Compensator design - Loop shaping

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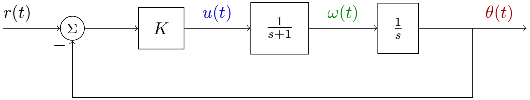
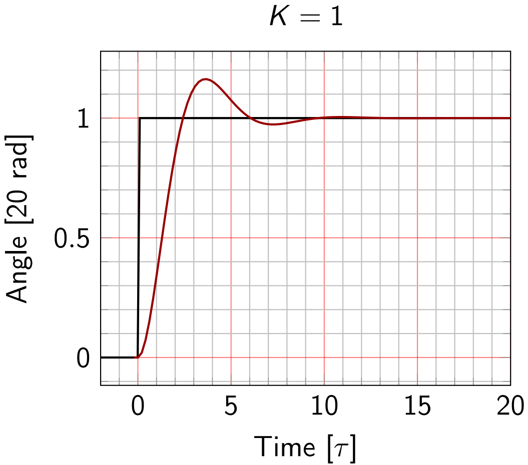
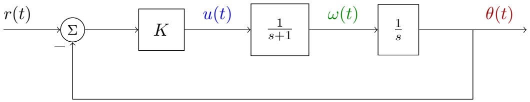
October 18, 2021

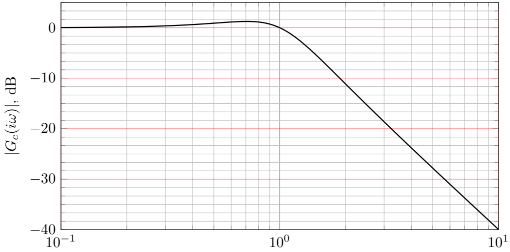
Proportional control of the normalized DC motor



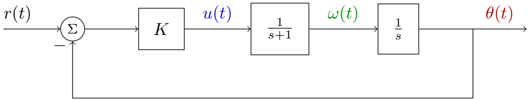


Proportional control of the normalized DC motor

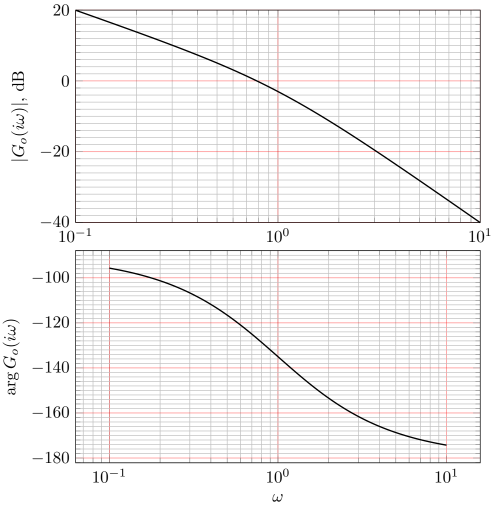




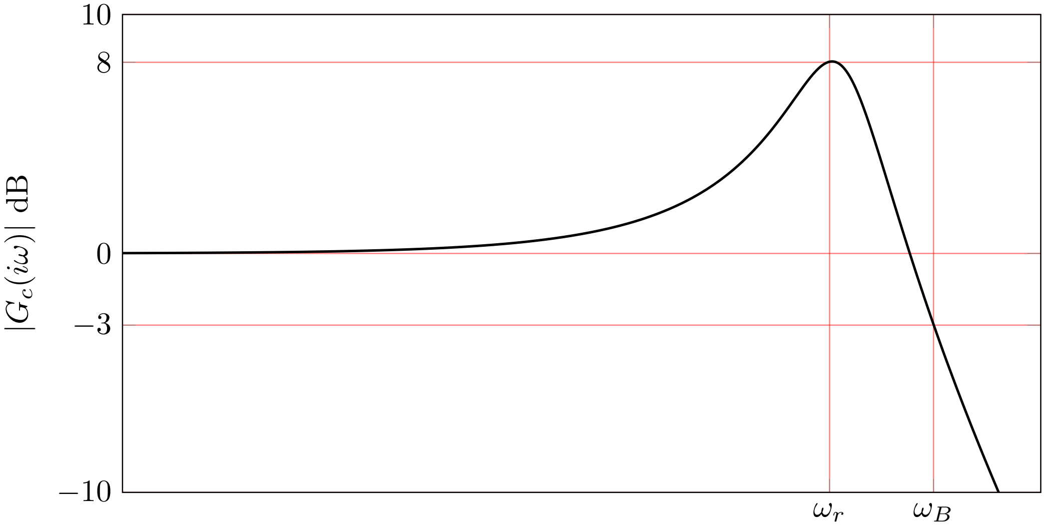
Proportional control of the normalized DC motor



Activity Determine the cross-over frequency and the phase margin.

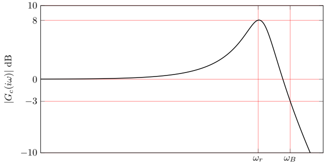


Specifications on the frequency properties of the closed-loop system

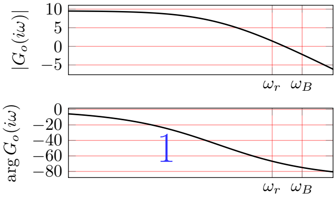


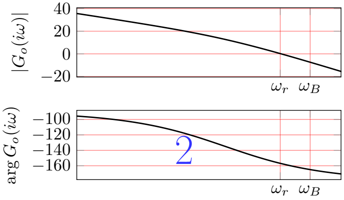
How to achieve the frequency-domain specifications

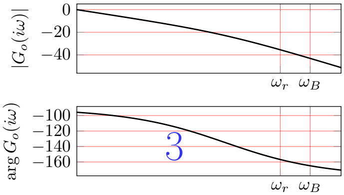
$$G_c(i\omega) = \frac{G_o(i\omega)}{1 + G_o(i\omega)}$$

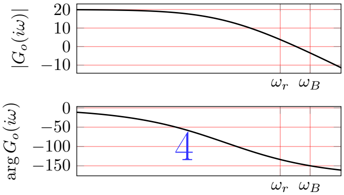


Activity Which of the Bode plots to the right shows the correct loop gain $G_o(i\omega)$?





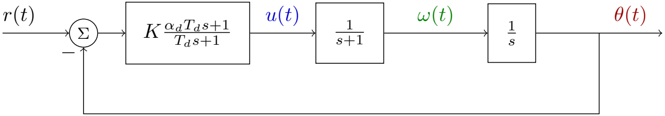




From specifications on G_c to specifications on G_o

Closed-loop specifications	Loop gain specifications
High bandwidth ω_B	High cross-over frequency ω_c
Low resonance peak M_p	Large phase margin φ_m
Static gain $G_c(0) \approx 1$	static gain $G_o(0)$ high

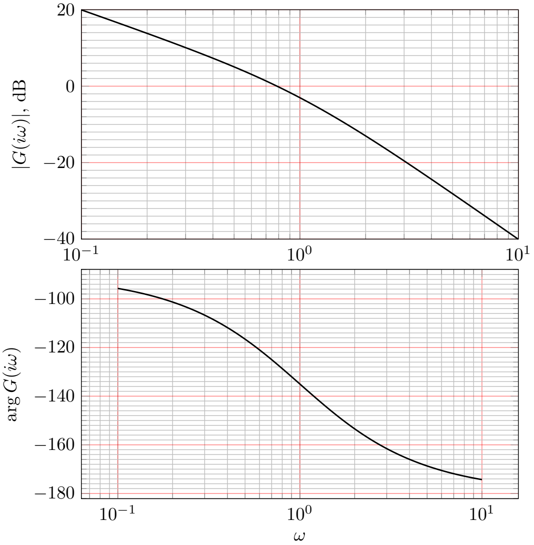
Position control of the DC motor



Specifications:

1. $\omega_B \approx \omega_c = 2\ \mathrm{rad/s}$
2. $\varphi_m > 60^\circ$

Position control of the DC motor



Specifications:

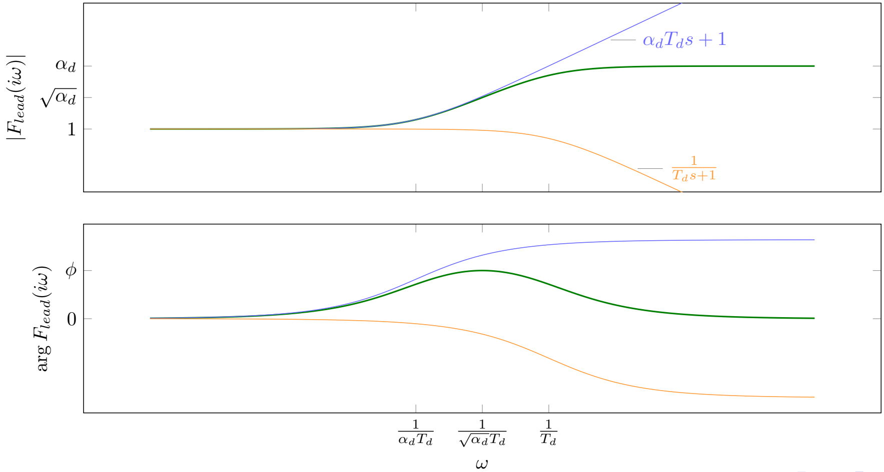
1. $\omega_B \approx \omega_c = 2\ \mathrm{rad/s}$
2. $\varphi_m =$
   $\arg G_o(i\omega_c) - (-180°) > 60°$

Activity

1. What is $|G(i\omega_c)|$?
2. What is $\arg G(i\omega_c)$?
3. What should $\arg G_o(i\omega_c)$ be to satisfy the phase margin requirement?
4. How much phase advance is needed at the desired cross-over frequency?

Position control of the DC motor - obtaining the phase advance

$$F_{lead}(s) = \frac{\alpha_d T_d s + 1}{T_d s + 1}$$



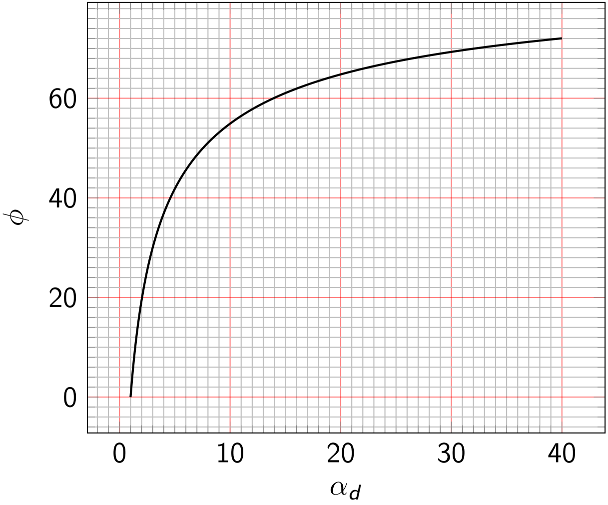
The maximum phase advance of the lead compensator

$$F_{lead}(s) = \frac{\alpha_d T_d s + 1}{T_d s + 1}$$

$$\phi = \max \arg F_{lead}(i\omega)$$

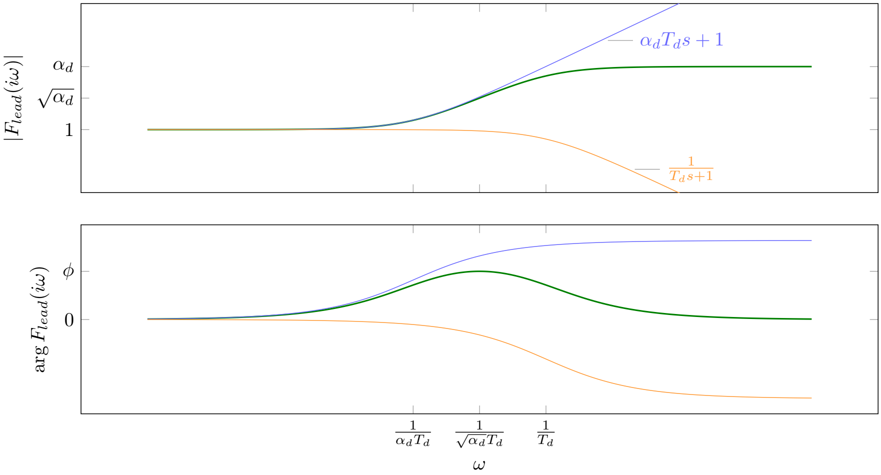
$$\sin\phi = \frac{\alpha_d - 1}{\alpha_d + 1} \quad \Leftrightarrow \quad \alpha_d = \frac{1 + \sin\phi}{1 - \sin\phi}$$

Activity Find the value of α_d that gives the necessary maximum positive phase $\arg F_{lead}(i\omega_c) = 34°$.



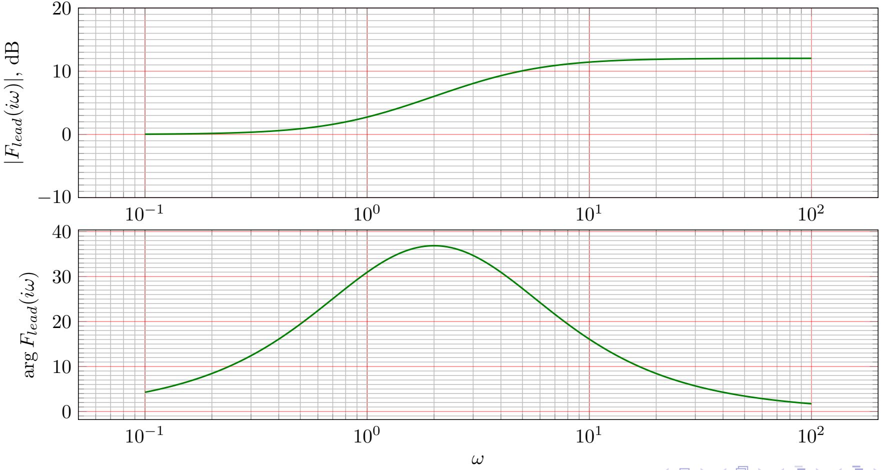
Position control of the DC motor - placing the phase peak

$$F_{lead}(s) = \frac{\alpha_d T_d s + 1}{T_d s + 1}$$

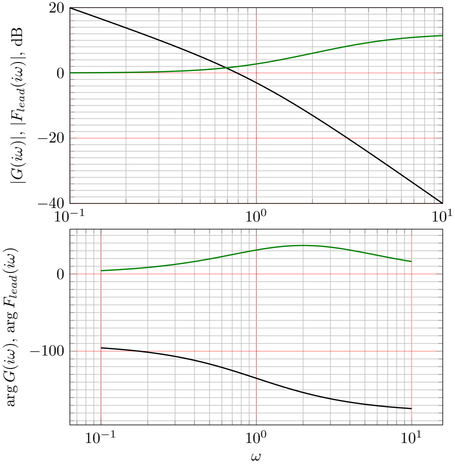


Position control of the DC motor - The resulting lead compensator

$$F_{lead}(s) = \frac{\alpha_d T_d s + 1}{T_d s + 1} = \frac{s + 1}{0.25s + 1}$$



Position control of the DC motor - Getting the gain right



Specifications

1. $\omega_B =\approx \omega_c = 2\ \mathrm{rad/s}$
2. $\varphi_m =$
   $\arg G_o(i\omega_c) - (-180^\circ) > 60^\circ$

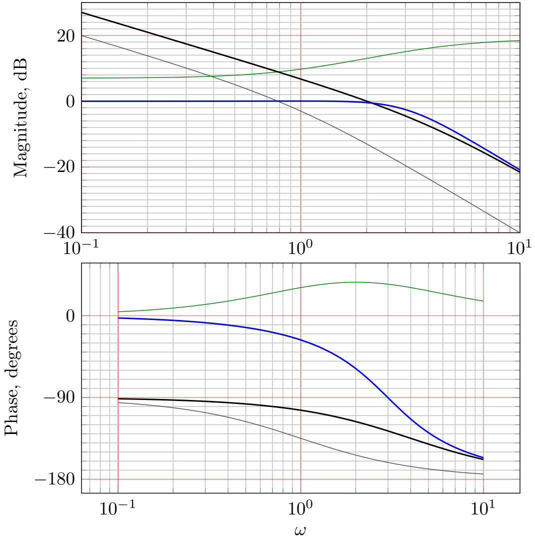
Activity

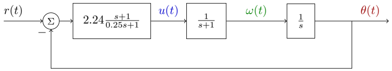
$$\begin{aligned} 20\log G_o(i\omega) &= 20\log KF(i\omega)G(i\omega) \\ &= 20\log K + 20\log F(i\omega) + 20\log G(i\omega) \end{aligned}$$

so, what should the gain K be to obtain

$$|G_o(i2)| = 1 = 0\text{dB}?$$

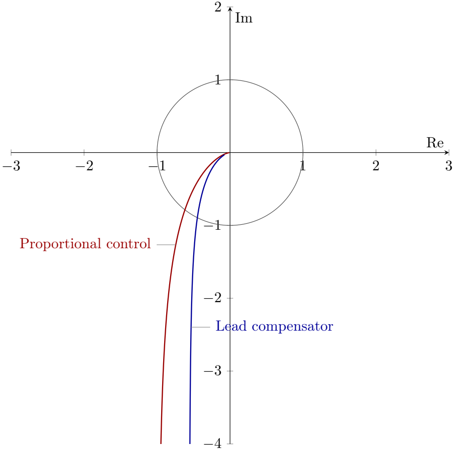
Position control of the DC motor - Results

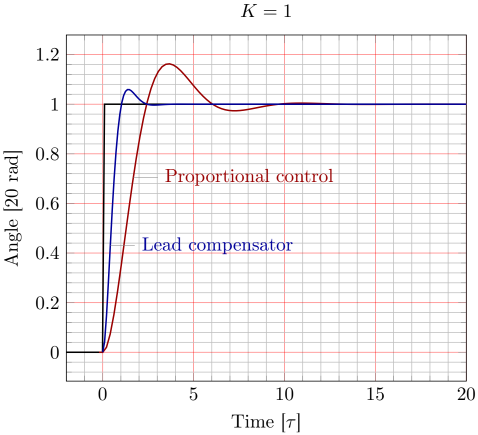




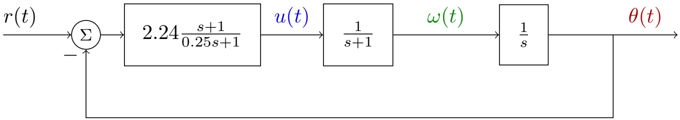
Activity Identify the frequency responses of: 1) The plant, 2) The compensator, 3) The loop gain, and 4) The closed-loop system.

Position control of the DC motor - Results

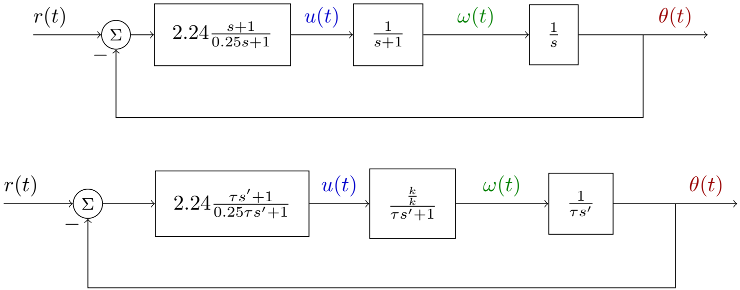




Applying the compensator design to a particular motor



Applying the compensator design to a particular motor



Applying the compensator design to a particular motor