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Identification for Control: From the Early Achievements to the Revival of Experiment Design*

Michel Gevers**

Center for Systems Engineering and Applied Mechanics (CESAME), Université Catholique de Louvain, Bâtiment Euler, 4 Avenue Georges Lemaître, B-1348 Louvain-la-Neuve, Belgium

This paper presents the author's views on the development of identification for control. The paper reviews the emergence of this subject as a specific topic over the last 15 years, at the boundary between system identification and robust control. It shows how the early focus on identification of control-oriented nominal models has progressively shifted towards the design of control-oriented uncertainty sets. This recent trend has given rise to an important revival of interest in experiment design issues in system identification. Some recent results on experiment design are presented.

Keywords: Closed-Loop Identification; Identification for Control

1. Introduction

1.1. Identification and Control: A Brief History

Identification has for a long time been the territory of mathematicians, statisticians, time-series analysts and econometricians. The history of system identification goes as far back as the work of Gauss and Legendre in the late 18th and early 19th century. An excellent presentation of the history of system identification can be found in Ref. [21].

Control theory has always been the territory of engineers. Until about 1960, control design was mostly based on the use of Bode, Nyquist or Ziegler–Nichols plots. It was applied to engineering fields in which reliable models were available on the basis of first-principles, such as is the case in electrical, mechanical or aerospace applications. In the process industries, very simple "first order plus time delay" models were used. The introduction of state-space models in 1960, together with the solution of optimal control and optimal filtering problems in a linear quadratic gaussian framework [47,48], gave birth to a tremendous development of model-based control design methods. Successful applications abounded, particularly in aerospace, where accurate models were readily available.

Even though parameter estimation techniques had been applied for some time to the control of systems with known structure but unknown or poorly known parameters, 1965 saw the start of identification of black-box models in the control community. The paper [41] set the stage for state space realization theory which, 25 years later, became the founding block for what is now called subspace identification. The paper [4] introduced into the control community the maximum likelihood framework for the identification of input–output models. This gave rise

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**E-mail: gevers@csam.ucl.ac.be

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to the celebrated prediction error (PE) framework [54] that has since proven so successful. Undoubtedly, the advent of identification theory was spurred by a desire to extend the applicability of model-based control design to broader and broader fields of applications, for which no reliable models could be obtained from first principles.

From 1965 to the late 1980s, model-based control was applied to ever growing classes of dynamical systems and processes, with models obtained through the newly emerging identification techniques. The prevailing habit at that time was to separate the identification step and the control design step. A model was identified first using the best available techniques; subsequently, a model-based control design was performed based on the "certainty equivalence principle", i.e. the model was treated as if it represented the true system. Dual control and adaptive control were two early attempts to address the issue of parametric uncertainty and model-based control design in a synergistic way. In dual control, the parameter estimation and the control design mechanism are obtained as the result of a single but complex optimization problem. In adaptive control, the parameter adjustment scheme is subsidiary to the control objective. Both schemes were developed for the case where the structure of the true system is assumed to be known, which severely limits their practical applicability. The solution of the dual control problem proved to be computationally intractable, even in the simplest cases. As for adaptive control, the major difficulty is that the parameters of the feedback control system change at every sampling instant, making the closed-loop dynamics nonlinear and their stability analysis extremely complex.

1.2. Identification for Control: It is A Design Problem

One of the main contributions of the control community to system identification theory was to consider identification as an exercise in estimating the *best possible approximate model* within some model set, rather than as a search for the *true system*. Together with this effort came the characterization of the approximate model in terms of bias error and variance error on the estimated transfer functions.

If the model of a system is exact, it is optimal for all applications. However, if the model is only an approximation of the "true system", then the quality of the model should be dependent on the intended application. It thus makes sense to tune the identification towards the objective for which the model is to be used, i.e. to ensure that the distribution of model error is such that it does not deteriorate the objective too much. This gave rise to the paradigm of *goal-oriented identification* and it led one to view identification as a *design problem*. Identification for control has been the major outlet for this new paradigm. The reasons for this are many: (i) control is very often the main motivation for model building; (ii) high performance control can often be achieved with very simple models, provided some basic dynamical features of the system are accurately reflected; (iii) a powerful robust control theory, based on nominal models and uncertainty sets, had been developed all through the eighties, but these models and uncertainty sets were not data-based for lack of a proper theory; (iv) identification for control research led to iterative model and controller tuning tools that were intuitive, practical and easy to implement by the process engineers.

One early approach to optimal identification design for control, which established a direct link between experimental conditions and controller performance, was obtained in Ref. [28] by considering variance errors only, i.e. the system was assumed to be in the model set. This approach consists, for a given certainty equivalence control design procedure, of computing the experimental conditions of the identification that minimize the average performance degradation that results from the fact that the controller is computed from an estimated (and hence random) model rather than from the exact true system. Like with all optimal experiment design methods, the optimal experiment depends on the unknown system: see Section 4. Hence, even though such results give useful insights, they do not provide an operational design method for the identification for control problem. In addition, they are based on a certainty equivalence controller design mechanism, rather than on a robust control design.

Except for the methods already mentioned (dual control, adaptive control, control-oriented optimal design), which are restricted to the case where the true system is in the model set, the first contributions in which identification and control design with restricted complexity models were looked upon as a combined design problem appeared only around 1990. The plenary [26] at the 1991 IFAC Symposium on System Identification (SYSID 1991) addressed many of the key issues; however, it was more an agenda for research than a presentation of solutions. Indeed, there was very little understanding at that time about the interplay between system identification and robust control. The two theories had been developed by separate communities with very little interaction. In the 1990s, the activity in identification for control surged.

In his SYSID 2003 plenary [36], H. Hjalmarsson estimated that ~1500 papers have appeared on the topic of identification for control after the SYSID 1991 plenary.

1.3. Where the Quality of the Model Depends on the Controller

When the application of a model is the design of a controller, then what really matters is the performance achieved by this model-based controller on the "true system", and not the intrinsic quality of the model. We illustrate this idea with a very simple example inspired by Ref. [69]. Let a 'true system' be represented by $G_0(s) = 1/(s+1)$. Then the 'model' $\hat{G}(s) = 1/s$ would clearly be deemed to be absurd as a model for G_0. However, if the objective is to design a high gain static output feedback controller, then $\hat{G}$ would be a perfectly acceptable model. Indeed, with a high gain static output feedback $u = -Ky$, the closed-loop transfer functions $K/(s+1+K)$ and $K/(s+K)$ become indistinguishable. Thus, whether or not a model is appropriate for control design depends as much on the controller that will be implemented as it depends on the plant/model mismatch.

In practice, the true system is unknown, the model is unknown at the identification design stage, and the controller that will be implemented is unknown because it depends on that model. What is typically known in control-oriented identification is the control performance objective. Some prior knowledge about the true system may also be available. Ideally, the design of a control-oriented identification procedure could then be formulated as follows: *Given a control performance objective, design the identification in such a way that the performance achieved by the model-based controller on the true system is as high as possible*. In a robust control design framework, this could be reformulated as follows: *Given a control performance objective, design the identification in such a way that the worst-case performance achieved by the model-based controller on the validated uncertainty set of models is as high as possible*. Note that this definition depends on several mechanisms that are closely interconnected: the identification design, the construction and validation of an uncertainty set, the control design mechanism. Identification design includes many choices: input data, feedback configuration (possibly), data length, model structure, identification criterion, validation criterion, etc. The identification design problem is impossible to solve in such generality. In order to get a handle on the problem, it has been customary to fix some of the choices; the number of data is usually taken to be fixed, and so is the model structure.

1.4. Control-Oriented Nominal Models by way of Iterative Schemes

In the first half of the 1990s, the research focused on the bias error distribution, assuming that low complexity models are being used for the controller design. It produced a string of results on the design of control-oriented nominal models [3,18,51,66,67,77]. The first and rather obvious result was to establish that a model is good for control design if the closed-loop system obtained by the feedback connection of that model with the designed controller is close to the system obtained by the feedback connection of the true system with that same controller. Since the 'to be designed controller' is not available at the identification stage, this led to the necessity of using iterative steps of controller updates and model updates obtained by closed-loop identification.

A second important result was that the identification criterion should be a function of the control performance criterion. This led to the observation that, for most control performance objectives, identification should be performed in closed loop [24,39]. This triggered a revival of interest for closed-loop identification, and the emergence of new identification methods specifically designed for this situation: see e.g. [23,34,72].

The iterative identification and control design schemes do not necessarily converge to a stationary point, corresponding to a stable closed-loop system. Thus, they must be applied with caution, and a lot of work has gone into developing tools for safe model and controller updates [2]. Even so, these iterative schemes have had a remarkably fast transfer into the world of applications. There are two main reasons for this:

- whereas much of the industrial world was still living with the belief that one should 'open the loop' to perform a valid identification experiment, here was a theory that showed the benefits of closed-loop identification; this came as welcome news to process control engineers who had never really liked the idea of opening the loop;
- in the process industry, thousands of measurements flow into the computer; here was a theory that showed how these data could be used for the design of a better controller.

As stated above, the work on control-oriented nominal models focused on the bias error distribution of the identified model, with the controller computed from the model in a certainty equivalence framework. Thus, that work did not incorporate the robust control concepts developed during the eighties. It focused

on the design of identification criteria that minimize a (control-oriented) measure of the model error.

1.5. Towards Control-Oriented Uncertainty Sets

The second half of the 1990s saw a shift towards the definition and estimation of *control-oriented uncertainty sets* [13,20,49,56] in order to put the control design into the framework of robust control design. The focus turned to shaping the distribution of the variance error of the identified models, i.e. on manipulating the shape of the uncertainty set. Indeed, the paradigm of robust control design is to compute a controller that achieves the best possible worst-case performance, i.e. the best possible performance over all models in an uncertainty set. Such best worst-case performance depends as much on the controller as it depends on the uncertainty set, and this set is directly dependent on the experimental conditions under which the identification is performed. The study of the interplay between experimental conditions of the identification and properties of the robust controller has been split up into two questions:

(1) What is the connection between a model uncertainty set and the properties of robust controllers computed from that set and, consequently, how should one define a control-oriented uncertainty set?
(2) How should one design the identification experiment in such a way that the uncertainty set around the identified model has such 'control-oriented' property?

Even though many new insights have been gained on the first question, there is, at this point, no clear view as to the most operational definition of a 'control-oriented uncertainty set'; we shall come back to this in Section 7. As for the second question, the search for models that have prespecified error quality distributions has led to a revival of activity in experiment design questions.

To summarize this historical account, the work on identification of the last 15 years has been essentially developed in four directions: optimal control-oriented experiment design for identification, the definition and computation of control-oriented nominal models, the connection between data-based uncertainty sets and the properties of robust controllers resulting from such sets, and the formulation and solution of control-oriented optimal experiment design problems. The new questions raised by the topic of identification for control have generated lots of parallel work on estimation and validation of uncertainty sets from data, closed-loop identification, experiment design, frequency domain identification. Some of the concepts that have emerged from these 15 years of work, such as the idea of improving the performance of an existing controller on the basis of closed-loop data collected with the presently operating controller, have quickly found their way into practice. However, many questions remain unsolved, particularly on the optimal tuning of uncertainty sets, and we are still far from an automatic data-based control design procedure. Let us mention in passing that the *automatic synthesis of control algorithms, with integrated validation and verification* is seen as one of the major future challenges in control by the panel for future directions in control [60].

In the remainder of this paper, we explain in some more detail the specific achievements and the remaining unsolved questions in the different subtopics that we have sketched: What is optimal control-oriented experiment design? What is a control-oriented nominal model? Why iterative design? What is a control-oriented uncertainty set? How can we match experiment design and uncertainty set? We have chosen to do the presentation of these ideas in a PE identification framework, because PE identification is by far the most successful and widely used identification method. Our presentation focuses on the ideas and issues, the chronology of developments, the motivation for the successive steps that were taken and problems that were addressed. The paper is probably written more for the systems and control generalist or the newcomer to the field who wants to get a good understanding of the key issues in identification for control than for the expert in this field. For a superb presentation of the technical issues in identification for control, we refer to [37], which is compelling reading for anyone who aims at becoming an expert in this difficult field. In particular, that paper contains fascinating new ideas on *near-optimal restricted complexity modelling* and on model validation.

In the next section, we first present the bare essentials of PE identifcation that are necessary to understand the remaining issues.

2. Basics on PE Identification

To simplify the presentation, we assume that the unknown true system can be represented by a single-input single-output linear time-invariant system:

$$\mathcal{S}: y_t = G_0(z)u_t + v_t = G_0(z)u_t + H_0(z)e_t, \quad (1)$$

where $G_0(z)$ is a linear time-invariant causal operator, y is the measured output, u is the control input and v is noise, assumed to be quasistationary, modelled as the output of a model $v_t = H_0(z)e_t$, where e is white noise. One considers a parametrized model set: $\mathcal{M} = \{(G(z, \theta), H(z, \theta), \theta \in D_\theta \subset \mathbb{R}^d\}$ where $G(z, \theta)$ and $H(z, \theta)$ are typically rational transfer functions, and D_θ is a subset of admissible values for the parameter vector θ. To every θ corresponds a one-step ahead predictor:

$$\hat{y}_{t|t-1}(\theta) = H^{-1}(z, \theta)G(z, \theta)u_t + [1 - H^{-1}(z, \theta)]y_t, \tag{2}$$

and hence a one-step ahead Prediction Error (PE):

$$\begin{aligned}\varepsilon_t(\theta) &\triangleq y_t - \hat{y}_{t|t-1}(\theta) \\ &= H^{-1}(z, \theta)[(G_0(z) - G(z, \theta))u_t + v_t].\end{aligned} \tag{3}$$

These PEs can, possibly, be filtered by a data filter $D(z)$, thus defining the filtered PEs $\varepsilon_t^f(\theta) = D(z)\varepsilon_t(\theta)$. The least squares PE estimate $\hat{\theta}_N$ based on N input–output data is then defined as

$$\hat{\theta}_N = \arg \min_{\theta \in D_\theta} V_N(\theta), \tag{4}$$

where the PE criterion is defined as

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}[\varepsilon_t^f(\theta)]^2. \tag{5}$$

The estimate $\hat{\theta}_N$ defines the model $\hat{G} = G(z, \hat{\theta}_N)$, $\hat{H} = H(z, \hat{\theta}_N)$. Under reasonable conditions, $\hat{\theta}_N \overset{N\to\infty}{\longrightarrow} \theta^*$, where $\theta^* \triangleq \arg\min_{\theta \in D_\theta} \bar{V}(\theta)$, with $\bar{V}(\theta) \triangleq E[\varepsilon_t^f(\theta)]^2$ [54].

Two different situations need to be considered. The first one is when the model structure $\mathcal{M}$ has been chosen sufficiently complex that the true system belongs to the model set. This is denoted $\mathcal{S} \in \mathcal{M}$, and means that there exists a value $\theta_0 \in D_\theta$ such that $G(z, \theta_0) = G_0(z)$ and $H(z, \theta_0) = H_0(z)$. In such a case, under reasonable conditions, $\theta^* = \theta_0$, which means that the PE estimates of the transfer functions converge to the true transfer functions: $G(z, \hat{\theta}_N) \overset{N\to\infty}{\longrightarrow} G_0(z)$, $H(z, \hat{\theta}_N) \overset{N\to\infty}{\longrightarrow} H_0(z)$. When $\mathcal{S} \in \mathcal{M}$, the parameter error converges to a Gaussian random variable:

$$(\hat{\theta}_N - \theta_0) \overset{N\to\infty}{\longrightarrow} N(0, P_\theta), \tag{6}$$

where

$$P_\theta = \frac{\sigma_e^2}{N}(\bar{E}(\psi(t, \theta_0)\psi(t, \theta_0)^{\mathrm{T}}))^{-1} \tag{7}$$

with $\psi(t, \theta) = -(\partial\varepsilon^f(t, \theta))/\partial\theta$. The asymptotic parameter covariance P_θ can be estimated from the data, and the true parameter vector θ_0 belongs to an ellipsoid:

$$U_\theta = \{\theta | (\theta - \hat{\theta}_N)^{\mathrm{T}} P_\theta^{-1}(\theta - \hat{\theta}_N) < \chi^2\} \tag{8}$$

with probability $\alpha(d, \chi^2) = \Pr(\chi^2(d) \le \chi^2)$, where $\chi^2(d)$ denotes the χ^2 distribution with d degrees of freedom. Thus, when the system is in the model set, PE identification delivers a nominal model $G(z, \hat{\theta}_N)$, $H(z, \hat{\theta}_N)$, together with an ellipsoidal confidence region in parameter space. This, in turn, defines an uncertainty region in the space of transfer functions:

$$\mathcal{D} = \{G(z, \theta) \,|\, \theta \in U_\theta\}. \tag{9}$$

In the more general situation where the system is not in the model set, $\lim_{N\to\infty} \hat{\theta}_N = \theta^* \neq \theta_0$. In such case, the transfer function error, $G_0(e^{j\omega}) - G(e^{j\omega}, \hat{\theta}_N)$, at a given frequency ω, can be decomposed as:

$$\begin{aligned}&G_0(e^{j\omega}) - G(e^{j\omega}, \hat{\theta}_N) \\ &\quad = \underbrace{G_0(e^{j\omega}) - G(e^{j\omega}, \theta^*)}_{\text{bias error}} + \underbrace{G(e^{j\omega}, \theta^*) - G(e^{j\omega}, \hat{\theta}_N)}_{\text{variance error}}.\end{aligned} \tag{10}$$

The bias error arises when the model structure is unable to represent the true system. The variance error is caused by the noise and the finiteness of the data set; by definition of θ^*, it goes to zero asymptotically. In the mid-1980s, Ljung produced some important formulas for the characterization of bias and variance errors of the identified transfer functions [53,75]. The bias was characterized implicitly by representing θ^* as the minimizing argument of a frequency integral. A variance error estimate for the estimated transfer functions was obtained under an assumption of model order going to infinity. More recent work has produced formulas for the estimation of an uncertainty set $\mathcal{D}$ around $\hat{G}$, with the property that $G_0 \in \mathcal{D}$ with probability α, where α is any desired level close to 1 (e.g. $\alpha = 0.95$) even in the case where the system is not in the model set, and for finite model orders: see Section 6. Finally, we note that the results described in this section are valid for both open-loop and closed-loop identification.

3. The Game and the Players

In identification for control, a typical situation is that we can perform experiments on the true system (1) with the purpose of designing a feedback controller.

The system may already be under feedback control, in which case the task is to replace the present controller by one that achieves better performance. This situation is representative of many practical industrial situations. We then denote the present controller by C_{id} and the reference signal, if any, by r_t:

$$u_t = C_{\mathrm{id}}(z)[r_t - y_t]. \tag{11}$$

The closed-loop system is then described by

$$\begin{aligned} y_t &= \frac{G_0(z)C_{\mathrm{id}}(z)}{1 + G_0(z)C_{\mathrm{id}}(z)} r_t + \frac{1}{1 + G_0(z)C_{\mathrm{id}}(z)} v_t \\ &= T_0(z) r_t + S_0(z) v_t. \end{aligned} \tag{12}$$

Using N data collected on the system, in open loop or in closed loop, we can compute a model $\hat{G}$ of the unknown G_0, and possibly also a noise model $\hat{H}$ of H_0 by PE identification. Since the complexity of a model-based controller is of the same order as that of the model, one often performs the identification with a low order model.

The traditional scenario in model-based robust control design was: *First estimate a model $\hat{G}$ and an uncertainty set $\mathcal{D}$, then design a new controller $C(z)$ that achieves closed-loop stability and meets the required performance with all models in $\mathcal{D}$, and hence with the unknown true system G_0.*

The objective in *identification for control* is to replace that traditional scenario by the following. *On the basis of the required performance, and of any knowledge of the unknown system, design a control-oriented identification experiment that produces a model $\hat{G}$ and an uncertainty set $\mathcal{D}$; then design a new controller C that achieves closed-loop stability and meets the required performance with all models in $\mathcal{D}$, and hence with the unknown true system G_0.* If necessary, repeat this design procedure, possibly with a more demanding performance criterion. In some scenarios, one first computes a class $\mathcal{C}(\hat{G}, \mathcal{D})$ of controllers, each of which achieves the required performance with all models in $\mathcal{D}$; the controller C is then chosen within this class in such a way as to have some additional nice features (e.g. low complexity).

The goal of the new scenario is to achieve the same or better performance based on models (and hence controllers) of lower complexity. The class $\mathcal{C}$ of controllers that achieve the required performance is larger if the model uncertainty set $\mathcal{D}$ can be tuned towards that aim. The players within this (iterative) identification and robust control design scenario are therefore:

- the unknown plant G_0,
- C^{opt} for G_0,
- the present controller C_{id} (if any),
- the present model $\hat{G}_{\mathrm{init}}$ (if any),
- the identified model $\hat{G} = G(z, \hat{\theta}_N)$,
- the uncertainty set of models $\mathcal{D}$ around $\hat{G}$,
- the set $\mathcal{C}$ of controllers that achieve the prescribed performance and
- the new model-based controller $C \in \mathcal{C}$

Except for the unknown plant G_0 and its corresponding optimal controller C_0, the designer has a handle on all other players. It is the complexity of the interplay between these players that makes the problem challenging and interesting. To illustrate the interplay, it is important to understand that one deals with five different feedback loops, which impact one another: see Figs 1–5.

In identification for control, the designer collects data on the experimental loop of Fig. 1, and estimates a model $\hat{G}$ such that the closed-loop system of Fig. 2 is "as close as possible" to the actual closed-loop system of Fig. 1. Sometimes, one of his design choices is the choice of a controller C_{id} and of a reference signal r_t in the experimental loop. On the basis of the identified model $\hat{G}$, possibly with an estimated uncertainty set $\mathcal{D}$,

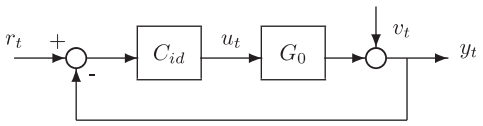

Fig. 1. Experimental loop.

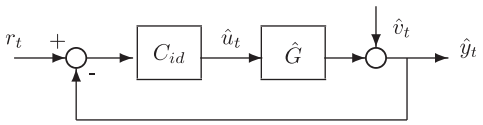

Fig. 2. Identified system.

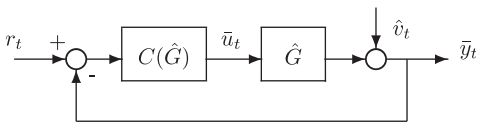

Fig. 3. Design loop.

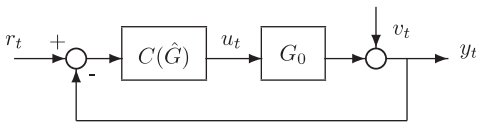

Fig. 4. Achieved loop.

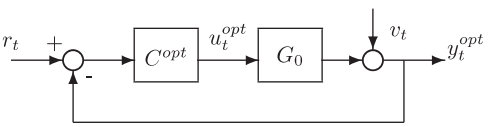

Fig. 5. Optimal loop.

the designer then computes the new controller $C(\hat{G})$, or $C(\hat{G}, \mathcal{D})$: this generates the designed loop of Fig. 3. However, what really matters is the performance achieved by this controller with the real system, i.e. the performance of the achieved loop of Fig. 4. Thus, if the identifier performs a good job, then the identified loop will be close to the experimental loop; but what is really desired is that the achieved performance on the loop of Fig. 4 is close to the designed performance on the loop of Fig. 3. If the experimental setup was such that C_{id} was identical (or at least close to) the "to-be-designed controller" C, then closeness of the loops of Figs 1 and 2 would entail closeness of the loops of Figs 3 and 4. This explains why it pays to have experimental conditions that closely match the conditions in which the "to-be-designed controller" will operate.

The first identification for control results, mentioned earlier, were based on a different approach. The aim was to find the optimal experiment conditions such that the output of the achieved loop of Fig. 4 is as close as possible to the output of the optimal loop of Fig. 5 that would be obtained if the true system were known exactly. Because of its historical precedence, and its intuitive appeal, we first present this approach in the next section.

4. Optimal Control-Oriented Identification Design

In this approach an identification experiment design is called "optimal" if the controller computed from the estimated model is one that minimizes the average performance degradation *vis-à-vis* the performance that would be achieved with the ideal controller. The ideal controller is the controller that would be computed if the true system were known. We now explain this in some more detail.

We denote by $J(G, H, C)$ the control design criterion, and by $C = c(G, H)$ the certainty equivalence mapping that maps a model (G, H) into the corresponding optimal controller by minimization of $J(G, H, C)$. In particular,

$$C^{\text{opt}} = c(G_0, H_0) = \arg\min_C J(G_0, H_0, C). \quad (13)$$

We consider PE identification of a parametric model from N data, and we assume that $\mathcal{S} \in \mathcal{M}$. The control design mapping then defines a controller $\hat{C}_N = c(\hat{G}_N, \hat{H}_N)$ for each model $(\hat{G}_N, \hat{H}_N)$.[1] The controller $\hat{C}_N$ is a random variable, because the estimated parameter vector $\hat{\theta}_N$ is random, and hence also the model. Applying $\hat{C}_N$ (rather than C^{opt}) to the true system results in an achieved cost $J(G_0, H_0, \hat{C}_N) \geq J(G_0, H_0, C^{\text{opt}})$. This results in a "performance degradation" $J^{\text{deg}} = J(G_0, H_0, \hat{C}_N) - J(G_0, H_0, C^{\text{opt}})$, which is again a random variable.

The problem statement of optimal identification design for control is then phrased as follows: "Find the experimental conditions $\mathcal{X}$ that minimize the average performance degradation".[2] In view of what precedes, this can be formulated as:

$$\min_{\mathcal{X}} EJ(G_0, H_0, c(\hat{G}_N, \hat{H}_N)). \quad (14)$$

The expected value is taken with respect to the noise, which affects the model estimate, and hence the controller estimate.

In the context of certainty equivalence control design, this is probably the most logical (and ideal) problem formulation for an *optimal identification for control design*. However, there are several difficulties with this formulation (see Chapter 9 in Ref. [1]), the main one being that the optimal experiment $\mathcal{X}$ defined by (14) necessarily depends on the unknown system (G_0, H_0). This is proper to all experiment design problems. It does not mean that such results are meaningless: they give useful guidelines for the identification design, and they may lead to iterative schemes that converge to the optimal experiment design (see e.g. [39]). We shall return to this optimal control-oriented experiment design problem in Section 8.

5. Iterative Design for the Nominal Model

In this section we discuss the control-oriented identification design of the nominal model, and we show why the pursuit of a control-oriented objective leads to iterative model and controller updates. This observation was made independently in the early 1990s by several research teams, who were using different combinations of identification method and control design criterion. To understand the need for iterative design, consider the closed-loop systems of Figs 3

[1] Here $\hat{G}_N$ is a shorthand notation for $G(z, \hat{\theta}_N)$, and similarly for $\hat{H}_N$.

[2] $\mathcal{X}$ denotes the set of all admissible experimental conditions that have an effect on the quality of the model estimates $(\hat{G}_N, \hat{H}_N)$, such as use of open-loop or closed-loop data, choice of input spectrum distribution, of regulator in the case of closed-loop identification, etc. By 'admissible' experimental conditions, we refer to conditions that obey possible constraints on signal powers or signal amplitudes.

and 4, and assume for simplicity that there is no noise ($v_t = 0$ and $\hat{v}_t = 0$), i.e. the control objective is reduced to a tracking performance objective.

The controller is designed on the basis of the model $\hat{G}$ and then applied to the system G_0. The achieved performance will therefore be close to the desired performance if the two closed-loop transfer functions, or a weighted version of these, are close to one another. Thus, we want the following error to be small[3] (see Figs 3 and 4):

$$\frac{G_0 C}{1 + G_0 C} - \frac{\hat{G} C}{1 + \hat{G} C} = (G_0 - \hat{G}) C S_0 \hat{S} \tag{15}$$

where $S_0 = 1/(1 + G_0 C)$ and $\hat{S} = 1/(1 + \hat{G}C)$. Now, closed-loop PE identification with a given model structure $\mathcal{M} = \{G(z, \theta) | \theta \in D_\theta\}$, and with a controller C_{id} in the loop, will asymptotically deliver a model $G(z, \theta^*)$, where $\theta^* = \arg \min_{\theta \in D_\theta} V(\theta)$, with $V(\theta)$ given by

$$V(\theta) = \int_{\pi}^{\pi} |G_0 - \hat{G}(\theta)|^2 |C_{\text{id}} S_0|^2 |D|^2 \Phi_r \, \mathrm{d}\omega. \tag{16}$$

Here $D(z)$ is the data filter that can be freely chosen by the user. Observe that, if $C_{\text{id}} = C$, and if the data filter was chosen such that $|D(\mathrm{e}^{j\omega})|^2 \Phi_r(\omega) = |\hat{S}(\mathrm{e}^{j\omega})|^2$, then the model $G(z, \theta^*)$ obtained asymptotically by such closed loop PE identification would make the error (15) small in an H_2 sense. By such design, the identification criterion would be matched with the control performance criterion. However, there are two difficulties with such design: (i) the controller C in the design loop of Fig. 3 is a function of the identified model, $C = C(\hat{G})$, and it is therefore impossible to choose $C_{\text{id}} = C(\hat{G})$ at the identification design stage; (ii) the sensitivity function $\hat{S}$ is also a function of the estimated model, $\hat{S} = \hat{S}(\hat{G})$. These observations have led to the concept of *iterative design*, where successive steps of closed-loop identification and model-based controller design are performed. Thus, at iteration k, where a model $\hat{G}_k$ has been obtained, select $C_{\text{id},k} = C(\hat{G}_k)$, and $|D_k|^2 \Phi_r = |\hat{S}_k|^2$, where $\hat{S}_k = 1/[1 + \hat{G}_k C(\hat{G}_k)]$.

The research work of the early nineties on the definition and computation of control-oriented nominal models led to several important guidelines, that can be summarized as follows.

- The identification criterion for the nominal model should aim at minimizing the distance between the achieved and the designed loop, where this distance is measured in a norm determined by the control performance criterion. An application of this principle to LQG control can be found in Ref. [77], to H_∞ control in Ref. [67] and to GPC in Ref. [68], the latter result being based on dual control ideas. In Ref. [25] a control-relevant parameter estimation criterion is also proposed that is based on a left fractional transformation of the closed-loop system. The method is applied to multivariable systems using a multivariable frequency-weighted identification approach developed in Ref. [7].
- One of the easiest ways to identify a control-relevant nominal model is to perform the identification in closed loop, with a specific data filter. The closed-loop experiment automatically brings into the identification criterion the frequency weighting by the unknown sensitivity S_0 of the actual closed-loop system.
- The data filter is model-dependent (i.e. θ-dependent). Thus, one has to resort to iterative model/controller updates for the practical implementation of this design.

In summary, in identification for control, the control performance objective shapes the bias error distribution of the nominal model. This means that the nominal model has a bias error that is small in the frequency areas where it needs to be small for the design of a better controller, typically around the present cross-over frequency.

Iterative identification and control schemes flourished in the 1990s, with various combinations of control criteria and identification criteria. The reader is referred to [8,27,73] for details and for a survey on such iterative schemes. Unfortunately, it was found [40] that these iterative schemes do not generically converge to the achievable minimum (within the model/controller set) of the control performance cost.

Despite this, the concept of iterative identification and control design was rapidly adopted in process control applications (see e.g. [17,19,42,46,64,67]). One reason is that it is typical in such applications that large numbers of closed-loop data are flowing into the control computer, and it then makes sense to use these data to replace the existing controller by one that achieves better performance. The practical impact of iterative model and controller redesign has been assessed in Ref. [50], where the following interesting observations are made on the distinction between this batch-like mode of operation and the more classical methods of adaptive control:

- most practical results have shown that the major improvement in closed-loop performance occurs after the first identification in closed-loop,

[3]For simplicity of notation, we omit all ω-dependent arguments whenever there is no risk of confusion.

which emphasizes the importance of closed-loop identification for control design;
- iterative identification with controller redesign can be viewed as as an indirect adaptive control scheme, but with a time-scale separation between identification in closed loop, and controller redesign; this allows a more performance-oriented analysis (locally) of the interaction between identification and controller design.

6. Model Uncertainty Sets and the Robust Control Paradigm

We have shown how control-oriented nominal models are obtained by minimization of an identification criterion that is determined by the overall control performance criterion. In establishing this result, one has assumed that the controller is computed from the estimated model $\hat{G}$ using the certainty equivalence paradigm. At no point has the model uncertainty due to the noise been taken into account. Conversely, the control design step that has led to the iterative design schemes developed above is not based on robust control principles. The paradigm of modern[4] robust control design can be briefly summarized as follows.

One wants to design a controller for an unknown system G_0. Some prior knowledge about G_0 allows one to assume that G_0 belongs to some *model uncertainty set* $\mathcal{D}$. Most often, a nominal model $\hat{G}$ of G_0 is available, typically at the centre of $\mathcal{D}$. Some performance objective is often given in the form of a criterion $J(G, C)$, to be minimized. If the true system were perfectly known, the optimal controller would then be defined as $C^{\text{opt}} = \arg\min J(G_0, C)$, where the minimization is performed over some predefined set of admissible controllers $\mathcal{C}$. In the robust control paradigm, the true system is unknown, but the information $(\hat{G}, \mathcal{D})$ is available. One then seeks a *robust controller* C with the following properties:

(1) C must stabilize all models in $\mathcal{D}$;
(2) the worst performance achieved by C on any model in $\mathcal{D}$ must be as high as possible.

The robust controller C is often selected as one that stablizes all models in $\mathcal{D}$ and achieves the best *worst-case performance* over $\mathcal{D}$, i.e. C is computed as

$$C = \arg \min_{C \in \mathcal{C}} \sup_{G \in \mathcal{D}} J(G, C). \tag{17}$$

[4]We refer to modern robust control design as the theory developed in the 1980s, that is based on model uncertainty sets; in contrast, the classical robust design theory relies on robustness margins expressed in Bode, Nyquist or Nichols plots.

The uncertainty model set $\mathcal{D}$ plays a central role in this design strategy. In the robust control theory developed through the 1980s and 1990s, this set is God-given in that it is based on so-called "prior assumptions" about the model and its uncertainty. Many different descriptions of model uncertainty have been considered. Most often they are expressed as frequency domain sets, containing both structured and unstructured components (see e.g. Refs [59,70,79]). The following are representative examples of commonly used uncertainty sets; for simplicity, we consider scalar transfer functions only.

Additive uncertainty set

$$\mathcal{D}_A = \{G_\Delta(z) | G_\Delta(z) = G(z) + \Delta(z), \text{ with } |\Delta(e^{j\omega})| < W(e^{j\omega}) \ \forall\omega\},$$

where $G(z)$ is a "nominal model" and $W(e^{j\omega})$ is a frequency weighting function.

Youla–Kucera uncertainty set

$$\mathcal{D}_{YK} = \{G_\Delta(z) | G_\Delta(z) = \frac{N_x + D_c\Delta}{D_x - N_c\Delta}, \quad \text{with } |\Delta(e^{j\omega})| < W(e^{j\omega}) \ \forall\omega\},$$

where N_x, D_x, N_c, D_c, Δ are stable, rational, proper transfer functions, $\hat{G} = N_x/D_x$ is a "nominal model", and $C = N_c/D_c$ is any stabilizing controller of G_o. This uncertainty set, dual of the Youla–Kucera set of all stabilizing controllers of a given system, was introduced for the description of model sets in [34].

The new approach, initiated around 1990, was to consider the estimation of uncertainty sets from data. Unfortunately, the available PE identification theory had rather little to offer to the existing robust control theory, for two reasons: (i) there were no adequate expressions for the estimation of the total error on an identified transfer function, or some upper bound on this error; the main difficulty was the estimation of the bias error, which could only be characterized by implicit integral expressions and (ii) the available uncertainty descriptions were not given as frequency domain sets. As a result, a wide range of new identification techniques were developed. New model assumptions, noise assumptions and identification criteria were introduced whose main merit was to deliver computable error bounds on the estimated models. The survey paper [62] is probably still one of the best presentations of these alternative methods. Some of these methods have been shown to lead to very conservative upper bounds on the uncertainty set. But clearly, this work has led to important advances in the characterization of bounds on transfer function error estimates.

One uncertainty set that was available in PE identification theory was the set (9) defined through the ellipsoidal set (8) in parameter space. For open-loop identification, this set can be described as follows.

PE uncertainty set

$$\mathcal{D}_{\mathrm{PE}} = \{G(z,\theta) | G(z,\theta) = \frac{N(z,\theta)}{D(z,\theta)} \quad \text{with } (\theta - \hat{\theta}_N)^{\mathrm{T}} R (\theta - \hat{\theta}_N) < 1\}.$$

Here N and D are polynomials parametrized by θ, and R is proportional to the inverse of the estimated covariance matrix of $\hat{\theta}_N$. The difficulty with this PE uncertainty set is that, in the early 1990s, it could be computed easily only in the case where the system is in the model set, and it did not connect with the available robust control theory and design tools. Thus, all through the 1990s, a lot of research was produced by the identification community to

1. extend the use of uncertainty sets in order to include also the bias error; this was achieved by either embedding the bias error in a stochastic framework [32,33], or by estimating the bias error through a validation step that uses a full order model [30,55]. In addition, one can now compute uncertainty sets in the frequency domain, derived from the ellipsoidal sets in parameter space, to which the true system is guaranteed to belong with *a priori* probability level α [9,71].
2. develop a robust control stability and performance theory for PE uncertainty sets characterized by ellipsoids in parameter space. We now have a necessary and sufficient condition for a given controller $C(z)$ to stabilize all systems within the uncertainty set $\mathcal{D}_{\mathrm{PE}}$ using μ-analysis; we have the computational tools to check whether this condition is satisfied. We can also compute the worst-case performance achieved by a given controller $C(z)$ with all systems in that set, for fairly general performance criteria [14,30].

Many important advances have thus been made, both in the characterization of bounds on transfer function error estimates obtained by PE identification, and in the development of a robust control analysis theory based on ellipsoidal uncertainty sets obtained by PE identification. This subject will undoubtedly remain an object of intense debate and activity for years to come. In addition, whereas the *robust analysis* theory based on PE uncertainty sets has made much progress, the results on *robust control design* based on such sets are still rather scarce.

7. Towards Control-Oriented Uncertainty Sets?

In the second half of the 1990s, one began to seriously study the interplay between model uncertainty sets and robust control objectives, in order to address the question of building *control-oriented uncertainty sets*. The motivation for this is based on the following two observations.

- It follows from the properties that define a robust controller (see the previous section) that satisfaction of these two properties hinges as much on the choice of the controller C as it does on the uncertainty set $\mathcal{D}$.
- The shape of a data-based uncertainty set depends very much on the experimental conditions under which it is estimated.

Combining these two observations leads to the idea of constructing control-oriented uncertainty sets by proper choice of experimental conditions.

7.1. A Gedanken Experiment for Motivation

To illustrate the connection between experiment conditions and model-based control properties, we consider a very simple gedanken experiment, presented in Ref. [29], where we focus attention only on robust stability. The idea of the gedanken experiment is to show that, for a particular control design objective, one set of experimental conditions delivers an uncertainty set that is much better suited than another set of experimental conditions.

Consider a "true system" G_0 described by the following simple ARX model:

$$(1 - 1.4z^{-1} + 0.45z^{-2})y_t = z^{-1}(1 + 0.25z^{-1})u_t + e_t,$$

where e is a unit variance white noise. With a constant gain feedback law $u_t = r_t - Cy_t$, the closed-loop system is stable for $C < C_{\max} = 2.2$. Consider now that we estimate the parameters of this ARX model by PE identification, using the exact structure, and that the objective of the identification exercise is to estimate the largest possible feedback gain, $\hat{C}_{\max}$ which, with probability 95%, will produce a stable closed loop with the true system.

With PE identification, we can estimate an ellipsoid in parameter space, to which the true parameters belong with probability 95%. We can then compute the proportional output feedback controller with the highest gain, $\hat{C}_{\max}$, that stabilizes all models $\hat{G}$ whose parameters lie in that 95% uncertainty set. Let us call

this controller the "optimal robust controller" for this problem. In order to show the effect of the experimental setup of the identification on the optimal robust controller, we have compared, by Monte Carlo simulations, two experimental conditions:

1. open-loop identification with a unit variance white noise input signal u;
2. closed-loop identification with a controller $u_t = r_t - y_t$ in the loop during data collection, and a white noise reference signal r with variance 18.38; this choice yields the same output variance as in the open-loop experiment.

Each of these two experiments was run 100 times, each time with 1000 input-output data. For each run, the 95% confidence ellipsoid was computed in parameter space, and the gain $\hat{C}^i_{\max}$ of the i-th run was computed as the largest gain that would stabilize all models in the corresponding model set. On the basis of 100 runs, we then computed the average $\hat{C}_{\max}$ and its variance for each of the two experimental conditions. The following results were obtained for open-loop (OL), respectively closed-loop (CL), identification:

$$\text{OL: } \hat{C}_{\max} = 1.36, \quad \sigma^2_{\hat{C}_{\max}} = 0.12$$
$$\text{CL: } \hat{C}_{\max} = 2.04, \quad \sigma^2_{\hat{C}_{\max}} = 0.02$$

Remember that, for the true system G_0, we have $C_{\max} = 2.2$. This example clearly shows that the experimental conditions used in the feedback experiment are more control-oriented than those in the open-loop experiment, since they lead to a much less conservative estimate of the limit gain for the robust controller.

7.2. On the Effect of Experimental Conditions on Control Performance

The example above serves to illustrate the effect of the identification experiment on the set of admissible controllers, through the model uncertainty set that results from the experiment. In a more realistic setup, the interplay between the design of the identification experiment, the corresponding model uncertainty set, the set of admissible controllers, and the worst-case control performance, is a lot harder to understand and analyze. In order to relate the identification design with the properties of a robust controller, one has resorted to splitting up the problem into its two components:

- understanding the interplay between the uncertainty set and the properties of the ensuing robust controller;
- understanding the effect of experimental conditions on model quality, or more precisely on the properties of the uncertainty set.

We elaborate on these two subproblems.

7.3. What Constitutes a Control-Oriented Uncertainty Set?

The central question for the first problem is: What constitutes a control-oriented uncertainty set? There is no clear consensus yet on a good definition. One possible view is to say that a model uncertainty set $\mathcal{D}$ is "control-oriented" if the corresponding set of admissible controllers, $\mathcal{C}$, is large.[5] Such track has been pursued in e.g. [22] where different uncertainty structures are compared, and in [30] where the worst-case ν-gap has been proposed as a control-oriented measure of size of $\mathcal{D}$, because it is related to the size of the corresponding set $\mathcal{C}$ of stabilizing controllers. One criticism one might level at this approach is that making the set of admissible controllers *large* does not necessarily make it easier to design a satisfactory controller.

Another approach to the definition of a control-oriented uncertainty set is as follows. Consider an uncertainty set $\mathcal{D}$ with center $\hat{G}$ containing the true G_0, and a controller $C = C(\hat{G}, \mathcal{D})$ with nominal stability margin $b(\hat{G}, C)$ and nominal performance $J(\hat{G}, C)$. Then $\mathcal{D}$ could be called "control oriented" if C stabilizes all models in $\mathcal{D}$, if the worst case stability margin, $\sup_{G\in\mathcal{D}} b(G, C)$, of C with all models in $\mathcal{D}$ is close to the nominal margin $b(\hat{G}, C)$, and if the worst case performance, $\sup_{G\in\mathcal{D}} J(G, C)$, is close to the nominal performance $J(\hat{G}, C)$. One criticism of this second approach is that the control-oriented quality of the uncertainty set depends very much on the choice of the particular controller C.

Thus, much work remains to be done on the definition of control-oriented uncertainty sets, and on the computation of the corresponding quality measures. This is certainly one area where one can expect a lot of research activity in the coming years.

As for the second question, it pertains to the connection between the experiment design conditions (length of the data set, choice of input signal, open-loop or closed-loop configuration, controller choice in the case of closed-loop identification, etc.) and the quality of the identified model. This topic lies squarely within the boundaries of identification theory, and a

[5] By admissible, we mean a controller that stabilizes all models in the set and achieves with all these models a prespecified performance level.

large body of results have been accumulated over the years that characterize the model quality as a function of the experimental conditions. However, the new focus on "control-oriented" identification has very naturally led the research community to also revisit the early experiment design results in the light of this new "control-oriented" objective. This has given rise to a surge of new activity, which we now describe.

8. The Rebirth of Experiment Design

8.1. The Early Work on Experiment Design

In the 1970s, optimal input design for system identification was an active area of research, with different quality measures of the identified model being used for this optimal design [31,57,78]. The questions at that time addressed open-loop identification only, and the objective functions that were minimized were various measures of the parameter covariance matrix P_θ. Important examples are D-optimal design which minimizes $\det(P_\theta)$, or L-optimal design which minimizes $\mathrm{tr}(WP_\theta)$, where W is a nonnegative weighting matrix. In open-loop identification, the dependence of the covariance matrix on the input spectrum is made apparent by the following expression, which can be easily derived using (7) and Parseval's formula (see e.g. [54]):

$$P_\theta^{-1} = \left(\frac{N}{\sigma_e^2}\frac{1}{2\pi}\int_{-\pi}^{\pi} F_u(\mathrm{e}^{j\omega}, \theta_0)F_u(\mathrm{e}^{j\omega}, \theta_0)^*\Phi_u(\omega)\,\mathrm{d}\omega\right) + \left(N\frac{1}{2\pi}\int_{-\pi}^{\pi} F_e(\mathrm{e}^{j\omega}, \theta_0)F_e(\mathrm{e}^{j\omega}, \theta_0)^*\,\mathrm{d}\omega\right). \tag{18}$$

Here, $F_u(z, \theta_0) = (\Lambda_G(z, \theta_0))/(H(z, \theta_0))$, $F_e(z, \theta_0) = (\Lambda_H(z, \theta_0))/(H(z, \theta_0))$, $\Lambda_G(z, \theta) = \partial G(z, \theta)/\partial\theta$ and $\Lambda_H(z, \theta) = \partial H(z, \theta)/\partial\theta$. The formula shows that the data length N and the input spectrum $\Phi_u(\omega)$ appear linearly in the expression of the information matrix P_θ^{-1}, and that, for a given data length N, the input spectrum is the only design quantity that can shape the parameter covariance matrix. Zarrop used Tchebycheff system theory to parametrize the input spectrum in terms of its so-called "moments" with respect to the system [78]. The information matrix $M_\theta \triangleq P_\theta^{-1}$ can then be expressed as a finite linear combination of these moments.

A classical open-loop input design problem would be to consider N fixed and to minimize $\det(P_\theta)$ with respect to $\Phi_u(\omega)$ subject to some constraint on $\Phi_u(\omega)$, typically of the form $\int_{-\pi}^{\pi}\Phi_u(\omega)\,\mathrm{d}\omega \le \alpha$ for some predefined α. This was called D-optimality; many other variants have been studied. An important result of this early work was to establish that the minimization of the classical objective functions of the covariance matrix, such as $\det(P_\theta)$ or $\mathrm{tr}(WP_\theta)$, subject to constraints on the input power spectrum led to a convex optimization problem, and that in addition an optimal solution could always be obtained in the form of a discrete power spectrum, i.e. the optimal input can always be generated as a finite linear combination of sinusoids (multisine). The number of sinusoids required depends on the particular model structure and on the constraints. For example, it was shown in Ref. [31] that, if a Box–Jenkins model structure is used with $G(z, \theta)$ containing $2n$ parameters, then an optimal input for the criterion $\det(P_\theta)$ under a constraint on the input power can be achieved with no more than $2n$ sinuoids.

8.2. Experiment Design Based on L_2 Control Performance Criteria

The classical experiment design results of the 1970s were all limited to open-loop identification, and they were based on parameter covariance formulas; see (18). The bias and variance formulas for the transfer function estimates of the mid-1980s [53,75] paved the way for the formulation of control-oriented experiment design problems. For example, for the variance of the input-output transfer function estimate $G(z, \hat{\theta}_N)$, the following approximations were obtained in [53], under the assumption that the model order n tends to infinity in some appropriate way when the data length N tends to infinity:

$$\mathrm{Var}(G(\mathrm{e}^{j\omega}, \hat{\theta}_N)) \approx \frac{n}{N}\frac{\Phi_v(\omega)}{\Phi_u(\omega)} \quad \text{in open-loop identification} \tag{19}$$

$$\mathrm{Var}(G(\mathrm{e}^{j\omega}, \hat{\theta}_N)) \approx \frac{n}{N}\frac{\Phi_v(\omega)}{\Phi_u^r(\omega)} \quad \text{in closed-loop identification} \tag{20}$$

where $\Phi_u^r(\omega) \triangleq |[C_{\mathrm{id}}(\mathrm{e}^{j\omega})]/[1 + C_{\mathrm{id}}(\mathrm{e}^{j\omega})G_0(\mathrm{e}^{j\omega})]|^2\Phi_r(\omega)$ is the part of the input spectrum that is caused by the reference signal (Fig. 1). These formulas explicitly contain the effect of the experimental conditions (e.g. number of data, input spectrum, noise spectrum, feedback configuration, feedback controller C_{id}, etc.) on the error measure. This yields the possibility of optimizing over some relevant experimental design variable in order to minimize a control-relevant measure of this error.

Except for dual control (see Section 1), the first application of optimal experiment design to

control-oriented identification was in [28], where an application to Minimum Variance control was treated in the framework presented in Section 4. The criterion (14) used for the optimal design was the squared error between the designed output of Fig. 3 and the output of the optimal loop of Fig. 5. The results showed that the optimal experiment consists of performing closed-loop identification with the unknown optimal Minimum Variance controller in the loop. The same approach was later extended to other control performance criteria in [24,39]. These results were all based on the variance formulas of [53] that were derived under the assumption that the model order tends to infinity, and it was observed in recent years that the use of these formulas for finite order models can sometimes lead to erroneous conclusions. This observation triggered a revival of interest in *variance expressions for finite order models*.

For the estimated parameter vector $\hat{\theta}_N$, various versions of the covariance expression (7) can be used for experiment design problems, one of which (18) is most relevant because the design variables appear in a transparent way. Note that (18) is a specialization of (7) to open-loop identification. Control-oriented L_2-optimal design criteria can then often be formulated as a *weighted trace optimal input design problem*, where the criterion to be optimized takes the form

$$\min_u \operatorname{tr}[W(\theta_0)P_\theta] \tag{21}$$

where $W(\theta_0)$ is a weighting function, depending on the true system, and which reflects the control objective. This is an L-optimal design problem, for which classical input design theory could offer a solution only in very special cases. In [16,17] a linear parametrization of $G(z, \theta)$ using orthogonal basis functions is used, and an L_2 control-oriented optimal input design problem of the form (21) is formulated, where the optimization is with respect to the finite time input sequence.

It often makes more sense to optimize over the input spectrum. One sensible open-loop optimal input design problem could be formulated as follows:

$$\min_{\Phi_u(w)} \operatorname{tr}[W(\theta_0)P_\theta] \text{ subject to } \int_{-\pi}^{\pi} \Phi_u(\omega)\, \mathrm{d}\omega \le \alpha, \text{ and } \Phi_u(\omega) \ge 0\ \forall\omega. \tag{22}$$

This is still a difficult, infinite dimensional optimization problem. However, by the use of Schur complement, the problem can be reformulated as a convex optimization problem under linear matrix inequality (LMI) constraints. The numerical solution of such problems became possible in the 1990s with the advent of interior point optimization methods [15,61]. The problem becomes finite dimensional if the input spectrum $\Phi_u(\omega)$ can be finitely parametrized.

In [52], the same performance criterion as in [28] was adopted for the optimal input design, but the authors used the finite order variance formula (18) for P_θ^{-1} and showed how to convert the problem to a weighted trace optimal design problem as above. The problem was made finite dimensional by restricting the input to be FIR filtered white noise. The coefficients of the FIR filter now become the design parameters.

8.3. Experiment Design for Robust Control

Robust stability and robust performance criteria are typically expressed as constraints on frequency weighted expressions of the variance of the transfer function error, rather than as L_2 performance criteria. For example, a robust stability constraint is typically formulated as

$$\operatorname{Var} G(\mathrm{e}^{j\omega}, \hat{\theta}_N) \le W(\mathrm{e}^{j\omega}) \tag{23}$$

where $W(\mathrm{e}^{j\omega})$ is a frequency weighting function that takes account of closed-loop properties (e.g. robust stability condition). In order to formulate optimal input design problems in terms of control-oriented quality measures on $G(\mathrm{e}^{j\omega}, \hat{\theta}_N)$ rather than in terms of L_2 criteria, without using the asymptotic (in model order) variance formulas, several approaches can be taken.

One approach is to derive finite order formulas for the variance of the transfer function estimate. For model structures that are linear in the parameter vector θ, such as FIR models or rational basis function models, the covariance P_θ for $\hat{\theta}_N$ and the parametric uncertainty set (8) directly translate to a variance expression for $G(\mathrm{e}^{j\omega}, \hat{\theta}_N)$ at any frequency ω. Another commonly used approach to go from parameter covariance to transfer function covariance is to use the following first order Taylor series approximation:

$$\operatorname{Var} G(\mathrm{e}^{j\omega}, \hat{\theta}_N) \approx \frac{\sigma_e^2}{N} \frac{\partial G^*(\mathrm{e}^{j\omega}, \theta_0)}{\partial\theta} P_\theta \frac{\partial G(\mathrm{e}^{j\omega}, \theta_0)}{\partial\theta}. \tag{24}$$

This approach was adopted in Ref. [43] where it is shown that several useful H_∞ design criteria can be reformulated as weighted trace optimal input design problems subject to LMI constraints. In order to obtain a finite dimensional problem, different ways of approximating the input spectrum by a linear

finite parametrization are proposed in that paper. An alternative to the use of the first order approximation (24) is to use the more accurate formulas that have recently been obtained for the variance of finite order transfer function estimates [63,76]. For example, for an Output Error model structure, the open-loop variance formula (19) is replaced by

$$\mathrm{Var}(G(e^{j\omega}, \hat{\theta}_N)) \approx \kappa_n(\omega)\frac{\Phi_v(\omega)}{\Phi_u(\omega)} \tag{25}$$

where $\kappa_n(\omega)$ depends on the poles of the true system. The use of these new transfer function variance formulas for input design has been advocated in [38], but one additional difficulty, as the authors point out, is that the function $\kappa_n(\omega)$ depends on the unknown system.

Another approach to optimal input design for robust control is to use the results in [14,30], which connect robust stability and robust performance measures directly to the parametric ellipsoidal uncertainty region (8) of a finite order model without the intermediate step of transfer function variance computation. Thus, no Taylor series approximation is required here. It is worth noting that ellipsoidal parametric uncertainty regions appear not only in PE identification, but also in set-membership identification; see e.g. [58]. The first open-loop optimal input design problem for robust control was formulated in [35] using the ellipsoidal parametric uncertainty set (8) and the corresponding transfer function set (9), without any approximation. The robust stability measure minimized in that paper, with respect to the input spectrum $\Phi_u(\omega)$, was the worst-case ν-gap $\delta_{\mathrm{WC}}(G(z, \hat{\theta}_N), \mathcal{D}_{\mathrm{PE}})$ between the identified model $G(z, \hat{\theta}_N)$ and all models in the PE uncertainty set $\mathcal{D}_{\mathrm{PE}}$ defined in Section 6:

$$\delta_{\mathrm{WC}}(G(z, \hat{\theta}_N), \mathcal{D}_{\mathrm{PE}}) = \sup_{G(z,\theta)\in\mathcal{D}_{\mathrm{PE}}} \delta_\nu(G(z, \hat{\theta}_N), G(z, \theta)) \tag{26}$$

where the ν-gap was defined in [74]. One of the merits of this measure is that it is directly related to the size of the set of its stabilizing controllers: the smaller the worst-case ν-gap of the uncertainty set $\mathcal{D}_{\mathrm{PE}}$, the larger is the set of controllers that stabilize all models in $\mathcal{D}_{\mathrm{PE}}$. The solution proposed in [35] uses Tchebycheff system theory: the input spectrum is parametrized in terms of its n moments with respect to the system, which appear linearly in the expression of the information matrix M_θ [78]. Thus, no approximation is required here for the parametrization of the input spectrum. The optimal solution can always be obtained as a multisine.

8.4. Optimal Experiment Design in Closed Loop

All the results discussed so far are for open-loop identification, whereas identification for control is typically performed in closed-loop, often in an iterative way. As it happens, the parameter covariance formula (18) can easily be extended to closed-loop identification as follows [10,45]:

$$P_\theta^{-1} = N\overbrace{\left(\frac{1}{\sigma_e^2}\frac{1}{2\pi}\int_{-\pi}^{\pi} F_r(e^{j\omega}, \theta_0)F_r(e^{j\omega}, \theta_0)^*\Phi_r(\omega)\,\mathrm{d}\omega\right)}^{\mathcal{P}_r^{-1}(\Phi_r(\omega),\theta_0,\sigma_e^2)} + N\overbrace{\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} F_e(e^{j\omega}, \theta_0)F_e(e^{j\omega}, \theta_0)^*\,\mathrm{d}\omega\right)}^{\mathcal{P}_v^{-1}(\theta_0)}. \tag{27}$$

Here, $F_r(z, \theta_0) = C_{\mathrm{id}}S_{\mathrm{id}}(\Lambda_G(z, \theta_0))/(H(z, \theta_0))$, $F_e(z, \theta_0)) = (\Lambda_H(z, \theta_0))/(H(z, \theta_0)) - C_{\mathrm{id}}S_{\mathrm{id}}\Lambda_G(z, \theta_0)$, $\Lambda_G(z, \theta) = (\partial G(z, \theta))/\partial\theta$ and $\Lambda_H(z, \theta) = (\partial H(z, \theta))/\partial\theta$. Note that P_θ^{-1} is made up of a part depending on $\Phi_r(\omega)$ and a part which does not depend on $\Phi_r(\omega)$. Both parts are linear in N and both parts depend on the operating controller C_{id}. For a given controller C_{id} and a fixed data length, we observe that the covariance matrix is again linear in the reference spectrum $\Phi_r(\omega)$, which is now the design object. Instead of using a fixed controller, and optimizing over the external reference spectrum $\Phi_r(\omega)$, closed-loop optimal design problems can also be formulated with respect to both the reference spectrum $\Phi_r(\omega)$ and the operating controller C_{id}. It turns out to be easier to use the input spectrum $\Phi_u(\omega)$ and the cross-spectrum $\Phi_{ue}(\omega)$ as design variables; note that there is a one-to-one relationship between the pair $\{\Phi_r(\omega), C_{\mathrm{id}}(e^{j\omega})\}$ and the pair $\{\Phi_u(\omega), \Phi_{ue}(\omega)\}$. Such approach has been proposed in [45].

8.5. Why Do More Work than is Needed?

The traditional approach to optimal input design has been to optimize some measure of the resulting uncertainty, subject to a constraint on the input signal power. Examples are a measure of the parameter covariance, or a weighted measure of the transfer function error that is related to stability robustness. However, in an identification for robust control setting, one should not spend more effort on the identification than what is needed for the design of a robust controller, under the constraint that this controller must achieve stability and a prespecified level of performance with all models in the uncertainty set. This idea has led to the recent concept of "least costly

identification for control": [12]. Instead of minimizing some measure of the uncertainty set, the objective now is to deliver an uncertainty set that is just within the bounds required by the robust control specifications, and to do so at the smallest possible cost. In [11] open-loop identification is considered and the cost is then defined as the total input signal power. In [10] a closed-loop disturbance rejection setup is considered (with no reference excitation signal in normal operation), and the identification cost is then defined as the additional penalty that occurs in the control performance cost when an excitation signal is added for the purposes of doing the identification. The idea of least costly (or minimum energy) identification experiment for control has been further developed in an open-loop framework in [44].

From a practical point of view, the cost of identification is an issue of major importance. This has been thoroughly discussed in [65] where the concept of "plant-friendly" identification is presented. It is often estimated that 75% of the cost associated to an advanced control project goes into model development. Even though the definition of the cost used in the recent work on "least costly identification for control" does by no means cover all the practical costs of modelling, the disruption caused to normal operation and the time required to arrive at a satisfactory model are considered to be very significant elements of this total modelling cost. These two costs are incorporated in the "least costly" criterion.

8.6. But is Optimal Design Really Worth the Effort?

One might wonder whether it pays to perform optimal input design computations, given that the optimal solution necessarily depends on the unknown system, which means that a preliminary model estimate must be obtained first before an approximately optimal input signal can be computed. This is sometimes referred to as *adaptive* (*or iterative*) *optimal input design*. In [6] the possible benefits of optimal input design for control have been quantified for two benchmark problems. One is a process control application where the time it takes to obtain a model of a prespecified quality is the cost to be minimized. The other is the control of a flexible mechanical structure, where the desired savings are in terms of the level of input excitation. In both cases, significant savings are obtained by the application of a two-step identification procedure, where the second step uses an optimally designed input signal computed from a preliminary model estimate.

While identification for control has for a long time focused on the selection of control-oriented identification criteria and on the definition of control-oriented uncertainty sets, the most recent focus has been on experiment design. This is a very natural evolution, since the experimental conditions have a direct impact on the quality of the model. It is fair to say that this area of research is still very much in its infancy. Many experiment design issues remain to be addressed, let alone solved. A fundamental issue is the fact that the optimal experiment depends on the unknown system. Thus, the practical implementation of optimal input design results requires that a preliminary model be estimated quickly on the basis of non-optimal inputs, after which an estimate of the optimal input can be computed. This raises the very important issue of the robustness of the optimal design to model errors, and of the convergence of such adaptive implementations. Some preliminary observations and recommendations on this robustness question issue have been made in [44].

Other important issues to be addressed are to formulate the input design problem directly in terms of the properties (robust stability and performance) of the controller that is designed from the identified model. Some preliminary results in this direction can be found in [5].

Finally, the new phase of research results that have been briefly described here are all based on variance results for finite order models, under the assumption that the true system is in the model set. At some point, one will need to address the situation where undermodelling is present; this will be a real challenge! As pointed out in [37], a proper choice of input is even more important when a restricted complexity model is used with a particular objective (e.g. control) in mind: it is then always better that the input excite only those parts of the system dynamics that need to be modelled.

9. Conclusions

We have attempted to explain the major issues related to the problem of identification for control, to present the successive solutions that have been brought to the overall problem, and to display some major remaining open problems. Most certainly, the major impact so far in terms of transfer of technology to the industrial world has been the iterative schemes of model and controller updates, which have provided a methodological background for iterative data-based controller performance enhancement. As for the synergy of robust control concepts and identification design concepts, we believe that most challenges are still

ahead of us. The main reason is that a full understanding of the interplay between identification design and robust control analysis and synthesis cannot bypass the role of the uncertainty set, and that the analysis of this problem is difficult and involves techniques from several different subdisciplines. Finally, it is interesting to observe that the most recent work on identification for robust control has seen a revival of interest for optimal experiment design. The optimal input design work of the 1970s was limited to open-loop identification and to quality criteria that did not take account of control objectives. Even in this simple framework, the solution of these open-loop design problems were computationally intractable at the time. With the powerful new techniques of convex optimization under linear constraints, it has now become realistic to address much more sophisticated control-oriented optimal design problems.

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