State feedback with observer

Kjartan Halvorsen

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State feedback with reconstructed states

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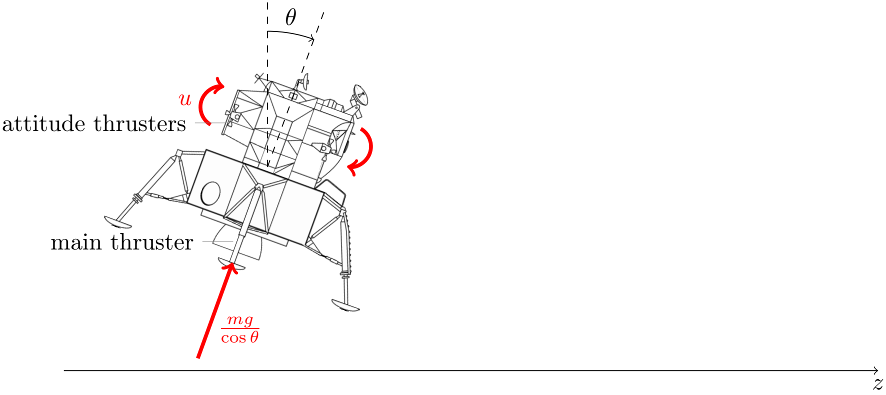

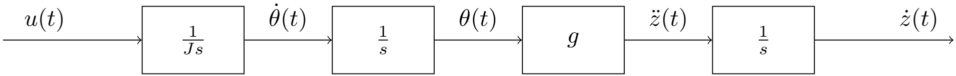

State feedback

Given

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \tag{1}$$

and measurements (or estimates) of the state vector x.

Linear state feedback is the control law

$$\begin{aligned}u = f\big((x, u_c\big) &= -l_1 x_1 - l_2 x_2 - \cdots - l_n x_n + l_0 u_c \\ &= -Lx + l_0 u_c,\end{aligned}$$

where

$$L = \begin{bmatrix} l_1 & l_2 & \cdots & l_n \end{bmatrix}.$$

Substituting the control law in the state space model (5) gives

$$\begin{aligned}\dot{x} &= (A - BL)x + l_0 Br \\ y(k) &= Cx\end{aligned} \tag{2}$$

Observer design

Given model

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

and measurements of the output signal y.

The observer is given by

$$\dot{\hat{x}} = \underbrace{A\hat{x} + Bu}_{\text{simulation}} + \underbrace{K\big(y - C\hat{x}\big)}_{\text{correction}} = (A - KC)\,\hat{x} + Bu + Ky$$

with poles given by the eigenvalues of the matrix $A_o = A - KC$

Rule-of-thumb Choose the poles of the observer (eigenvalues of $A - KC$) at least twice as fast as the poles (eigenvalues) of $A - BL$.

Observer design

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Control by feedback from reconstructed states

The design problem can be separates into two problems

1. Determine the gain vector L and the gain l_0 of the control law

$$u = -L\hat{x} + l_0 r$$

   so that the closed-loop system has good reference tracking.

2. Determine the gain vector K of the observer

$$\dot{\hat{x}} = A\hat{x} + Bu + K\left(y - C\hat{x}\right)$$

   to get a good balance between disturbance rejection and noise attenuation.

Computing the observer gain

A matrix M and its transpose M^{T} have the same eigenvalues. Hence, the problem of determining the gain K to obtain desired eigenvalues of

$$A - KC$$

is equivalent to determining the gain K in

$$(A - KC)^{\mathrm{T}} = A^{\mathrm{T}} - C^{\mathrm{T}} K^{\mathrm{T}}.$$

The last problem has the exact same form as the problem of determining L to obtain desired eigenvalues of

$$A - BL$$

So, the same matlab function can be used for both problems.

Computing the observer gain

1. Ackerman's method

   ```
   K = acker(Phi', C', po)'
   ```

2. More numerically stable method

   ```
   K = place(Phi', C', pd)'
   ```