TTK4130 - Exercise 1

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Problem 1

a

Know that:

$$Ni = \phi(R_a + R_c + R_b + R_r)$$

where

$$R_a = \frac{z}{A\mu_0}$$

Under the assumption that R_c and R_b are negligible

$$Ni = \phi(R_a + R_r)$$

And the return path is constant denoted z_0

$$R_r = \frac{z_0}{A\mu_0}$$

Meaning the total magnetomotive force can be expressed as:

$$Ni = \phi\frac{z + z_0}{A\mu_0}$$

b

Given the inductance

$$L(z) = \frac{N^2 A\mu_0}{z + z_0}$$

We find that

$$\frac{dL}{dz} = -\frac{N^2 A\mu_0}{(z + z_0)^2}$$

Which gives the magnetic force

$$F = -\frac{i^2}{2}\frac{N^2 A \mu_0}{(z+z_0)^2}$$

Which agains make the total force on the ball

$$\sum F = G + F = mg - \frac{i^2}{2}\frac{N^2 A \mu_0}{(z+z_0)^2}$$

Netwons 2. law states

$$\sum F = m\ddot{z}$$

Combining this gives the following equation of motion

$$\ddot{z} = g - \frac{1}{m}\frac{i^2}{2}\frac{N^2 A \mu_0}{(z+z_0)^2}$$

c

Our equation is on the form

$$\ddot{z} = f(z, i)$$

A linearization at (z_d, i_d) can then be expressed as

$$\ddot{z} = f(z_d, i_d) + (z - z_d)\left.\frac{df}{dz}\right|_{(z_d, i_d)} + (i - i_d)\left.\frac{df}{di}\right|_{(z_d, i_d)}$$

where

$$f(z_d, i_d) = g - \frac{i_d^2}{2m}\frac{N^2 A \mu_0}{(z_d+z_0)^2}$$

$$\left.\frac{df}{dz}\right|_{(z_d, i_d)} = \frac{i_d^2}{m}\frac{N^2 A \mu_0}{(z_d+z_0)^3}$$

$$\left.\frac{df}{di}\right|_{(z_d, i_d)} = -\frac{i_d}{m}\frac{N^2 A \mu_0}{(z_d+z_0)^2}$$

combining equations give the linearized system

$$\ddot{z} = g - \frac{i_d}{m}\frac{N^2 A \mu_0}{(z_d+z_0)^2}\left[\frac{i_d}{2} + (z - z_d)\frac{i_d}{z_d + z_0} - (i - i_d)\right]$$

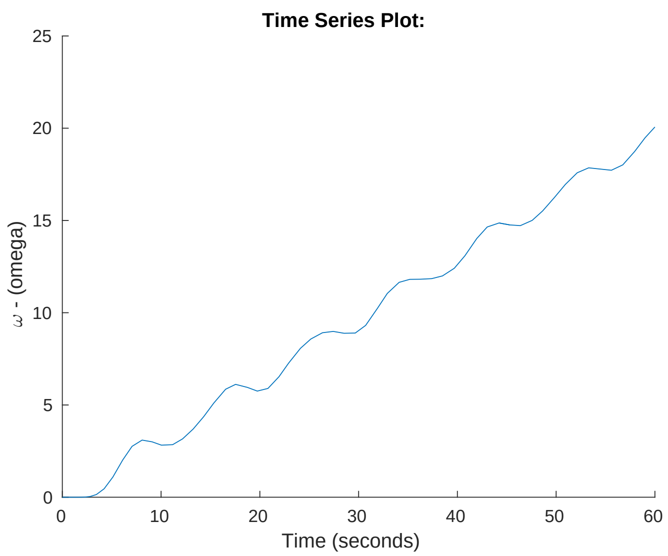
Problem 2

b

For it to fit with the motor model computationaly it needs ω_{i-1} as input and to fit with next load it needs ω_i as output. It also need to have T_{i-1} as output and T_i as input. For energy flow it would possibly be more natural to have T_{i-1} and ω_{i-1} as inputs and T_i and ω_i as outputs.

d

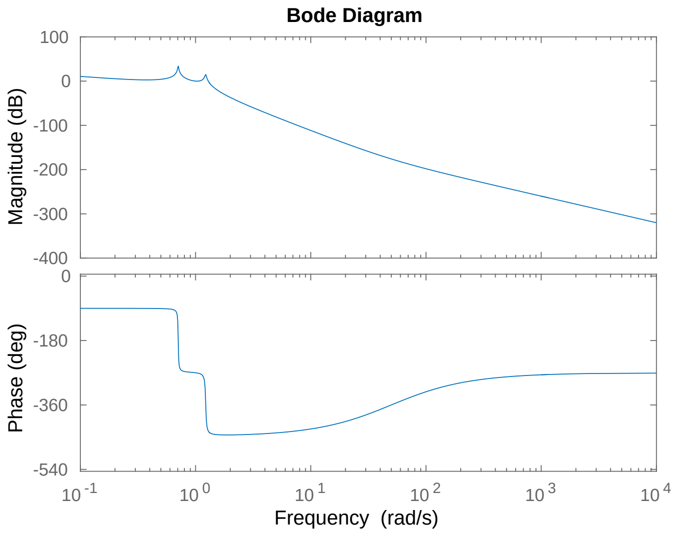
When simulating it seems like the last load will just increase in speed until infinity. This is not the simulations fault but our simplified motor model without any back-emf or similar.



Figur 1: Rotational speed of last load in matlab

e

The most obvious thing to comment is that the system is unstable.

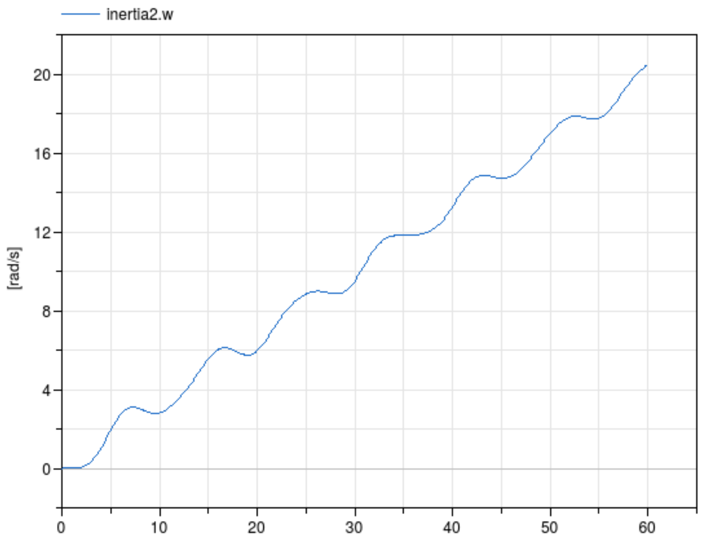


Figur 2: Bode plot done with matlab

Problem 3

a

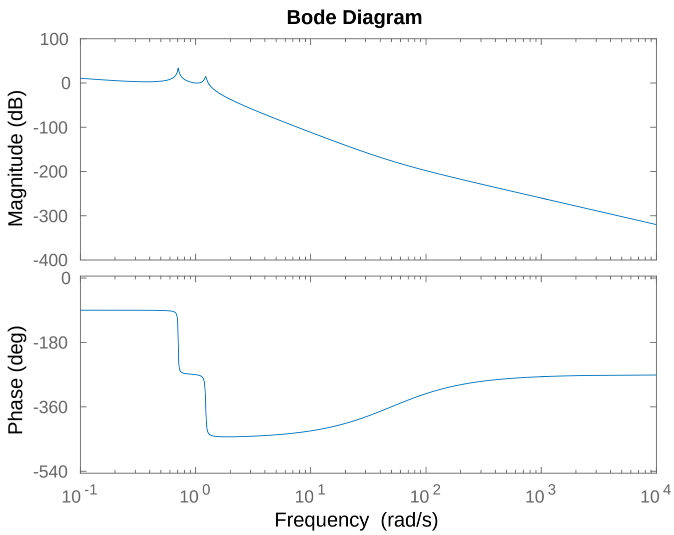
The plot of omega seems pretty much the same. Dymola gives a much more flexible user experience than matlab.



Figur 3: Rotational speed of last load in dymola

c

The bode plots seems identical.



Figur 4: Bode plot done with dymola