TTK4130 - Exercise 2

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Problem 1

a

There are 5 variables, so 5 equations should be implemented.

Problem 2

For a rational transfer function to be positive real it must satisfy:

1. All the poles of $H(s)$ have real parts less than or equal to zero.
2. $Re(H(j\omega)) \geq 0$ for all ω so that $j\omega$ is not a pole of $H(s)$.
3. If $j\omega_0$ is pole in $H(s)$, then it is a simple pole, and
$$Res_{s=j\omega_0}[H(s)] = \lim_{s \to j\omega_0} (s - j\omega_0)H(s)$$
is real and positive. If $H(s)$ has a pole at inifinity, then it is a simple pole, and
$$R_\infty = \lim_{\omega \to \infty} \frac{H(j\omega)}{j\omega}$$

a

In regards to the transfer function:

$$H_1(s) = \frac{1}{1 + Ts}$$

It is obvious that all poles have real parts less then or equal to zero as long as $T > 0$. The frequency response

$$H_1(j\omega) = \frac{1 - j\omega T}{1 + (\omega T)^2}$$

has positive real parts for all ω

$$Re(H_1(j\omega)) = \frac{1}{1+(\omega T)^2}$$

And it got no poles on the imaginary axis. Thus H_1 is real positive.

The other transfer function:

$$H_2(s) = \frac{s}{s^2+\omega_0^2}$$

Has poles only on the imaginary axis, $j\omega$ and $-j\omega$. The first and second criteria is thus satisfied but the third criteria remains.

$$Res_{s=\pm j\omega_0} H(s) = \frac{1}{2}$$

Thus H_2 is real positive.

b

Consider the transfer function

$$H_3(s) = \frac{s+a}{(s+b)(s+c)}$$

Since $c > 0$ and $b > 0$ the first criteria is satisfied and the thrid criteria is not applicable. Considering the second criteria:

$$Re(H_3(j\omega)) = \frac{abc - a\omega^2 + b\omega^2 + c\omega^2}{(\omega^2+b^2)(\omega^2+c^2)}$$

If $a \leq b+c$ the the criteria is satisfied, and if $a > b+c$ it fails. Meaning it is positive real for all $a \leq b+c$

c

Considering the transfer function

$$H_4(s) = \frac{s^2+a^2}{s(s^2+\omega_0^2)}$$

The first criteria is satisfied since all poles are on the imaginary axis. The second criteria is also satsified since $Re(H_4(j\omega)) = 0$. The third criteria requires that the following are real and positive:

$$Res_{s=0}[H_4(s)] = \frac{a^2}{\omega_0^2}$$

$$Res_{s=\pm j\omega_0}[H_4(s)] = \frac{\omega_0^2 - a^2}{2\omega_0^2}$$

Thus if $a < \omega_0$ then the transfer function is positive real.

d

By chosing the storage function

$$V = \frac{1}{2}Ty^2$$

i get

$$\dot{V} = Ty = uy - y^2$$

Which obviously satisfy

$$\dot{V} = uy - g(y)$$

where $g(y) = y^2$ which is positive definite.

e

It's already given that all poles are in the left half plane thus the first criteria is satisfied and the third is not applicable. So wether $H(s)$ is positive real is the same as if the second criteria is satisfied, namely if $Re[H(j\omega)] \geq 0$ for all $\omega \neq 0$