Project 3 FYS4150

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Abstract

The program used in this project can be found at Github.

Test: Energy conservation, modulus "position" (lengde vektor) bevart Alle vectorer samme str.

Printe + plotte energi stabilitet mellom euler og verlet.

To do: OBS Unit tests

3b: Forklare objektorientering, hvorfor deler kan generaliseres.

3c: Plotte ulike dt-er Plott energi som funk av ulike dt Referere til convergens - funksjonen. Vise at angulærmoment bevart: Oppdatere til L = mr x v i Planet + egen verdi tilhørene hver planet Vise tid ulike algoritmer.

3d: Exact løsning escape vel Plots ulike init.hastigheter Bytte gravitasjonskrefter...

3e- 3body 3 ulike masser Plotte alle banene Stabilitet: Energi-plot

3f Hvordan velge origio???????? IKKE GJORT NOENITING FORELØPIG

3g: IKKE GJORT NOENITING FORELØPIG FLOPS euler/Verlet

Konklusjon: Kunne strukturert annerledes: Solver class - kun euler, verlet Egen klasse for å kjøre algoritmer i (tidsteg, skrive fil)

Contents

1 Introduction

2 Theory

Utgangspunkt - oppgavetekst Også relativistisk effekter!

scaling, enheter

Nevne approksimasjon med barrycentre v sol i sentrum v pos sol @ t=0: Vi har ingen beregninger med sol i sentrum, men bruker sol ved t=0 som posisjon.

Hvorfor bør egentlig være barrycentre til univers?

Can the observed perihelion precession of Mercury be explained by the general theory of relativity?

2.1 Classical Solar system

In the classical description of the solar system, there is only a single force working:

$$F_G = -G\frac{M_1 M_2}{r^2} \tag{1}$$

By applying Newtons 2.law on component form we achieve two more equations, in the case of a two dimensional system.

$$\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{\vec{F}_G}{M_2} \tag{2}$$

Equation 2 is in reality two seperate, independent equations, one for the x directon and the other for the y direction.

Sentrifugal: $a = \frac{v^2}{r}$.

3 Units and scaling

A computer has a limited bit-resolution and the distances and timescale are large when computing the solar system. This means that it is important to use appropriate units. The distance between the sun and the earth is defined as 1 Astronomical unit (1 Au) and the timescale used in this project will be in units of 1 year.

We know that the earth needs one year to orbit the sun, meaning that $v = \frac{2\pi r}{1 \text{ year}}$. This can be rewritten with $v = \tilde{v}v_0$ and $r = \tilde{r}r_0$. The units of r and v are contained in $v_0 = \frac{1\text{Au}}{1 \text{ year}}$ and $r_0 = 1$ Au, giving that $\tilde{v}^2\tilde{r} = 4\pi^2$. In the same way $t = \tilde{t}t_0$, with $t_0 = 1$ year. For the rest of the paper, we will assume all variables to be dimensionless.

For the case of the earth - sun system one can assume that the sun is stationary, as $M_{sun} >> M_{earth}$. The force experienced by the earth is thus centrifugal, which means that $a = \frac{v^2}{r}$, with $v = 2\pi r/1year$. Combining this with equations 1 and 2 it is possible to scale the equations in the following manner:

$$a_E = \frac{F_E}{M_E} = G\frac{M_{sun}}{r^2} = \frac{v^2}{r} \tag{3}$$

$$GM_{sun} = v^2 r = 4\pi^2\frac{(1\text{ Au})^3}{(1\text{ year})^2} \tag{4}$$

$$\frac{\mathrm{d}\tilde{v}}{\mathrm{d}\tilde{t}} = \frac{4\pi^2}{\tilde{r}^2} \tag{5}$$

In the two dimensional system $r = (x, y) = (r\cos\theta, r\sin\theta)$. Using the notation $\dot{p} = \frac{\mathrm{d}p}{\mathrm{d}t}$, this gives the following coupled differential equations:

$$\dot{v}_x = \frac{4\pi^2 r\cos\theta}{r^3} = \frac{4\pi^2 x}{r^3} \tag{6}$$

$$\dot{v}_y = -\frac{4\pi^2 r\sin\theta}{r^3} = \frac{4\pi^2 y}{r^3} \tag{7}$$

$$\dot{x} = v_x \tag{8}$$

$$\dot{y} = v_y \tag{9}$$

$$\tag{10}$$

3.1 Energy considerations

As with any physical system, the total energy has to be conserved. The potential energy $E_p = \int_{r'}^{\infty}\vec{F}(r')\cdot d\vec{r}' = -\frac{GM_1M_2}{r}$, while the kinetic energy is $E_K = \frac{1}{2}mv^2$. This gives a total energy of:

$$E_{tot} = \frac{1}{2}mv^2 - \frac{GM_1M_2}{r} \tag{11}$$

In addition, the angular momentum of the system has to be preserved. This is because there are no additional sources of torque (τ) once the system has been initialized and $\tau = \frac{\mathrm{d}L}{\mathrm{d}t} = 0 \Rightarrow L =$ constant. **For a circular motion the angular momentum $\vec{L} = m(\vec{r}\times\vec{v})$ can be described as:**

$$L = Iv^2 r \tag{12}$$

For pointparticles: $I = r^2 m$ and for sircular motion $v = \frac{v}{r}$.

In order to maintain in the gravitational field of another object, the distance between them needs to be smaller than ∞. However, with a large enough velocity it is possible to escape, with the lowest velocity given when setting eguation 11 equal to 0:

$$\frac{1}{2}mv^2 - \frac{GM_1M_2}{r} = 0 \tag{13}$$

$$v_{escape} = \sqrt{\frac{2GM_1M_2}{mr}} \tag{14}$$

Any planet with a constant velocity equal to or grater than v_{escape} will escape

4 Method

Euler Velocity verlet FLOPS ulike metoder

UNIT TESTS

Classes Instanser deklerasjon friend class

5 Result

6 Discussion

7 Conclusion

References