Project 3 FYS4150

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Abstract

The program used in this project can be found at Github.

Test: Energy conservation, modulus "position" (lengde vektor) bevart Alle vectorer samme str.

Printe + plotte energi stabilitet mellom euler og verlet.

To do: OBS Unit tests

3b: Forklare objektorientering, hvorfor deler kan generaliseres.

3c: Plotte ulike dt-er Plott energi som funk av ulike dt Referere til convergens - funksjonen. Vise at angulærmoment bevart: Oppdatere til L = mr x v i Planet + egen verdi tilhørene hver planet Vise tid ulike algoritmer.

3d: Exact løsning escape vel Plots ulike init.hastigheter Bytte gravitasjonskrefter...

3e- 3body 3 ulike masser Plotte alle banene Stabilitet: Energi-plot

3f Hvordan velge origio???????? IKKE GJORT NOENITING FORELØPIG

3g: IKKE GJORT NOENITING FORELØPIG FLOPS euler/Verlet

Konklusjon: Kunne strukturert annerledes: Solver class - kun euler, verlet Egen klasse for å kjøre algoritmer i (tidsteg, skrive fil)

SPR: is the force, work positive/negative?????? PLAN: Torsdag: Kjetil: Teori, metode, (intro?), Fikse angulærmoment Vilde: Plots, andre resultater (CPU-tid...), (skrive resultater)

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1 Introduction

2 Theory

2.1 Classical Solar system

In the classical description of the solar system, there is only a single force working:

$$F_G = -G\frac{M_1 M_2}{r^2} \tag{1}$$

By applying Newtons 2.law on component form we achieve two more equations, in the case of a two dimensional system.

$$\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{\vec{F}_G}{M_2} \tag{2}$$

Equation 2 is in reality two seperate, independent equations, one for the x-direction and the other for the y-direction.

2.2 Units and scaling

A computer has a limited bit-resolution and the distances and timescale are large when computing the solar system. This means that it is important to use appropriate units. The distance between the sun and the earth is defined as 1 Astronomical unit (1 Au) and the timescale used in this project will be in units of 1 year.

For simplicity we will define a new unite of energy: $J_{ast} = \frac{m_{planet}}{|M_{sun}|}\left(\frac{Au}{year}\right)^2$, where $|M_{sun}|$. In reallity this is simply Joule with a prefactor. This prefactor can be obtained by inserting the values into J_{ast}:

$$\begin{aligned} J_{ast} &= \frac{m_{planet}}{|M_{sun}|}\left(\frac{Au}{year}\right)^2 && (3)\\ &= \frac{m_{planet}}{2\cdot 10^{30}}\left(|v|\frac{1.5\cdot 10^{11}m}{360*24*60*60s}\right)^2 && (4)\\ &\simeq 1.163\cdot 10^{-23}J && (5)\end{aligned}$$

The dimensionality of the variables are as follows: $[v] = m/s$ and $[m_{planet}] = kg$.

We know that the earth needs one year to orbit the sun, meaning that $v = \frac{2\pi r}{1\text{ year}}$. This can be rewritten with $v = \tilde{v}v_0$ and $r = \tilde{r}r_0$. The units of r and v are contained in $v_0 = \frac{1\text{Au}}{1\text{ year}}$ and $r_0 = 1$ Au, giving that $\tilde{v}^2\tilde{r} = 4\pi^2$. In the same way $t = \tilde{t}t_0$, with $t_0 = 1$ year.

For the case of the earth - sun system one can assume that the sun is stationary, as $M_{sun} >> M_{earth}$. The force experienced by the earth is thus centrifugal, which means that $a = \frac{v^2}{r}$, with $v = 2\pi r/1year$. Combining this with equations 1 and 2 it is possible to scale the equations in the following manner:

$$\begin{aligned} a_E = \frac{F_E}{M_E} = G\frac{M_{sun}}{r^2} = \frac{v^2}{r} && (6)\\ GM_{sun} = v^2 r = 4\pi^2\frac{(1\text{ Au})^3}{(1\text{ year})^2} && (7)\\ && (8)\end{aligned}$$

This gives that the dimensionless expression can be stated as:

$$\frac{\mathrm{d}\tilde{v}}{\mathrm{d}\tilde{t}} = \frac{4\pi^2}{\tilde{r}^2} \tag{9}$$

In the two dimensional system $r = (x,y) = (r\cos\theta, r\sin\theta)$. Using the notation $\dot{p} = \frac{\mathrm{d}p}{\mathrm{d}t}$, this gives the following coupled differential equations:

$$\begin{aligned} \dot{v}_x &= \frac{4\pi^2 r\cos\theta}{r^3} = \frac{4\pi^2 x}{r^3} && (10)\\ \dot{v}_y &= -\frac{4\pi^2 r\sin\theta}{r^3} = \frac{4\pi^2 y}{r^3} && (11)\\ \dot{x} &= v_x && (12)\\ \dot{y} &= v_y && (13)\\ & && (14)\end{aligned}$$

2.3 Energy considerations

As with any physical system, the total energy has to be conserved. The potential energy $E_p = \int_{r'}^{\infty}\vec{F}(r')\cdot d\vec{r}' = -\frac{GM_1M_2}{r}$, while the kinetic energy is $E_K = \frac{1}{2}mv^2$. This gives a total energy of:

$$E_{tot} = \frac{1}{2}mv^2 - \frac{GM_1M_2}{r} \tag{15}$$

In addition, the angular momentum ($\vec{L}$) of the system has to be preserved. This is because there are no additional sources of torque ($\vec{\tau}$) once the system has been initialized and $\vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = 0$. As a result all the absolute value of $\vec{L}$ hast to be constant.

In order to maintain in the gravitational field of another object, the distance between them needs to be smaller than ∞. However, with a large enough velocity it is possible to escape. By setting equation 15 equal to 0 one obtains that the escape velocity v_{esc} is:

$$\begin{aligned} \frac{1}{2}mv_{esc}^2 - \frac{GM_1M_2}{r} = 0 && (16)\\ v_{esc} = \sqrt{\frac{2GM_1M_2}{mr}} && (17)\end{aligned}$$

Any planet with a constant velocity equal to or grater than v_{esc} will escape the other mass object.

Nevne approksimasjon med barrycentre v sol i sentrum v pos sol @ t=0: Vi har ingen beregninger med sol i sentrum, men bruker sol ved t=0 som posisjon.

Hvorfor bør egentlig være barrycentre til univers?

Can the observed perihelion precession of Mercury be explained by the general theory of relativity?

3 Method

Euler Velocity verlet FLOPS ulike metoder

UNIT TESTS

Classes Instanser deklerasjon friend class

4 Result

with steps per year: 7*3600*360 Running velocity verlet Perihelion position after 100 years: 0.307498, -0.000933806 Perihelion angle after 100 years: -626.38 arc seconds CPU time: 3248.07

with steps per year: 2*7*3600*360

5 Discussion

6 Conclusion

References