Compensator design - Loop shaping

Kjartan Halvorsen

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Specifications on the frequency properties of the closed-loop system

The design procedure - overview

$$\boxed{\text{Specifications on the closed-loop system } G_c(i\omega)}$$

$$\downarrow$$

$$\boxed{\text{Specifications on the loop gain } G_o(i\omega)}$$

$$\downarrow$$

$$\boxed{\text{Determine } F(i\omega) \text{ in } G_o(i\omega) = G(i\omega)F(i\omega)}$$

From specifications on G_c to specifications on G_o

Closed-loop specifications	Loop gain specifications
Bandwidth ω_B	cross-over frequency ω_c
Resonance peak M_p	phase margin φ_m
Static gain $G_c(0) \approx 1$	static gain $G_o(0)$ high

$$e_0 = |G_c(0) - 1| = \left| \frac{G_o(0)}{1 + G_o(0)} - 1 \right| = \left| \frac{1}{1 + G_o(0)} \right| < \epsilon$$

$$\Rightarrow$$

$$G_o(0) > ?$$

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$$\Rightarrow$$

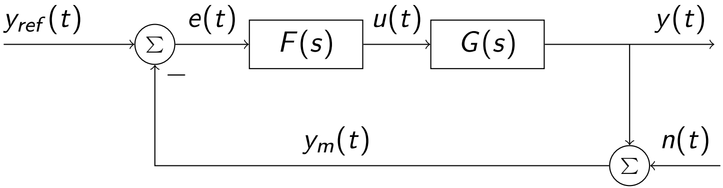
$$G_o(0) > \frac{1}{\epsilon} - 1$$

Design procedure in detail

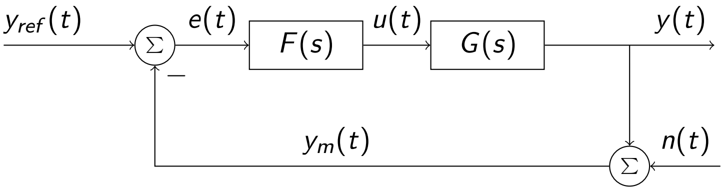
Given $G(i\omega)$ and specifications on $G_o(i\omega)$: ω_c, φ_m, steady-state error e_0.

The problem with a PD-controller

The problem with a PD-controller, contd

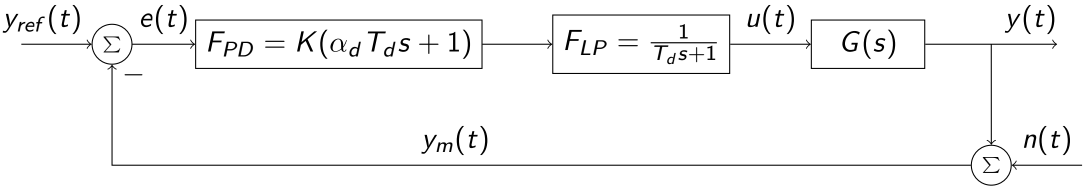


The problem with a PD-controller, contd



High frequency measurement noise entering the system is amplified in the PD-controller $F(s)$

PD-controller + Low-pass filter = lead compensator + gain



$$F(s) = KF_{lead} = K\frac{\alpha T_d s+1}{T_d s+1}$$

The lead- and lag filters/compensators

$$F_{lead} = \frac{\alpha_d T_d s+1}{T_d s+1},\ \alpha_d > 1 \qquad F_{lag} = \frac{1}{\alpha_i} \cdot \frac{\alpha_i T_i s+1}{T_i s+1},\ \alpha_i < 1 \text{ or } F_{lag} = \frac{T_i s+1}{T_i s}$$

Position control of a radar antenna

Nyquist plot of the plant

Will proportional control work? (The open-loop system is stable)