Issue #15
Entanglement Witness Using Classical Shadow

Kwangjun Choi

Gyungmin Cho

Eungi Min

Qiskit

Motivation of Classical Shadow

$$\langle E \rangle = \langle \psi(\theta_{opt}) | H | \psi(\theta_{opt}) \rangle$$

Motivation of Classical Shadow

Destructive nature of measurement in QM

-> an original state is unknown to us

To calculate the expectation value of a certain observable, we should reconstruct the original state.

Definition

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"Shadow tomography is the task of predicting properties

without fully characterizing the quantum state" [2]

Procedure

- Pick a unitary from the ensemble, and rotate the state

$$(\rho \mapsto U\rho U^{\dagger})$$

- Measure by a computational basis

$$n\text{-bit measurement outcome } |\hat{b}\rangle \in \{0,1\}^n,$$

Procedure

* For the states by the computational measurement, re-rotate them and get an expectation.

* We can consider M as an ‘operator’ (can be inversed)

$$\mathbb{E}\left[U^{\dagger}\left|\hat{b}\right\rangle\!\left\langle\hat{b}\right|U\right]=\mathcal{M}(\rho)\;\Rightarrow\;\rho=\mathbb{E}\left[\mathcal{M}^{-1}\left(U^{\dagger}\left|\hat{b}\right\rangle\!\left\langle\hat{b}\right|U\right)\right]$$

Procedure

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For the state re-rotated, if we apply the inverse-M, we can get a shadow of the original state.

$$S(\rho; N) = \left\{ \hat{\rho}_1 = \mathcal{M}^{-1}\left(U_1^\dagger \middle| \hat{b}_1 \middle\rangle\middle\langle \hat{b}_1 \middle| U_1 \right), \ldots , \hat{\rho}_N = \mathcal{M}^{-1}\left(U_N^\dagger \middle| \hat{b}_N \middle\rangle\middle\langle \hat{b}_N \middle| U_N \right) \right\}$$

Procedure

Algorithm 1. Median of means prediction based on a classical shadow S(ρ, N)

1 **function** LINEARPREDICTIONS(O_1, ..., O_M, S(ρ; N), K)
2 Import $\mathrm{S}(\rho; N) = \left[\hat{\rho}_1, \ldots, \hat{\rho}_N\right]$ ▻ Load classical shadow
3 Split the shadow into K equally-sized parts and set ▻ Construct K estimators of ρ
$\hat{\rho}_{(k)} = \frac{1}{\lfloor N/K \rfloor} \sum_{l=(k-1)\lfloor N/K \rfloor+1}^{k\lfloor N/K \rfloor} \hat{\rho}_l$
4 **for** $i = 1$ to M **do**
5 Output $\hat{o}_i(N, K) = \mathrm{median}\left\{\mathrm{tr}\left(O_i\hat{\rho}_{(1)}\right), \ldots, \mathrm{tr}\left(O_i\hat{\rho}_{(K)}\right)\right\}$. ▻ Median of means estimation

Random n-qubit Clifford Circuits (Clifford Measurements)

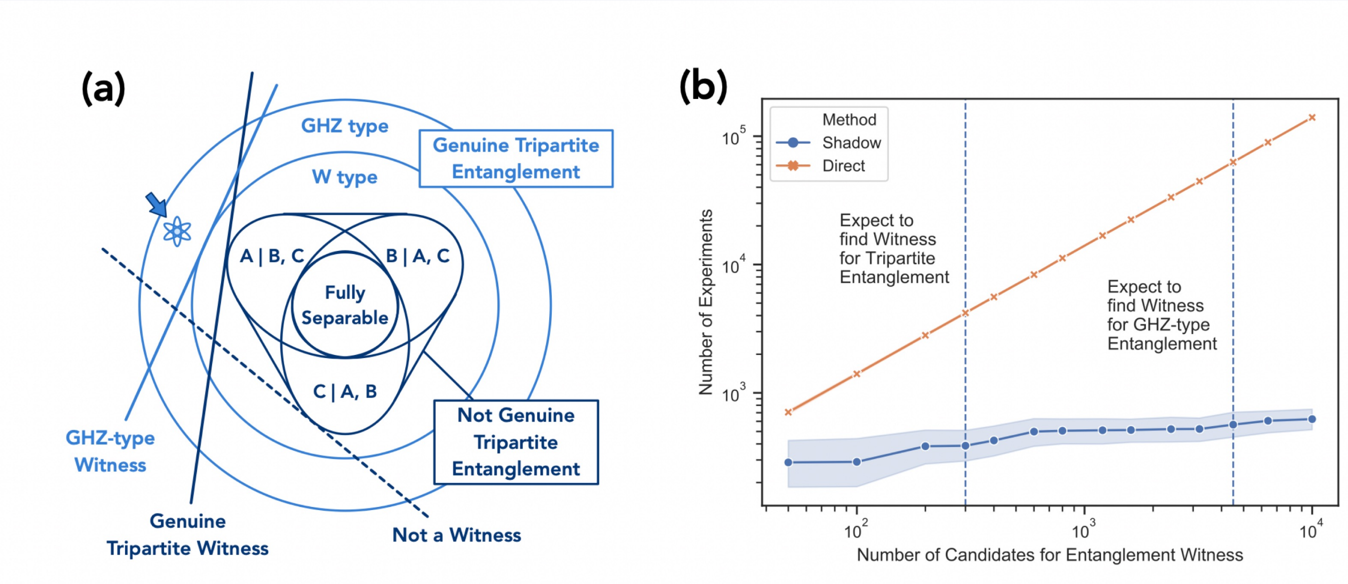
The Clifford group, generated by CNOT, H, S [2]

$$|\mathcal{C}_n| = 2^{n^2+2n} \prod_{j=1}^{n} (4^j - 1).$$

$$\hat{\rho} = (2^n + 1) U^\dagger |\hat{b}\rangle\langle\hat{b}| U - \mathbb{I}$$

`qiskit.quantum_info.random_clifford`

Detecting Genuine Entanglement



Detecting Genuine Entanglement

$$|\psi\rangle = U_A \otimes U_B \otimes U_C |\psi^+_{\mathrm{GHZ}}\rangle \quad \text{where } U_A, U_B, U_C \text{ are random single-qubit rotations.}$$

$$O := O(V_A, V_B, V_C) = V_A \otimes V_B \otimes V_C |\psi^+_{\mathrm{GHZ}}\rangle\langle\psi^+_{\mathrm{GHZ}}| V_A^\dagger \otimes V_B^\dagger \otimes V_C^\dagger.$$

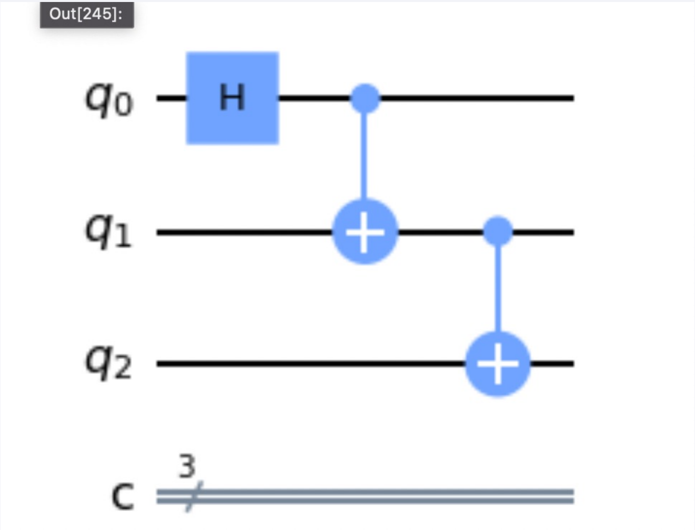
Detecting Genuine Entanglement

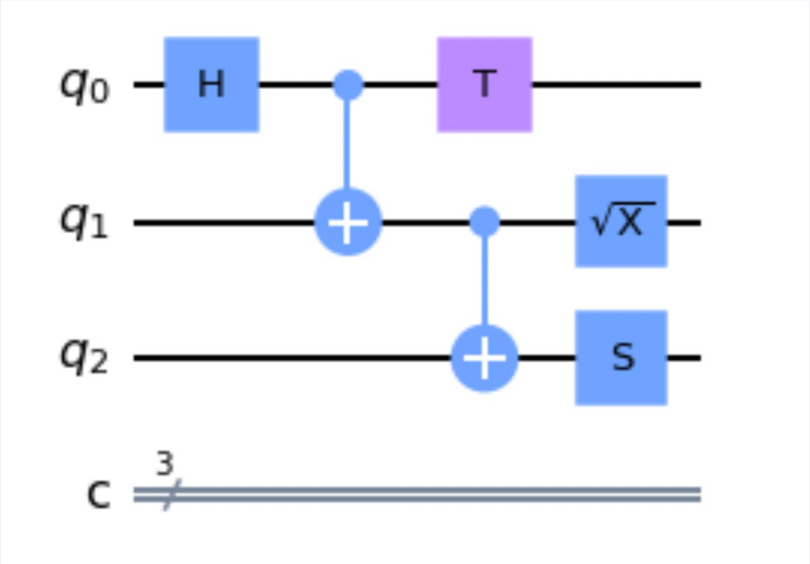
any state ρ_s with only bipartite entanglement, $\mathrm{tr}\,(O\rho_s) \leq .5$,

for any state ρ_s with at most W-type

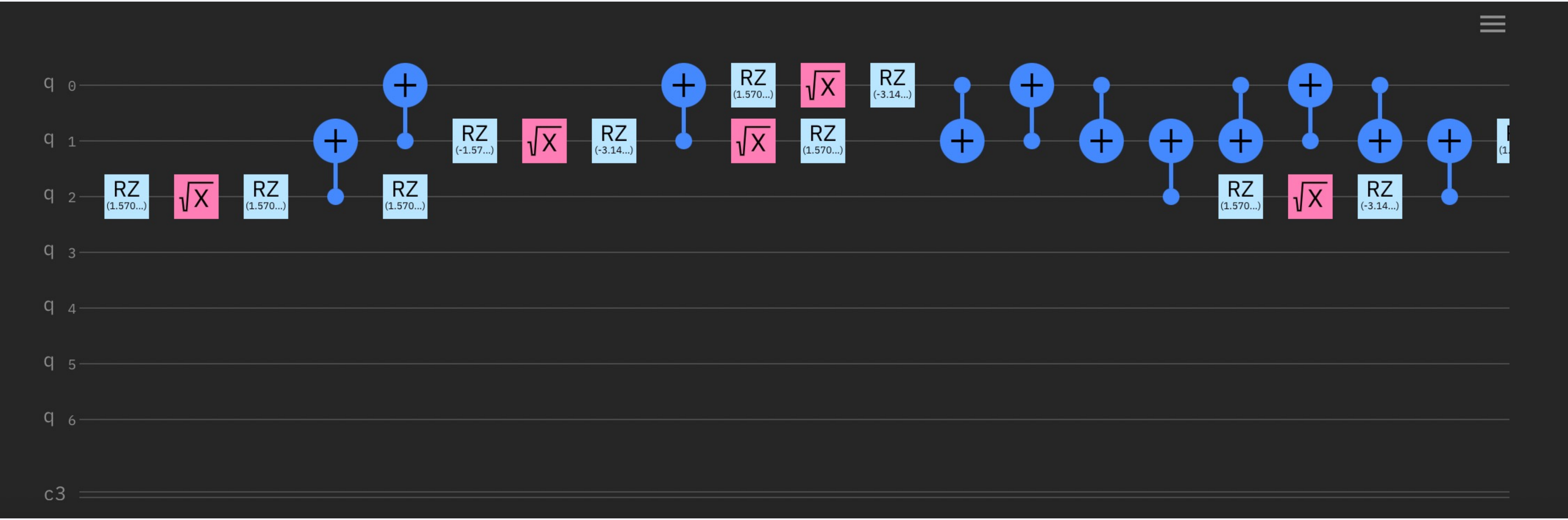
$\mathrm{tr}\,(O\rho) > .75$ certifies that ρ has GHZ-type entanglement.

State Preparation





Transpiled Cirucit on IBMQJakarta with Random Clifford Measurement



Trace Distance

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We used a Fake backend named as 'FakeJakarta'

	#shadow = 5000	#shadow = 1000
GHZ	0.2635582983193262,	0.3841866197183080З
Rotated GHZ	0.1962142857142854,	0.2459181338028158

Entanglement Witness

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```
ideal_ghz_fidelity_list = []

for w in rw_list_ghz:
    fid = witness_check(w, ghz_arr)
    ideal_ghz_fidelity_list.append(fid)
ideal_ghz_fidelity_array = np.array(ideal_ghz_fidelity_list)
np.where(ideal_ghz_fidelity_array>0.5)
```

```
(array([ 51, 188, 244, 283, 549, 583, 937]),)
```

#witness = 1000

Entanglement Witness

```
ghz_fidelity_list_jak_1000 = []

for w in rw_list_ghz:
    for ghz_shadow in ghz_jak_1000_split_shadow_witness:
        fid = witness_check(w, ghz_shadow)
    med_fid = np.median(fid)
    ghz_fidelity_list_jak_1000.append(med_fid)
ghz_fidelity_jak_1000_array = np.array(ghz_fidelity_list_jak_1000)
np.where(ghz_fidelity_jak_1000_array>0.5)
```

```
(array([], dtype=int64),)
```

#shadow = 1000

Entanglement Witness

```
ghz_fidelity_list_jak = []

for w in rw_list_ghz:
    for ghz_shadow in ghz_jak_split_shadow_witness:
        fid = witness_check(w, ghz_shadow)
    med_fid = np.median(fid)
    ghz_fidelity_list_jak.append(med_fid)
ghz_fidelity_jak_array = np.array(ghz_fidelity_list_jak)
np.where(ghz_fidelity_jak_array>0.5)
```

```
(array([583]),)
```

#shadow = 5000

Entanglement Witness

```
ideal_rot_ghz_fidelity_list = []
for w in rw_list_rot_ghz:
    fid = witness_check(w, rot_ghz_arr)
    ideal_rot_ghz_fidelity_list.append(fid)
ideal_rot_ghz_fidelity_array = np.array(ideal_rot_ghz_fidelity_list)
np.where(ideal_rot_ghz_fidelity_array>0.5)
```

```
(array([4830]),)
```

#witness = 10000

Entanglement Witness

```
rot_ghz_jak_1000_fidelity_list = []

for w in rw_list_rot_ghz:
    for ghz_shadow in rot_ghz_jak_1000_spli
        fid = witness_check(w, ghz_shadow)
    med_fid = np.median(fid)
    rot_ghz_jak_1000_fidelity_list.append(m
rot_ghz_jak_1000_fidelity_array = np.array(
rot_ghz_jak_1000_fidelity_array[rot_ghz_jak
```

```
array([], dtype=complex128)
```

#shadow = 1000

```
rot_ghz_jak_5000_fidelity_list = []

for w in rw_list_rot_ghz:
    for ghz_shadow in rot_ghz_jak_5000_spl
        fid = witness_check(w, ghz_shadow)
    med_fid = np.median(fid)
    rot_ghz_jak_5000_fidelity_list.append(
rot_ghz_jak_5000_fidelity_array = np.array
rot_ghz_jak_5000_fidelity_array[rot_ghz_ja
```

```
array([], dtype=complex128)
```

#shadow = 5000

More Qubits?

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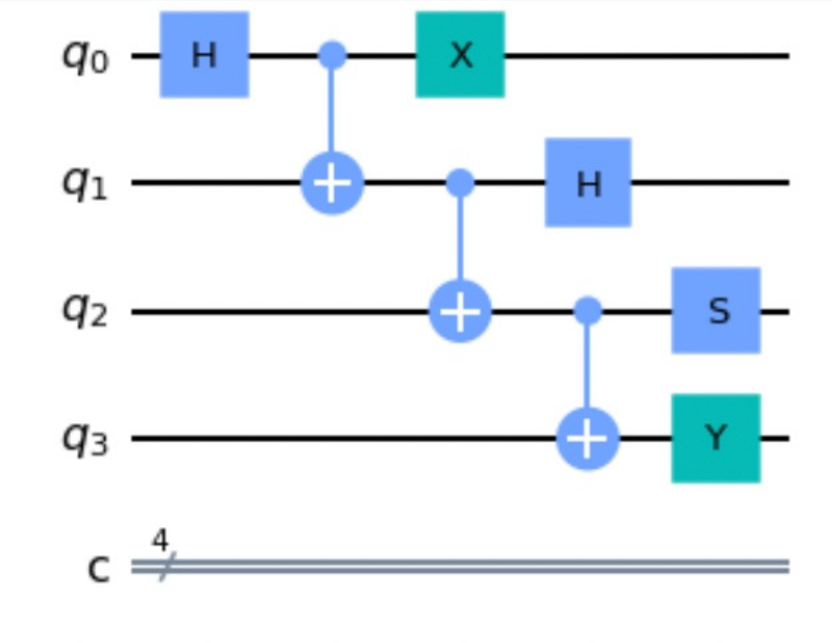

0.5305592750000007

#shadow = 5000

Future Direction

1. To apply this method on the efficient way to measure fidelity such as multiple quantum coherence for GHZ state.

However, using Clifford measurements is required to construct many entangling gates.

2. Some condition observable-based (Bipartite entanglement or non-locality and based on randon pauli measurement)

- More optimal p3-PPT method
- Mermin inequality (for non-locality)

Thanks for Listening!