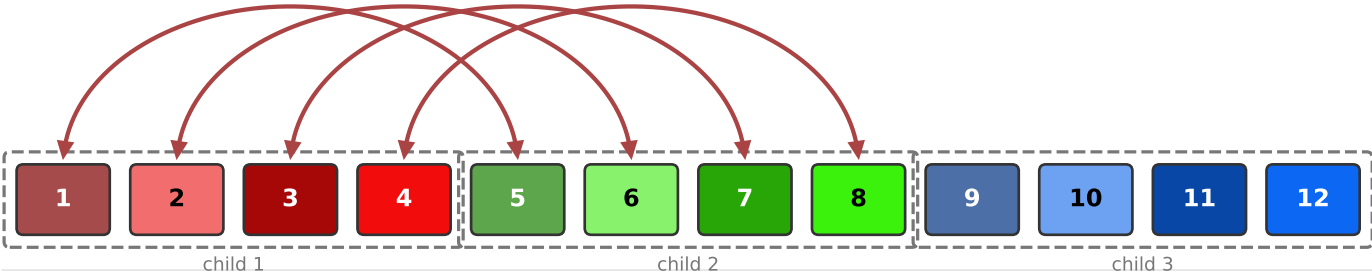
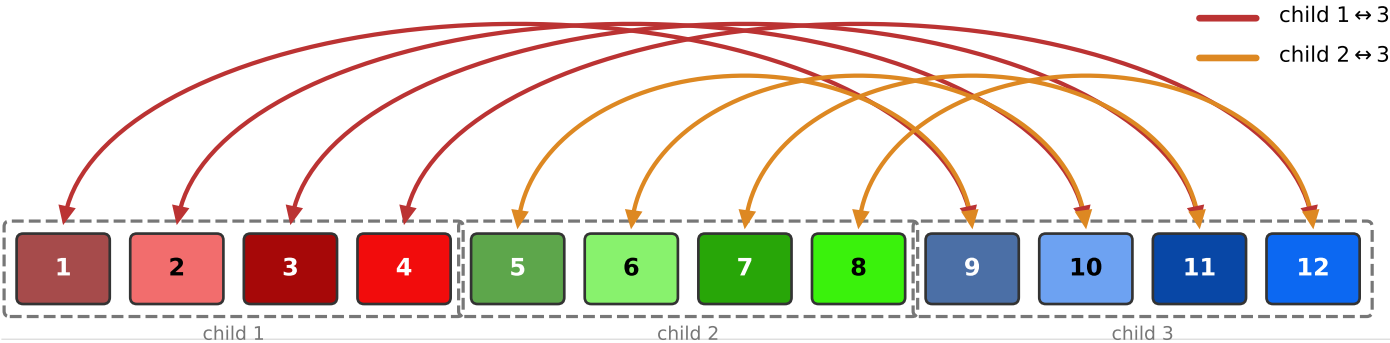


**Hierarchical exchange communication pattern for  $N_{\text{cpu}} = 12$  ( $= 3 \times 2 \times 2$ )**

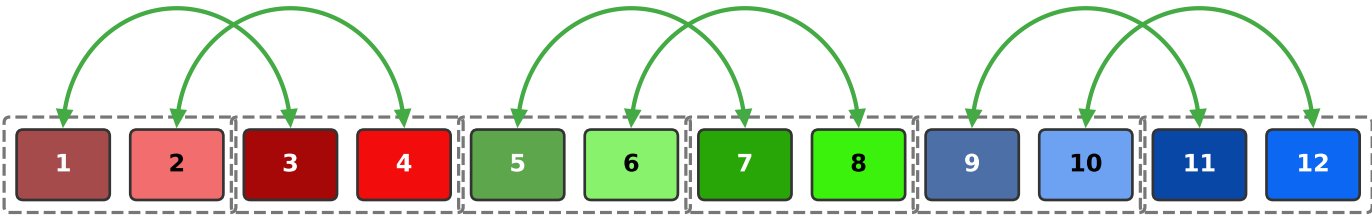
Level 1, step 1  
( $k_1 = 3$ , siblings  $1 \leftrightarrow 2$ )



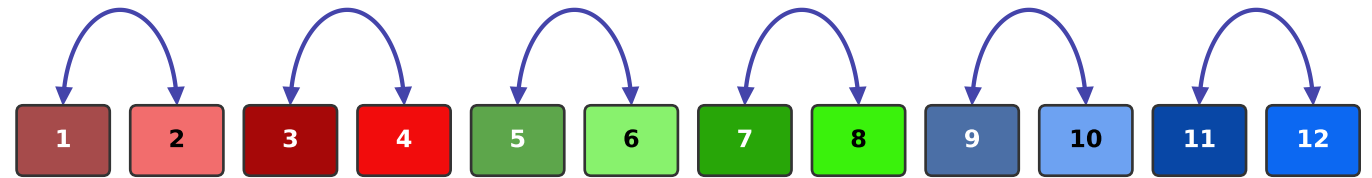
Level 1, step 2  
( $k_1 = 3$ , siblings  $\{1,2\} \leftrightarrow 3$ )



Level 2  
( $k_2 = 2$ )



Level 3  
( $k_3 = 2$ )



$$N_{\text{msg}} = (k_1 - 1) + (k_2 - 1) + (k_3 - 1) = 2 + 1 + 1 = 4 \text{ messages per rank per exchange}$$