Traversing graphs

Most basic algorithms on graphs will be applications of graph traversal.

- Printing or validating each edge/vertex.
- Copying a graph or converting between representations.
- Counting the number of edges/vertices.
- Identifying connected components.
- Finding paths between two vertices, or cycles.

Efficiency and correctness

- **Efficiency:** Don't loop or visit vertices repeatedly.
- **Correctness:** Don't miss any vertex.

We need to mark vertices as we traverse the graph.

1. Undiscovered: the initial state, before we've seen it.
2. Discovered: we've seen the vertex but not all of its incident edges.
3. Finished: all incident edges have been visited.

Order of exploration

The order in which we explore vertices depends on the container used for storing *discovered* but not *finished* vertices.

There are two types of containers used:

- **Queue:** leads to so called breadth-first search.
- **Stack:** leads to so called depth-first search.

Breadth-first search

- Properties
- The algorithm
- Example

Breadth-first search properties

- **Input:** A directed/undirected graph with source vertex s.
- **Output:** The shortest distance from s.
- Only visits vertices reachable from s.
- **Running time:** $\Theta(V+E)$

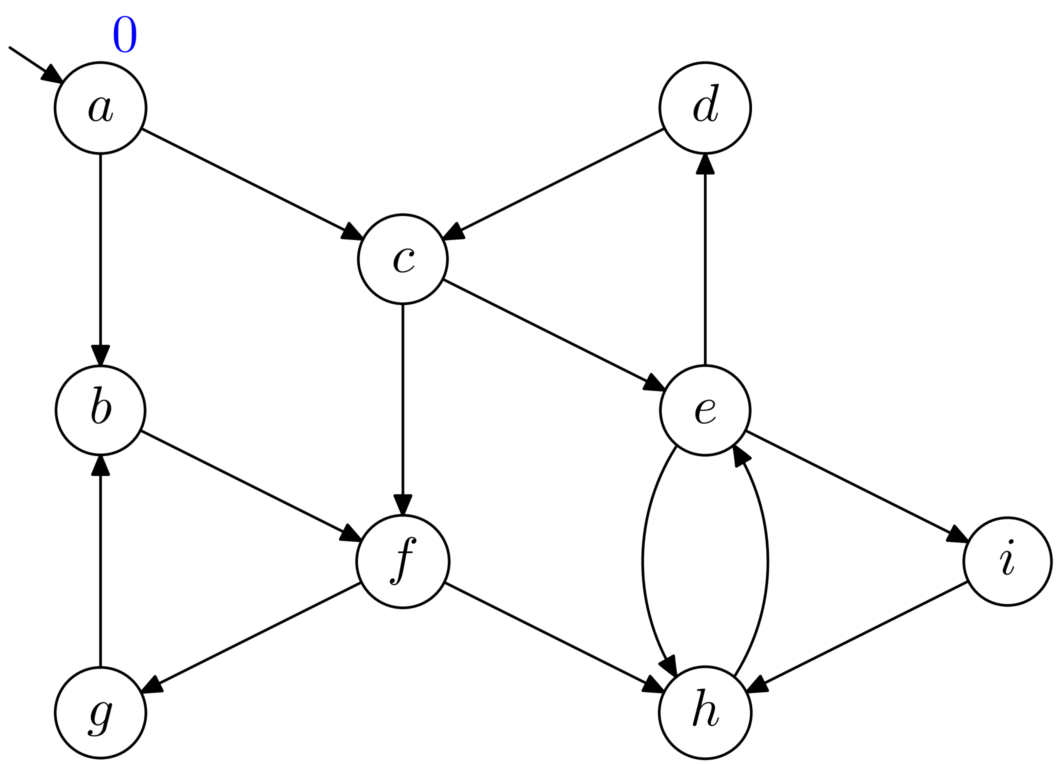
  Linear time with respect to adjacency list.

Breadth-first search algorithm

$\mathrm{BFS}(s)$

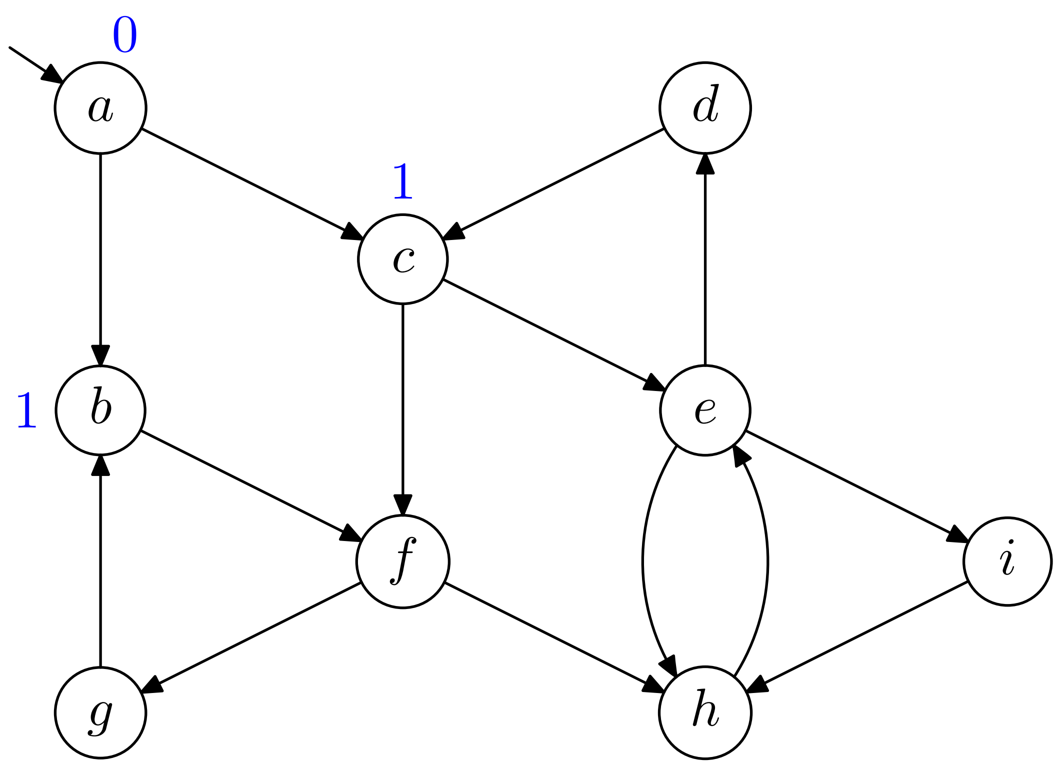
for each $u \in V - \{s\}$
    do $d[u] \leftarrow \infty$
$d[s] \leftarrow 0$
$Q \leftarrow \{s\}$
while $Q \neq \emptyset$
    do remove u from Q
        for each $v \in Adj[u]$
            do if $d[v] = \infty$
                then $d[v] \leftarrow d[u] + 1$
                    put v onto Q

Breadth-first search example



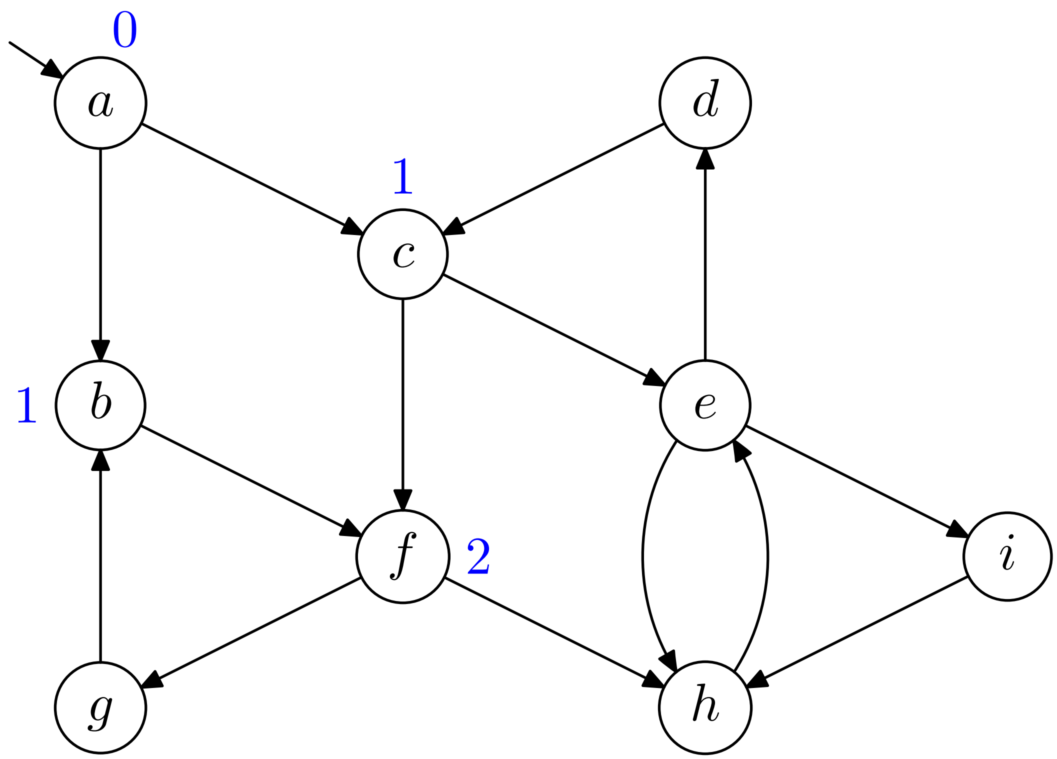
$$Q = \{a\}$$

Breadth-first search example



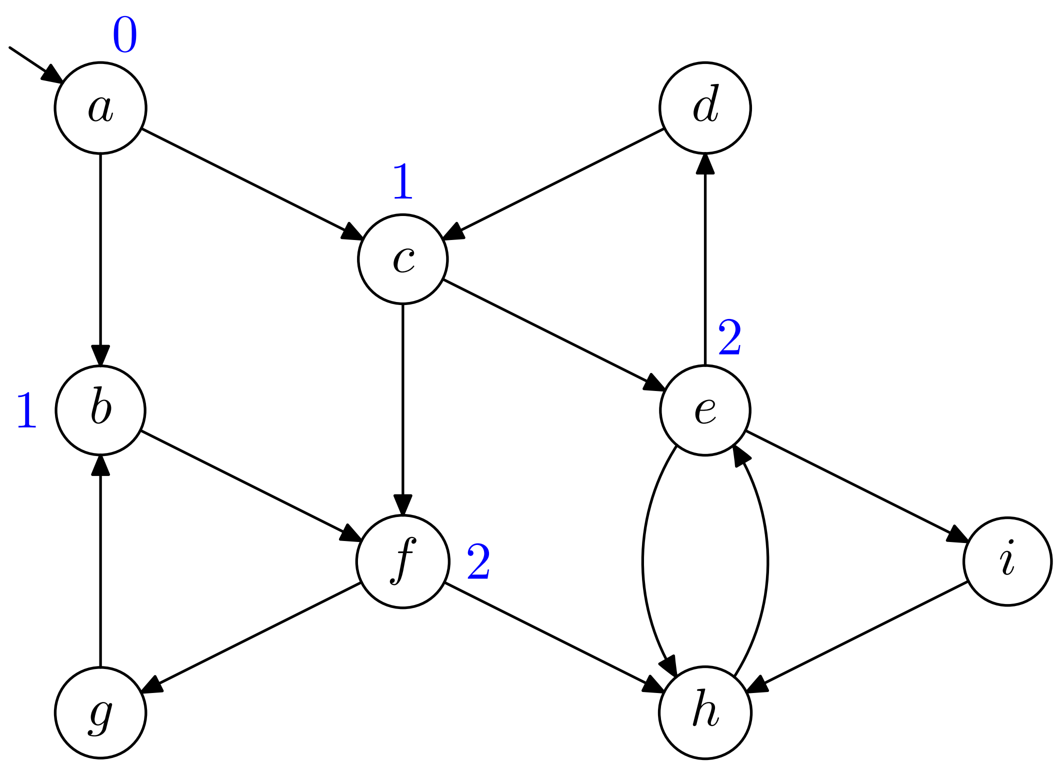
$$Q = \{ \cancel{a}, b, c \}$$

Breadth-first search example



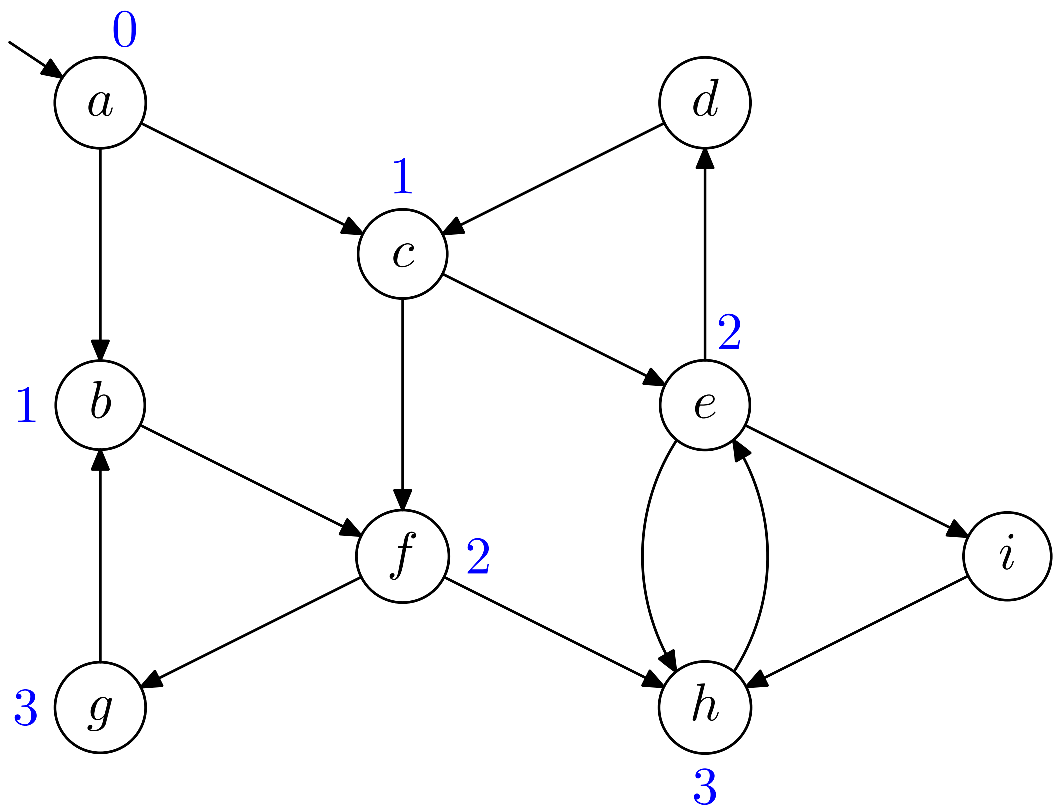
$$Q = \{\,\cancel{a},\ \cancel{b}, c, f\,\}$$

Breadth-first search example



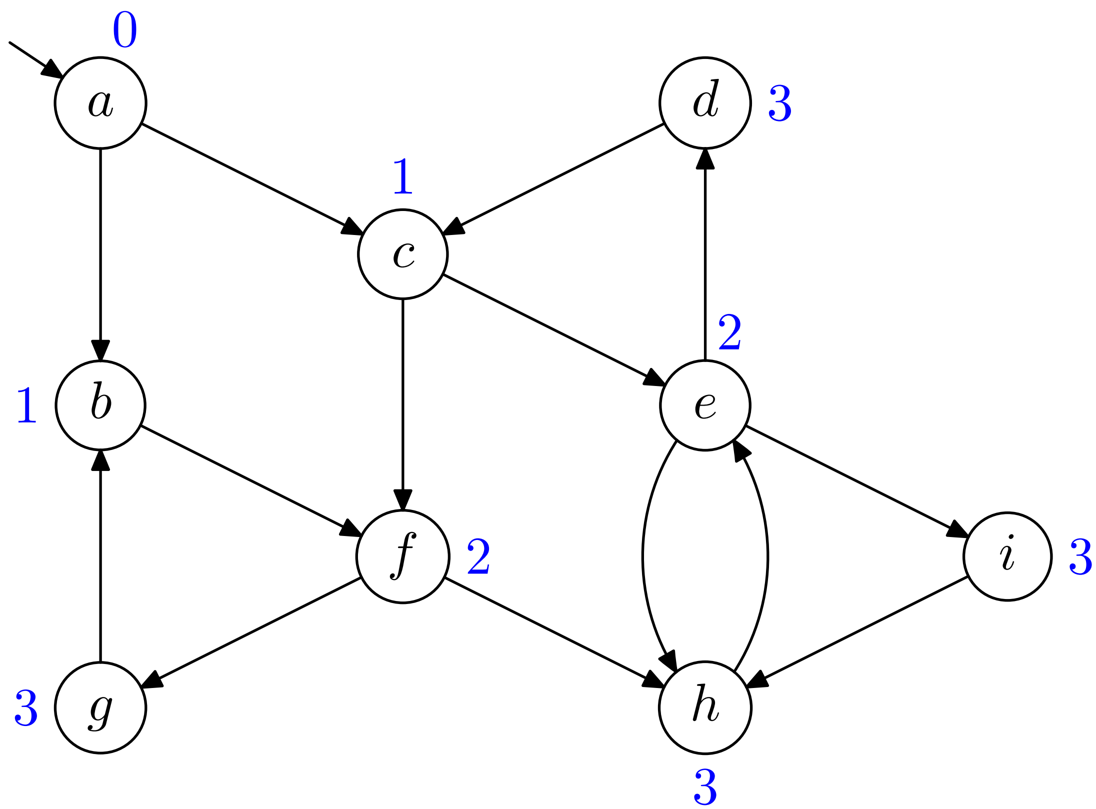
$$Q = \{\, \cancel{a},\ \cancel{b},\ \cancel{c}, f, e\}$$

Breadth-first search example



$$Q = \{\, \cancel{a},\ \cancel{b},\ \cancel{c},\ \cancel{f}, e, g, h\}$$

Breadth-first search example



$$Q = \{\, \cancel{a}, \cancel{b}, \cancel{c}, \cancel{f}, \cancel{e}, g, h, d, i \}$$

Depth-first search

- Properties
- The algorithm
- Classification of vertices
- Classification of edges
- Example

Depth-first search properties

- **Input:** A directed/undirected graph.
- **Output:**
  - ▷ Vertices are time-stamped: *discover* and *finish*.
  - ▷ Edges $\in$ { *tree*, *back*, *forward or cross edge* }.
- Visits all vertices.
- **Running time:** $\Theta(V+E)$

  Linear time with respect to adjacency list.

Depth-first search: algoritmen

DFS

for each $u \in V$

  do $color[u] \leftarrow \text{White}$

$time \leftarrow 0$

for each $u \in V$

  do if $color[u] = \text{White}$

    then $\text{DFS-Visit}(u)$

$\text{DFS-V{\scriptsize ISIT}}(u)$

$color[u] \leftarrow \text{G{\scriptsize RAY}}$ $\triangleright$ u has just been discovered

$time \leftarrow time + 1$

$d[u] \leftarrow time$

for each $v \in Adj[u]$

do if $color[v] = \text{W{\scriptsize HITE}}$

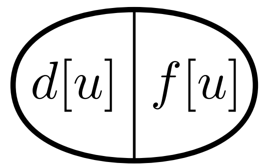
then $\text{DFS-V{\scriptsize ISIT}}(v)$

$color[u] \leftarrow \text{B{\scriptsize LACK}}$ $\triangleright$ finished with u

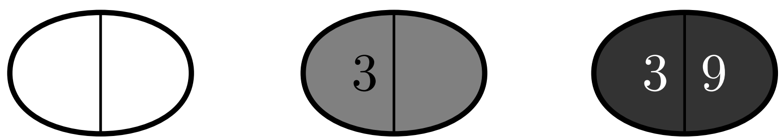
$f[u] \leftarrow time \leftarrow time + 1$

Depth-first search: vertex classification

Time-stamp vertices when they are discovered/finished:



Then vertex colors are equivalent to the following cases:



WHITE GRAY BLACK

Depth-first search: edge classification

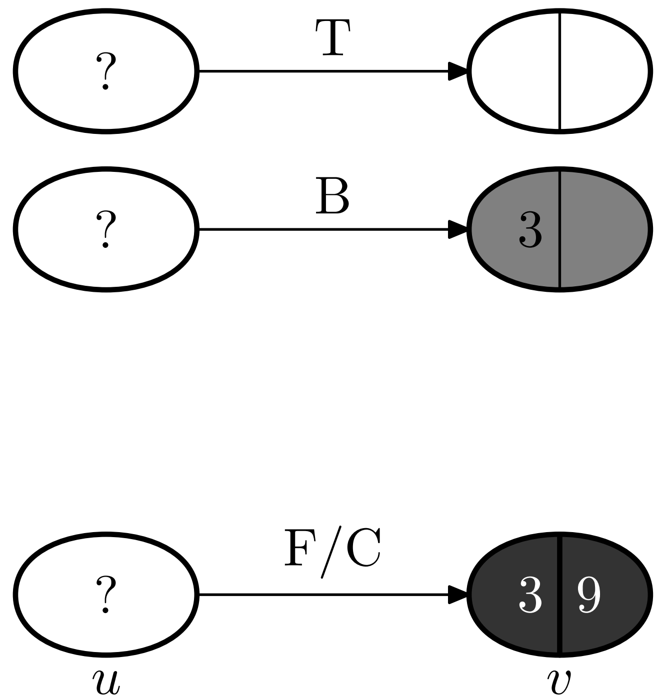
Edges are classified according to the following cases:

Tree edge

Back edge

$d[u] < d[v]$: Forward edge (F)

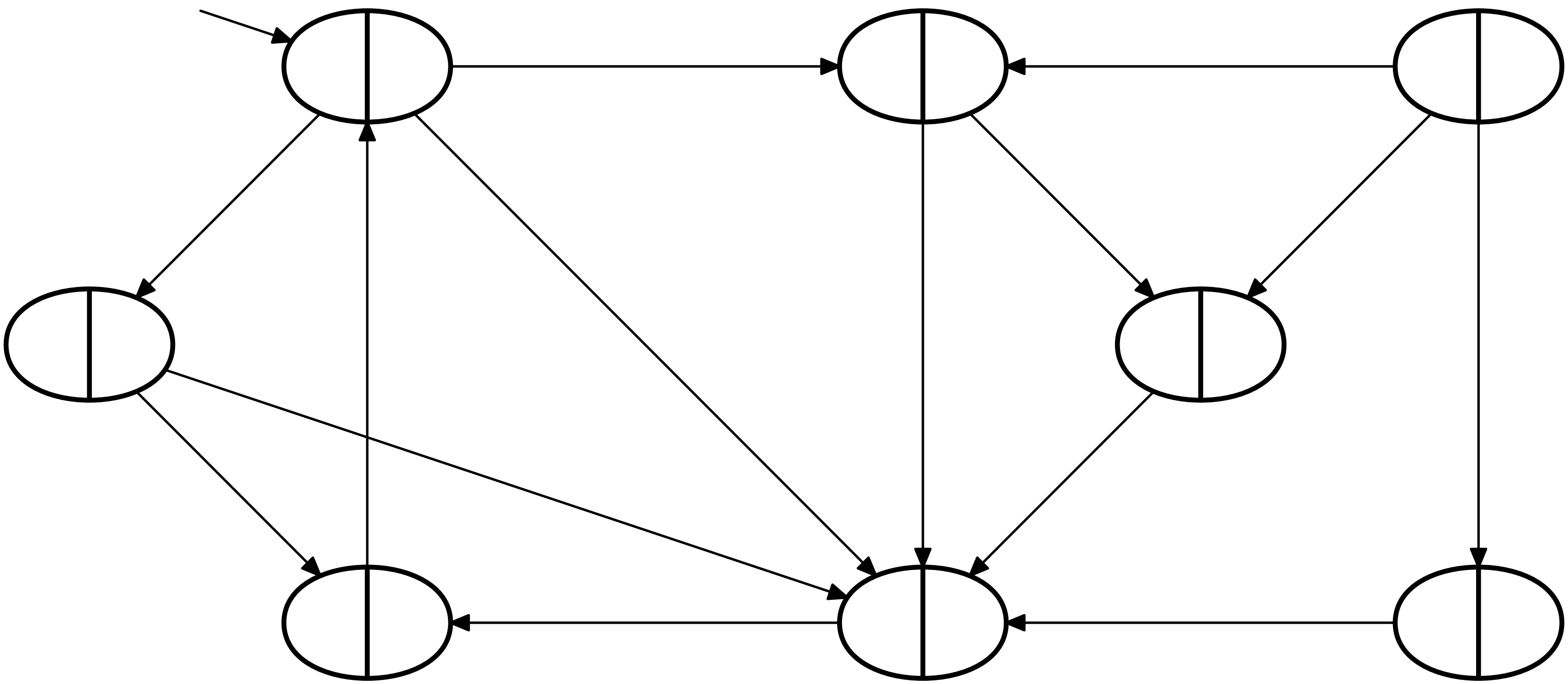
$d[u] > d[v]$: Cross edge (C)



$\Leftarrow$

MÄLARDALENS HÖGSKOLA

Depth-first search example



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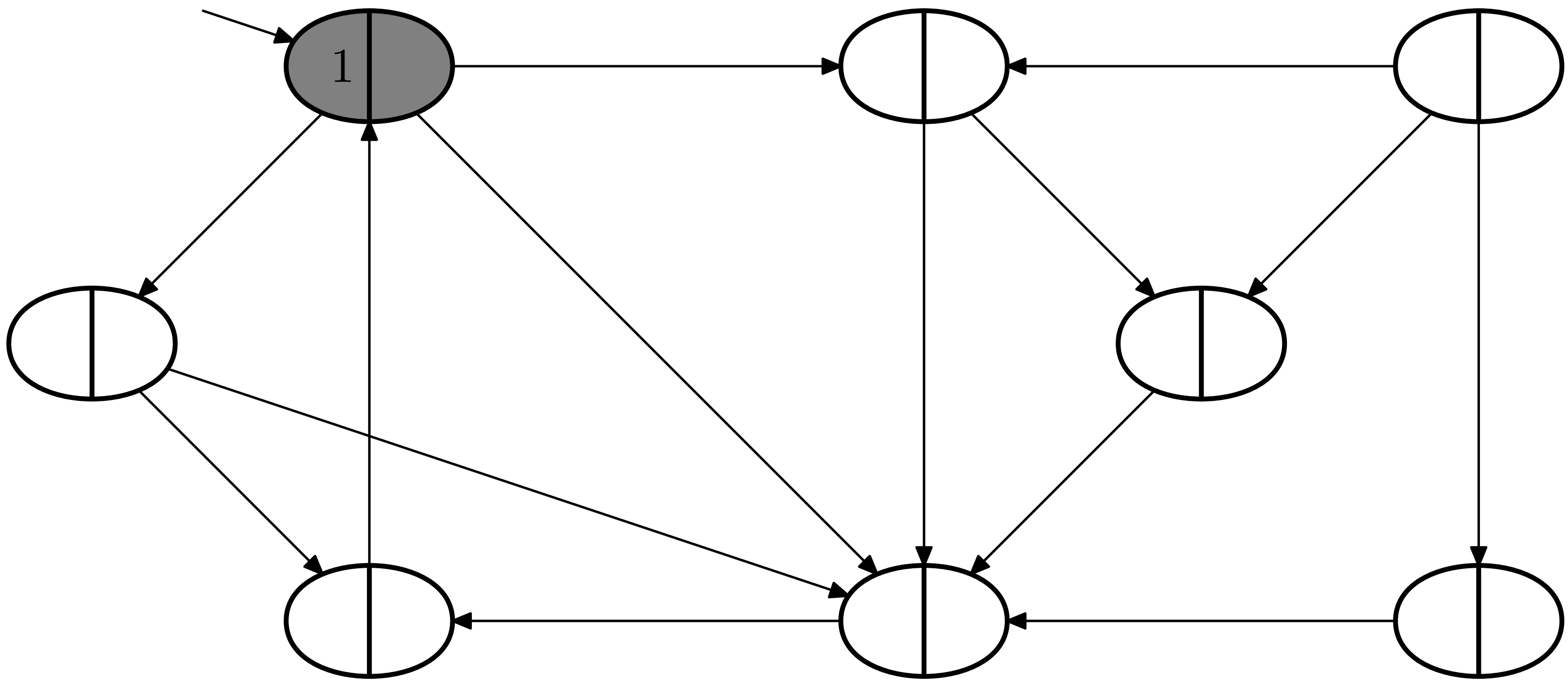

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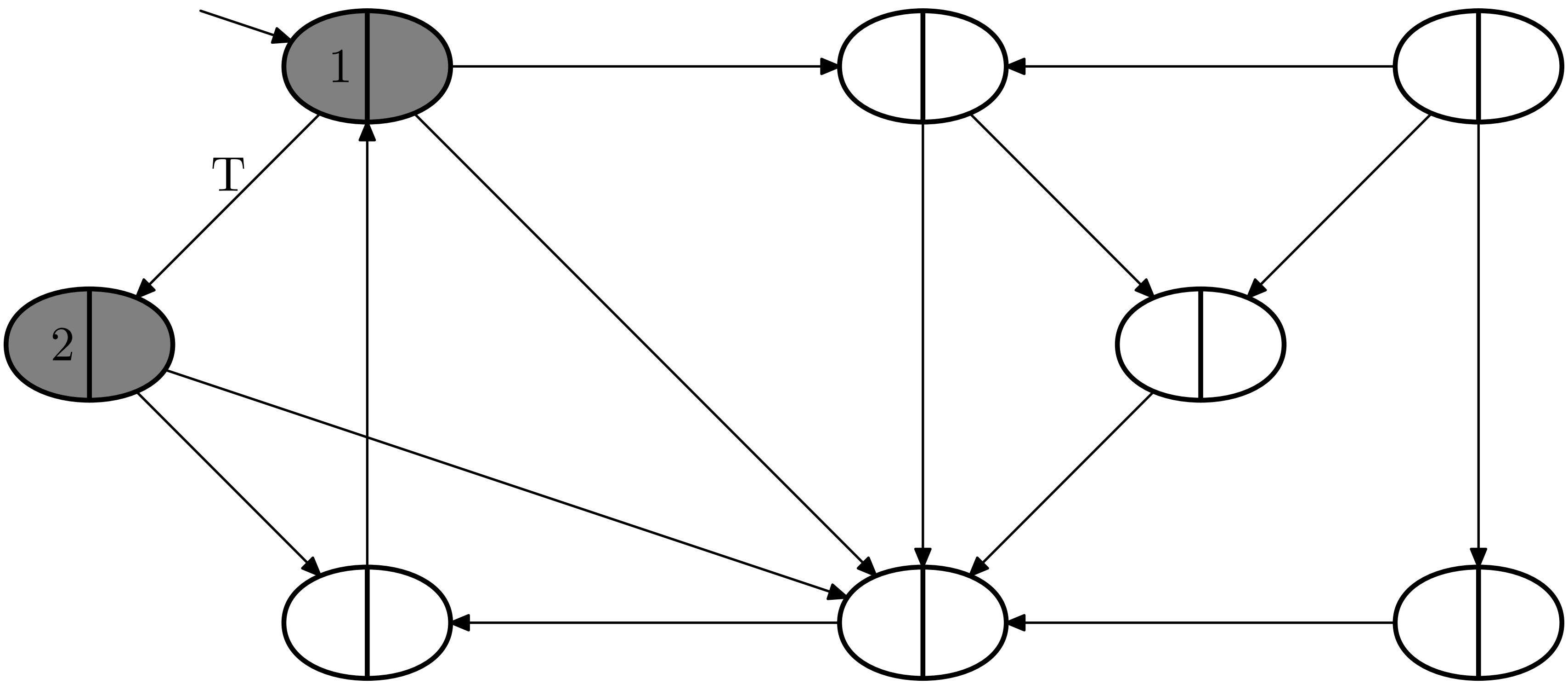

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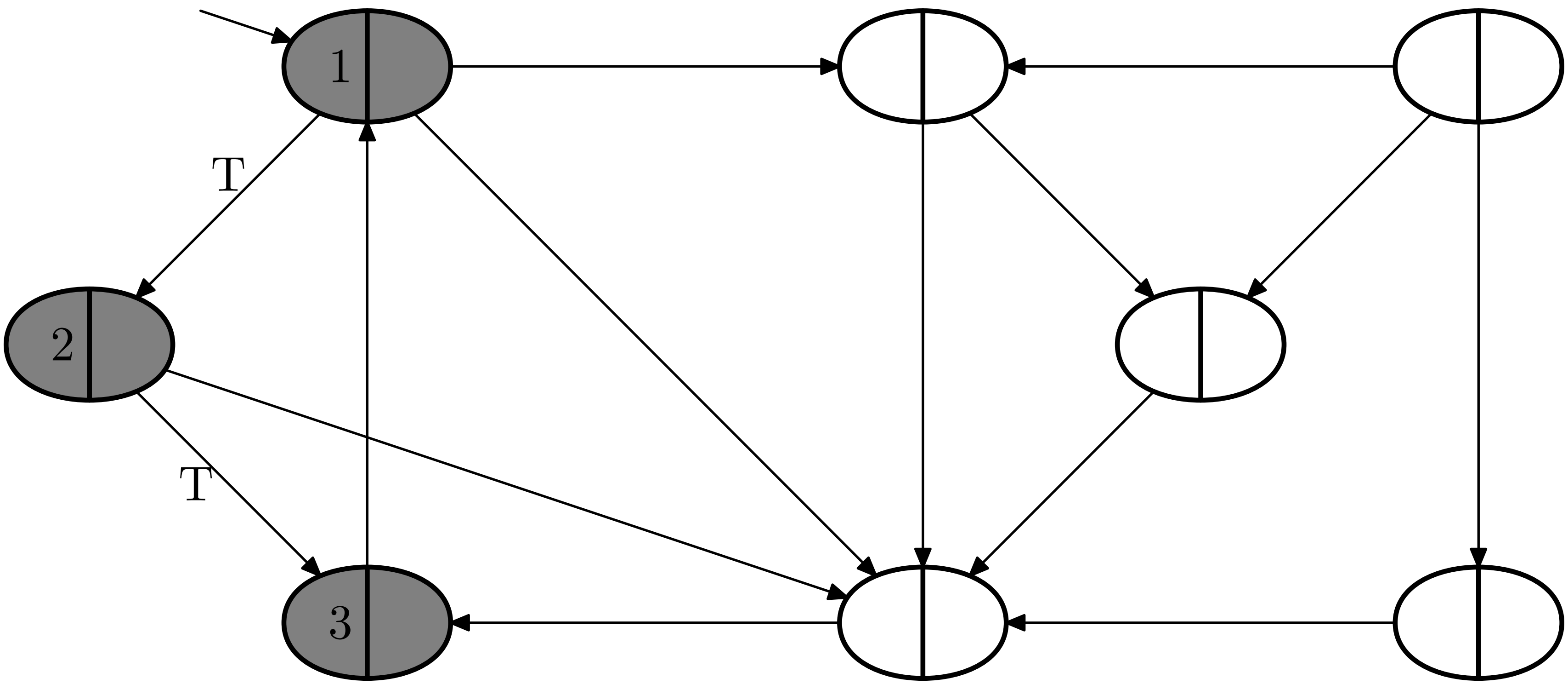

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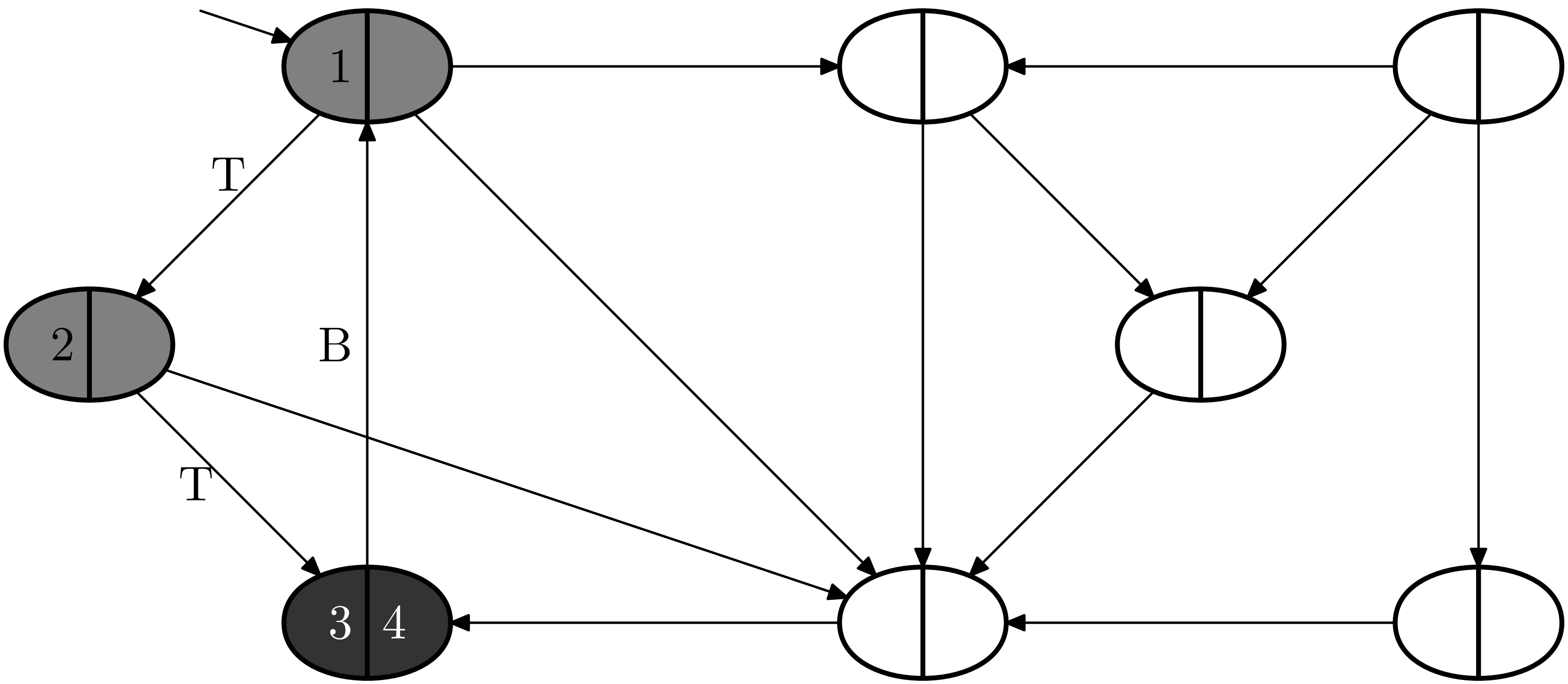

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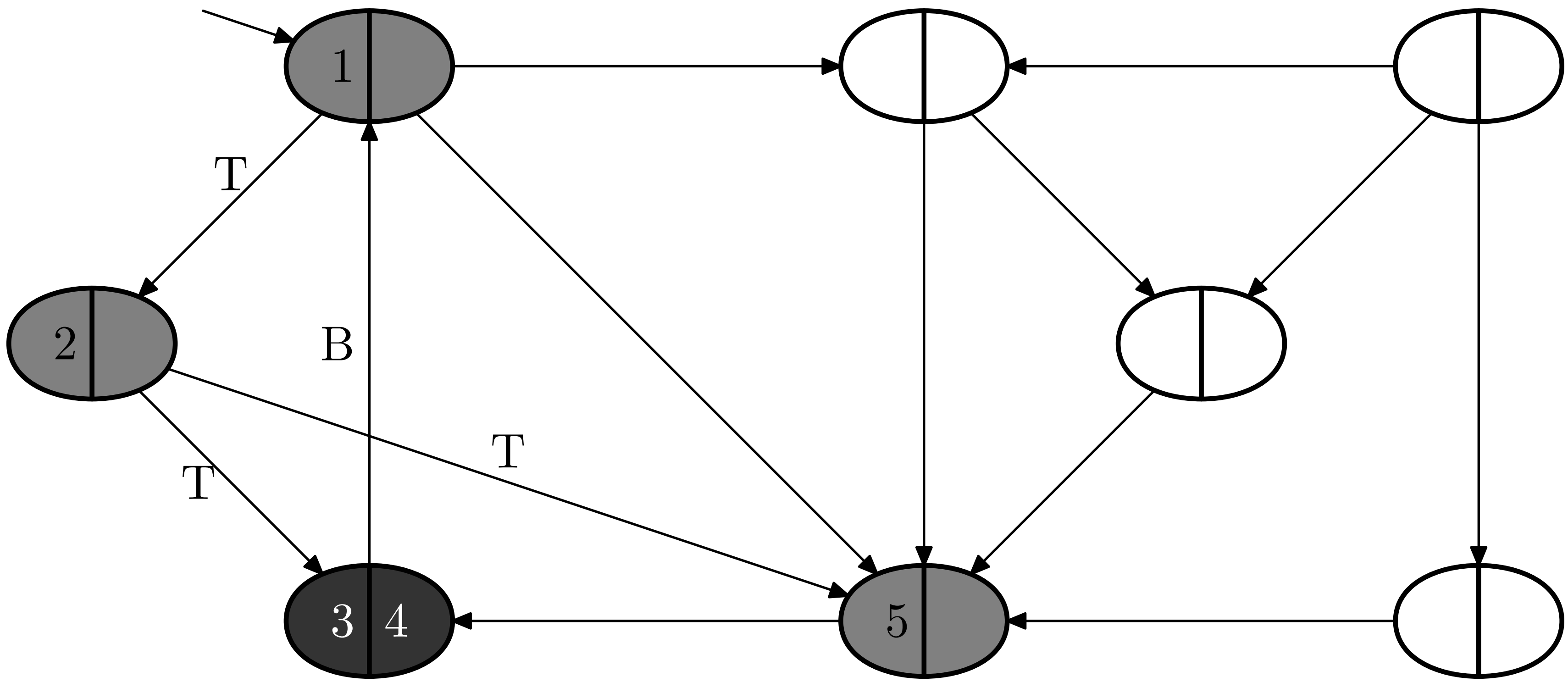

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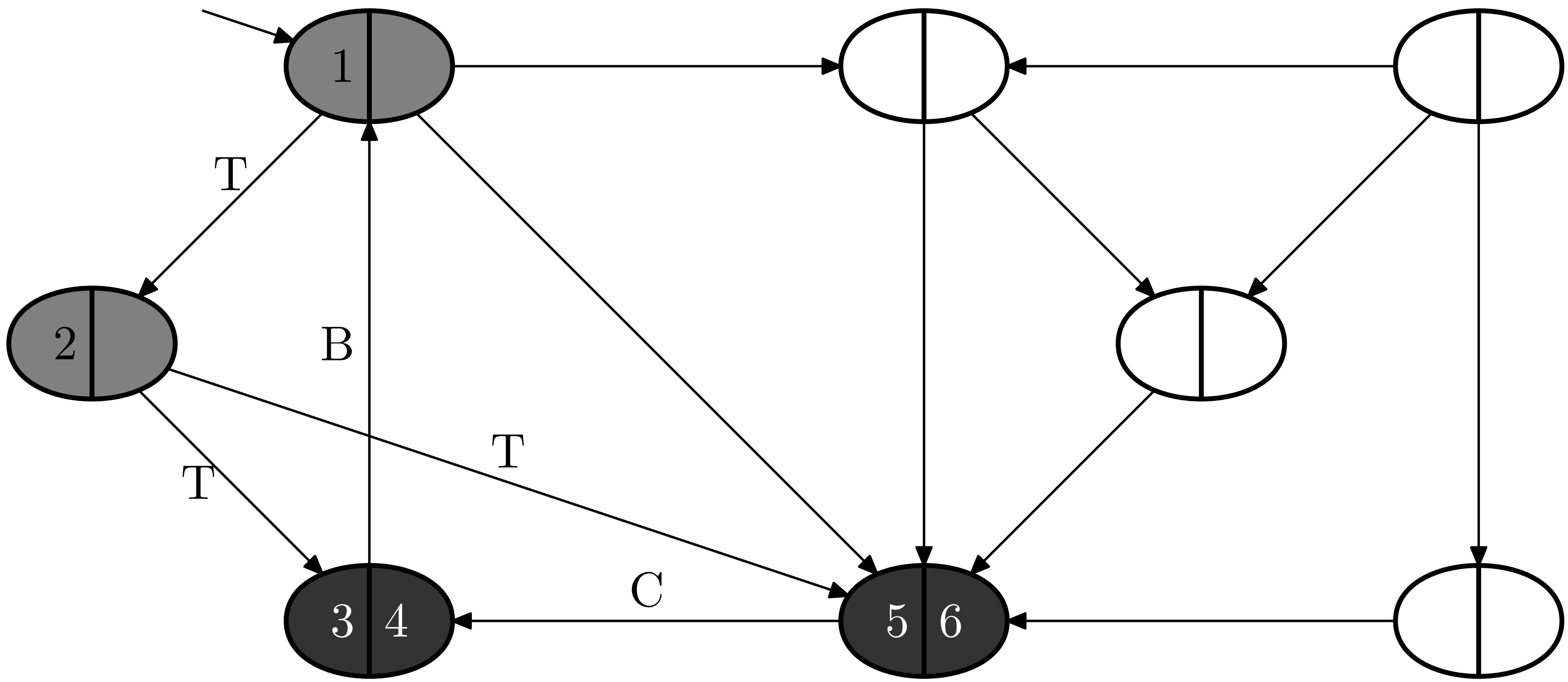

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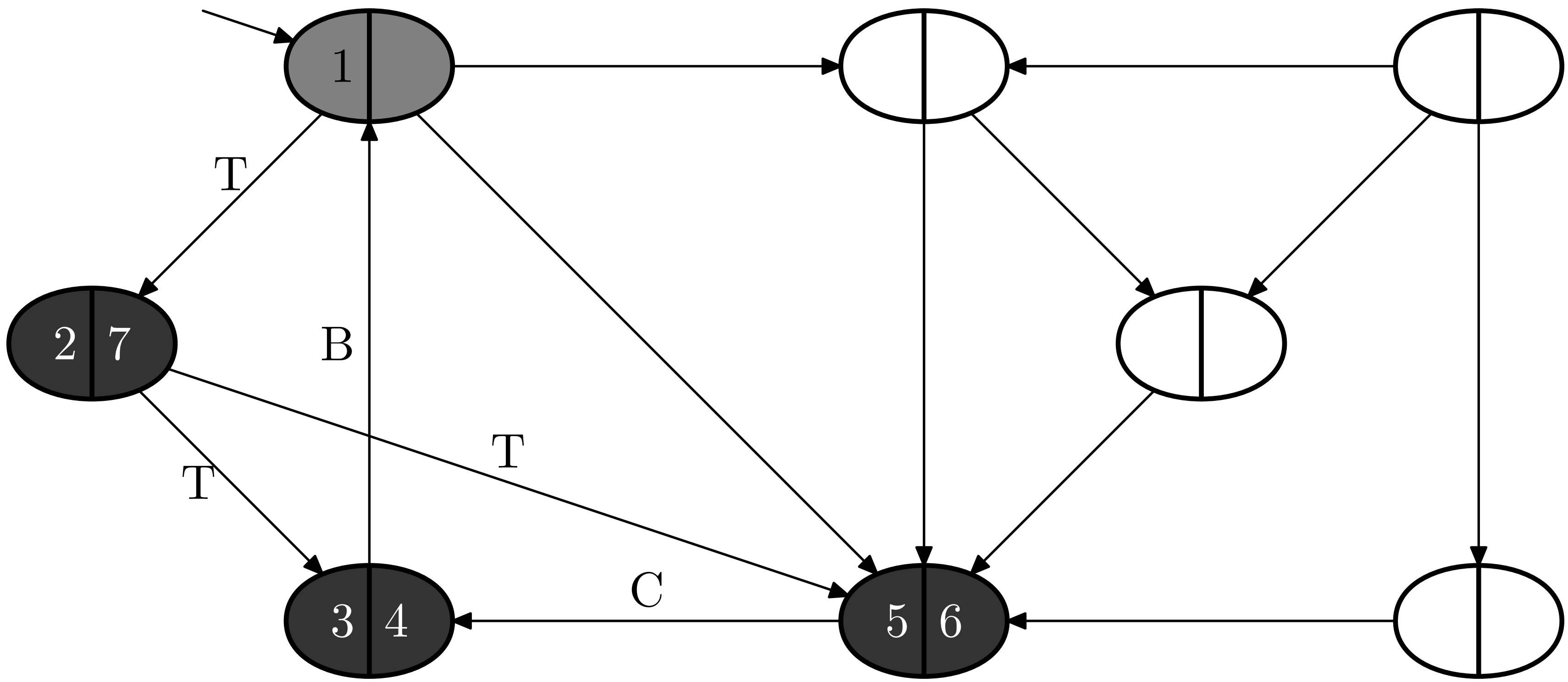

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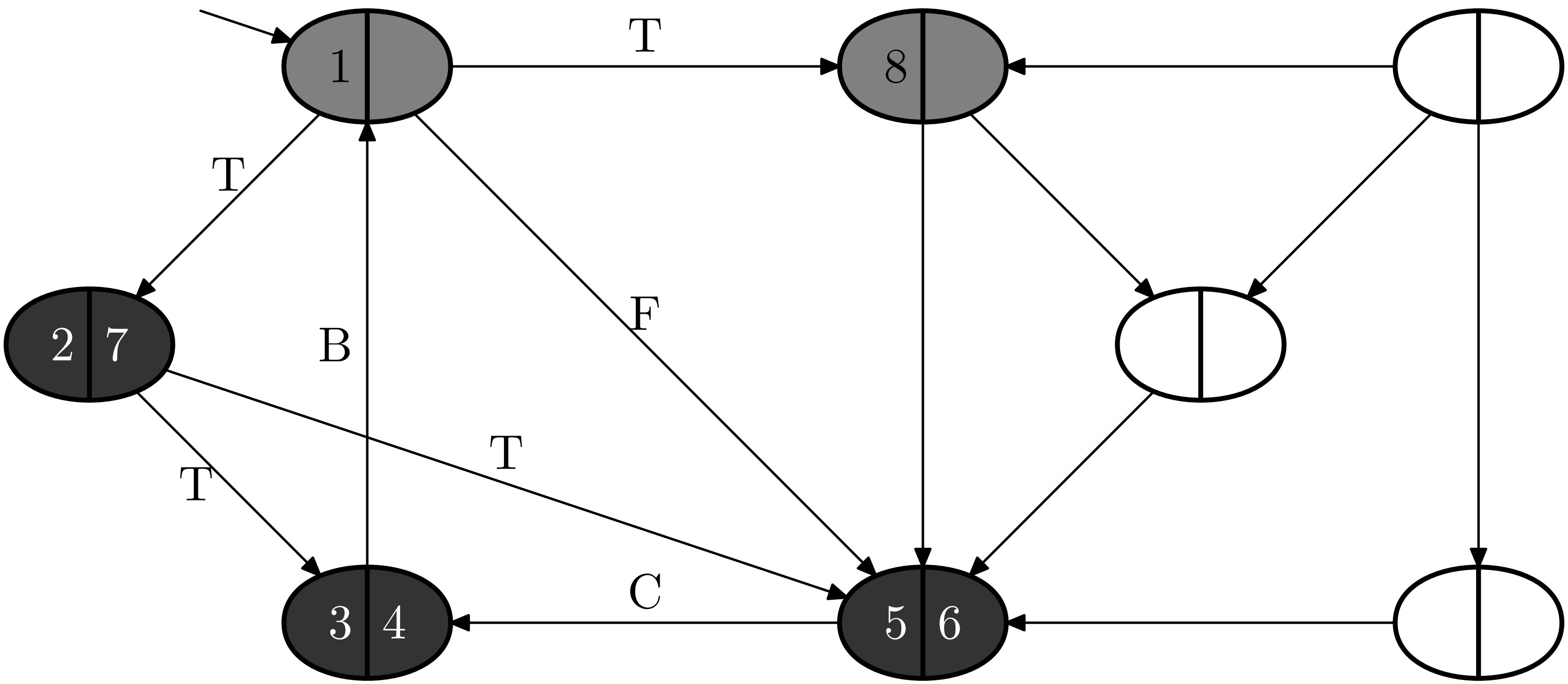

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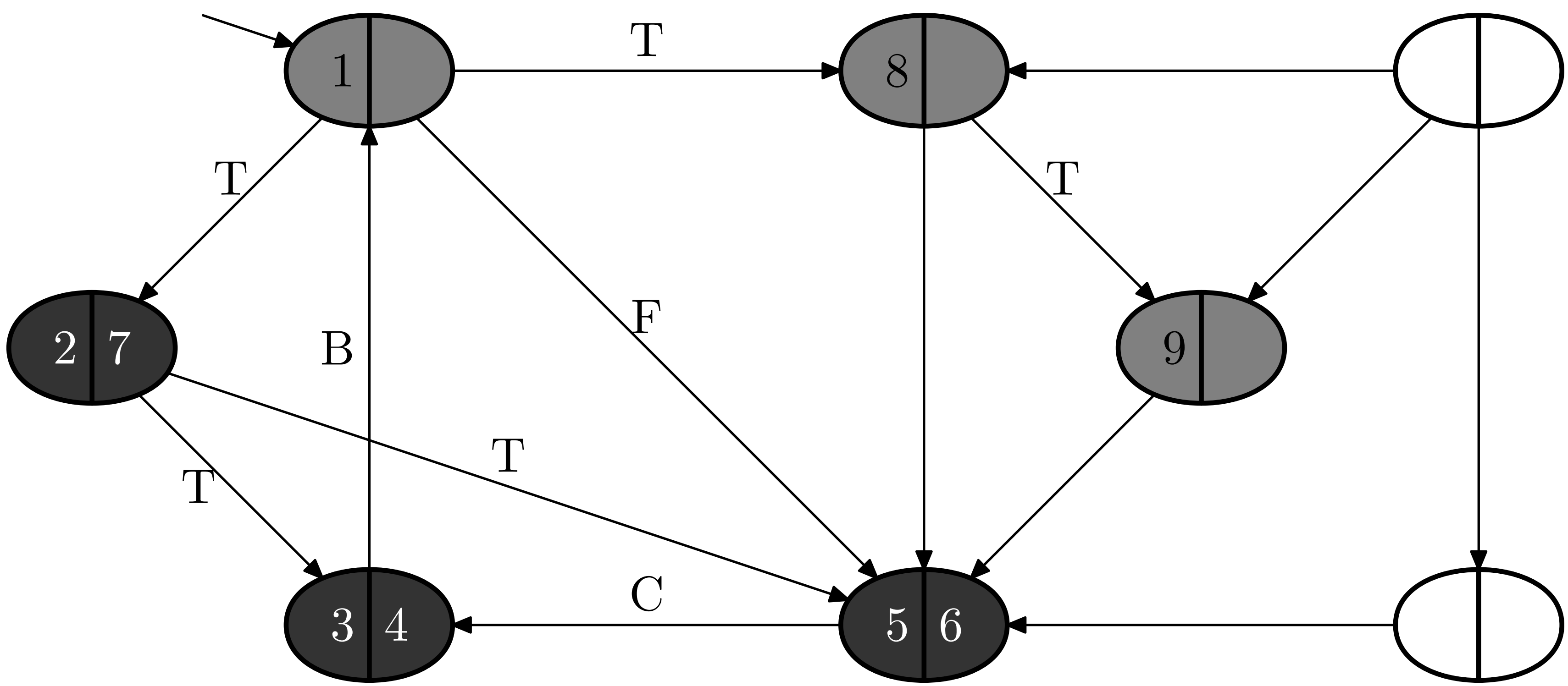

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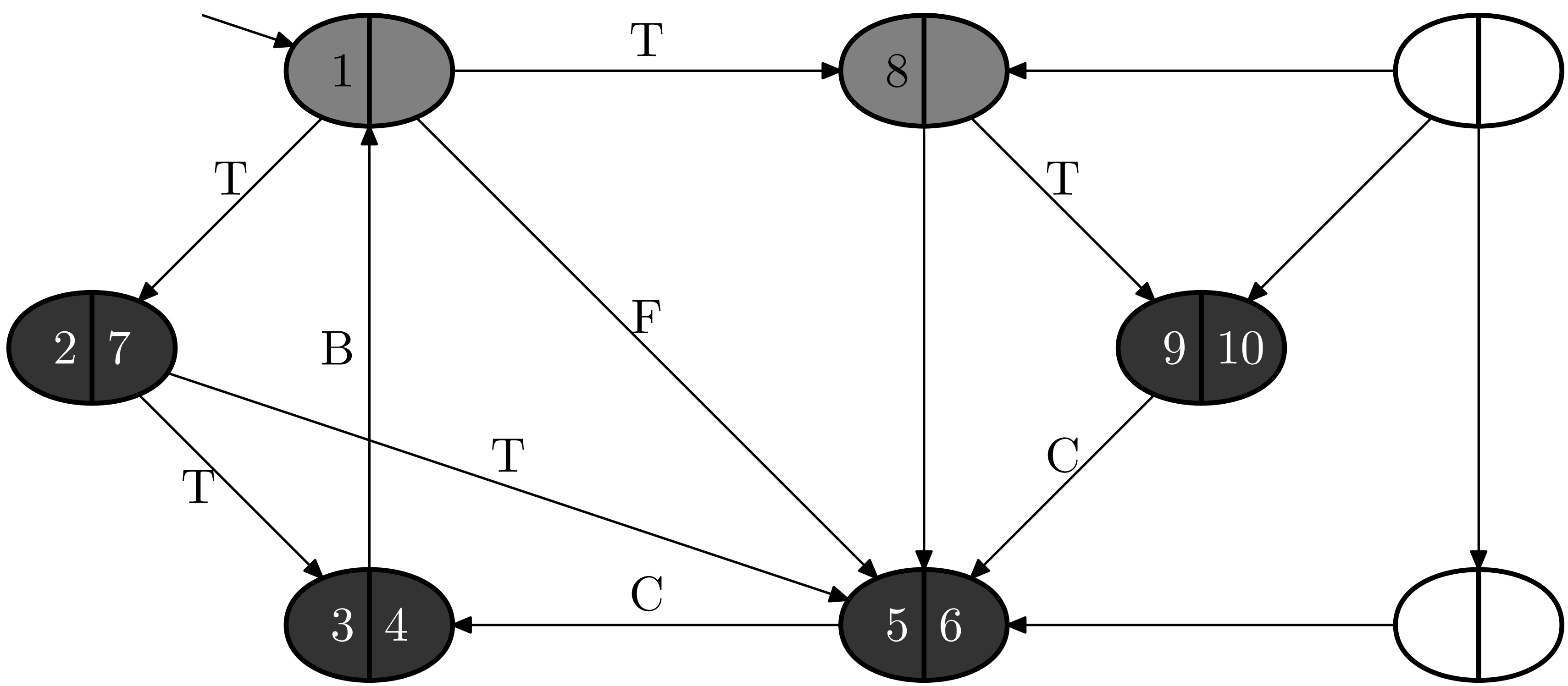

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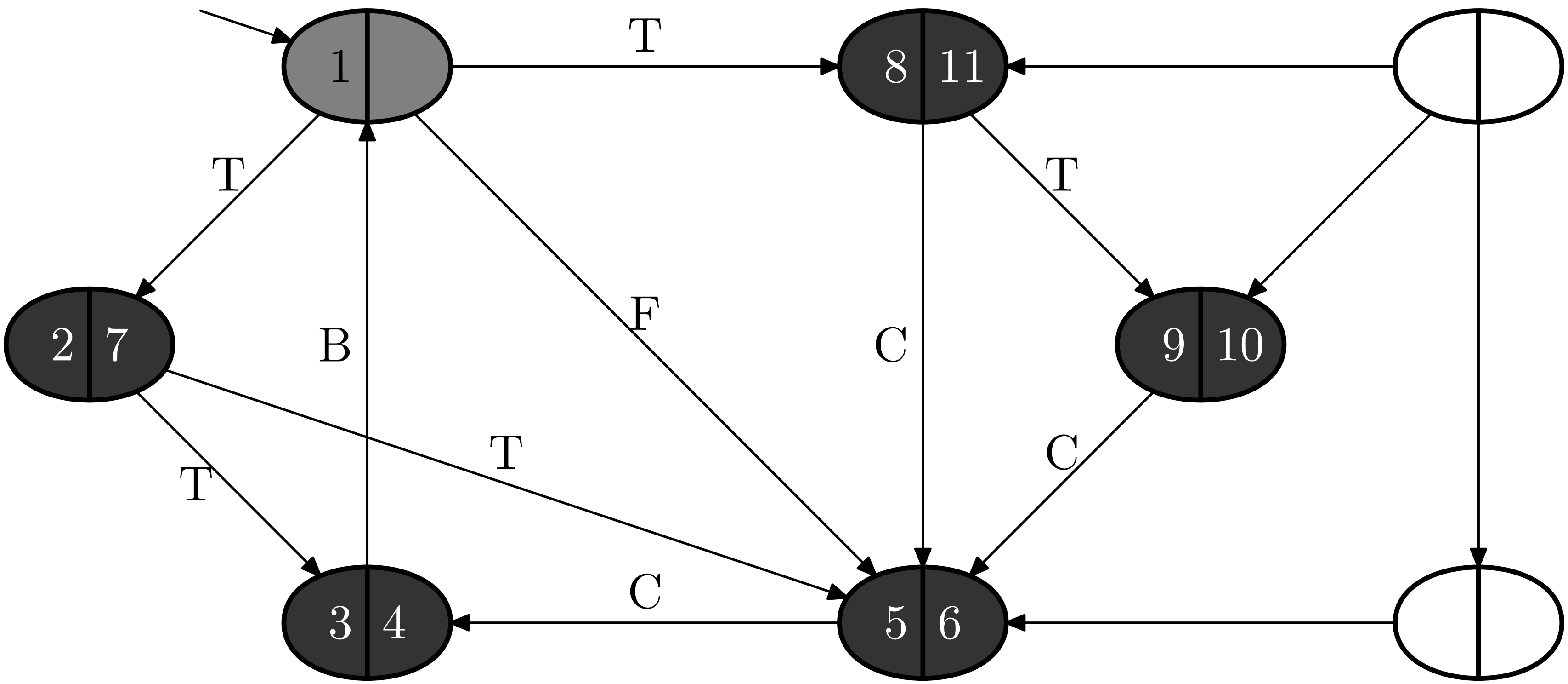

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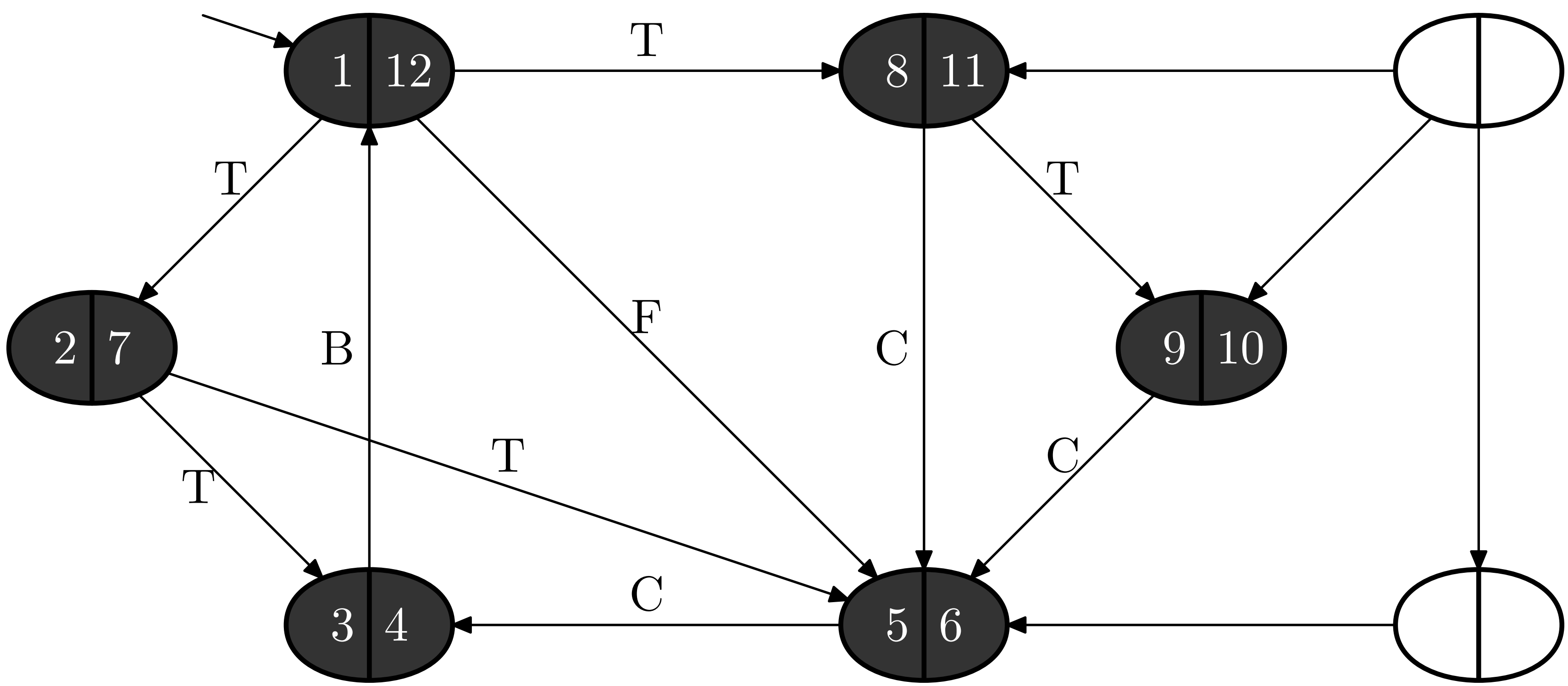

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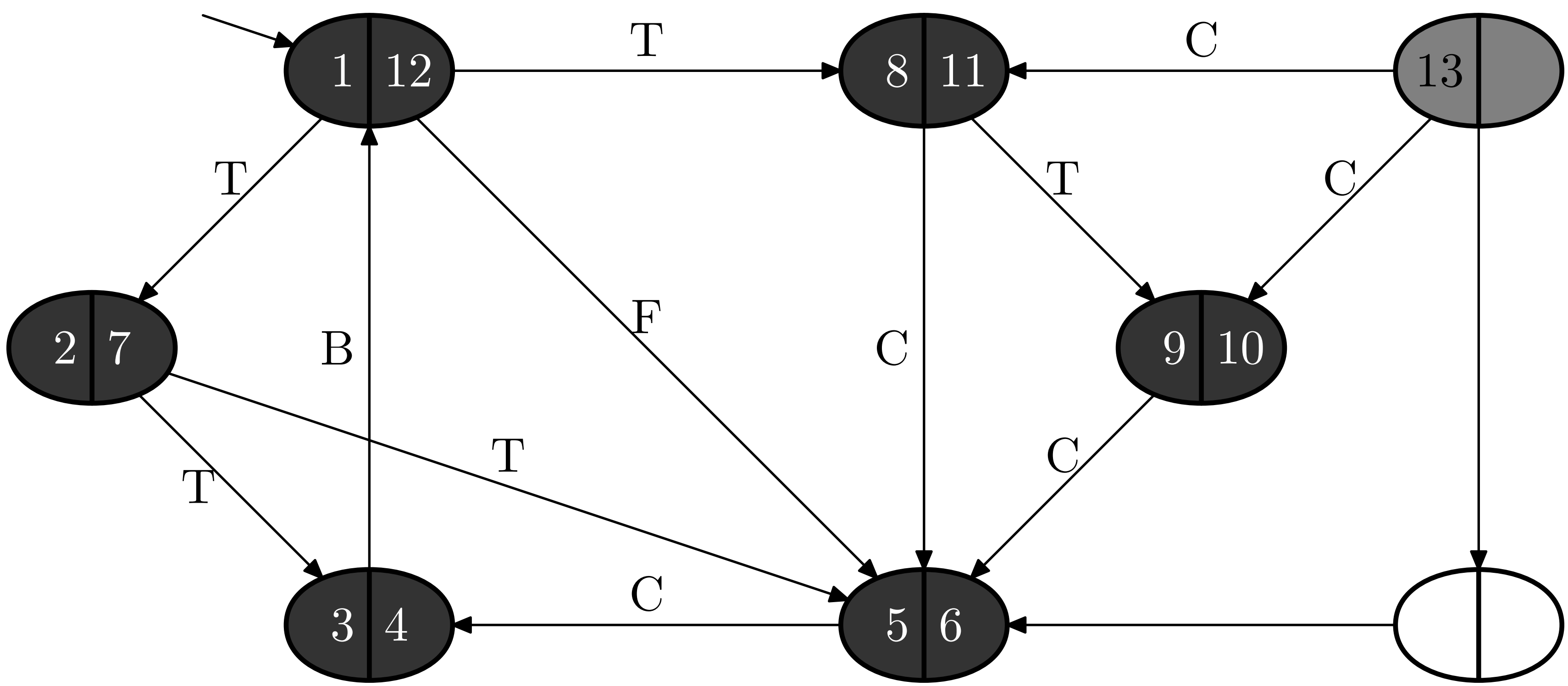

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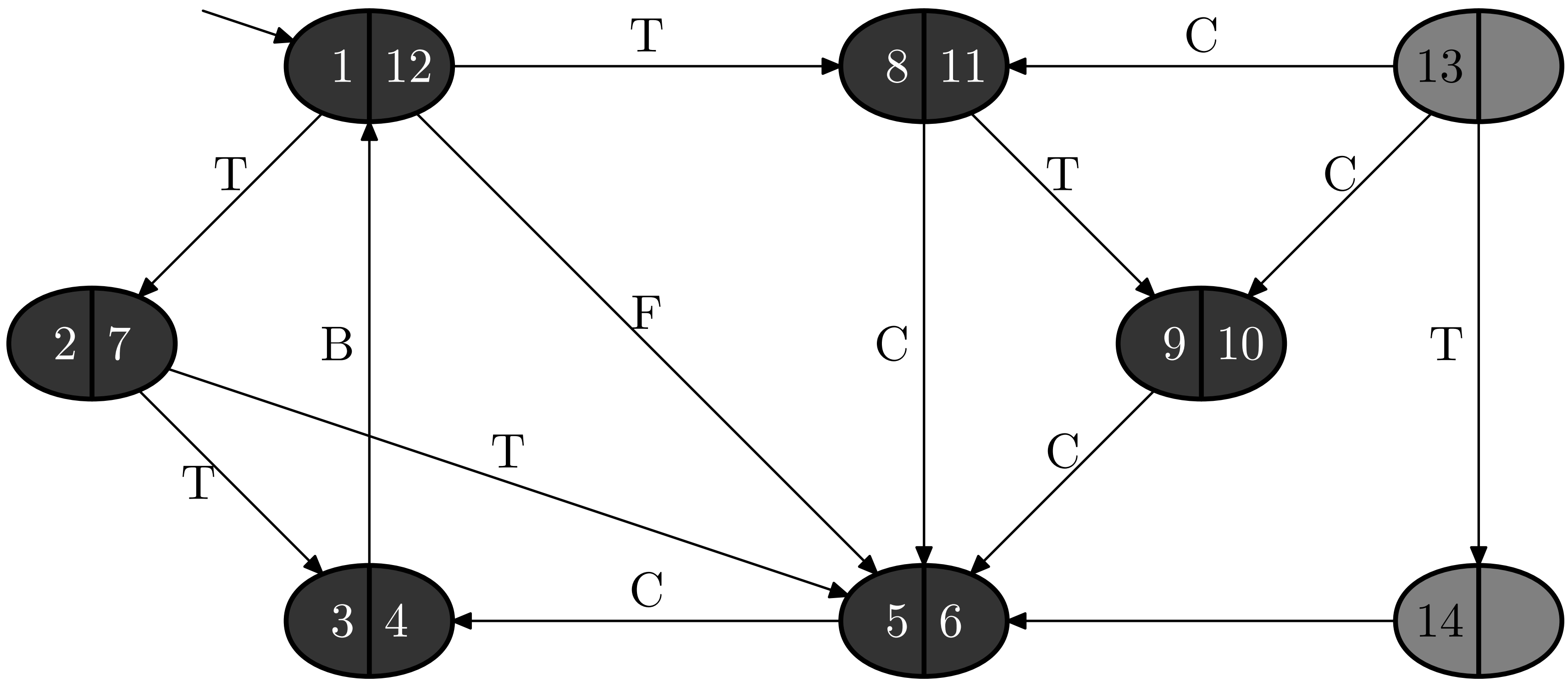

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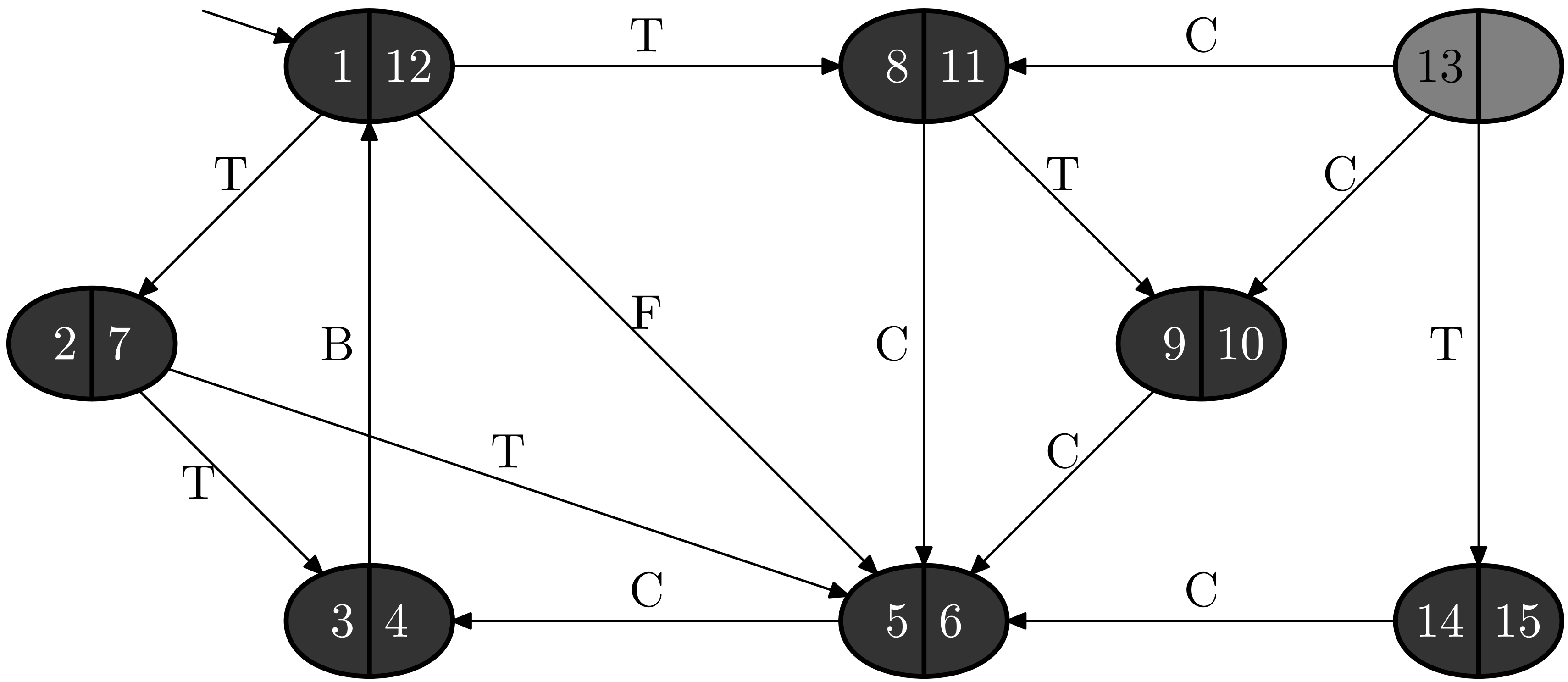

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