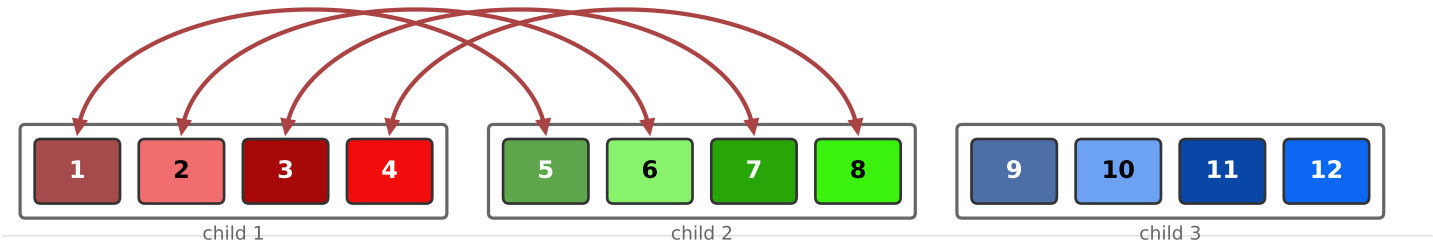
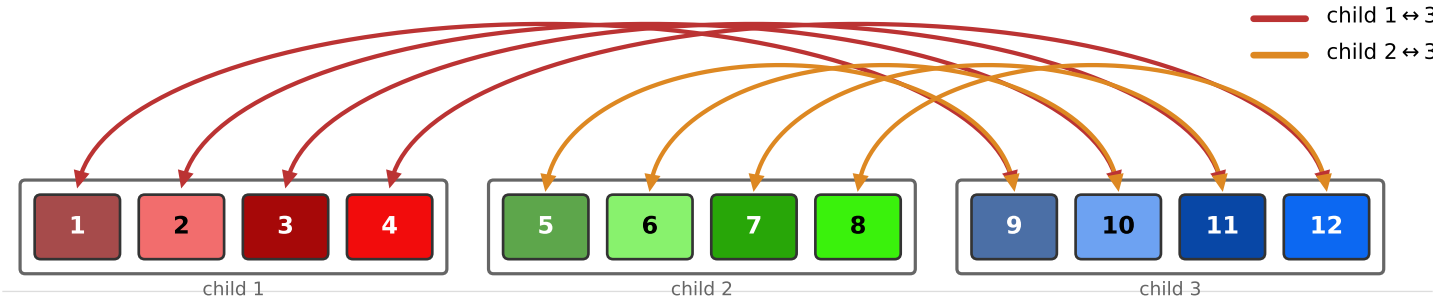


Hierarchical exchange communication pattern for $N_{\text{cpu}} = 12$ ($= 3 \times 2 \times 2$)

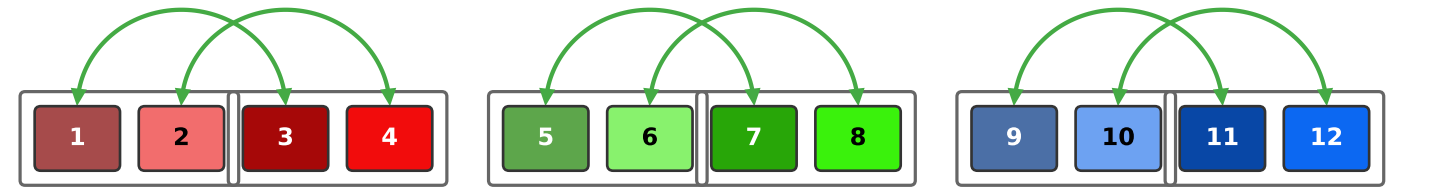
Level 1, step 1
($k_1 = 3$, siblings $1 \leftrightarrow 2$)



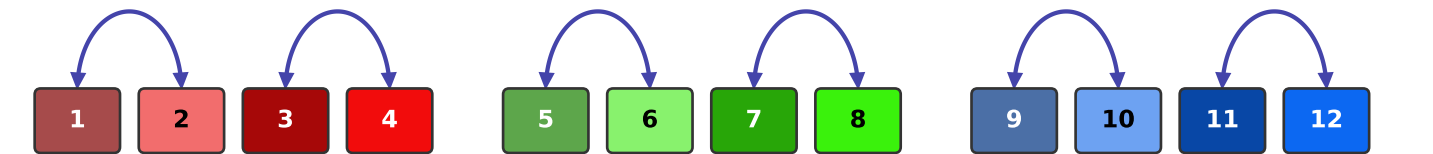
Level 1, step 2
($k_1 = 3$, siblings $\{1,2\} \leftrightarrow 3$)



Level 2
($k_2 = 2$)



Level 3
($k_3 = 2$)



$$N_{\text{msg}} = (k_1 - 1) + (k_2 - 1) + (k_3 - 1) = 2 + 1 + 1 = 4 \text{ messages per rank per exchange}$$