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## Abstract

*Keywords:*

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## 1. Introduction

The search for alternate energy sources which could be categorised under the "green" label has become important area of research in the modern world. Solar, wind power and wave power are some of the examples of these sources. Recently, a new branch of research has been developing to extract energy from flow induced vibrations (Bernitsas et al. (2008)). It has been hypothesized that this technique may work efficiently in areas where regular turbines cannot.

A simple structure that is susceptible to flow-induced vibrations that are suitable for energy extraction are slender structures, such as cylinders, elastically mounted perpendicular to a fluid stream. With regards to a slender body two common types of flow induced vibrations are Vortex Induced Vibrations (VIV) and aeroelastic galloping. Significant research has been carried out by Bernitsas and his team on extracting useful energy from VIV. Some of their significant work includes investigating the influence of physical parameters such as mass ratio (the ratio of the mass of the cylinder and the displaced fluid), Reynolds number, mechanical properties (Raghavan and Bernitsas (2011), Lee and Bernitsas (2011)) and location (effect of the bottom boundary) (Raghavan et al. (2009)). However, the possibility of extracting energy using aeroelastic galloping has not been thoroughly investigated. Some theoretical work was carried out by (Barrero-Gil et al. (2010)). Utilizing galloping may be a more viable method to harness energy from flow induced vibrations as it is not bounded by a "lock-in" range of reduced velocities (ratio between the freestream velocity and the product of the natural frequency of the system and the characteristic length). Therefore it is preferable to investigate further the possibility of harnessing energy from flow induced vibrations using aeroelastic galloping.

Real life energy harvesting systems use high damping ratios where the energy generator (e.g electrical generator) puts a significant amount of damping into the system. Therefore it is crucial to investigate the behaviour of aeroelastic galloping scenarios at high damping ratios in order to optimise the system to obtain a acceptable power output. Hence the focus of this paper is concentrated on investigating the mechanical power output of high-damped galloping systems in laminar flow.

According to Païdoussis et al. (2010), Glauert (1919) has provided a criterion for galloping by considering the auto-rotation of an aerofoil. Den Hartog (1956) has provided a theoretical explanation for galloping for iced electric transmission lines. A non-linear

theoretical aeroelastic model to predict the response of galloping was developed by Parkinson and Smith (1964) based on the quasi-steady state (QSS) theory. Experimental lift and drag data on a fixed square prism at different angles of attack were used as an input for the theoretical model. It essentially used a curve fit of the transverse force to predict the galloping response. The study managed to achieve a good agreement with experimental (wind tunnel) data. Joly et al. (2012) have observed that finite element simulations shows a sudden change in amplitudes below a critical values of the mass ratio, which the (QSS) model fails to reproduce. The Parkinson's equation was essentially modified to account for the vortex shedding and managed to produce the effects to the amplitude at low mass ratios. Barrero-Gil et al. (2010) have investigated the possibility of extracting power from vibrations caused by galloping using quasi-steady state theory. In the conclusions of that paper it was pointed out that in order to obtain a high power to area ratio the mass-damping ( $m^*\zeta$ ) parameter should be kept low as well as the frequency of oscillations should be carefully matched **have a good agreement with the size of the cross section**. Another interesting conclusion was that energy conversion systems which uses galloping could operate over a large range of flow velocities unlike VIV energy harvesting systems where the factor of energy conversion has a strong dependence on the incoming flow velocity.

**Nomenclature**

$a_1, a_3, a_5, a_7$	coefficients of the polynomial to determine $C_y$
$F_y$	force due to $C_y$
$\rho$	fluid density
$m$	mass of the body
$m_a$	added mass
$c$	damping constant/damping factor
$k$	spring constant
$U$	freestream velocity
$y, \dot{y}, \ddot{y}$	transverse displacement, velocity and acceleration
$A$	displacement amplitude
$\mathcal{A}$	cross sectional area
$F_0$	force due to shedding
$\omega_s$	vortex shedding frequency
$t$	time
$P_{mean}$	mean power
$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$	natural frequency of the system
$\omega_n = 2\pi f$	natural frequency of the system
$D$	characteristic length of the body
$m^* = \frac{m}{\rho \times \text{Volume of the body}}$	mass ratio
$U^* = \frac{U}{f \times D}$	reduced velocity
$\zeta = \frac{c}{2m\omega_n}$	damping ratio
$P_t$	power transferred to the body by the fluid
$P_d$	power dissipated due to mechanical damping
$\theta = \tan^{-1}(\frac{\dot{y}}{U})$	instantaneous angle

## 2. Background theory

### 2.1. Mathematical model (Quasi-steady )

One of the widely used mathematical model to predict the system response under galloping is the Quasi-steady state (QSS) model, incorporated by Parkinson and Smith (1964) for a square cross section. The equation of motion of the body under galloping is given by Eq. (1). The forcing term  $F_y$  is given by Eq.(2).

$$(m + m_a)\ddot{y} + c\dot{y} + ky = F_y \quad (1)$$

$$F_y = \frac{1}{2}\rho U^2 C_y \quad (2)$$

In the QSS model  $C_y$  is determined by an interpolating polynomial based on the stationary lift and drag data. The order of the interpolation polynomial has varied from study to study (e.g.. $7^{th}$  order was used in Parkinson and Smith (1964) and  $3^{rd}$  order polynomial

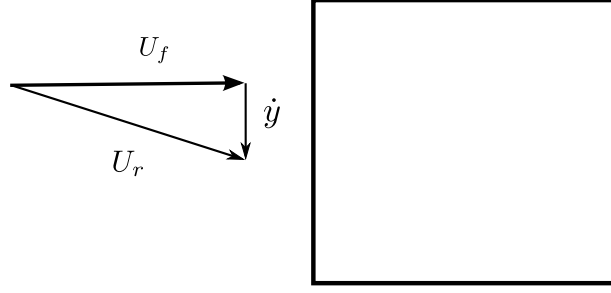


Figure 1:

was used in Barrero-Gil et al. (2009). Ng et al. (2005) concluded that using a 7<sup>th</sup> order polynomial is sufficient and a polynomial higher than that of 7<sup>th</sup> order polynomial neither results in a result significantly better result nor does it exhibit an additional amplitudes of oscillation. Thus A 7<sup>th</sup> order interpolating polynomial is incorporated in this present study.

$$C_y(\alpha) = a_1 \left( \frac{\dot{y}}{U} \right) + a_3 \left( \frac{\dot{y}}{U} \right)^3 + a_5 \left( \frac{\dot{y}}{U} \right)^5 + a_7 \left( \frac{\dot{y}}{U} \right)^7 \quad (3)$$

Joly et al. (2012) used a sinusoidal forcing function to the RHS of the oscillator model (Eq. (1)) in order to represent forcing due to VIV. This method provided satisfactory results with the numerical simulations obtained at low mass ratios. This study, the forcing due to VIV is incorporated using a sinusoidal forcing function  $F_0 \sin \omega_s t$  added to the RHS.  $\omega_s$  and  $F_0$  represents shedding frequency and the maximum force due to shedding respectively. Thus, the final equation is represented by Eq. (4).

$$(m+m_a)\ddot{y}+c\dot{y}+ky=\frac{1}{2}\rho U^2 A \left( a_1 \left( \frac{\dot{y}}{U} \right) + a_3 \left( \frac{\dot{y}}{U} \right)^3 + a_5 \left( \frac{\dot{y}}{U} \right)^5 + a_7 \left( \frac{\dot{y}}{U} \right)^7 \right) + F_0 \sin(\omega_s t) \quad (4)$$

This equation could be solved by time integration methods. In this study “Ode 45” routine in MATLAB was used to obtain the solutions.

## 2.2. Calculation of average power

The dissipated power due to the damper could be expressed as the harvested power output assuming that the other power dissipation due to internal damping such as friction of the system is negligible. Therefore the mean power output could be given by Eq. (5).

$$P_{mean} = \frac{1}{T} \int_0^T (c\dot{y})\dot{y}dt \quad (5)$$

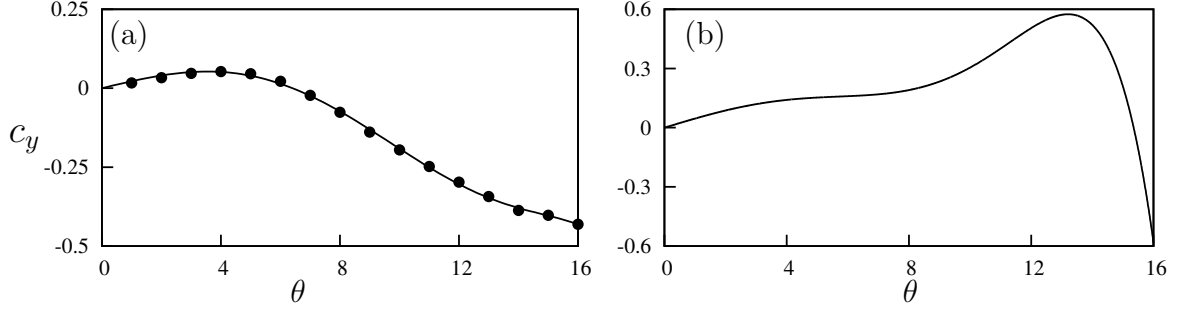


Figure 2: Lift coefficient,  $C_y$ , as a function of incidence angle  $\theta$ , for a static square cross section at  $Re=165$  by (a). Points ( $\bullet$ ) are measurements from simulations. The data obtained by Parkinson and Smith (1964) at  $Re=22300$  are represented by (b). The solid line is a 7th-order polynomial fit that has been used to calculate the right-hand side of equation 4 throughout this study

### 2.3. Parameters used

The stationary data and the FSI data were obtained using a higher order spectral element code which simulates 2D laminar flow. The Reynolds number was kept at 165 as it was pointed out by Sheard et al. (2009) and Tong et al. (2008) that the 3 dimensional transition for a square cylinder occurs at approximately  $Re=160$ .  $F_0$  was kept at 0.4937 which was obtained by using a simple linear interpolation on the data of Joly et al. (2012).  $\omega_s$  was set to 0.98 which was obtained by a power spectral analysis of the stationary data at  $0^\circ$ . Stationary  $C_y$  data were obtained at different angles of attack ranging from  $0^\circ$  to  $16^\circ$ . The average power was obtained by using Eq. (5) with data sets consisting substantial amount of peaks. In order to obtain a comparison with high Reynolds number power data was obtained using Parkinson and Smith (1964)  $C_y$  data.

FSI data were obtained for the oscillating (free-vibration) scenario. The Navier-Stokes equations were solved using an accelerated frame of reference using the previously mentioned code. A three-step time splitting scheme together with high-order Lagrangian polynomials were used to obtain the solution. The details of the method could be found in Thompson et al. (2006, 1996). This code was incorporated in Leontini et al. (2011, 2007) where it was employed in a fluid-structure interaction problems.

The computational domain consists of 690 quadrilateral macro elements (refer figure) where majority of the elements were concentrated near the square section. A freestream condition was given to the inlet, top and bottom boundaries and the normal velocity gradient was set to zero at the outlet. A convergence study was performed by changing the order of the polynomial ( $p$ -refinement) at  $U^* = 40$  and  $Re = 165$ . A 9th order polynomial together with a time step of  $\frac{\Delta t U}{D} = 0.001$  was sufficient to ensure an accuracy of 2% with regards to amplitude of oscillation.

## 3. Results

### 3.1. Displacement, velocity and power output as a function of reduced velocity

The quasi-steady analysis data reveals that the displacement amplitude tend to grow with increasing  $U^*$  Fig.3 (a) and (b). The onset of galloping is delayed with increasing

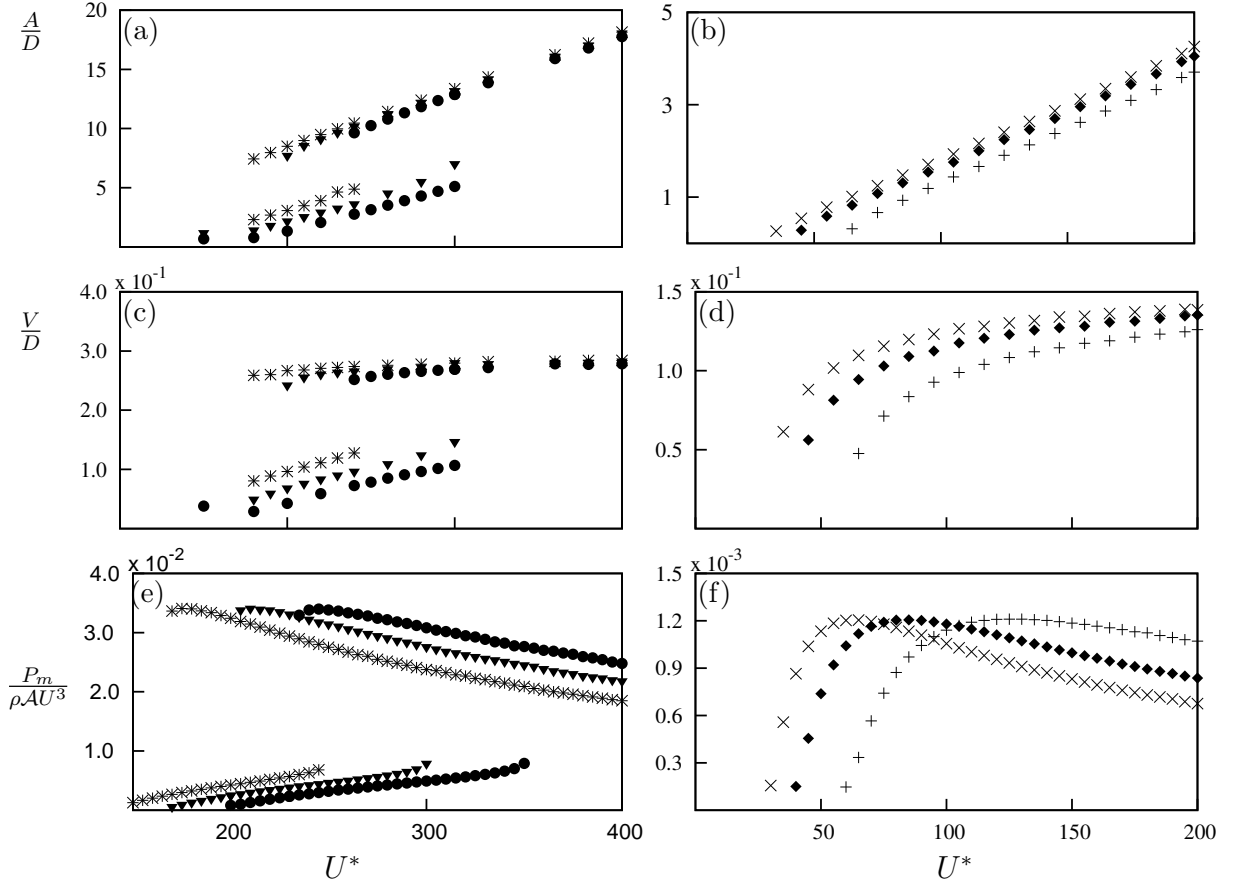


Figure 3: Velocity and displacement amplitude and mean power as a function of  $U^*$ . (a), (c) and (e) are calculated using input  $C_y$  data at  $Re=22300$  obtained by Parkinson and Smith (1964) and present data at three different damping ratios:  $\zeta = 0.0125$  (\*),  $\zeta = 0.015$  (▼) and  $\zeta = 0.0175$  (●). (b), (d) and (f) are from  $C_y$  data at  $Re=165$  and are calculated from the fixed body simulations and present data at three different different damping ratios:  $\zeta = 0.075$  (×),  $\zeta = 0.1$  (◆) and  $\zeta = 0.15$  (+). The multiple branches for the higher  $Re$  are due to the hysteresis between two solutions.

$\zeta$  for both high and low Reynolds numbers. This echoes the findings of previous studies Parkinson and Smith (1964) and Barrero-Gil et al. (2010). Hysteresis could be observed at higher Reynolds numbers.

#### *Power vs $U^*$*

The mean power grows, peaks and then reduces as  $U^*$  is increased (Fig.3 (e) and (f)) for each value of  $\zeta$ . A shift of the peak power could be observed as  $\zeta$  increases. However, the magnitude of the peaks remain constant for all the values of  $\zeta$ . A similar observation could be made from the results of Barrero-Gil et al. (2010). It could be observed that unlike VIV the system has no preferred frequency. The onset of galloping and the peak power occurs at different  $U^*$  at when the damping ratio is changed. The peak power remains constant regardless of  $U^*$ .

#### *3.2. Galloping response and natural frequency*

If the oscillator equation Eq.(4) is considered from a power perspective (disregarding the shedding term as the net effect is negligible !!! this should be elaborated further possibly in an earlier section as to why. I suggest when you talk about this when you discuss the shedding parameters and discuss the findings from Joel.

Only negligible when system oscillates at natural frequency which is far from shedding frequency), it could be seen that the forcing term on RHS of the equation is only dependent on transverse velocity( $\dot{y}$ ) which is essentially the input power of the system. On the RHS, the mechanical damping or system damping is the only term that takes out power at any instant by the product of damping force and the velocity ( $P_d$ ). The inertia and the stiffness terms govern the frequency of the system but the forces associated by those terms are conservative forces i.e there is zero net energy in or out of the system when averaged over a period. Therefore it appears that the system is governed by the transverse velocity rather than the natural frequency.

Using  $U^*$  and  $\zeta$  assumes that the system has a preferred frequency. The effect of fixing  $\zeta$  and increasing  $U^*$  actually decreases damping constant for a fixed free-stream velocity. ( $U^* = \frac{U}{f \times D}$ ,  $\zeta = \frac{c}{2m\omega_n}$ ). Both these effects lead to the multiple lines that are horizontally transposed when  $\zeta$  is increased (Fig.3 (e) and (f)). Therefore the effect of  $\zeta$  essentially scales up the damping coefficient for a fixed  $U^*$ .

Therefore a single set of results for a given instantaneous lift ( $C_y$ ) could be obtained if we were to plot displacement, velocity and power as a function of damping constant  $c$  (Fig ??, ??). A similar maximum velocity could be obtained for a given 'c'. Fig.?? clearly shows the validity of this argument.

Power could be expressed as the product of force and velocity. Therefore the transferred power from fluid-to-body could be expressed as  $P_t = F_y \dot{y}$ . Similarly the dissipated power due to the mechanical damping could be expressed as  $P_d = (c\dot{y})\dot{y}$ . The time average of these two quantities should be equal when mechanical friction is neglected due to energy

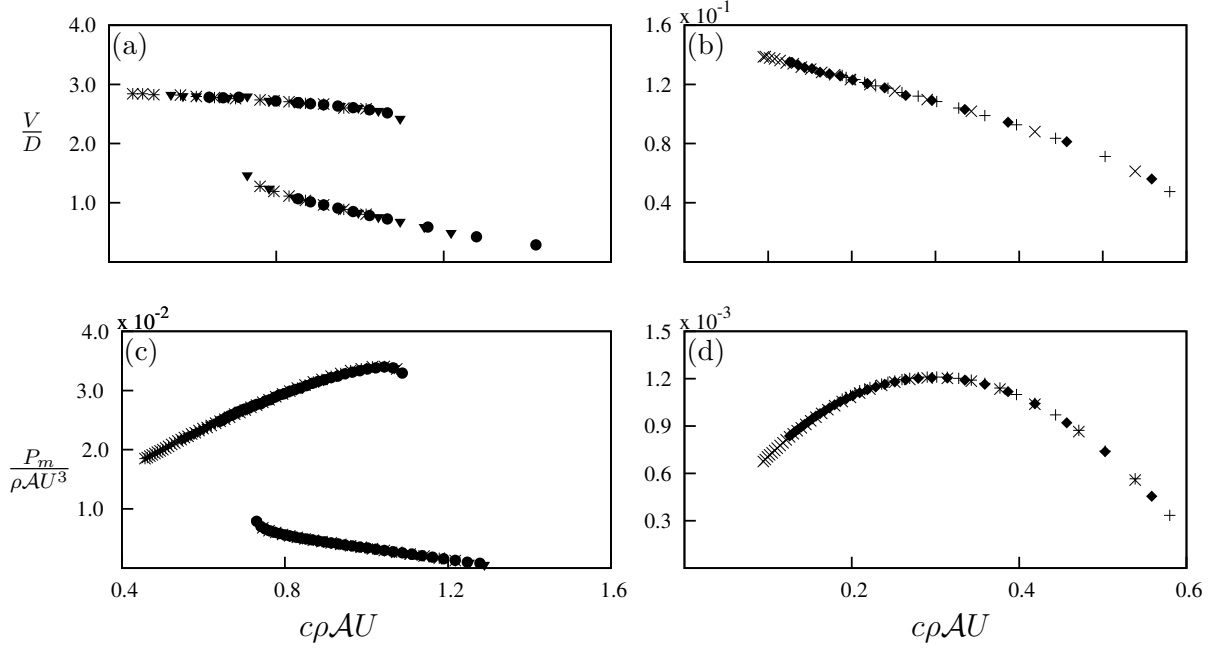


Figure 4: Velocity amplitude and mean power as a function of ‘ $c$ ’ (damping constant). (a) and (c) are calculated using input  $C_y$  data at  $Re=22300$  obtained by Parkinson and Smith (1964) and present data at three different damping ratios:  $\zeta = 0.0125$  (\*),  $\zeta = 0.015$  (▼) and  $\zeta = 0.0175$  (●). (b) and (d) are at  $Re=165$  are calculated from the fixed body simulations and present data from three different damping ratios:  $\zeta = 0.075$  (×),  $\zeta = 0.1$  (◆) and  $\zeta = 0.15$  (+). The collapsed data implies that there is no frequency selection and the tuning parameter of the mechanical side of the system is the damping constant to obtain an optimum power output.

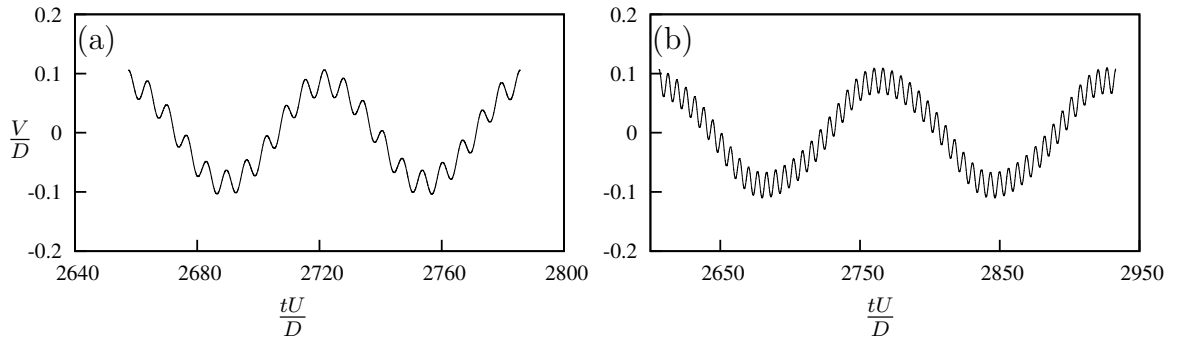


Figure 5: Time histories of velocity at two different  $\zeta$  and  $U^*$  which produces same mean power ( $1.2 \times 10^{-3}$ ). Data presented using QSS assumption, (a) at  $U^*=60$  and  $\zeta = 0.075$ , (b) at  $U^*=165$  and  $\zeta = 0.175$  at  $Re=165$ . Shedding is present in both signals as noise but the average velocity amplitude remain constant in both cases.



conservation. The analysis of time histories of  $P_t$  and  $P_d$  at key regions (Fig.6) on the mean power vs  $U^*$  provides a detailed explanation for the varying power output when the reduced velocity is increased. It has been established earlier that the damping factor is a function of  $U^*$ . It could be derived that  $U^*$  is inversely proportional to damping coefficient.

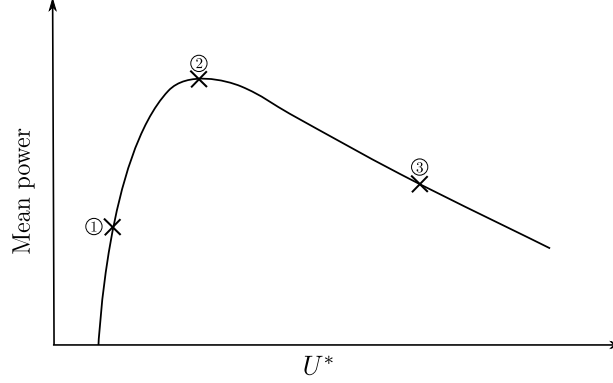


Figure 6: Three key regions taken into account to analyse the time histories of power in a typical the Mean power vs.  $U^*$  curve at Re 165

From Fig 3 (a) shows that the instantaneous force rises until  $4^0$  where it peaks and then falls and at round  $6^0$  becomes negative. Maximum amount of power could be transferred within the peak region. At the region where the instantaneous force becomes negative it will be opposing the velocity  $\dot{y}$ .

$U^* = 90$  (region 1) the damping constant is high and therefore a clear sinusoidal signal could be observed for both  $P_d$  and  $P_t$  Fig. 7 (a). Fig.7 (d) and (g) shows that  $\theta$  is in line or in phase with  $F_y$ . Hence both  $P_d$  and  $P_t$  becomes sinusoidal. However, due to the higher damping  $\theta$  does not go to the region where the peak power is produced.

At region 2 where the mean power output is at its maximum( $U = 165$ ),  $P_t$  is not a pure sinusoidal signal. However, the signal remains periodic. From the time history graph of  $P_t$ , two 'peaks' are present in a single half cycle (Fig 7 (b)). In this case at certain point in time  $\theta$  arrives at the region where  $F_y$  decreases but does not become negative when the angle is increased. Therefore, the force  $F_y$  and  $P_t$  reduces as the velocity further increases since  $\theta = \tan^{-1}(\frac{\dot{y}}{U})$ . As the velocity  $\dot{y}$  is sinusoidal,  $\theta$  recovers back and this leads to two 'peaks' in a single half cycle.

At region 3 ( $U^* = 400$ ) ' $c$ ' is low in comparison with region 1 and 2 which leads to a low mean power output. From Fig.7 (c) .b  $P_t$  becomes negative over some portion of the cycle. This is because  $\theta$  passes the point where both  $\theta$  and  $c_y$  (therefore  $c_y$ ) are positive. As the force opposes the direction of travel and the power becomes negative. On the other hand from an energy perspective we could see that the mechanical damping is not sufficient to dissipate out the energy transferred from the fluid to the structure during the part of the cycle when this occurs(as ' $c$ ' is substantially low), therefore part of this energy is transferred back to the fluid in the remaining part of the cycle.

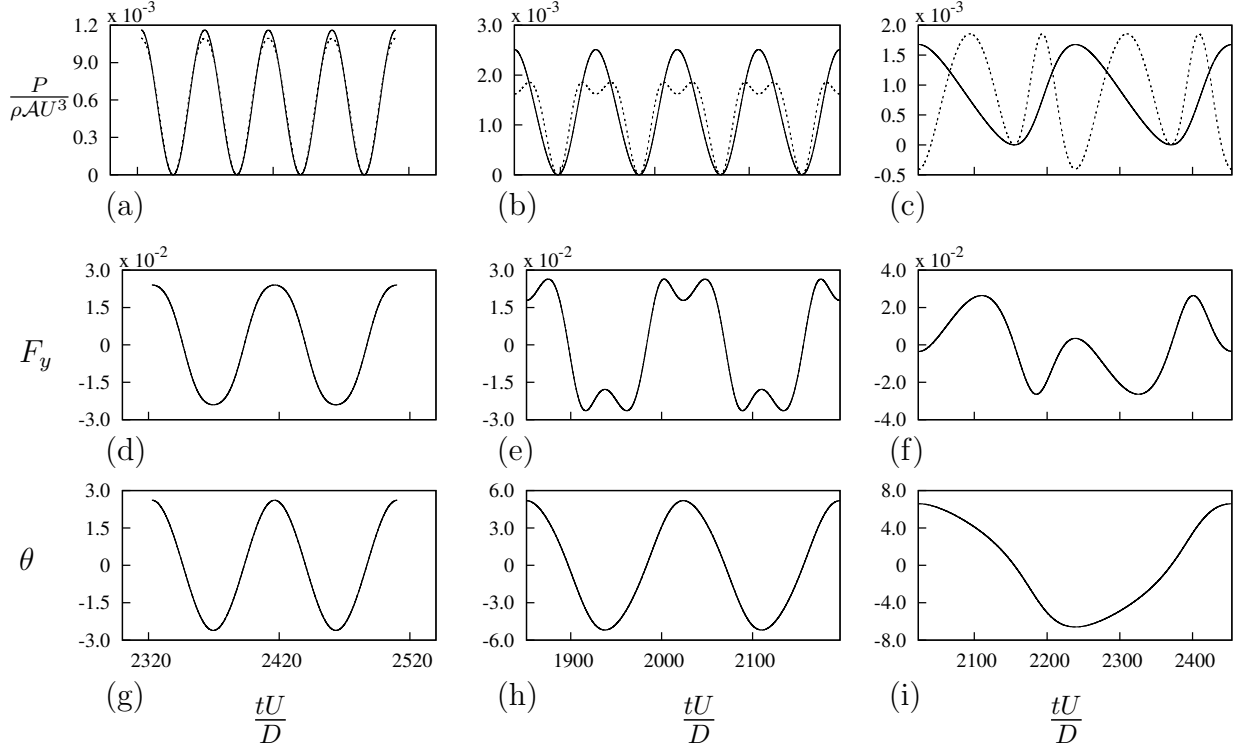


Figure 7: Time histories of  $P_t, P_d, F_y$  and  $\theta$  at  $U^* = 90, 165$  and  $400$  where the mean power dissipated due to mechanical was calculated using equation [equation](#). Data was obtained at  $\zeta = 0.1$  and  $m^* = 40$ . The time histories of  $P_t$  (—) and  $P_d$  (---) are presented in (a), (b) and (c) where  $U^*$  is 90, 165 and 400 respectively. (d), (e) and (f) shows time histories of the instantaneous force  $F_y$  and (g), (h) and (i) shows the time history of the instantaneous angle  $\theta$  of the corresponding power plots above

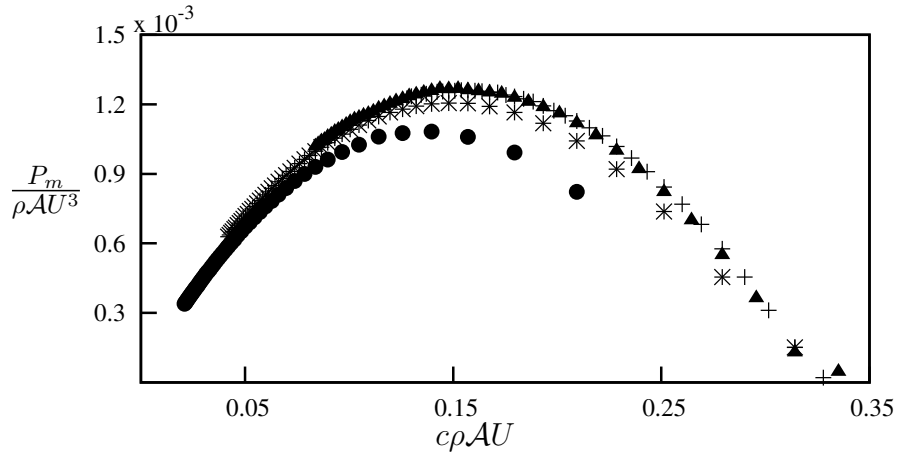


Figure 8: Mean power as a function of damping factor. Data are presented at  $m^* = 10$  (●),  $m^* = 20$  (\*),  $m^* = 40$  (▲),  $m^* = 60$  (+) at  $\text{Re} = 165$  and  $\zeta = 0.1$ . The reduction of maximum mean power could be observed when  $m^* < 40$

### 3.3. Effect of $m^*$

The maximum mean power at different  $m^*$  Fig.?? was constant beyond  $m^* = 30$ . However, at  $m^* \leq 30$  an effect of  $m^*$  it could be observed that the mean power curve reduces with  $m^*$ . This may be due to the fact that shedding dominates as the inertia of the system is reduced.

!! Are you sure?? I am sure if you run cases without shedding you will still get less power. Shedding has very little influence when the galloping is sustained and the frequencies are far apart. I believe when  $m^*$  is low, there is not enough inertia (stored kinetic energy in the first term of the oscillatory equation) in the system to carry it over the cycle. We could add a little value to this paper by investigating this a little further since everything is already set up. Turn off shedding and run at low  $m^*$ . eg.  $M^*=1$  or 2. You will get nothing. So you cannot say shedding suppresses galloping. Galloping is suppressed by low  $m^*$  and if there is no galloping then all you see is the response to shedding if shedding forces are present. !!

This was also reported by Joly et al. (2012) where an influence of vortex shedding was present on galloping amplitude at low mass ratios.

### 3.4. Comparison with FSI simulations

Similar trends are captured for both displacement and velocity amplitudes between QSS and FSI simulations (Fig. 9(a) and 9(b)). Quantitatively a large discrepancy (average of 30%) could be observed between QSS and FSI data. Therefore the power also becomes significantly low (Fig.9(c)). However, the FSI data (Fig.9 (c)) was able to produce the main the rise and the fall of mean power as  $U^*$  is increased. The reasoning behind this the fact is that galloping is weak at  $Re$  165 and therefore fluid damping has a significant effect. It was reported by Barrero-Gil et al. (2009) that galloping only starts to occur at  $Re \geq 159$ . As power is function of  $(\dot{y})^2$  the error between QSS and FSI power becomes significantly large.

Put conclusion here

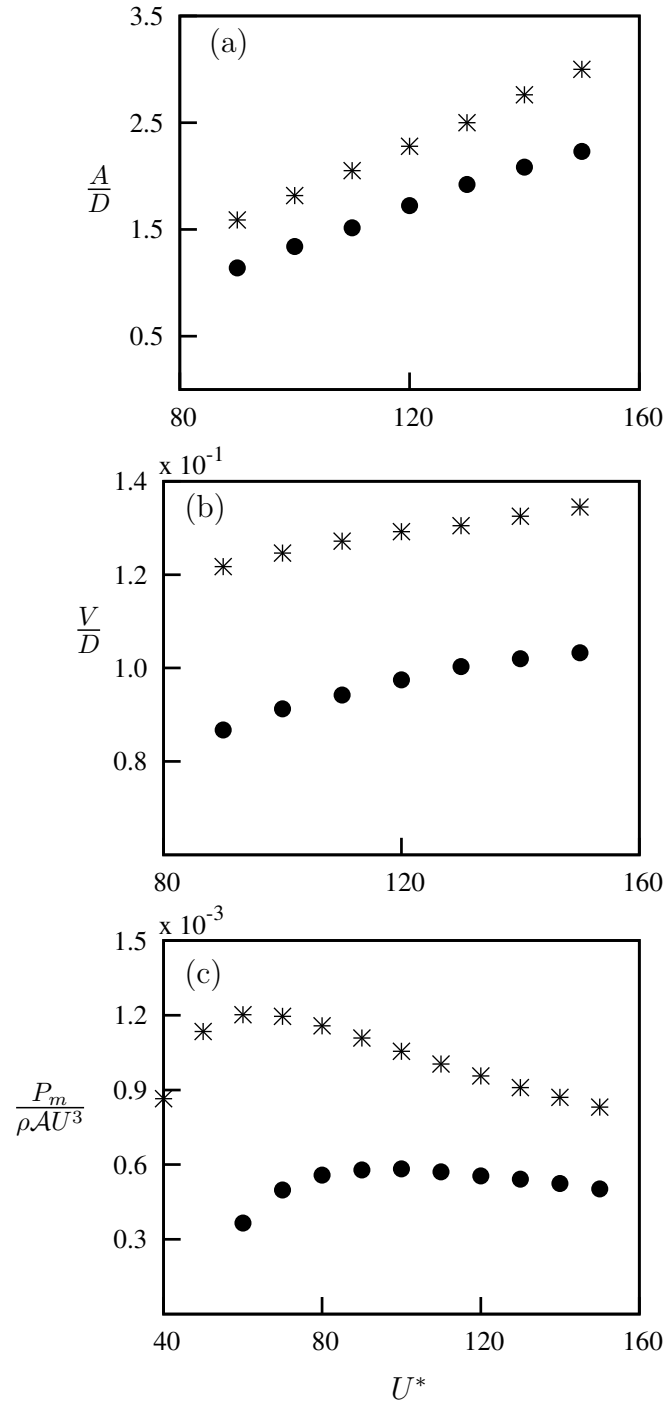


Figure 9: Comparison of QSS (\*) and FSI (●) data of displacement amplitude, velocity amplitude and mean power as a function of  $U^*$  represented by (a), (b) and (c) respectively. Data were obtained at  $Re=165$  and  $\zeta = 0.075$ . An average error of 34% could be observed for both displacement and velocity amplitude. Essential physics i.e the rise and fall of mean power could be captured of the FSI data

## References

- Barrero-Gil, A., Alonso, G., Sanz-Andres, A., Jul. 2010. Energy harvesting from transverse galloping. *Journal of Sound and Vibration* 329 (14), 2873–2883.
- Barrero-Gil, a., Sanz-Andrés, A., Roura, M., Oct. 2009. Transverse galloping at low Reynolds numbers. *Journal of Fluids and Structures* 25 (7), 1236–1242.
- Bernitsas, M. M., Raghavan, K., Ben-Simon, Y., Garcia, E. M. H., 2008. VIVACE (Vortex Induced Vibration Aquatic Clean Energy): A new concept in generation of clean and renewable energy from fluid flow. *Journal of Offshore Mechanics and Arctic Engineering* 130 (4), 041101–15.
- Den Hartog, J. P., 1956. *Mechanical Vibrations*. Dover Books on Engineering. Dover Publications.
- Glauert, H., 1919. The rotation of an aerofoil about a fixed axis. Tech. rep., Advisory Committee on Aeronautics R and M 595. HMSO, London.
- Joly, A., Etienne, S., Pelletier, D., Jan. 2012. Galloping of square cylinders in cross-flow at low Reynolds numbers. *Journal of Fluids and Structures* 28, 232–243.
- Lee, J., Bernitsas, M., Nov. 2011. High-damping, high-Reynolds VIV tests for energy harnessing using the VIVACE converter. *Ocean Engineering* 38 (16), 1697–1712.
- Leontini, J. S., Lo Jacono, D., Thompson, M. C., Nov. 2011. A numerical study of an inline oscillating cylinder in a free stream. *Journal of Fluid Mechanics* 688, 551–568.
- Leontini, J. S., Th, M. C., Hourigan, K., Apr. 2007. Three-dimensional transition in the wake of a transversely oscillating cylinder. *Journal of Fluid Mechanics* 577, 79.
- Ng, Y., Luo, S., Chew, Y., Jan. 2005. On using high-order polynomial curve fits in the quasi-steady theory for square-cylinder galloping. *Journal of Fluids and Structures* 20 (1), 141–146.  
URL <http://linkinghub.elsevier.com/retrieve/pii/S0889974604001215>
- Païdoussis, M., Price, S., de Langre, E., 2010. *Fluid-Structure Interactions : Cross-Flow-Induced Instabilities*. Cambridge University Press.
- Parkinson, G. V., Smith, J. D., 1964. The square prism as an aeroelastic non-linear oscillator. *The Quarterly Journal of Mechanics and Applied Mathematics* 17 (2), 225–239.
- Raghavan, K., Bernitsas, M., Apr. 2011. Experimental investigation of Reynolds number effect on vortex induced vibration of rigid circular cylinder on elastic supports. *Ocean Engineering* 38 (5-6), 719–731.

- Raghavan, K., Bernitsas, M. M., Maroulis, D. E., 2009. Effect of Bottom Boundary on VIV for Energy Harnessing at  $8 \times 10^3 < Re < 1.5 \times 10^5$ . *Journal of Offshore Mechanics and Arctic Engineering* 131 (3), 031102.
- Sheard, G. J., Fitzgerald, M. J., Ryan, K., Jun. 2009. Cylinders with square cross-section: wake instabilities with incidence angle variation. *Journal of Fluid Mechanics* 630, 43.
- Thompson, M., Hourigan, K., Sheridan, J., Feb. 1996. Three-dimensional instabilities in the wake of a circular cylinder. *Experimental Thermal and Fluid Science* 12 (2), 190–196.  
URL <http://linkinghub.elsevier.com/retrieve/pii/0894177795000984>
- Thompson, M. C., Hourigan, K., Cheung, A., Leweke, T., Nov. 2006. Hydrodynamics of a particle impact on a wall. *Applied Mathematical Modelling* 30 (11), 1356–1369.  
URL <http://linkinghub.elsevier.com/retrieve/pii/S0307904X06000321>
- Tong, X., Luo, S., Khoo, B., Oct. 2008. Transition phenomena in the wake of an inclined square cylinder. *Journal of Fluids and Structures* 24 (7), 994–1005.  
URL <http://linkinghub.elsevier.com/retrieve/pii/S088997460800025X>