Project 3 FYS4150

Kjetil Karlsen and Vilde Mari Reinertsen

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Abstract

In this report we have modelled the solar system using Newtons law of gravitation and, for the case of mercury, with a relativistic correction. In order to integrate the dynamics of the solar system both the Euler and the Velocity Verlet methods of integration has been used. Benchmarks and comparisons of energy and angular momentum conservation shows that the Velocity Verlet method is the more stable alternative. The program used in this project can be found at Github.

To do: OBS Unit tests

3c: - Find out which initial velocity that gives a circular motion (plot) - Test stability (energy-stability) as function of dt (both Verlet and Euler) - Plot the earth orbiting the sun - Check (for the circular orbit) that the energy is conserved (plot - both kin and pot separated and together?) - Check that angular moment is conserved

- Discuss differences between Euler and Verlet - number of FLOPS + CPU time

* Plotte ulike dt-er * Plott energi som funk av ulike dt * Referere til convergens - funksjonen. * Vise at angulærmoment bevart

3d: - Find escape velocity (plot) - Compare with numerical results(Result? or Discussion?) - Find exact escape velocity (theory?) - Changing beta (plot) - Comment result + What happens when beta -> 3 ?

Exact løsning escape vel Plots ulike init.hastigheter Bytte gravitasjonskrefter...

3e: - How much does Jupiter alter Earth's orbit? - Position of Jupiter and Earth (plot) - Plot Earth's motion for increased mass of Jupiter (3 masses) - - Discuss stability of velocity verlet (3 body)

* 3 ulike masser * Plotte alle banene * Stabilitet: Energi-plot

3f: - Find center off mass - use as origin - Give sun initial velocity so momentum is zero (origin is fixed) - Compare with 3e) - Extend to all planets (plot) - Discuss difference 3e) and 3f) (3 body) - Discuss result of all planets

3g: - Find perihelion for both relativistic and non-relativisitc (table) - Relativistic - should be a few magnitudes smaller. - Can the observed perihelion precession of Mercury be explained by the general theory of relativity?

FLOPS euler/Verlet Result: - Find out which initial velocity that gives a circular motion (plot) - Test stability (energy-stability) as function of dt (both Verlet and Euler) - Plot the earth orbiting the sun - Check (for the circular orbit) that the energy is conserved (plot - both kin and pot separated and together?) - Check that angular moment is conserved

- Find escape velocity (plot) - Compare exact and numerical results - Find exact escape velocity (theory?) - Changing beta (plot) - Comment result + What happens when beta -> 3 ? (last part in discussion? - How much does Jupiter alter Earth's orbit? - Position of Jupiter and Earth (plot) - Plot Earth's motion for increased mass of Jupiter (3 masses) - Find center off mass - use as origin - Give sun initial velocity so momentum is zero (origin is fixed) - Compare with 3e) - Extend to all planets (plot) - Find perihelion for both relativistic and non-relativisitc (table) - Relativistic - should be a few magnitudes smaller. - Can the observed perihelion precession of Mercury be explained by the general theory of relativity?

Discussion: - Discuss differences between Euler and Verlet - number of FLOPS + CPU time - Discuss stability of velocity verlet (3 body) - Discuss difference 3e) and 3f) (3 body) - Discuss

result of all planets

Contents

1 Introduction

There are many physical problems that deals with differential equations. A simple example of this is the ordinary differential equations that governs how objects move in relations to each other, for example the solar system. The aim of this project is to model our solar system using a relatively simple model, relativistic model for the movement of the planets and the sun. In order to do this efficiently to different integration methods will be explored, the Euler method and the Velocity Verlet method.

This report starts by the explaining basic theoretical physics that governs the movement of the objects in the solar system. These equations can be scaled and implemented in a general algorithm for solving 2. order differential equations. The two different integration models will then be discussed alongside how we chose to implement these in our experiment.

The results of benchmark and stability tests of the different methods will be discussed together with more general results of how the planets are moving. This is all wrapped up in a conclusion at the end.

2 Theory

2.1 Classical Solar system

In the classical description of the solar system, there is only a single force working:

$$F_G = -G\frac{M_1 M_2}{r^2} \tag{1}$$

By applying Newtons 2.law on component form, we achieve two more equations, in the case of a two dimensional system.

$$\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{\vec{F}_G}{M_2} \tag{2}$$

Equation 2 is in reality two separate, independent equations, one for the x-direction and the other for the y-direction.

2.2 Units and scaling

A computer has a limited bit-resolution and the distances and time scales are large when computing the solar system. This means that it is important to use appropriate units. The distance between the sun and the earth is defined as 1 Astronomical unit (1 Au) and the time scale used in this project will be in units of 1 year.

For simplicity we will define a new unite of energy: $J_{ast} = \frac{m_{planet}}{|M_{sun}|}\left(\frac{Au}{year}\right)^2$, where $|M_{sun}|$. In reallity this is simply Joule with a prefactor. This prefactor can be obtained by inserting the values into J_{ast}:

$$J_{ast} = \frac{m_{planet}}{|M_{sun}|}\left(\frac{Au}{year}\right)^2 \tag{3}$$

$$= \frac{m_{planet}}{2\cdot 10^{30}}\left(|v|\frac{1.5\cdot 10^{11}m}{360*24*60*60s}\right)^2 \tag{4}$$

$$\simeq 1.163\cdot 10^{-23}J \tag{5}$$

The dimensionality of the variables are as follows: $[v] = m/s$ and $[m_{planet}] = kg$.

We know that the earth needs one year to orbit the sun, meaning that $v = \frac{2\pi r}{1\text{ year}}$. This can be rewritten with $v = \tilde{v}v_0$ and $r = \tilde{r}r_0$. The units of r and v are contained in $v_0 = \frac{1\text{Au}}{1\text{ year}}$ and $r_0 = 1$ Au, giving that $\tilde{v}^2\tilde{r} = 4\pi^2$. In the same way $t = \tilde{t}t_0$, with $t_0 = 1$ year.

For the case of the two body earth - sun system one can assume that the sun is stationary, as $M_{sun} >> M_{earth}$. The force experienced by the earth is thus centrifugal, which means that $a = \frac{v^2}{r}$, with $v = 2\pi r/1year$. Combining this with equations 1 and 2 it is possible to scale the equations in the following manner:

$$a_E = \frac{F_E}{M_E} = G\frac{M_{sun}}{r^2} = \frac{v^2}{r} \tag{6}$$

$$GM_{sun} = v^2 r = 4\pi^2\frac{(1\text{ Au})^3}{(1\text{ year})^2} \tag{7}$$

$$\tag{8}$$

This gives that the dimensionless expression can be stated as:

$$\frac{\mathrm{d}\tilde{v}}{\mathrm{d}\tilde{t}} = \frac{4\pi^2}{\tilde{r}^2} \tag{9}$$

In the two dimensional case $r = (x, y) = (r\cos\theta, r\sin\theta)$. Using the notation $\dot{p} = \frac{\mathrm{d}p}{\mathrm{d}t}$, this gives the following coupled differential equations:

$$\dot{v}_x = \frac{4\pi^2 r\cos\theta}{r^3} = \frac{4\pi^2 x}{r^3} \tag{10}$$
$$\dot{v}_y = -\frac{4\pi^2 r\sin\theta}{r^3} = \frac{4\pi^2 y}{r^3} \tag{11}$$
$$\dot{x} = v_x \tag{12}$$
$$\dot{y} = v_y \tag{13}$$
$$\tag{14}$$

2.3 Energy considerations

As with any physical system, the total energy has to be conserved. The potential energy $E_p = \int_{r'}^{\infty} \vec{F}(r') \cdot d\vec{r}' = -\frac{GM_1M_2}{r}$, while the kinetic energy is $E_K = \frac{1}{2}mv^2$. This gives a total energy of:

$$E_{tot} = \frac{1}{2}mv^2 - \frac{GM_1M_2}{r} \tag{15}$$

In addition, the angular momentum $(\vec{L})$ of the system has to be preserved. This is because there are no additional sources of torque $(\vec{\tau})$ once the system has been initialized and $\vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = 0$. As a result all the absolute value of $\vec{L}$ has to be constant.

In order to maintain in the gravitational field of another object, the distance between them needs to be smaller than ∞. However, with a large enough velocity it is possible to escape. By setting equation 15 equal to 0 one obtains the lowest escape velocity v_{esc}:

$$\frac{1}{2}mv_{esc}^2 - \frac{GM_1M_2}{r} = 0 \tag{16}$$
$$v_{esc} = \sqrt{\frac{2GM_1M_2}{mr}} \tag{17}$$

Nevne approksimasjon med barrycentre v sol i sentrum v pos sol @ t=0: Vi har ingen beregninger med sol i sentrum, men bruker sol ved t=0 som posisjon.

Hvorfor bør egentlig være barrycentre til univers?

2.4 Perihelion precession

With a two body problem, only one force working on the planet, Mercury's elliptical orbit should be fixed if there where only non-relativistic Newtonian forces working on it. Mercury's orbit has been observed to rotate even when the forces from the other planets are subtracted away. If we add a relativistic part to the force, the rotation of the elliptical orbit can be explained. The gravitational force with the relativistic part is Equation 18.

$$F_G = \frac{GMm}{r^2}\left[1 + \frac{3l^2}{r^2c^2}\right] \tag{18}$$

Here l is the magnitude of the angular momentum, $l = |\mathbf{r} \times \mathbf{v}|$, of the planet per unit mass and c is the speed of light. To be able to see the rotation, we look at the movement of the perihelion, the position where Mercury is closest to the sun.

3 Method

In this project we tested two numerical integration techniques, the Euler method and the velocity Verlet method. This section is largely based on Hjort-Jensen [1]

3.1 The Euler method

When evaluating the function $x(t)$ in the interval between t and $t + h$ it is natural to do a Taylor expansion:

$$x(t + h) = x(t) + \sum_{n=1}^{\infty} \frac{h^n}{n!} \frac{\mathrm{d}^n x(t)}{\mathrm{d}t^n} \tag{19}$$

By choosing a small h, it is sufficient to truncate the sum after n=1, giving $x(t + h) = x(t) + h\frac{\mathrm{d}x}{\mathrm{d}t} + O(h^2)$. The term$O(h^2)$ contains the rest of the infinite sum, often called the truncation error. In order for a computer to use this method it is necessary to discretise the expression, substituting $x(t) \to x(t_i) \to x_i$. The Euler method can thus be expressed as

$$x_{i+1} = x_i + h\dot{x}_i + O(h^2) \tag{20}$$

Combining this with equations 10 and 12 we get an algorithm for doing a 1 dimentional integration for the solar system:

$$a_i = F(t_i) \tag{21}$$
$$v_{i+1} = v_i + ha_i \tag{22}$$
$$x_{i+1} = x_i + hx_i \tag{23}$$

This gives in total 4 floating points operations per iteration, per dimension. For our two dimentional system this results in a total of 8N FLOPS per iteration, with $N = \frac{1}{h}$.

3.2 The Velocity Verlet method

The velocity Verlet can be derived from the same Taylor expansion as the Euler method, equation 19. As with the Euler method, we will truncate the taylor series for position after n=1. However, for the velocity there exist a similar Taylor series: $v_{i+1} = v_i + h\dot{v}_i + \frac{h}{2}\ddot{v}_i + O(h^3)$. Unfortunately, there is no expression for $\ddot{v}_i$, but it can be approximated by $h\ddot{v}_i \simeq \dot{v}_{i+1} - \dot{v}_i$. Adding this to the expression for v_{i+1} and doing some simple algebra one get the final expression for the velocity:

$$v_{i+1} = v_i + \frac{h}{2}\left(\dot{v}_{i+1} + \dot{v}_i\right) + O(h^3) \tag{24}$$

Looking a bit closer at equation 24, there is a immediate problem. In order to calculate v_{i+1} one need to already know v_{i+1}. This problem can be solved by updating the velocity in two steps, which gives the following algorithm:

$$v_{i+\frac{1}{2}} = v_i + \frac{h}{2}a_i \tag{25}$$
$$x_{i+1} = x_i + v_{i+\frac{1}{2}} \tag{26}$$
$$a_{i+1} = \frac{F_{i+1}}{m} \tag{27}$$
$$v_{i+1} = v_{i+\frac{1}{2}}. + \frac{h}{2}a_{i+1} \tag{28}$$
$$\tag{29}$$

The numerical cost of the algorithm can be understood when analysing the number of floating points operations. In the case of the velocity verlet method there are 8 FLOPS per iteration and dimension. As the value $\frac{h}{2}$ a constant, it can be calculated in advance of the loop, thus reducing the FLOPS to 6. This gives a total of 12 FLOPS per iteration for the two dimensional system.

3.3 Object orientation

In order to simplify the calculation of an ensemble of planets, each with velocities, positions, energies and angular momentum, it is useful to generalize the code in an object oriented way. We chose to create one class for the planets, where all the internal dynamics (energies, position, velocity, ...) were stored. The forces experienced by the planets are specific for this project and we kept this in the Planet class, so that the second class, the Solver class, could be more general and easier reused.

This Solver class is were all the technicalities are located, including the different integration methods. For each time step one need the location of every planet in the system and Solver includes therefore a function that for each time step loop over all the planets. In order to update their position it is necessary to again loop over all the different planets in order to find the total gravitational force exerted on the current planet.

When all the calculations are taken care of by the solver class and all the properties of the different planets are stored in each planet object. This means that the main part of the program only needs to initialize the different planetary instances , adding these to the a instance of the Solver class which is initialized according to what output we want to achieve, see the snippet below for how this looks in 'main'.

```
    Planet earth("Initializing inputs");
            Planet sun("Initializing
inputs");
    Solver verlet("Initializing inputs")
;
    verlet.add(earth);
    verlet.add(sun);
    verlet.add(mercury);
    verlet.algorithm("input variables");
```

3.4 Choice of origin

The choice of origin is important, as it effects the physics of the system. As the sun is massively more massive than the earth and any other object in the solar system, it would be a good approximation to have the sun fixed in the origin. For the two-body simulations we chose to define the initial position of the sun as the origin instead, allowing the sun to move freely. However, this opens up for the sun to move away from the origin which affects the orbital of the earth.

For the simulations of the three-body solar system it was necessary too choose the barycentre of the universe as origin. This position, $\vec{R}$ is chosen so that the mass-weighted position of all the planets relative to R is zero:

$$\sum_{i=i}^{N} m_i(r_i - R) = 0 \tag{30}$$

In addition the sun had to be initialized with velocities so that the total momentum of the system was $\vec{p}_{system} = 0$, or else the system would slowly move further and further away from origin. Thus the initial velocity of the sun must equal $\vec{v}_{sun} = \frac{1}{M}\sum_{i=0}^{N} m_i v_i$ for N planets in the sun-planet system and a total mass of the system equal to M. With this choice of initial velocities the barycentre will be in the origin throughout the simulations, given that the momentum is conserved numerically.

When simulating the full solar system we used positions and velocities given by NASA [2], with the barycentre as origin.

4 Result

The simulations in this section are done with the velocity verlet unless stated other-ways. All calculations were also non-relativistic, except for the case of the Sun-mercury two-body case.

4.1 Two body system

We started out with a two body system consisting of the sun and the earth. The origin of the system was the sun's position at time = 0. We

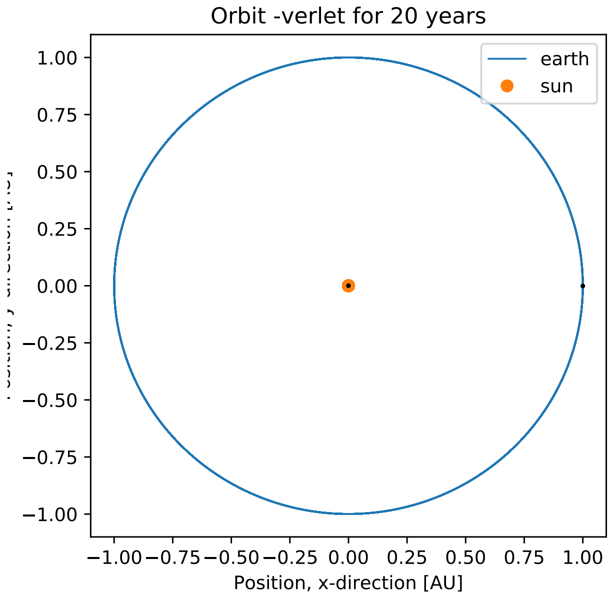

Figure 4.2: Simulation of the earth-sun system with velocity Verlet over 20 years, with initial velocity = 2π

wanted to find out what initial velocity the earth needed to have, for the orbit of the earth to be circular. From figure 4.1 it is clear that is the initial velocity that gives the perfect circular orbital is between 6.24 and 6.33. This fits with our expectations as the numerical value of 2π is 6.28.

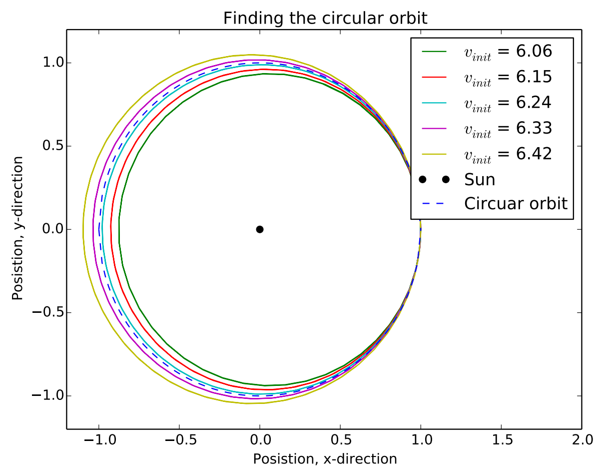

Figure 4.1: Plot of the orbit with different initial velocities for earth. The dotted line indicates where the perfect circular orbital is.

As mentioned in the method part of this report, we used two different methods to solve the differential equations, to simulate the motion, Euler's method and the Velocity Verlet method. We started out by plotting the circular motion of the earth with different time steps (see Figure 4.3 and 4.4)to be able to evaluate the two methods.

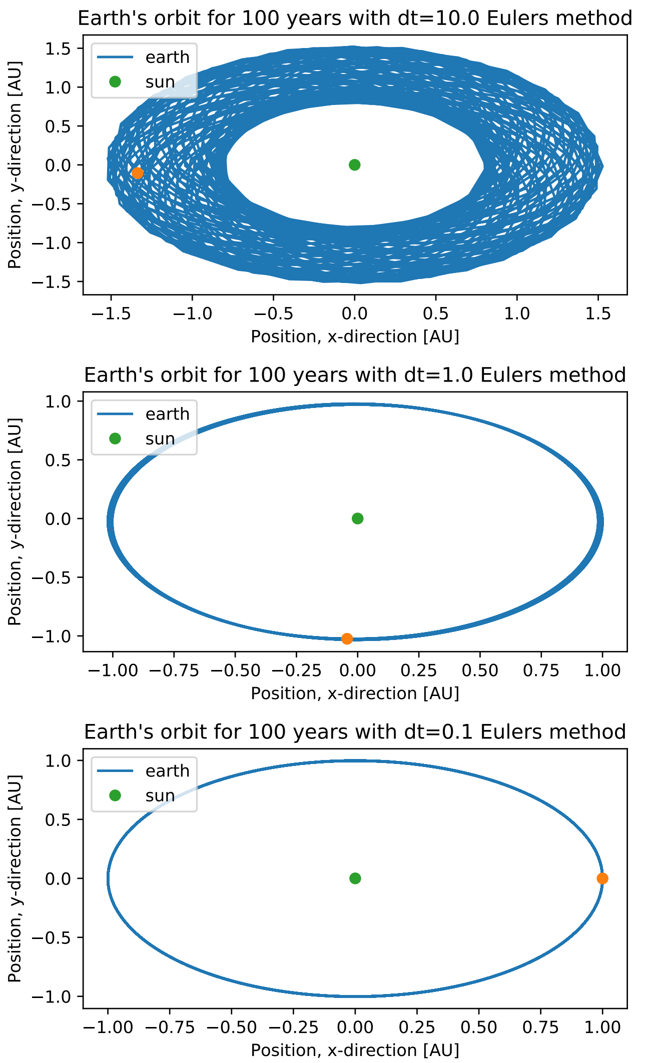

Figure 4.3: Plot of the earth's orbit for different dt's for 100 years using Euler's method.

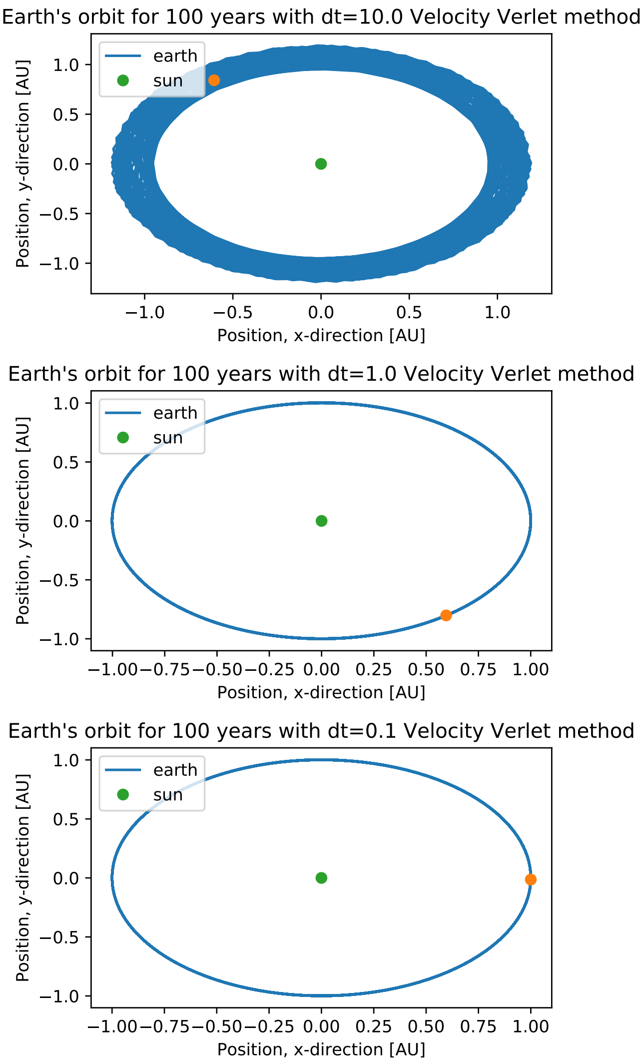

Figure 4.4: Plot of the earth's orbit for different dt's for 100 years using velocity Verlet

Another way to compare two methods are looking at the CPU time. In Table 4.1 the CPU time for the two method are listed. We can see that the difference between them are small, but the velocity Verlet algorithm is slower than the Euler method.

Table 4.1: Comparison of CPU time of Euler's method and the Velocity Verlet method (VV). Calculations were performed with 1000 time steps per year. As the CPU-time also calculate the time spent on updating energies and calling several functions, one will not be able to read out the CPU time of the euler/velocity verlet loop alone.

Years	time Euler (ms)	time VVerlet (ms)	$\Delta time$ (ms)
100	88.728	93.646	4.912
1000	812.486	852.949	40.46
10000	8145.7	8504.06	358.36

4.2 Conservation

First we checked if the angular momentum of the two body system was conserved. Figure 4.5 and 4.6 shows the results. The angular momentum is not totally conserved. The values are oscillating and there is an increase in the angular momentum when we use the Velocity Verlet method and a decrease when we use Euler's method. The scale of the change is small and on occurs after 10 leading decimal points for Euler's and after 11 leading decimals for Velocity Verlet. That is small compared to the value of each individual point which is proportional to 10^{-5}.

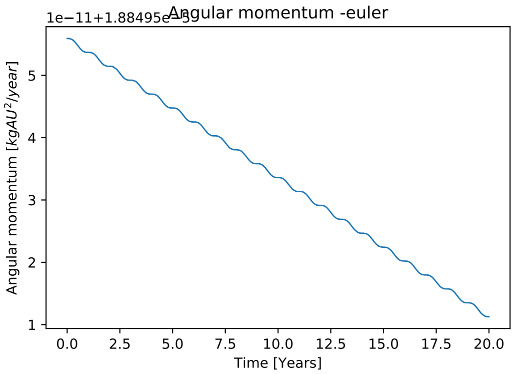

Figure 4.5: This is a plot of the angular momentum of the sun-earth system for 20 years using Euler's method, with 1000 timesteps per year. Notice that the scale of the axis is a sum.

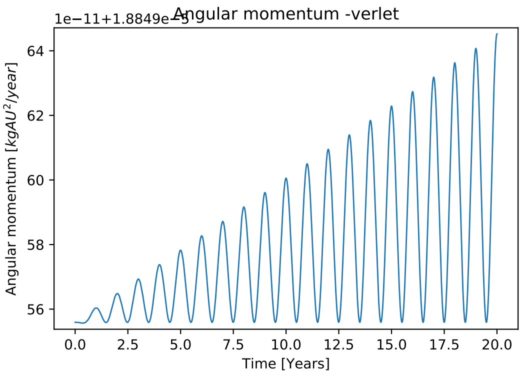

Figure 4.6: This is a plot of the angular momentum of the sun-earth system for 20 years using the velocity Verlet method, with 1000 timesteps per year. Notice that the scale of the axis is a sum.

The other important parameter that needs to be conserved is the energy. Figure 4.7 shows the energy development of the two methods. The total energy is oscillating like the angular momentum and slightly changing, but the scales are small here too. Note that the velocity Verlet method oscillates with a much smaller amplitude than Euler. Although a bit difficult to see, the total energy decreases over several years with the Euler method and increase slightly with the velocity verlet. Table 4.2 lists the difference in the oscillation amplitudes of the two methods.

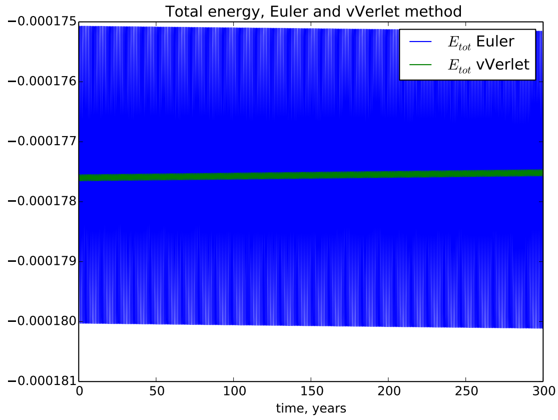

Figure 4.7: Plot of the total energy in the system for the Euler and the velocity Verlet method.

Table 4.2: This is a table that list how the energy oscillates in the two different method. These are only approximate values that are read form the plots of the energies (Figure ?? and ??). Plot of kinetic and potential energies can be found in the appendix.

Oscillation of:	Euler's:	Velocity Verlet:
Kinetic energy [J_{ast}]:	$0.8 \cdot 10^{-6}$	$1.5 \cdot 10^{-9}$
Potential energy [J_{ast}]:	$1.5 \cdot 10^{-6}$	$3 \cdot 10^{-9}$
Total energy [J_{ast}]:	$5 \cdot 10^{-6}$	$2 \cdot 10^{-9}$

4.3 Checking gravitational forces

To check the gravitational force working on the earth from the sun in our two body system, we started by testing earth's necessary initial velocity to escape the sun, the escape velocity. Figure 4.8 show some of the velocities near the exact escape velocity. The velocities $v = 7.73$ and $v = 7.84$ does not have a linear motion when it moves away from the sun, but rather an elliptical form. The full ellipse is not shown due to a relatively short simulation.

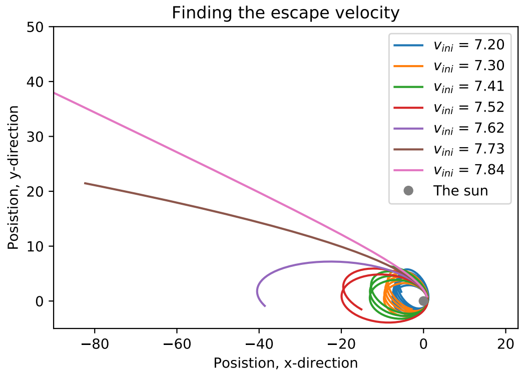

Figure 4.8: This is a plot of the sun-earth system for different initial velocities. The plot shows initial velocities near the exact escape velocity, $2\pi\sqrt{2} \approx 8.88$.

Next we wanted to see what happened if we changed the gravitational force. The gravitational force is given by $F = \frac{GM_E M_S}{r^\beta}$ where β is 2. In Figure 4.9 we plotted the motion of the earth with different β between 2 and 3.

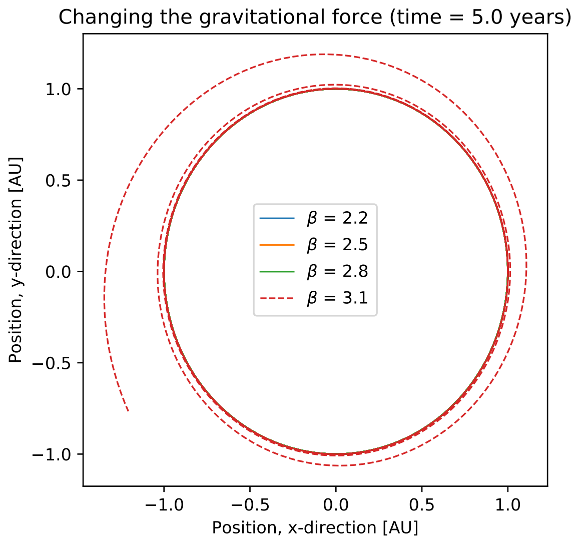

Figure 4.9: This is a plot of the sun-earth system for different gravitational forces. The gravitational force is given by $F = \frac{-GM_E M_S}{r^\beta}$ and in the plot β is changed from it's normal value $\beta = 2$ to $\beta = 3.1$.

4.4 Three body system

VILDE: Kommentere before and after barycentre (see new figs = 3bodynobary and 3bodybary) VILDE: Fikse masses Jupiter.

Next we added a planet, Jupiter, to our system and got a three body problem. Figure 4.10 shows the motion of the planets.

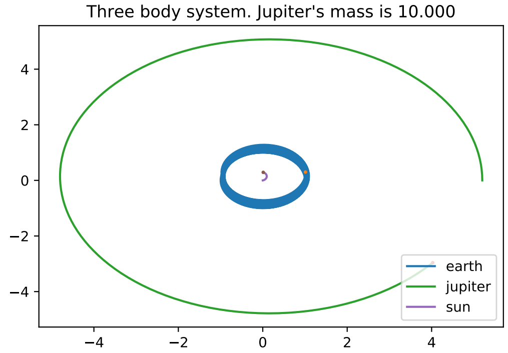

Figure 4.10: This is a plot of the three body system, Jupiter, Earth and Sun, with the origin in sun at time = 0.

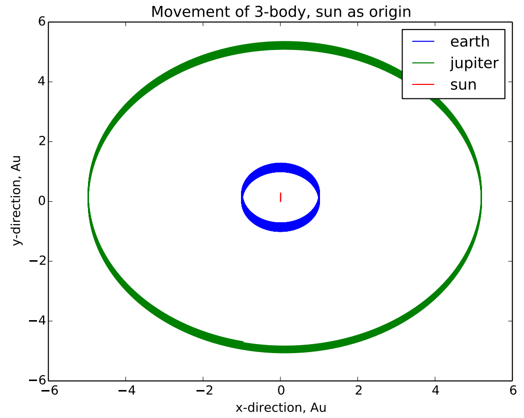

Figure 4.11: Simulation of the 3-body system with the initial position of the sun as origin. The system is simulated for 100 years

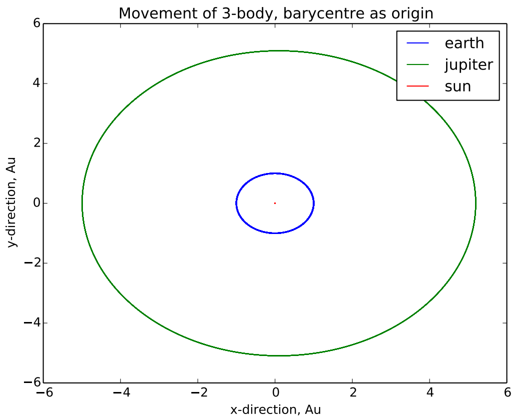

Figure 4.12: Simulation of the 3-body system with the barycentre as the origin and the total momentum equal to 0. The system is simulated for 100 years

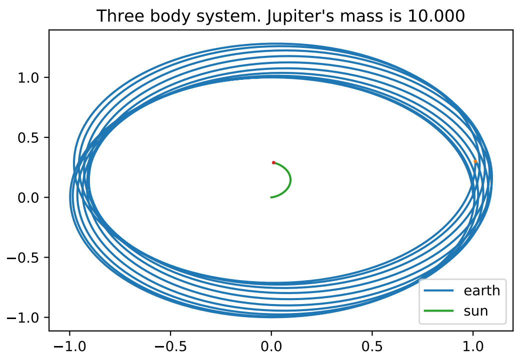

Figure 4.13: Plot of the 3-body system with the mass of Jupiter = 10 m_{sun}

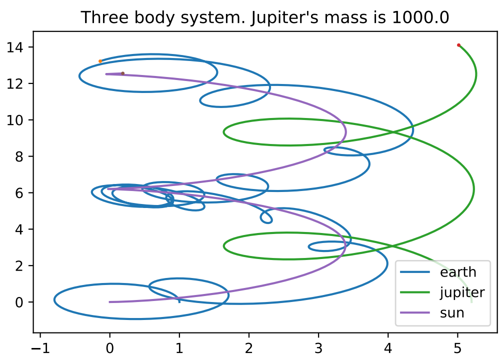

Figure 4.14: Plot of the 3-body system with the mass of Jupiter = 1000 m_{sun}

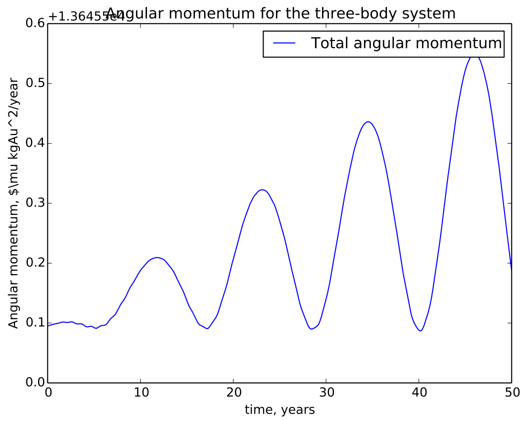

Figure 4.15

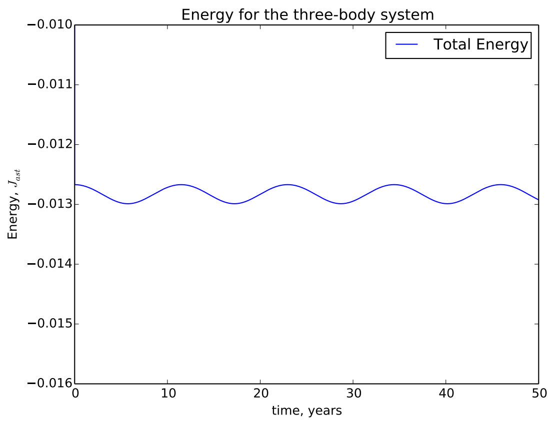

Figure 4.16

Ok - if initial values are correct.

- Plot Earth's motion for increased mass of Jupiter (3 masses)

same.

- Find center off mass - use as origin

Found and incorporated.

- Give sun initial velocity so momentum is zero (origin is fixed)

How? What momentum? velocity?

- Compare with 3e) - Extend to all planets (plot)

4.5 All planets

At last we added all the planets with initial conditions form NASA [2]. Figure 4.17 shows the orbits of all the planets and Figure 4.18 shows the inner planet's orbits, because they are difficult to distinguish in the other plot.

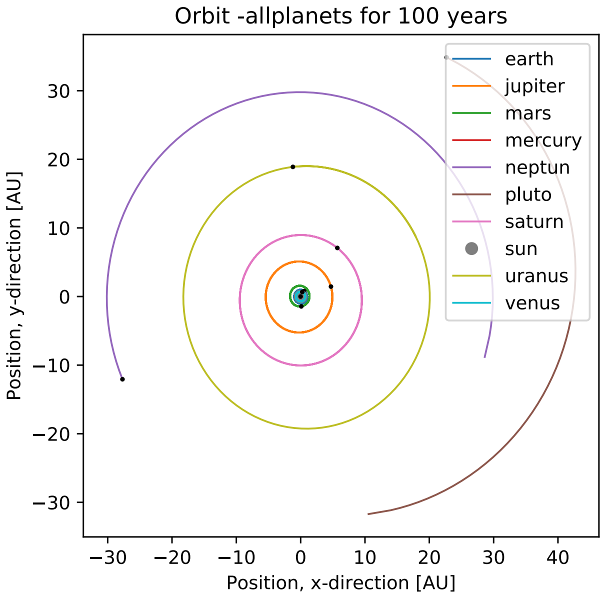

Figure 4.17: This is a plot of the Solar System after 100 years of motion.

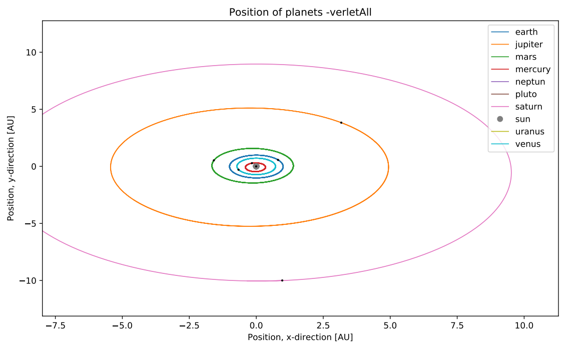

Figure 4.18: This is a plot of planets closest to the sun in the Solar System after 30 years of motion.

4.6 Considering relativistic force

As a last point we wanted to see how relativistic forces affect the orbits. We used the perihelion precession of Mercury to explore this. Table 4.3 shows the perihelion position of Mercury after 100 years of orbiting the sun alone with no other planets affecting the movement. The table lists the position with non-relativistic forces and relativistic forces.The perihelion precession is observed to be 43 arc seconds and, as we can see, this fits well with our result. However, the pure newtonian force with no relativistic correction has an periphelion that is far away from the observed values.

Table 4.3: Table with the Perihelion information about Mercury after one century, 100 years.

	Position [AU]	Angle [arcs]
Newtonian	$(0.307, -1.52 \cdot 10^{-7})$	-0.102
Relativistic	$(0.307, 6.447 \cdot 10^{-5})$	43.247

5 Discussion

5.1 Stability of the methods

As we see from the figures describing the movement of the earth-sun system, figures 4.3 4.4, the velocity Verlet seems to conserve the motion of the earth around the sun better with a increase in dt.

However, the plots 4.6 and 4.5 indicates that the angular momentum is not well conserved in either integration method used in this project. For a circular motion the angular momentum should oscillate, but with the same amplitude and around a constant value. The Euler method plot shows a constant amplitude, but the maximum and minimum is decreasing with time, indicating that system looses angular momentum and thus not conserving it. The Verlet plot shows an increasing amplitude, but the minimum of the oscillation is not changing. This can be explained by the fact that the calculations were conducted with a movable sun and a origin fixed at the sun's position at $t = 0years$. As the sun is moving the total angular momentum of the system changes, as the barycentre of the sun and earth moves around. It is thus impossible to claim that the total angular momentum is conserved or not based on this plot alone.

The total energy also needs to be conserved and by investigating figures **??** and 4.7 it is clear that the energies coming from the Euler-simulation oscillates with a larger amplitude than the velocity verlet method does. The development of the total energy is very similar to the development of the angular momentum. Thought it is difficult to see, the total energy of the Euler-simulation decreases more than the Verlet simulation increases the total energy.

Although the Euler method consist is faster the velocity Verlet method, owing to the fact that it has two fewer FLOPS per time iteration, figure

4.1 shows that the Verlet method works better for smaller time-resolution, which allows for fewer iterations per year and a possible increase in speed of the overall program.

5.2 Effects of initial values

The effect of initial conditions is obvious when considering figures 4.1 and 4.8. If the initial velocity is greater than $v = 2\pi$, the orbit will have an elliptical shape and the larger the initial velocity, the longer the orbit will be. In our simulations we were unable to simulate an infinite distance away from the sun (requirement for an escaped planet) and it is thus a bit difficult to numerically determine the exact escape velocity. Velocities lower than that required to keep an perfect circular orbit will also be elliptical, but with the initial starting position as the point furthest away from the sun.

The origin is another choice of initial values which effects the physics at hand. For the case of figure 4.11 it is clear that the all the objects in the 3-body simulation is moving and the barycentre is not in the origin throughout the simulation. As the total momentum of this system is not 0, we see that the entire system is moving in the y-direction, most clearly indicated by the sun. This effects the other planets, as the mass of the sun is many times larger than the other masses. When we compensated the suns initial velocity so that the total momentum was 0 and placed origin in the barycentre, see figure 4.12, the interactions between the planets are easier to see. However, this is a relatively small effect as the system in both figures were simulated for 100 years.

In order to see the effect Jupiter has on the orbit of the earth it is useful to compare figure 4.2 to figure 4.12. When Jupiter is included, the orbit of the earth changes from a circular to an elliptical shape. This trend is seen even clearer in figure 4.13, where the mass of jupiter is $10m_{sun}$. Here the motion of the sun is much more substantial and the earth is not only following an elliptical orbit, but the orbit seems to shift with the position of the sun and jupiter.

As the mass of jupiter is increased to $1000m_{sun}$ in figure 4.14, the entire system gets very chaotic. As the sun is attracted to Jupiter, it follows Jupiters orbit. This again effects the orbit of Jupiter. While the sun is chasing Jupiter the earth seems to orbit follow the sun, as this is the closest object and the force of the gravitation is proportional to $1/r^2$ with r being the distance between two objects. The earth also indicates were the sun and jupiter are closest, as it experience the force from Jupiter stronger at certain points in the plot than others.

5.3 The full solar system

Figure 4.17 shows that the outer planets in the solar system takes a long time to orbit the sun. However, over a period of 30 years, the massive, outer planets does not move much. In figure 4.18 this shows in the elliptical orbits of the inner planets.

However, for mercury the simple newtonian force is not enough. The observed periphelion angle of 43 arc seconds is coherent with the results of the simulation with the relativistic correction. Simulations using only Newtonian gravitational force should produce a closed orbital, as seen in ie. figure 4.12.

The closed orbitals that we expect from a pure Newtonian gravitational force, as indicated in figure 4.1 for the case of the initial velocity equal to 2π.

VILDE: Discussion: - Discuss stability of velocity verlet (3 body) - Discuss difference 3e) and 3f) (3 body)

6 Conclusion

The fact that the velocity Verlet method has a smaller amplitude in the oscillations for both the angular momentum and energy and has a smaller deviation of the centre of the oscillations in the energy-plot than the Euler-simulation, indicates that Verlet method is more stable and that it conserves energy better.

comment on code

References

[1] Morten Hjorth-Jensen. Computational physics: Lecture notes fall 2015. Department of Physics, University of Oslo, 8 2015. Chapter 2 and 6.

[2] Solar system dynamics. https://ssd.jpl.nasa.gov/horizons.cgi#top. Accessed: 2017-10-28.

Appendix

appendix

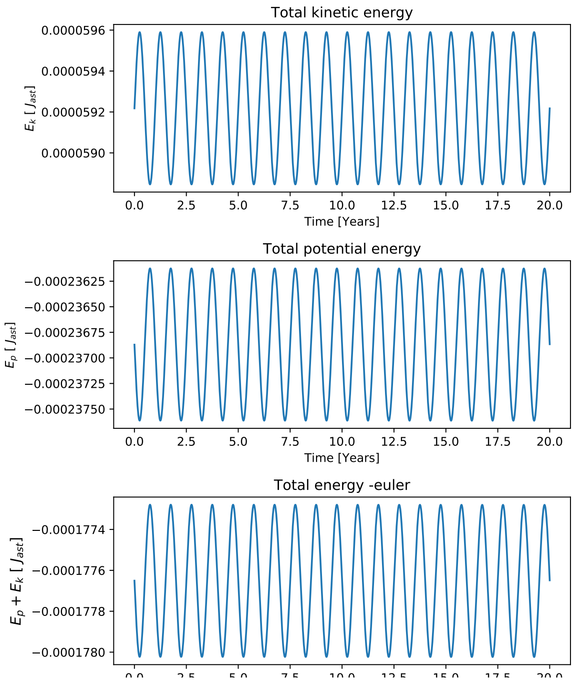

Figure 6.1: Plots of the total energies of the sun-earth system for 20 years using Euler's method, with 1000 timesteps per year.

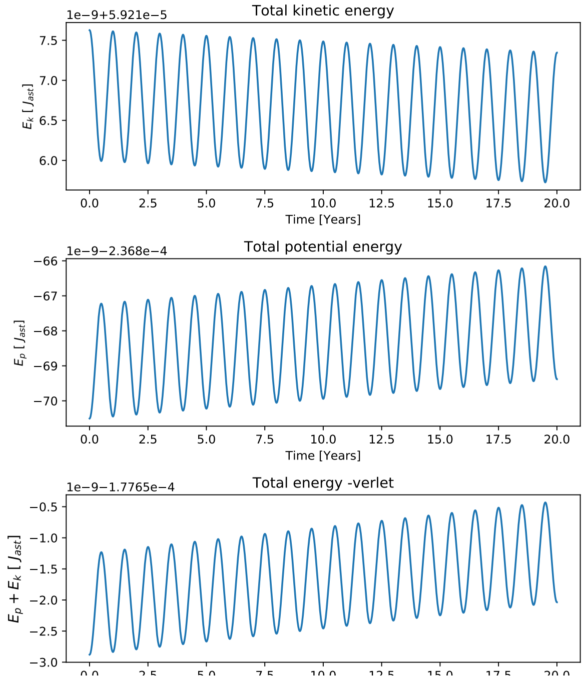

Figure 6.2: Plot of the total energies of the sun-earth system for 20 years using the velocity Verlet method, with 1000 timesteps per year.