Statistical Mechanics-inspired Framework for Studying the Effects of Mixed Traffic Flows on Highway Congestion

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***Abstract*— Intelligent vehicles equipped with adaptive cruise control (ACC) technology have the potential to significantly impact the traffic flow dynamics on highways. Prior work in this area has sought to understand the impact of intelligent vehicle technologies on traffic flow by making use of mesoscopic modeling that yields closed-form solutions. However, this approach does not take into account the self-organization of vehicles into clusters of different sizes. Consequently, the predicted absence of a large traffic jam might be inadvertently offset by the presence of many smaller clusters of jammed vehicles. This study – inspired by research in the domain of statistical mechanics – uses a modification of the Potts model to study cluster formation in mixed traffic flows that include both human-driven and ACC-enabled vehicles. Speciifcally, the evolution of self-organized traffic jams is modeled as a non-equilibrium process in the presence of an external field and with repulsive interactions between vehicles. Monte Carlo simulations of this model at high vehicle densities suggest that traffic streams with low ACC penetration rates are likely to result in larger clusters. Vehicles spend significantly more time inside each cluster for low ACC penetration rates, as compared to streams with high ACC penetration rates.**

I. Introduction

Technologies that modify the movement of individual vehicles in a traffic stream, such as adaptive cruise control (ACC) systems, have the potential to significantly impact macroscopic traffic flow dynamics. Unfortunately, few analyses and simulation tools exist to understand how such technologies may affect traffic flow, particularly under mixed human/algorithm-driven situations. Recent reports suggest that there are increasing disparities between highway capacity and vehicular population [1], and that highway congestion is on the rise [2]. In such a scenario, it may be ill-advised to alter the vehicular population characteristics through the introduction of ACC-enabled vehicles without first evaluating their systemic effects on traffic flow dynamics.

Several analytical approaches may be used to study the changes in traffic flow parameters as a consequence of increasing ACC penetration rates. Traditionally, microscopic numerical simulations have served well to provide some insight into traffic flow dynamics as a function of driver behaviors [3]. These simulation approaches utilize models that represent the driving behavior of humans to varying degrees of accuracy. However, some approaches, such as the cellular automata (CA) models [4][5], may significantly simplify the driver behavior to rules that are independent of physical interpretation of model parameters. Other approaches, such as those that use detailed car-following models (e.g. General Motors [6] or optimal velocity models [7]), may cause the simulations to be computationally expensive and memory intensive. Consequently, it is desirable to have a simulation approach that is more detailed than CA models and builds upon physical principles, while simultaneously being computationally amenable to parametric studies. The generalized Ising model provides such an approach.

This paper builds upon the work presented in [8] [9] and draws from the principles of statistical mechanics that have been successfully used to analyze several disparate systems such as ferromagnets [10], adaptation in autonomous systems [11], and neurons [12]. Specifically, the work utilizes the simple, yet immensely popular Ising model, and its generalized forms, to study traffic flow and vehicle cluster dynamics as a function of ACC penetration. The next section discusses some of the recent studies on mixed traffic flows as well as relevant statistical mechanics-based traffic analyses.

II. Literature Review

The transportation research community has shown keen interest in evaluating the effects of introducing adaptive cruise control-enabled vehicles into the traffic stream. Early research in the field focused on the effects of introducing tightly controlled platoons into the traffic stream [13], and the effects of lead vehicle maneuvers on the stability of the platoon [14][15]. However, as ACC-enabled vehicles become available to the everyday consumer, it is expected that the traffic flow scenario will evolve into one where ACC-enabled vehicles are randomly interspersed in a stream of human-driven vehicles, i.e. a randomly mixed traffic scenario. Realizing the importance of this transitional period where the highway population changes from primarily human-driven vehicles to primarily autonomous vehicles, researchers have begun to focus on the randomly mixed traffic scenario.

Recent studies of such randomly mixed traffic flow that include ACC-enabled vehicles have produced some interesting results. Analytical approaches have shown that the critical density at which self-organized traffic jams form increases as ACC penetration increases [16]. The approaches presented in [16] are limited to the study of self-organized traffic jams at a mesoscopic scale only and hence do not convey results about vehicle cluster distributions. Other approaches that utilize numerical simulations with car-following models have shown that traffic capacity potentially increases with increasing ACC penetration [3]. These simulations provide a more

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detailed description by recording the states of all vehicles at each time step, but with an obvious computational burden associated with this detail. New modeling approaches are needed to meet the challenges of obtaining sufficient model fidelity to consider mixed traffic flows, but with minimal computational complexity. Additionally, the new modeling approach should be able to provide deeper insights into the statistics of self-organized traffic jams and their relation to increasing ACC penetration.

In this regard, the recent statistical mechanics-based work by Sopasakis and colleagues provides a good starting point [8][9]. The origins of the use of statistical mechanics-based techniques for traffic flow analysis can be traced back to the works of Prigogine and his colleagues, who attempted to study traffic flow using gas kinetic theory [17][18]. Recently, statistical mechanics-based techniques have seen a resurgence, with the prototypical Ising model being used as a model for different traffic related problems [8][9][19]. However, in the context of analyzing changes in traffic flow dynamics as a consequence of driver behavior, these statistical mechanics-based techniques have been limited to environments of single species [8]. The presented work extends these techniques to represent a traffic stream that consists of both human-driven and ACC-enabled vehicles by using a generalized Ising model (or more specifically, the Potts model). The novel use of the Potts model may help provide insight into traffic flow dynamics without significant computational expenditure. Additionally, the Potts model approach maintains reasonable simulation fidelity in a way that model parameters can be associated with their counterparts in the physical world. The next section provides a short description of the physical system under consideration, which is a traffic stream with both human-driven and ACC-enabled vehicles.

III. Traffic System Description

The system under consideration in the presented work is a single-lane closed-ring road of length L with M vehicles. The idealization of a closed-ring road is preferable, because it circumvents an open-system representation of a highway which may include on- and off-ramps [16]. The ring road is divided into N discrete cells or sites that can each contain at most one vehicle. The length of a single site is the spatial extent of a vehicle d and some safe spacing d_0. In the presented work, these values have been assumed to be 5.5 m and 2.5 m respectively, so that the spatial extent of each site is $D = d + d_0 = 8$ m. Fig. 1 indicates the physical system under consideration, with a mixture of human-driven and ACC-enabled vehicles in the traffic stream.

The parameters of the traffic system used hereafter are taken as typical values of a single-lane highway segment, and are known to be representative of a single lane highway system [20]. The free flow velocity (v_f), which is a measure of the speed of the vehicles at low densities, is assumed to be 25 m/s (or 90 km/h). The maximum flow of vehicles in the lane or lane capacity (q_{max}) is assumed to be 1800 veh/h. The backward wave speed (u), which represents the speed

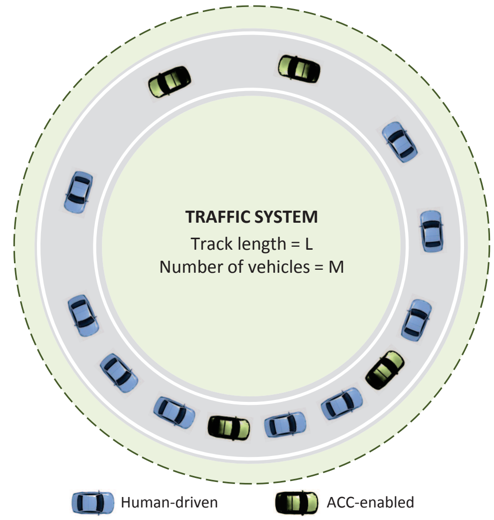

Fig. 1. Description of traffic system under consideration. Ring road of length L meters is occupied by M vehicles, a fraction of which are human-driven vehicles (light blue) and the remaining are ACC-enabled vehicles (dark green).

at which a signal travels backwards in a traffic stream, is assumed to be -6 m/s (-19.6 km/h). The steady state speed (v_{ss}) is a function of the vehicular density ρ. The parameter values assumed here are fairly representative of a single lane highway system [20]. The fidelity of the numerical simulation of the traffic flow dynamics is substantiated in part by calibrating the Potts model parameters to yield the aforementioned values. The next section discusses the Potts model framework for traffic flow dynamics.

IV. Generalized Ising (or Potts) Model Approach

This section discusses the statistical mechanics-based framework used to model traffic dynamics in a mixed traffic flow that consists of both human-driven and ACC-enabled vehicles. Details about state transition probabilities and their use in a Monte Carlo framework for simulating traffic dynamics are also provided.

A. Limitations of the Ising Model

Previous work by Sopasakis and his colleagues on the modeling of traffic flow using the Ising model provides an excellent starting point for studying multi-species traffic flow [8]. However, the Ising model allows a site to be in one of only two states (i.e. $q = 2$). With regards to traffic flow, the site can either be vacant, or be occupied by a vehicle. The structure of the Ising model makes it incapable of distinguishing between vehicles of different species, such as human-driven and ACC-enabled vehicles. As a result, the Ising model cannot be used to analyze mixed traffic flows which are the subject of this work. Fortunately, extensions of the Ising model – such as the Potts model which allows a specific site to assume one of three distinct states (i.e. $q = 3$) – can be used for modeling mixed traffic flow dynamics.

B. Space Partitioning and Lattice Structure of Traffic System

The closed ring road described in Section III is partitioned into indvidual sites so as to span the entire length L of the

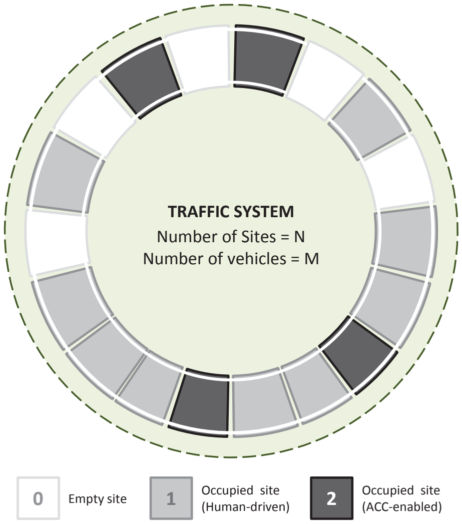

Fig. 2. Discretized version of traffic system shown in Fig. 1 for statistical mechanics-based numerical analysis. Three states are possible for each site in accordance with the Potts model approach.

road, while avoiding overlap with other sites. In other words, elements $S_i (i = 1, 2, ..., N)$ of a partition S of the road create a mutually exclusive and exhaustive set of sites. Each site S_i in the partition S can contain at most one vehicle. The collection of sites that represents the ring road forms a one-dimensional lattice, and the setup is referred to as the lattice structure of the system. The terminology is borrowed from physical systems such as lattice gases, whose analysis often relies on statistical mechanics techniques [21].

Next, consider the set of states $\{\sigma_i\}$ that a particular site S_i can assume. Since the goal of this work is to analyze mixed traffic flow, a specific site can be in one of three states, i.e. $\sigma_i \in \Sigma = \{0, 1, 2\}$, where the set Σ is referred to as the alphabet of the system. It is assumed that $\sigma_i = 0$ represents a site that is vacant, $\sigma_i = 1$ represents a site that is occupied by a human-driven vehicle, and $\sigma_i = 2$ represents a site that is occupied by an ACC-enabled vehicle. A schematic representation of the partitioning of the ring road is included in Fig. 2. In this discretized version of the road, white sites indicate that no vehicle is present, filled sites (gray) indicate that a human-driven vehicle currently occupies that site, and filled sites (black) indicate that an ACC-enabled vehicle currently occupies that site. As a result of the space partitioning of the road into S_i sites $(i = 1, 2, ..., N)$ and with a known alphabet $\Sigma = \{0, 1, 2\}$ of the system, the total number of microstates that the traffic system can assume is given by $|\Sigma|^{|S|} = 3^N$. The microstate contains complete information of the entire system at any given instant of time. The next subsection builds upon the lattice structure discussed here to create a framework for simulating the traffic system using the Potts model.

C. Potts Model Formulation

Drivers in traffic, either humans or ACC algorithms, are privy to only local information up to a certain 'look-ahead' distance. As a result, in the current context of analyzing traffic flow dynamics, the Hamiltonian for the Potts model is based on local energy around a particular site. The expression for the Hamiltonian is given by:

$$H(\sigma_i) = -B\sigma_i - \sum_{j \in \mathcal{N}(i)} J_{ij}\sigma_i\sigma_j \tag{1}$$

where $H(\sigma_i)$ refers to the local energy or the Hamiltonian for the Potts model around the site σ_i, B represents an external driving field acting on the entire system, J_{ij} represents the strength of interaction between the sites i and j, and $\mathcal{N}(i)$ represents the forward-looking neighborhood of site i, i.e.

$$\mathcal{N}(i) = \{j : j - i < d_l\} \tag{2}$$

where d_l represents the forward 'look-ahead' distance in terms of the number of forward sites that a driver takes into account while making driving decisions.

Analogous to Prigogine's kinetic theory of gases-based interpretation of traffic flow [18], competing forces are at play in traffic flow dynamics. The external field B corresponds to Prigogine's 'relaxation' term and is the driving force and 'pushes' vehicles forward. The stronger the external field, the greater the tendency of the vehicles to keep moving forward. On the other hand, the interaction strength J_{ij} corresponds to the 'collision' term and is a measure of 'repulsion' faced by vehicles as they approach downstream vehicles. The greater the magnitude of the interaction term between a vehicle entering a jam and a preceding one already inside a jam, the slower that vehicle will enter the jam. In order to model the interaction term as a repulsive force acting on the vehicles and slowing them down, the local energy is calculated with $J_{ij} < 0$. The interaction strength between sites is assumed to fall off according to an inverse-square law as follows:

$$J_{ij} = \begin{cases} J_0/((j-i)D)^2, & \text{if } j - i < d_l \\ 0, & \text{else} \end{cases} \tag{3}$$

where the interaction coefficient $J_0 < 0$ is a constant, and $D = d + d_0$ is the size of a single site. Since accelerating and decelerating behaviors of vehicles may be different, the interaction coefficient J_0 is postulated to be:

$$J_0 = \begin{cases} J_{\text{IN}}, & \text{if } \rho_i - \rho_{i+1} \le 0, j - i < d_l \\ J_{\text{OUT}}, & \text{if } \rho_i - \rho_{i+1} > 0, j - i < d_l \\ 0, & \text{else} \end{cases} \tag{4}$$

where ρ_i is an indicator of locally perceived density at site S_i, $J_{\text{IN}} < 0$ is a constant representing the interaction coefficient J_0 for vehicles entering a region of higher local density ($\rho_i \le \rho_{i+1}$), and $J_{\text{OUT}} < 0$ is a constant representing the interaction coefficient J_0 for vehicles exiting a region of higher local density ($\rho_i > \rho_{i+1}$). Mathematically, for a given look-ahead distance d_l, the perceived local density may be defined as:

$$\rho_i = \sum_{j \in \mathcal{N}(i)} \mathbb{1}_{\sigma_j} \tag{5}$$

where $\mathbb{1}_{\sigma_j}$ is the indicator function and (5) represents the total number of vehicles present within the neighborhood of vehicle i. Fig. 3 clarifies how the perceived density is modified for a vehicle entering or exiting a traffic jam.

Additionally, since the goal is to study mixed traffic flows, the expression for the interaction coefficient J_0 presented in (4) also has to account for differences between various vehicle species. Specifically, the interaction coefficients J_0^{H} and J_0^{ACC} are distinguished by the expected differences in behaviors of human-driven and ACC-enabled vehicles. The ideal behavior for ACC-enabled vehicles is assumed to be the avoidance of traffic jams, and if stuck in one, to try to exit it as soon as possible. Mathematically, this behavior translates to increased repulsion for ACC-enabled vehicles when approaching a traffic jam so that $|J_{\mathrm{IN}}^{\mathrm{ACC}}| > |J_{\mathrm{IN}}^{\mathrm{H}}|$, and reduced repulsion for ACC-enabled vehicles when exiting a traffic jam so that $|J_{\mathrm{OUT}}^{\mathrm{ACC}}| < |J_{\mathrm{OUT}}^{\mathrm{H}}|$. Intuitively, this behavior of ACC-enabled vehicles is expected to reduce the number and size of traffic jams formed on highways. The external field $B > 0$ indicates a positive forward driving field and is applied equally to all vehicles irrespective of whether they are human-driven or ACC-enabled. Note that this interpretation of the terms in the Hamiltonian is different from the interpretations presented in the prior work by Sopasakis [8]. The next subsection discusses the calculation of transition probability rates that dictate traffic dynamics, using the expression for the Hamiltonian given in (1).

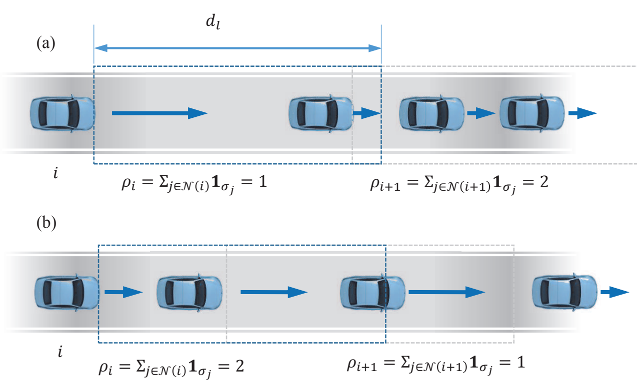

Fig. 3. Interaction coefficients for vehicles calculated by using differences in locally perceived density. It is assumed that this information is available from an infrastructure-to-vehicle (I2V) system. (a) Difference in perceived local density is negative for a vehicle i entering a traffic jam. (b) Difference in perceived local density is positive for a vehicle i exiting a jam.

D. Transition Probability Rates and Exchange Dynamics

Parallels can be drawn between the problem of modeling of traffic dynamics using the Potts model, and the study of driven Ising lattice gases [5][21]. Driven systems typically operate far-from-equilibrium (i.e. in a state of non-equilibrium) and while such system may reach a steady state, such a state is in continuous flux. Specifically, the microstates of the far-from-equilibrium system cannot be described by a stationary probability distribution. This is evident in traffic jams on a ring road, and especially so in a 'stop-and-go' waves, where the probability distribution of finding a system in a particular microstate in non-stationary. A consequence of the system operating far-from-equilibrium is that the condition for detailed balance is no longer inviolate [22]. The potential violation of the detailed balance condition allows for selection of transition probability rates based on observations of specific physical phenomena in the system being studied. For example, in a traffic system, microstate transitions always occur due to the forward motion of vehicles on the road, and never due to their backward motion. These observations allow reasoned estimates about potential transition probability rates to be made.

In order to determine the traffic flow dynamics using the Potts model, the microstate transition probability rates are calculated. Since the total number of vehicles on the ring road is conserved, the microstate transitions are limited to *exchange dynamics* [23]. Exchange dynamics imply that for a given state ($\boldsymbol{\sigma}$), the only possible means of system evolution is through an exchange of states between any two adjacent sites in the lattice structure, σ_i and σ_{i+1}. This evolution *may* occur if

$$\sigma_i \neq 0 \text{ and } \sigma_{i+1} = 0 \tag{6}$$

i.e., if the site ahead of a vehicle is unoccupied, it may move to the empty site based on the transition probability rates calculated using (7):

$$\begin{aligned} w(\boldsymbol{\sigma} \to \boldsymbol{\sigma'}) &= exp(-\beta H) \\ &= exp\left(\beta B\sigma_i + \beta \sum J_{ij}\sigma_i\sigma_j\right) \end{aligned} \tag{7}$$

where $\beta = \rho_i$, and the system microstates $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma'}$ differ by a pair of sites whose states have been exchanged. The parameter β is assumed to be proportional to local density and is conceptually analogous to its interpretation in physical systems such as magnets. In ferromagnets, a high value of $\beta = 1/k_BT$ corresponds to low temperature, wherein magnetic moments are forced to align with their neighboring moments. For a traffic system, a high value of $\beta = \rho_i$ corresponds to high local density, wherein vehicles are forced to stop if their neighboring vehicles are stationary.

The expression for the transition probability rates is similar to Arrhenius-type dynamics with an external field, but without an explicit diffusion coefficient. The probability of a randomly selected vehicle jumping to an empty site immediately ahead of it is thus given by:

$$P(\boldsymbol{\sigma}_{k+1} = \boldsymbol{\sigma'} | \boldsymbol{\sigma}_k = \boldsymbol{\sigma}) = min\{1, w(\boldsymbol{\sigma} \to \boldsymbol{\sigma'})\} \tag{8}$$

which is similar to the expression for the acceptance probability employed in the Metropolis-Hastings algorithm [24]. The remainder of the paper deals with using the Potts model and the associated transition probability rates as inputs for the Monte Carlo simulation to study traffic flow dynamics in the presence of human-driven and ACC-enabled vehicles.

E. Monte Carlo Simulations

Monte Carlo simulations make use of the Hamiltonian to calculate the local energy of the system about a particular site using (1), before utilizing (7) to calculate the microstate transition probability rates. The microstate transition probability directly relates to the probability that the traffic system will move from one particular configuration to another configuration and hence describes the traffic dynamics.

Before the Monte Carlo simulations can be carried out, the model parameters, i.e. the external field and the interaction coefficients, must be determined to ensure that the resulting behavior mimics the behavior observed in real-life traffic. In order to ensure the fidelity of the simulation, the typical traffic flow parameters discussed in Section III are used. The calibration procedure begins with using an initial configuration or microstate of the system in which human-driven vehicles are stuck in a queue. As vehicles begin to move out of the queue, they discharge at capacity, i.e. they exit the queue at free flow velocity (v_f). On the other hand, a signal can also be observed moving back through the queue at the backward wave speed (u). The calibration procedure consists of experimenting with several combinations of values of the external field B and the interaction coefficients $J^{\mathrm{H}}_{\mathrm{IN}}$ and $J^{\mathrm{H}}_{\mathrm{OUT}}$ for human-driven vehicles. After few trials, the approximate values of the parameters are found to be $B = 60$ m, $J^{\mathrm{H}}_{\mathrm{IN}} = -14,000$ m^3, and $J^{\mathrm{H}}_{\mathrm{OUT}} = -320,000$ m^3. Fig. 4 shows the results of the calibration that yield the correct typical traffic flow parameters, i.e. the free flow speed is found to be approximately 25 m/s (or 55 mph) and the backward wave speed is found to be approximately -6 m/s (or -13 mph).

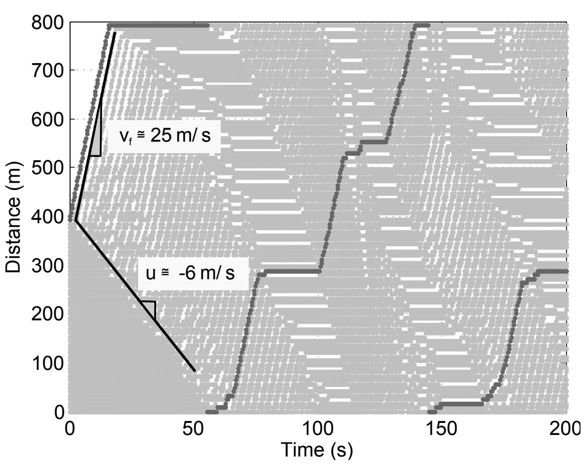

Fig. 4. Calibration for an initial configuration of the traffic system; v_f and u are respectively the free flow velocity and the backward wave speed of a vehicle exiting the queue. The choice of values of model parameters B and J^H_{ij} mimics real-life traffic behavior. Typical traffic flow patterns are obtained for $B = 60$ m, $J^{\mathrm{H}}_{\mathrm{IN}} = -14,000$ m^3, and $J^{\mathrm{H}}_{\mathrm{OUT}} = -320,000$ m^3.

The calibration performed above yields parameters B, $J^{\mathrm{H}}_{\mathrm{IN}}$ and $J^{\mathrm{H}}_{\mathrm{OUT}}$ only. The interaction coefficients for ACC-enabled vehicles can be determined using some intuition regarding their behavior while entering and exiting a traffic jam as discussed in Section IV-C. For the purposes of the Monte Carlo simulation, the parameter $J^{\mathrm{ACC}}_{\mathrm{IN}}$ is set to $-28,000$ m^3 and the parameter $J^{\mathrm{ACC}}_{\mathrm{OUT}}$ is set to $-20,000$ m^3. These parameters enable us to carry out the Monte Carlo simulations to mimic traffic flow dynamics in a mixed traffic flow. It must be noted that these parameter values provide us with only a qualitative assessment of the effects of increased ACC penetration. Future work will focus on estimating the

Algorithm 1 : Simulating traffic flow dynamics with Monte Carlo algorithm

Uniformly distribute M vehicles across N available sites to set up initial microstate $\boldsymbol{\sigma}$
while time $\leq$ ENDTIME **do**
  for all sites $S_i \in S$ **do**
    if $\sigma_i \neq 0$ and $\sigma_{i+1} = 0$ **then**
      Calculate local energy $H(\sigma_i)$ at site S_i
      Calculate probability $w = exp(-\beta H(\sigma_i))$
      Select a random number $r \sim U(0,1)$
      if $r < min(1, w)$ **then**
        Exchange σ_i and σ_{i+1}
        Accept new microstate
      end if
    end if
  end for
  time $\leftarrow$ time + 1
end while

appropriate values of interaction coefficients that provide a more accurate description of ACC car-following behavior. The pseudo code for the Monte Carlo algorithm used in the study is included in Algorithm 1.

V. Results and Conclusions

The Monte Carlo simulations were run for 12,000 time steps each, yielding an equivalent time of 1 hour per simulation. The simulations were run for two cases: low-density and high-density traffic. The normalized density for low-density traffic was chosen to be 0.1, whereas the normalized density for high-density traffic was chosen to be 0.6. Each density level was simulated for several different ACC penetration levels: ranging from low penetration (33% of vehicles are ACC-enabled), to complete penetration (100% of vehicle are ACC-enabled).

The results generated from the Monte Carlo simulations were used to determine the cumulative probability distribution $F_T(\cdot)$ of a randomly chosen vehicle having a maximum duration T of being inside a single cluster. The Monte Carlo simulations for low-density yielded expected and uninteresting results – no major cluster formation events were observed and the effect of introduction of ACC-enabled vehicles was negligible. On the other hand, the Monte Carlo simulations for high density traffic yielded more interesting results. The simulation results in Fig. 5 indicate that for higher ACC penetration rates the mean time a vehicle spends stuck in a cluster or jam is much smaller than for lower ACC penetration rates. Specifically, the lower mean time spent by an individual vehicle in a jam or cluster also translates to smaller cluster sizes, indicating that high ACC penetration rates lead to formation of several smaller clusters as opposed to fewer large clusters for low ACC penetration rates.

To aid in understanding the values of the mean time spent by a vehicle in a single cluster are calculated for the high-density scenario. The results are included in Table I and indicate what is visually evident from the distribution – that high ACC penetration rates on an average lead to less time spent by an individual vehicle in a single cluster, and by extension small-sized vehicle clusters.

TABLE I

MEAN TIME SPENT BY A VEHICLE IN A SINGLE CLUSTER FOR HIGH DENSITY TRAFFIC STREAM

% ACC	**Mean duration in single cluster (s)**
33	26.31
50	13.04
66	7.75
100	3.74

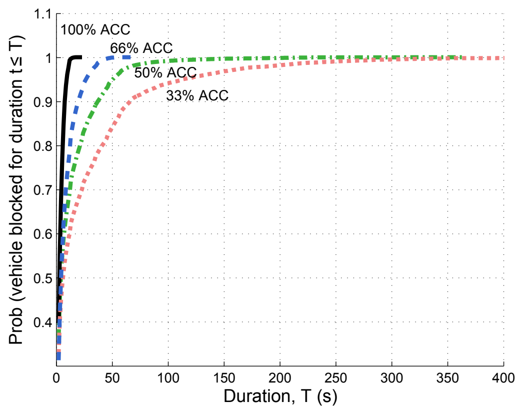

Fig. 5. Cumulative probability distribution $F_T(\cdot)$ of a randomly chosen vehicle having a maximum duration T of being inside a single cluster for traffic system operating at normalized density = 0.6 (high density) and at varying penetration levels of adaptive cruise control (ACC). Distributions are only shown up to the highest duration observed for an individual vehicle stuck inside a single cluster.

In conclusion, it was found that the Potts model could be used satisfactorily to simulate the traffic dynamics of mixed traffic flows. Additionally, it was found that introduction of ACC-enabled vehicles into high density traffic streams results in reduced time spent by vehicles in a single cluster, as well as formation of smaller clusters. The implications of this observation are detailed below. From a traffic control perspective, it is easier to handle and design highway elements to counter localized bottlenecks as in the case of high density-low ACC penetration traffic. On the other hand, having several small clusters that may appear at random locations and may be distributed across a large swathe of highway may require the development of more elaborate traffic control techniques due to the non-localized occurrence of these jams.

Additionally, this work implies that the inclusion of ACC-enabled vehicles into a mixed traffic stream might increase the risk of collisions. The presence of many more, but smaller, jams may lead to an increase in the amount of "stop-and-go" behavior that vehicles encounter on the roadway. Each of these stopping maneuvers carries the potential for a collision or disruption of traffic flow, especially from human drivers. Additionally, each stop-and-go maneuver acts as a nucleus for further disruption or collisions and could result in a decreased roadway capacity, mitigating the capacity increases engineered by the presence of ACC vehicles. Furthermore, increased numbers of acceleration and deceleration cycles may lead to an increase in the release of greenhouse gases and harmful emissions into the environment.

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