Sensitivity and robustness

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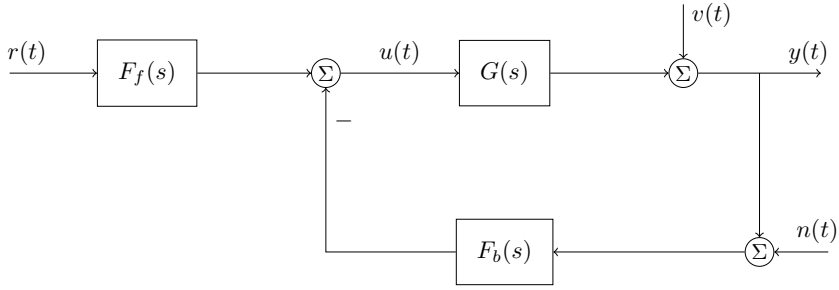
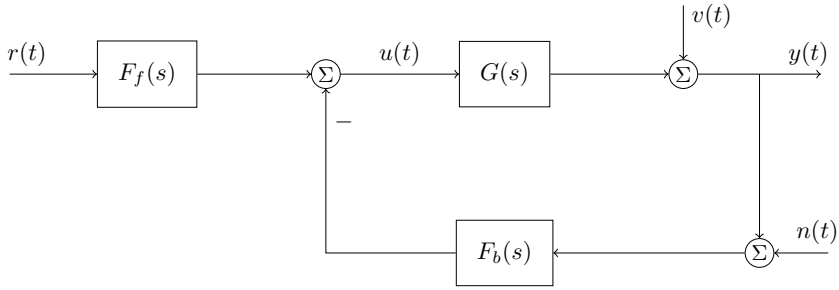
Sensitivity and complementary sensitivity

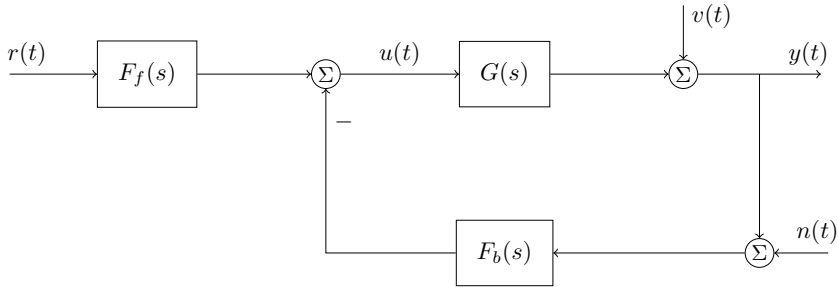


Sensitivity and complementary sensitivity



1. Determine the closed-loop transfer function $G_v(s)$ from $v(t)$ to $y(t)$ and the transfer function $G_n(t)$ from $n(t)$ to $y(t)$.

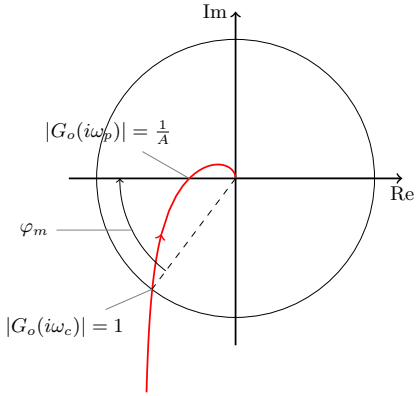
Sensitivity and complementary sensitivity



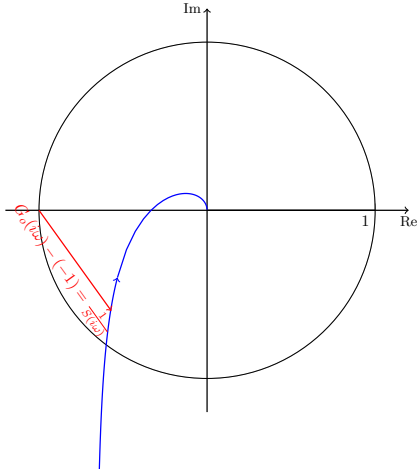
1. Determine the closed-loop transfer function $G_v(s)$ from $v(t)$ to $y(t)$ and the transfer function $G_n(t)$ from $n(t)$ to $y(t)$.

2. Show that if $F_{fb}(s)$ and/or $G(s)$ contains an integrator (pole in the origin), then $G_v(0) = 0$ and $G_n(0) = -1$. This means that constant disturbances are completely eliminated, but a constant measurement error (sensor bias) is passed unattenuated to the output.

The Nyquist plot and stability margins

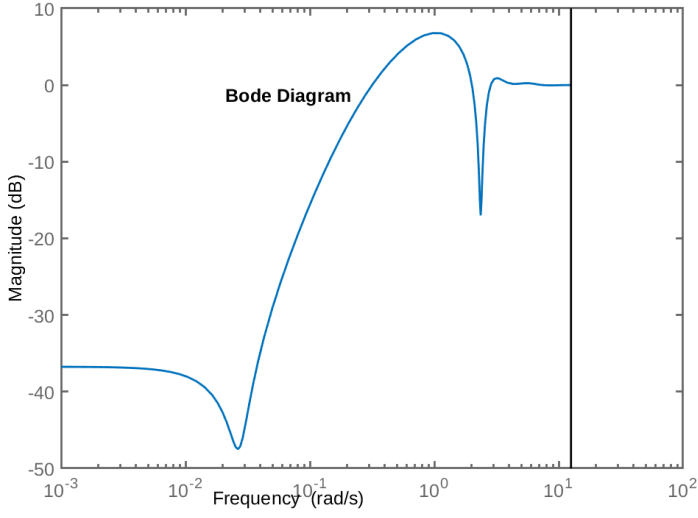


The sensitivity function



$$S(i\omega) = G_v(i\omega) = \frac{1}{1 + G_o(i\omega)} = \frac{1}{1 + G(i\omega)F_{fb}(i\omega)}$$

The sensitivity function

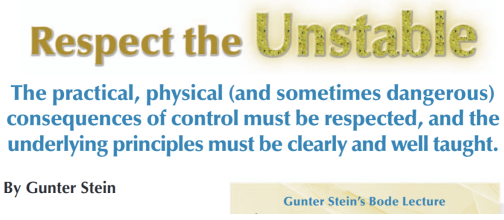


Interpretations of the sensitivity function

Several important interpretations of the sensitivity function $S(s)$

1. $S(i\omega)$ tells us how well our closed-loop system attenuates disturbances of different frequencies
2. Its maximum value is a measure of how close the closed-loop system is to being unstable.
3. $S(i\omega)$ tells us how modelling errors or modelling variations of the plant influences the closed-loop system

An important limitation



An important limitation

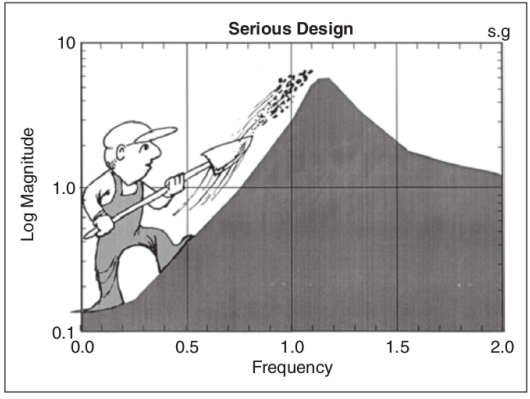


Figure 3. *Sensitivity reduction at low frequency unavoidably leads to sensitivity increase at higher frequencies.*

$$\int_0^\infty \ln|S(i\omega)|d\omega = \int_0^\infty \ln\left|\frac{1}{1+G_o(i\omega)}\right| d\omega = \pi \sum Re(p_k)$$

An important limitation