

Practical work on Non-negative matrix factorization

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The files necessary for this practical work are available on the ATIAM Moodle. You can load a sound file in Matlab by typing the command [x,Fs] = wavread('file.wav'). To listen to it, you can type soundsc(x,Fs). The template of a Python notebook NMF_TP.ipynb is also provided.

1 Non-Negative Matrix Factorization with β -divergence

In this practical work, we will deal with non-negative matrix factorization (NMF) with the β -divergence. The problem that we aim to solve is defined as follows:

$$(\mathbf{W}^{\star}, \mathbf{H}^{\star}) = \operatorname{argmin}_{\mathbf{W} \ge 0, \mathbf{H} \ge 0} \sum_{f=1}^{F} \sum_{t=1}^{T} d_{\beta}(x_{ft} || \hat{x}_{ft}), \tag{1}$$

where x_{ft} is an element of $\mathbf{X} \in \mathbb{R}_+^{F \times T}$, that is the *non-negative* data matrix, and $\mathbf{W} \in \mathbb{R}_+^{F \times R}$ and $\mathbf{H} \in \mathbb{R}_+^{R \times T}$ are the unknown *non-negative* factor matrices. We also define $\hat{x}_{ft} = \sum_r w_{fr} h_{rt}$. The cost function that we are minimizing is called the β -divergence, which is defined as follows:

$$d_{\beta}(x||\hat{x}) = \frac{x^{\beta}}{\beta(\beta - 1)} - \frac{x\hat{x}^{\beta - 1}}{\beta - 1} + \frac{\hat{x}^{\beta}}{\beta}.$$
 (2)

When $\beta = 2$, we obtain the squared Euclidean distance (EUC), when $\beta = 1$, we obtain the Kullback-Leibler (KL) divergence, when $\beta = 0$ we obtain the Itakura-Saito (IS) divergence.

One of the most popular algorithms for NMF is called the multiplicative update rules (MUR). The MUR algorithm has the following update rules :

$$\mathbf{W} \leftarrow \mathbf{W} \circ \frac{(\mathbf{X} \circ \hat{\mathbf{X}}^{\beta-2})\mathbf{H}^{\top}}{\hat{\mathbf{X}}^{\beta-1}\mathbf{H}^{\top}}$$
(3)

$$\mathbf{H} \leftarrow \mathbf{H} \circ \frac{\mathbf{W}^{\top} (\mathbf{X} \circ \hat{\mathbf{X}}^{\beta-2})}{\mathbf{W}^{\top} \hat{\mathbf{X}}^{\beta-1}},\tag{4}$$

where o denotes element-wise multiplication and / and i denote element-wise division.

By following the technique that we used in the lecture, derive the MUR algorithm by yourselves.

2 Variations on a simple example

The aim here is to program the NMF as described above. For that, we will rely on the script test_nmf.m which manages the display part.

2.1 Construction of the simple example

We propose to synthesize the NMF of a simple musical example : a C, followed by an E, followed by a C/E chord, which we will note C-E-C/E.

This example is obtained by constructing the matrix W of the spectral basis and the matrix H of the activations in a synthetic way. No sound is calculated here. For this we can for example :

- use the sampling frequency $F_e = 8000$ Hz, the DFT order : $N_{\rm fft} = 512$.
- calculate the fundamental frequencies F_0 , of C and E located under A 440.

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- for each note r, create a (non-negative) harmonic spectrum of the form $\mathbf{W}_r(f) = \sum_{k=1}^{K_r} a_k w(f kF_{0,r})$ where w is a usual window (e.g. Hann of width 80 Hz) and a_k describes the amplitude of the spectral lines (e.g. decreasing, of the form $a_k = exp(-k/K_r)$). K_r is fixed using the Shannon criterion. Deduce the spectral matrix \mathbf{W}_s .
- for the activations, we will use a function $h_r(t) = \sum_p b_{r,p} h(t p\Delta T)$ where $h(t) = e^{-t/\tau}$ for $t \in [0, \Delta T]$ with ΔT of the order of 0.5 seconds and h(t) = 0 elsewhere. b_p is 0 or 1. Deduce the matrix \mathbf{H}_s . Attention, to put ourselves in realistic conditions, we will suppose that the representation is obtained with frames of length $N = N_{\text{fit}}$ and an overlap of 75%, i.e. a sampling rate of $F_e/(N_{\text{fit}}/4)$ for the temporal activations. τ will be selected nearby $\Delta T/3$.
- then construct the time-frequency matrix $\mathbf{X}_s = \mathbf{W}_s \mathbf{H}_s$.

2.2 Standard NMF

- Use the multiplicative update rules to factorize matrix X_s into a product of non-negative matrices. In order to avoid that the NMF indeterminacies lead to numerical problems, at the end of every iteration we renormalize the columns of W and we multiply the rows of H by the appropriate factors in order to keep the product WH unchanged. The matrices W and H will be initialized with random values.
- Draw the graph of the cost function as a function of the iteration. What do you notice?

2.3 Variations

Test your algorithm by varying the data and parameters:

- by repeating with several draws of initialization (what differences do you notice on the results?)
- with addition of noise (be careful to respect the non-negativity, use rand)
- with different values of β (0, 1 and 2 typically)
- with more notes and indeterminations like C/E, C/G, C/E/G
- by making synthetic notes (waveforms) as sum of sinusoids and by calculating the STFT of the obtained temporal signal.

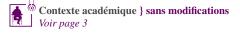
3 Automatic transcription using semi-supervised NMF

The isolated notes of a piano are provided, as well as a piece of music played with the same instrument. The semi-supervised NMF consists in defining the spectral matrix **W** using the isolated notes (provided in the form of xxx.wav where xxx is the MIDI number, 21-108 for the tessitura of the piano), by simply calculating an estimate of the associated power spectral density. The only remaining update is therefore that of **H**. We will work on signals sub-sampled at 22 kHz.

The provided Matlab script or Python function build_signal allows, with the help of a simple syntax, to build a piece from notes defined by their MIDI number, their attack time, their duration and the associated velocity.

- 1. Build the basis **W** corresponding to the 88 notes of the piano.
- 2. Synthesize a test signal corresponding to the C3-C4 scale.
- 3. Program the semi-supervised NMF using the results of the previous part and apply it to the test signal. Observe the temporal activations obtained.

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4. Program an attack detection function to be applied to the activations in order to estimate the times when the notes are played.



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