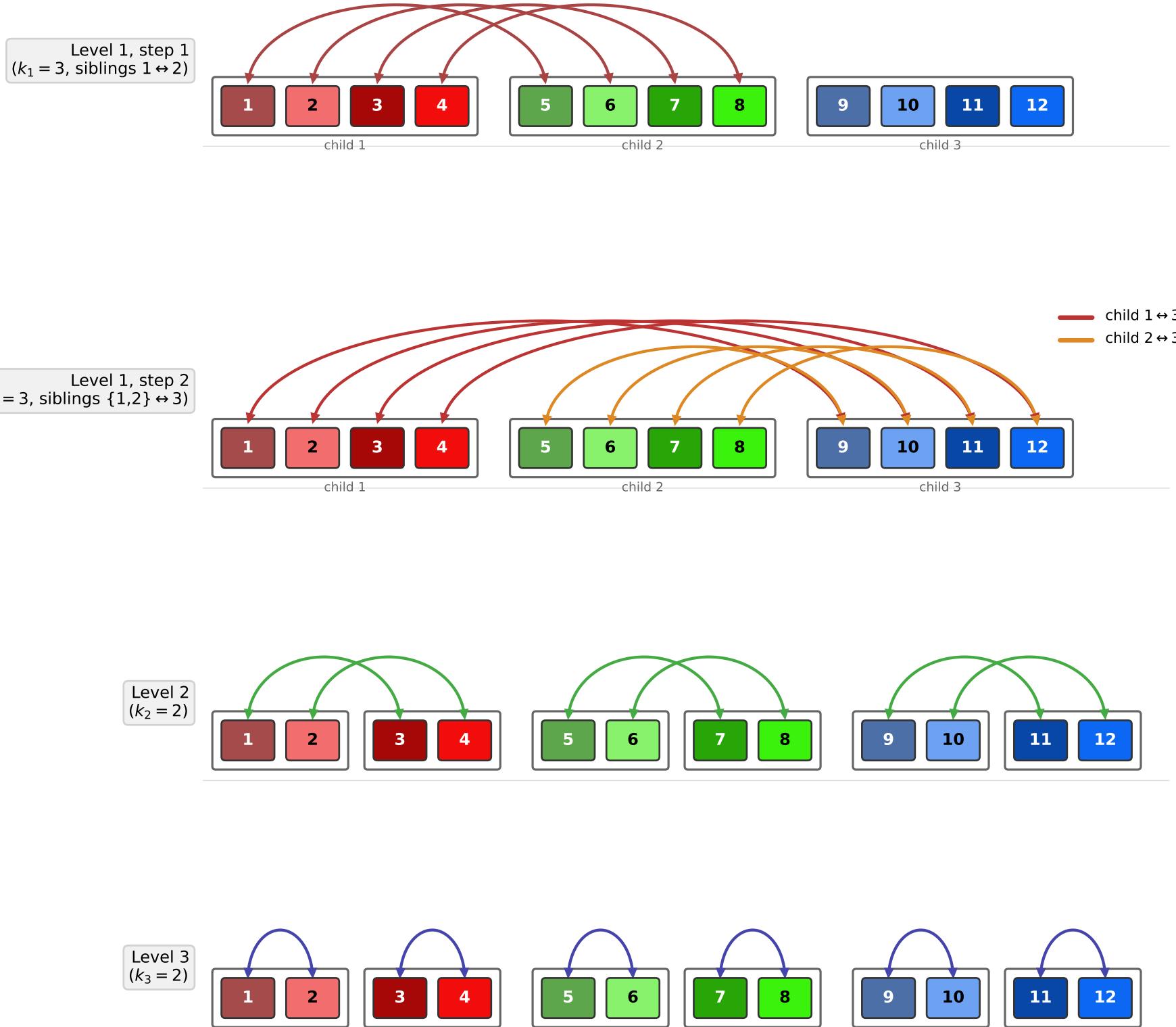


Hierarchical exchange communication pattern for $N_{\text{cpu}} = 12$ ($= 3 \times 2 \times 2$)



$$N_{\text{msg}} = (k_1 - 1) + (k_2 - 1) + (k_3 - 1) = 2 + 1 + 1 = 4 \text{ messages per rank per exchange}$$