Characterization of a DC motor

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Modeling

ODE

The two coupled differential equations that govern the behavior of the DC motor with constant stator magnetic field are

$$L\frac{d}{dt}i(t) = u(t) - Ri(t) - kg_r\omega(t), \tag{1}$$
$$J\frac{d}{dt}\omega(t) = kg_r i(t) - T_l(t) \tag{2}$$

with variables

$i(t)$ The armature current,

$\omega(t)$ The angular velocity of the wheel,

$u(t)$ The armature voltage,

$T_l(t)$ The load torque,

and parameters

L The armature inductance,

R The armature resistance,

k The motor constant (in Nm/A or V/(rad/s)),

g_r The gear ratio

J The moment of inertia.

Transfer functions

Taking the Laplace transform of (1) and (2) (and ignoring the initial values) gives

$$sLI(s) = U(s) - RI(s) - kg_r\Omega(s), \tag{3}$$
$$sJ\Omega(s) = kg_r I(s) - T_l(s). \tag{4}$$

Solving (**??**) for $I(s)$ gives

$$I(s) = \frac{1}{sL+R}U(s) - \frac{kg_r}{sL+R}$$

and inserting in (4) gives

$$\begin{aligned}
sJ\omega(s) &= \frac{kg_r}{sL+R}U(s) - \frac{(kg_r)^2}{sL+R}\Omega(s) - T_l(s) \\
sJ(sL+R)\Omega(s) &= kg_rU(s) - (kg_r)^2\Omega(s) - (sL+R)T_l(s) \\
\big(sJ(sL+R) + (kg_r)^2\big)\Omega(s) &= kg_rU(s) - (sL+R)T_l(s) \\
\Omega(s) &= \frac{kg_r}{sJ(sL+R)+(kg_r)^2}U(s) - \frac{sL+R}{sJ(sL+R)+(kg_r)^2}T_l(s)
\end{aligned} \tag{5}$$