
A NUMERICAL INVESTIGATION OF THE ENERGY
TRANSFER OF A BODY UNDER FLUIDELASTIC
GALLOPING

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STATEMENT OF ORIGINALITY

This thesis contains no material that has been accepted for the award of a degree or diploma in this, or any other, university. To the best of the candidates knowledge and belief, this thesis contains no material previously published or written by another person except where due reference is made in the text.

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ABSTRACT

The potential of energy harvesting through fluid-elastic galloping is explored through studying the energy transfer between the body and the fluid. This study identified a need for new scaling parameters to better represent fluid-elastic galloping as the parameters used currently (i.e. the traditional Vortex Induced Vibration parameters) were not providing a satisfactory collapse. After reviewing earlier works, this study proposed and tested a hypothesis which asserts that delaying the shear layer reattachment would lead to a higher power output. To meet the identified requirements, this study was divided into two main phases. Phase one aims to study the underpinning mechanical parameters while phase two was to understand the fluid mechanics of the system and attempt to control the fluid flow to gain a higher power output.

This fundamental study is carried out using theoretical modelling and numerical simulations. The Quasi-Steady State (QSS) model and Direct Numerical Simulations (DNS) on both stationary and oscillatory cases are carried out to obtain the data.

Phase 1 was initiated by formulating new governing non-dimensional parameters for galloping namely, the combined mass-stiffness, Π_1 and the combined mass-damping Π_2 . These parameters were formulated using the natural time scales of the linearised QSS equation. The formulated dimensionless groups provided a good collapse for the predicted power output in comparison with the classical VIV parameters which have been traditionally used, i.e. U^* , m^* , and ζ , reinforcing the statement of Paidoussis et al. (2010) that galloping is a “velocity dependent damping controlled” system. A comparison between the quasi-steady state and direct numerical simulation data, revealed that the quasi-steady state model provides a good approximation of the power output at high Π_1 . However, the QSS approximation deviates from the DNS predictions at low values of Π_1 because the QSS model does not model vortex shedding which becomes more significant as Π_1 decreases. Be

that as it may, the QSS model does provide a reasonable prediction of the value of Π_2 at which maximum power is produced. Both the error in predicted maximum power between the QSS and the DNS models and the relative power of the vortex shedding have been quantified and scale approximately to $1/\sqrt{\Pi_1}$.

To completely describe the system in terms of Π_1 and Π_2 , a frequency study was carried out. An expression for the frequency based on Π_1 and Π_2 was formulated. This expression was formulated using the eigenvalues of the linearised QSS model and hence was termed the linear frequency, f_{lin} . Two regions of frequency response were identified namely, the region where a linear frequency is predicted and the region where f_{lin} does not exist. Both frequency data obtained using QSS model and DNS agreed well with f_{lin} within the boundaries of the DNS simulations conducted, where lower boundary of Π_1 was limited to $\Pi_1 = 10$ because the galloping signal got weaker when $\Pi_1 < 10$.

QSS frequency was scaled with the undamped natural frequency of the system f , in the region where a f_{lin} could not be defined. This revealed that it was within $0.55 \leq \frac{f_{QSS}}{f} \leq 0.75$ in the range of $0.06 \leq \Pi_1 \leq 0.1$ and dropped further as Π_1 reduces.

The mere existence of this region is questionable as no DNS data could be obtained in this region due to the fact that galloping signal was weak and the techniques used to obtain the frequency was not sensitive enough to capture the weak signals. There is scope for further study to corroborate or otherwise the QSS prediction using experiments or DNS.

Be that as it may, The linear expression provided a excellent prediction within the boundaries of data obtained through DNS and therefore confirming that frequency predicted by linearised oscillator model expressed in Π_1 and Π_2 is accurate.

The second phase of this study initiated by testing the hypothesis of gaining higher power output by delaying the flow re-attachment. In order to investigate test this, a square cross section was systematically tapered off from the top and bottom of the cross section.

A negative region of the C_y vs. θ curve beyond $\frac{d}{l} \leq 0.25$ could be observed in this study. Therefore, as a consequence, there is a loss of power in a certain portion of the galloping cycle which is a result of the velocity and the transverse forcing F_y being out of phase.

The maximum mean power increases as $\frac{d}{l}$ is decreased until $\frac{d}{l} = 0.25$. However, further analysis revealed that the maximum power at $\frac{d}{l} = 0.25$ was greater than $\frac{d}{l} = 0$ which was found out to be a direct result of the size of the negative region of the C_y vs. θ curve.

Further investigation of the surface pressure data and the velocity magnitude data revealed that the initial negative region was created as a result of the uneven flow distribution due to the profile and the positioning of the geometry, which generated C_y similar to the generation of lift of an aerofoil. As the incidence angle was further increased this mechanism was suppressed by the force created due to the relative proximity of shear layers to the wall, which is typically associated with the positive region of the C_y vs. θ curve.

Comparison of QSS maximum power data and FSI data provided similar trends of maximum power being increased as $\frac{d}{l}$ was decreased proving the initial hypothesis. However, the difference between the QSS and FSI maximum power data increased exponentially as $\frac{d}{l}$ reduced. Investigations carried out using time averaged flow-filed data concluded that the mean flow of the FSI simulations had a significant deviation with the corresponding stationary DNS data. This was a result of the incurred higher traverse velocities as $\frac{d}{l}$ was decreased. As a result significant non-linear forcing was present, which resulted in a deviation from the quasi-steady assumption. Be that as it may, as concluded in phase one of this study, the QSS model could be used as a tool to obtain initial qualitative approximations to design galloping energy harvesting systems.

Obtaining a good balance between the negative and the positive regions of the of the C_y vs. θ curve is a key design consideration in obtaining an optimum cross section for energy harvesting purposes. Although delaying the shear layer reattachment has it's advantages, the negative region of the C_y vs. θ curve has an adverse effect on power transfer.

One of the areas for research in future is investigating ways to reduce the negative region of the C_y curve by making alterations to the geometry. Changing the proximity of the shear layers by rounding the edges of the separation points could be one possible approach which can be investigated as future research.

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DEDICATION

“ To Mrs. Malin Bamunuarachchi; whom without, this work would never have seen the light of day. Thank you madam; for your prayers, blessings, guidance, kind words of encourgaement and above all, believeing in me and giving me strength to get back up, when I myslef have given up hope....”

A LIST OF PUBLICATIONS RELATED TO THIS THESIS

JAYATUNGA, H. G. K. G., TAN, B.T & LEONTINI, J.S. 2015 A study on the energy transfer of a square prism under fluid-elastic galloping *Journal of Fluids and Structures* **55**,384–397.

LEONTINI, J.S., JISHENG, Z., JAYATUNGA, H. G. K. G., LO JANCONO, D., TAN B. T., & SHERIDAN, J. 2014 Frequency selection and phase locking during aeroelastic galloping *19th Australasian Fluid Mechanics Conference, Melbourne, Australia*

NOMENCLATURE

Symbol	Description
a_1, a_3, a_5, a_7	Coefficients of the polynomial to determine C_y
A	Displacement amplitude
c	Damping constant
D	Characteristic length (side length) of the cross section of the body
El	Subscript denoting integration over a single element
$f = \sqrt{k/m}/2\pi$	Natural frequency of the system
f_g	Frequency of galloping
f_s	Frequency of vortex shedding
f_{QSS}	Frequency predicted by the QSS model
f_{lin}	Linear frequency
f_{DNS}	Frequency predicted by DNS simulations
F_y	Instantaneous force normal to the flow
F_0	Amplitude of the oscillatory force due to vortex shedding
\mathcal{F}	Fourier transform of velocity
g	Index of the data points inside each element in the ξ -direction
h	Variable indicative of resolution of macro-element mesh
i	Index of the data point being considered during construction of the Lagrange polynomial in the ξ -direction
\mathbf{J}	Jacobian operator for coordinate transformation
j	Data point index in computational space in η -direction

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Symbol	Description
k	Spring constant
m	Mass of the body
m_a	Added mass
n	Timestep count to the current timestep
\mathbf{n}	Unit vector in the normal direction to a boundary
P_d	Power dissipated due to mechanical damping
$P_{in} = \rho U^3 D/2$	Energy flux of the approaching flow
P_m	Dimensionless mean power
P_t	Power transferred to the body by the fluid
P_s	Surface pressure
P_{trial}	Trial solution for pressure
q	Data point index in computational space in ξ -direction
\mathbf{R}	Residual formed when substituting trial solution into governing equations
s	Data point index in computational space in η -direction
t	Time
U	Freestream velocity
U_i	Induced velocity
V_m	velocity magnitude of the flow
\mathbf{V}	Non-dimensional velocity vector, \mathbf{u}/U
\mathbf{V}_{trial}	Trial solution for velocity
\mathbf{V}^*	Intermediate normalised velocity vector at the end of the advection sub-step
\mathbf{V}^{**}	Intermediate normalised velocity vector at the end of the pressure sub-step
\mathbf{V}_{cyl}	Transverse velocity of the cylinder, \dot{y}/U
$\mathbf{V}_{cyl}^{(n+1)\dagger}$	First approximation of \mathbf{V}_{cyl} at the end of the timestep during the elastically-mounted cylinder convection substep

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Symbol	Description
$\mathbf{V}_{cyl}^{(n+1)\ddagger}$	Second approximation of \mathbf{V}_{cyl} at the end of the timestep during the elastically-mounted cylinder convection substep
$\mathbf{V}_{cyl}^{(n+1)'} $	Approximation of \mathbf{V}_{cyl} at the end of the timestep after relaxation during the elastically-mounted cylinder convection substep
$\mathbf{V}^{(n)}$	Normalised velocity vector at timestep n
$\mathbf{V}^{(n+1)}$	Normalised velocity vector at timestep $n + 1$
$\widehat{\mathbf{V}}^*$	Vector of \mathbf{V}^* at the node points
v	Normalised component of velocity in the y -direction
\mathbf{v}_{cyl}	Instantaneous transverse cylinder velocity
x	Cartesian coordinate in the freestream flow direction, positive downstream
y	Cartesian coordinate transverse to the flow direction and span direction
y_{cyl}	Transverse cylinder displacement
$y_{cyl}^{(n+1)\dagger}$	A first approximation to y_{cyl} at the end of the timestep during the elastically-mounted cylinder convection substep
y, \dot{y}, \ddot{y}	Transverse displacement, velocity and acceleration of the body/cylinder
$\Delta \mathbf{V}_{cyl}$	Change in \mathbf{V}_{cyl} over one timestep
$\Delta \mathbf{V}_{cyl}^\dagger$	First approximation of change in \mathbf{V}_{cyl} over one timestep during the elastically-mounted cylinder convection substep
$\Delta \tau$	The non-dimensional timestep
ϵ	Under-relaxation parameter used during the elastically-mounted cylinder convection substep
η	Coordinate axis in computational space
ξ	Coordinate axis in computational space
$\mathcal{A} = DL$	Frontal area of the body

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Symbol	Description
λ	Inverse time scale of a galloping dominated flow
$\lambda_{1,2}$	Eigenvalues of linearised equation of motion
ρ	Fluid density
$\omega_n = 2\pi f$	Natural angular frequency of the system
ω_s	Vortex shedding angular frequency
$c^* = cD/mU$	Non-dimensionalised damping factor
$C_y = F_y/0.5\rho U^2 DL$	Normal (lift) force coefficient
$m^* = m/\rho D^2 L$	Mass ratio
Re	Reynolds number
$U^* = U/fD$	Reduced velocity
$Y = y/D$	Non-dimensional transverse displacement
$\dot{Y} = m^* \dot{y}/a_1 U$	Non-dimensional transverse velocity
$\ddot{Y} = m^{*2} D \ddot{y}/a_1^2 U^2$	Non-dimensional transverse acceleration
$\Gamma_1 = 4\pi^2 m^{*2}/U^{*2} a_1^2$	First dimensionless group arising from linearised, Non-dimensionalised equation of motion
$\Gamma_2 = c^* m^*/a_1$	Second dimensionless group arising from linearised, Non-dimensionalised equation of motion
$\zeta = c/2m\omega_n$	Damping ratio
$\theta = \tan^{-1}(\dot{y}/U)$	Instantaneous angle of incidence (angle of attack)
$\Pi_1 = 4\pi^2 m^{*2}/U^{*2}$	Combined mass-stiffness parameter
$\Pi_2 = c^* m^*$	Combined mass-damping parameter

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CHAPTER 1

INTRODUCTION

The review of published literature reveals that fluid-elastic galloping has a potential to be used as a mechanism for energy extraction. Thus, the following questions emerged. What are the optimum parameters for energy transfer in a galloping system? How do they influence galloping?

Over the years, VIV has been the popular research problem studied on flow induced vibrations. As a result, the parameters used to describe VIV problems (i.e m^* , ζ and U^*) has been incorporated to describe galloping, which could be observed throughout the current literature (Barrero-Gil et al., 2009, 2010; Parkinson and Smith, 1964).

However, data presented using this classical VIV parameters mean harvested power in particular (Barrero-Gil et al., 2010), does not provide a good collapse.

Therefore it is hypothesised that more suitable parameters which could provide a better collapse of power output data could be obtained from the relevant time scales of galloping.

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