PID for industrial control

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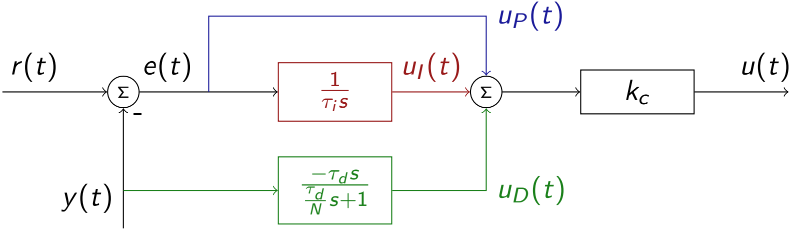
The PID - practical form



The parameter N is chosen to limit the influence of noisy measurements. Typically,

$$3 < N < 20$$

PID tuning

Method by Smith & Corripio using table by Ziegler-Nichols

Given process model (fitted to response of the system)

$$G(s) = K\frac{\mathrm{e}^{-s\theta}}{\tau s + 1}$$

and PID controller

$$F(s) = k_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right)$$

Choose the PID parameters according to the following table (Ziegler-Nichols, 1943)

Controller	k_c	τ_i	τ_d
P	$\frac{\tau}{\theta K}$		
PI	$\frac{0.9\tau}{\theta K}$	$\frac{\theta}{0.3}$	
PID	$\frac{1.2\tau}{\theta K}$	2θ	$\frac{\theta}{2}$

Gives good control for

$$0.1 < \frac{\theta}{\tau} < 0.6.$$

The delay

$$\xrightarrow{\;u\;} \boxed{G(s)} \xrightarrow{\;y\;}$$

Let $u(t) = \sin \omega_1 t$. Then, after transients have died out,

$$y(t) = |G(\omega_1)| \sin\left(\omega_1 t + \arg G(i\omega_1)\right).$$

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Activity Identify $|G(i\omega)|$ and $\arg G(i\omega)$ for the pure delay.

Analytical PID design

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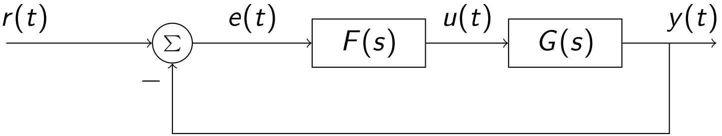

Activity Solve for $F(s)$ in the closed-loop transfer function

$$G_c(s) = \frac{G(s)F(s)}{1 + G(s)F(s)}$$

Analytical PID design - Solution

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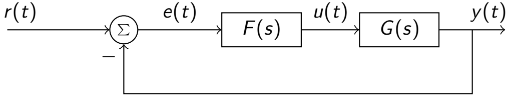
Solving for $F(s)$ in the closed-loop transfer function $G_c(s) = \frac{G(s)F(s)}{1+G(s)F(s)}$

$$\left(1 + G(s)F(s)\right) G_c(s) = G(s)F(s)$$

$$G_c(s) = \left(1 - G_c(s)\right) G(s)F(s)$$

$$F(s) = \frac{\frac{G_c(s)}{G(s)}}{1 - G_c(s)}$$

Analytic PID tuning - first-order with delay

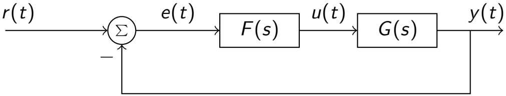


Given model $G(s) = K\frac{\mathrm{e}^{-s\theta}}{\tau s+1}$ of the process and desired closed-loop transfer function $G_c(s) = \frac{\mathrm{e}^{-s\theta}}{\tau_c s+1}$

$$F(s) = \frac{\frac{G_c(s)}{G(s)}}{1 - G_c(s)} = \frac{\frac{\mathrm{e}^{-s\theta}}{\tau_c s+1}\frac{\tau s+1}{K\mathrm{e}^{-s\theta}}}{1 - \frac{\mathrm{e}^{-s\theta}}{\tau_c s+1}} = \frac{1}{K}\left(\frac{\tau s + 1}{\tau_c s + 1 - \mathrm{e}^{-s\theta}}\right)$$

$$\approx \frac{1}{K}\left(\frac{\tau s + 1}{(\tau_c + \theta)s}\right)$$

Analytic PID tuning - first-order with delay



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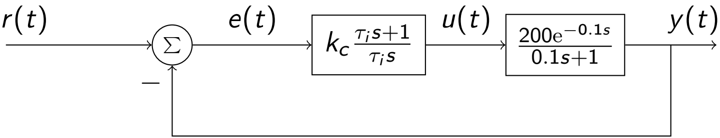
$$\approx \frac{1}{K}\left(\frac{\tau s + 1}{(\tau_c + \theta) s}\right)$$

Activity A PI-controller can be written $F(s) = k_c\frac{\tau_i s+1}{\tau_i s}$. Determine k_c and τ_i in terms of the parameters K, θ, τ and τ_c.

Example



$k_c = \frac{\tau}{K(\tau_c+\theta)}$ and $\tau_i = \tau$.

Activity Determine the controller for the choice $\tau_c = \tau$

The PID - practical aspects

Åström & Hägglund (1988) *PID controllers: Theory, design and tuning, 2nd ed* Instrument Society of America.

Approximating nonlinear systems with linear models

- Model is accurate only in neighborhood of operating point for which the system is approximated.
- Solution: Divide operating range into many regions, with separate PID parameters for each region

Approximating high-order systems with low-order models

- Only accurate for low frequencies
- Beware of behavior for high-frequency input to the closed-loop system

The PID - practical aspects, contd

When do PID controllers work well?

- The plant dynamics can be well approximated with low-order model
- Demands on performance not too high

More sophisticated control needed when

- Higher order dynamics
- Oscillatory modes
- Long deadtime

The PID - practical aspects, contd

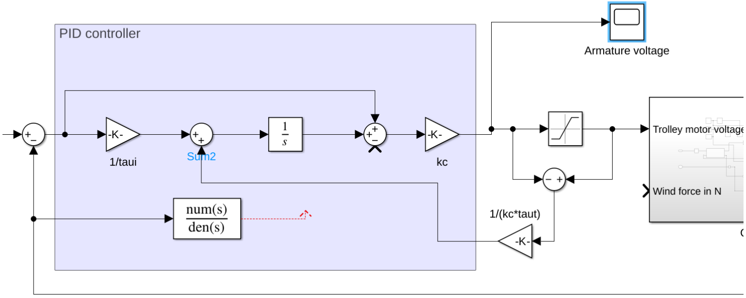
Choice of controller

1. P-controller if damping and steady-state error satisfied
2. PI-controller if steady-state error must be zero (often 1st order dynamics)
3. PID-controller if PI does not give sufficient damping (often 2nd order dynamics)
4. Tuning parameter τ_c for SIMC tuning method:
    - Smaller (=faster) than τ if sufficiently damped and limitations on input signal not violated.
    - larger (=slower) than τ if more damping required or smaller input signal required.

Integral windup

Video by Tomás Alejandro Lugo Salinas (MTY)

Anti-windup using back-calculation



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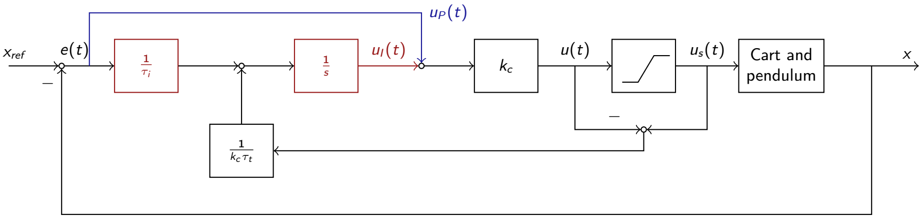

Activity Assume $e(t) = 0$ and determine the transfer function from $u_s(t)$ to $u(t)$.