Project 3 FYS4150

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Abstract

The program used in this project can be found at Github.

Test: Energy conservation, modulus "position" (lengde vektor) bevart Alle vectorer samme str.

Printe + plotte energi stabilitet mellom euler og verlet.

To do: OBS Unit tests

3b: Forklare objektorientering, hvorfor deler kan generaliseres.

3c: - Find out which initial velocity that gives a circular motion (plot) - Test stability (energy-stability) as function of dt (both Verlet and Euler) - Plot the earth orbiting the sun - Check (for the circular orbit) that the energy is conserved (plot - both kin and pot separated and together?) - Check that angular moment is conserved

- Discuss differences between Euler and Verlet - number of FLOPS + CPU time

* Plotte ulike dt-er * Plott energi som funk av ulike dt * Referere til convergens - funksjonen. * Vise at angulærmoment bevart

3d: - Find escape velocity (plot) - Compare with numerical results(Result? or Discussion?) - Find exact escape velocity (theory?) - Changing beta (plot) - Comment result + What happens when beta -> 3 ?

Exact løsning escape vel Plots ulike init.hastigheter Bytte gravitasjonskrefter... (hvordan loope for å få til det????)

3e: - How much does Jupiter alter Earth's orbit? - Position of Jupiter and Earth (plot) - Plot Earth's motion for increased mass of Jupiter (3 masses) - - Discuss stability of velocity verlet (3 body)

* 3 ulike masser * Plotte alle banene * Stabilitet: Energi-plot

3f: - Find center off mass - use as origin - Give sun initial velocity so momentum is zero (origin is fixed) - Compare with 3e) - Extend to all planets (plot) - Discuss difference 3e) and 3f) (3 body) - Discuss result of all planets

3g: - Find perihelion for both relativistic and non-relativisitc (table) - Relativistic - should be a few magnitudes smaller. - Can the observed perihelion precession of Mercury be explained by the general theory of relativity?

FLOPS euler/Verlet Result: - Find out which initial velocity that gives a circular motion (plot) - Test stability (energy-stability) as function of dt (both Verlet and Euler) - Plot the earth orbiting the sun - Check (for the circular orbit) that the energy is conserved (plot - both kin and pot separated and together?) - Check that angular moment is conserved

- Find escape velocity (plot) - Compare exact and numerical results - Find exact escape velocity (theory?) - Changing beta (plot) - Comment result + What happens when beta -> 3 ? (last part in discussion? - How much does Jupiter alter Earth's orbit? - Position of Jupiter and Earth (plot) - Plot Earth's motion for increased mass of Jupiter (3 masses) - Find center off mass - use as origin - Give sun initial velocity so momentum is zero (origin is fixed) - Compare with 3e) - Extend to all planets (plot) - Find perihelion for both relativistic and non-relativisitc (table) - Relativistic - should be a few magnitudes smaller. - Can the observed perihelion precession of Mercury be explained by the general theory of relativity?

Discussion: - Discuss differences between Euler and Verlet - number of FLOPS + CPU time - Discuss stability of velocity verlet (3 body) - Discuss difference 3e) and 3f) (3 body) - Discuss

result of all planets

Contents

1 Introduction

2 Theory

Sentrifugal: $a = \frac{v^2}{r}$.

We know that the earth needs one year to orbit the sun, meaning that $v = \frac{2\pi r}{1 \text{ year}}$. This can be rewritten with $v = \tilde{v}v_0$ and $r = \tilde{r}r_0$. The units of r and v are contained in $v_0 = \frac{1\text{Au}}{1 \text{ year}}$ and $r_0 = 1$ Au, giving that $\tilde{v}^2\tilde{r} = 4\pi^2$. In the same way $t = \tilde{t}t_0$, with $t_0 = 1$ year.

$$a_E = \frac{F_E}{M_E} = -G\frac{M_{sun}}{r^2} \tag{1}$$

$$= \frac{v^2}{r} \tag{2}$$

$$GM_{sun} = v^2 r = 4\pi^2 \frac{(1 \text{ Au})^3}{(1 \text{ year})^2} \tag{3}$$

$$\frac{\mathrm{d}\tilde{v}}{\mathrm{d}\tilde{t}} = -\frac{4\pi^2}{\tilde{r}^2} \tag{4}$$

For the rest of the paper we will assume all variables to be dimensionless. In a two dimensional system $r = (x, y) = (r\cos\theta, r\sin\theta)$. This gives the following parametrized relations :

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = -\frac{4\pi^2 r\cos\theta}{r^3} = -\frac{4\pi^2 x}{r^3} \tag{5}$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = -\frac{4\pi^2 r\sin\theta}{r^3} = -\frac{4\pi^2 y}{r^3} \tag{6}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_x \tag{7}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = v_y \tag{8}$$

$$\tag{9}$$

Angular momentum:

$$L = I\omega \tag{10}$$

$$= mrv \tag{11}$$

For pointparticles: $I = r^2 m$ and for sircular motion $v = \frac{v}{r}$.

3 Method

4 Result

with steps per year: 7*3600*360 Running velocity verlet Perihelion position after 100 years: 0.307498, -0.000933806 Perihelion angle after 100 years: -626.38 arc seconds CPU time: 3248.07

Result: - Find out which initial velocity that gives a circular motion (plot)

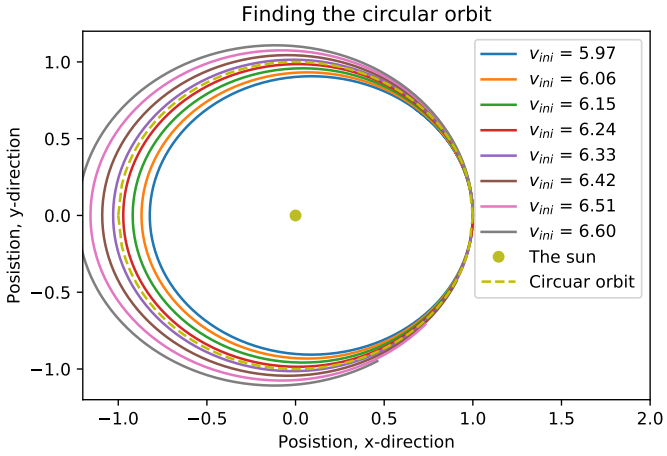

Figure 4.1: This is a plt of the orbit with different initial velocities. The circular orbit has a velocity between 6.24 and 6.33. $2\pi = 6.28$ and that is the initial velocity that gives a circular orbit.

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5 Discussion

6 Conclusion

References