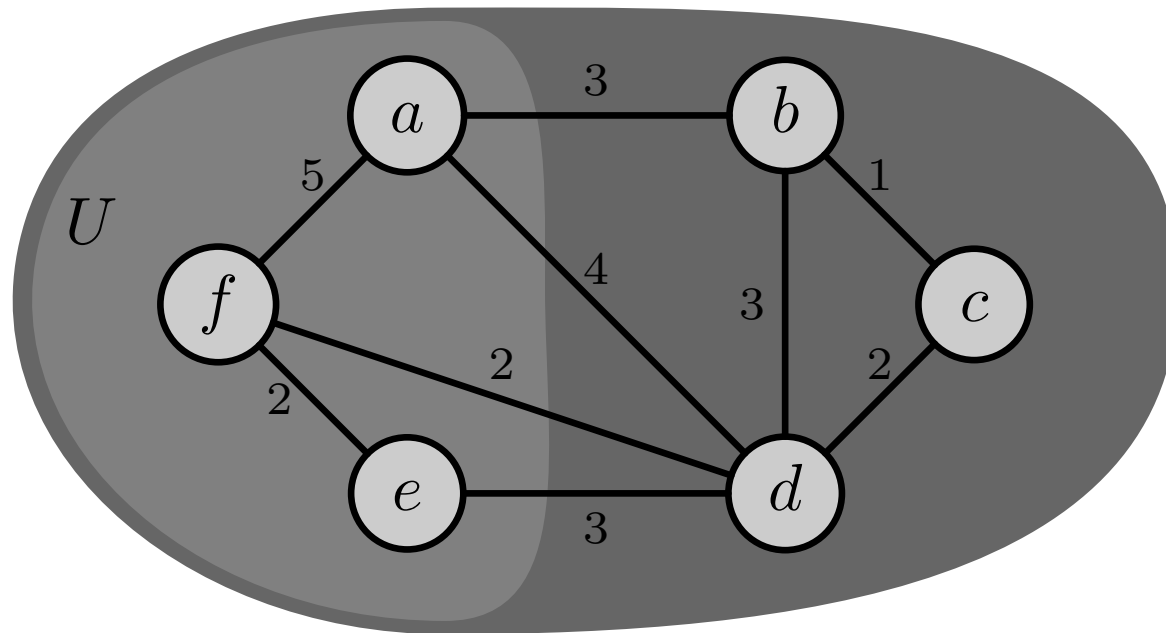


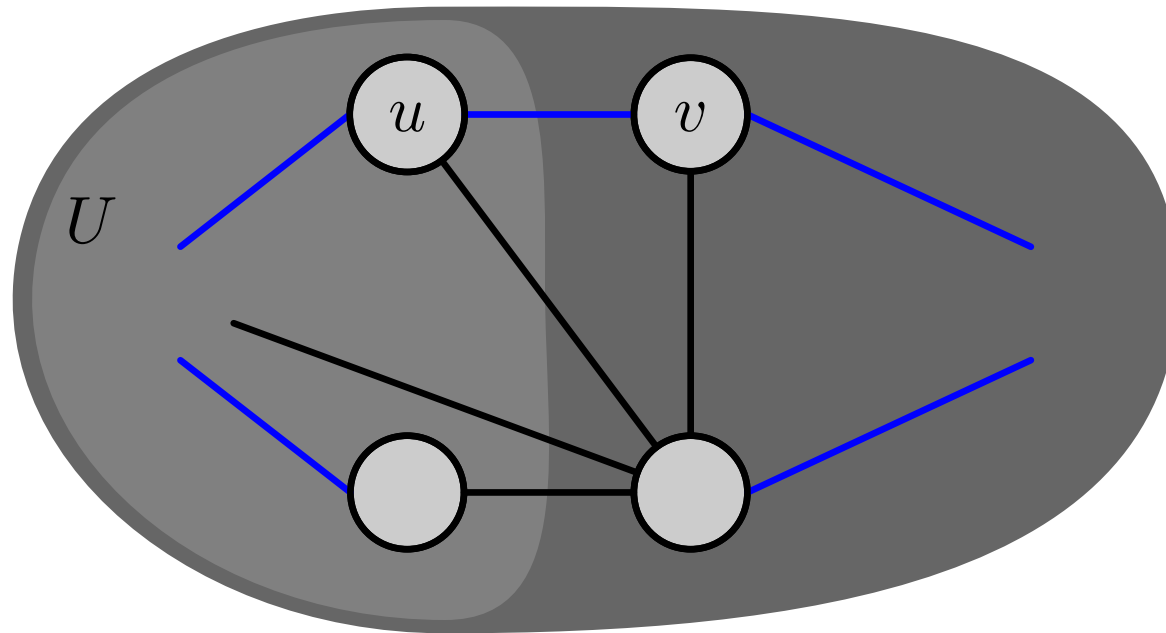
# Why does Prim's algorithm work?

Let  $G = (V, E)$  be a connected graph with weights.

Let  $(U, V - U)$  partition  $V$  into two disjoint sets.



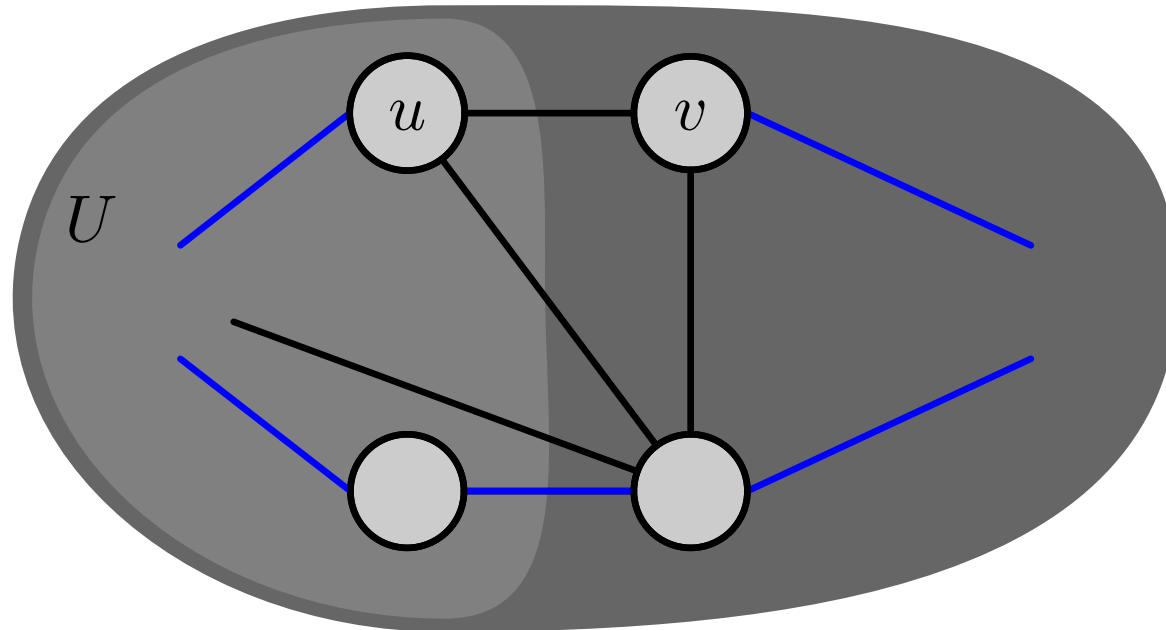
Let  $(u, v)$  be the *cheapest* edge from  $U$  into  $V - U$ .



**Claim:** then  $(u, v)$  *must be* part of the MST for  $G$ .

## Proof (by contradiction)

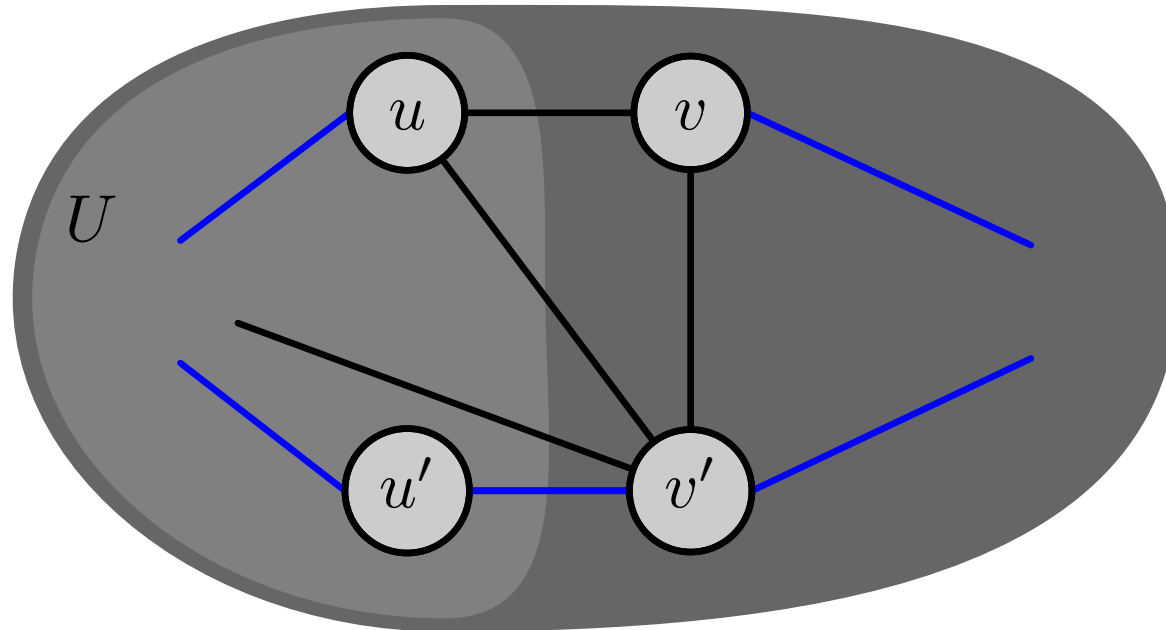
Let  $T$  be a MST that does *not* include  $(u, v)$ .



Adding  $(u, v)$  to  $T$  produces a cycle.

Why? Because  $T$  is a free tree.

There must be another  $(u', v') \in T$  from  $U$  into  $V - U$ .



Why? Otherwise we couldn't get a cycle.

In that case,  $T - (u', v') + (u, v)$  is a cheaper MST.