The DC motor

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Force acting on an electric conductor in a magnetic field

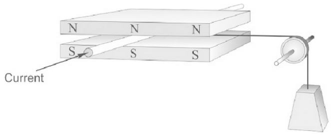


FIG. 1.14 Primitive linear d.c. motor.

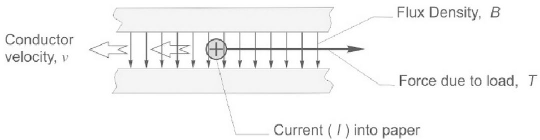


FIG. 1.15 Diagrammatic sketch of primitive linear d.c. motor.

Source: Hughes and Drury "Electric motors and drives"

$$F = k_m I = (B l_m) I,$$

Rotating motor

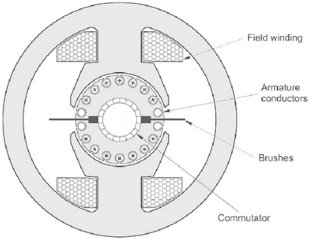


FIG. 3.1 Conventional (brushed) d.c. motor.

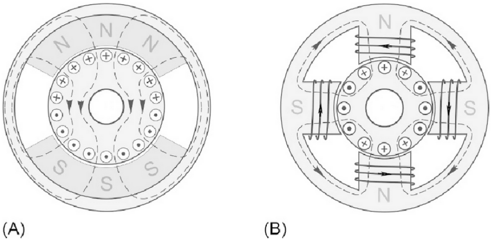


FIG. 3.2 Excitation (field) systems for d.c. motors (A) two-pole permanent magnet; (B) four-pole wound field.

Source: Hughes and Drury "Electric motors and drives"

Magnetic force and electro-motive force

The magnetic force on a current-carrying conductor

$$F(t) = k_f i(t) \quad \Leftrightarrow \quad T(t) = k_f r i(t) = k_m i(t)$$

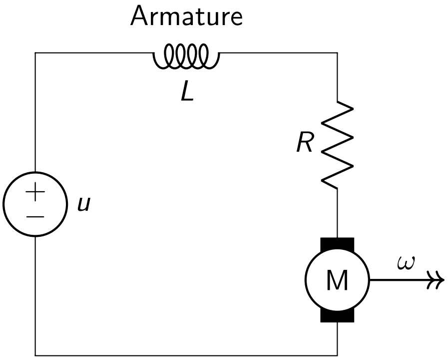
Voltage generated in a conductor moving in a magnetic field

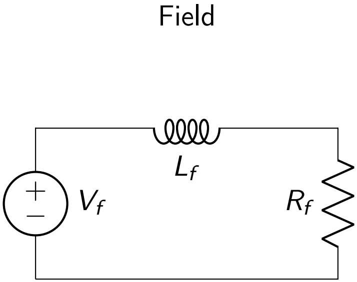
$$e(t) = k_v v(t) \quad \Leftrightarrow \quad e(t) = k_v r \omega(t) = k_e \omega(t)$$

$e(t)$ is called *Back electro-motive force (Back e.m.f.)*.
In the SI-system units $k_m = k_e = k$.

Equivalent circuit

Consider a DC motor with separate excitation

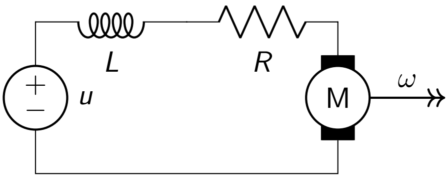




$$L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = u$$

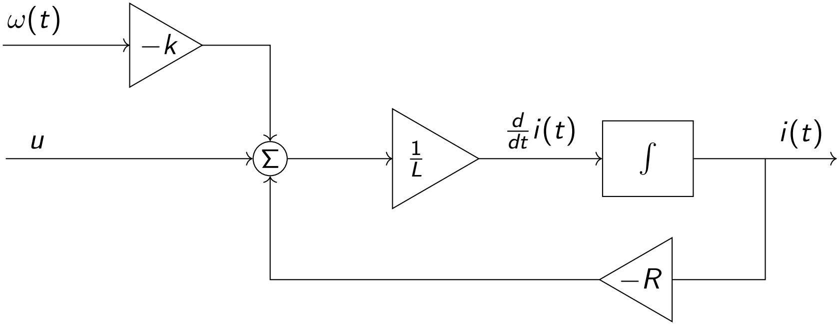
Newton: $J\frac{d}{dt}\omega(t) = ki(t) - T_l(t)$

Modeling the DC motor



$$L\frac{d}{dt}i(t) + Ri(t) + k\omega(t) = u$$

$$\frac{d}{dt}i(t) = \frac{1}{L}\Big(- Ri(t) - k\omega(t) + u\Big)$$



Transmission

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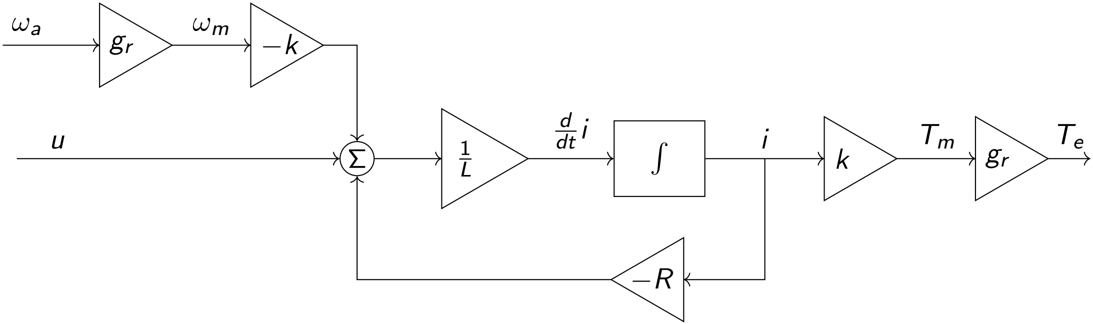
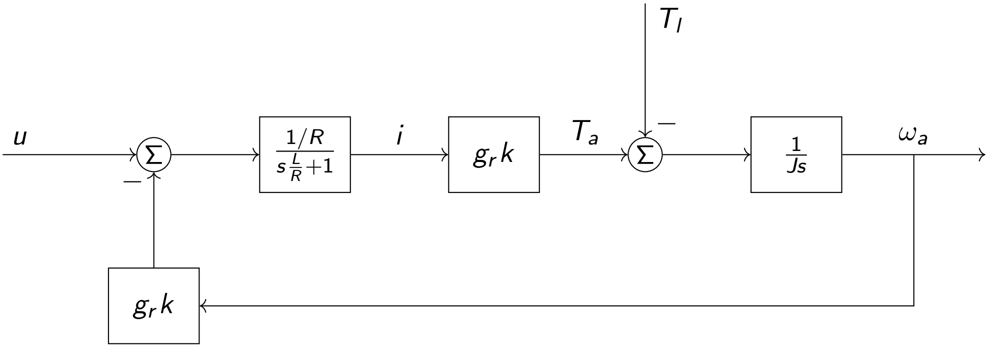


Ignoring losses in the transmission:

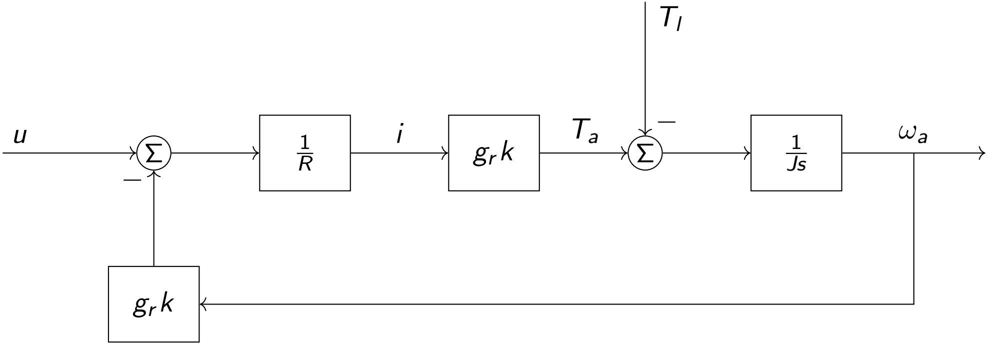
$$\underbrace{T_m\omega_m}_{\text{Power in}} = \underbrace{T_e\omega_a}_{\text{Power out}}$$

DC motor driving a load



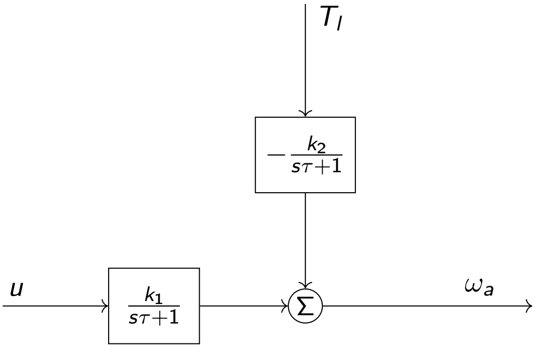
DC motor driving a load

Assuming the inductance to be negligable.



Activity What is the transfer function from the voltage input $u(t)$ to the angular velocity $\omega_a(t)$?

DC motor driving a load



$$\tau = \frac{JR}{(g_r k)^2}, \quad k_1 = \frac{1}{g_r k}, \quad k_2 =$$