

# Stability

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# Hard disk drive arm model



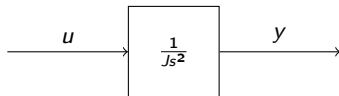
"Laptop-hard-drive-exposed" by Evan-Amos - Own work. Licensed under CC BY-SA 3.0 via Commons

## Sampling the hard disk drive arm model

$$H(z) = \frac{z-1}{z} \mathcal{Z} \left\{ \mathcal{L}^{-1} \left\{ \frac{G(s)}{s} \right\} \right\}$$



$$J\ddot{y} = u$$

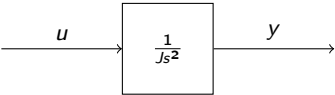


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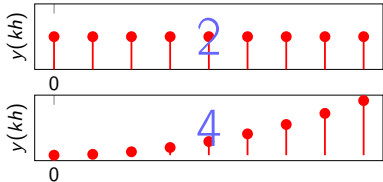
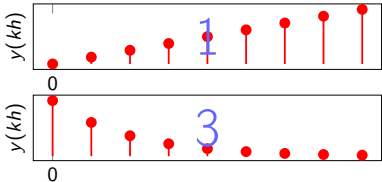
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Which of the below graphs show the sampled **step-response** of the system?

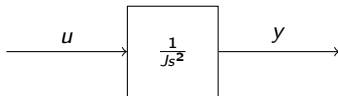


## Sampling the hard disk drive arm model

$$H(z) = \frac{z-1}{z} \mathcal{Z} \left\{ \mathcal{L}^{-1} \left\{ \frac{G(s)}{s} \right\} \right\}$$



$$J\ddot{y} = u$$



Sampled step-response:  $y(kh) = \frac{1}{2J}(kh)^2$

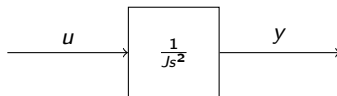
$$k^2 \quad \xleftrightarrow{\mathcal{Z}} \quad \frac{z(z+1)}{(z-1)^3}$$

# Sampling the hard disk drive arm model

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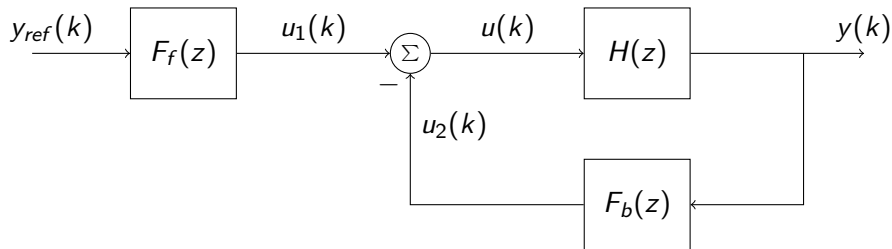
$$k^2 \quad \xleftrightarrow{\mathcal{Z}} \quad \frac{z(z+1)}{(z-1)^3}$$

Which of the below pulse-transfer functions corresponds to the discretized hard disk drive model?

$$\begin{array}{ccc} 1 & 2 & 3 \\ H(z) = \frac{h^2 z}{2J(z+1)^2} & H(z) = \frac{h^2(z+1)}{2Jz^2} & H(z) = \frac{h^2(z+1)}{2J(z-1)^2} \end{array}$$

## Block-diagram algebra

Same rules as in the continuous-time case!



With

$$U(z) = U_1(z) - U_2(z) = F_f(z)Y_{ref}(z) - F_b(z)Y(z), \quad \text{and}$$

$$Y(z) = H(z)U(z), \quad \text{we obtain}$$

$$Y(z) = \underbrace{\frac{F_f(z)H(z)}{1 + F_b(z)H(z)}}_{H_c z} Y_{ref}(z).$$

## Block-diagram algebra - steps in detail

With

$$U(z) = U_1(z) - U_2(z) = F_f(z)Y_{ref}(z) - F_b(z)Y(z), \quad \text{and}$$

$$Y(z) = H(z)U(z), \quad \text{we obtain}$$

$$Y(z) = H(z)U(z) = H(z)(F_f(z)Y_{ref}(z) - F_b(z)Y(z))$$

Move all terms with  $Y$  to the left side:

$$Y(z) + H(z)F_b(z)Y(z) = H(z)F_f(z)Y_{ref}(z)$$

$$Y(z)(1 + H(z)F_b(z)) = H(z)F_f(z)Y_{ref}(z)$$

$$Y(z) = \frac{H(z)F_f(z)}{1 + H(z)F_b(z)} Y_{ref}(z)$$



## Stability for the closed-loop system

$$Y(z) = \frac{F_f(z)H(z)}{1 + F_b(z)H(z)} Y_{ref}(z).$$

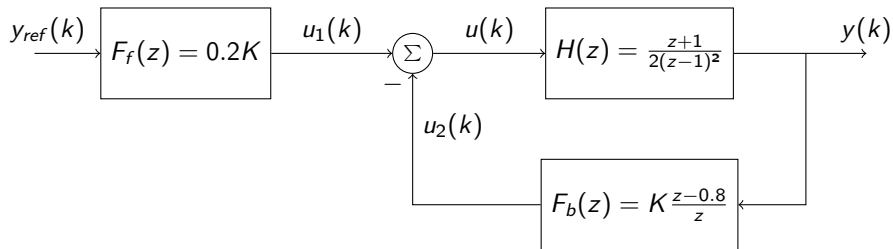
Stability requires that all poles of the system, i.e. all solutions to the characteristic equation

$$1 + F_b(z)H(z) = 0$$

are located inside the unit circle of the z-plane.

## Stability for the disk drive arm

Case  $\frac{h^2}{J} = 1$ . Suggested controller:

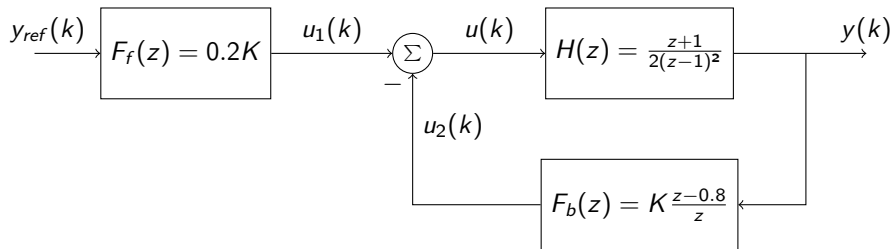


Characteristic equation

$$\begin{aligned} 1 + H(z)F_b(z) &= 0 \\ 1 + \frac{z+1}{2(z-1)^2} K \frac{z-0.8}{z} &= 0 \\ (z-1)^2 z + \frac{K}{2} (z+1)(z-0.8) &= 0 \end{aligned}$$

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Characteristic equation

$$\begin{aligned} 1 + H(z)F_b(z) &= 0 \\ 1 + \frac{z+1}{2(z-1)^2} K \frac{z-0.8}{z} &= 0 \\ (z-1)^2 z + \frac{K}{2} (z+1)(z-0.8) &= 0 \end{aligned}$$

Is the system stable for all gains  $K$ ?

## Stability for the disk drive arm

**Pair activity** Complete the root-locus below

$$(z - 1)^2 z + \frac{K}{2}(z + 1)(z - 0.8) = 0$$

