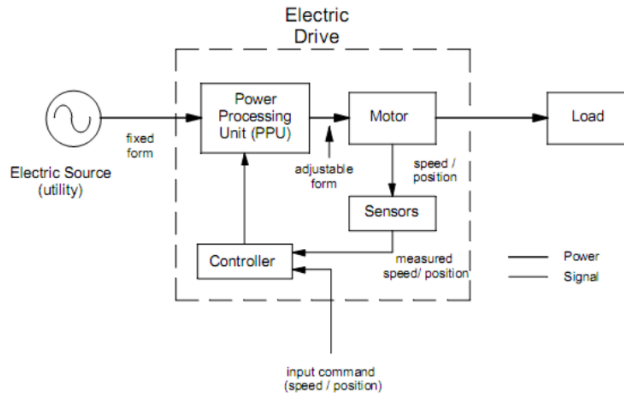
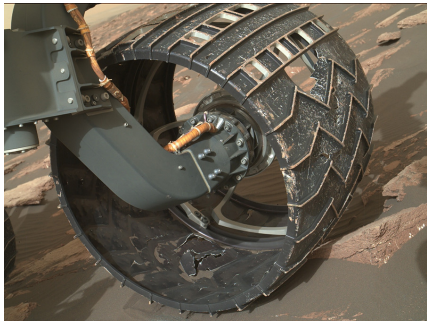


- ▶ Mass 900 kg
- ▶ Six wheels, each with an electric motor
- ▶ Front- and rear-wheel steering
- ▶ Rocker-bogie suspension system

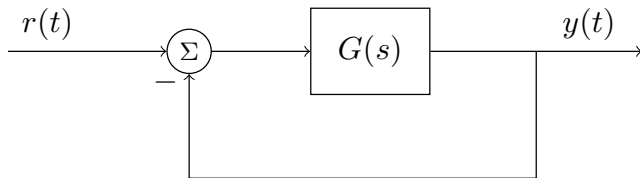
The problem situation



The problem situation



Block diagram algebra



Transfer function from $r(t)$ to $y(t)$:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

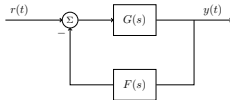
Block diagram algebra

Activity Pair the block-diagram with the correct closed-loop transfer function!

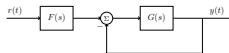
A



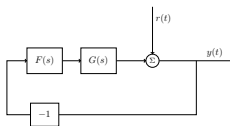
B



C



D



I

$$\frac{Y(s)}{R(s)} = \frac{G(s)F(s)}{1+G(s)}$$

II

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)F(s)}$$

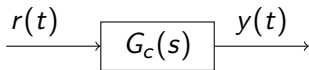
III

$$\frac{Y(s)}{R(s)} = \frac{1}{1+G(s)F(s)}$$

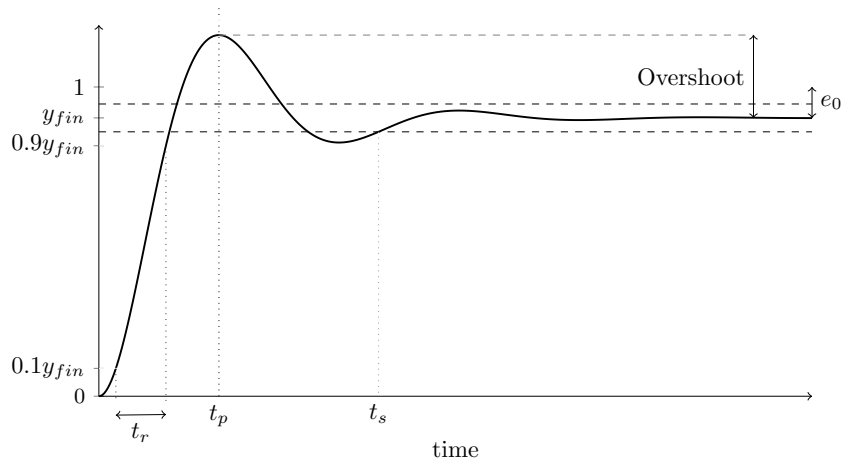
IV

$$\frac{Y(s)}{R(s)} = \frac{G(s)F(s)}{1+G(s)F(s)}$$

Performance requirements



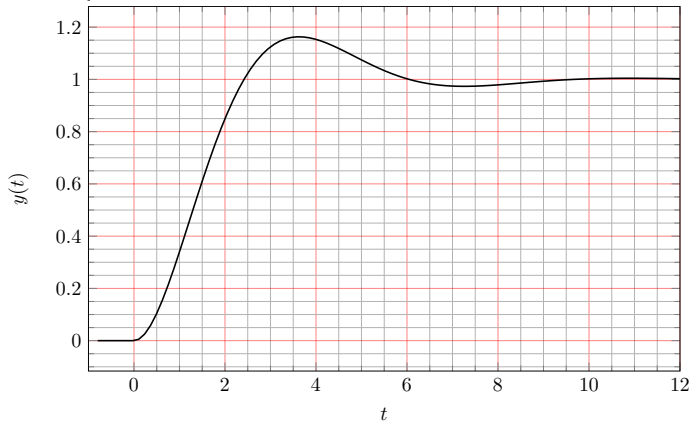
Performance requirements - time domain



Performance requirements - time domain

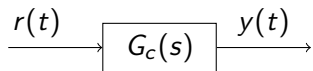
Activity Does the system satisfy the requirements?

Rise time $< 1.5\text{s}$
Overshoot $< 18\%$



Performance requirements - frequency domain

Response of LTI systems to sinusoids

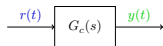


Let $r(t) = \sin \omega_1 t$. Then, after transients have died out,

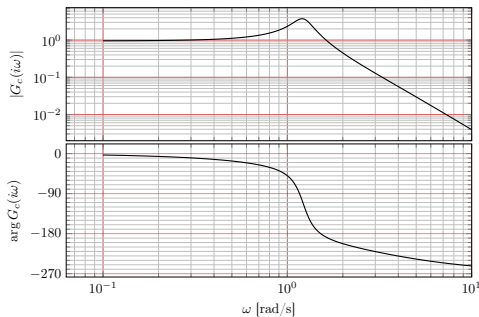
$$y(t) = |G_c(i\omega_1)| \sin(\omega_1 t + \arg G_c(i\omega_1)).$$

The Bode diagram shows the frequency properties of a dynamical system

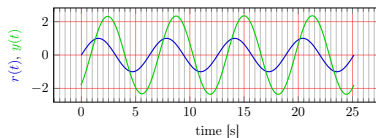
$$y(t) = \underbrace{|G_c(i\omega_1)|}_{\text{amplification}} \sin(\omega_1 t + \underbrace{\arg G_c(i\omega_1)}_{\text{phase shift}})$$



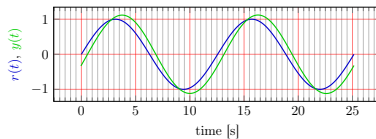
Bode plot



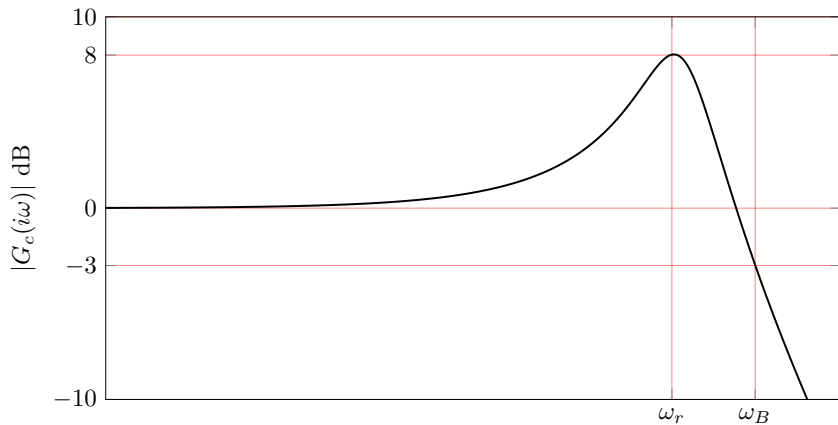
$$r(t) = \sin(\omega t), \omega = 1$$



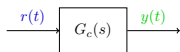
$$r(t) = \sin(\omega t), \omega = 0.5$$



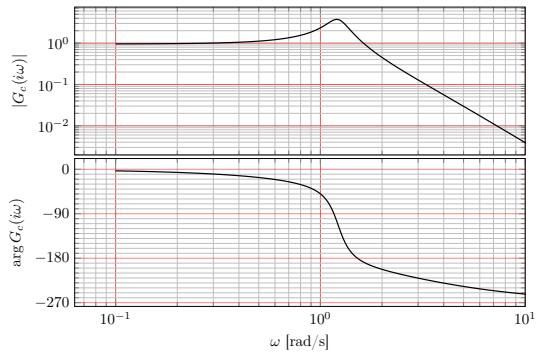
Performance requirements - frequency domain



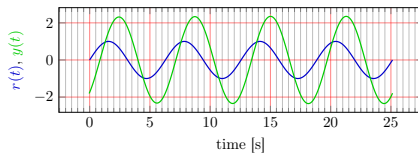
Performance requirements - frequency domain



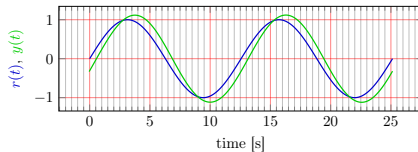
Bode plot



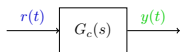
$r(t) = \sin(\omega t)$, $\omega = 1$



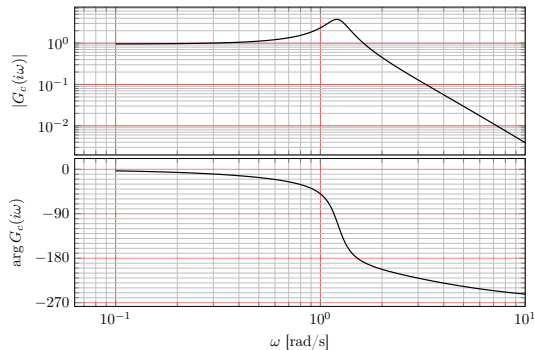
$r(t) = \sin(\omega t)$, $\omega = 0.5$



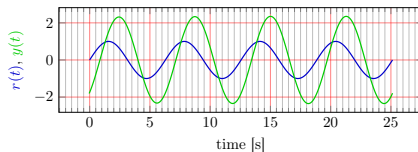
Performance requirements - frequency domain



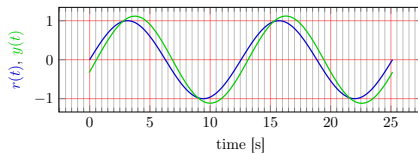
Bode plot



$r(t) = \sin(\omega t)$, $\omega = 1$

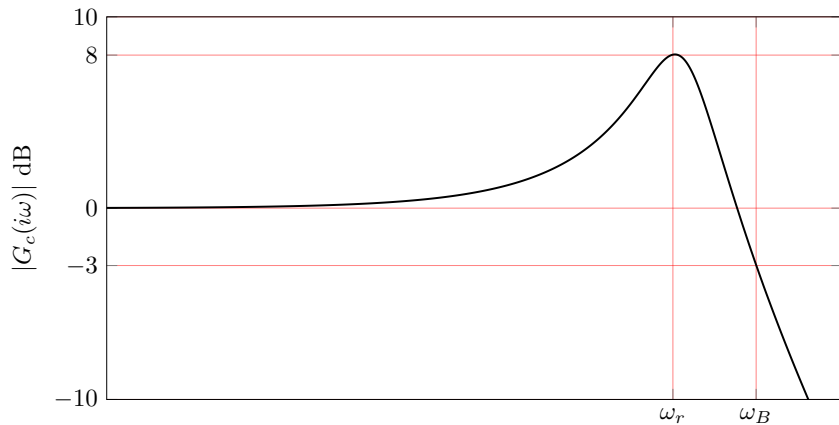


$r(t) = \sin(\omega t)$, $\omega = 0.5$



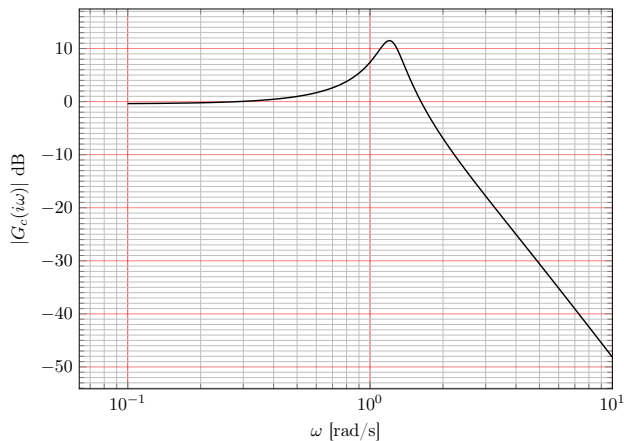
Activity What is the gain and phase shift at $\omega = 2$ rad/s?

Performance requirements - frequency domain



Performance requirements - frequency domain

Activity Does the system satisfy the requirements?



Bandwidth > 3 rad/s
Resonance peak < 9 dB