INFLUENTIAL SUBPACES OF CONNECTED VEHICLES IN HIGHWAY TRAFFIC

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ABSTRACT

This work introduces the novel concept of an influential subspace, with focus on its application to highway traffic containing connected vehicles. In this context, an influential subspace of a connected vehicle is defined as the region of a highway where the connected vehicle has the ability to positively influence the macrostate, i.e. the traffic jam, so as to dissipate it within a specified time interval. Analytical expressions for the influential subspace are derived using the Lighthill-Whitham-Richards theory of traffic flow. Included results describe the span of the influential subspace for specific traffic flow conditions and pre-specified driving algorithms of the connected vehicles.

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INTRODUCTION

In recent years, there have been significant developments in the ability to inform drivers about nearby traffic conditions, which often leads to the questions: can an individual driver use such information to affect traffic flow? And which drivers in a traffic network have the most influence on traffic flow, i.e. where and to whom should the information be delivered? This work introduces the concept of an *influential subspace* of a Connected Vehicle (*1*), which is defined as the region of a highway where the connected vehicle has the ability to drive the macroscopic state of traffic flow to a desired state within a pre-specified time. This concept is applicable to several physical, biological and engineered systems, and a general formulation will be presented in future publications. In this paper, analysis of the influential subspace is conducted specifically for a connected vehicle entering a self-organized traffic jam, using basic postulates of the Lighthill-Whitham-Richards (LWR) model of traffic flow (*2*) (*3*).

To better understand the concept, consider the following example with reference to Figure 1. Given a single-lane highway segment where no passing is allowed, assume that a spontaneous traffic jam has formed on one section so that the macroscopic state (or simply *macrostate*) of traffic flow in that region is the jammed state *J*. Next assume that the desired macrostate is free flow (state *A*), with known flow and density, that currently exists upstream of the traffic jam. Vehicles in this state are assumed to travel at the maximum permissible velocity, i.e. the free flow velocity (v_f), and cannot travel any faster. Now, as a thought experiment, consider the impact that a connected vehicle receiving information on downstream traffic conditions could have on the jammed state, for each of the four regions outlined in Figure 1.

In region 1 of Figure 1, a connected vehicle is sufficiently far from the jammed state so that its actions (such as slowing down to avoid the jam) have no positive effect on the jammed state – the jam would have dissipated by the time the connected vehicle of region 1 moves downstream. In region 2, a connected vehicle could slow down to avoid the traffic jam, and this action could result in fewer vehicles entering the jammed state. As a result, *region 2 represents the influential subspace*. However, as a connected vehicle moves closer to the jammed state *J*, its influence decreases, and in region 3, the connected vehicle cannot escape the jam by slowing down and its actions have no positive influence on the jammed state. Finally, in region 4, a vehicle may decide to exit the jam slower rather than at the free flow velocity, resulting in a negative influence and a persistent jammed state. The outcomes of this thought experiment are validated using the LWR model in later sections.

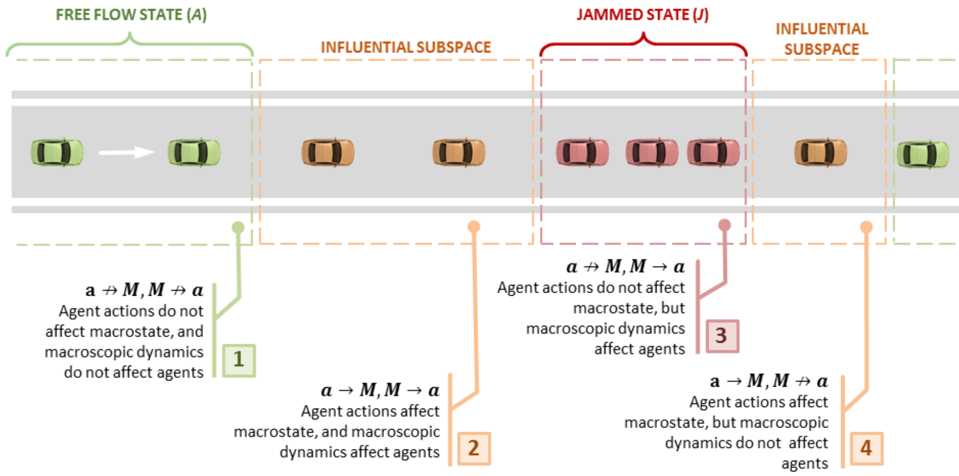

FIGURE 1 Thought experiment for understanding the concept of influential subspaces of connected vehicles in highway traffic. White arrow indicates direction of travel.

As Connected Vehicles technology becomes sufficiently advanced and begins to enter the mainstream, it is imperative that the research community helps fully realize its potential and efficacy. Prior work on connected vehicles has primarily focused on communication protocols and vehicular network topologies. While this research is important, it produces few research insights into the potential impact of connected vehicles on traffic flow. Recent work on the impact of mixed traffic on self-organized jams (*4*), effect of individual driving strategies on traffic flow (*5*), cooperative adaptive cruise control (*6*), and cooperative highway driving (*7*) have all briefly touched on various aspects of how individuals affect macroscopic traffic flow dynamics. However, these research efforts do not address the traffic system from the perspective of influential subspaces of connected vehicles. The following section presents the framework within which the concept of influential subspaces will be introduced.

PROBLEM SETUP

The problem is setup as a single-lane highway where no passing is allowed. Representative values of traffic flow parameters such as maximum flow ($q_{max} = 1800$ veh/hr), jam density ($k_j = 110$ veh/km), and free flow velocity ($v_f = 90$ km/hr) are used, assuming a triangular relationship between flow and density. The analysis uses standard results of the LWR model by drawing time-space diagrams to identify the time taken for the traffic flow to reach a desired macrostate, viz. one where the traffic system is operating in a free flow state.

To keep the analysis simple, only two connected vehicles are considered in the presented work. At time $t = 0$, the first connected vehicle (*CV1*) enters the jam and sends an alert signal indicating a jammed state to the connected vehicle upstream, which receives the signal instantaneously. The reception of the alert signal from *CV1* causes an event-triggered control action in *CV2*, which slows down to a pre-determined speed v_s as selected by the driver or dictated by an inbuilt cruise control algorithm. When *CV1* exits the traffic jam at time $t = t_{EXIT}$, it sends another alert signal upstream.

This alert results in a second event-triggered control action in *CV2* due to which it speeds up to free flow velocity v_f. Depending on the location of the second connected vehicle *CV2*, its control policy, i.e. the combined event-triggered actions of slowing down and speeding up, may or may not have an effect on the macrostate. The next section discusses several explanatory cases similar to the ones described in Figure 1 that make the problem setup clearer.

INFLUENTIAL SUBSPACES OF CONNECTED VEHICLES

For the following example, the traffic system is assumed to be operating at traffic state A given by $q_A = 900$ veh/hr and $k_A = 10$ veh/km. Without loss of generality, it may be assumed that the first connected vehicle *CV1* enters the spontaneous traffic jam and sends the alert signal at time $t = 0$. Upon receiving the signal, the second connected vehicle *CV2* is assumed to slow down to a predetermined speed $v_s = 10$ km/hr in order to avoid the traffic jam. This results in a slow-moving state S given by $q_S \approx 733$ veh/hr and $k_S = 73$ veh/km.

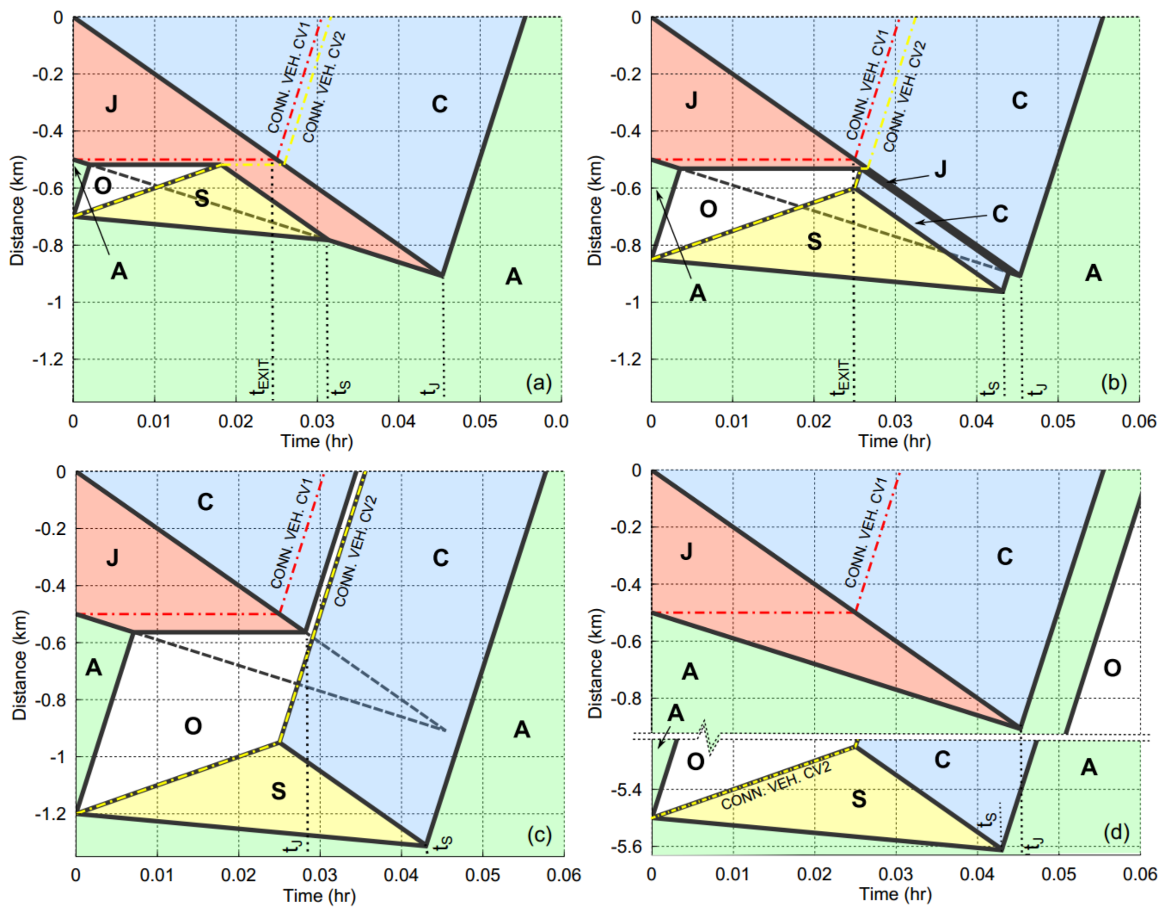

FIGURE 2 Time-space diagram when distance between the connected vehicles *CV1* and *CV2* is (a) 200 m, (b) 350 m, (c) 700 m, and (d) 5000 m, for $v_s = 10$ km/hr. In cases (a) and (b), the actions of *CV2* have no positive impact on time taken for traffic to return to macrostate *A*. In case (c), the slowing down by *CV2* causes a more rapid return to macrostate *A* (e.g. jam-free traffic flow) – *CV2* is in its influential subspace. In case (d), the vehicle *CV2* has no positive impact on the macrostate – the jam has already dissipated. Dashed line indicates jam evolution without connected vehicles. Dash-dotted lines are vehicle trajectories of connected vehicles.

Interpretation of Time-space Diagrams

With reference to Figures 1 and 2, cases (a) and (b) correspond to region 3 in Figure 1. In these cases, the actions of the vehicle *CV2* have no positive effect on the time it takes to return to the desired macrostate *A*. In both cases, the jammed state *J* dissipates at time t_J, independent of the presence of connected vehicles in the traffic stream. Case (c) in Figure 2 corresponds to region 2 in Figure 1, where the actions of vehicle *CV2* cause the traffic system to reach the desired macrostate *A* faster. Specifically, the slow-moving state *S* vanishes at time t_S, whereas the jammed state vanishes at time $t_J < t_S$. Thus, there is a net reduction in the time taken for the traffic flow to return to the desired macrostate *A*. Finally, the case (d) in Figure 2 corresponds to region 1 of Figure 1, where the actions of vehicle *CV2* have no positive impact on the time taken to return to macrostate *A*, since the jammed state *J* dissipates of its own accord.

Analytical Solution of Influential Subspaces

Mathematically, the time taken for the traffic system to reach the desired macrostate *A* is given by:

$$t_A = \max\{t_J, t_S\} \qquad [1]$$

where t_J denotes the time taken for the jammed state *J* to dissipate, and t_S represents the time taken for the slow-moving traffic state *S* to vanish. In other words, the time taken to reach the desired macrostate *A* is governed by which of state *J* or *S* persists for a longer period of time. The mathematical expressions for t_J and t_S can be calculated from geometric considerations of Figure 2.

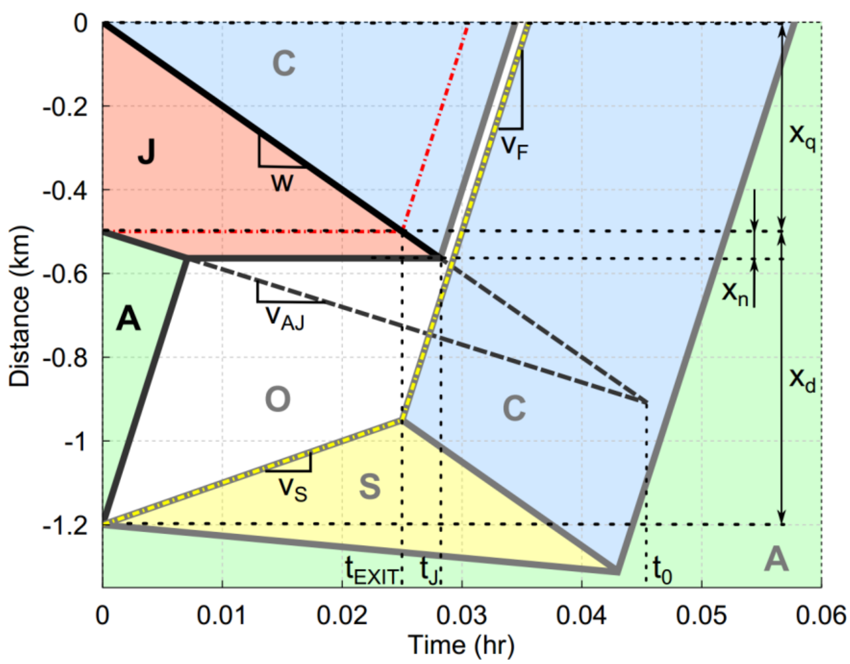

FIGURE 3 Evaluation of t_J using space-time diagram. Only relevant quantities needed for deriving analytical solution are labeled.

Expression for Dissipation Time t_J of Jammed State J

Specifically, first consider the evaluation of t_J with reference to Figure 3 (or Figure 2(c)). In this scenario, the time taken for the jammed state *J* to dissipate is a function of the original queue length x_q at time $t = 0$, the distance between the connected vehicles x_d at time $t = 0$, and the traffic state *A* that exists upstream of the jammed state *J*. The expression for t_J in Figure 3 is given by [2] as follows:

$$t_J = \frac{x_q + x_n}{w} \tag{2}$$

where x_n is the length of the roadway occupied by new vehicles entering the jammed state *J* after time $t = 0$, and w is the backward wave speed obtained from the triangular fundamental diagram. The quantity x_n is determined by assuming that the number of vehicles is conserved on the roadway. Specifically, under this assumption, the number of vehicles between the two connected vehicles *CV1* and *CV2* can be calculated to be:

$$\text{Number of vehicles between } CV1 \text{ and } CV2 = x_d k_A = x_n k_J \;\Rightarrow x_n = \frac{x_d k_A}{k_J} \tag{3}$$

where x_d is the distance between the connected vehicles *CV1* and *CV2*, and k_A and k_J represent the densities of traffic flow in states *A* and *J*, respectively. Consequently, the expression in [2] can be expanded to yield:

$$t_J = \frac{x_q + x_d k_A / k_J}{w} \tag{4}$$

However, this expression is correct only for a specific region of the roadway. The analytical expressions demarcating this specific region can be found by a careful analysis of Figure 3. Note that the expression for t_J in [4] becomes valid in situations similar to Figure 3, when the second connected vehicle just manages to avoid the jammed state *J*, and stays valid till situations similar to Figure 2(d), when the last vehicle ahead of the vehicle *CV2* just manages to avoid the jammed state *J*. To evaluate the lower spatial limit, i.e. in the case when the second connected vehicle just manages to avoid the jammed state *J*, the expression [4] becomes valid if:

$$x_d - x_n \geq v_s t_{EXIT} + v_f (t_J - t_{EXIT}) \tag{5}$$

where v_s represents the speed that the second connected vehicle *CV2* slows down to, v_f represents the free flow velocity, and t_{EXIT} $(= x_q / w)$ represents the time at which the first connected vehicle *CV1* exits the jammed state *J*. The expression in [5] may be simplified to be written as:

$$x_d - x_n \geq v_s \frac{x_q}{w} + v_f \left(\frac{x_q + x_d k_A / k_J}{w} - \frac{x_q}{w} \right) \tag{6}$$

or,

$$x_d - x_n \geq v_s \frac{x_q}{w} + \frac{v_f}{w} \left(\frac{x_d k_A}{k_J} \right) \tag{7}$$

or,

$$x_d - x_d \frac{k_A}{k_J} - x_d \left(\frac{v_f k_A}{w k_J} \right) \geq v_s \frac{x_q}{w} \tag{8}$$

or,

$$x_d \geq \left\{ 1 - \left(1 + \frac{v_f}{w} \right) \frac{k_A}{k_J} \right\}^{-1} \left(v_s \frac{x_q}{w} \right) \tag{9}$$

The upper spatial limit for the validity of expression [4] is evaluated in the scenario when the second connected vehicle *CV2* is sufficiently upstream so that last vehicle just ahead of *CV2* reaches the jammed state at time t_0, i.e. when the jam is just about to dissipate of its own accord. Thus, the upper spatial limit is given simply by:

$$x_q + x_n \leq wt_0 \tag{10}$$

or,

$$\frac{k_A}{k_J} x_d \leq wt_0 - x_q \tag{11}$$

or,

$$x_d \leq \frac{k_J}{k_A}\left(wt_0 - x_q\right) \tag{12}$$

On the other hand, in Figures 2 (a), (b), and (d), the expression for t_J is obtained quite simply from the original jam dissipation time t_0 evaluated in the absence of any connected vehicles. The jam evolution trajectory is indicated using dashed lines in Figures 2 and 3. In these scenarios, the jam dissipation time $t_J = t_0$ and is found as follows:

$$\text{Distance traveled} = \ wt_0 = x_q + v_{AJ}t_0 \Rightarrow t_0 = \frac{x_q}{w - v_{AJ}} \tag{13}$$

where v_{AJ} represents the interface speed between traffic states *A* and *J*. Consequently, the expression for time taken for dissipation of the jammed state *J* is given by combining the expressions in [4], [9], [12], and [13] to yield:

$$t_J = \begin{cases} \dfrac{1}{w}\left\{x_q + x_d\left(\dfrac{k_A}{k_J}\right)\right\}, & \text{if } \left\{1 - \left(1 + \dfrac{v_f}{w}\right)\dfrac{k_A}{k_J}\right\}^{-1}\left(v_s\dfrac{x_q}{w}\right) \leq x_d \leq \dfrac{k_J}{k_A}\left(wt_0 - x_q\right) \\ \dfrac{x_q}{w - v_{AJ}}, & \text{else} \end{cases} \tag{14}$$

Expression for Dissipation Time t_S of Slow-moving State S

Similar geometric arguments can be used to determine the expression for the time taken for the slow-moving traffic state *S* to dissipate. Specifically, consider Figure 4 (or Figure 2(a)) in order to ascertain the analytical expressions.

If the second connected vehicle *CV2* is too close to the first one, as depicted in Figure 4 (or Figure 2(a)), it enters the jam and the dissipation time for state S is governed by this distance. In alternative scenarios, when the vehicle *CV2* is further upstream, the dissipation time is constant, as evinced by Figures 2 (b), (c), and (d). In Figure 4, the dissipation time of the slow-moving state can be evaluated by geometric calculations as follows:

$$\text{Distance} = \ v_S t_{HIT} - w(t_S - t_{HIT}) = v_{AS}t_S \ \Longrightarrow t_S = \left(\frac{v_S + w}{v_{AS} + w}\right) t_{HIT} \tag{15}$$

where t_{HIT} is the time at which the vehicle *CV2* first enters the jammed state *J*, and v_{AS} is the interface speed between the states *A* and *S*. The expression for t_{HIT} can be found using geometric considerations to be:

$$t_{HIT} = \frac{x_d - x_n}{v_S} = \frac{x_d}{v_S}\left(1 - \frac{k_A}{k_J}\right) \tag{16}$$

so that the dissipation time of state *S* when *CV2* is close to the jam is given by:

$$t_S = \left(\frac{v_S + w}{v_{AS} + w}\right)\left(1 - \frac{k_A}{k_J}\right)\frac{x_d}{v_S} \qquad [17]$$

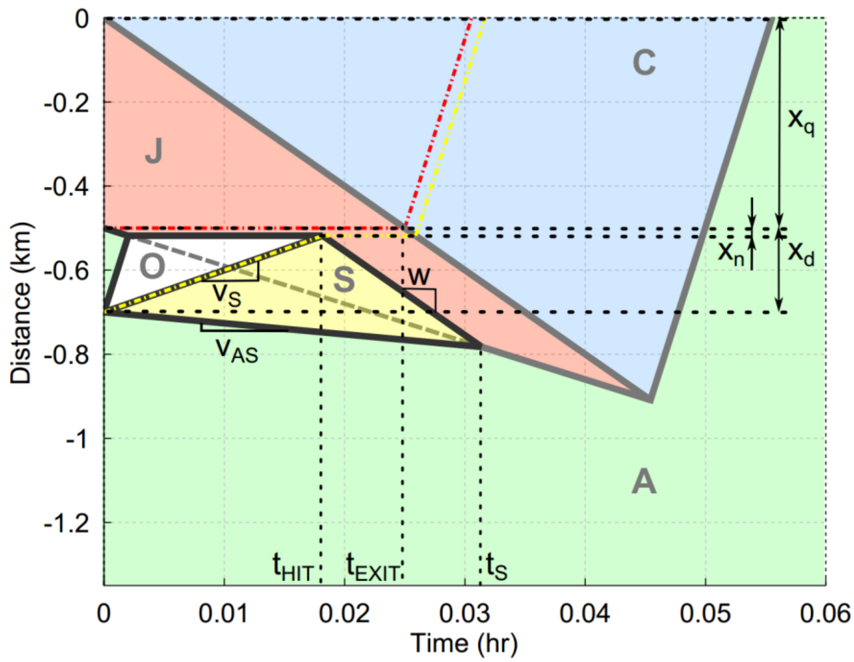

FIGURE 4 Evaluation of t_S using space-time diagram. Only relevant quantities needed for deriving analytical solution are labeled.

On the other hand, in Figures 2 (b), (c), and (d), where the vehicle *CV2* is further upstream, the dissipation time for the state *S* can be calculated similarly as follows:

$$\text{Distance} = v_S t_{EXIT} - w(t_S - t_{EXIT}) = v_{AS} t_S \implies t_S = \left(\frac{v_S + w}{v_{AS} + w}\right) t_{EXIT} \qquad [18]$$

where t_{EXIT} is the time at which the first connected vehicle exits the jammed state *J*, and which can be found using geometric considerations to be:

$$t_{EXIT} = \frac{x_q}{w} \qquad [19]$$

so that the dissipation time of state *S* when *CV2* is close to the jam is given by:

$$t_S = \left(\frac{v_S + w}{v_{AS} + w}\right)\frac{x_q}{w} \qquad [20]$$

Consequently, by observing the nature of t_S across the various parts of Figure 2, it is realized that the expression for the dissipation time for the slow-moving state *S* is:

$$t_S = \min\left\{\left(\frac{v_S + w}{v_{AS} + w}\right)\left(1 - \frac{k_A}{k_J}\right)\frac{x_d}{v_S}, \left(\frac{v_S + w}{v_{AS} + w}\right)\frac{x_q}{w}\right\} \qquad [21]$$

To recapitulate the major result of this work, the time taken for the traffic system to reach the desired macrostate *A*, is given by:

$$t_A = \max\{t_J, t_S\} \tag{22}$$

where the expressions for t_J and t_S are provided in [14] and [21], respectively.

RESULTS

An influential subspace is defined for a specific agent or vehicle in a multi-agent system. The influential subspace is defined by the ability of the specific agent or vehicle to drive the system to a desired macrostate (A) within a predetermined time (t_{DES}). In this example, the time taken for the traffic system to reach the macrostate A is calculated for varying distances between the connected vehicles *CV1* and *CV2*. If the goal is to reach the macrostate A within, say t_{DES} =160 s, then the influential subspace for *CV2* is situated between 0.5 km and 4.3 km from the vehicle *CV1*, as indicated in Figure 5. On the other hand, if the goal is to reach the macrostate A within, say t_{DES} = 100 s, then it can be said that the influential subspace is empty, or it does not exist.

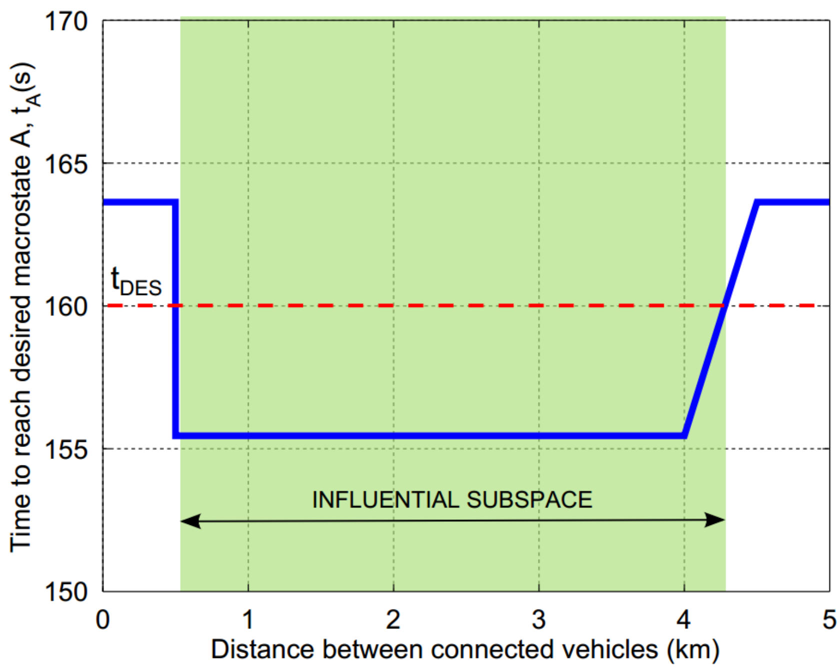

FIGURE 5 Influential subspace of the second connected vehicle, given the desired macrostate A and pre-specified time t_{DES}.

Knowledge of the influential subspace is a critical element for the efficient implementation of connected vehicles technology. Implementation of connected vehicles technology will have to deal with, among other things, issues such as bandwidth limitations and packet transmission ranges. Consequently, knowledge of the influential subspace can help ensure that bandwidth is not wasted by transmitting packets to vehicles that are not in their influential subspaces. Additionally, the same knowledge can help optimally route packets to vehicles within the influential subspaces and reduce power requirements for transmission equipment. Further, the concept of influential subspaces has significant potential applications in other areas such as cooperative adaptive cruise control, where formation, merging, and splitting of platoons can benefit from the use of this novel concept.

REFERENCES

(1) Research and Innovative Technology Administration, U.S. Department of Transportation, *Connected Vehicle Research*, www.its.dot.gov/connected_vehicle/connected_vehicle.htm, Accessed Mar. 24, 2014.

(2) Lighthill, M. J., and Whitham, G.B. On Kinematic Waves. II. A Theory of Traffic Flow on Long Crowded Roads. *Proceedings of the Royal Society A*, Vol. 229, No. 1178, 1955, pp. 317-345.

(3) Richards, P. Shock waves on the highway. *Operations Research*, Vol. 4, No. 1, 1956

(4) Jerath, K., and Brennan, S. N. Analytical Prediction of Self-organized Traffic Jams as a Function of Increasing ACC Penetration. *IEEE Transactions on Intelligent Transportation Systems*, Vol. 13, No. 4, 2012, pp. 1782-1291.

(5) Nishi, R., Tomoeda, A., Shimura, K., Nishinari, K. Theory of jam-absorption driving. Transportation Research Part B: Methodological. Vol. 50, 2013, pp. 116-129.

(6) Shladover, S., et al. *Effects of cooperative adaptive cruise control on traffic flow*. California PATH Program, University of California, Berkeley, 2009.

(7) Monteil, J., Billot, R., Sau, J., Armetta, F., Hassas, S., and El Faouzi, N-E. Cooperative Highway Traffic. In Transportation Research Record: Journal of the Transportation Research Record, Vol. 2391, Transportation Research Board of the National Academies, Washington, D.C., 2014.