Mandatory 2

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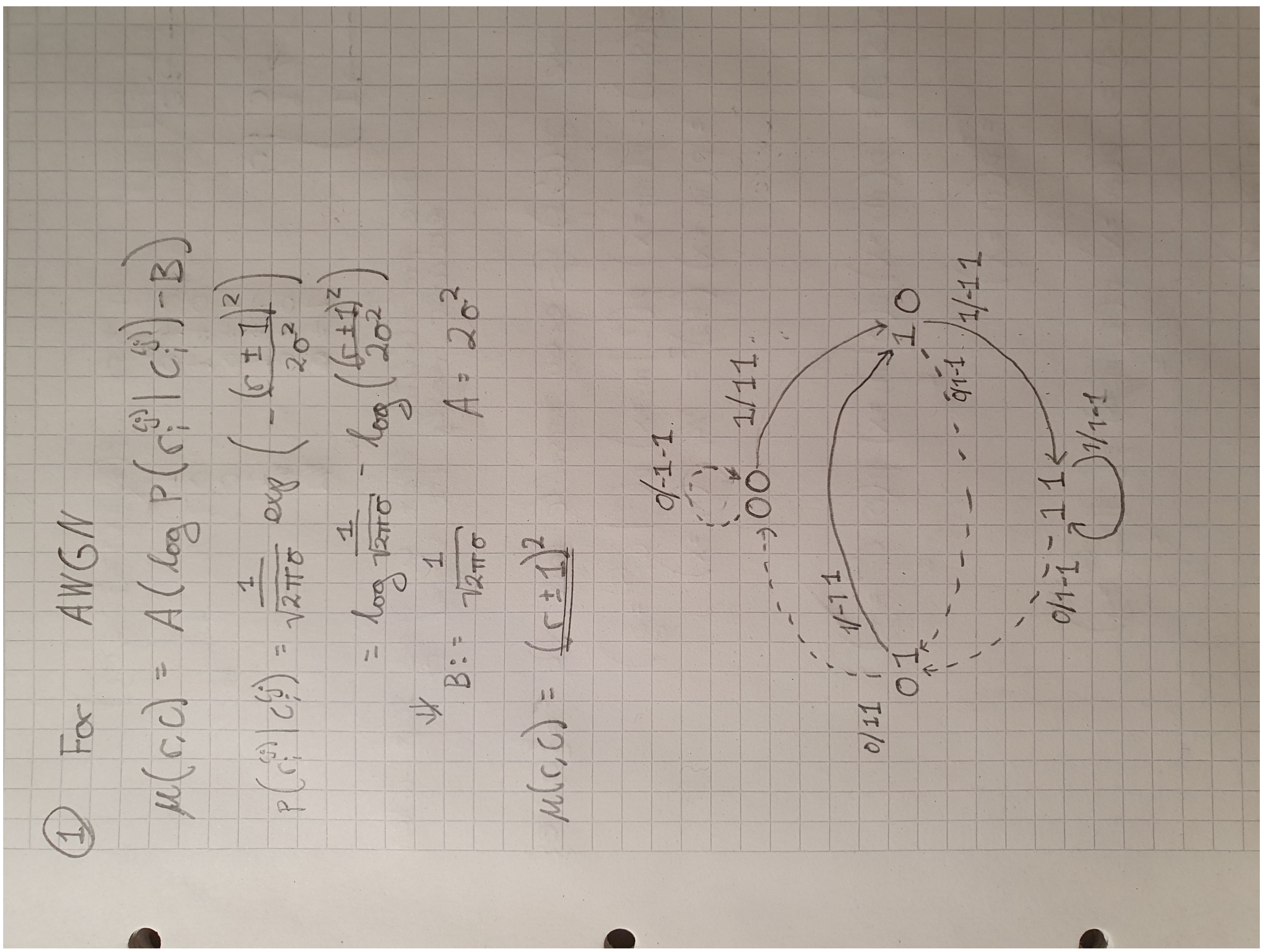
Remark: Some of the picture rotated after downloading the latex file, I will include all the picture in the zip folder.

Exercise 1

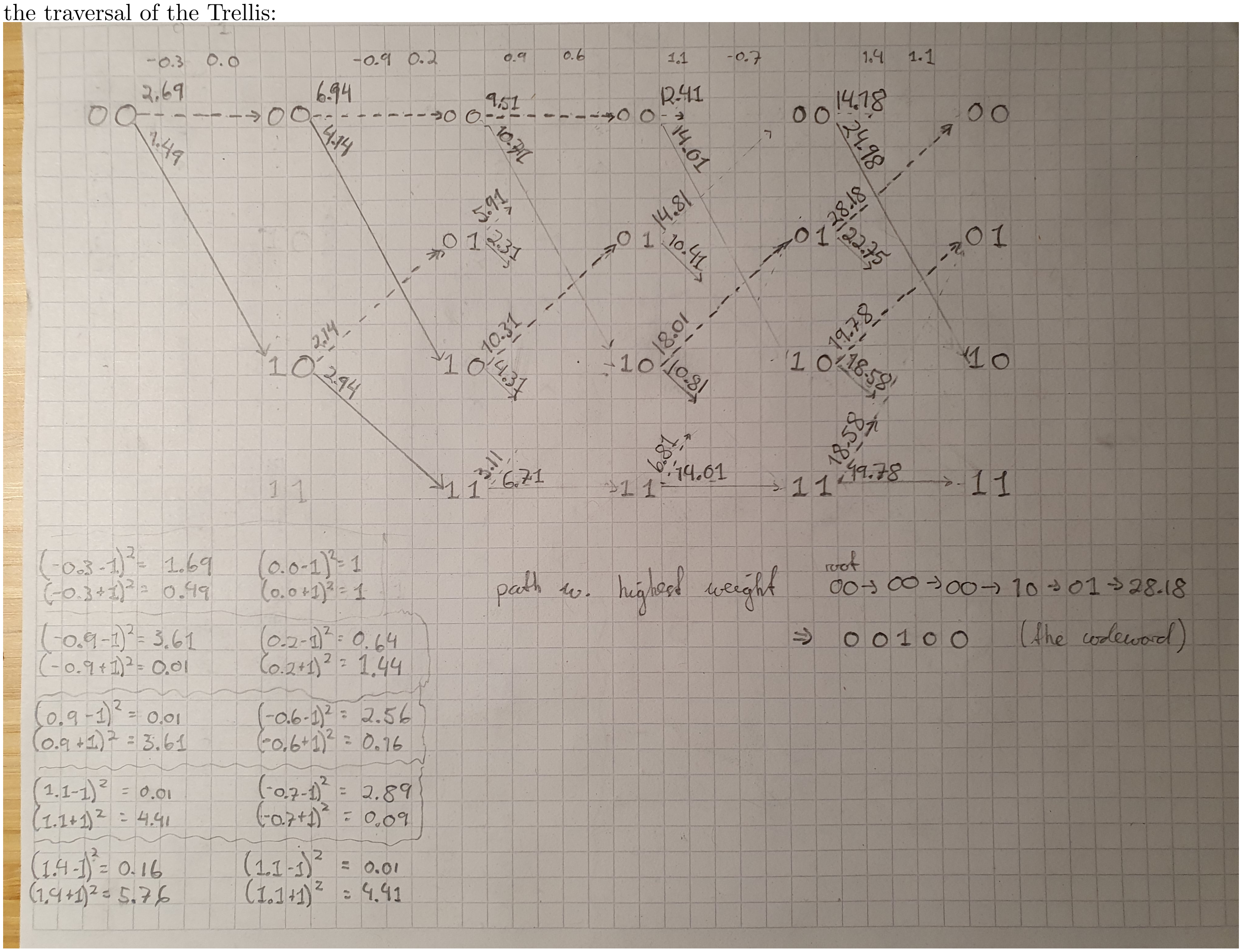
Below is a picture of $\gamma(r,c) = A(logP(r_i^{(j)}|c_i^{(j)}) - B)$ and the state diagram of $G(x)$ when its output 0 is mapped to -1, and 1 is mapped to 1.
When, in the Trellis, the correctness of going from one state to another is the received signal pluss or



minus the output from the diagram going from one state to another. example at depth 0:
$00 \rightarrow 00 = (-0.43-1)^2 + (0.0-1)^2 = 2.69$

$00 \to 10 = (-0.3+1)^2 + (0.0+1)^2 = 1.49$
when two paths enter the same node/state I kept the path with the highest value, the path most likeley to be correct.

the traversal of the Trellis:

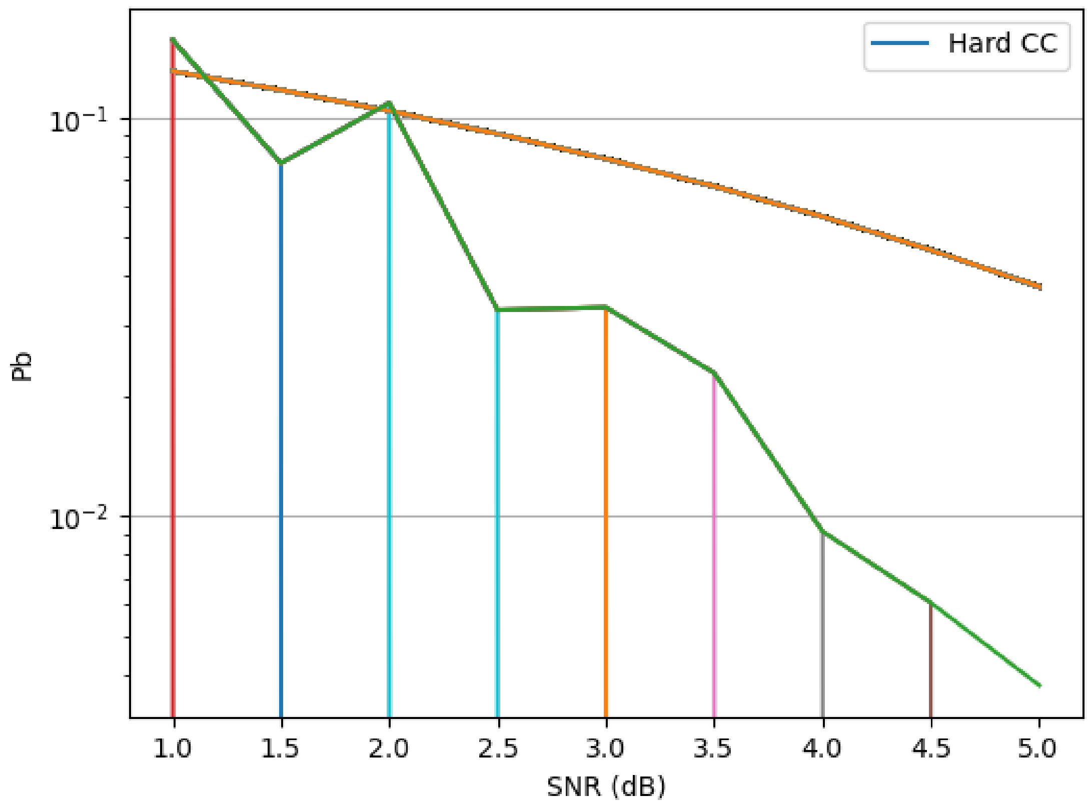


The received signal was decoded to "00100".

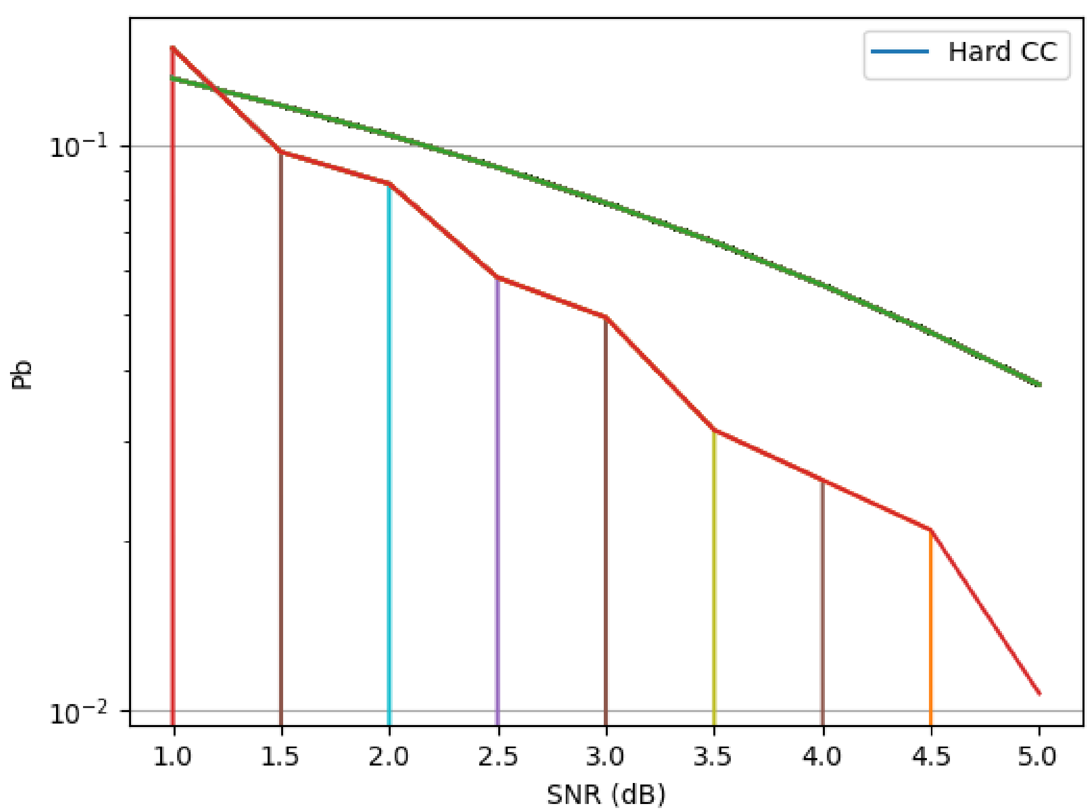
Exercise 2

Picture 1 is Monte Carlo simulation for $G(x) = (1 + x + x^2 1 + x^2)$

Picture 2 is Monte Carlo simulation for $G(x) = (1 + x + x^2 x^2)$



Hard CC
10^{-1}
10^{-2}
Pb
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
SNR (dB)



Hard CC
10^{-1}
10^{-2}
Pb
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
SNR (dB)

It looks like the two codes behave very similarly, but it looks like $G(x) = (1+x+x^2x^2)$ has, overall, better performance since the graph is more steep.

Exercise 3

The sequence after permutation:
m_pi = [1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1]
the output of the Turbo Encoder:
C=[1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0]

Exercise 4

$$\gamma(s',s) = p(r_i^{(0)} \mid m_i = x)\, p(r_i^{(1)} \mid c_i^{(1)} = y)\, P(m_i = x)$$

for <u>BSC</u>

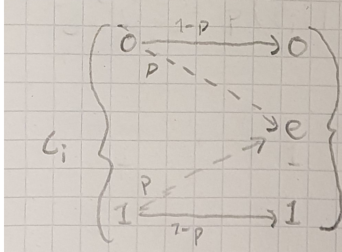
$$\gamma(s',s) = p(m_i = x)\,(1-p)\left(\frac{p}{1-p}\right)^{d_H(r_i^{(0)},x)}\,(1-p)\left(\frac{p}{1-p}\right)^{d_H(r_i^{(1)},y)}$$

for $d_H(a,b) = a \oplus \text{sign}(b)$

where $\text{sign} = \begin{cases} 1 & \text{if } b > 0 \\ 0 & \text{otherwise } (b \leq 0) \end{cases}$

$$\gamma(s',s) = p(m_i = x)\,(1-p)^2 \left(\frac{p}{1-p}\right) \exp\left(d_H(r_i^{(0)},x) + d_H(r_i^{(1)},y) \right)$$

for BEC



$c_i \to r_i$	$p(r_i \mid c_i = \{0,1,e\})$
$0 \to 0$	1
$0 \to 1$	0 } does not happen
$1 \to 0$	0 } does not happen
$1 \to 1$	1
$0 \to e$	1/2
$1 \to e$	1/2

↑ The probability that c_i was sent when receiver reads the signal r_i

Constructing one $\gamma(s', s)$ for every possible case, i.e $r = \{0, 1, e\}$

$$r_i = ? \rightarrow \gamma(s', s) = p(m_i = x)\, p(? \mid m_i = x)\, p(? \mid c_i^{(2)} = g)$$
$$= p(m_i = x)\, (1/2)^2 = 1/4\, p(m_i = x)$$

$$r_i = 0 \rightarrow \gamma(s', s) = p(m_i = x)\, p(0 \mid \underset{\underset{1}{\shortparallel}}{m_i} = x)\, p(0 \mid \underset{\underset{1}{\shortparallel}}{c_i^{(2)}} = g)$$

$$= p(m_i = x)$$

$$r_i = 1 \rightarrow \gamma(s', s) = p(m_i = x)\, p(1 / \underset{\underset{1}{\shortparallel}}{m_i} = x)\, p(0 \mid \underset{\underset{1}{\shortparallel}}{c_i^{(2)}} = g)$$

$$= p(m_i = x)$$