The Apollo LM state feedback assignment

Kjartan Halvorsen

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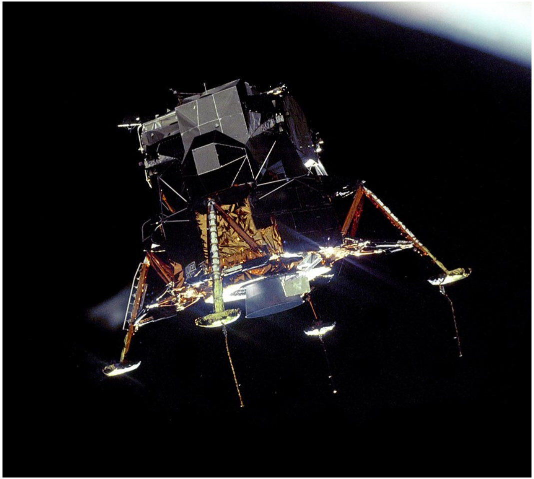
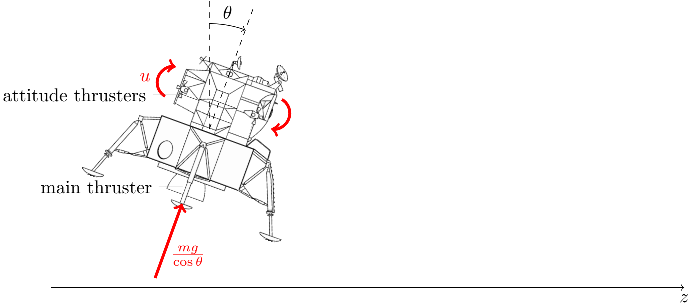
The Apollo lunar module

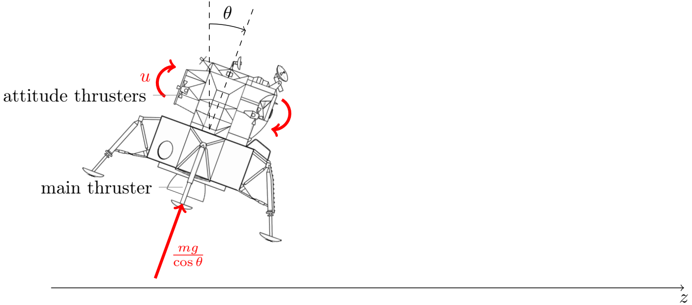


The Apollo lunar module



$$u(t) \longrightarrow \boxed{\frac{1}{Js}} \xrightarrow{\dot{\theta}(t)} \boxed{\frac{1}{s}} \xrightarrow{\theta(t)} \boxed{g} \xrightarrow{\ddot{z}(t)} \boxed{\frac{1}{s}} \longrightarrow \dot{z}(t)$$

The Apollo lunar module



$$u(t) \longrightarrow \boxed{\frac{1}{Js}} \xrightarrow{\dot{\theta}(t)} \boxed{\frac{1}{s}} \xrightarrow{\theta(t)} \boxed{g} \xrightarrow{\ddot{z}(t)} \boxed{\frac{1}{s}} \xrightarrow{\dot{z}(t)}$$

Activity Which is the transfer function of the system?

$$1: \;\; G(s) = \frac{\frac{g}{J}}{s^2} \qquad 2: \;\; G(s) = \frac{\frac{g}{J}}{s(s^2+1)} \qquad 3: \;\; G(s) = \frac{\frac{g}{J}}{s^3}$$

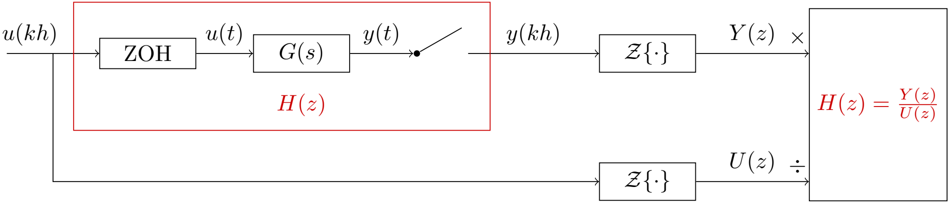
Plan

1. Obtain discrete-time pulse transfer function for the LM.
2. Convert transfer function to discrete-time state space model.
3. Design a state feedback controller $u(k) = Lx(k) + l_0 r(k)$ to obtain good reference response.
4. Design an observer $\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K\big(y(k) - C\hat{x}(k)\big)$ and studying two cases: Slow observer vs fast observer.

1. Discretize the continuous-time model

$$G(s) = \frac{\frac{g}{J}}{s^3}$$

The idea is to sample the continuous-time system's response to a step input, in order to obtain a discrete approximation which is exact (at the sampling instants) for such an input signal.



Step-invariant sampling (zero order hold): $u(kh) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$

2. Convert to discrete-time state space

Using, for instance, the observable canonical form

$$H(z) = \frac{b_1 z^{n-1} + \cdots + b_{n-1} z + b_n}{z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_n}$$

can be represented on state-space form as

$$x(k+1) = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ -a_3 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_n & 0 & 0 & \cdots & 0 \end{bmatrix} x(k) + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} x(k)$$

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$$y(k) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} x(k)$$

Activity Determine the discrete-time state-space model of the LM on observable canonical form