1) Skal vise at

$$\frac{\partial(b^T a)}{\partial a} = b.$$

hvor a og b er kolonnevektorer, og b^T er en radvektor.

$\alpha = b^T a$, hvor α er en skalar.

$$\alpha = \sum_{i=0}^{n-1} b_i a_i$$

$$\frac{\partial(b^T a)}{\partial a} = \frac{\partial \alpha}{\partial a}$$

Deriverer med hensyn på enhver a_k for alle $k = 0, 1, 2, \ldots, n-1$.

$$\frac{\partial \alpha}{\partial a_k} = \frac{\partial}{\partial a_k} \sum_{i=0}^{n-1} b_i a_i = \sum_{i=0}^{n-1} b_i = b$$

Alle ledd i summen blir 0 når $k \neq i$, men når $k = i$ blir leddet b_i.

Skal vise at:

$$\frac{\partial(a^T A a)}{\partial a} = a^T(A + A^T)$$

Vet at $a^T A a = \alpha$ hvor α er en skalar.

Vet også at skalarer er sine egne transponerte.

$$\alpha = \alpha^T = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} a_i A_{ij} a_j$$

Deriverer for en vilkårlig a_k:

$$\frac{\partial \alpha}{\partial a_k} = \frac{\partial}{\partial a_k}\left(\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} a_i A_{ij} a_j\right)$$

Separerer dobbelsummen inn i ledd hvor k inntreffer ved forskjellige i og j.

$$= \frac{\partial}{\partial a_k}\left(\sum_{\substack{i=0\\k\neq i}}^{n-1} \sum_{\substack{j=0\\k\neq j}}^{n-1} a_i A_{ij} a_j + \sum_{\substack{i=0\\k\neq i}}^{n-1} a_i A_{ik} a_k + \sum_{\substack{j=0\\k\neq j}}^{n-1} a_k A_{kj} a_j + a_k^2 A_{kk}\right)$$

$$= 0 + \sum_{\substack{i=0\\k\neq 0}}^{n-1} a_i A_{ik} + \sum_{\substack{j=0\\k\neq j}}^{n-1} A_{kj} a_j + \underbrace{2 a_k A_{kk}}$$

Innlemmer

$$\Rightarrow \sum_{i=0}^{n-1} a_i A_{ik} + \sum_{j=0}^{n-1} A_{kj} a_j$$

$$= a^T A + A a = a^T A + a^T A^T = \underline{\underline{a^T(A + A^T)}}$$

Skal vise at

$$\frac{\partial (x-As)^T(x-As)}{\partial s} = -2(x-As)^T A$$

hvor x og s er vektorer og A er en matrise.

Skriver om:

$$x - As = \sum_{i=0}\sum_{j=0}(x_i - A_{ij}s_i)$$

$$\Rightarrow \frac{\partial}{\partial s}\sum_{i=0}\sum_{j=0}\left((x_j - A_{ji}s_j)(x_i - A_{ij}s_i)\right)$$

$$= \frac{\partial}{\partial s}\sum_{i=0}\sum_{j=0}(x_i x_j - x_j A_{ij}s_i - x_i A_{ji}s_j + A_{ji}s_j A_{ij}s_i)$$

Deriverer for en hver s_K

$\forall K = 0,1,2,\ldots,n-1$

$$\Rightarrow \frac{\partial}{\partial s_K}\sum_{\substack{i=0\\K\neq i}}(x_i x_K - x_K A_{iK}s_i - x_i A_{Ki}s_K + A_{Ki}s_K A_{iK}s_i)$$

$$+ \frac{\partial}{\partial s_K}\sum_{\substack{j=0\\K\neq j}}(x_K x_j - x_j A_{Kj}s_K - x_K A_{jK}s_j + A_{jK}s_j A_{Kj}s_K)$$

$$+ \frac{\partial}{\partial s_K}(x_K^2 - x_K A_{KK}s_K - x_K A_{KK}s_K + A_{KK}^2 s_K^2)$$

$$= \sum_{\substack{i=0\\K\neq i}}(-x_i A_{Ki} + A_{Ki}A_{iK}s_i)$$

$$+ \sum_{\substack{j=0\\K\neq j}}(-x_j A_{Kj} + A_{jK}s_j A_{Kj})$$

$$+ (-2x_K A_{KK} + 2A_{KK}^2 s_K)$$

$$\left.\begin{aligned} &= -\sum_{i=0}(x_i - A_{iK}s_i)A_{Ki} \\ &\quad -\sum_{j=0}(x_j - A_{jK}s_j)A_{Kj} \end{aligned}\right\}$$ i og j blir samme indeks $\Rightarrow j \to i$

$$= -2\sum_{i=}(x_i - A_{iK}s_i)A_{Ki}$$

$$= \underline{-2(x - As)^T A}$$

Videre, den dobbelderiverte

men nå er:

$s \to \beta$, $A \to X$, $x \to y$

$$\frac{\partial^2}{\partial s^2}\left((x-As)^T(x-As)\right) \to \frac{\partial^2}{\partial \beta^2}\left((y-X\beta)^T(y-X\beta)\right)$$

$$\Rightarrow \frac{\partial}{\partial \beta}(-2(y-X\beta)^T X)$$

$$= -2\frac{\partial}{\partial \beta}\left((y^T - \beta^T X^T)X\right)$$

$$= -2\frac{\partial}{\partial \beta}(y^T X - \beta^T X^T X)$$

Deriverer for en hver β_K:

$$-2\frac{\partial}{\partial \beta_K}\sum_{i=0}(y_K X_{iK} - \beta_K X_{Ki}X_{iK})$$

$$= 2\frac{\partial}{\partial \beta_K}\beta_K\sum_{i=0}X_{Ki}X_{iK}$$

$$= \underline{2\,\mathbf{X}^T\mathbf{X}}$$