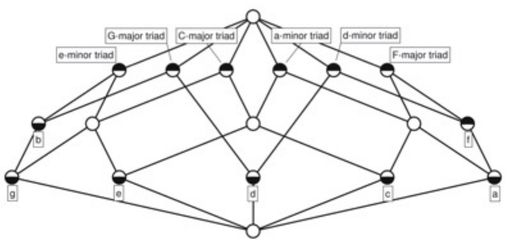
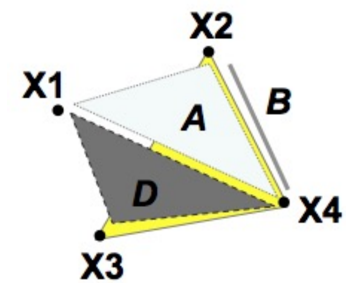
PARCOURS DU MASTER SCIENCES ET TECHNOLOGIES
UPMC • IRCAM • TELECOM PARISTECH

Algebraic and geometrical models in computational musicology



2024-2025
(part I)



Moreno Andreatta
CNRS / IRMA / Université de Strasbourg
CNRS / IRCAM / Sorbonne Université
www.morenoandreatta.com

Fondamentaux pour ATIAM – Musicologie Computationnelle
http://www.atiam.ircam.fr/planning/

2024-2025

9/9 : 9h30-12h30 + 14h30-16h30 (salle Stravinsky) – **François-Xavier Féron** : Musique et sciences – histoire de l'Ircam et illusions auditives (**5h**). Séance ouverte aux Ircamien.ne.s

10/9 (10h-13h) + 16/9 (14h30-17h30) + 23/9 (14h30-17h30)
Moreno Andreatta : Méthodes algébriques et topologiques en analyse musicale computationnelle (**9h**)

12/9 (14h-17h) - **Pierre Couprie** : Transcription, représentation et analyse de la musique électroacoustique (**3h**)

13/9 (14h30-17h30) - **Karim Haddad** : composition assistée par ordinateur (**3h**)
,

18/9 (9h30-12h30+14h-15h) - **Marc Chemillier** : Grammaires formelles, théorie des automates et improvisation assistée par ordinateur (**4h**)

19/9 (14h30-16h30) – **Clément Canonne** : musicologie empirique et cognitive (**2h**) -

25/9 (9h30-12h30 + 14h30-15h30) - **Angelo Orcalli** : Modèles épistémologiques en musicologie computationnelle (**4h**)

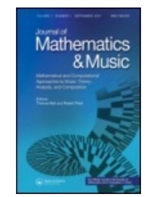
Computational Musicology in academic research

Conferences of the SMCM:

- 2007 Technische Universität (Berlin, Allemagne)
- 2009 Yale University (New Haven, USA)
- 2011 IRCAM (Paris, France)
- 2013 McGill University (Canada)
- 2015 Queen Mary University (Londres)
- 2017 UNAM (Mexico City)
- 2019 Universidad Complutense de Madrid (Spain)
- 2022 Georgia State University (Atlanta, USA)
- 2024 Coimbra

Official Journal and MC code (00A65: Mathematics and Music)

- *Journal of Mathematics and Music*, Taylor & Francis (Editors: E. Amiot & J. Yust)



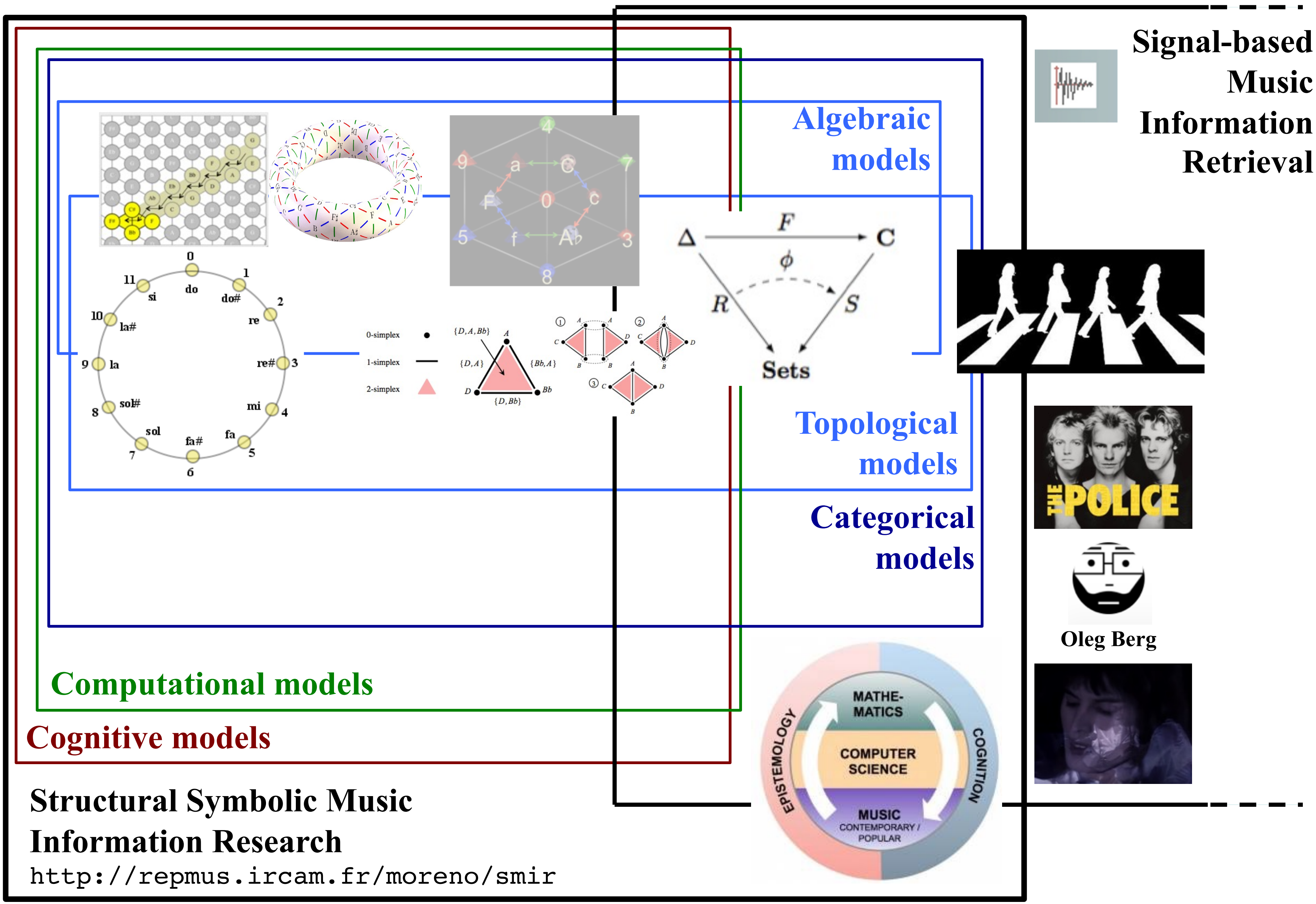
Books Series:

- *Computational Music Sciences* Series, Springer (G. Mazzola & M. Andreatta eds. – 19 books published (since 2009)
- Collection *Musique/Sciences*, Ircam-Delatour France (J.-M. Bardez & M. Andreatta dir. – 16 books published (since 2006)

Some examples of PhD on maths & music

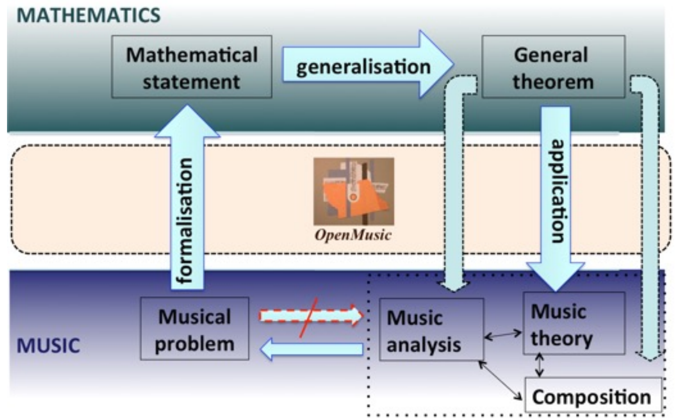
- **Christophe Weis,** *Geometric Models of Harmony as Tools for Computer Assisted Composition and Improvisation*, PhD in **music research**, **Hochschule für Musik Karlsruhe** (supervised by Marlon Schumacher and Moreno Andreatta).
- **Riccardo Giblas,** *Persistent homology and music analysis,* PhD in **maths** in cotutelle agreement, **University of Padova** (L. Fiorot & Alberto Tonolo) / **Université de Strasbourg** (M. Andreatta), 2024.
- **Paul Lascabettes,** *Mathematical Models for the Discovery of Musical Patterns and Structures, and for Performances Analysis,* PhD in **maths** at **SU**), 2023.
- **Gonzalo Romero**, *Morphologie mathématique et signal/symbolique*, PhD in **computer science** at **SU**), 2023.
- **Victoria Callet**, *Modélisation topologique de structures et processus musicaux*, PhD in **maths**, Université de Strasbourg (supervised by Pierre Guillot and Moreno Andreatta, IRMA), 2023.
- **Matias Fernandez Rosales,** *Mathematical models in Computer-assisted composition,* PhD in **composition and research**, **HEAR/University of Strasbourg** (supervision: Daniel D'Adamo, Xavier Hascher, Moreno Andreatta)
- **Greta Lanzarotto**, *Fuglede Spectral Conjecture, Musical Tilings and Homometry*, PhD in **maths** in cotutelle agreement, **University of Pavia** (L. Pernazza) / **Université de Strasbourg** (M. Andreatta), 2021.
- **Alessandro Ratoci**, Vers l'hybridation stylistique assistée par ordinateur, PhD in music **composition & research**, Sorbonne University / IRCAM (cosupervised with Laurent Cugny), ongoing
- **Sonia Cannas**, *Représentations géométriques et formalisations algébriques en musicologie computationnelle*, PhD in **maths** in cotutelle agreement, **University of Pavia** (L. Pernazza) / **Université de Strasbourg** (A. Papadopoulos & M. Andreatta), 2019.
- **Grégoire Genuys**, *Théorie de l'homométrie et musique*, PhD in **maths**, **Sorbonne University** / IRCAM (cosupervised with Jean-Paul Allouche), 2017.
- **Hélianthe Caure**, *Pavages en musique et conjectures ouvertes en mathématiques*, PhD in **computer science**, **Sorbonne University** (cosupervised with Jean-Paul Allouche), 2016.
- **Mattia Bergomi**, *Dynamical and topological tools for (modern) music analysis,* PhD in **maths** in a cotutelle agreement **Sorbonne University** / **University of Milan** (with Goffredo Haus, 2015).
- **Charles De Paiva**, *Systèmes complexes et informatique musicale*, thèse de doctorat, Programme Doctoral International « Modélisation des Systèmes Complexes », PhD in **musicology** in a cotutelle agreement, **Sorbonne University** / **UNICAMP**, Brésil, 2016.
- **Louis Bigo**, *Représentation symboliques musicales et calcul spatial*, PhD in **computer science**, **University of Paris Est Créteil / IRCAM**, 2013 (with Olivier Michel and Antoine Spicher)
- **Emmanuel Amiot**, *Modèles algébriques et algorithmiques pour la formalisation mathématique de structures musicales*, PhD in, **Sorbonne University** / **IRCAM**, 2010 (cosupervised with Carlos Agon) **computer science**
- **Yun-Kang Ahn**, *L'analyse musicale computationnelle*, PhD in **computer science**, **Sorbonne University** / **IRCAM**, 2009 (cosupervised with Carlos Agon)

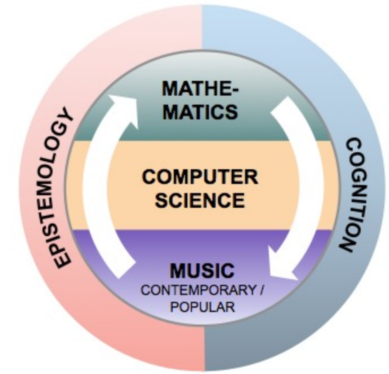
The SMIR Project: Structural Music Information Research

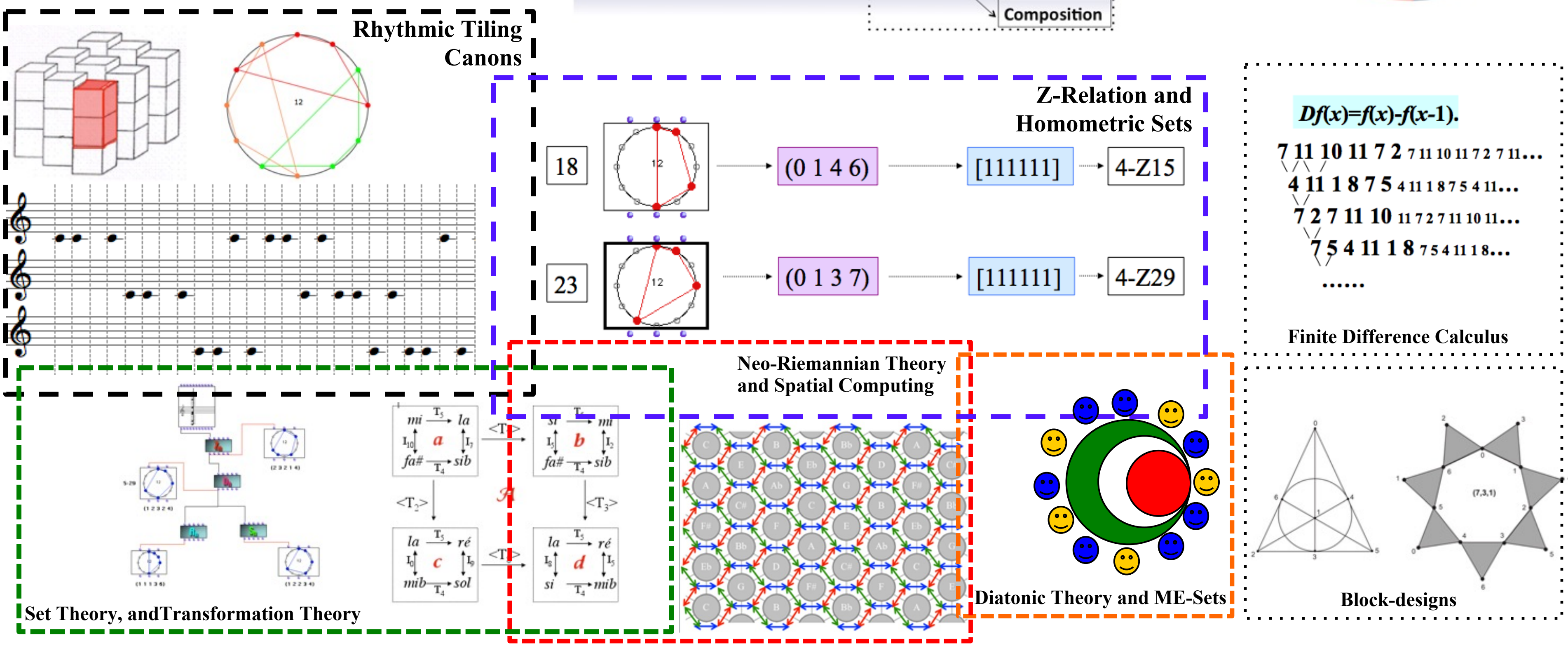


Some musically-driven mathematical problems

- Tiling Rhythmic Canons
- *Z* relation and homometry
- Transformational Theory
- Music Analysis, SC and FCA
- Diatonic Theory and ME-Sets
- Periodic sequences and FDC
- Block-designs in composition

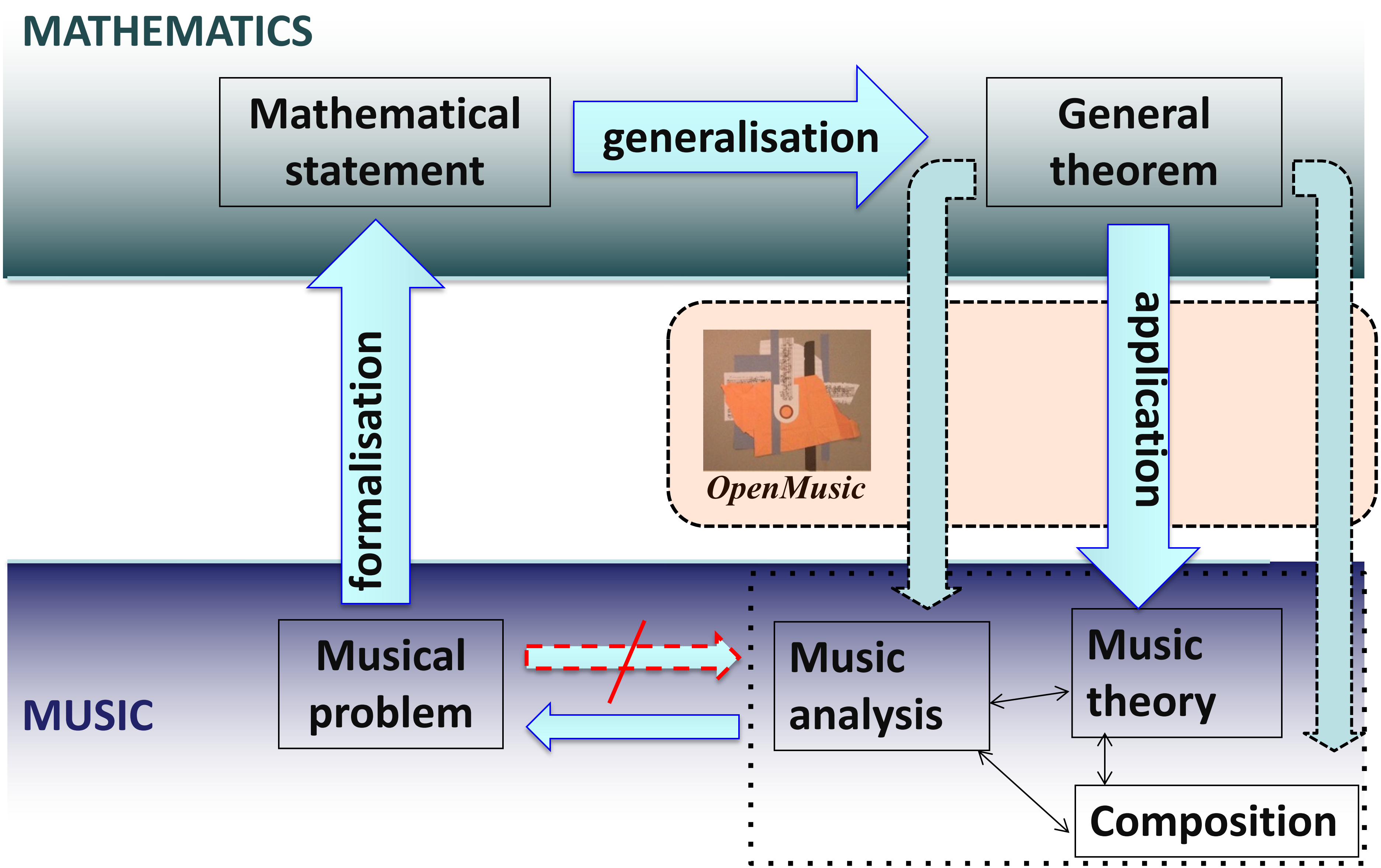






➔ M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

The double movement of a ‘mathemusical’ activity



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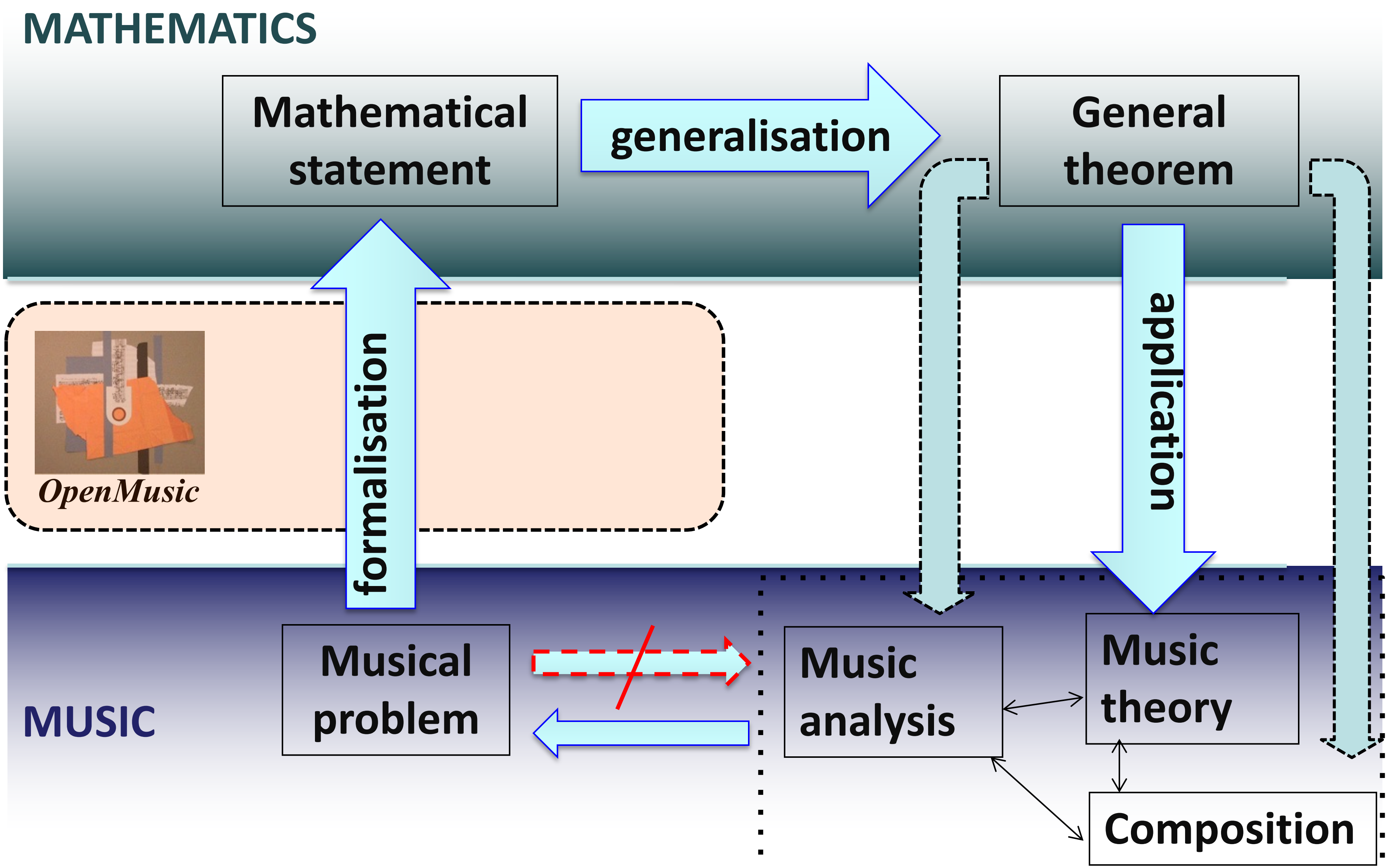

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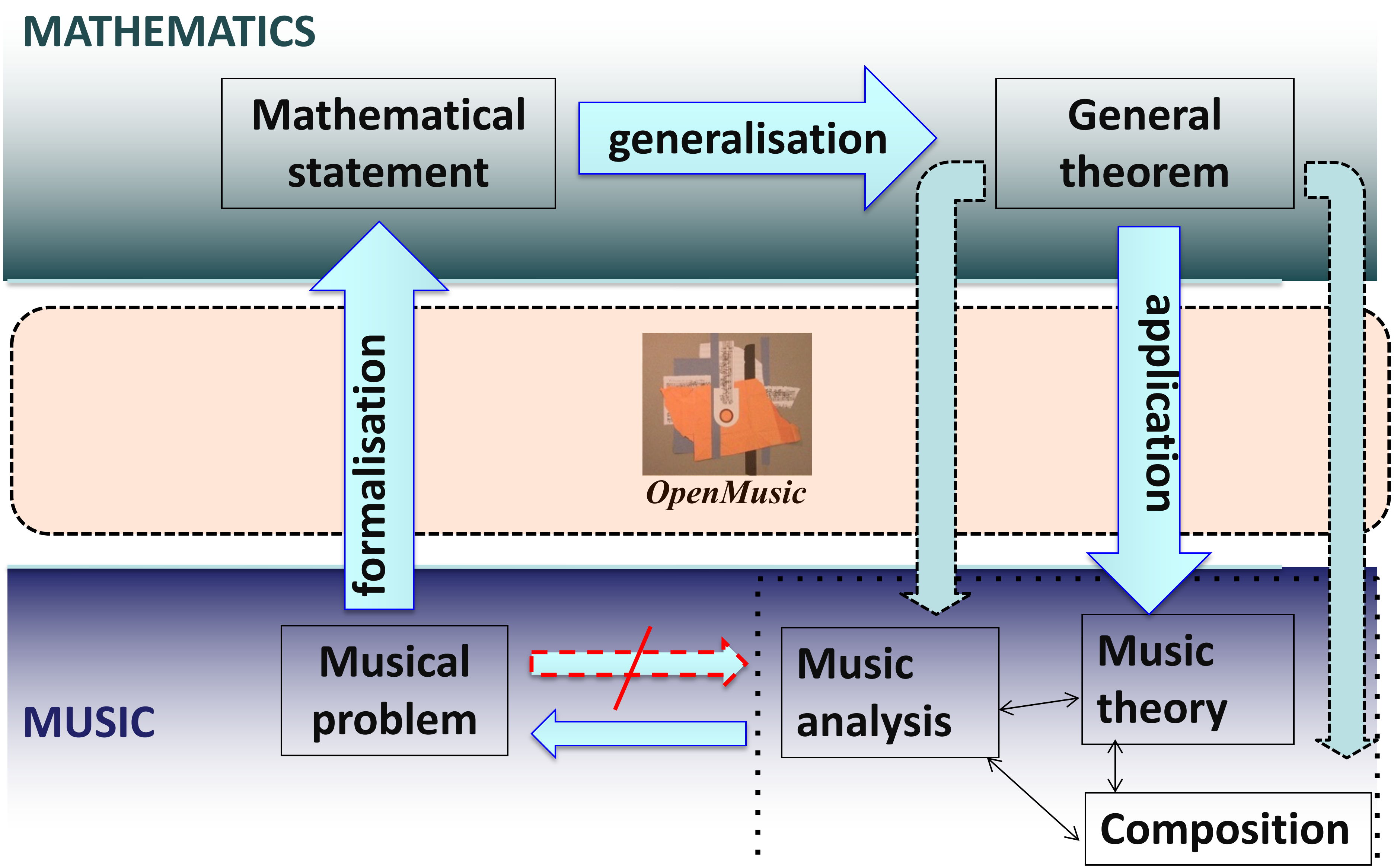

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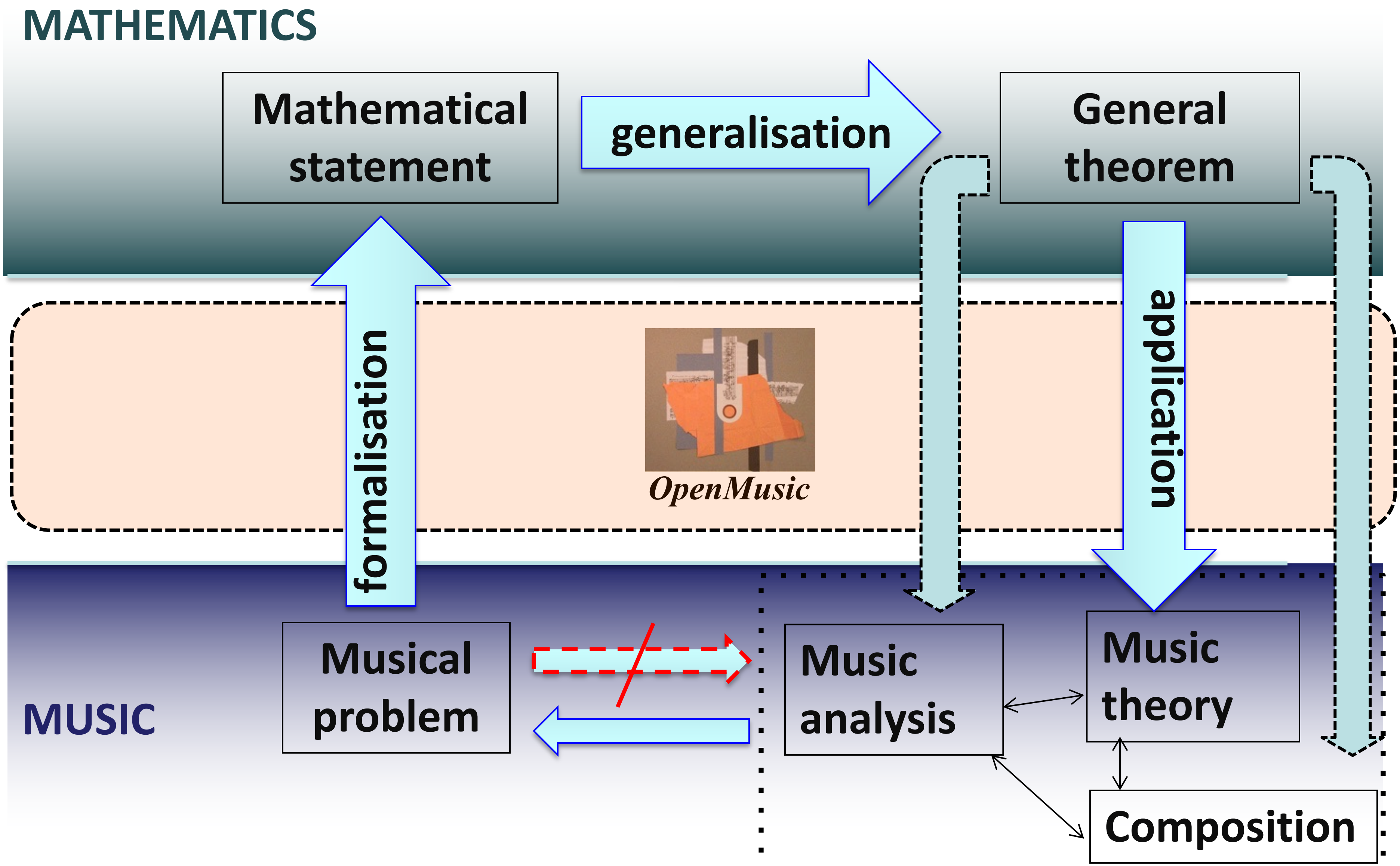
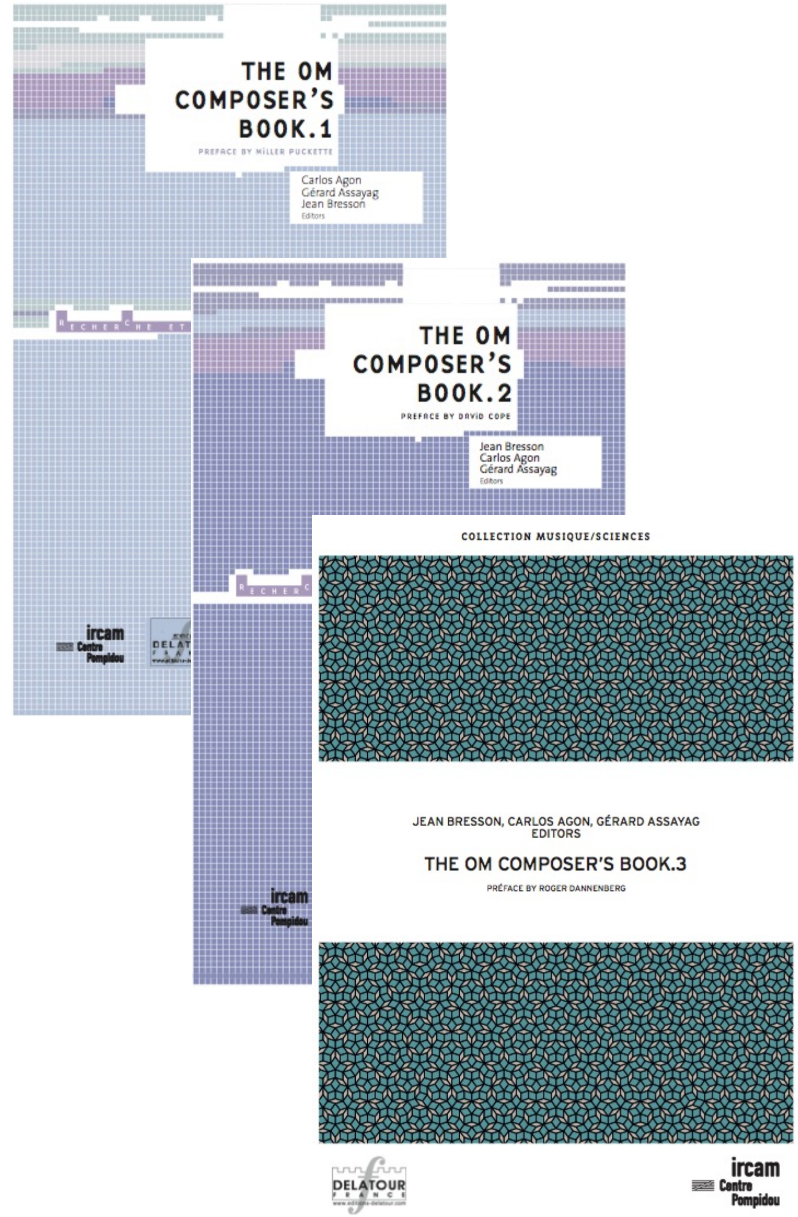


OpenMusic, a Visual Programming Language for computer-aided composition

www.repmus.ircam.fr/openmusic/home

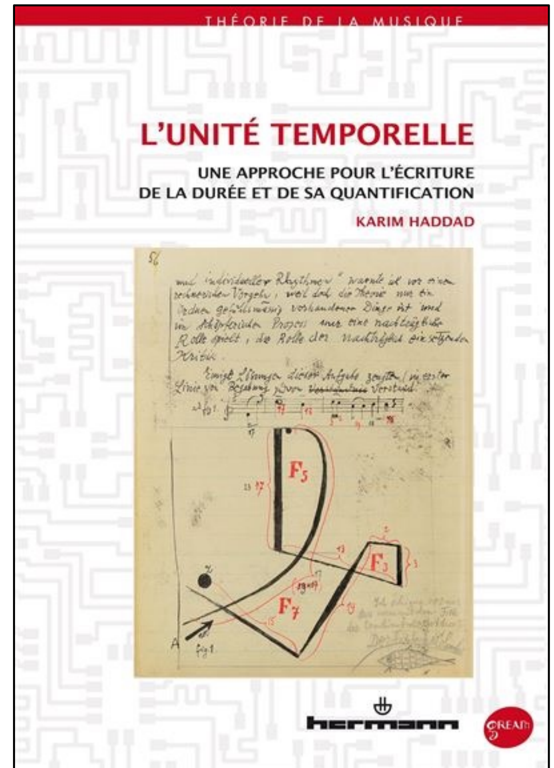


C. Agon, G. Assayag and J. Bresson, *The OM Composer's Book* (3 volumes)
"Musique/Sciences" Series, Ircam/Delatour, 2006, 2007 and 2016

OpenMusic, a Visual Programming Language for computer-aided composition

www.repmus.ircam.fr/openmusic/home

Karim Haddad (né en 1962)

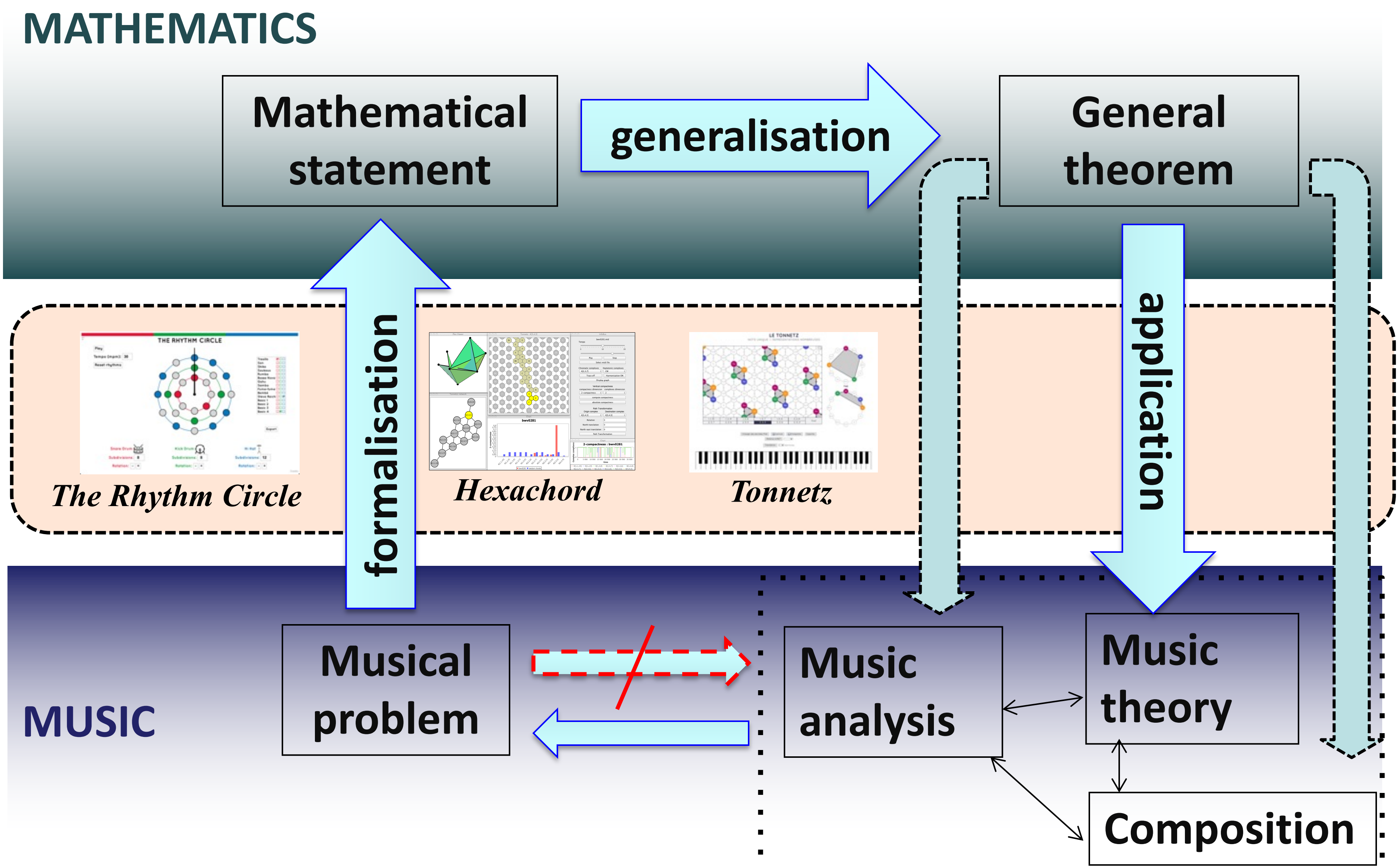


K. Haddad, *L'unité temporelle. Une approche pour l'écriture de la durée et de sa quantification*, Hermann, collection GREAM, Strasbourg, 2024.

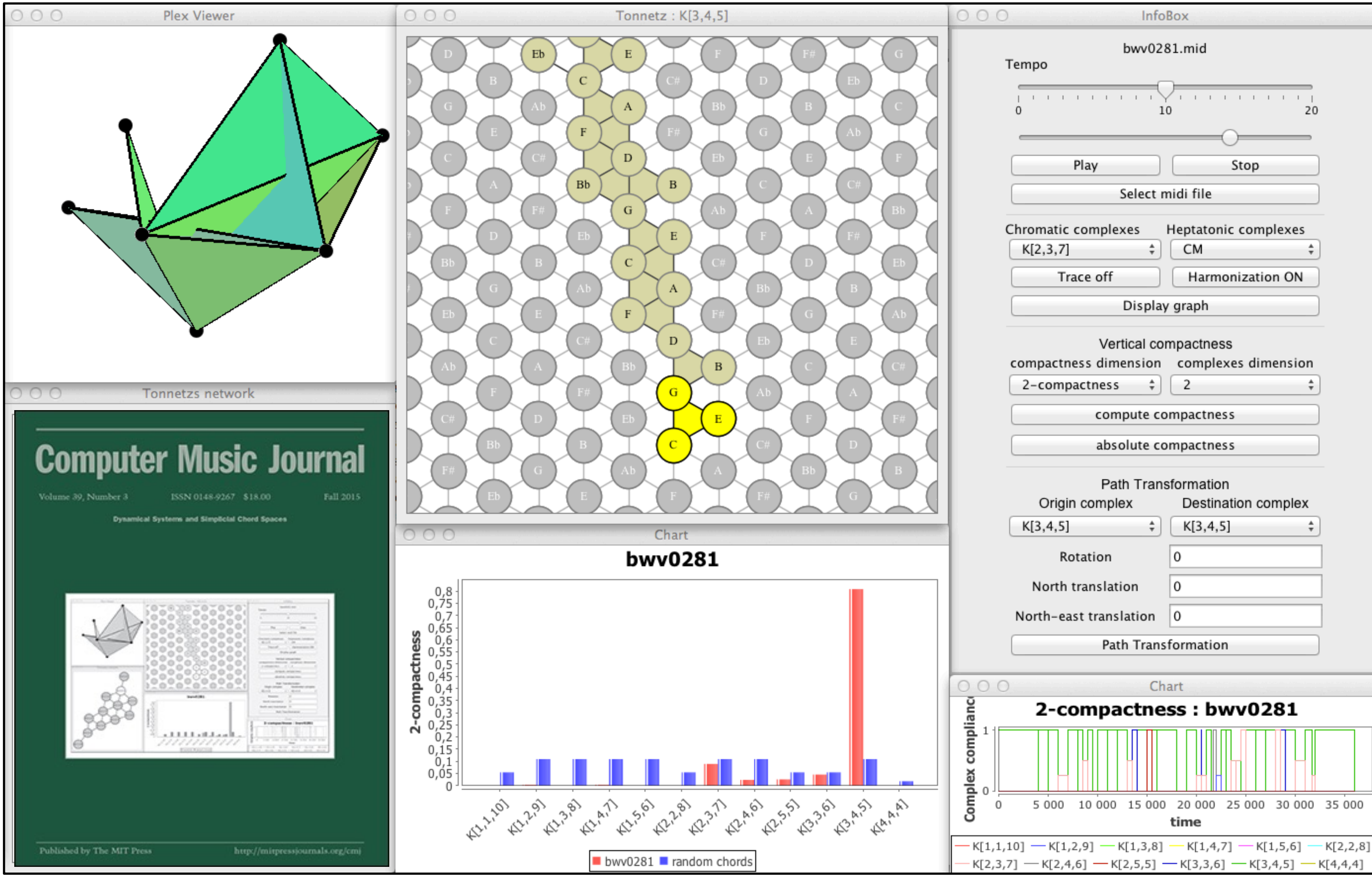
ATIAM

13/9 (14h30-17h30)

The double movement of a ‘mathemusical’ activity



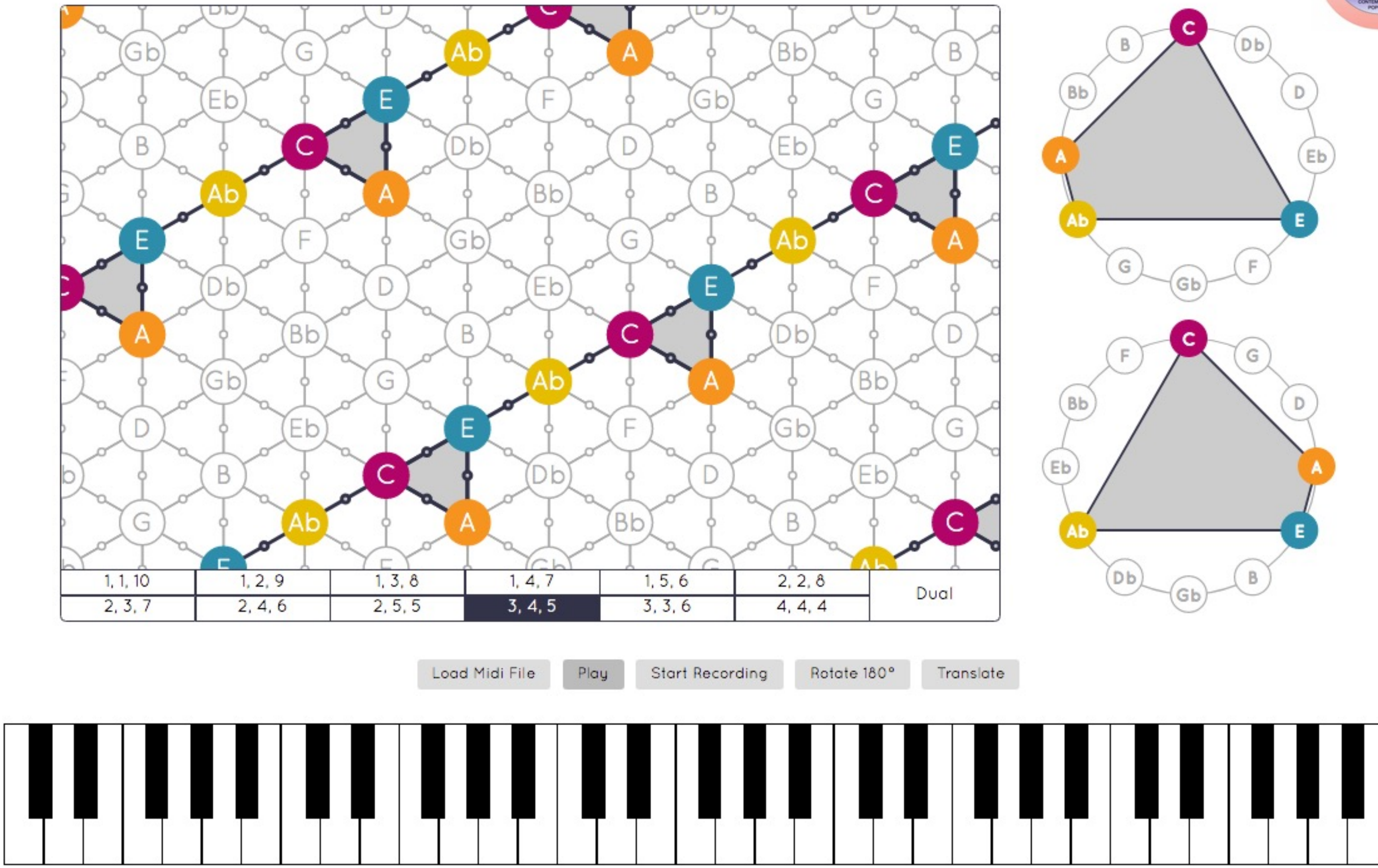
Spatial music analysis via *Hexachord*



→http://www.lacl.fr/~lbigo/hexachord

THE TONNETZ

ONE KEY – MANY REPRESENTATIONS



➔ **https://thetonnetz.com**

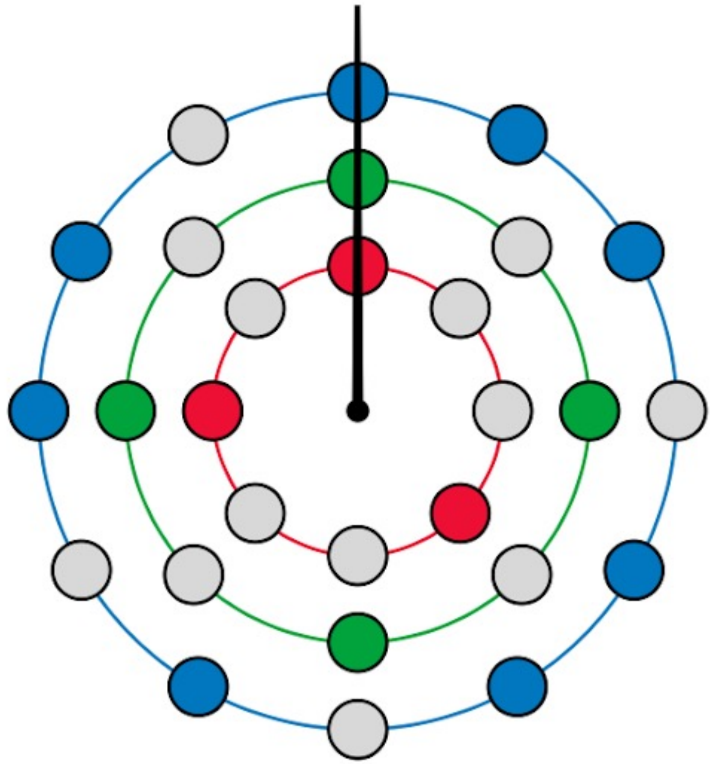
en
fr

THE RHYTHM CIRCLE

Play

Tempo (mpm): 30

Reset rhythms



➔ www.youtube.com/@mathemusique

Tresillo
Son
Shiko
Soukous
Rumba
Bossa Nova
Gahu
Samba
Fume-fume
Bembé
Steve Reich
Basic 1
Basic 2
Basic 3
Basic 4

Export

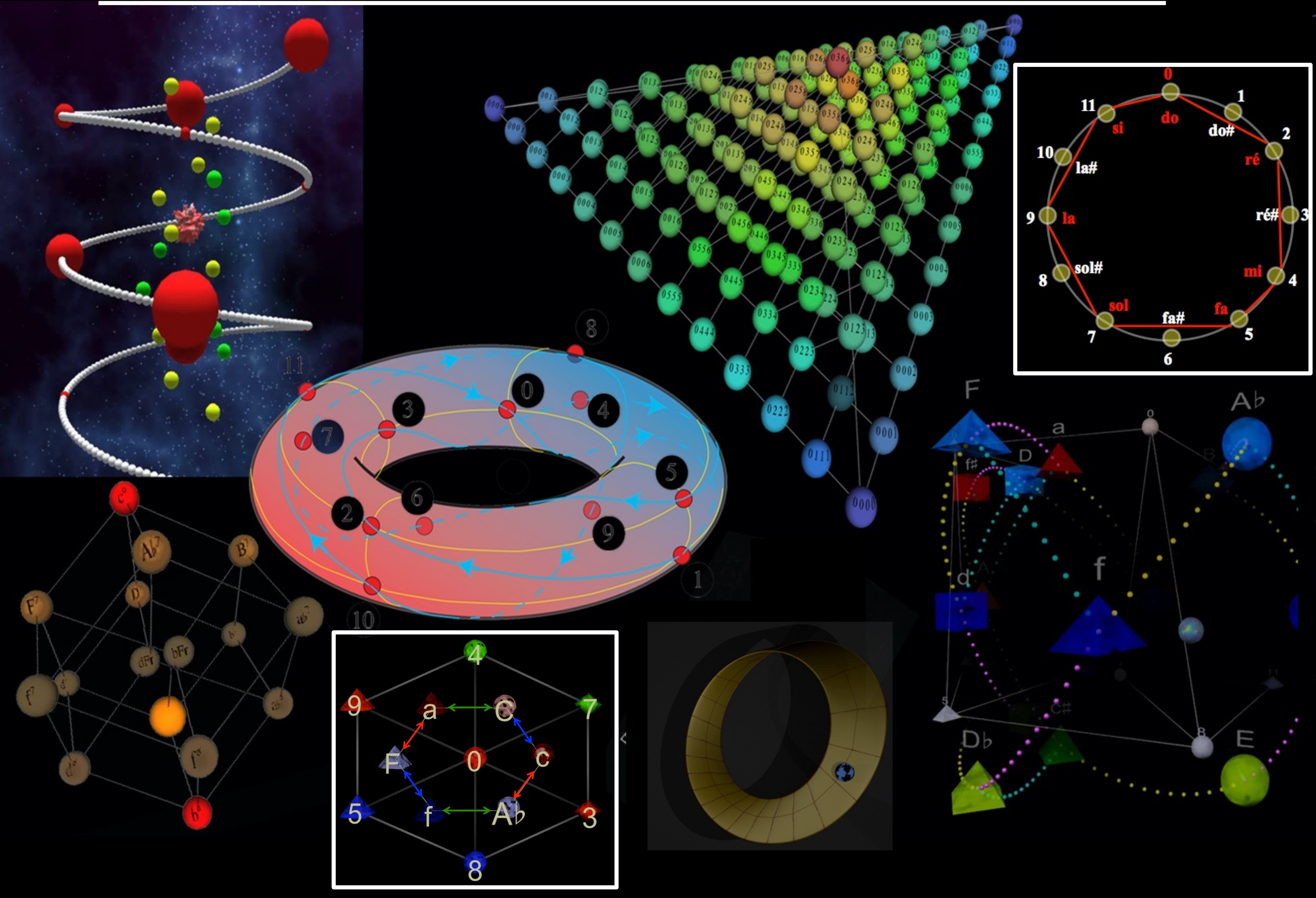
Snare Drum
Subdivisions: 8
Rotation: - +

Kick Drum
Subdivisions: 8
Rotation: - +

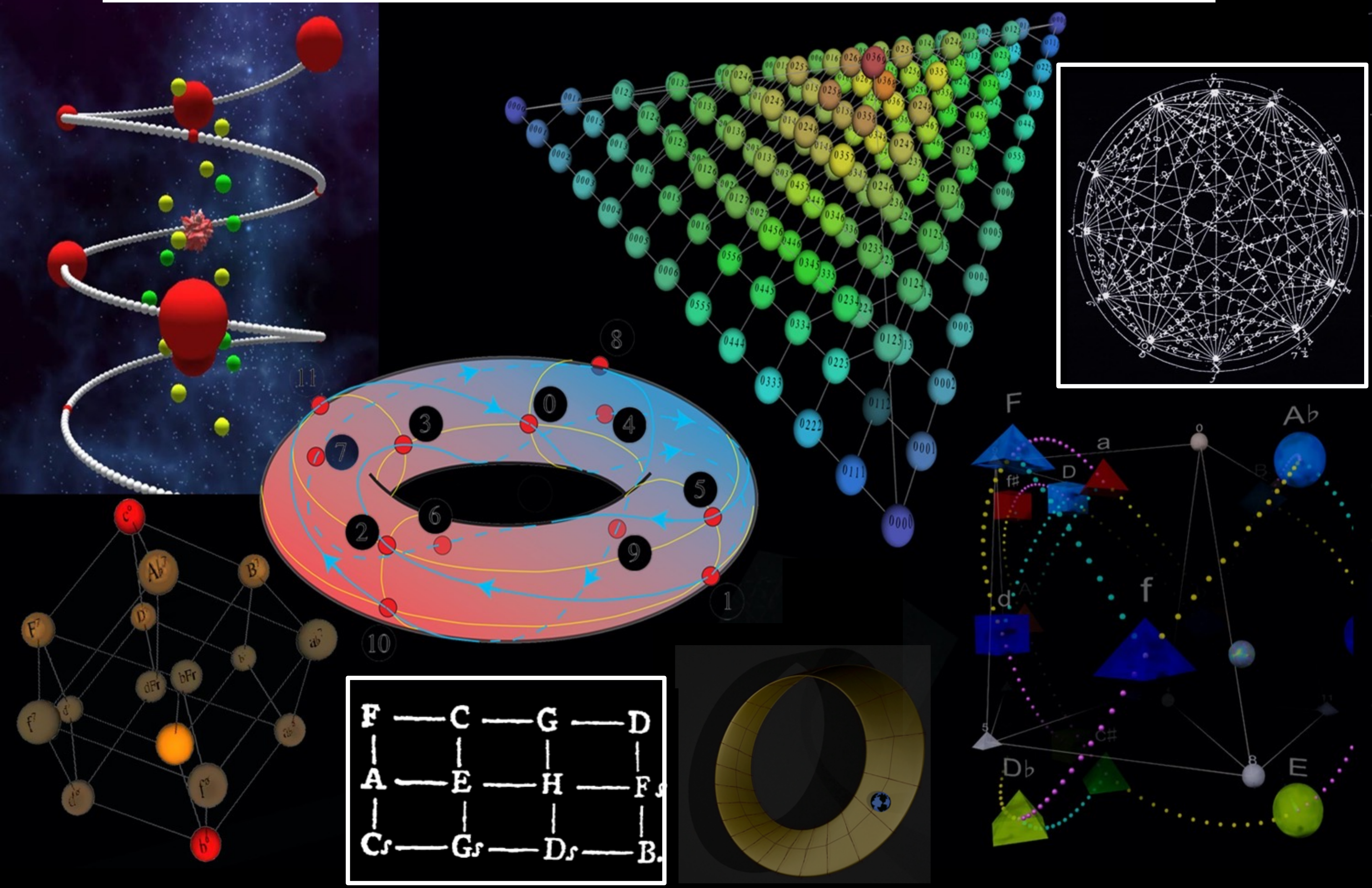
Hi Hat
Subdivisions: 12
Rotation: - +

➔ https://rhythm-circle.com

The galaxy of geometrical models at the service of music



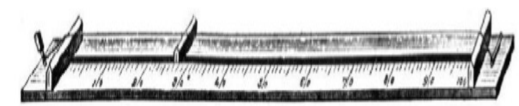
The galaxy of geometrical models at the service of music



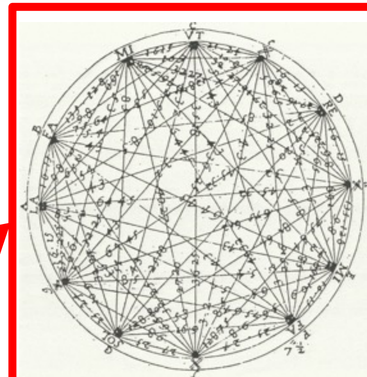
Music and mathematics: « prima la musica »!

MUSIC	MATHS
500 B.C. Pitches and lengths of strings are related. Here **music** gives a marvelous thrust to **number theory** and **geometry**.	Discovery of the fundamental importance of **natural numbers** and the invention of **fractions**.
No correspondence in music.	Positive irrational numbers [...]
300 B.C. [...] Music theory highlights the discovery of the **isomorphism between the logarithms** (musical intervals) and **exponentials** (string lengths) more than 15 centuries before their discovery in mathematics; also a **premonition of group theory** is suggested by Aristoxenos.	No reaction in mathematics. [...]
1000 A.D. Invention of the **two-dimensional spatial representation of pitches** linked with time by means of staves and points [...] seven centuries (1635-37) before the magnificent **analytical geometry** of Fermat and Descartes.	*No parallel in mathematics.*
1500 *No response or development of the preceding concepts.*	Zero and negative numbers are adopted. Construction of the set of rationals.
1600 *No equivalence, no reaction.*	The sets of real numbers and of logarithms are invented.
1648 Invention of **musical combinatorics** by Marin Mersenne (*Harmonicorum Libri*)	Probability theory by Bernoulli (*Ars Conjectandi*, 1713)
1700 [...] The **fugue**, for example, is an **abstract automaton** used two centuries before the birth of the science of automata. Also, there is an **unconscious manipulation of finite groups** (Klein group) in the four variations of a melodic line used in counterpoint.	Number theory is ahead of but has no equivalent yet in temporal structures. [...]
1773 A first **geometric** and **graph-theoretic** representation of pitches (*Speculum Musicum*)	Invention of graph theory
1900 Liberation from the tonal yoke. First acceptance of the neutrality of chromatic totality (Loquin [1895], Hauer, Schoenberg).	The infinite and transfinite numbers (Cantor). Peano axiomatics. [...] The beautiful measure theory (Lebesque, ...)
1920 First radical formalization of macrostructures through the serial system of Schoenberg.	No new development of the number theory.
1929 and **1937-1939** Susanne K. Langer and Ernst Krenek on the role of axioms in music	David Hilbert, *Die Grundlage der Geometrie* (1899)
1946 Milton Babbitt on group theory and integral serialism	Rudolf Carnap, *The Logical Syntax of Language* (1937)

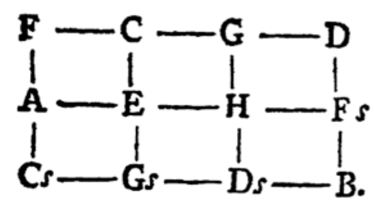
I. Xenakis



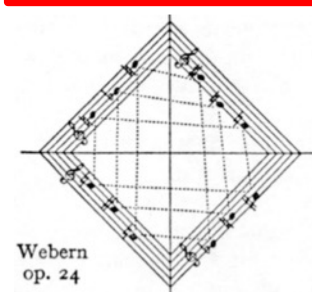
Pythagoras and the monochord, VIth-Vth Century B.C.

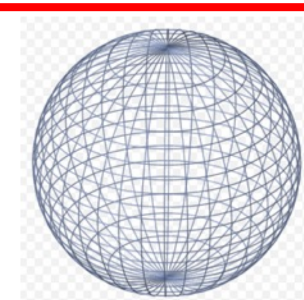


Mersenne and the 'musical clock', 1648

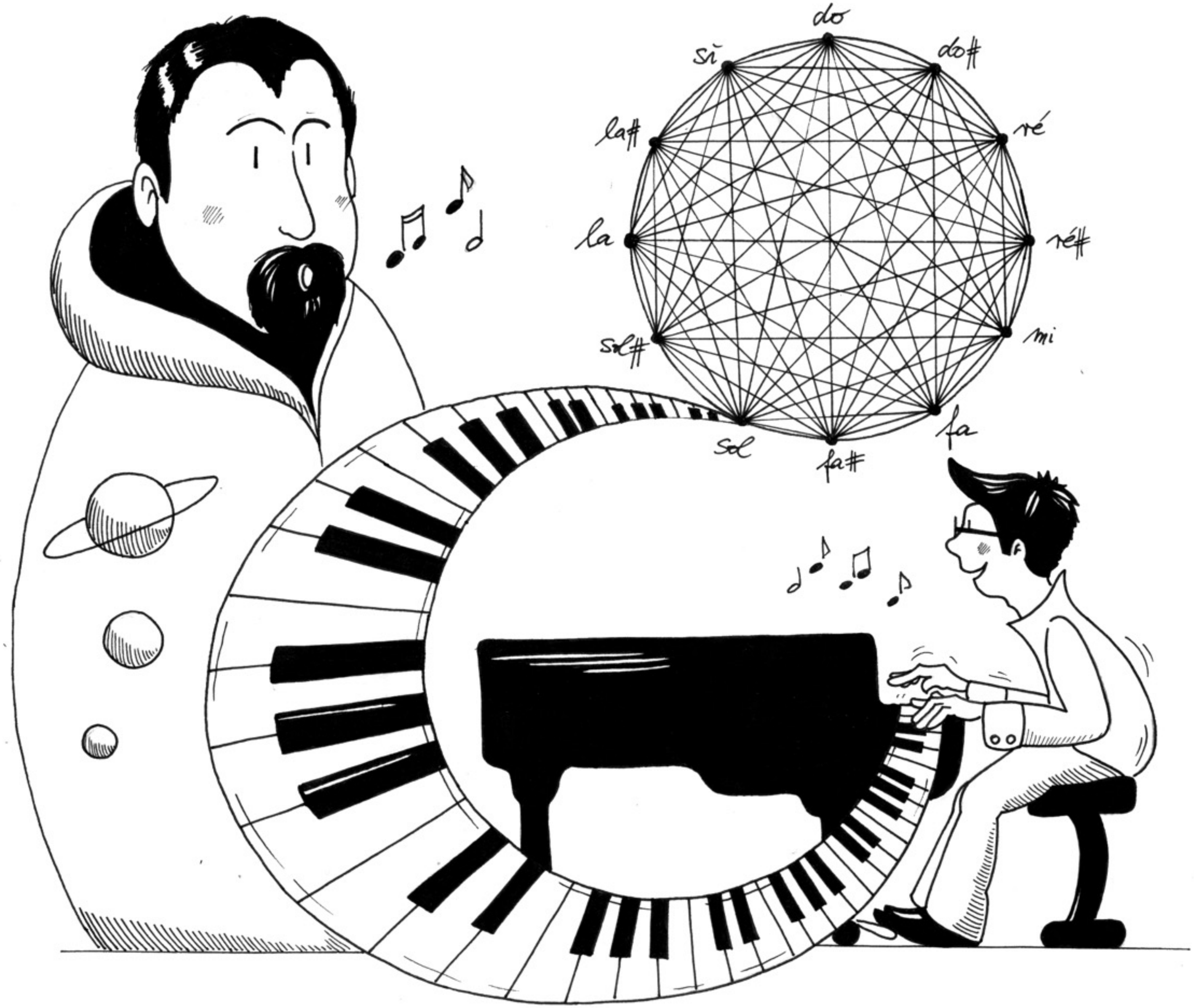


Euler and the *Speculum musicum*, 1773



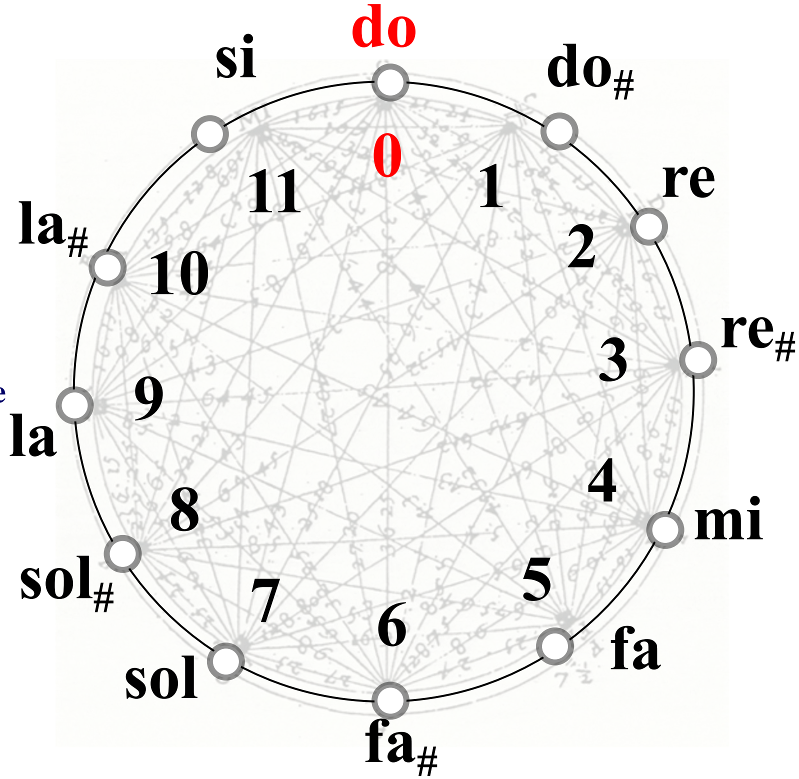


Marin Mersenne, the father of combinatorics

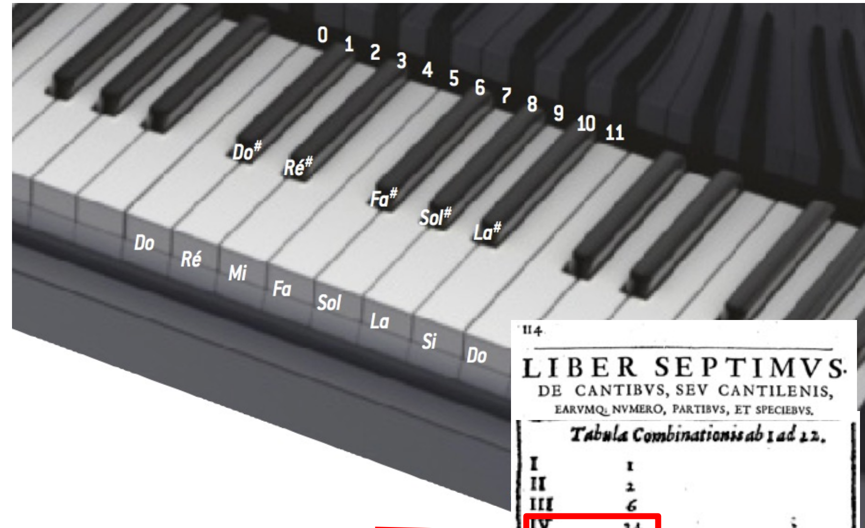


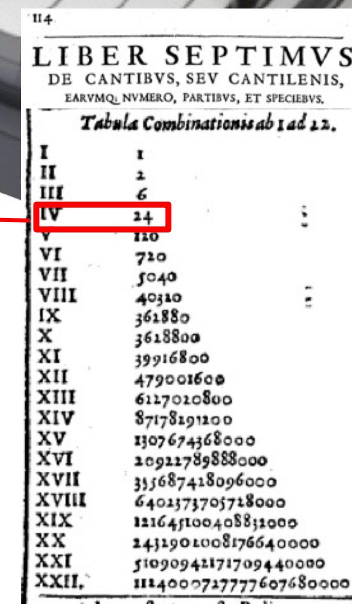
The circular representation of the pitch space

Marin Mersenne



Harmonicorum Libri XII, 1648





114

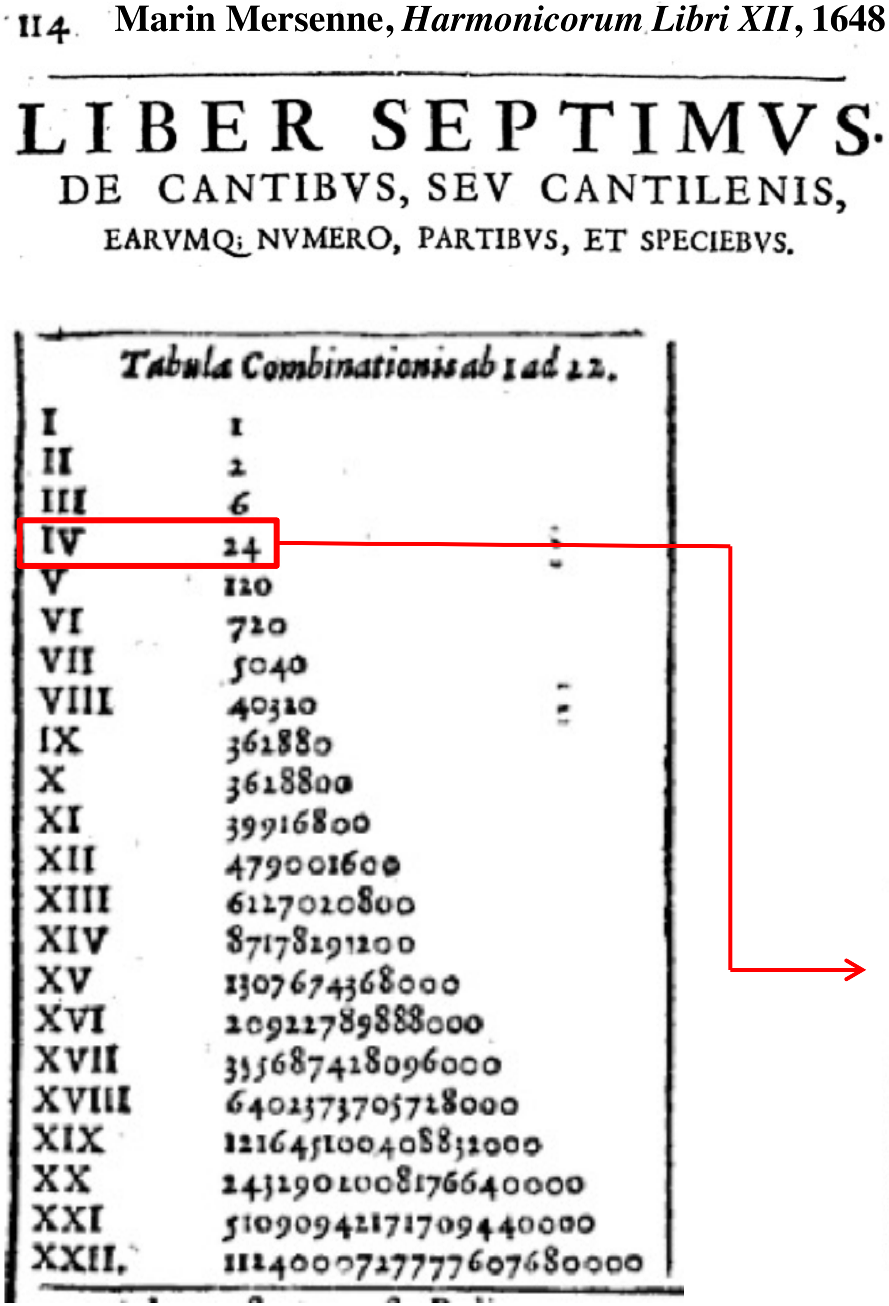
LIBER SEPTIMVS.
DE CANTIBVS, SEV CANTILENIS, EARVMQ. NVMERO, PARTIBVS, ET SPECIEBVS.

Tabula Combinationis ab 1 ad 22.

I	1
II	2
III	6
IV	24
V	120
VI	720
VII	5040
VIII	40320
IX	362880
X	3628800
XI	39916800
XII	479001600
XIII	6227020800
XIV	87178291200
XV	1307674368000
XVI	20922789888000
XVII	355687428096000
XVIII	6402373705728000
XIX	121645100408832000
XX	2432902008176640000
XXI	51090942171709440000
XXII.	1124000727777607680000

Permutational strategies in contemporary music

Marin Mersenne, *Harmonicorum Libri XII*, 1648



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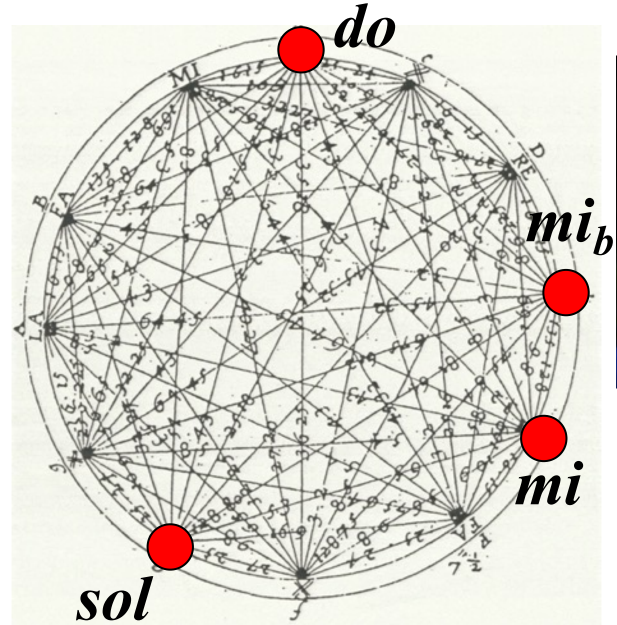

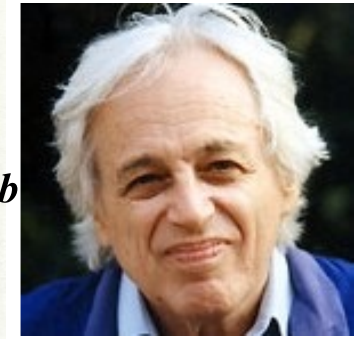

Six Bagatelles
(G. Ligeti, 1953)

Permutational strategies in contemporary music

114 Marin Mersenne, *Harmonicorum Libri XII*, 1648

LIBER SEPTIMVS.

DE CANTIBVS, SEV CANTILENIS,

EARVMQ; NVMERO, PARTIBVS, ET SPECIEBVS.

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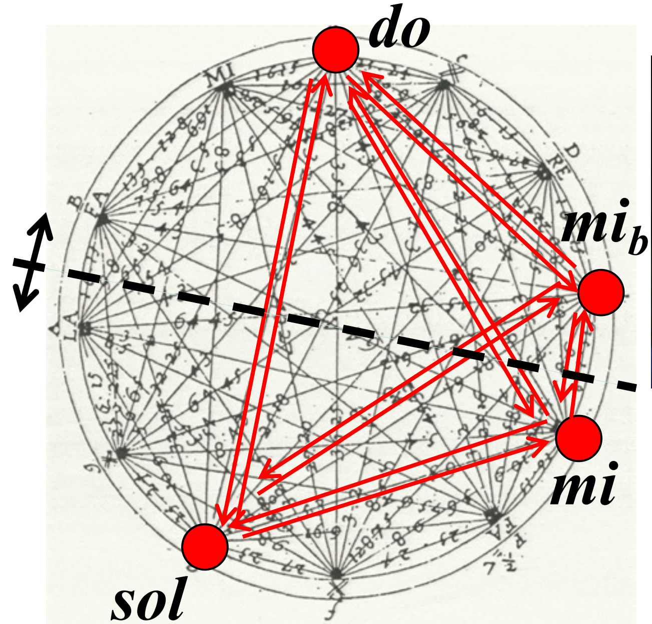
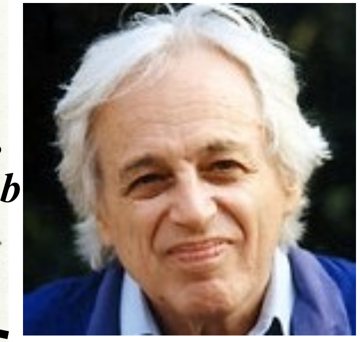




Six Bagatelles
(G. Ligeti, 1953)

Varietas, seu Combinatio quatuor notarum.

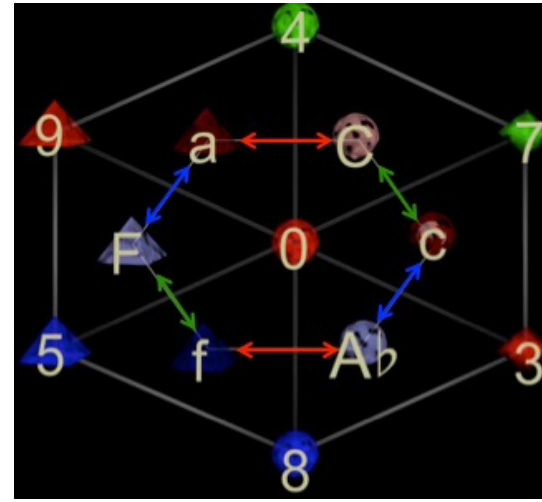
Permutational melodies in song writing

Se telefonando, 1966 (Maurizio Costanzo/Ennio Morricone) / Mina

(min. 0’53”)

Ennio Morricone

The harmonic space

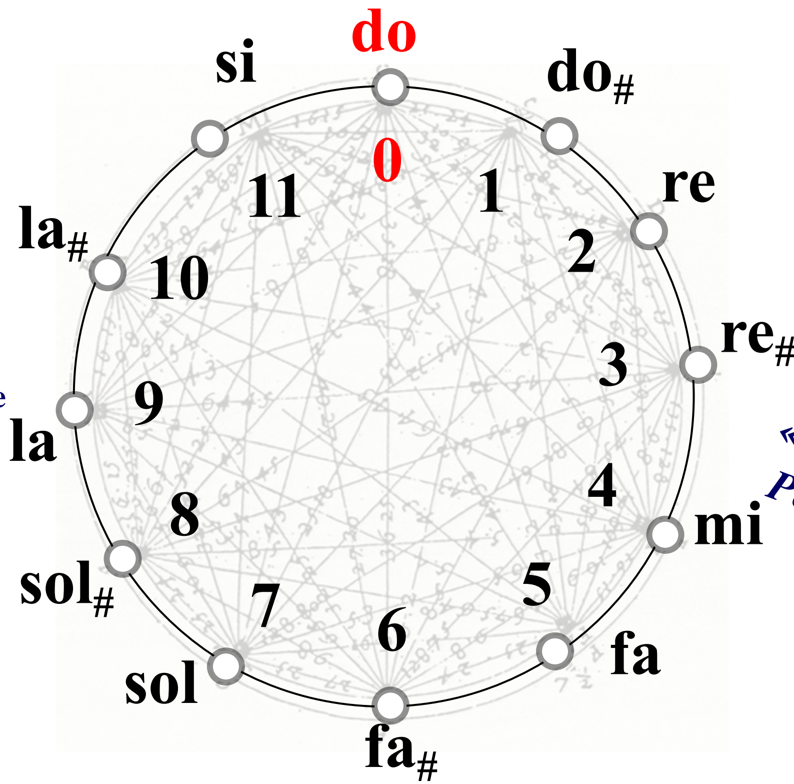


C	c	**$C_\#$**	$c_\#$	D	**d**
E_b	**e_b**	E	e	**F**	f
$F_\#$	$f_\#$	G	**g**	$G_\#$	$g_\#$
A	a	**B_b**	**b_b**	**B**	b

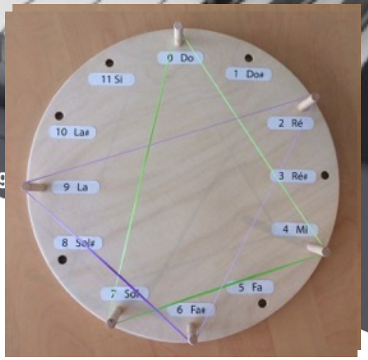
Chord enumeration

The circular representation of the pitch space

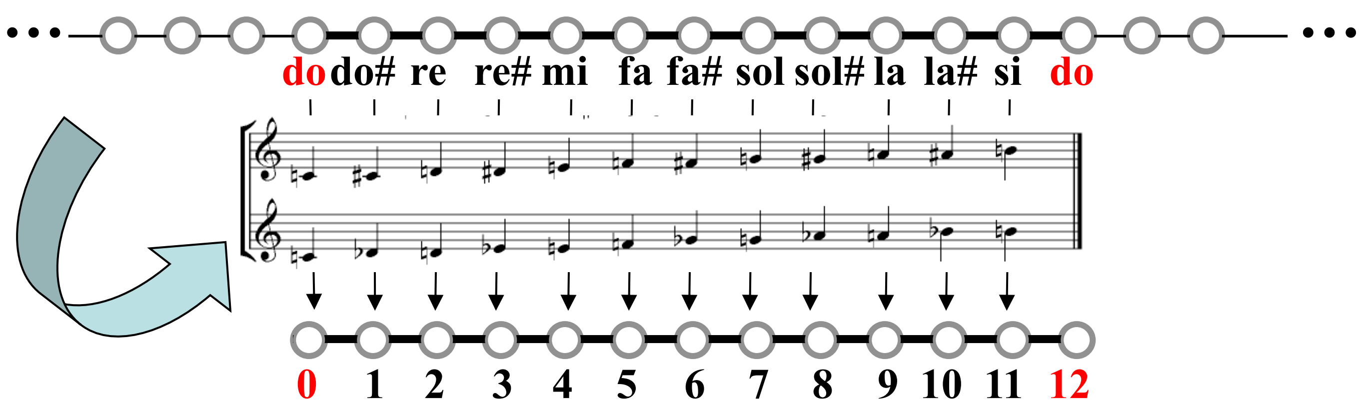
Marin Mersenne



Harmonicorum Libri XII, 1648

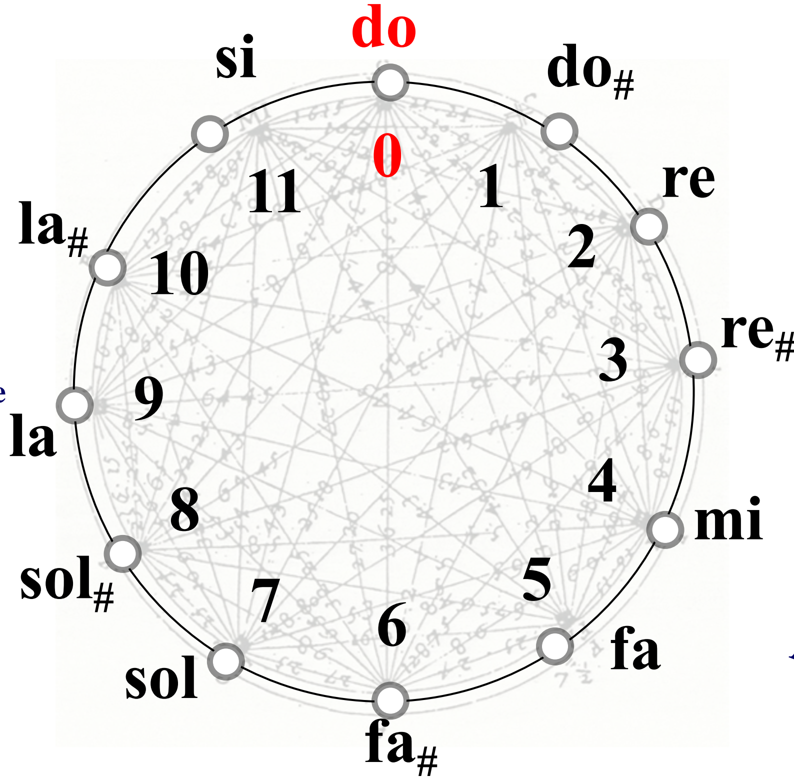


M. Andreatta, C. Agon, «La musique mise en algèbre», *Pour la Science*, 2008

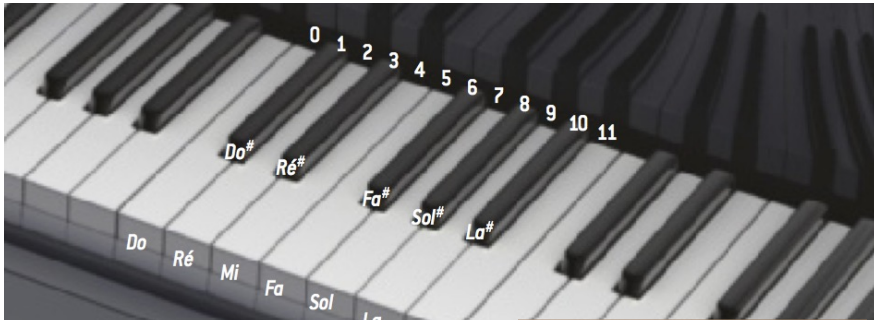


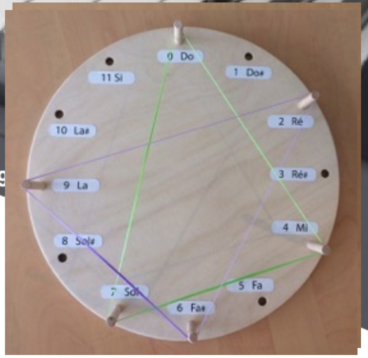
The circular representation of the pitch space

Marin Mersenne

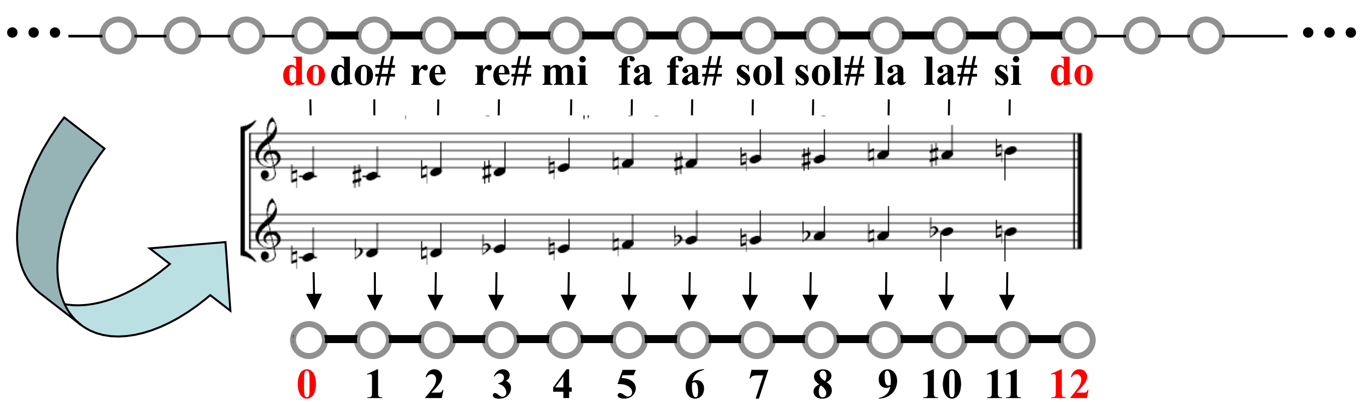


Harmonicorum Libri XII, 1648

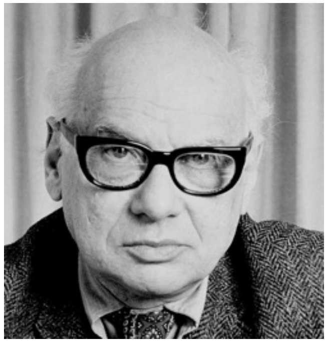




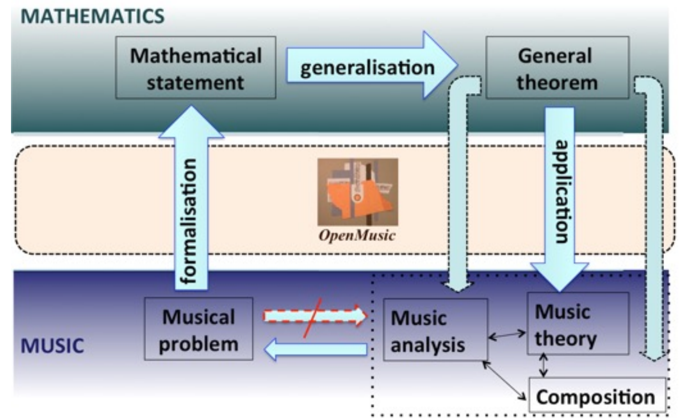
M. Andreatta, C. Agon, « Algèbre et géométrie : sont-elles inscrites dans le cerveau ? », ***Pour la Science*****, 2018**

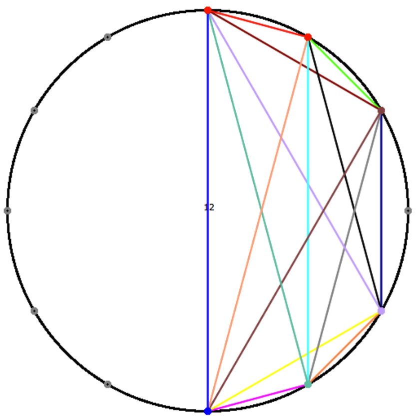


A historical example of "mathemusical" problem



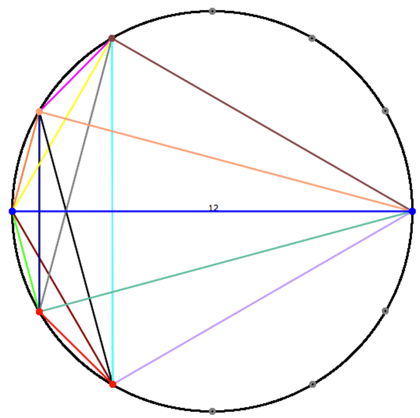
M. Babbitt





?

Hexachord Theorem



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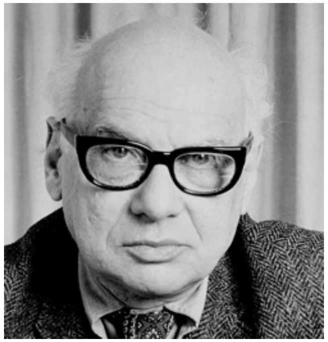

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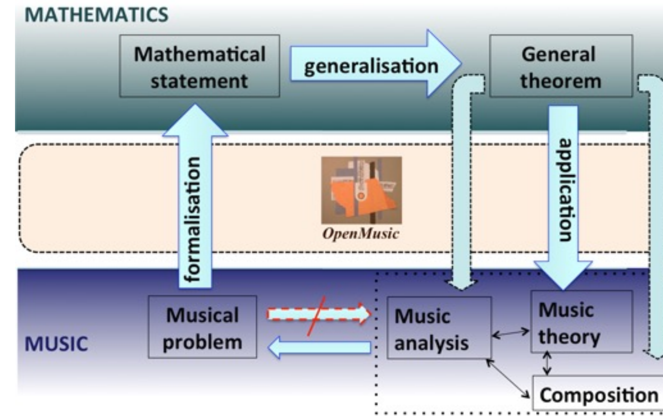
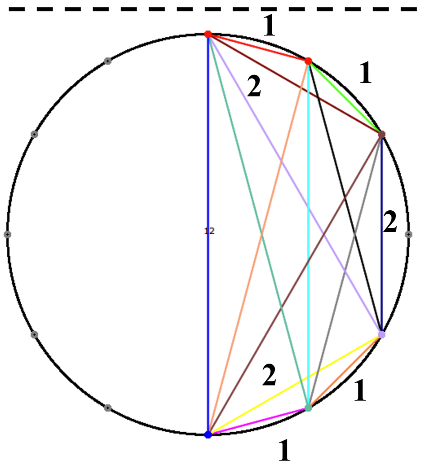
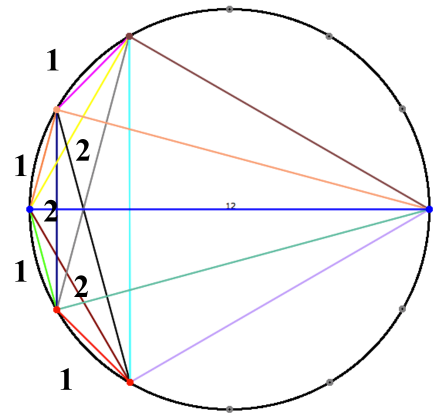
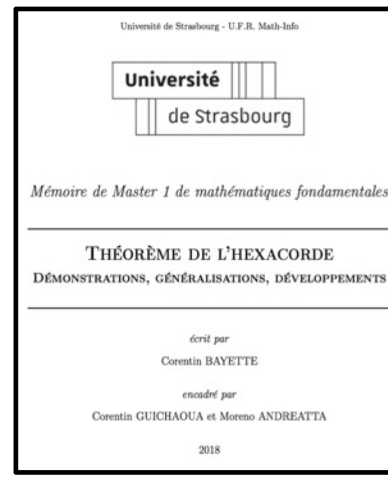




HOMOMETRY

Hexachord Theorem





Université de Strasbourg - U.F.R. Math-Info

Université de Strasbourg

Mémoire de Master 1 de mathématiques fondamentales

THÉORÈME DE L'HEXACORDE

DÉMONSTRATIONS, GÉNÉRALISATIONS, DÉVELOPPEMENTS

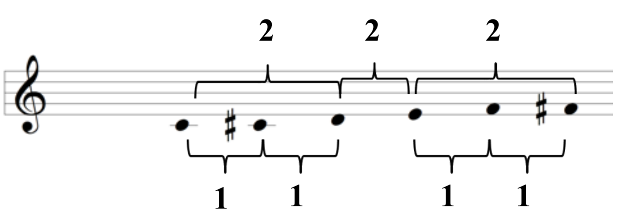
écrit par

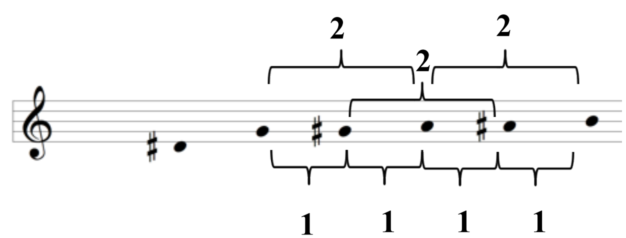
Corentin BAYETTE

encadré par

Corentin GUICHAOUA et Moreno ANDREATTA

2018

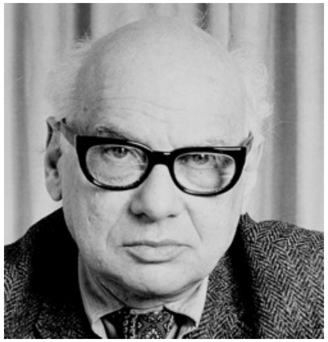




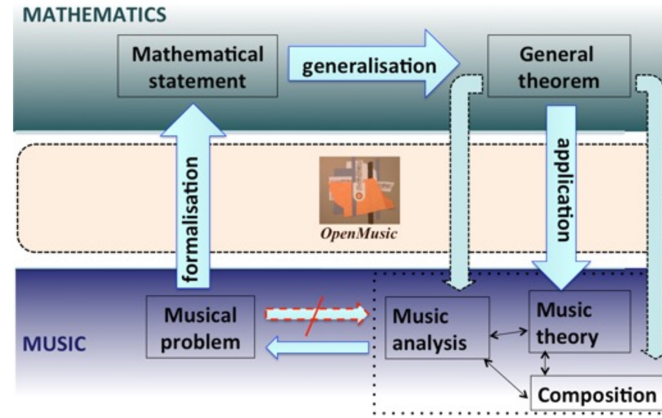
IMAGES DES MATHÉMATIQUES

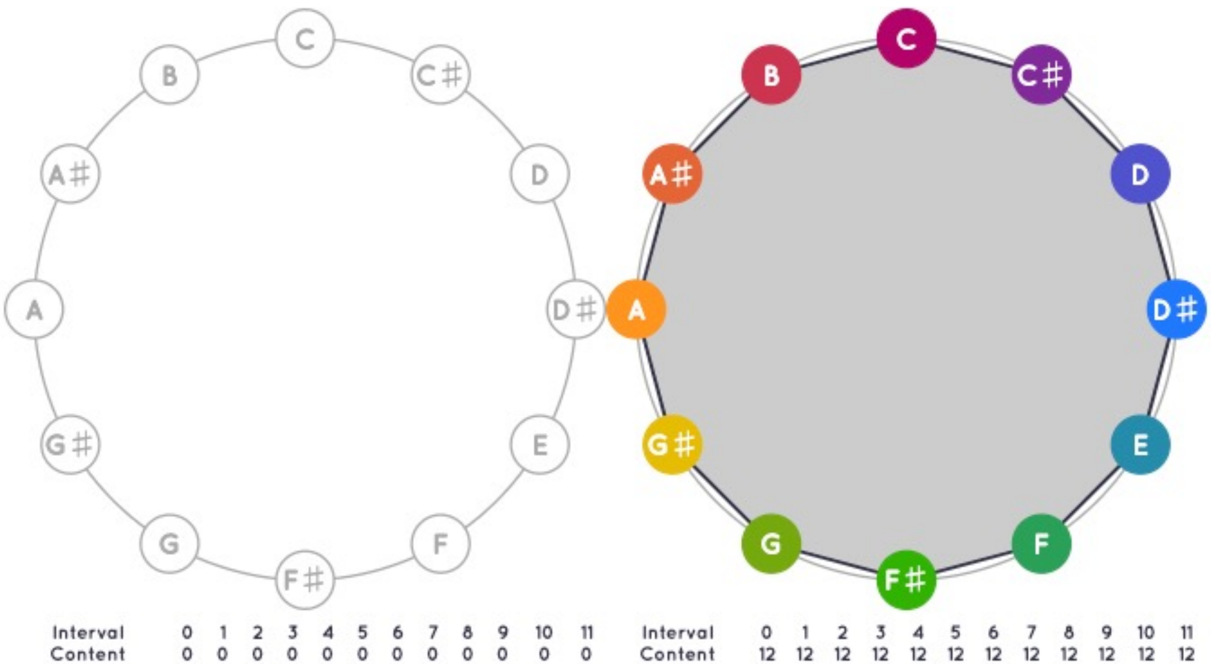
➔ **Corentin Bayette, Théorème de l'Hexacorde, Master 1 dissertation, University of Strasbourg, 2018**

A historical example of "mathemusical" problem



M. Babbitt



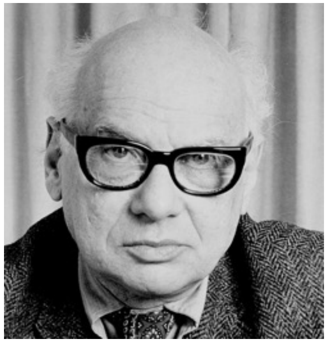


DEMO

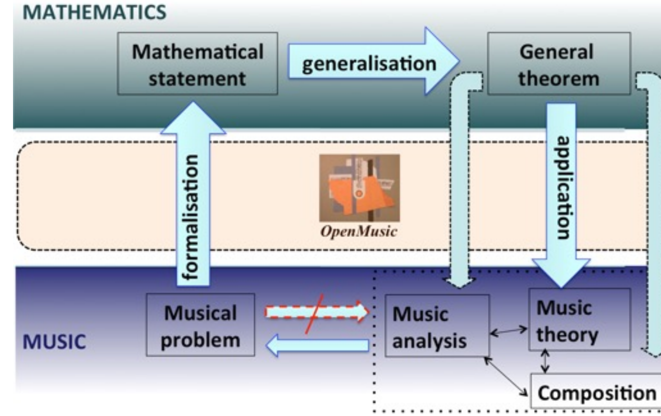
➔ Corentin Bayette, Théorème de l'Hexacorde, Master 1 dissertation, University of Strasbourg, 2018

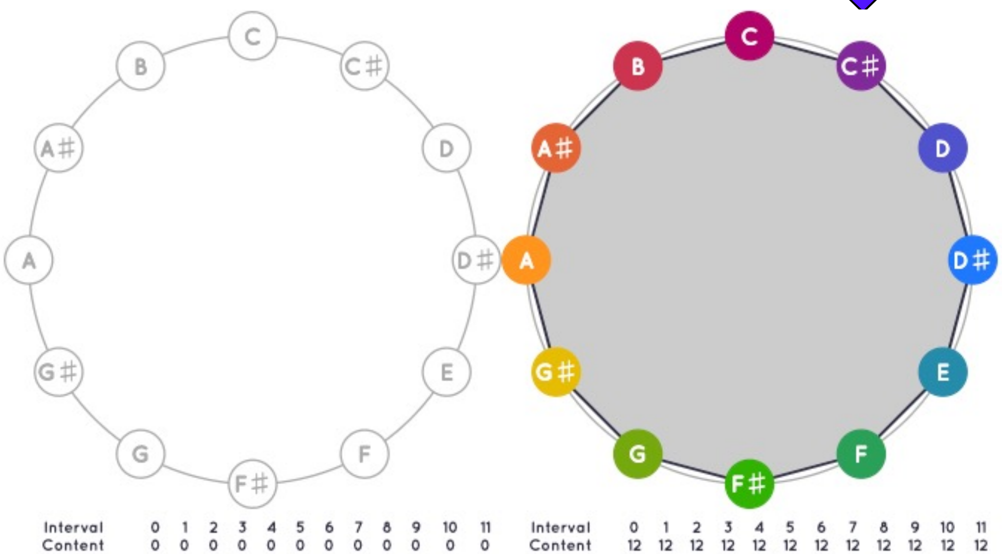
➔ https://guichaoua.gitlab.io/web-hexachord/hexachordTheorem

A historical example of “mathemusical” problem



M. Babbitt



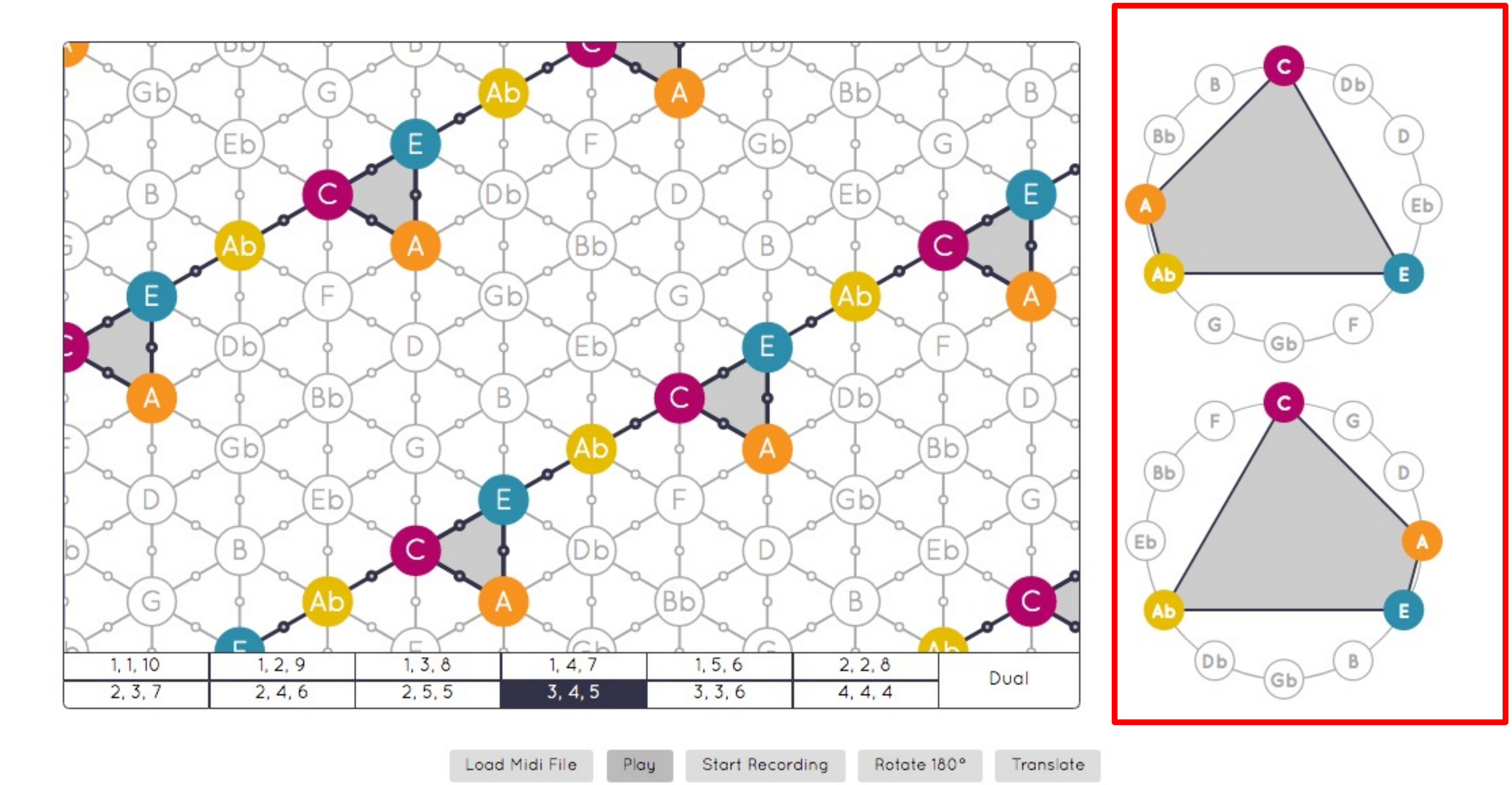


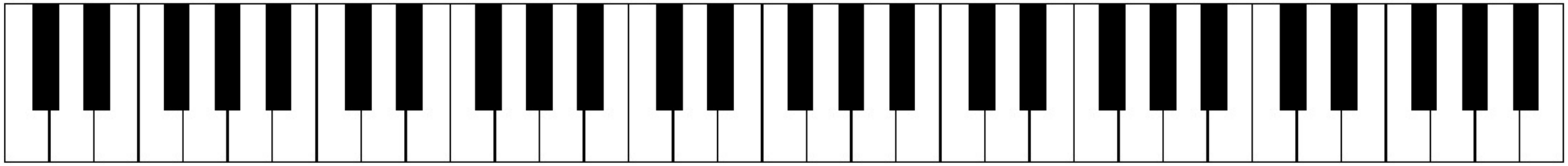
- ➔ Corentin Bayette, Théorème de l'Hexacorde, Master 1 dissertation, University of Strasbourg, 2018
- ➔ https://guichaoua.gitlab.io/web-hexachord/hexachordTheorem
- ➔ M. Andreatta et al., New hexachordal theorems in metric spaces with a probability measure", Rendiconti Univ Padova, 2023
- ➔ M. Andreatta et al., Taking music seriously: on the dynamics of ‘mathemusical’ research with a focus on Hexachordal Theorems", SIGMA (Symmetry, Integrability and Geometry: Methods and Applications), 2024

Symmetry, Integrability and Geometry: Methods and Applications

THE TONNETZ

ONE KEY – MANY REPRESENTATIONS

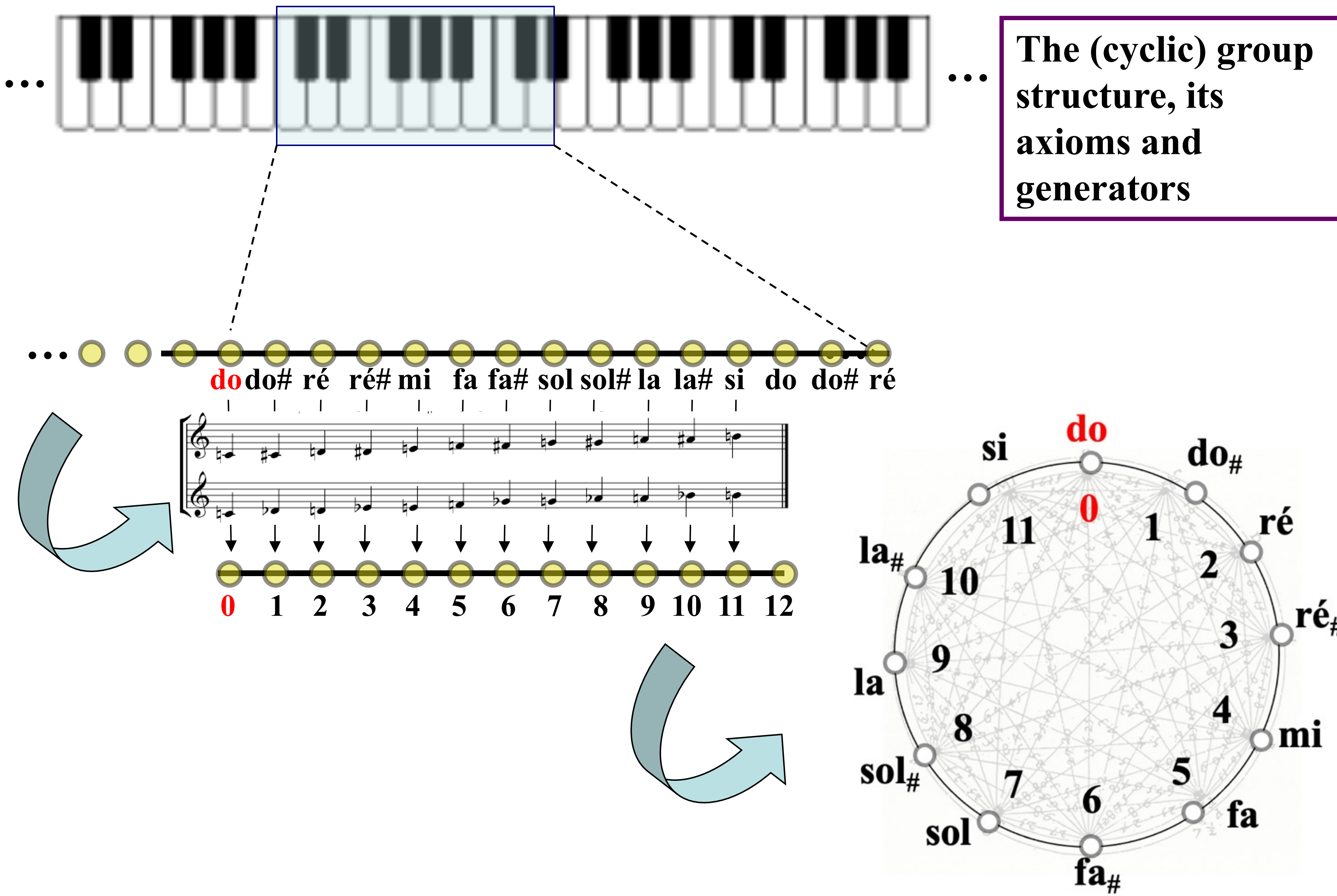




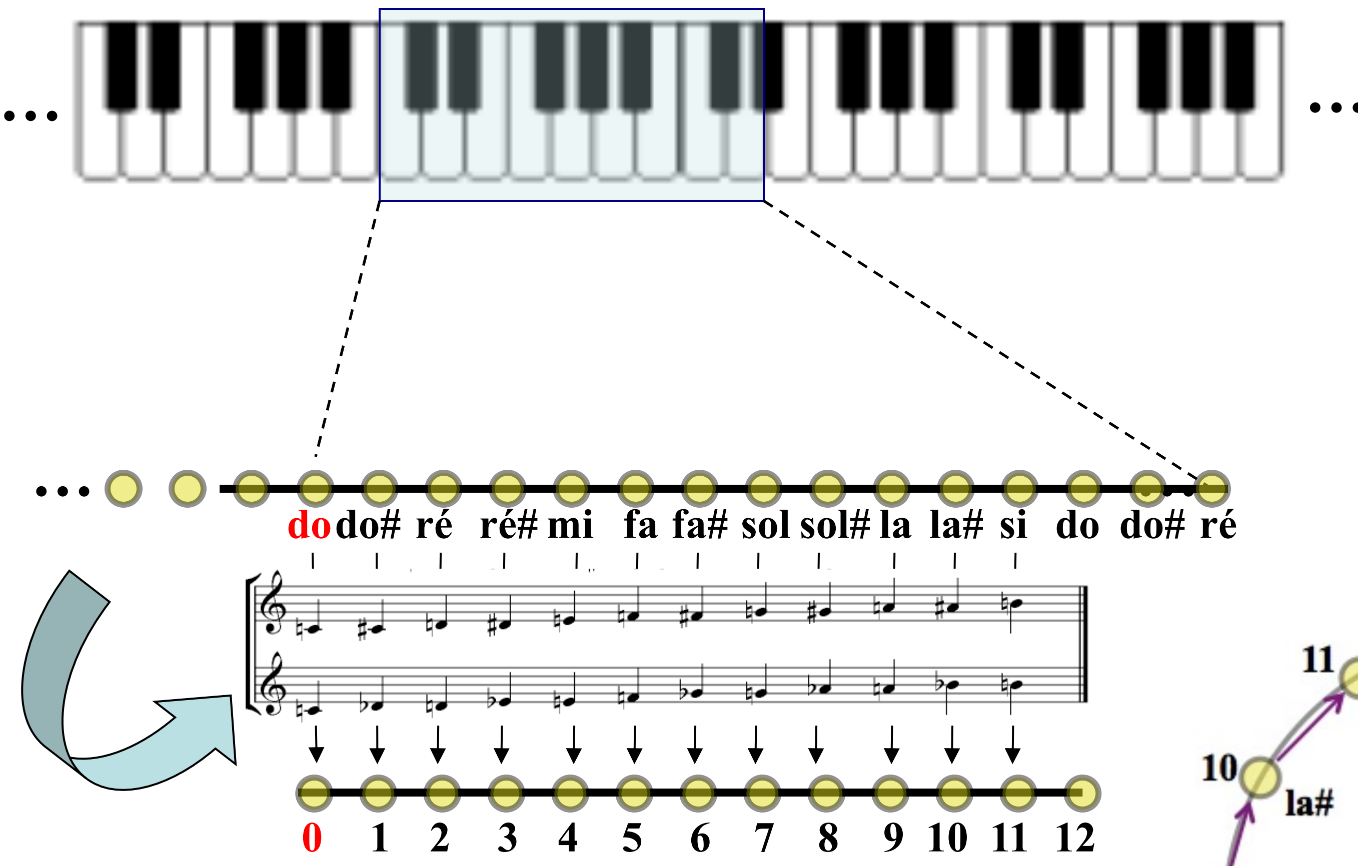
➔ **https://thetonnetz.com** **DEMO**

The equal tempered space is a cyclic group

The (cyclic) group structure, its axioms and generators



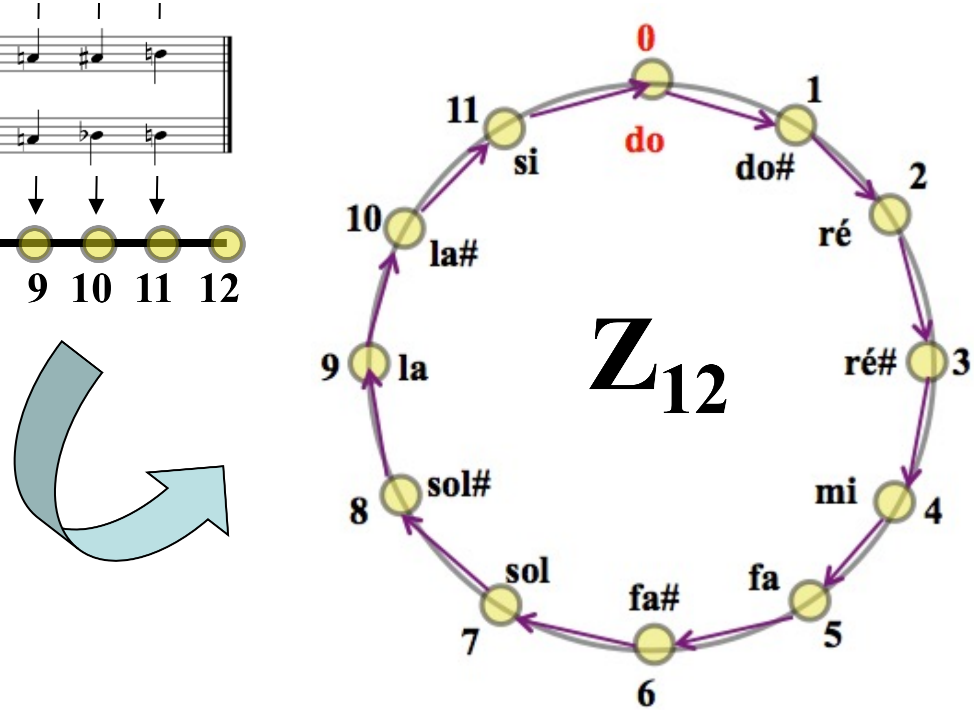
The equal tempered space is a cyclic group



The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11} where

T_k: $x \rightarrow x+k \bmod 12$

$$Z_{12} = < T_1 \mid (T_1)^{12} = T_0 >$$



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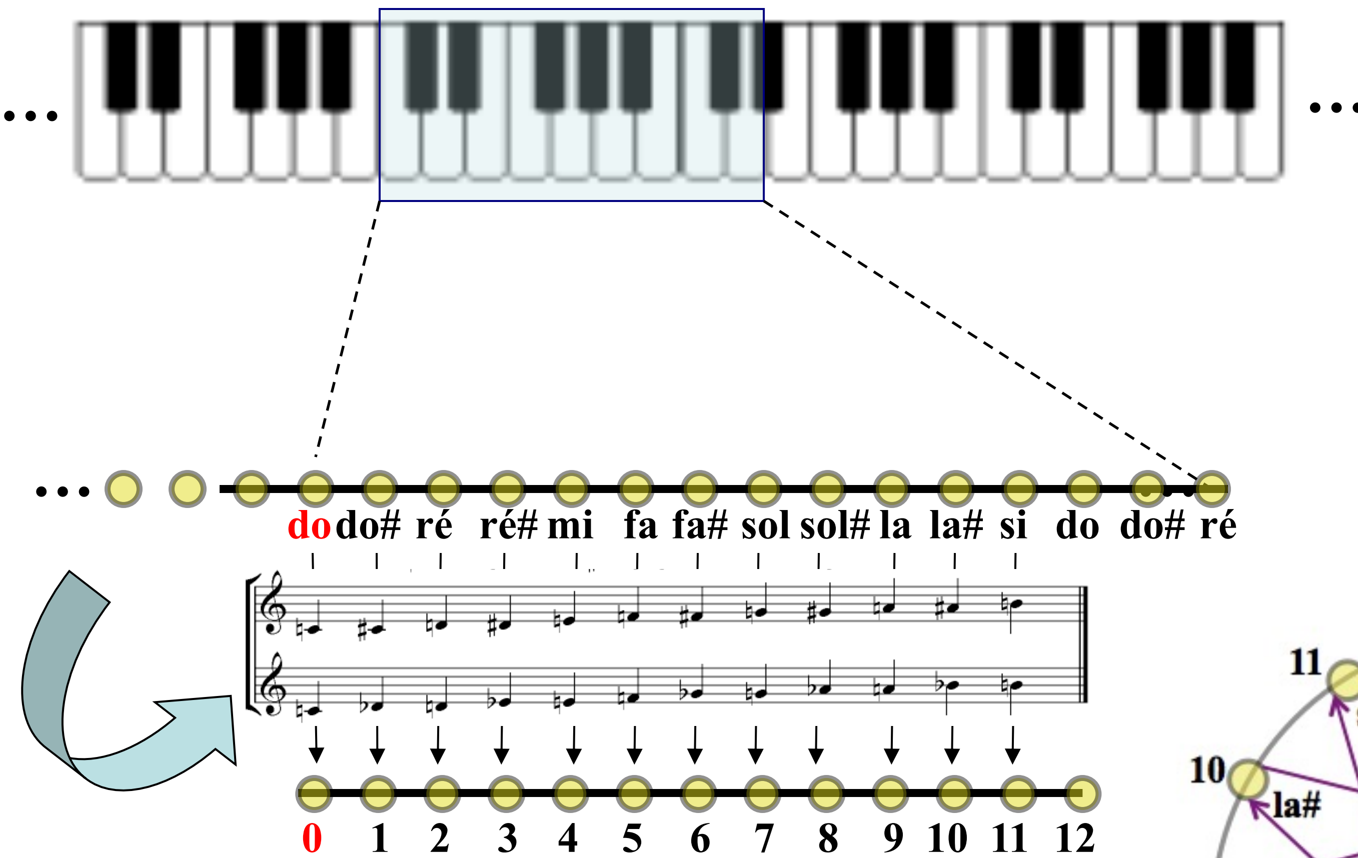
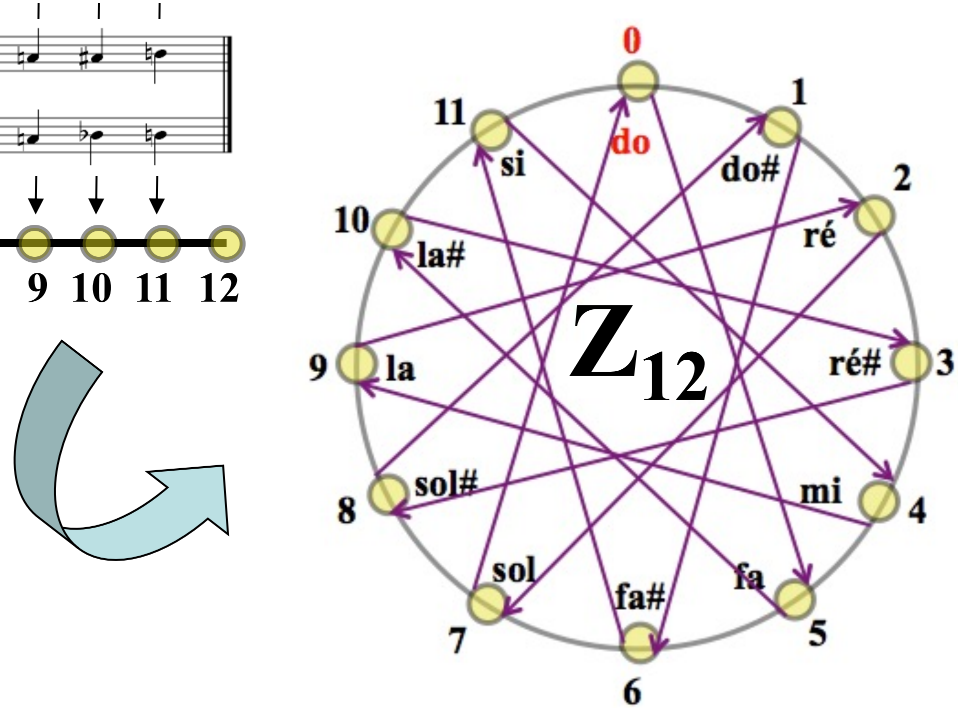


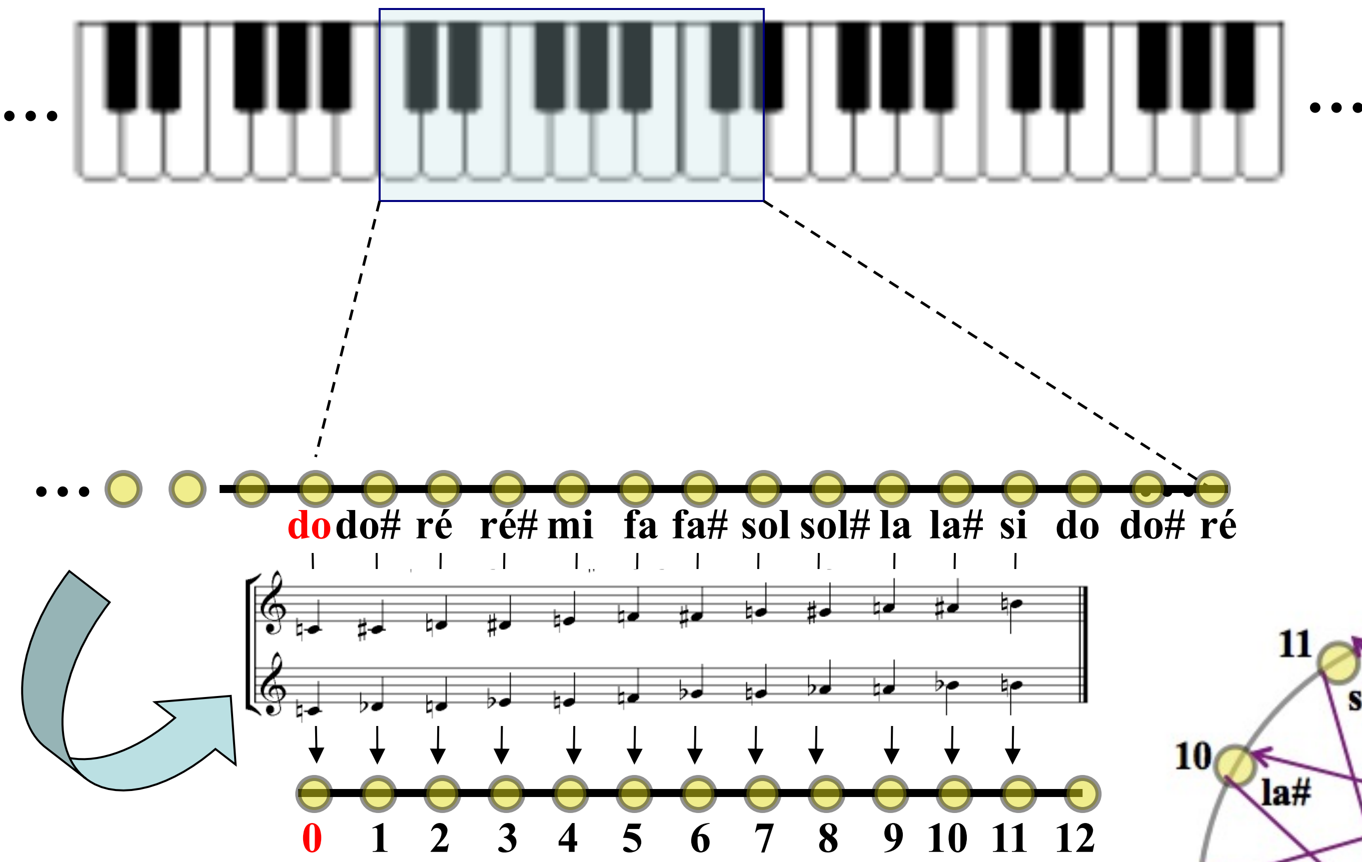
The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11} where

$T_k: x \rightarrow x+k \bmod 12$

$$\mathbf{Z_{12}} = <T_1 \mid (T_1)^{12} = T_0> =$$
$$= <\mathbf{T_5} \mid (\mathbf{T_5})^{12} = \mathbf{T_0}>$$



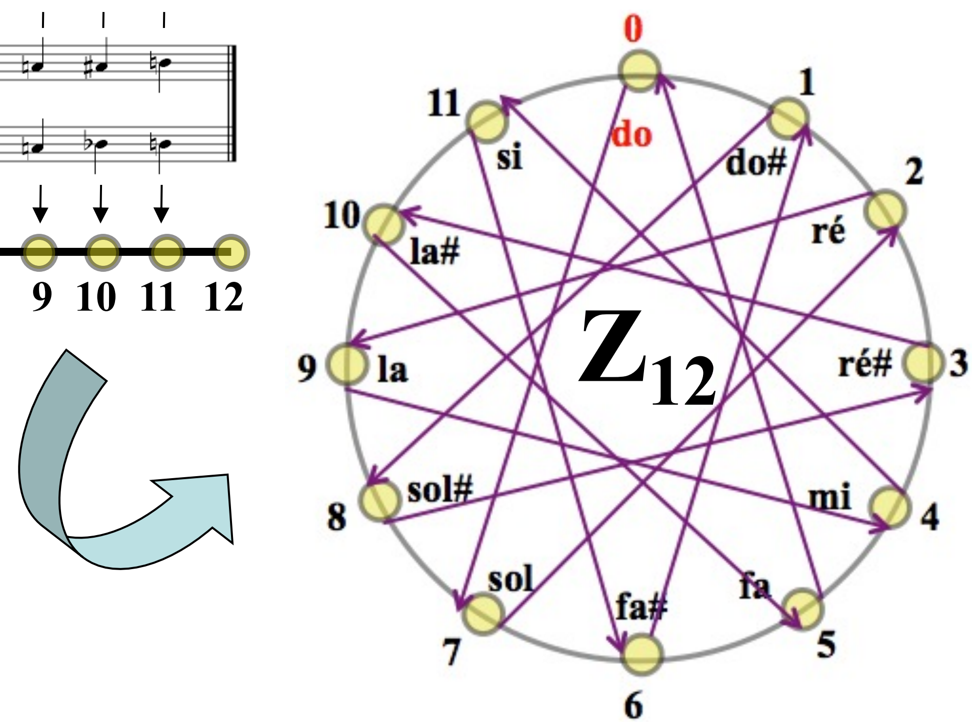
The equal tempered space is a cyclic group



The generators of the cyclic group of order 12 are the transpositions T_1, T_5, T_7 et T_{11} where

$T_k: x \rightarrow x+k \bmod 12$

$$\mathbf{Z_{12}} = \langle T_1 \mid (T_1)^{12} = T_0 \rangle = \langle T_5 \mid (T_5)^{12} = T_0 \rangle = \boldsymbol{\langle T_7 \mid (T_7)^{12} = T_0 \rangle}$$



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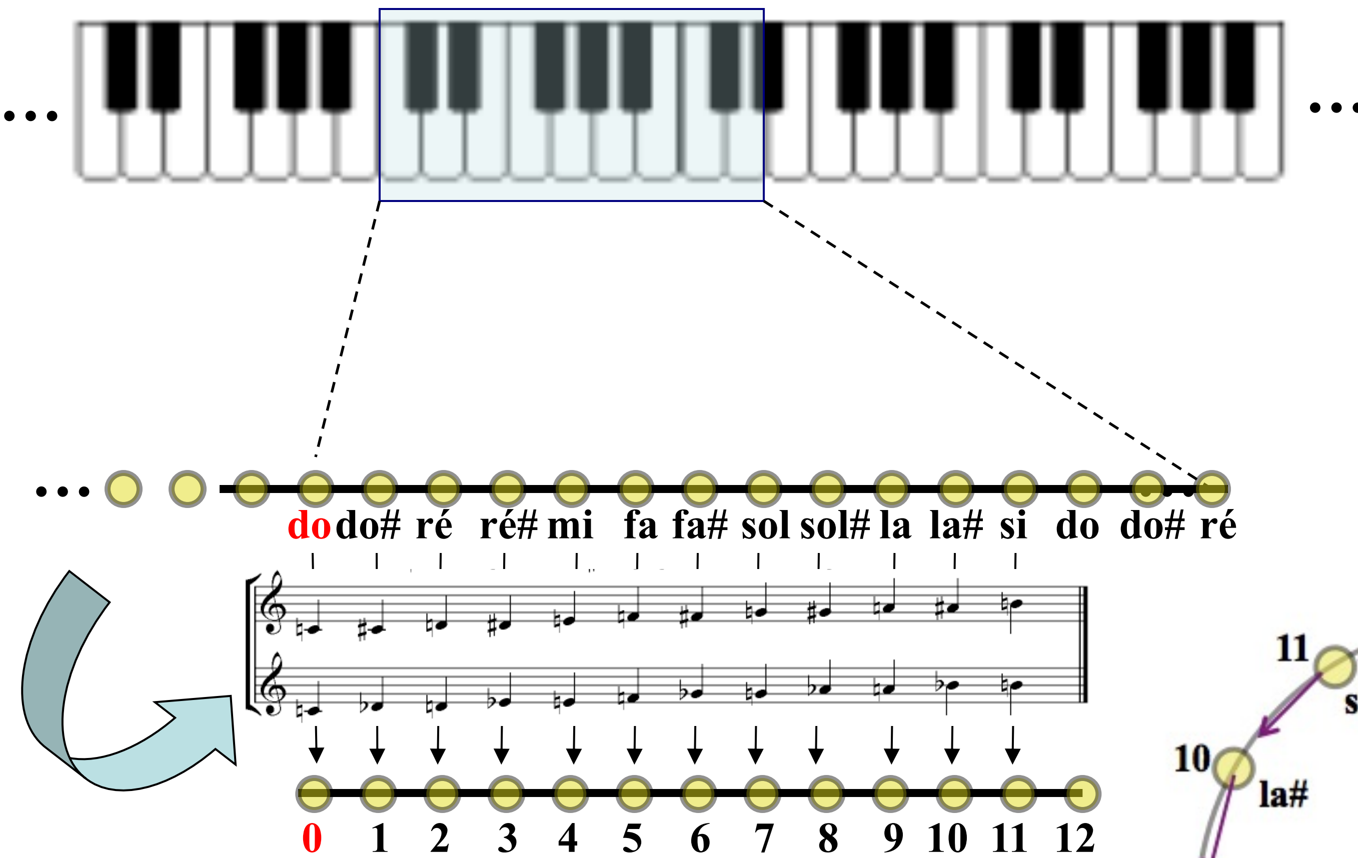

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T_k: $x \rightarrow x+k \bmod 12$

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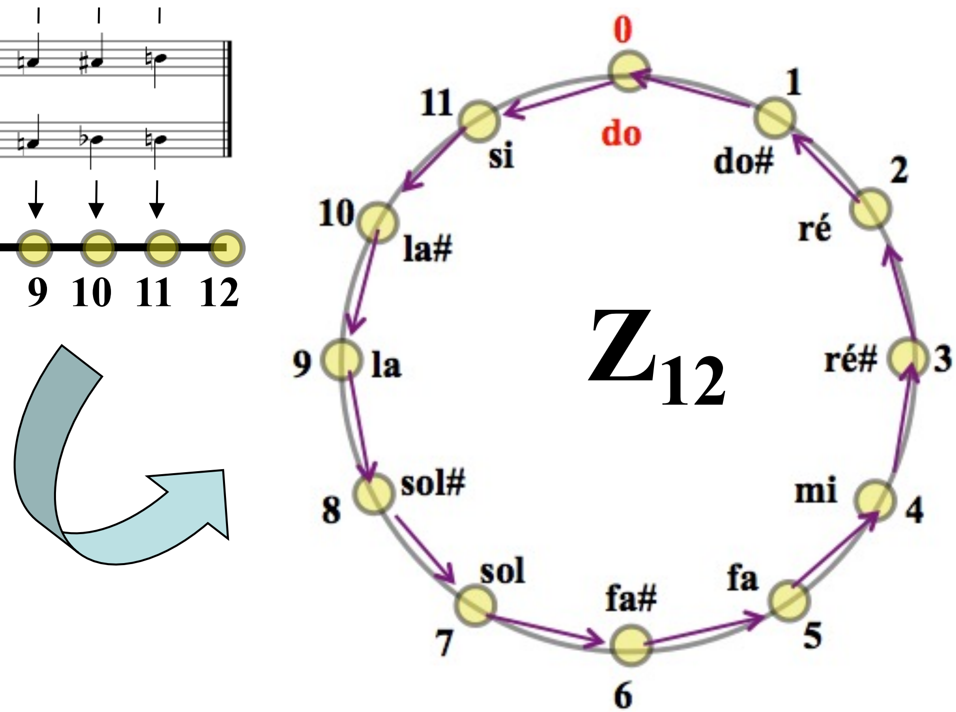
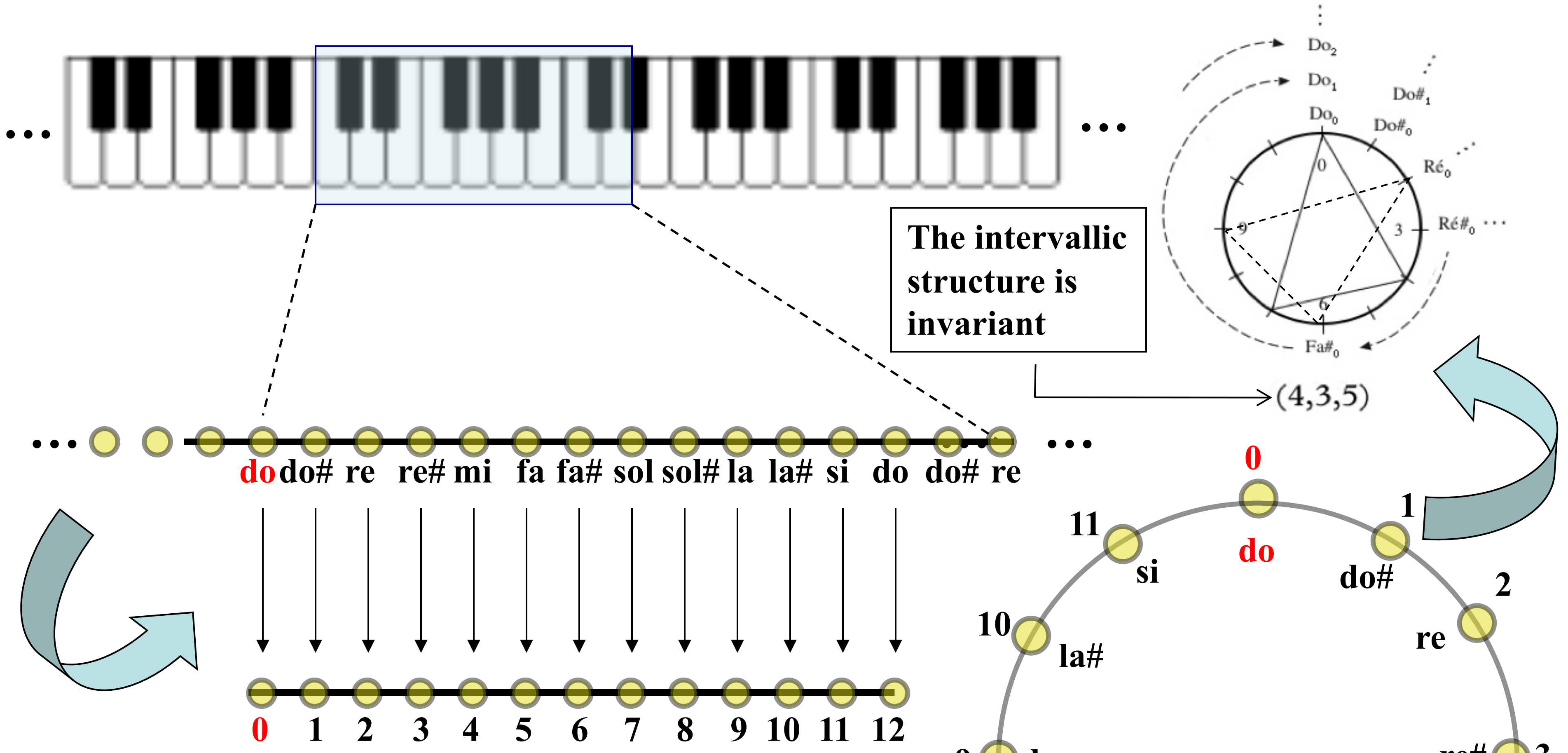
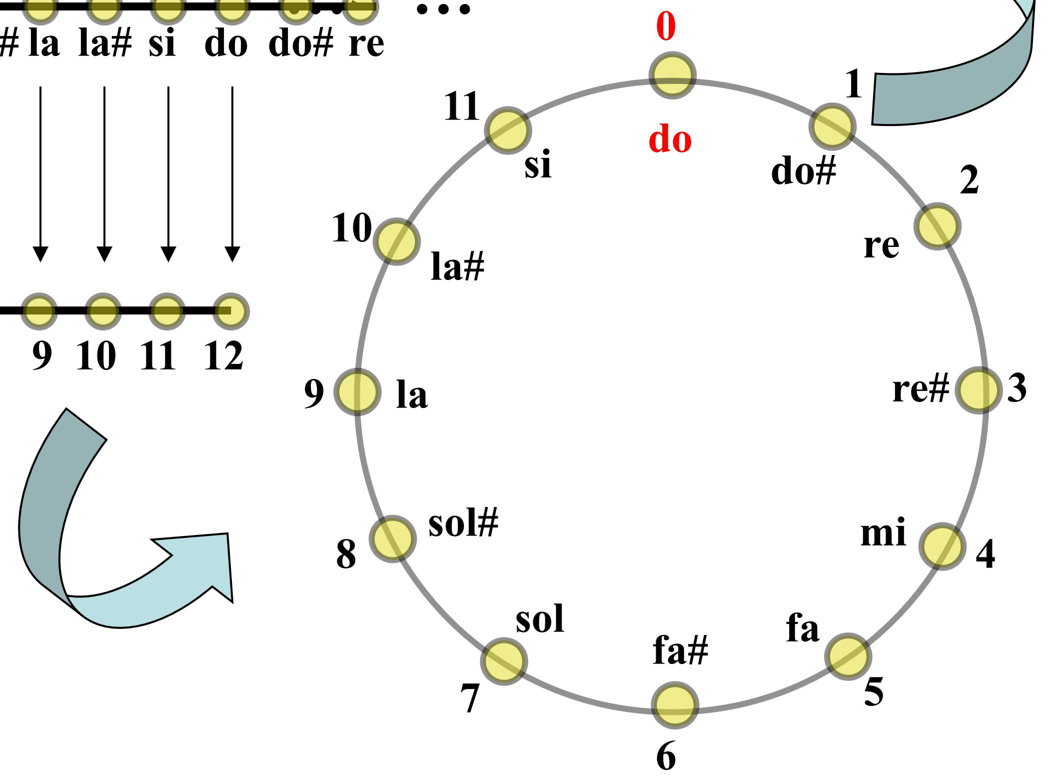
$$= < T_5 \mid (T_5)^{12} = T_0 > =$$
$$= < T_7 \mid (T_7)^{12} = T_0 > =$$
$$\mathbf{= < T_{11} \mid (T_{11})^{12} = T_0 >}$$



Circular representation and intervallic structure



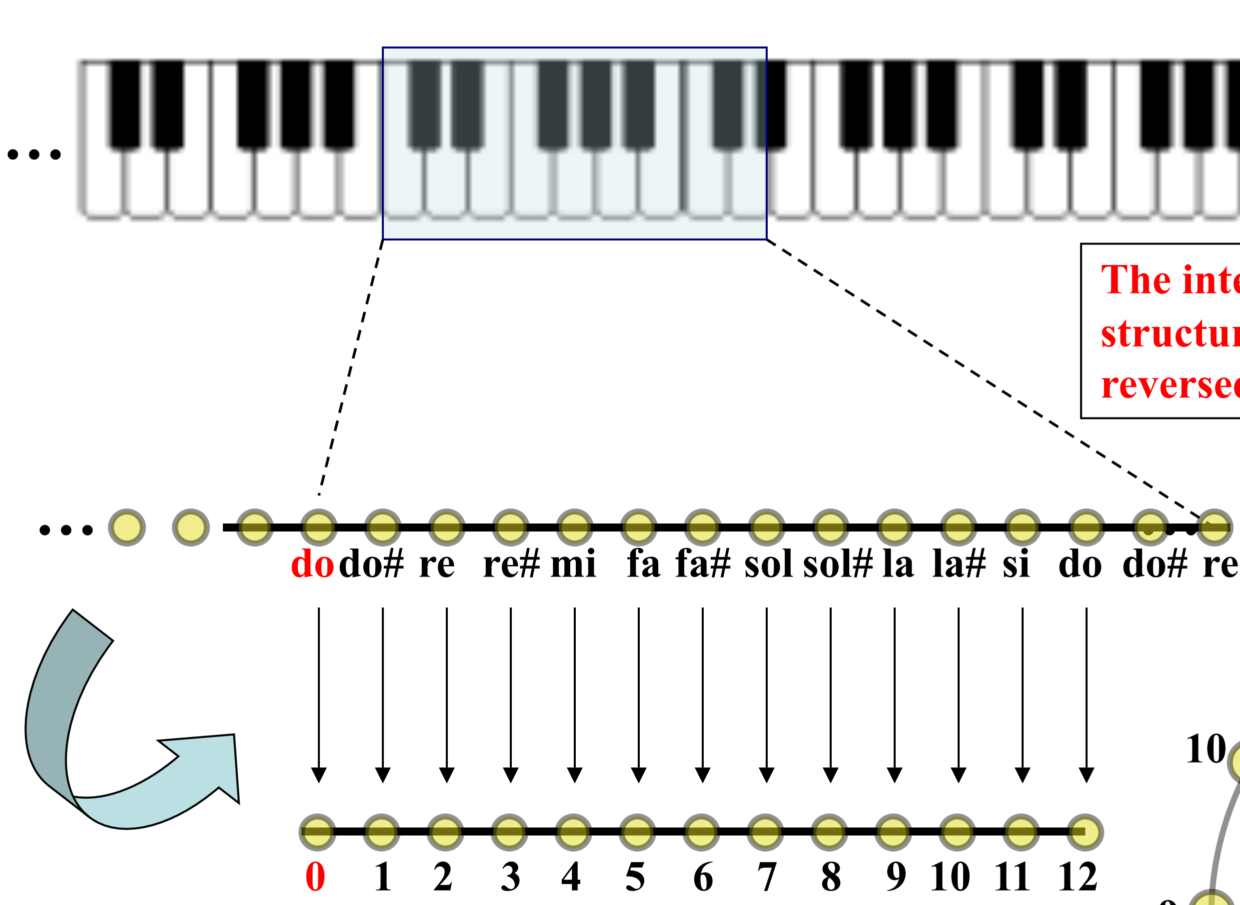


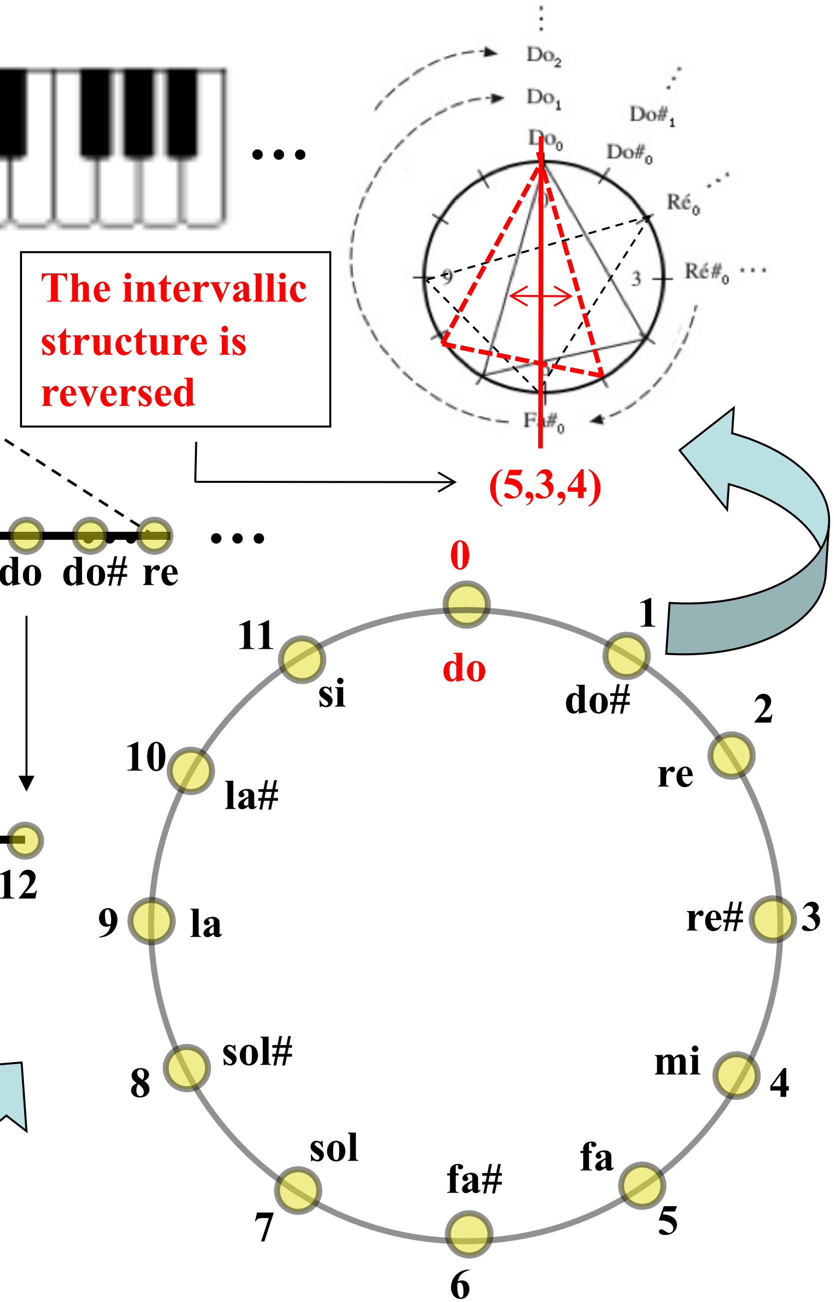
The two basic musical applications

$T_k : x \rightarrow x+k \bmod 12$ (transposition)

$I : x \rightarrow -x \bmod 12$ (inversion)

Circular representation and intervallic structure





The two basic musical applications

$T_k : x \rightarrow x+k \bmod 12$ (transposition)

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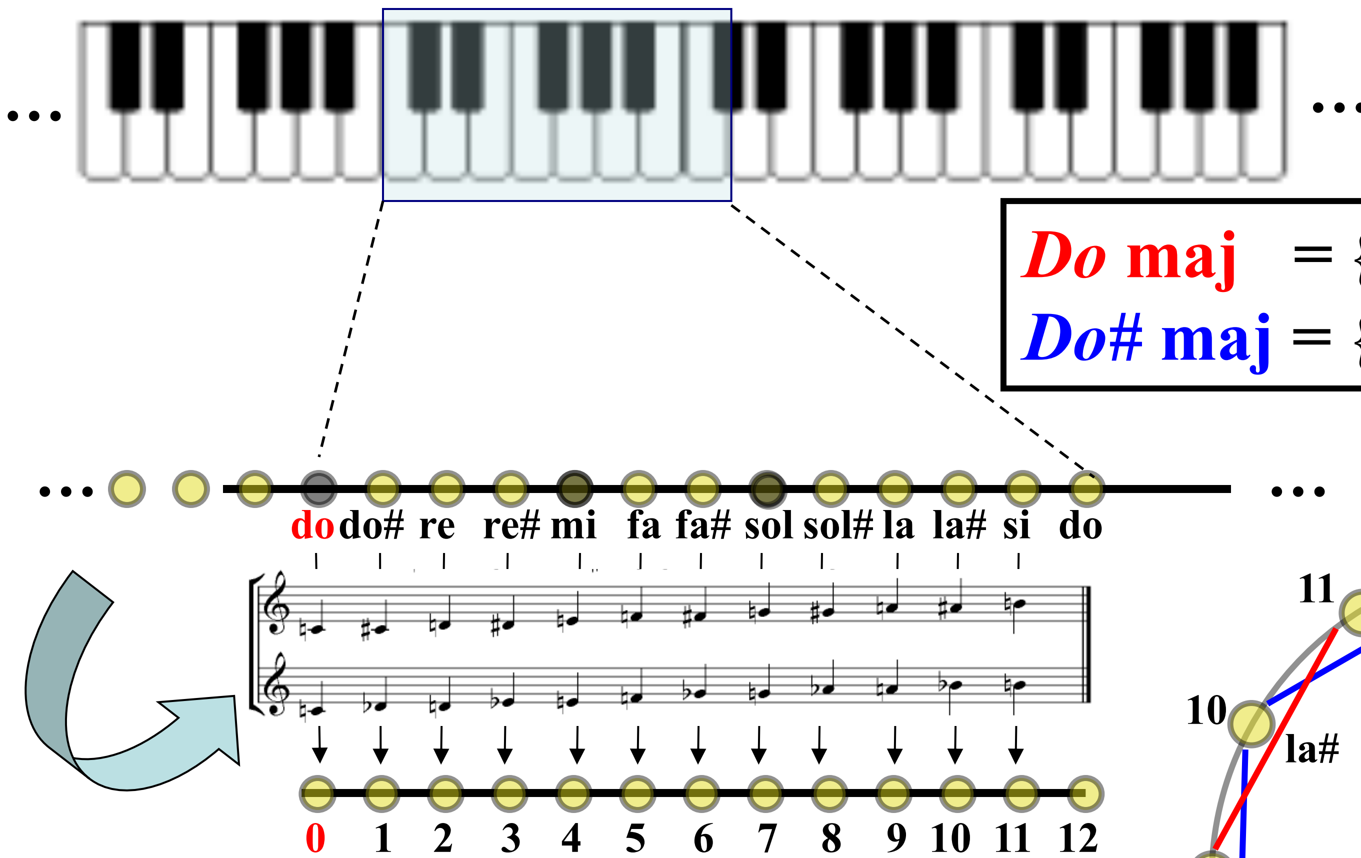
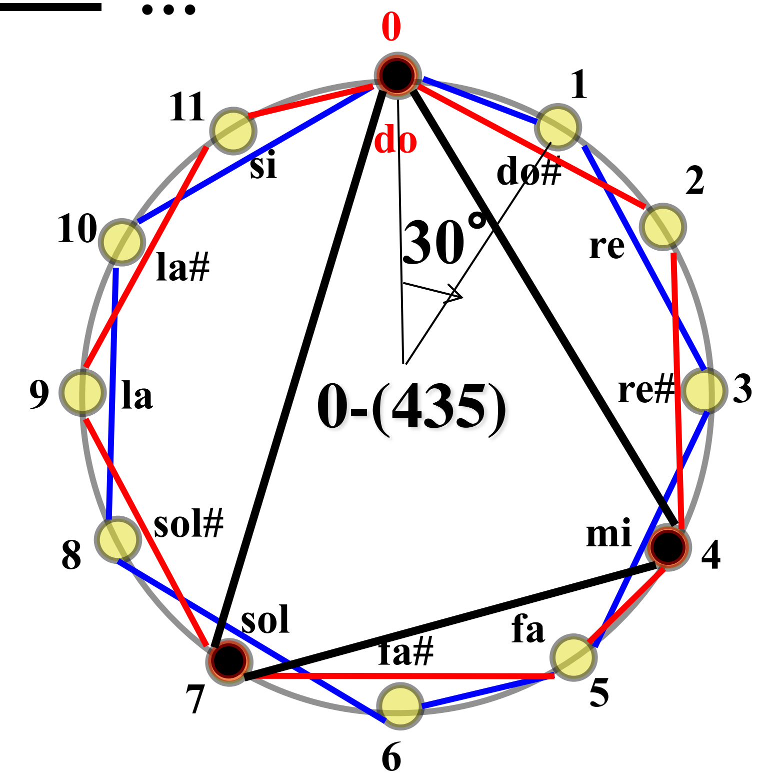
Musical transpositions are additions...

$T_k : x \rightarrow x+k \bmod 12$

Do maj $= \{0,2,4,5,7,9,11\}$ +1

$Do\#$ maj $= \{1,3,5,6,8,10,0\}$

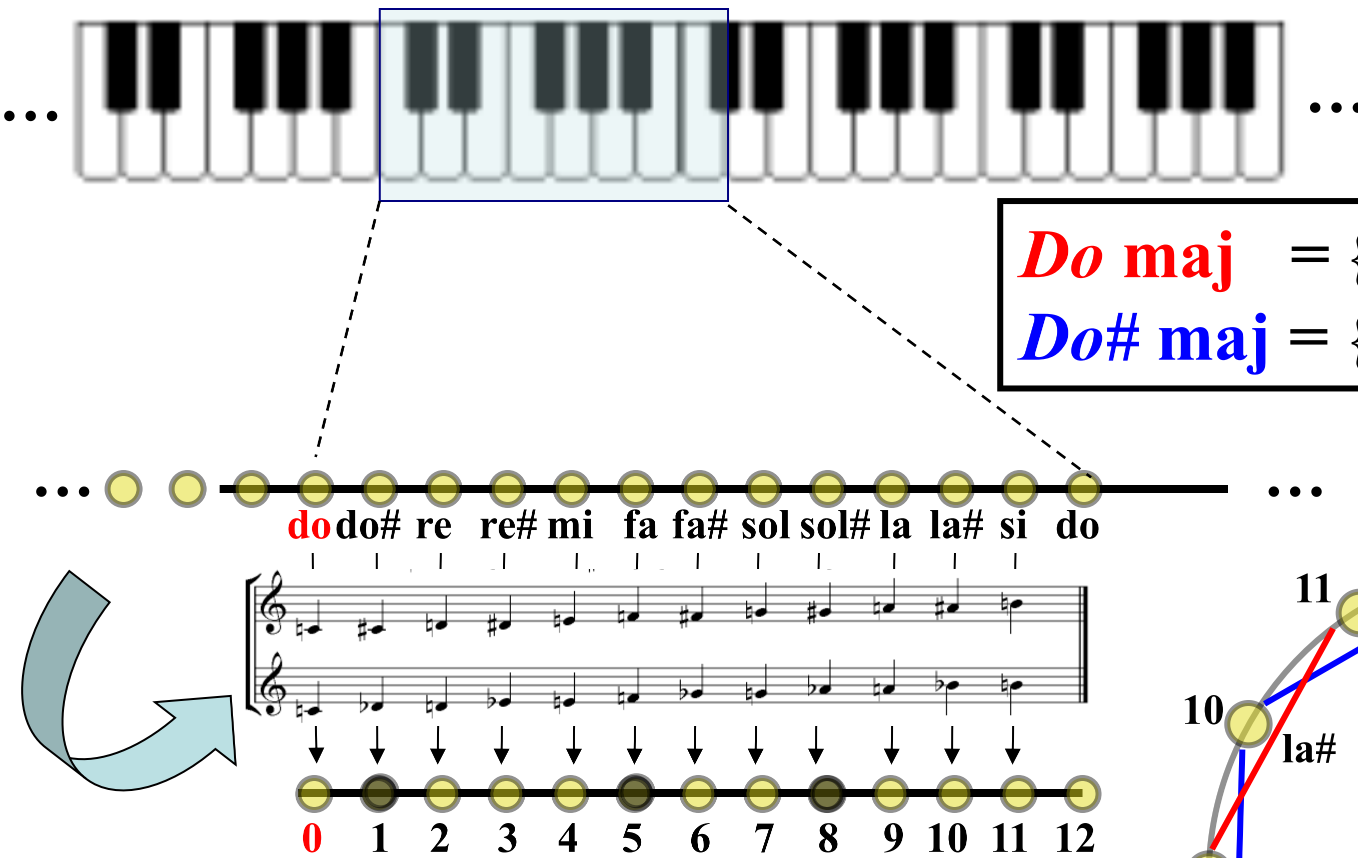




...or rotations !

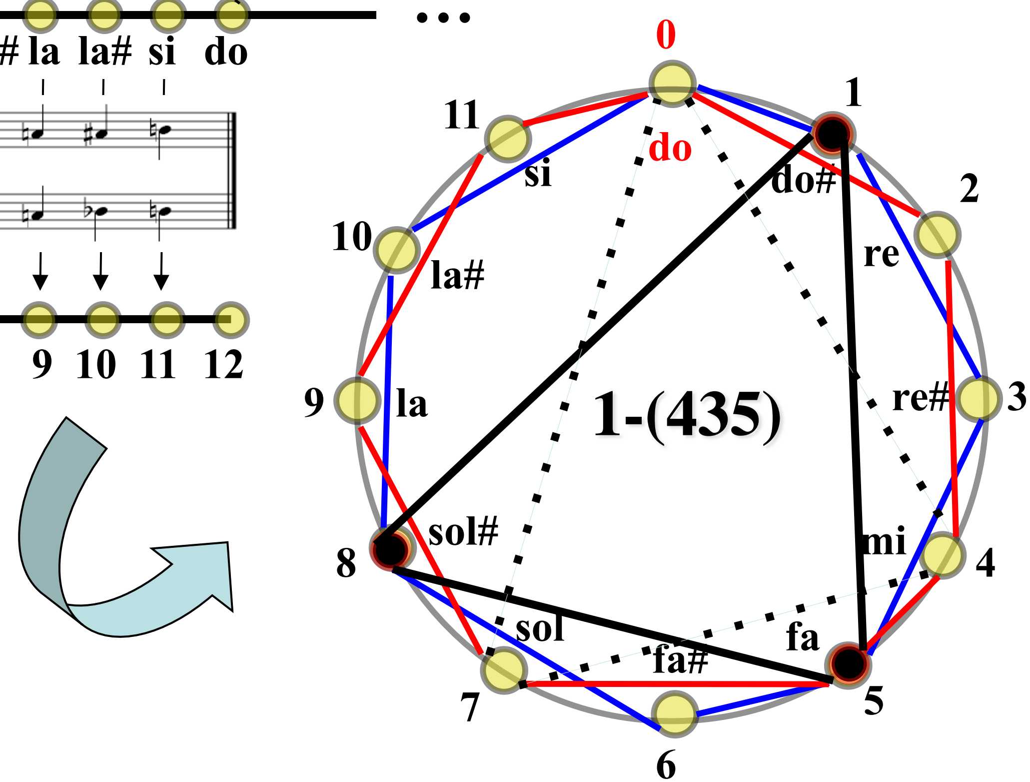
Musical transpositions are additions...

$T_k : x \to x+k \bmod 12$



Do **maj** = {0,2,4,5,7,9,11} +1

Do# **maj** = {1,3,5,6,8,10,0}



...or rotations !

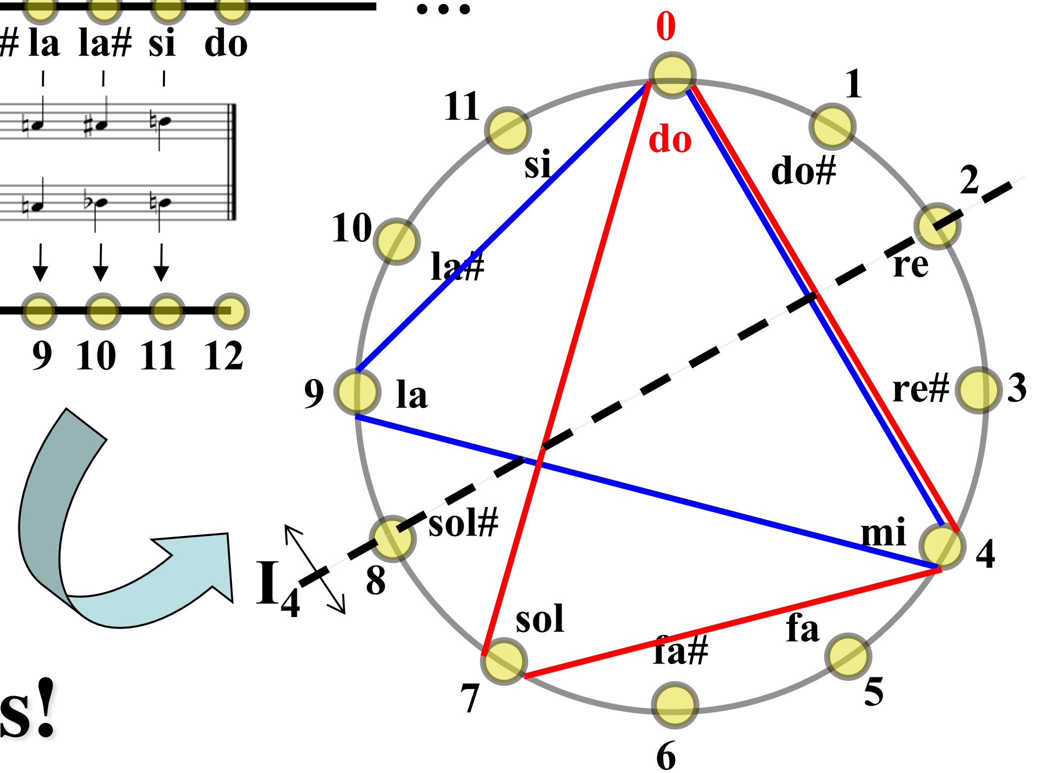
Musical inversions are differences...

$\mathrm{I} : x \to -x \bmod 12$

Do maj $= \{0,4,7\}$

La min $= \{0,4,9\}$

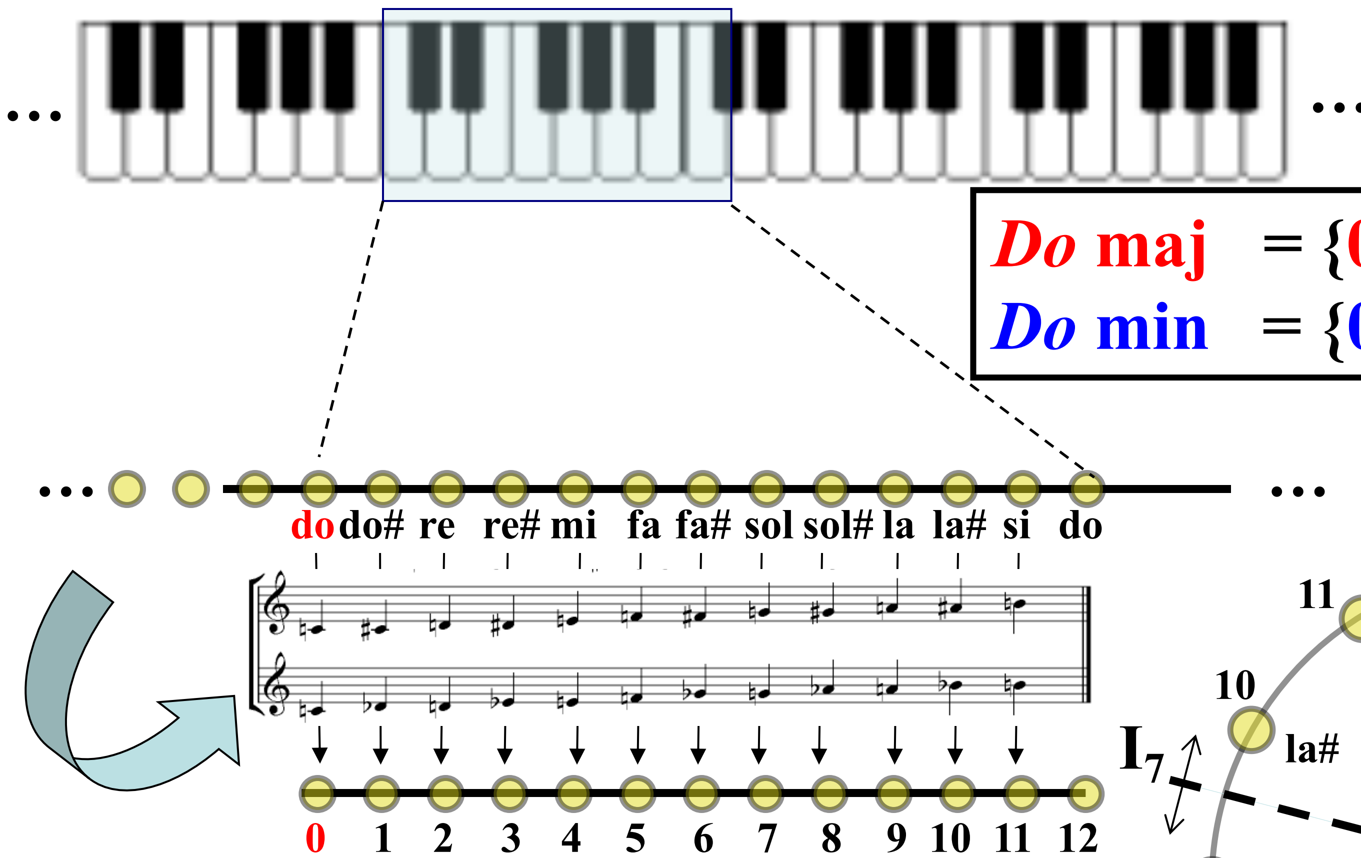
$\mathrm{I}_4(x)=4-x$



... or axial symmetries!

Musical inversions are differences...

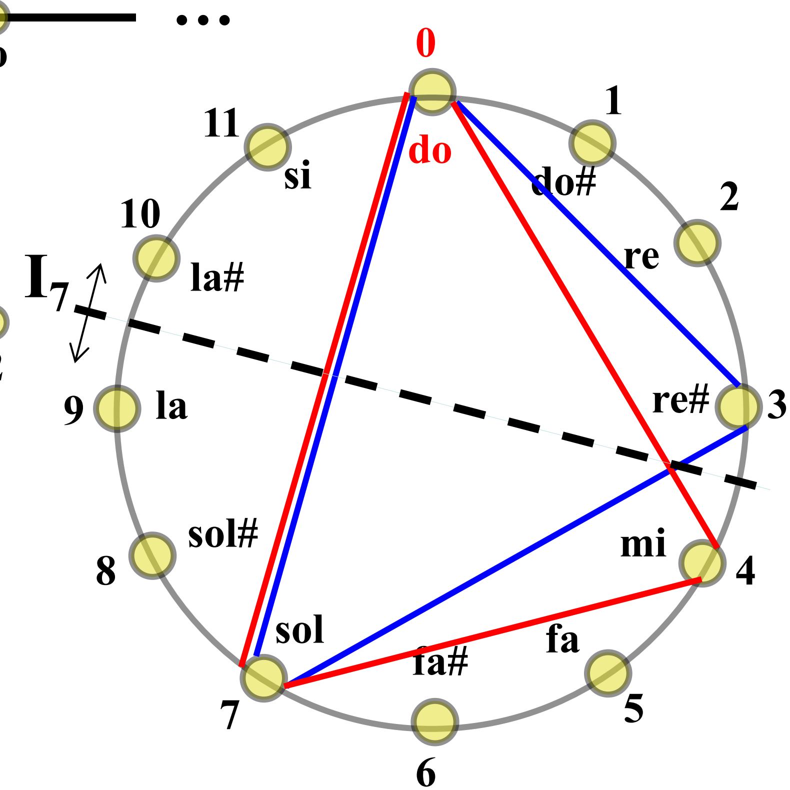
$\mathrm{I} : x \rightarrow -x \bmod 12$



Do maj $= \{0,4,7\}$

Do min $= \{0,3,7\}$

$\mathrm{I}_7(x)=7-x$



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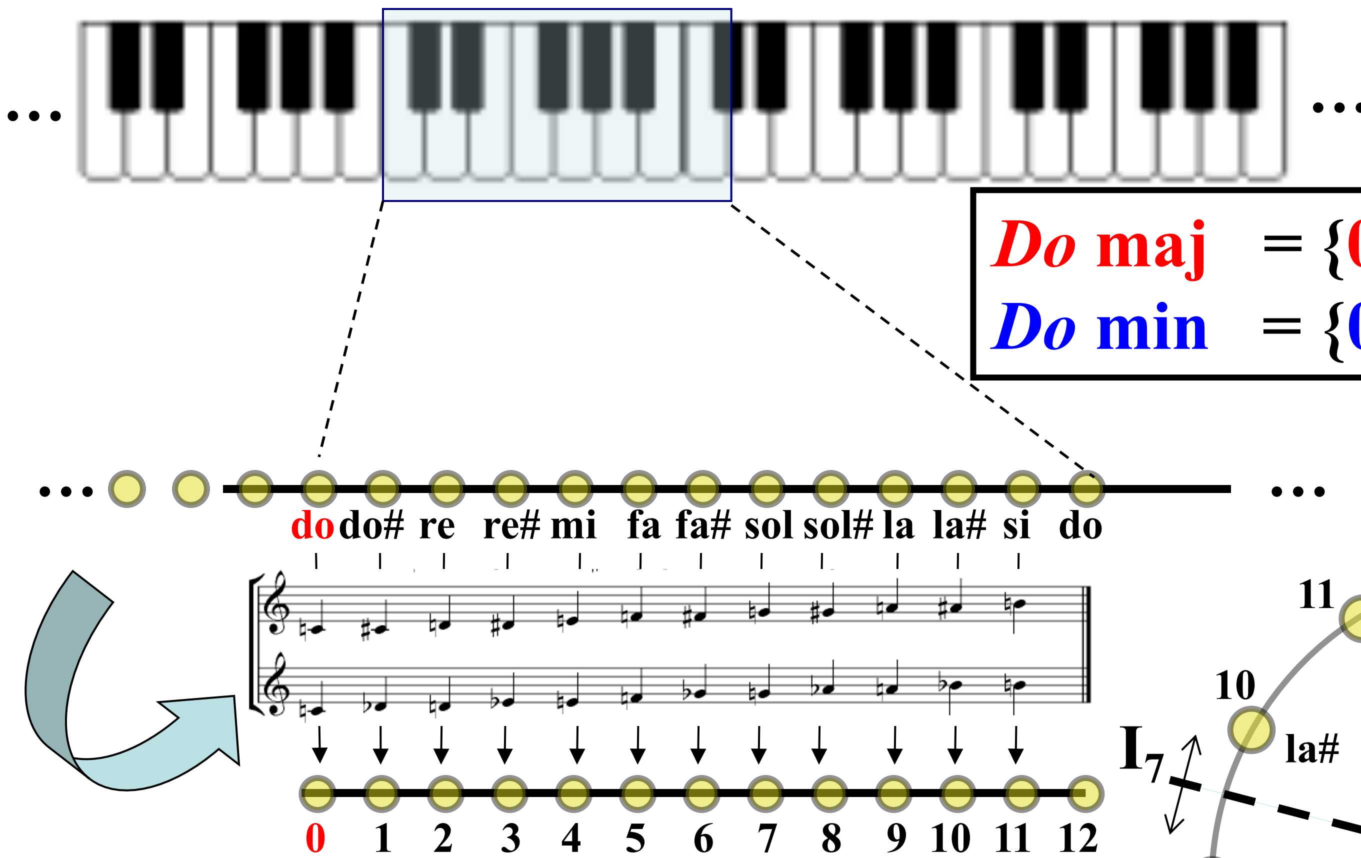
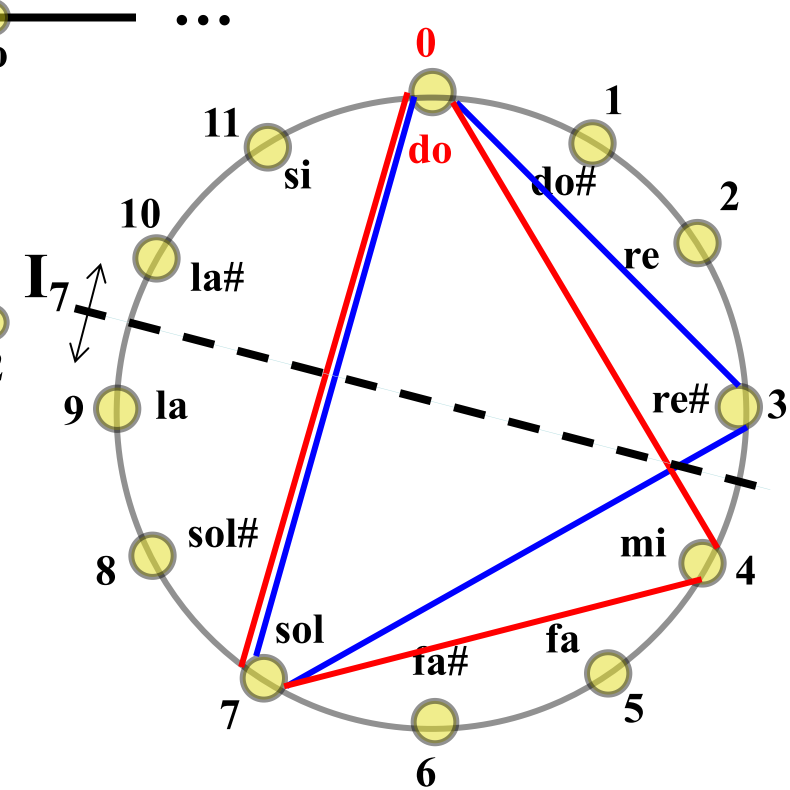
Musical inversions are differences...

$I : x \to -x \bmod 12$



Do maj $= \{0,4,7\}$
Do min $= \{0,3,7\}$

$I_7(x)=7-x$



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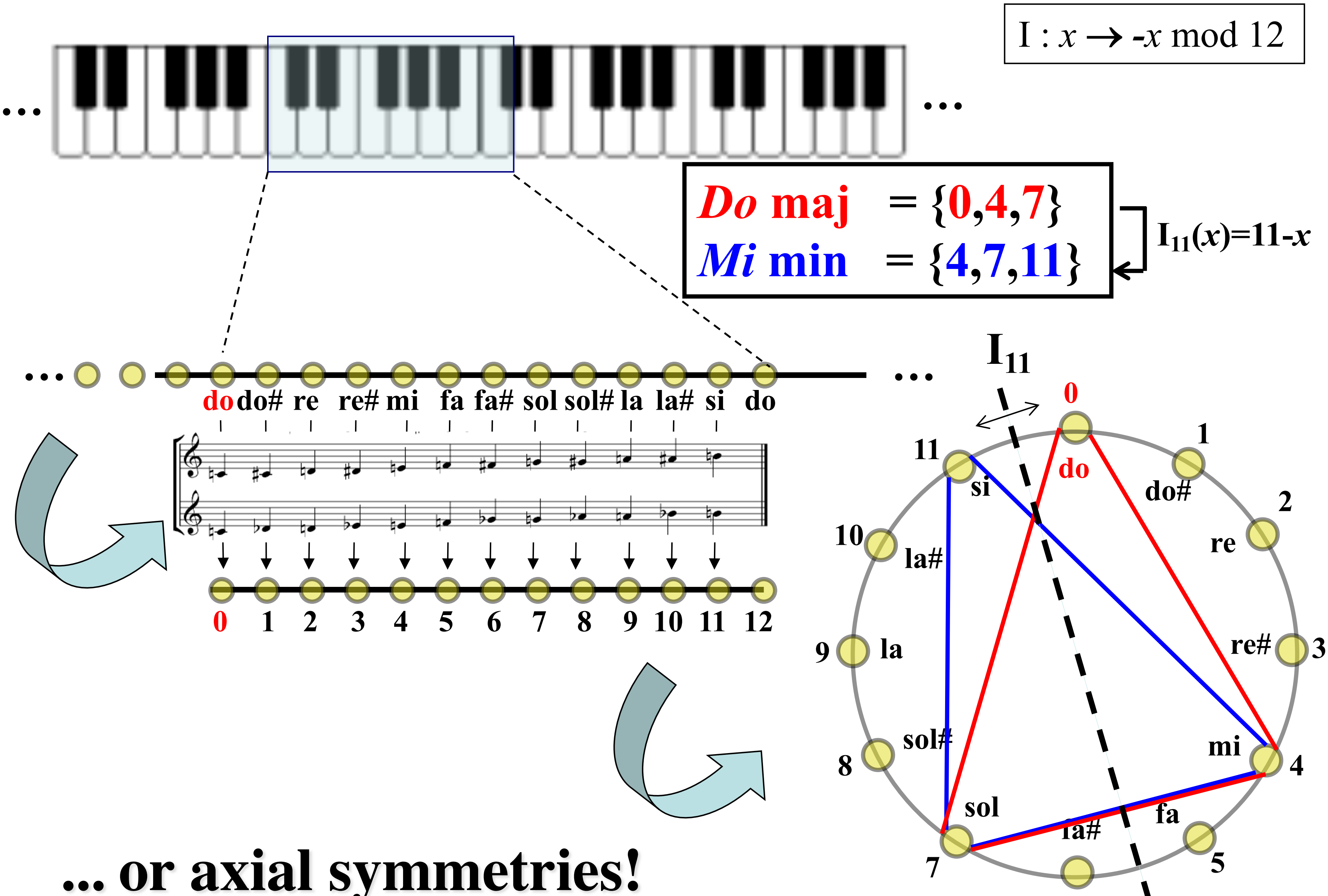
Musical inversions are differences...

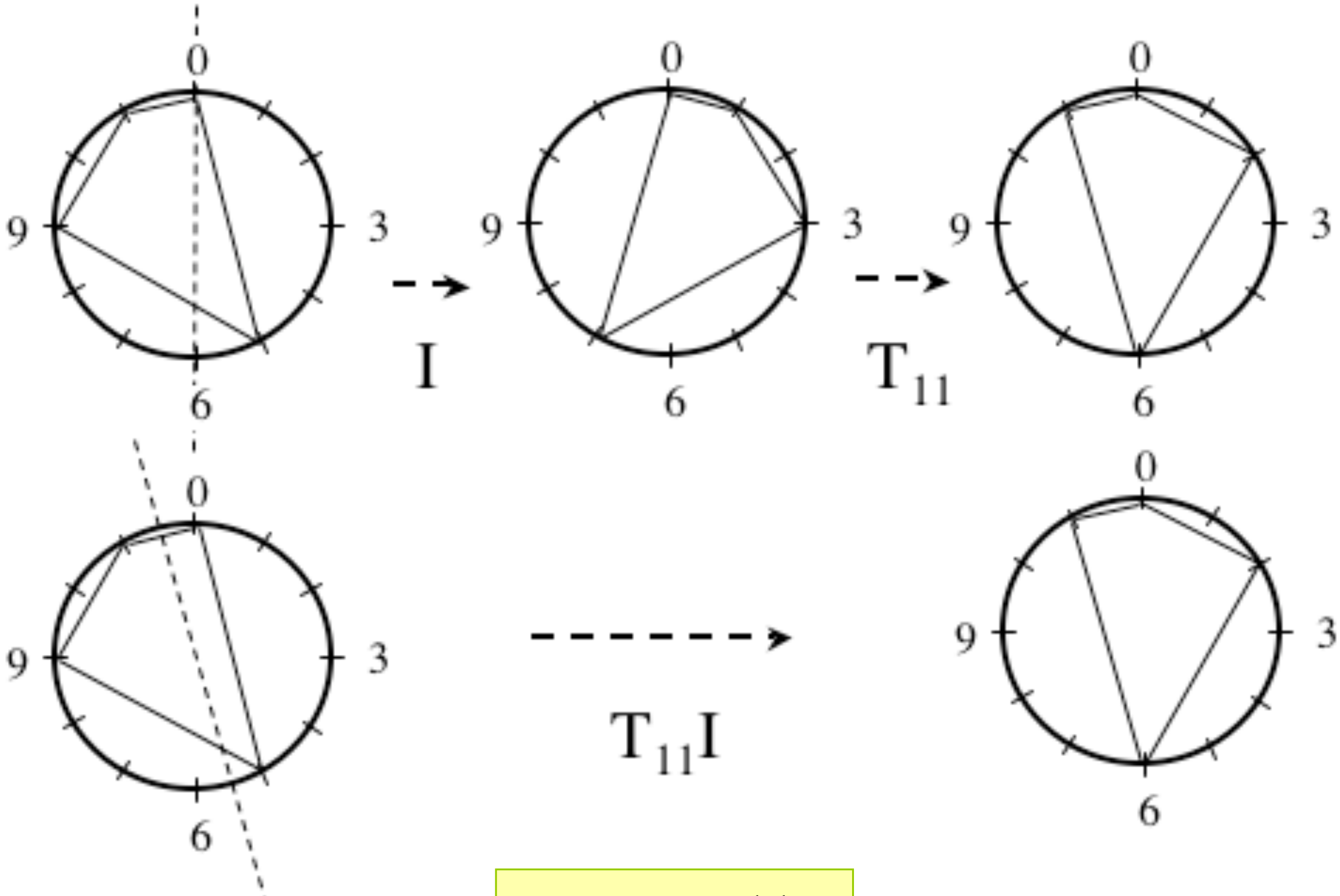


... or axial symmetries!

Transposition and Inversion

I: $x \to 12-x$

T_k: $x \to k+x$



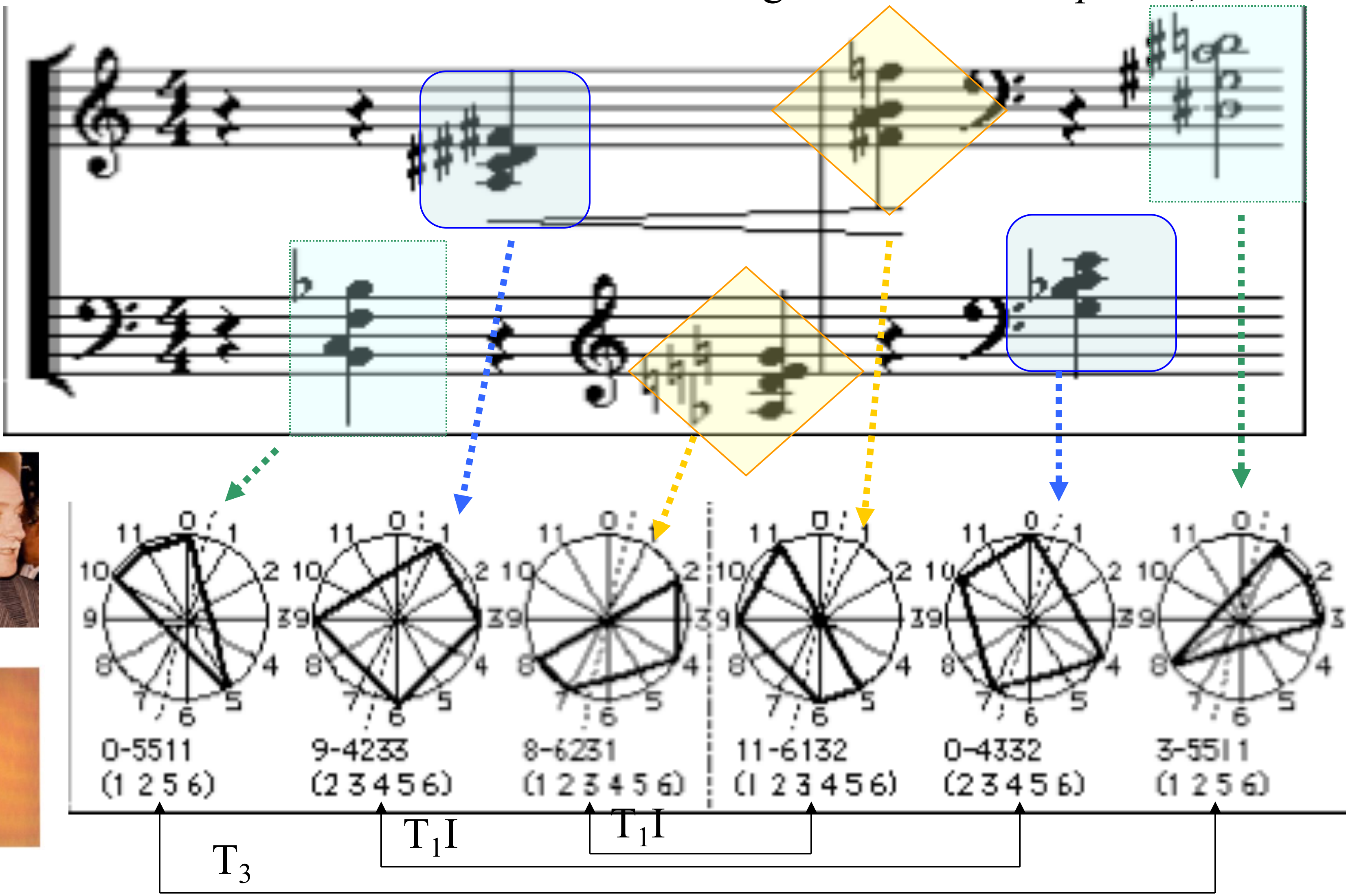
$T_{11}I$: $x \to 11-x$

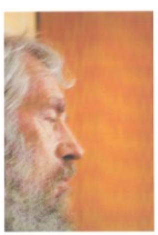
$$\{0, 5, 9, 11\} \longrightarrow \{11, 6, 3, 0\}$$

« Entités formelles pour l'analyse musicale »

Marcel Mesnage (1998)

A. Schoenberg : *Klavierstück Op. 33a*, 1929





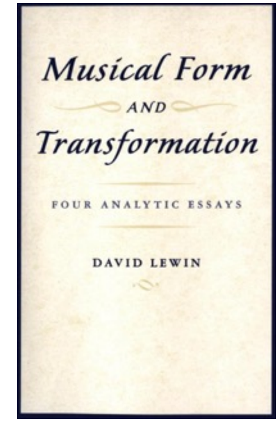
« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

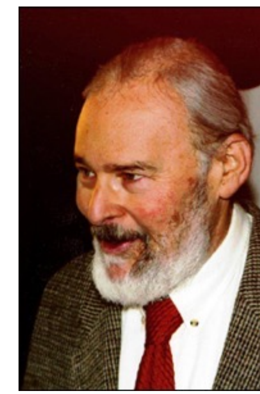
Three interpretations:

Henck

Kontarsky

Tudor





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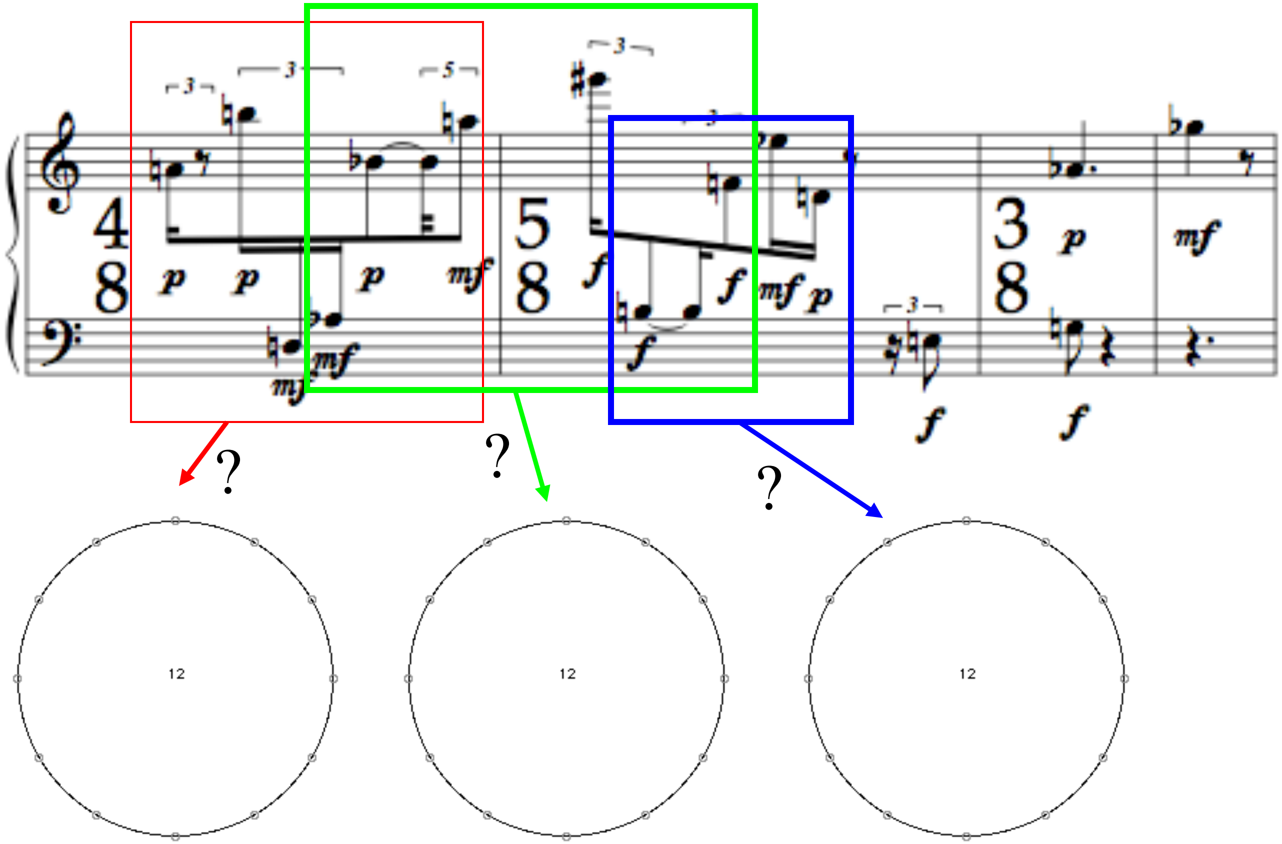
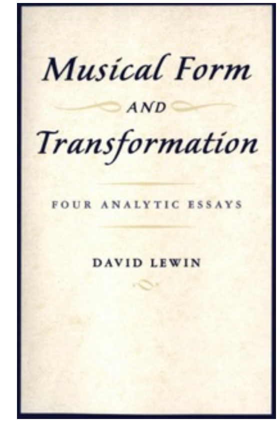
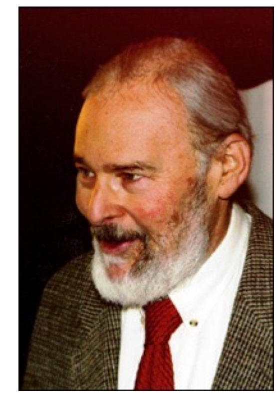




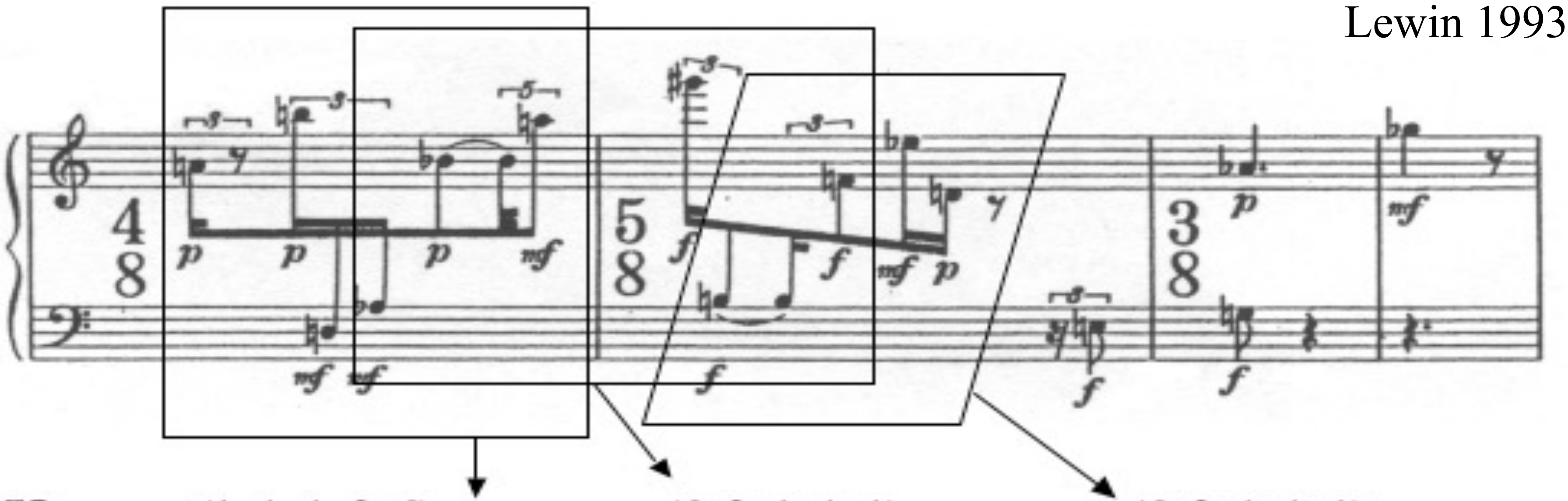


« The most ‘theoretical’ of the four essays, it focuses on the forms of one pentachord reasonably ubiquitous in the piece. A special **group of transformations** is developed, one suggested by the musical interrelations of the pentachord forms. Using that group, the essay arranges **all pentachord forms** of the music into a **spatial configuration** that illustrates network structure, for this particular phenomenon, over the entire piece. »

David Lewin, *Musical Form and Transformation,* YUP 1993

« *Making* and *Using* a Pcset Network for Stockhausen's Klavierstück III »

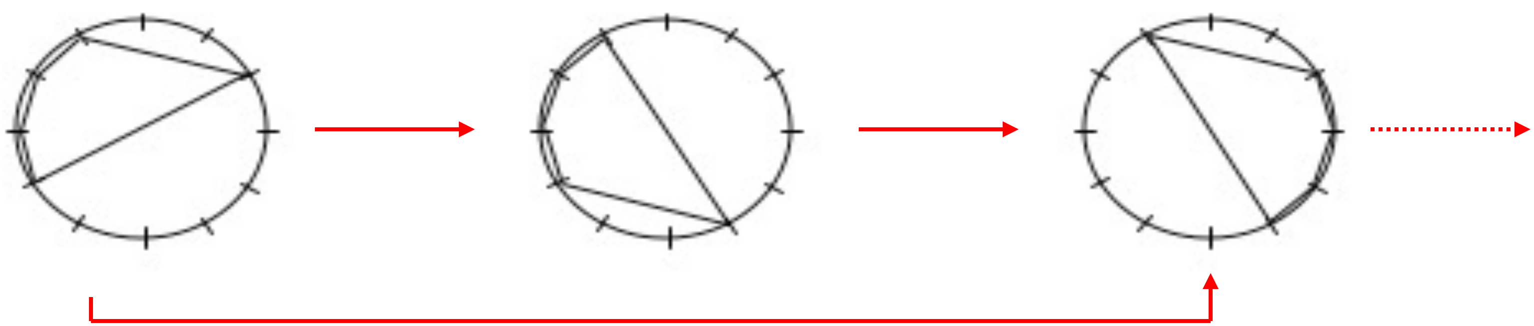
Lewin 1993



SI: (1, 1, 1, 3, 6) (6, 3, 1, 1, 1) (6, 3, 1, 1, 1)

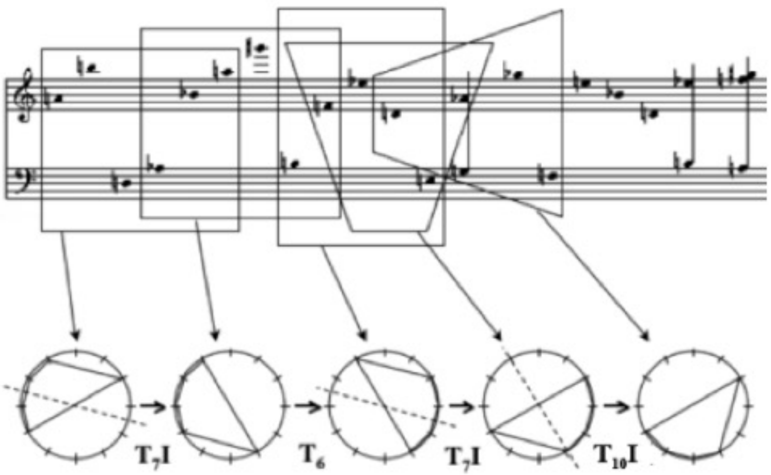
IFUNC: [5 3 2 2 1 1 1 1 1 2 2 3] [5 3 2 2 1 1 1 1 1 2 2 3] [5 3 2 2 1 1 1 1 1 2 2 3]

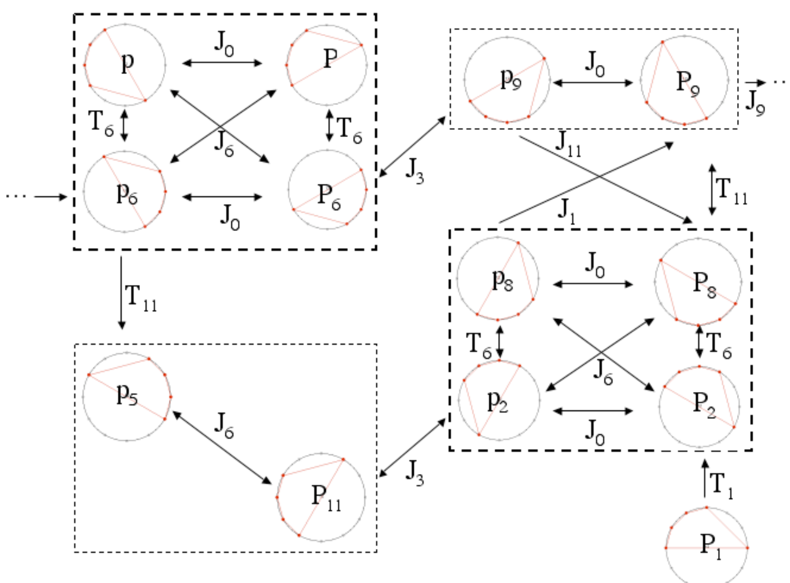
VI: [3 2 2 1 1 1] [3 2 2 1 1 1] [3 2 2 1 1 1]

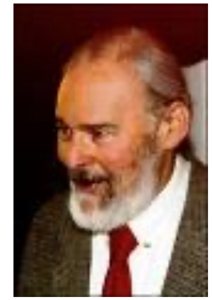


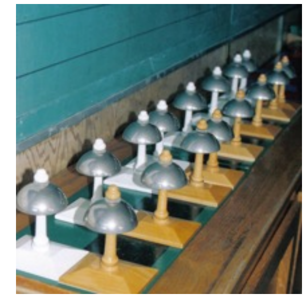
Transformational Networks and Music Cognition

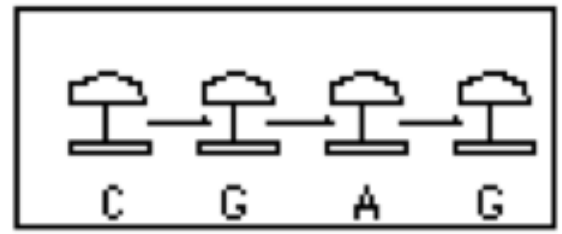
Bamberger, J. (1986). Cognitive issues in the development of musically gifted children. In *Conceptions of giftedness (eds., R. J. Sternberg, & J. E. Davidson), pp. 388-413. Cambridge University Press, Cambridge*

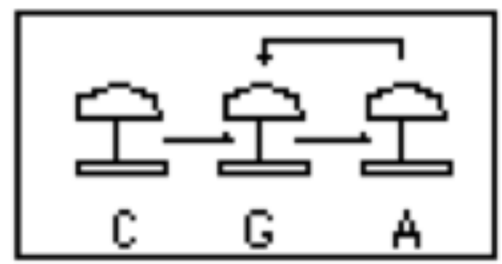








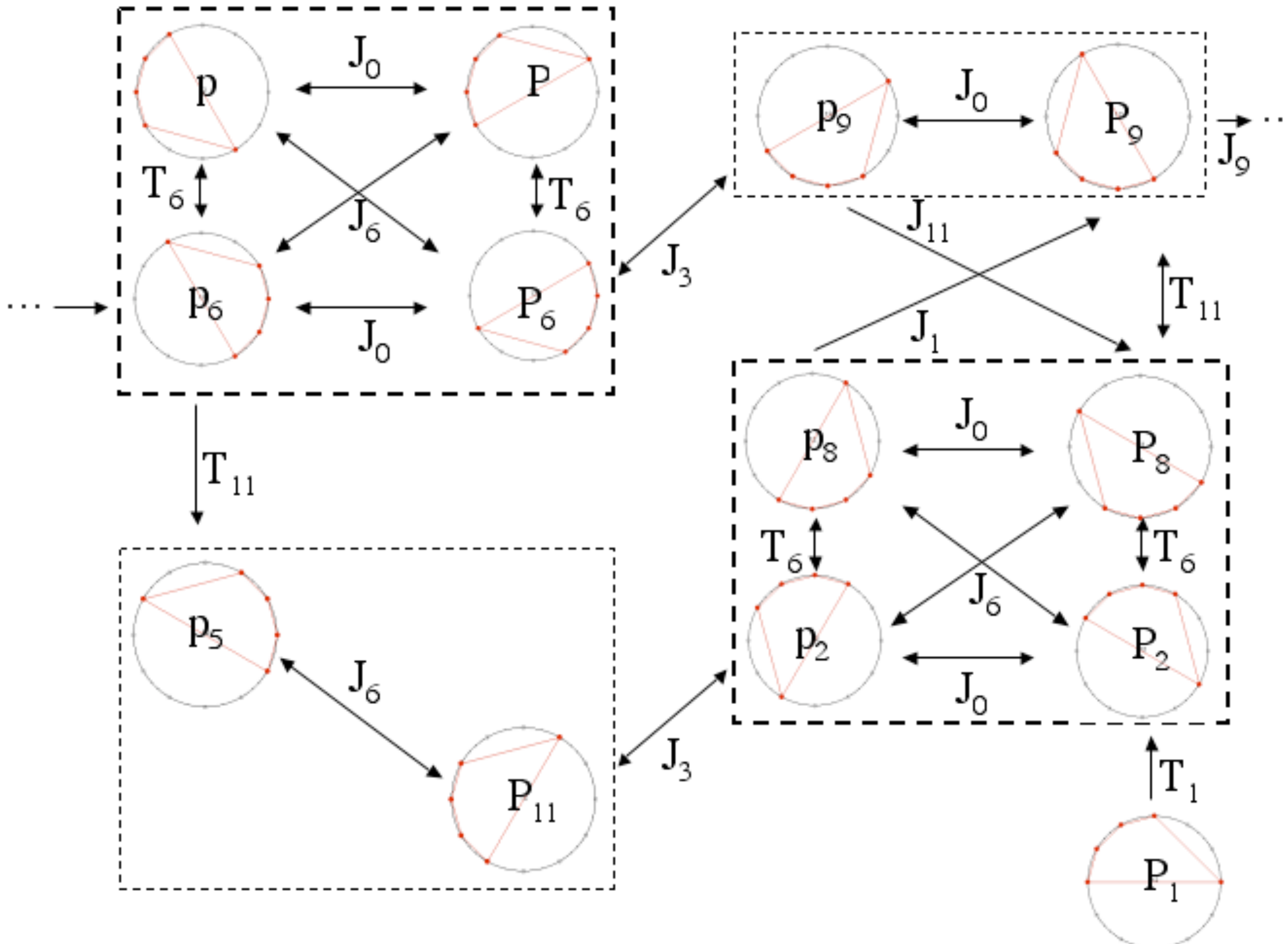




Bamberger, J. (2006). "What develops in musical development?" In G. MacPherson (ed.) *The child as musician: Musical development from conception to adolescence.* Oxford, U.K. Oxford University Press.

Transformational Networks and isographies

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)

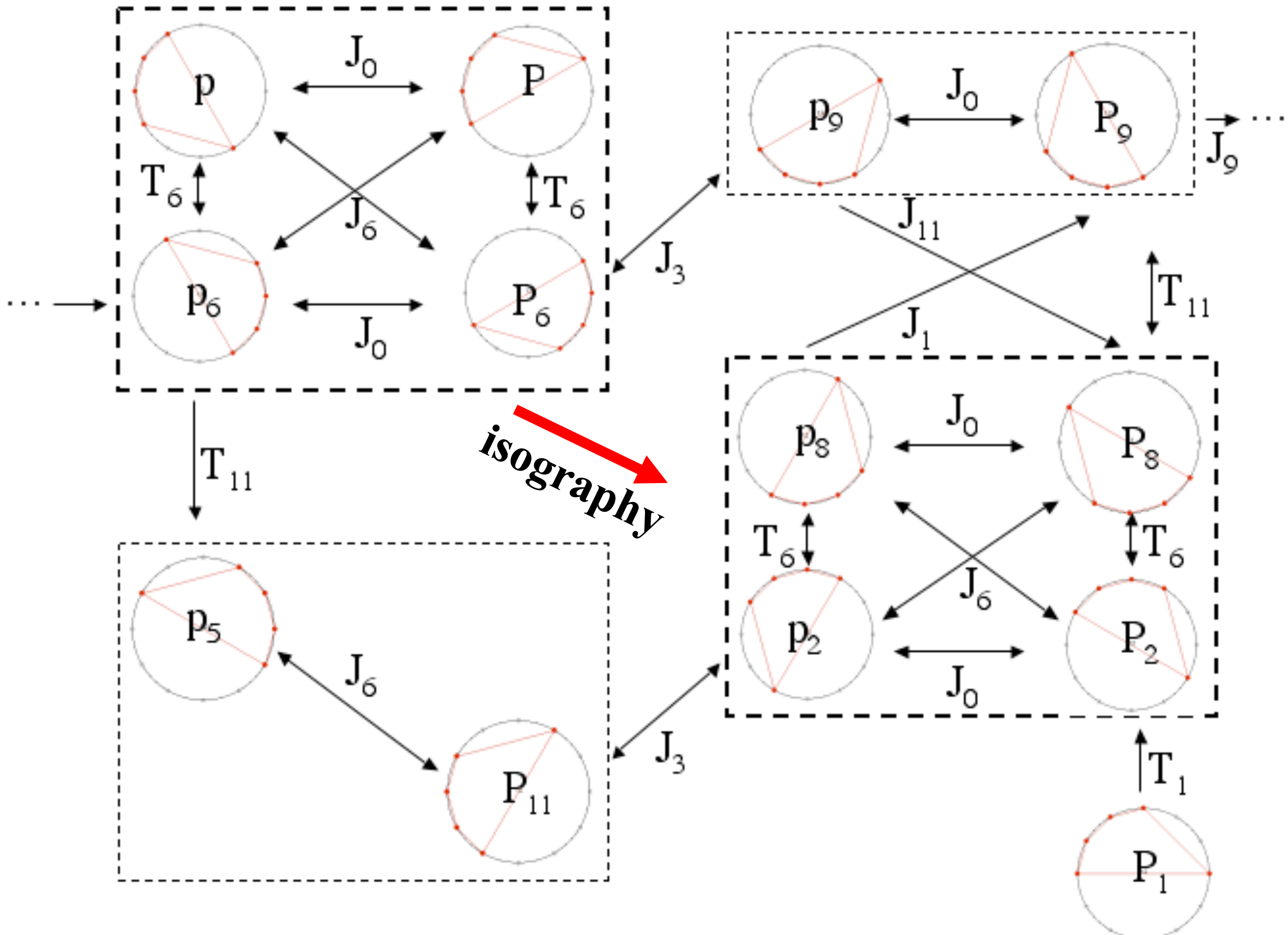


« Rather than asserting a network that follows pentachord relations one at a time, according to the chronology of the piece, I shall assert instead a network that displays all the pentachord forms used and all their **potentially functional interrelationships**, in a very compactly organized little **spatial configuration**. »

« […] the sequence of events moves within a clearly defined world of possible relationships, and because - in so moving - **it makes the abstract space of such a world accessible to our sensibilities**. That is to say that the story projects what one would traditionally call *form*. »

Transformational Networks as spatial configurations

Stockhausen: *Klavierstück III* (Analyse de D. Lewin)



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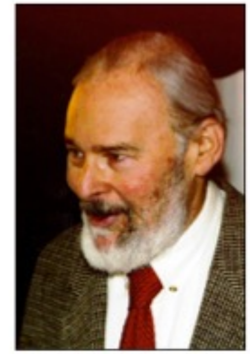
K-nets as a transformational construction

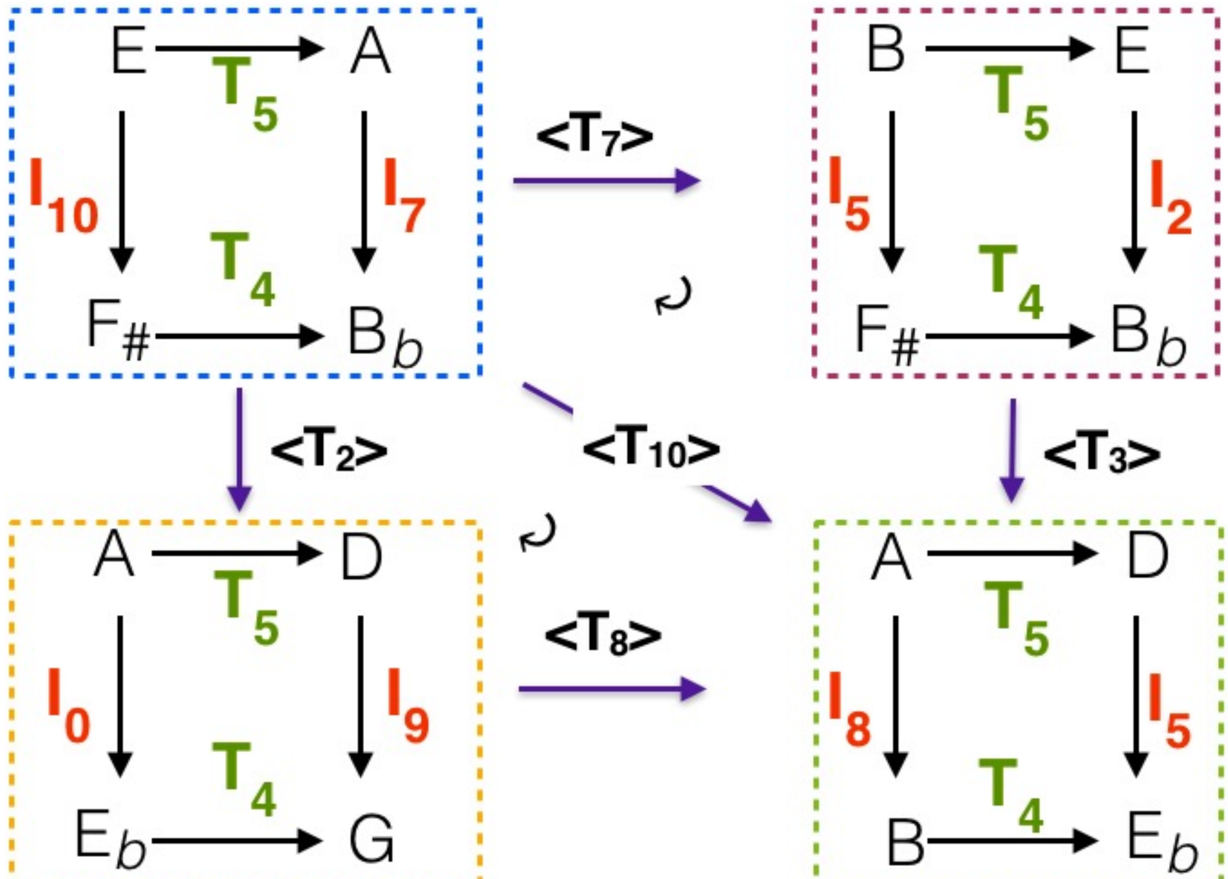
D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994



D. Lewin

H. Klumpenhouwer

$$\boxed{\begin{aligned}\langle T_k\rangle : T_m &\rightarrow T_m\\ I_m &\rightarrow I_{k+m}\end{aligned}}$$



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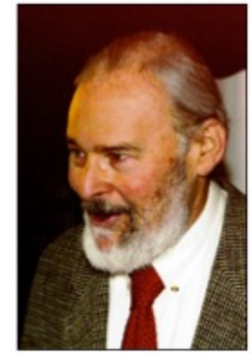
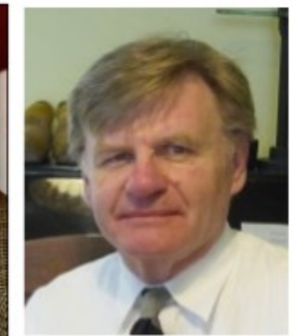


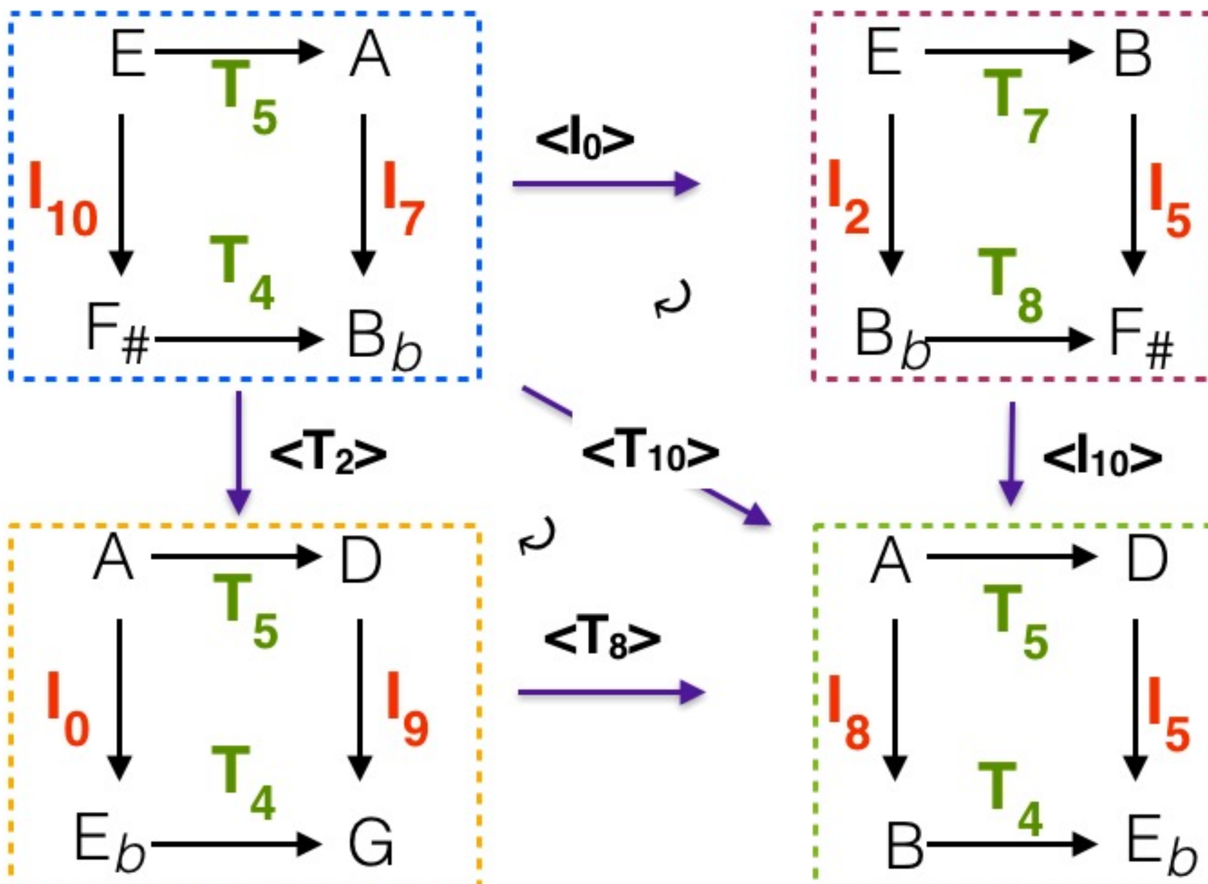
D. Lewin



H. Klumpenhouwer

$$\langle T_k\rangle : \begin{aligned} T_m &\to T_m \\ I_m &\to I_{k+m} \end{aligned}$$

$$\langle I_k\rangle : \begin{aligned} T_m &\to T_{-m} \\ I_m &\to I_{k-m} \end{aligned}$$



K-nets as a transformational construction

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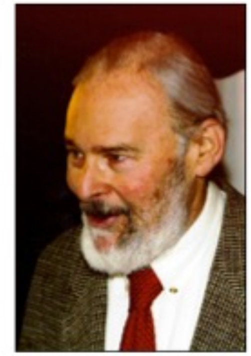

D. Lewin

H. Klumpenhouwer

$$\langle T_k \rangle : T_m \to T_m \qquad I_m \to I_{k+m}$$

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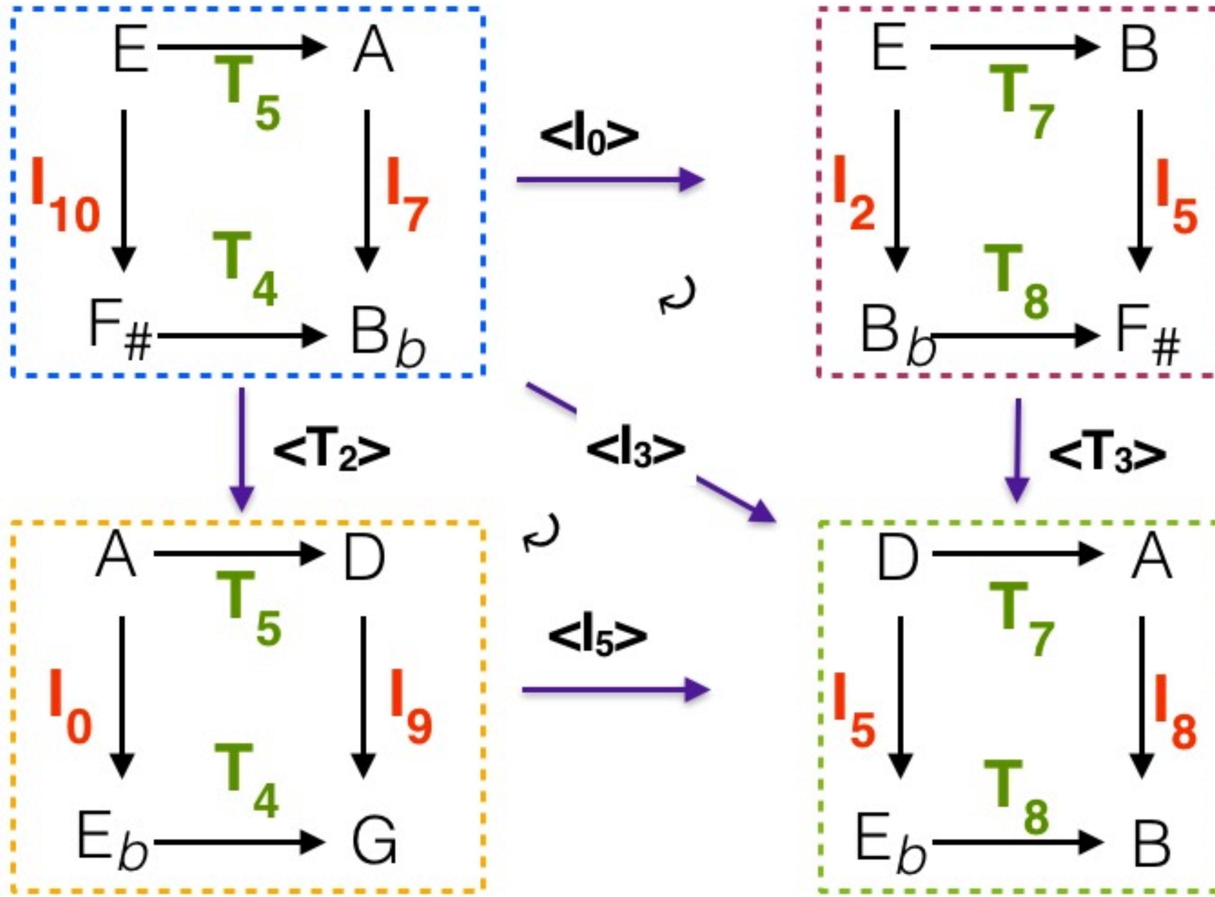
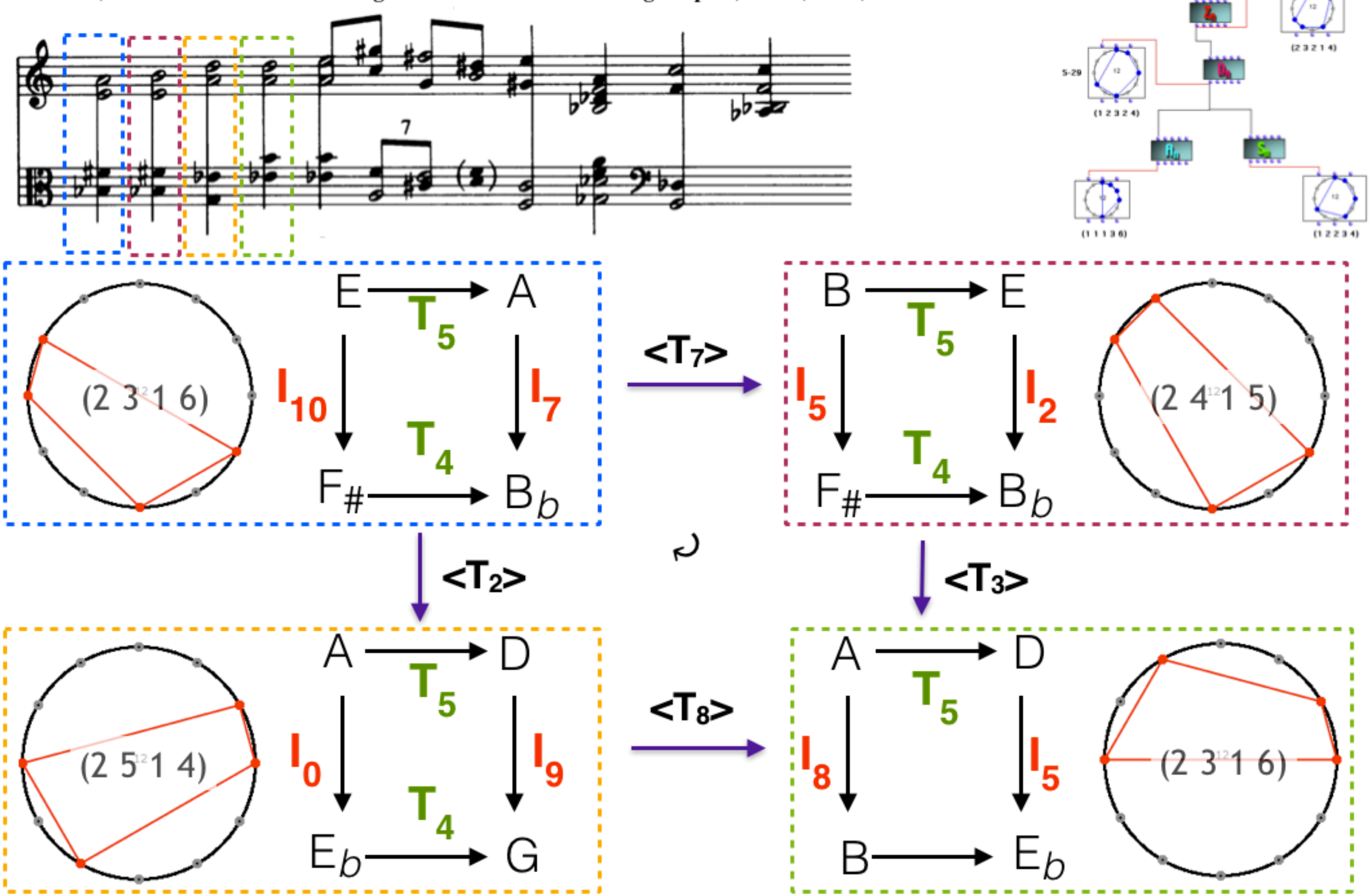


Transformational *vs* set-theoretical approaches

D. Lewin, "A Tutorial on K-nets using the Chorale in Schoenberg's Op.11, N°2 », *JMT*, 1994



From K-Nets to category-based PK-Nets

$$
\begin{tikzcd}
C \arrow[r, "T_3"] \arrow[d, leftrightarrow, "I_2"'] & E\flat \arrow[d, "T_7M_7"] \\
D \arrow[r, "T_2"'] & E
\end{tikzcd}
\quad
\begin{tikzcd}
E\flat \arrow[r, "T_3"] \arrow[d, leftrightarrow, "I_2"'] & F\sharp \arrow[d, "T_7M_7"] \\
B \arrow[r, "T_2"'] & C\sharp
\end{tikzcd}
\quad
\begin{tikzcd}
F\sharp \arrow[r, "T_3"] \arrow[d, leftrightarrow, "I_2"'] & A \arrow[d, "T_7M_7"] \\
G\sharp \arrow[r, "T_2"'] & B\flat
\end{tikzcd}
\quad
\begin{tikzcd}
A \arrow[r, "T_3"] \arrow[d, leftrightarrow, "I_2"'] & C \arrow[d, "T_7M_7"] \\
F \arrow[r, "T_2"'] & G
\end{tikzcd}
\quad \Rightarrow \quad
\begin{tikzcd}
\square \arrow[rr, "F"] \arrow[dr, "R"'] & \arrow[d, phantom, "\phi"] & \square \arrow[dl, "S"] \\
& \square &
\end{tikzcd}
$$

Definition 1 *Let* **C** *be a category, and* S *a functor from* **C** *to the category* **Sets**. *Let* Δ *be a small category and* R *a functor from* Δ *to* **Sets**. *A PK-net of form* R *and of support* S *is a 4-tuple* (R, S, F, ϕ), *in which*

- F *is a functor from* Δ *to* **C**,
- *and* ϕ *is a natural transformation from* R *to* SF.

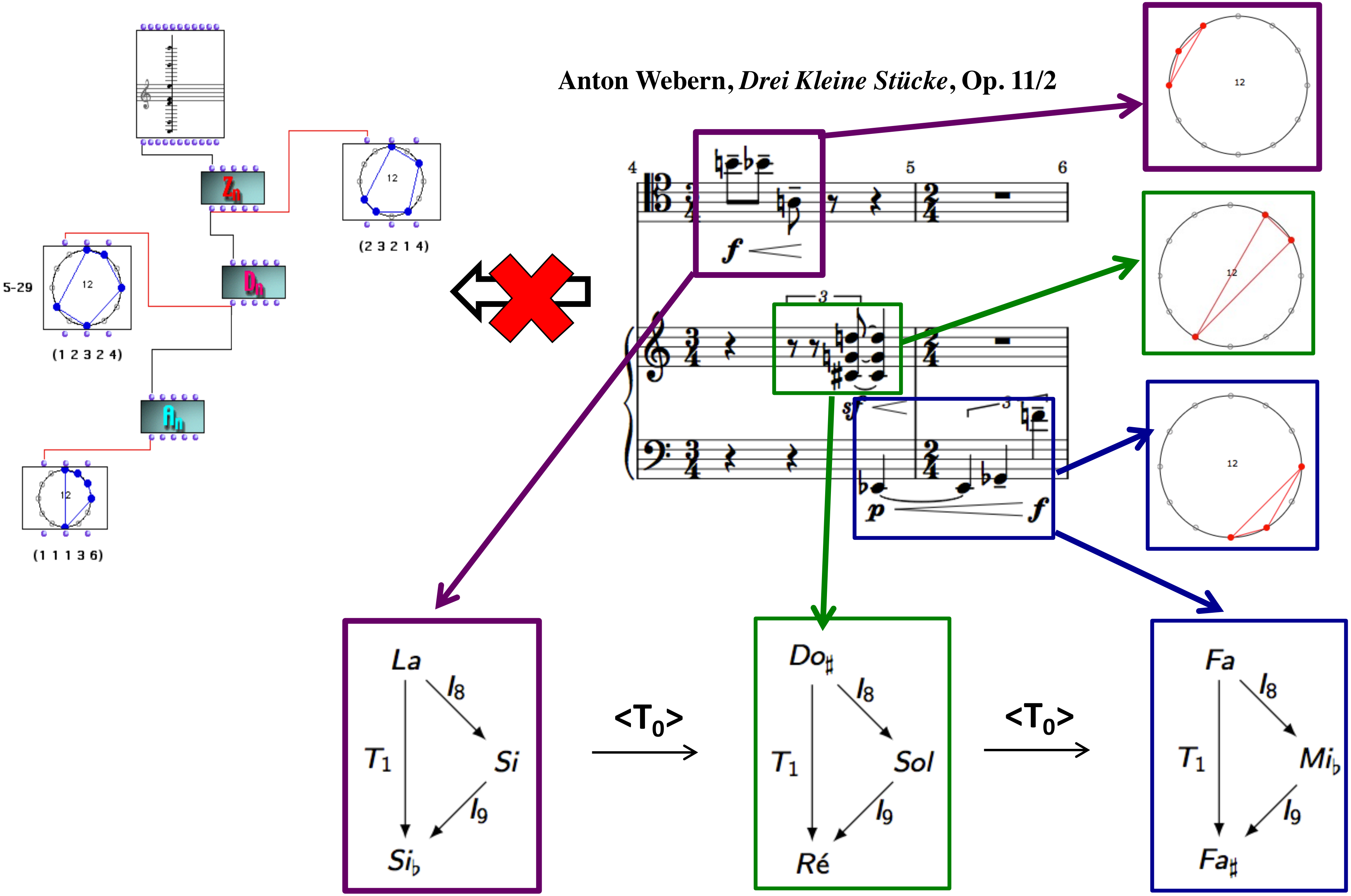
The definition of a PK-net is summed up by the following diagram:

$$
\begin{tikzcd}
\Delta \arrow[rr, "F"] \arrow[dr, "R"'] & \arrow[d, phantom, "\phi"] & \mathbf{C} \arrow[dl, "S"] \\
& \mathbf{Sets} &
\end{tikzcd}
$$

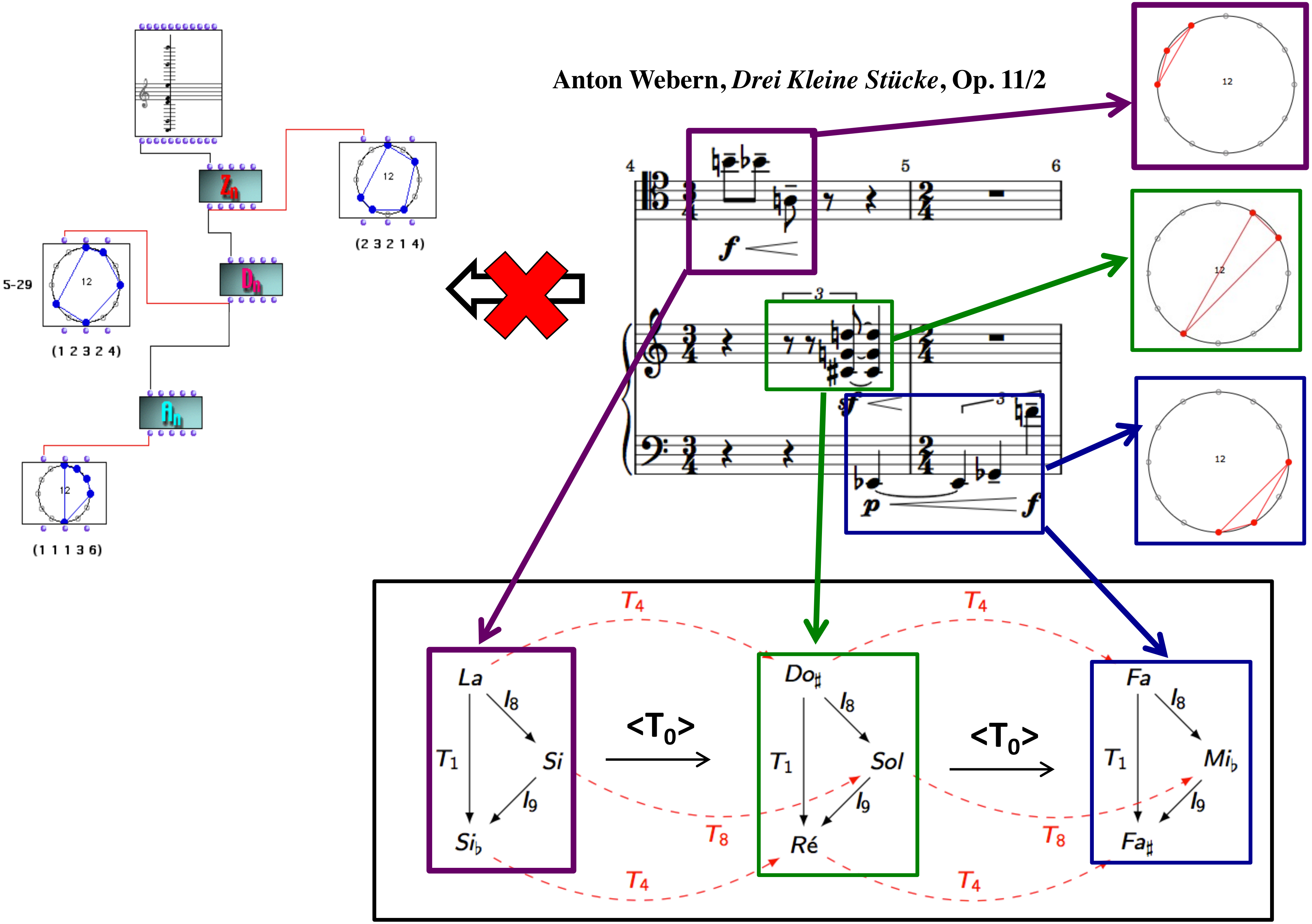
Popoff A., M. Andreatta, A. Ehresmann, « A Categorical Generalization of Klumpenhouwer Networks », MCM 2015, Queen Mary University, Springer, p. 303-314

MCM 2015 London 22-25 June 2015
Mathematics and Computation in Music

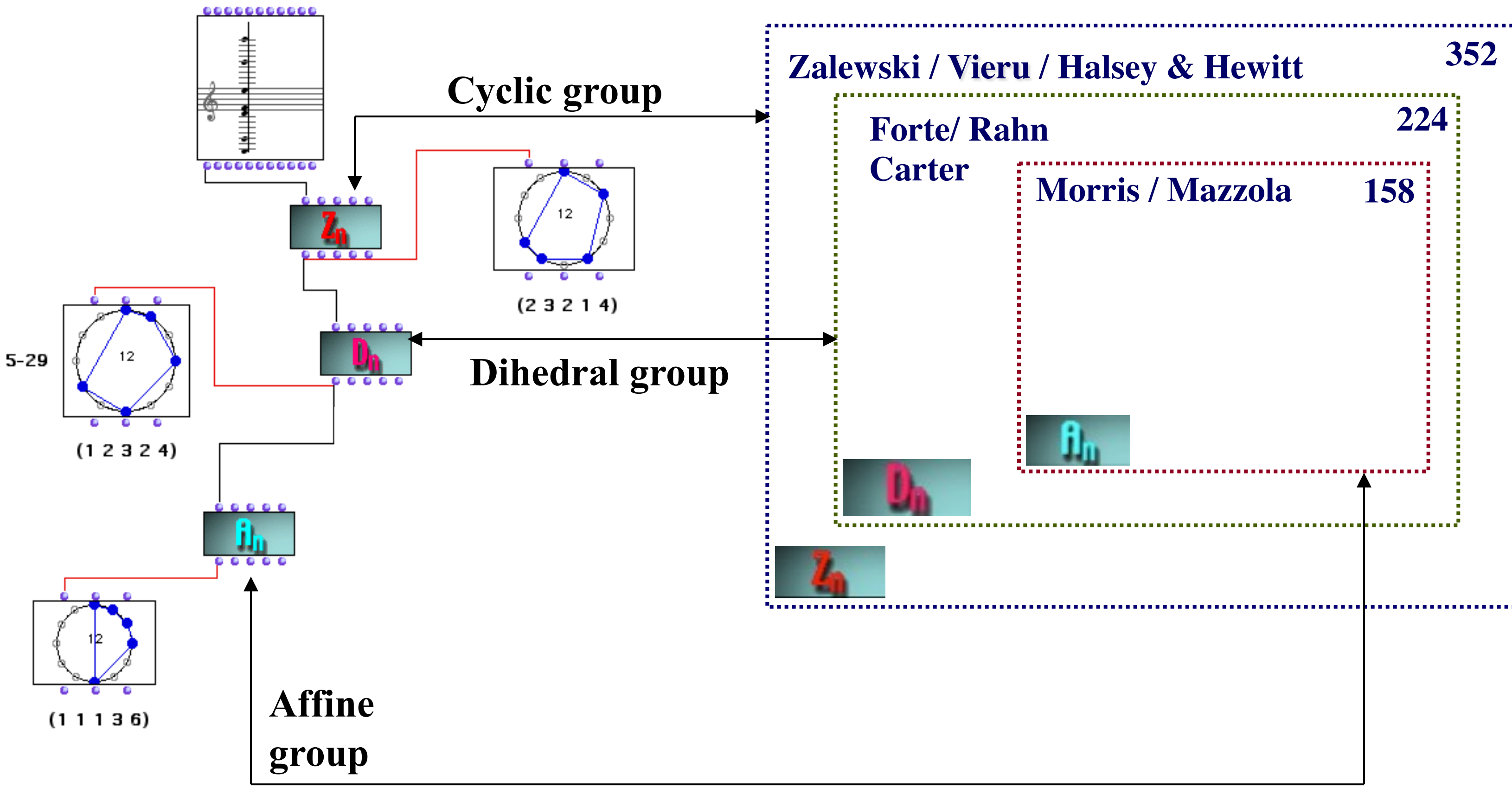
K-Nets and the paradigmatic approach



The categorical vs paradigmatic action-based approach



The interplay between Algebra and Geometry



F. Klein

W. Burnside

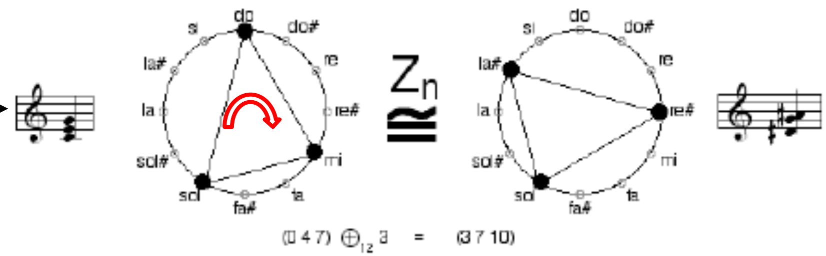
G. Polya

E. Cassirer

The nature of a given geometry is […] defined by the *reference* to a determinate group and the way in which spatial forms are related within that type of geometry. [*Cf. Felix Klein Erlangen Program - 1872*][…] We may raise the question whether there are any concepts and principles that are, although in different ways and different degrees of distinctness, necessary conditions for both the *constitution* of the **perceptual world** and the construction of the universe of **geometrical thought**. It seems to me that the **concept of group** and the **concept of invariance** are such principles.

E. Cassirer, "The concept of group and the theory of perception", 1944

Equivalence classes of musical structures
(up to a group action)

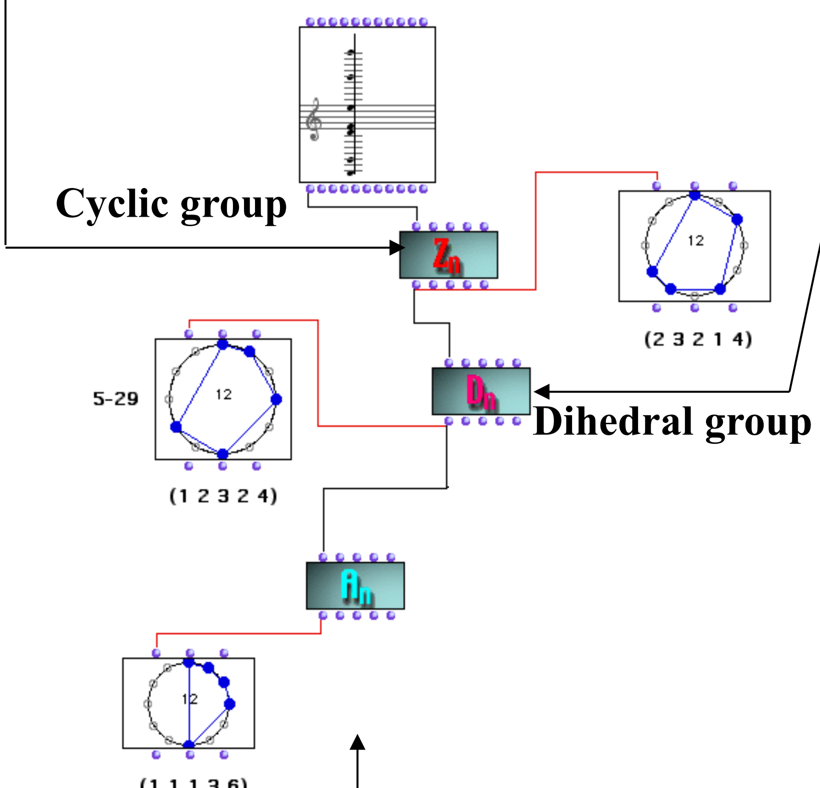


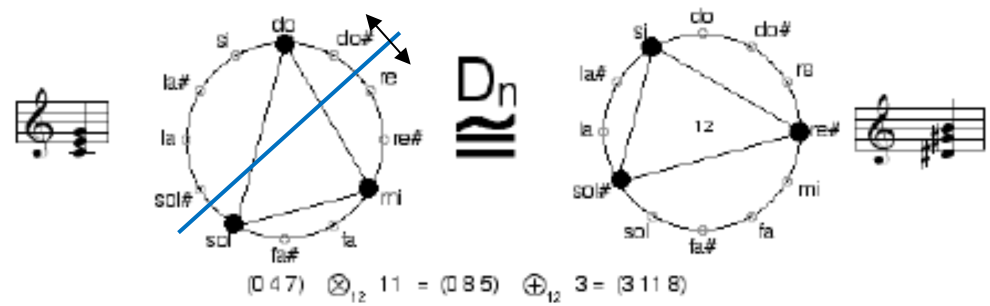
$\mathbf{Z_{12}} = < \mathrm{T_k} \mid (\mathrm{T_k})^{12} = \mathrm{T_0} >$ where $\mathrm{T_k}(x)=x+1$

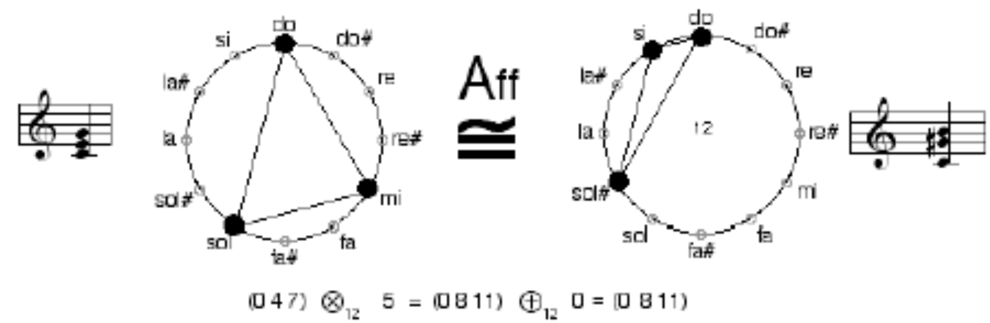
$\mathbf{D_{12}} = < \mathrm{T_k}, \mathrm{I} \mid (\mathrm{T_k})^{12} = \mathrm{I}^2 = \mathrm{T_0}, \mathrm{ITI}=\mathrm{I}(\mathrm{IT})^{-1} >$

where $\mathrm{I}(x)=-x$

$\mathbf{Aff} = \{ f \mid f(x)=ax+b,\ a \in (\mathbf{Z_{12}})^*,\ b \in \mathbf{Z_{12}} \}$







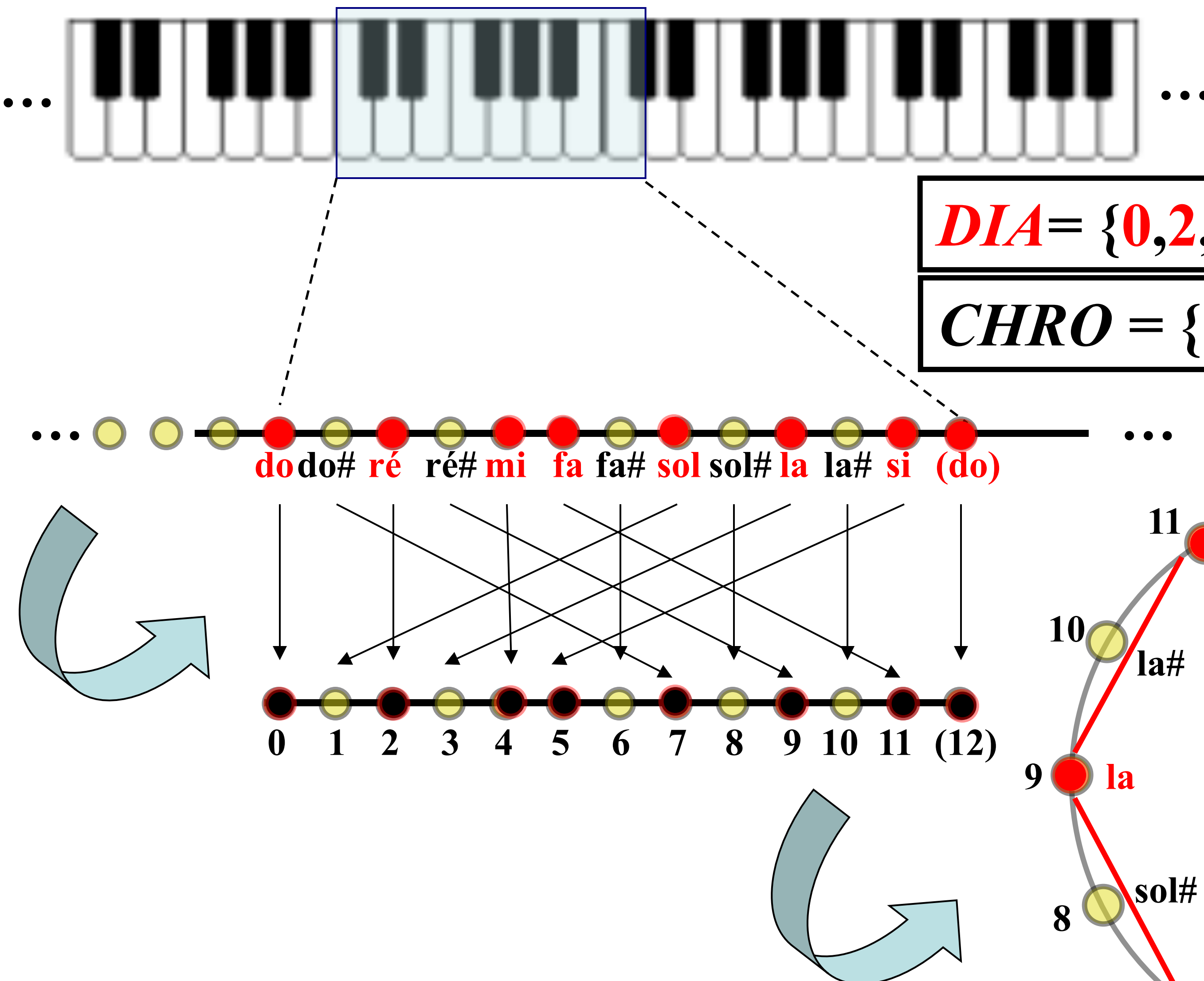
Affine group

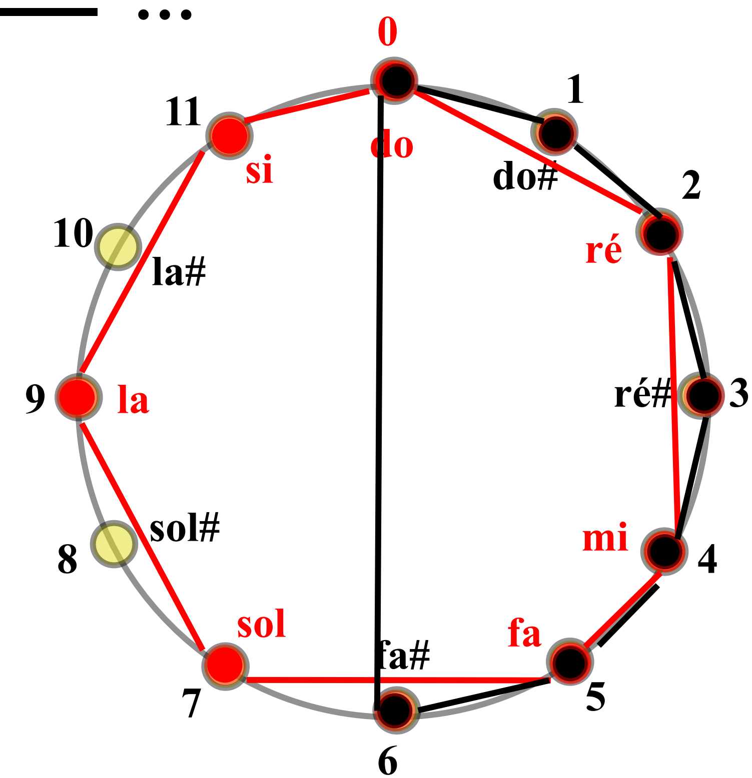
Paradigmatic architecture

Augmentations are multiplications...

$DIA = \{0,2,4,5,7,9,11\}$

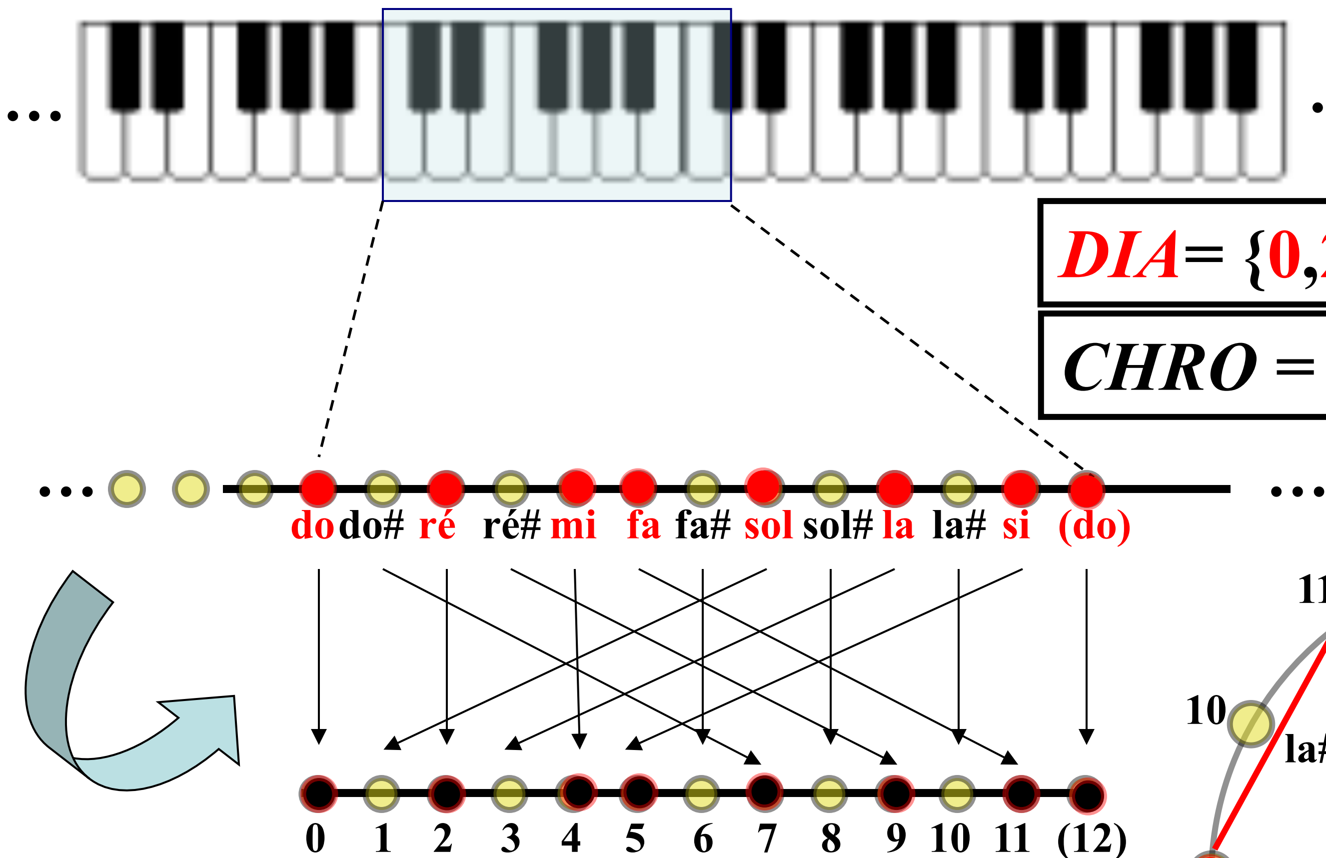
$CHRO = \{0,1,2,3,4,5,6\}$





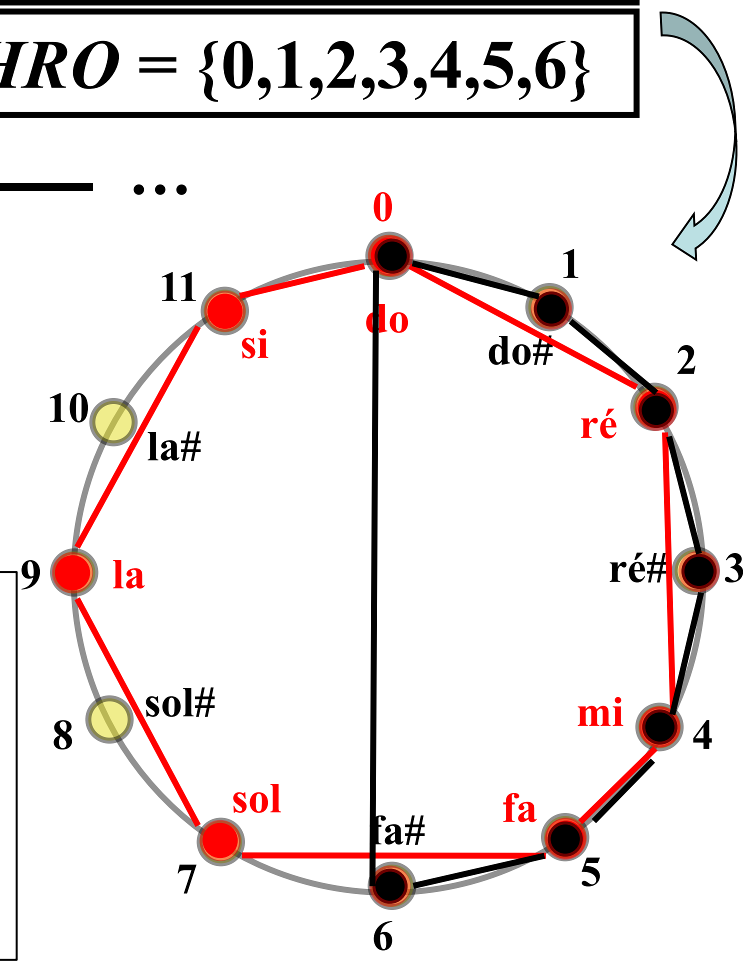
... or affine transformations!

Affine transformations and DIA/CHRO duality



$DIA = \{0,2,4,5,7,9,11\}$

$CHRO = \{0,1,2,3,4,5,6\}$



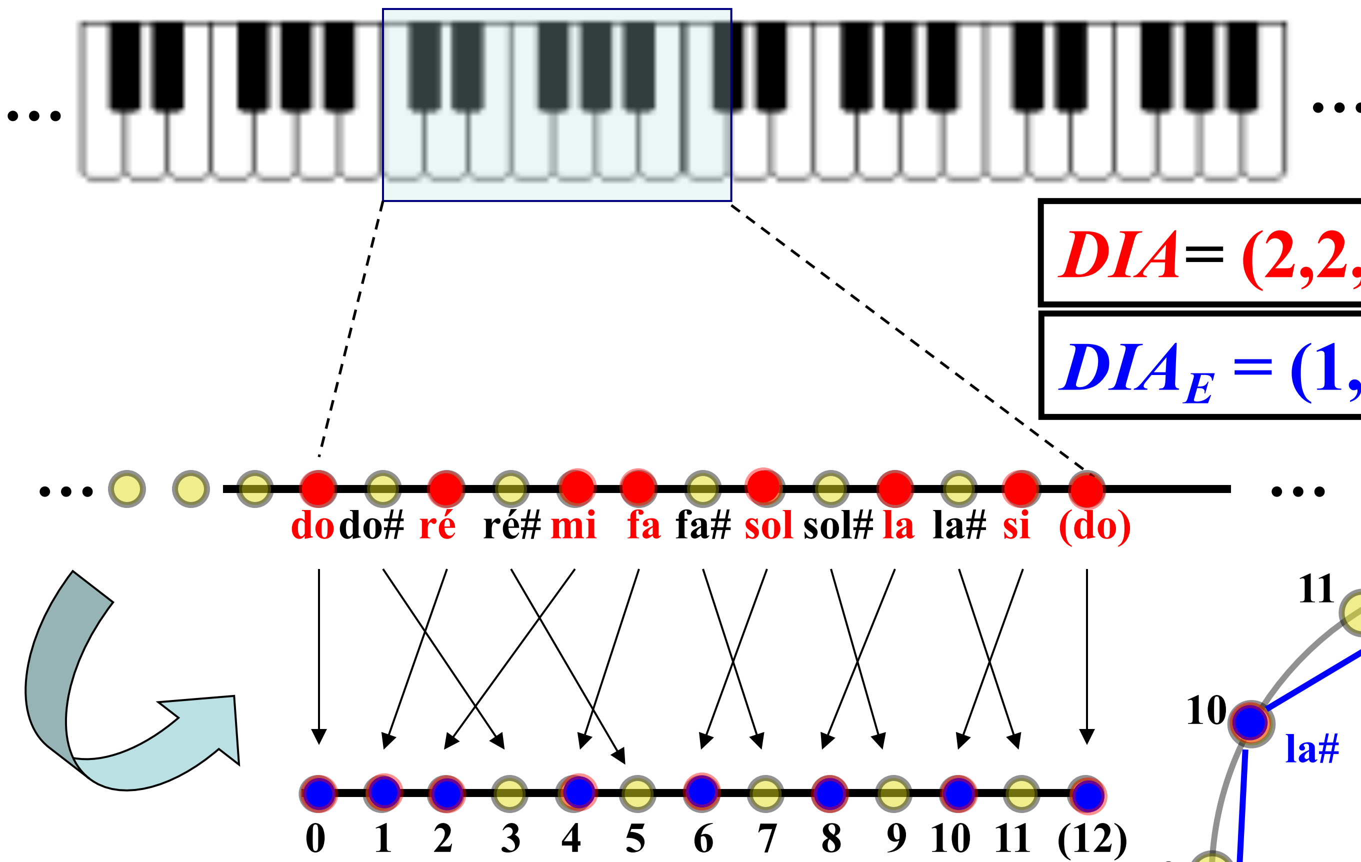
« Le **diatonisme** et le **chromatisme** ne peuvent pas être envisagés en termes de simplicité ou de complexité, comme on le pensait jadis. Il s'agit plutôt d'une question **d'unité des contraires** dans le groupe **Z/12Z** »

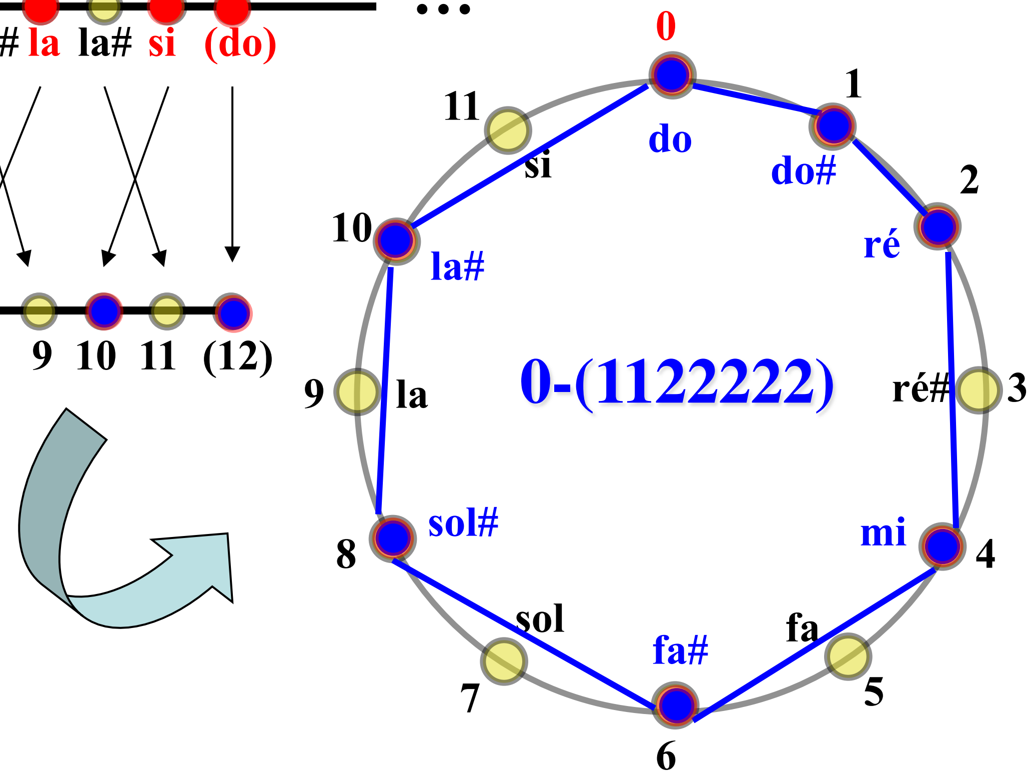
(A. Vieru, « The Musical Signification of Multiplication by 7. Diatonicity and Chromaticity », *Muzica,* 1995)

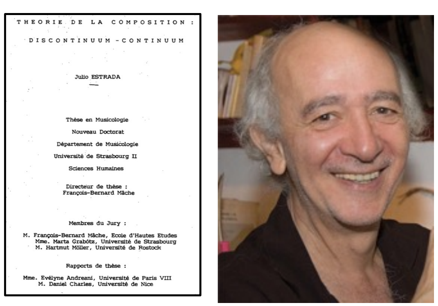
Classifying chords up to permutations of intervals

$DIA = (2,2,1,2,2,2,1)$

$DIA_E = (1,1,2,2,2,2,2)$



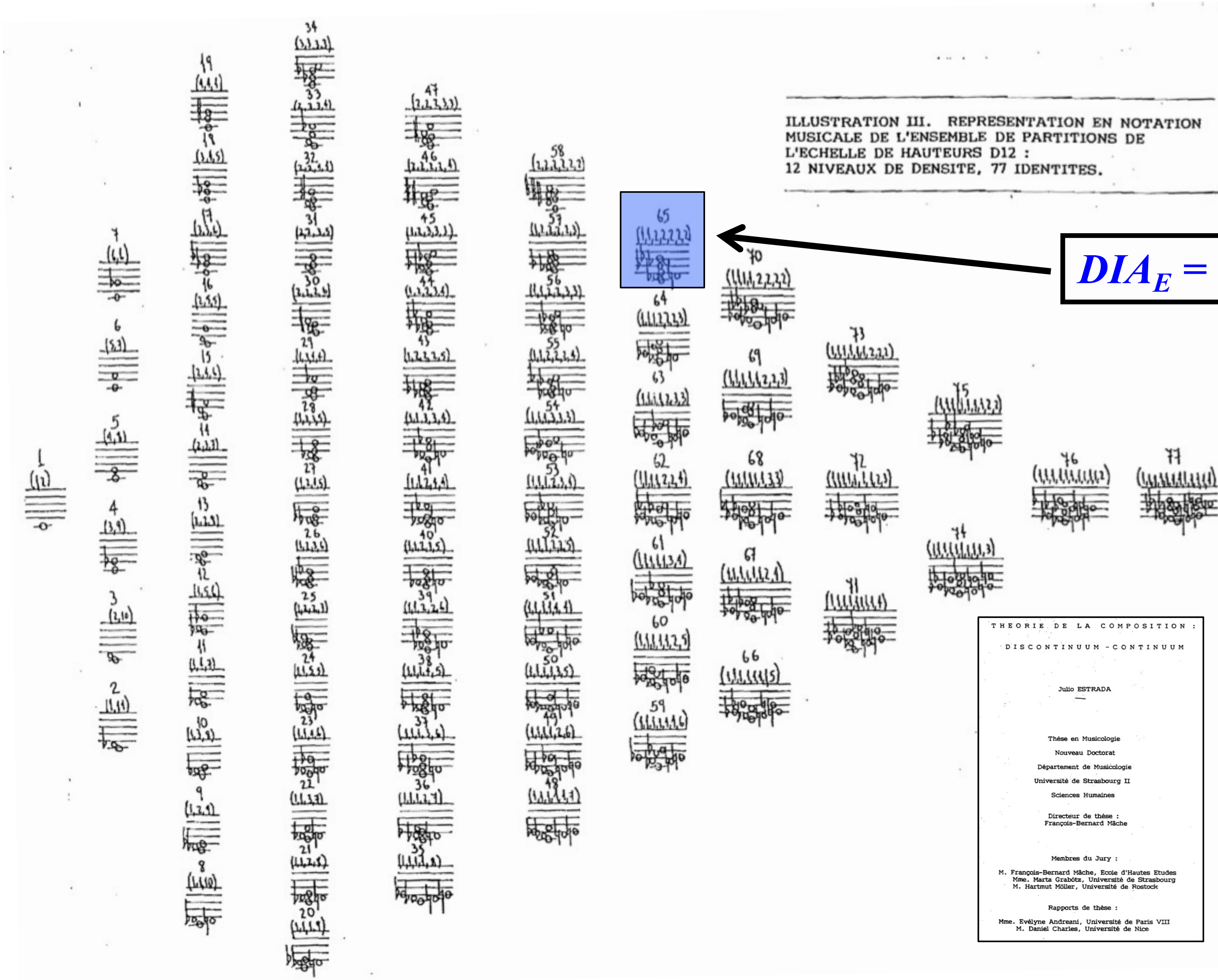




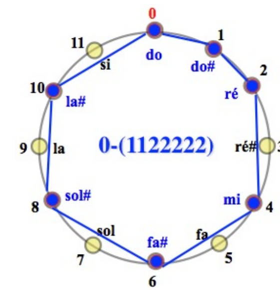
Julio Estrada, *Théorie de la composition : discontinuum – continuum,* université de Strasbourg II, 1994

The permutohedron as a combinatorial space

Julio Estrada, *Théorie de la composition : discontinuum – continuum*, université de Strasbourg II, 1994



DIA_E **$= (1,1,2,2,2,2,2)$**



THEORIE DE LA COMPOSITION :

DISCONTINUUM - CONTINUUM

Julio ESTRADA

Thèse en Musicologie

Nouveau Doctorat

Département de Musicologie

Université de Strasbourg II

Sciences Humaines

Directeur de thèse :
François-Bernard Mâche

Membres du Jury :

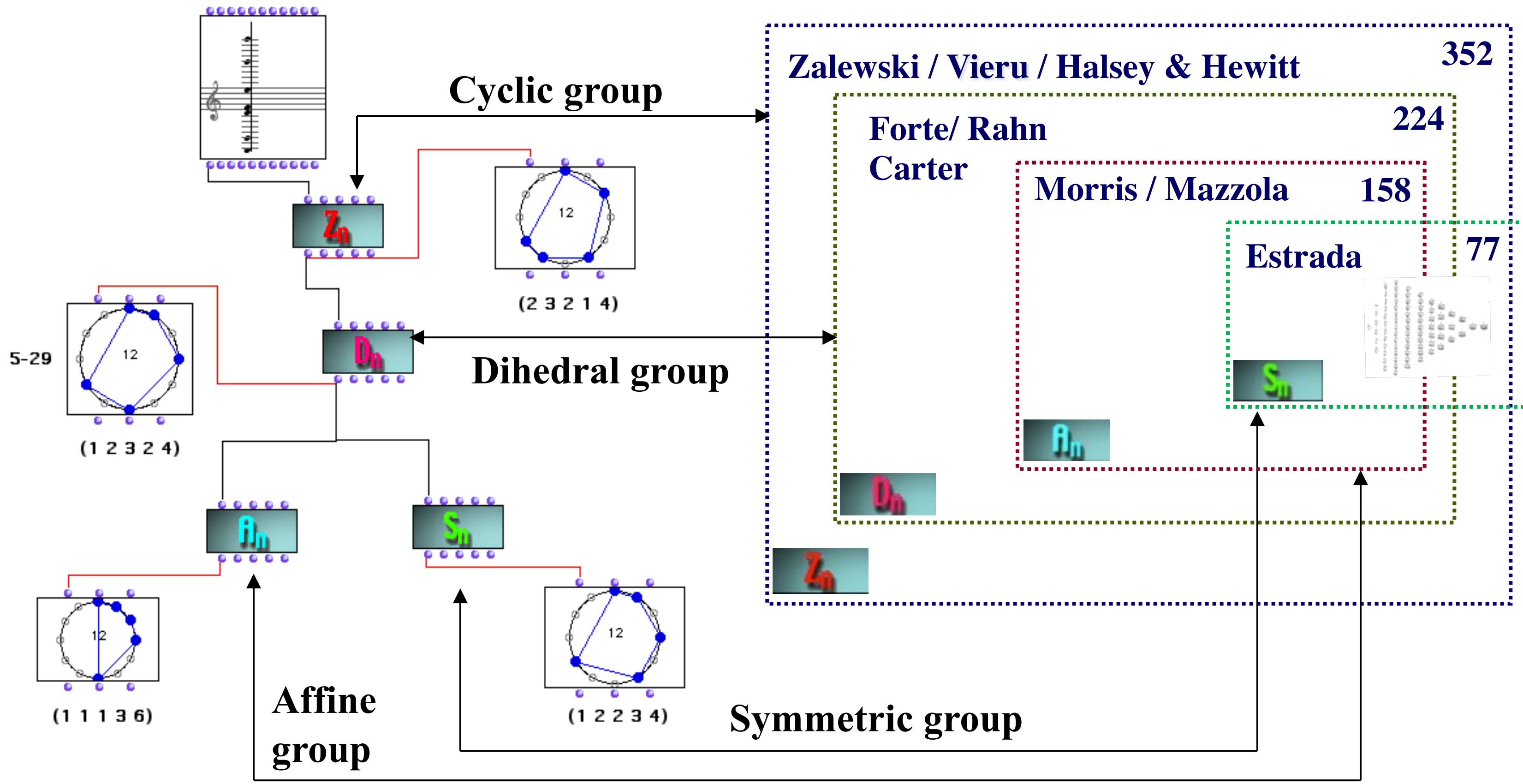
M. François-Bernard Mâche, Ecole d'Hautes Etudes
Mme. Marta Grabócz, Université de Strasbourg
M. Hartmut Möller, Université de Rostock

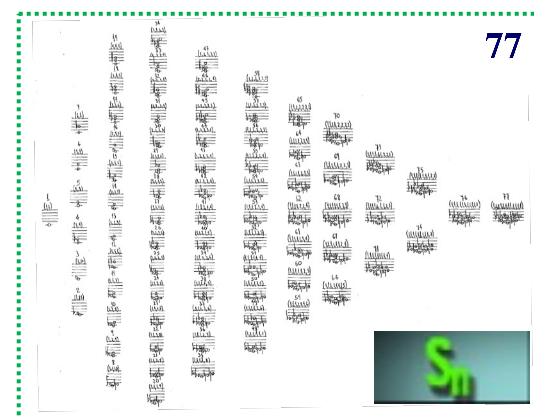
Rapports de thèse :

Mme. Evélyne Andreani, Université de Paris VIII
M. Daniel Charles, Université de Nice

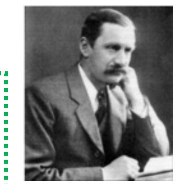
J. Estrada

Algebraic Combinatorics and Music Enumeration





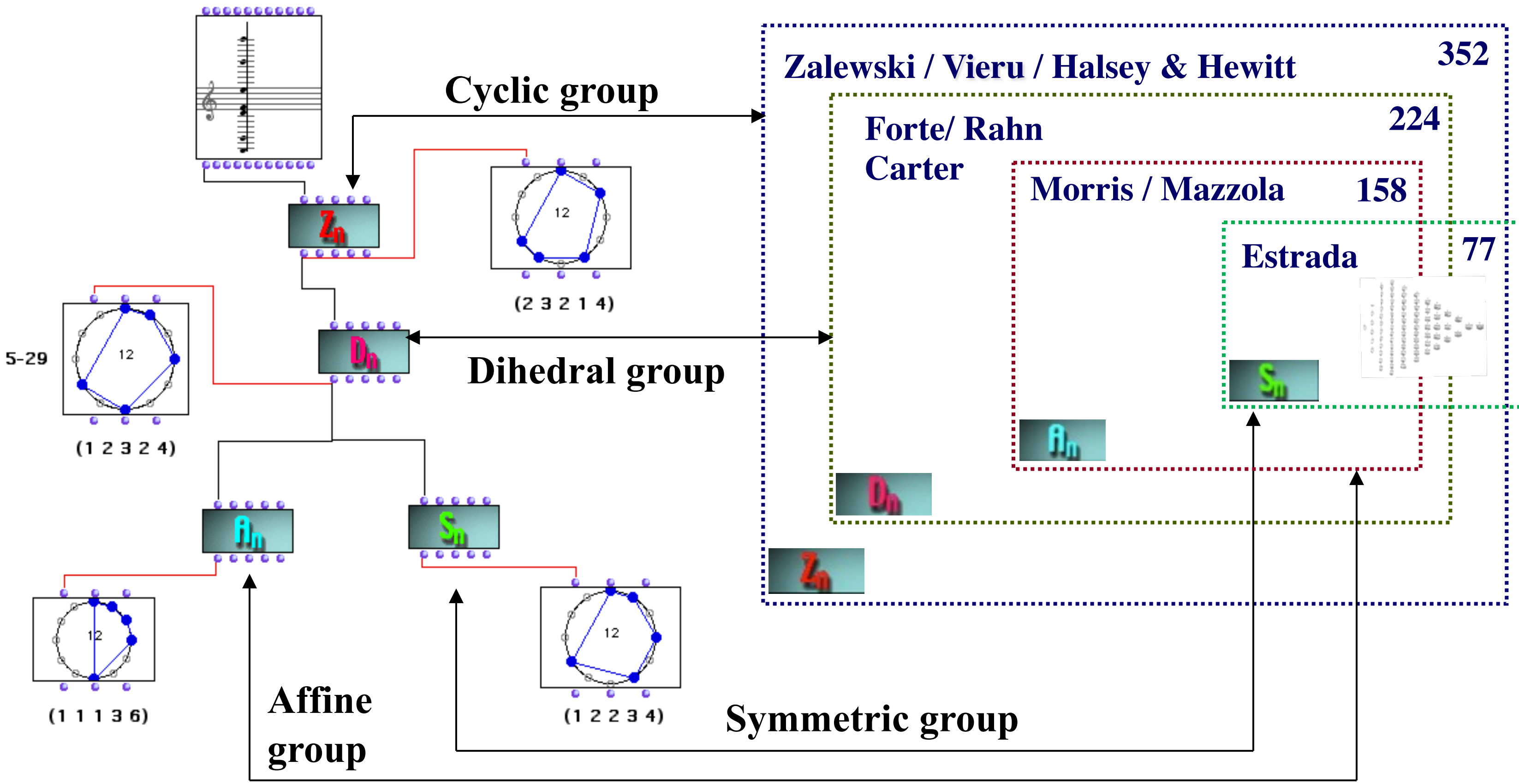

F. Klein


W. Burnside

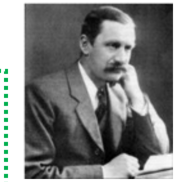

G. Polya


Julior Estrada

Algebraic Combinatorics and Music Enumeration



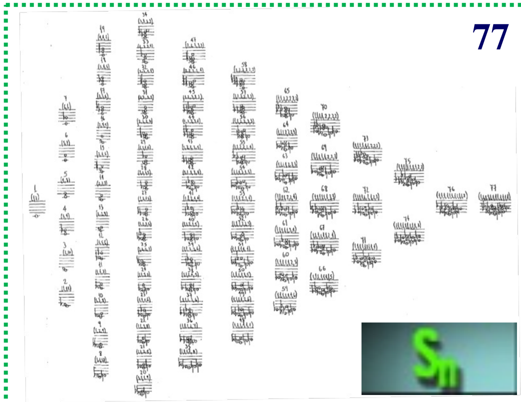

F. Klein


W. Burnside

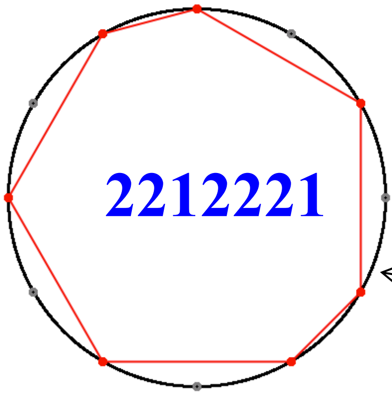

G. Polya

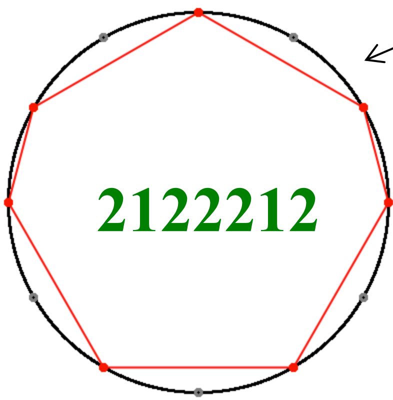

Julior Estrada

	1	2	3	4	5	6	7	8	9	10	11	12
Z_n	1	6	19	43	66	80	66	43	19	6	1	1
D_n	1	6	12	29	38	50	38	29	12	6	1	1
A_n	1	5	9	21	25	34	25	21	9	5	1	1
S_n	***1***	***6***	***12***	***15***	***13***	***11***	***7***	***5***	***3***	***2***	***1***	***1***



The pitch-rhythm *cognitive* isomorphic correspondence





...

cyclic permutations

TABLE I

Comparison of M = 7, L = 12 patterns for pitch (scales) and rhythm (time-lines)

pattern	pitch domain name and notation (in C)	rhythm domain notation	examples from West Africa	references
1. 2212221	major scale (Ionian) CDEFGAB	♩ ♩ ♪♩ ♩ ♩ ♪	Ewe (Atsiabek , Sogba, Atsia) also Yoruba	Jones (1959), C. K. Ladzekpo, S. K. Ladzekpo and Pantaleoni, Locke
2. 2122212	Dorian CDE♭FGAB♭	♩ ♪♩ ♩ ♩ ♪♩	Bemba—Northern Rhodesia	Jones (1965), (Ekwueme)
3. 1222122	Phrygian CD♭E♭FGA♭B♭	♪♩ ♩ ♩ ♪♩ ♩	——	——
4. 2221221	Lydian CDEF#GAB	♩ ♩ ♩ ♪♩ ♩ ♪	Ga-Adangme (common) also common Haitian pattern, Akan (Ab fo)	C. K. Ladzekpo, Combs (1974), R. Hill, Asiama
5. 2212212	Mixolydian CDEFGAB♭	♩ ♩ ♪♩ ♩ ♪♩	Yoruba sacred music from Ekiti	King
6. 2122122	Aeolian CDE♭FGA♭B♭	♩ ♪♩ ♩ ♪♩ ♩	Ashanti (Ab fo , Mpre)	Koetting
7. 1221222	Locrian CD♭E♭FG♭A♭B♭	♪♩ ♩ ♪♩ ♩ ♩	Ghana*	Nketia (1963a)
8. 2121222	(#2 Locrian) CDE♭FG♭A♭B♭	♩ ♪♩ ♪♩ ♩ ♩	Ashanti (Asedua)	C. K. Ladzekpo
9. 2112123	—— CDD#EF#GA	♩ ♫♩ ♪♩ ♩.	Akan (juvenile song)	Nketia (1963b)

* clap pattern
† mute stroke on bell

J. Pressing, "Cognitive isomorphisms between pitch and rhythm in world musics: West Africa, the Balkans and Western tonality", *Studies in Music*, 17, p. 38-61

The pitch-rhythm isomorphic correspondence

"It is well known that there exists an isomorphic relation between pitch and rhythm, which several authors have pointed out from time to time."

(G. Toussaint, *The geometry of musical rhythm. What makes a "Good" Rhythm Good?* CRC Press, 2013, p. xiii)

The *Euclidean* Algorithm Generates Traditional Musical Rhythms

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School of Computer Science, McGill University
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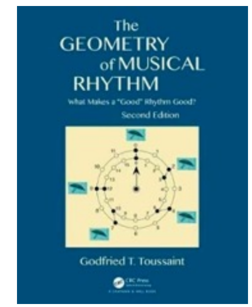
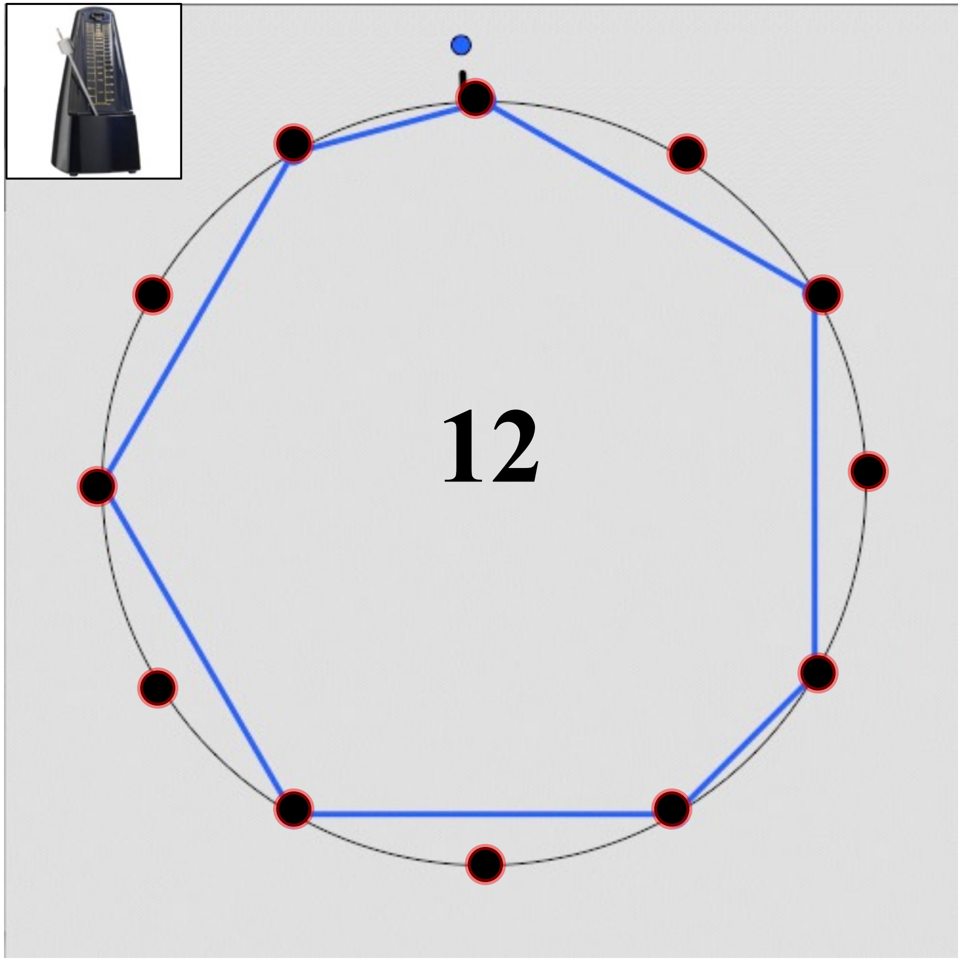


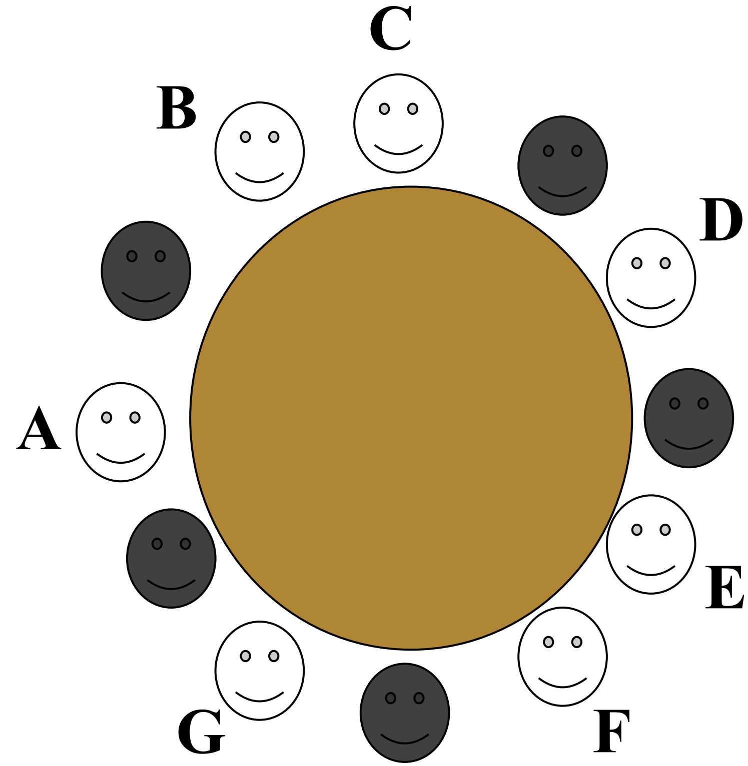
Abstract

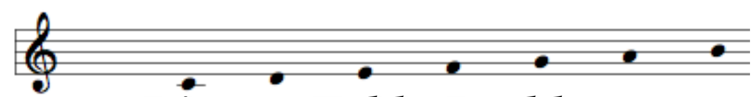
The *Euclidean* algorithm (which comes down to us from Euclid's *Elements*) computes the greatest common divisor of two given integers. It is shown here that the structure of the Euclidean algorithm may be used to generate, very efficiently, a large family of rhythms used as timelines (*ostinatos*), in sub-Saharan African music in particular, and world music in general. These rhythms, here dubbed *Euclidean* rhythms, have the property that their onset patterns are distributed as evenly as possible. *Euclidean* rhythms also find application in nuclear physics accelerators and in computer science, and are closely related to several families of words and sequences of interest in the study of the combinatorics of words, such as Euclidean strings, to which the *Euclidean* rhythms are compared.

G. T. Toussaint, The Euclidean algorithm generates traditional musical rhythms, Proceedings of BRIDGES, 2005, pp. 47–56.

African-Cuban Maximally-Even rhythms

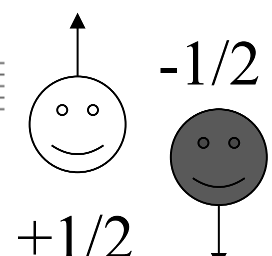






Dinner Table Problem

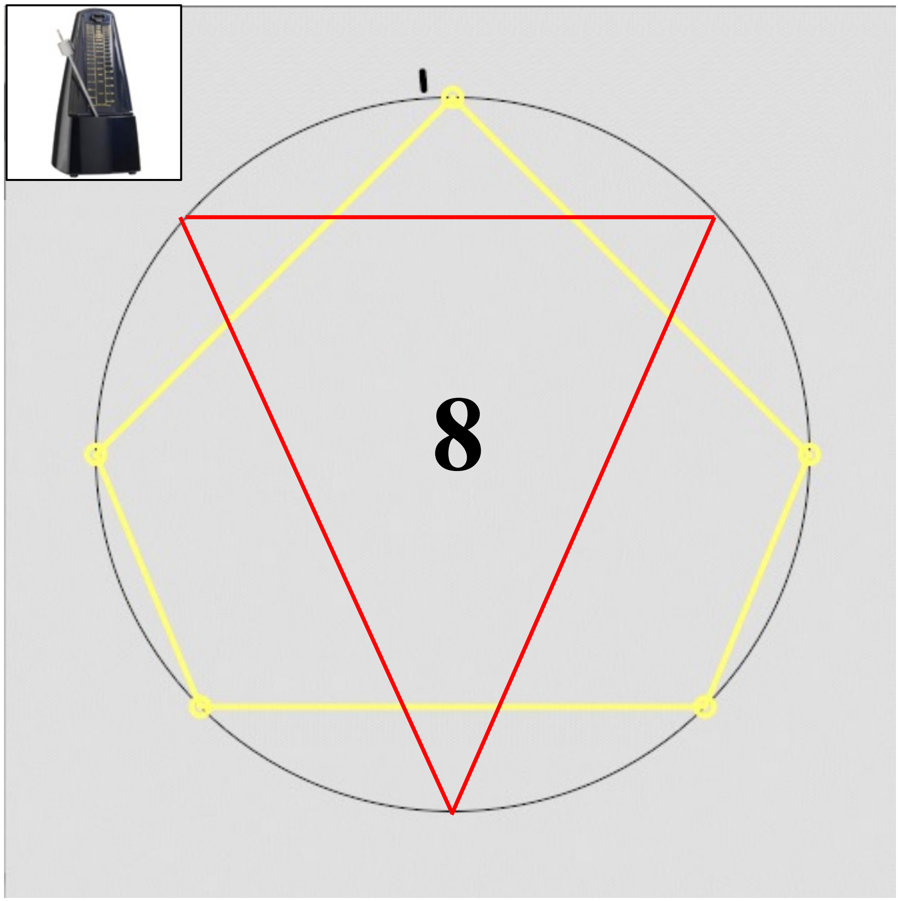
Abadja ou Bembé



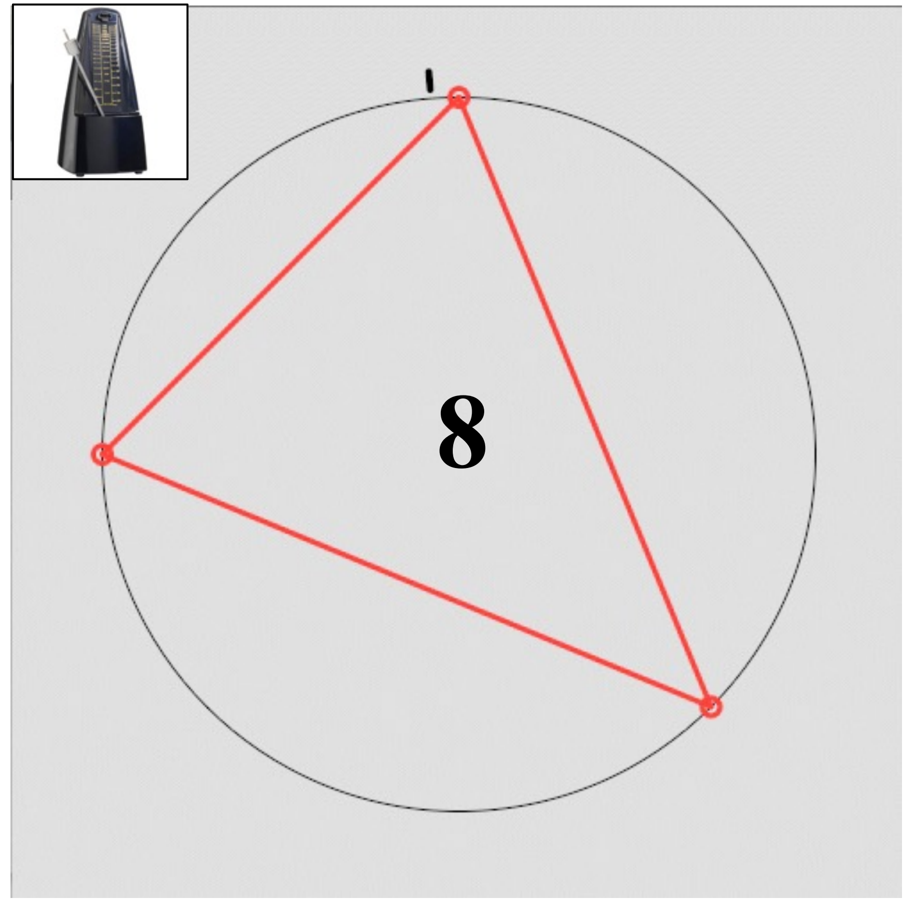
J. Douthett & R. Krantz, "Energy extremes and spin configurations for the one-dimensional antiferromagnetic Ising model with arbitrary-range interaction", *J. Math. Phys.* 37 (7), July 1996

African-cuban ME-rhythms

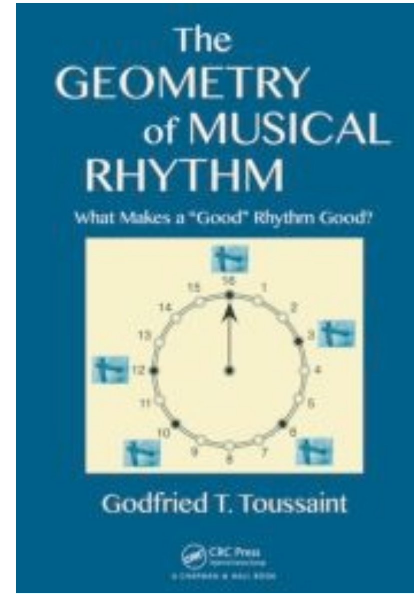
El cinquillo



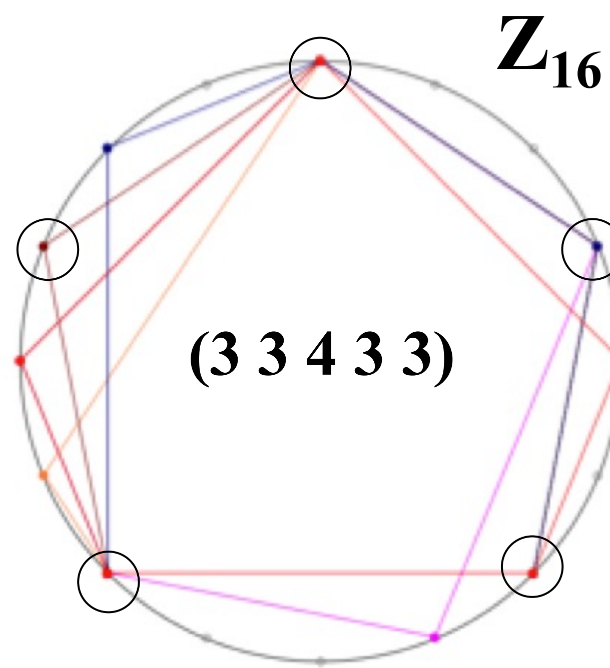
El trecillo



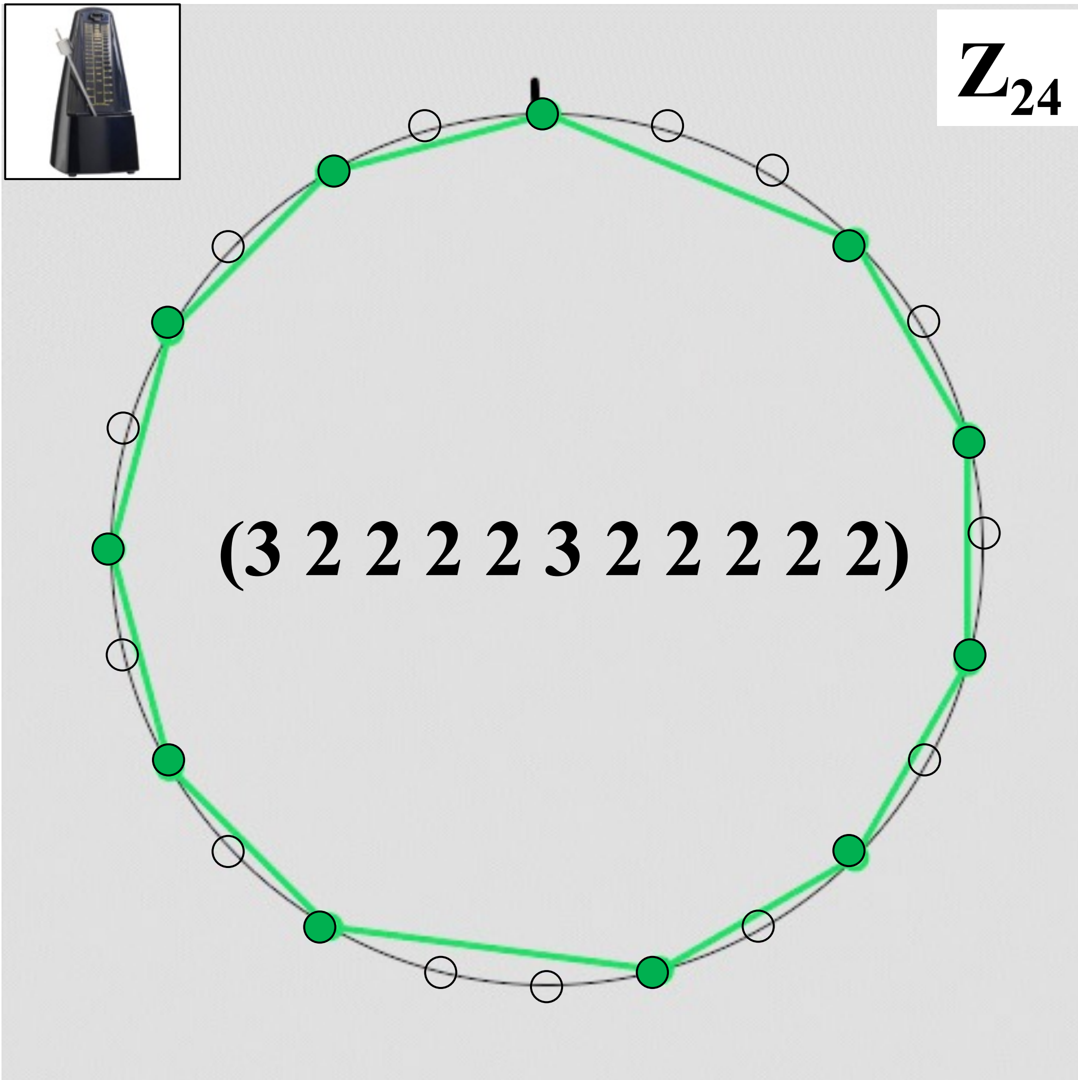
The geometry of African-Cuban rhythms



Shiko	●				●		●				●		●			
Son	●			●			●				●		●			
Soukous	●			●			●				●	●				
Rumba	●			●				●			●		●			
Bossa	●			●			●				●			●		
Gahu	●			●			●				●				●	

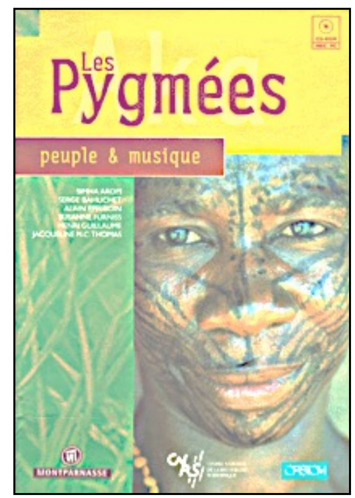


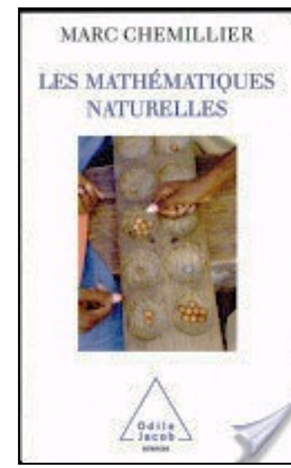
Odditive property of orally-trasmitted practices



Simha Arom

Marc Chemillier





musimédiane

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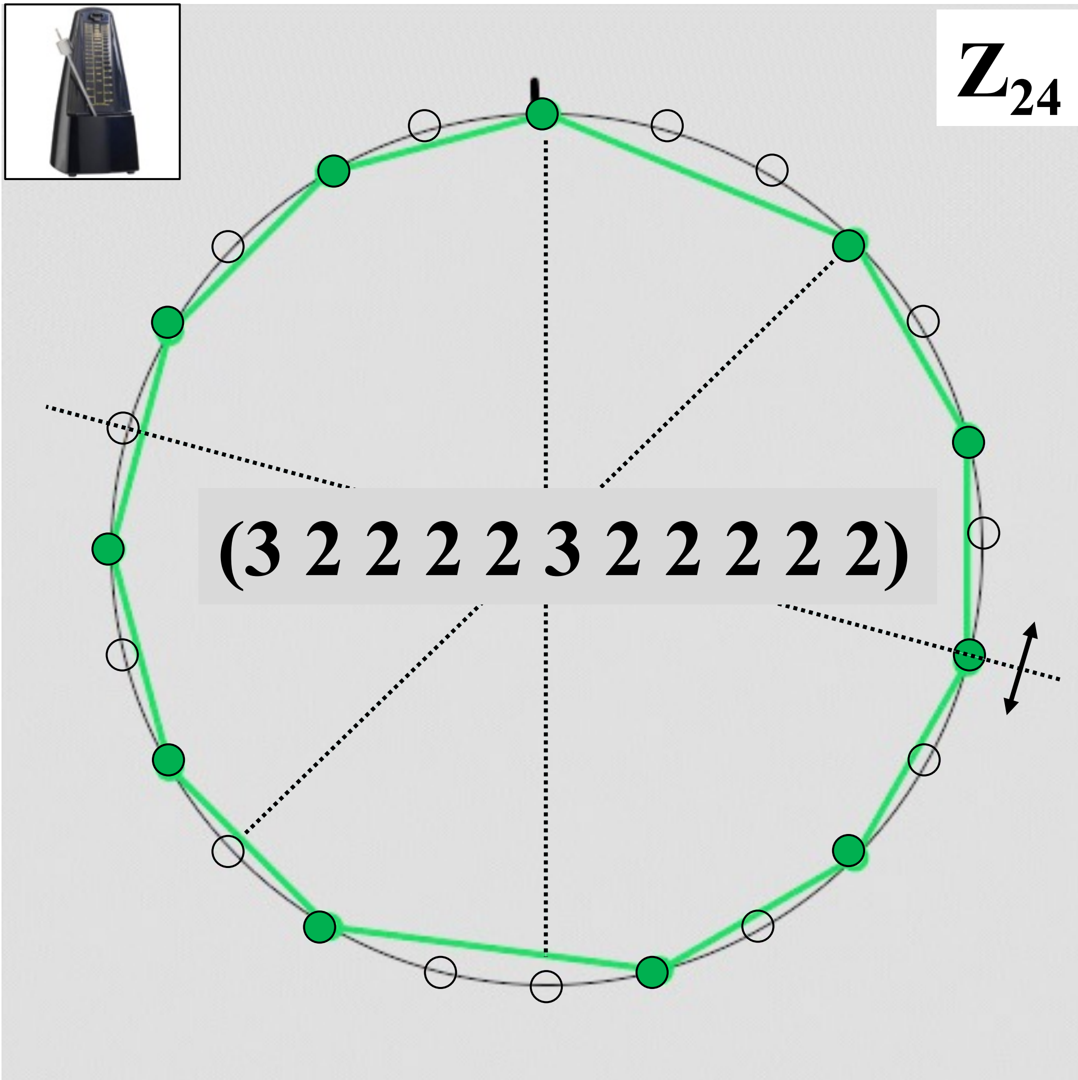
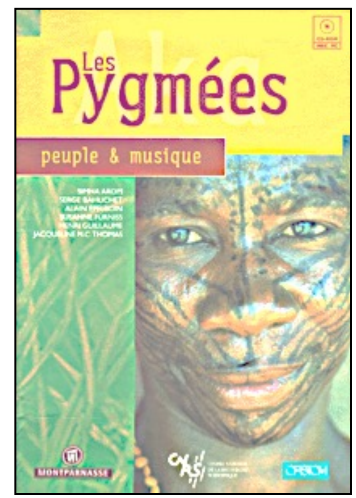
revue audiovisuelle et multimédia d'analyse musicale

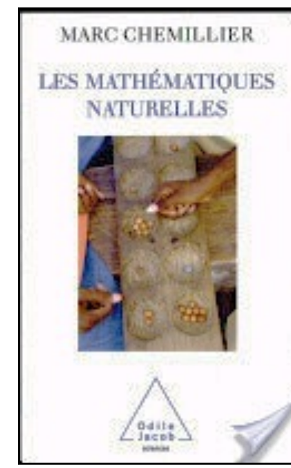
Odditive property of orally-trasmitted practices




Simha Arom


Marc Chemillier





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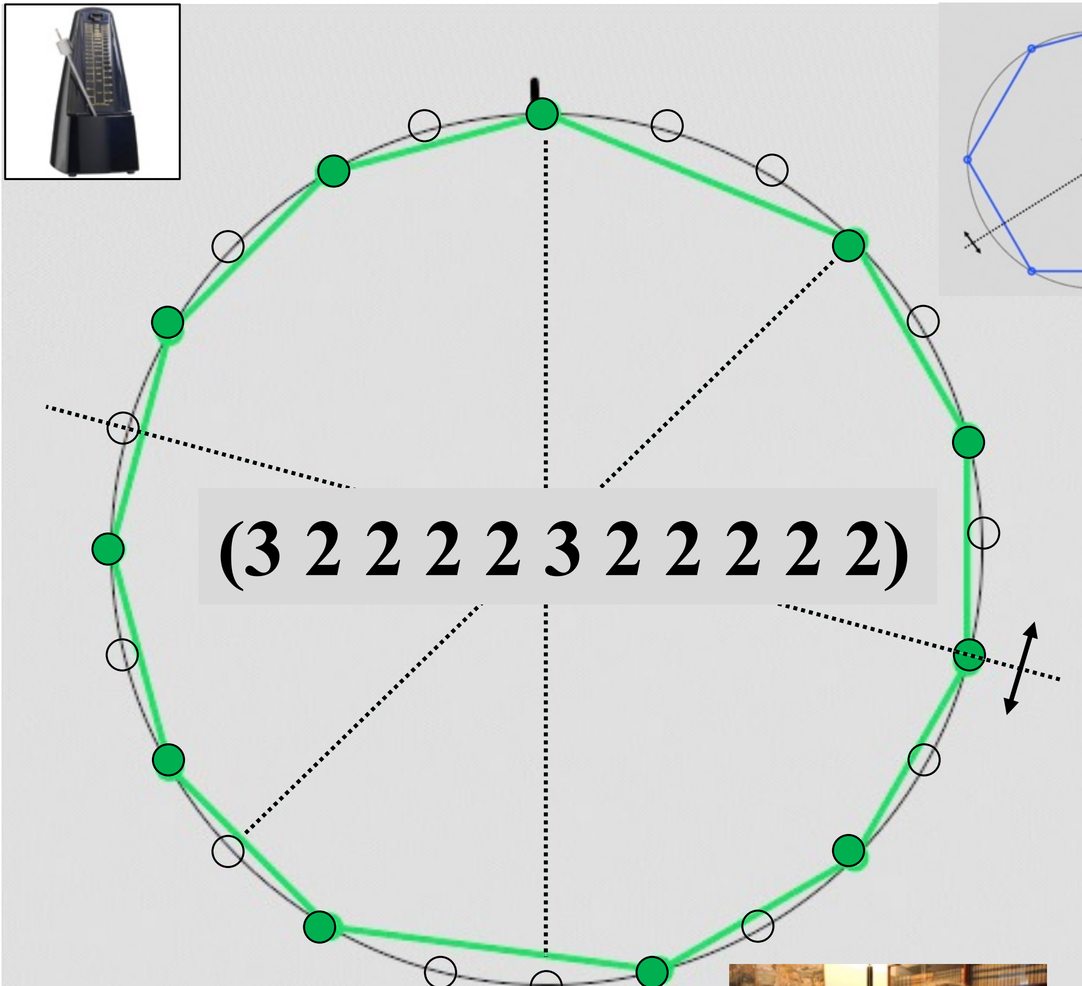
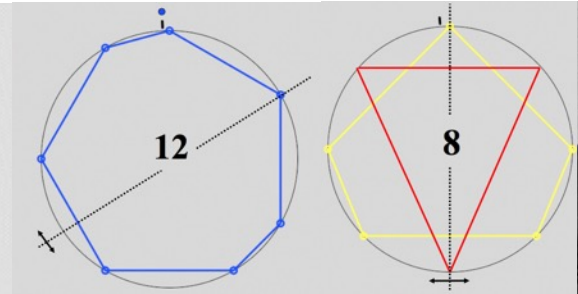
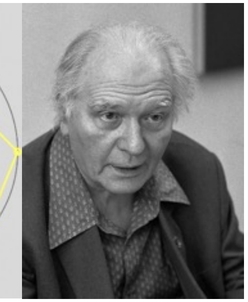
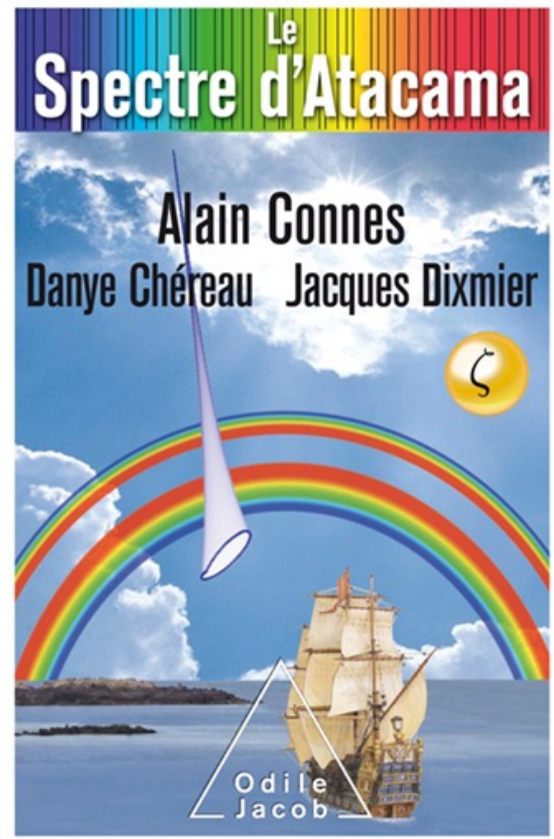
Olivier Messiaen’s non-invertible rhythms





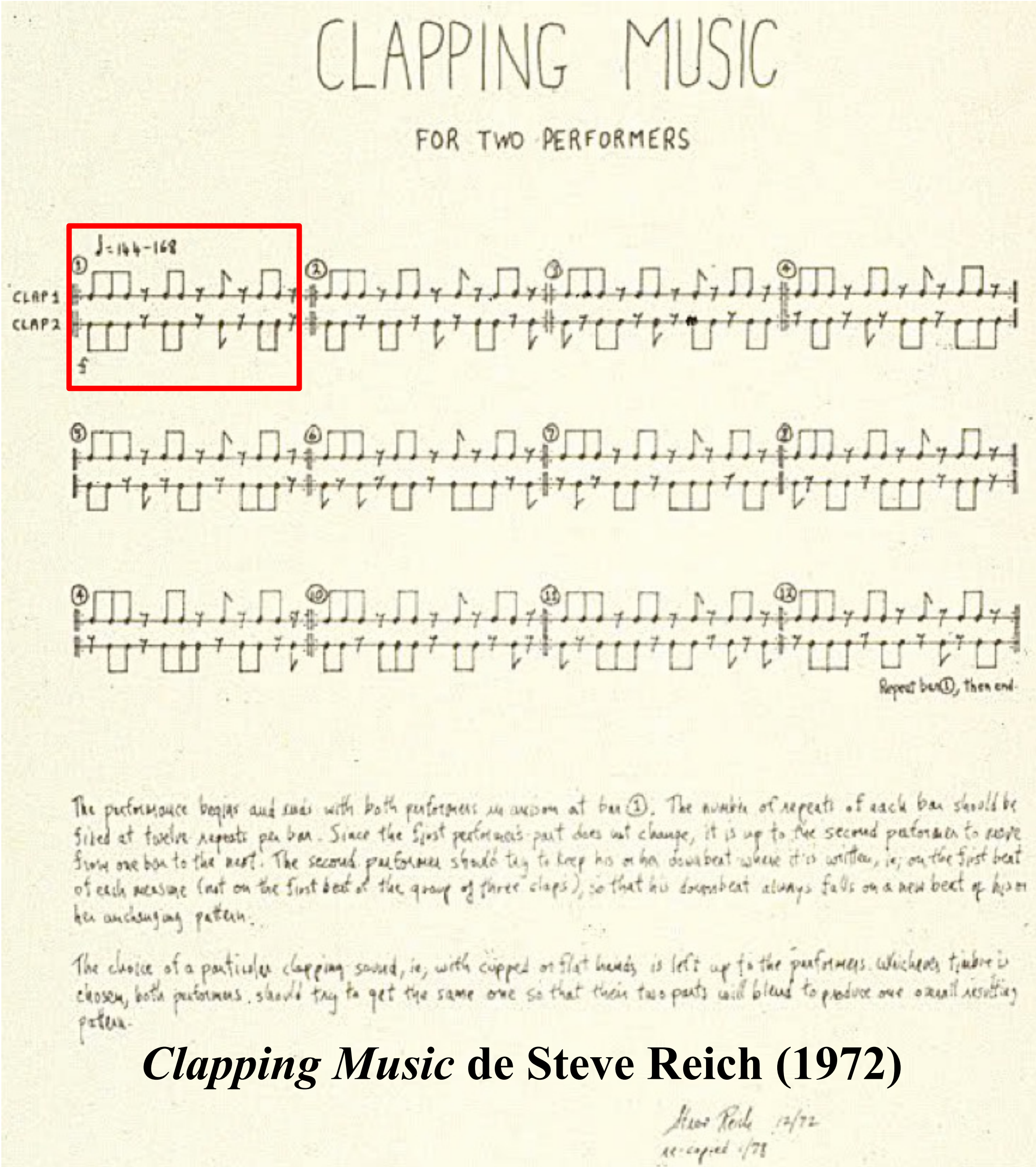


Olivier Messaien

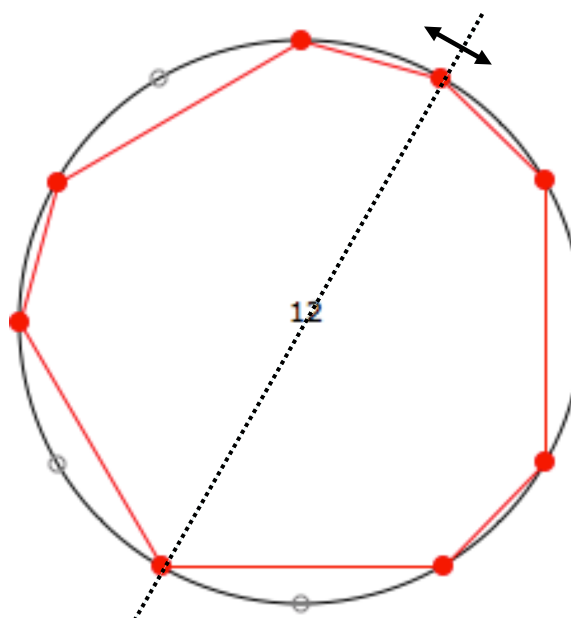


Alain Connes

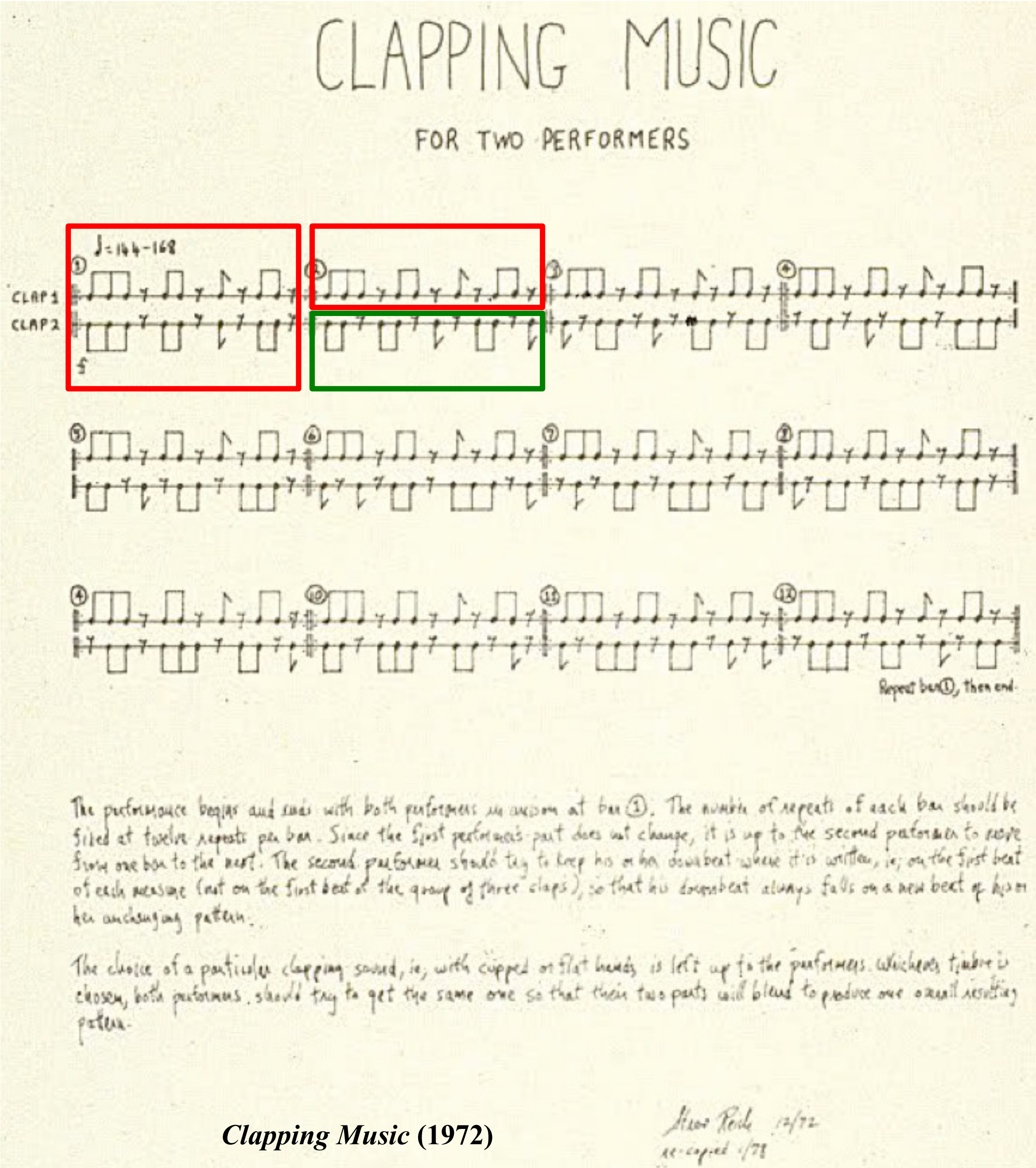
Palindromic structures in Steve Reich's music



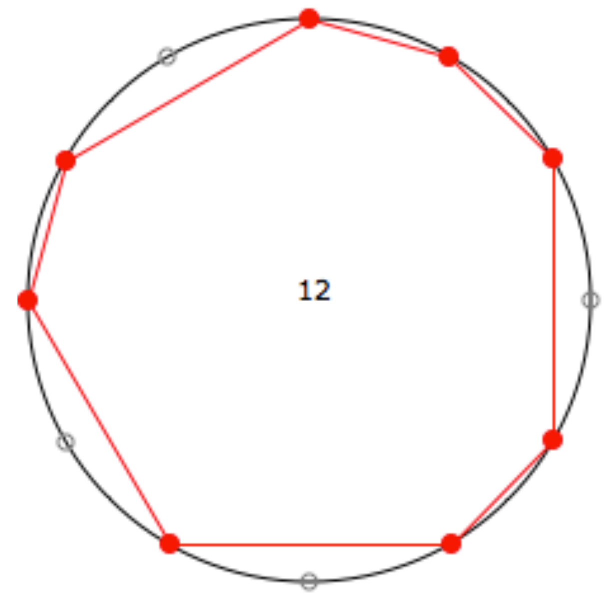
Clapping Music de Steve Reich (1972)

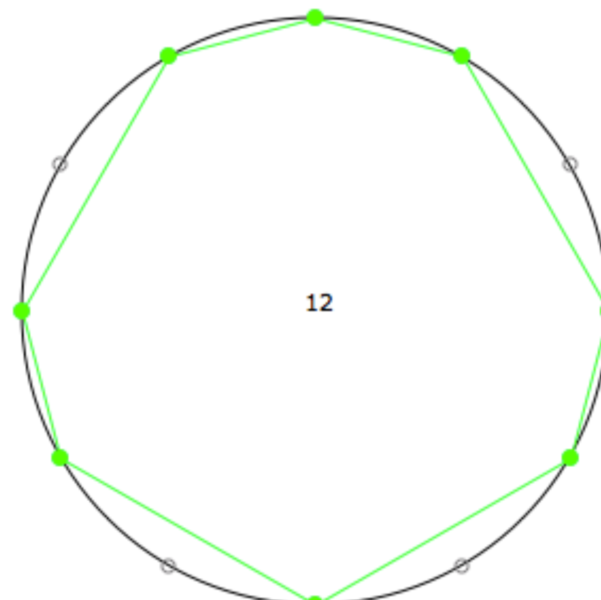


The circle and its ‘canonic’ rotations



Clapping Music (1972)





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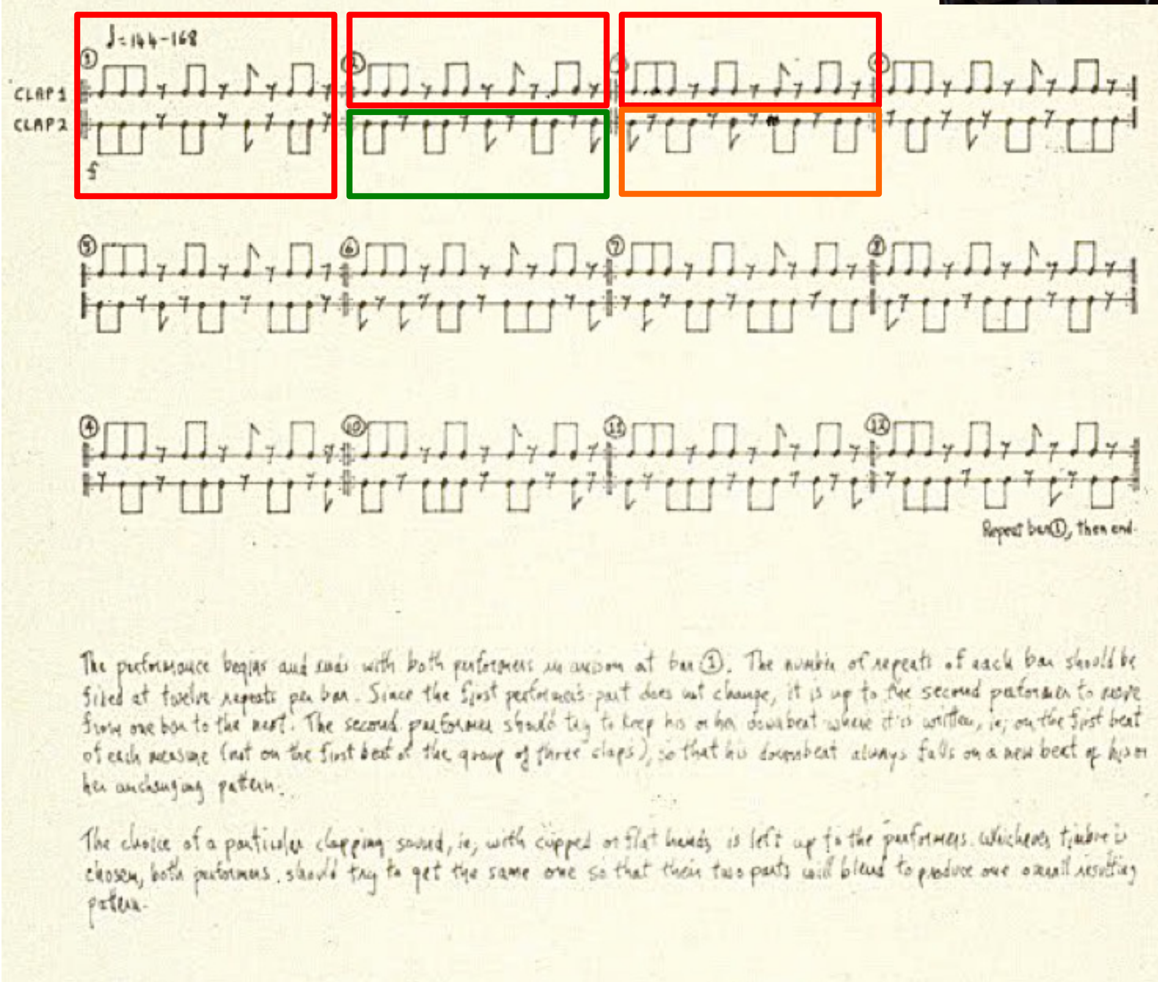

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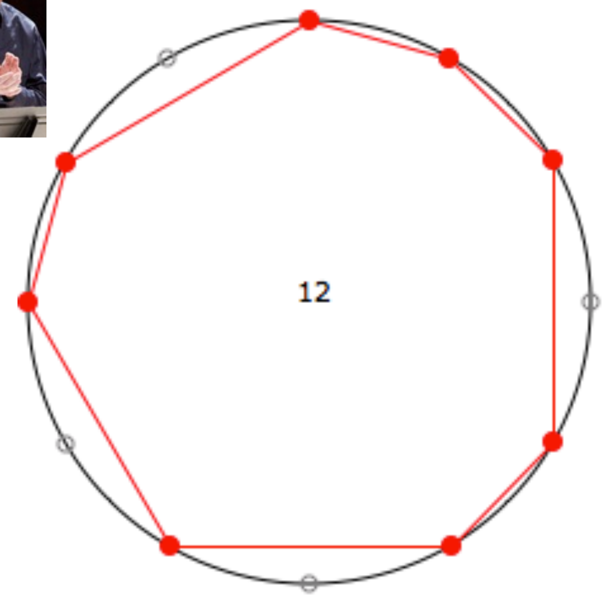
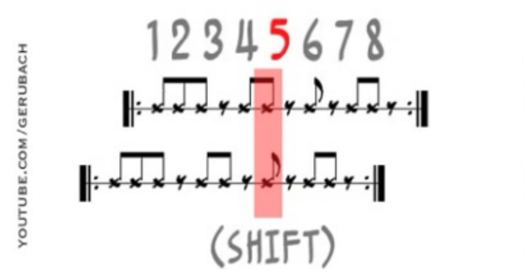
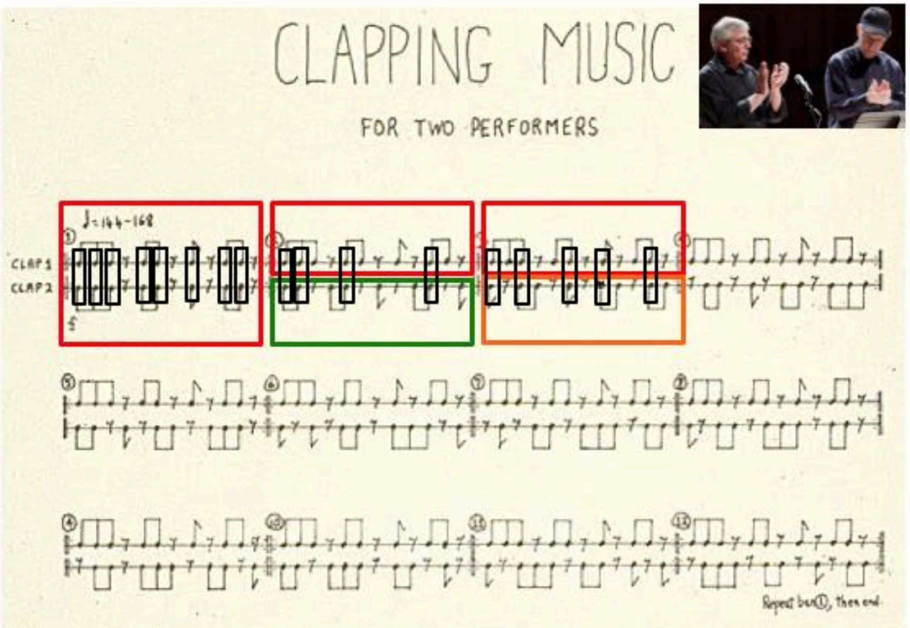


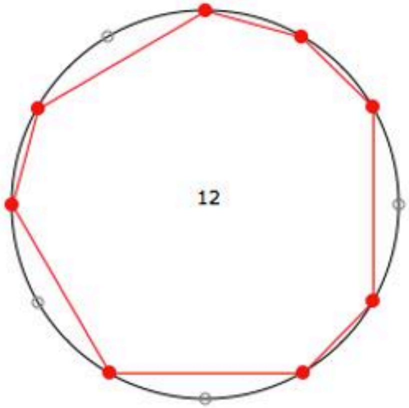


Gerubach's Scrolling Score Project
http://www.gerubach.com

FpA 2022-2023 – Un exemple de devoir maison

En vous appuyant sur les outils présentés pendant le cours – ainsi que sur les slides additionnels – discuter le problème musical suivant. Dans la pièce *Clapping Music* de Steve Reich, les deux « clappeurs » commencent à l'unisson. Autrement dit, au début de la pièce, tous les points d'attaque entre les deux musiciens coïncident. Dans la deuxième mesure, les deux « clappeurs » n'ont que quatre instants où ils jouent à l'unisson, c'est-à-dire quatre points d'attaque qui coincïdent. Ces points d'attaque sont cinq dans la troisième mesure, etc. (cf. figure ci-dessous).





1. Discuter le comportement du nombre d'unissons tout au long des douze mesures de la pièce
2. Toujours en gardant la périodicité du pattern rythmique égale à 12, appliquer le même procédé compositionnel aux deux rythmes euclidiens (ou ME-sets ou ensembles « bien repartis ») de cardinalité respectivement 7 et 5 et caractériser le nombre d'unissons dans les deux nouvelles pièces.
3. Quel lien y a-t-il entre le nombre d'unissons des deux nouvelles pièces ?
4. (Optionnel) En utilisant un logiciel de notation (Musescore, Finale, Sibelius ou autres logiciels à votre gré), produire les partitions des deux nouvelles variantes de la composition originale. Ca sera votre double composition (à jouer au plaisir entre deux « clappeurs »… ;-)

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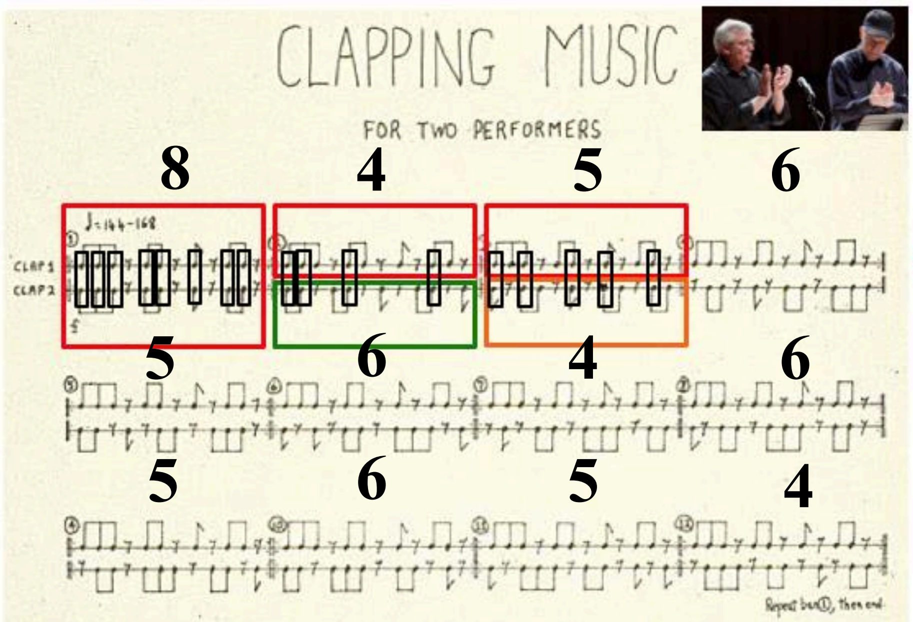

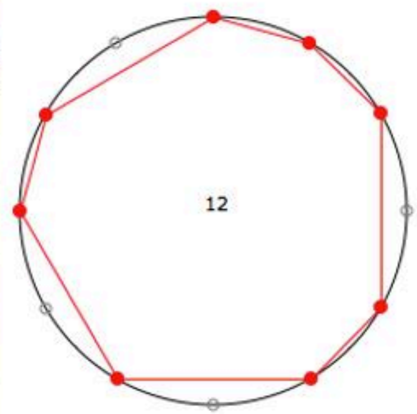

1. Discuter le comportement du nombre d'unissons tout au long des douze mesures de la pièce
2. Toujours en gardant la périodicité du pattern rythmique égale à 12, appliquer le même procédé compositionnel aux deux rythmes euclidiens (ou ME-sets ou ensembles « bien repartis ») de cardinalité respectivement 7 et 5 et caractériser le nombre d'unissons dans les deux nouvelles pièces.
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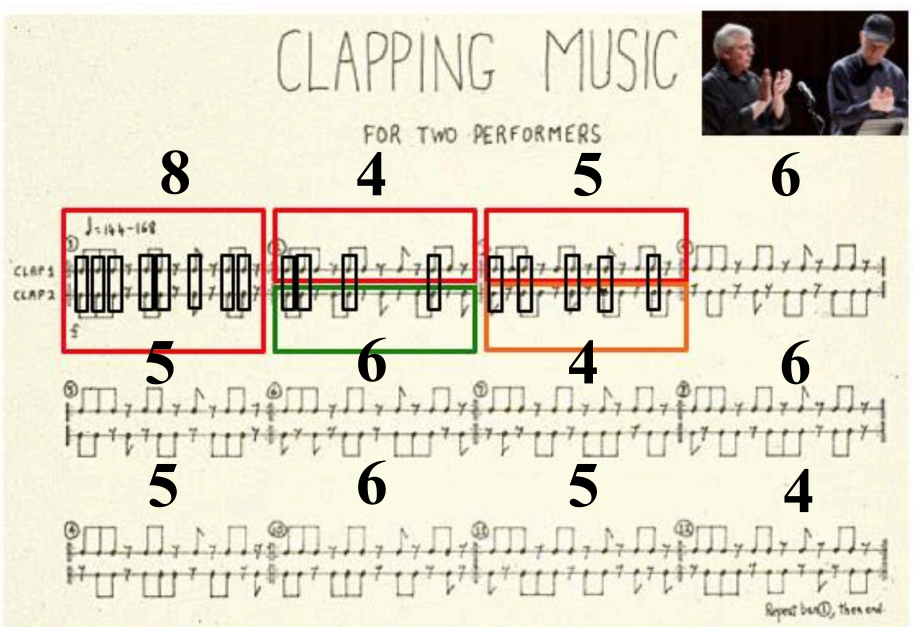

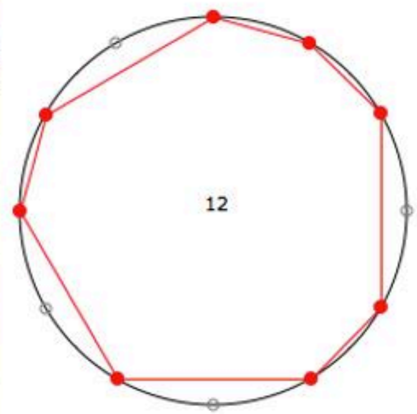

[8,4,5,6,5,6,4,6,5,6,5,4]

1. Discuter le comportement du nombre d'unissons tout au long des douze mesures de la pièce
2. Toujours en gardant la périodicité du pattern rythmique égale à 12, appliquer le même procédé compositionnel aux deux rythmes euclidiens (ou ME-sets ou ensembles « bien repartis ») de cardinalité respectivement 7 et 5 et caractériser le nombre d'unissons dans les deux nouvelles pièces.
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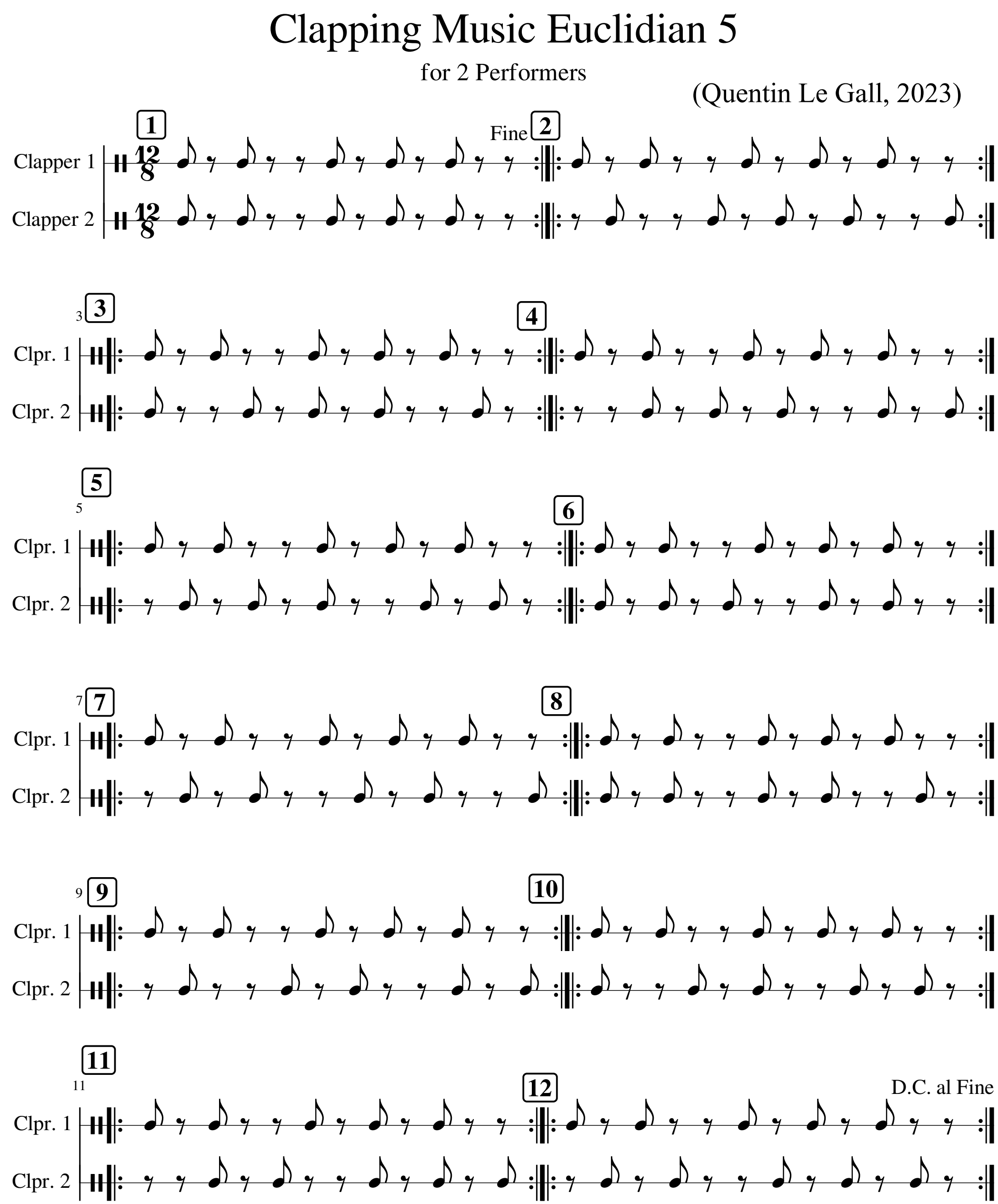
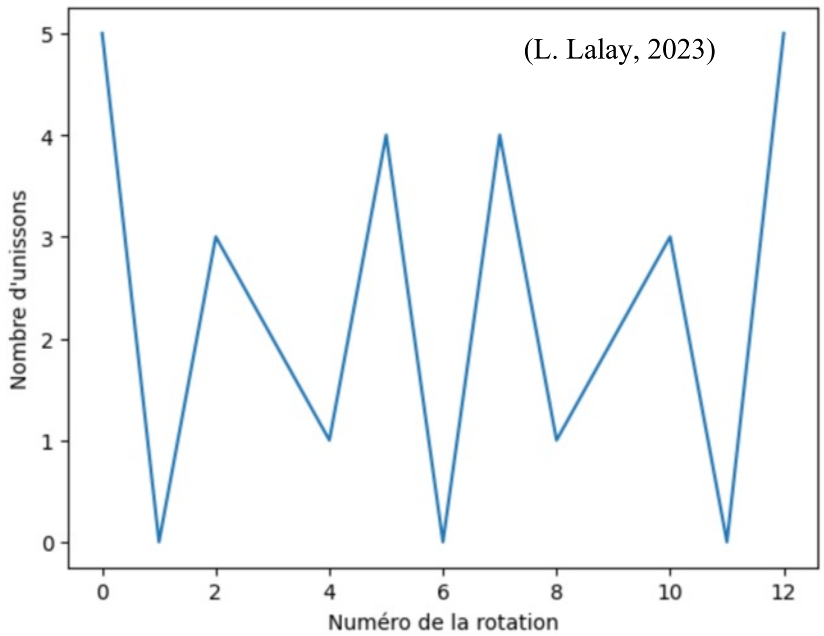
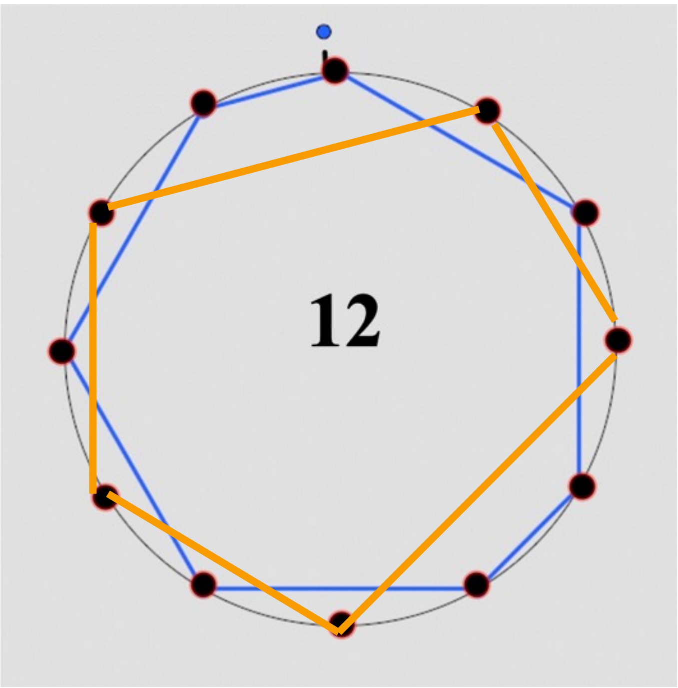
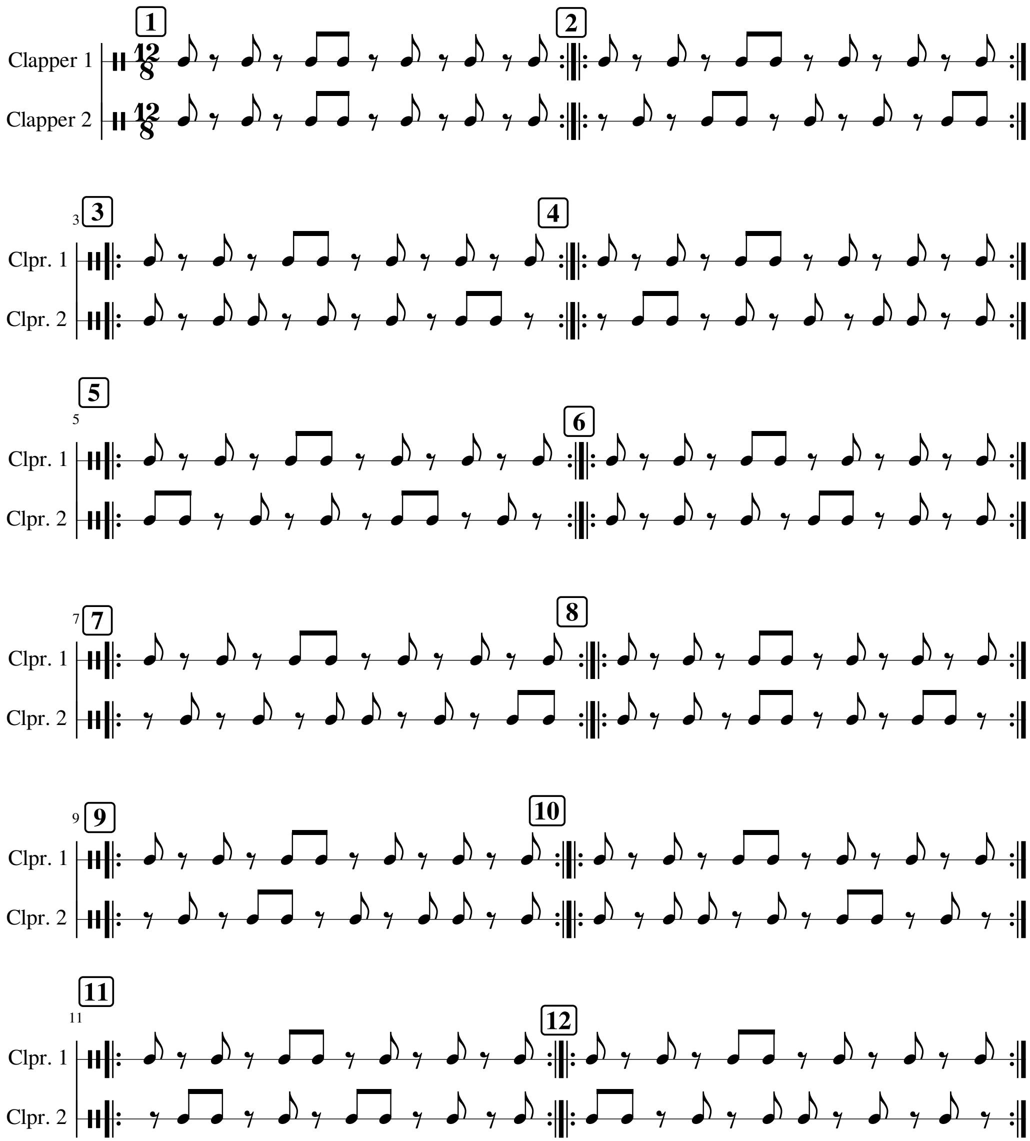

Clapping Music Euclidian 5
for 2 Performers
(Quentin Le Gall, 2023)
Clapper 1
Clapper 2
Fine
Clpr. 1
Clpr. 2
D.C. al Fine

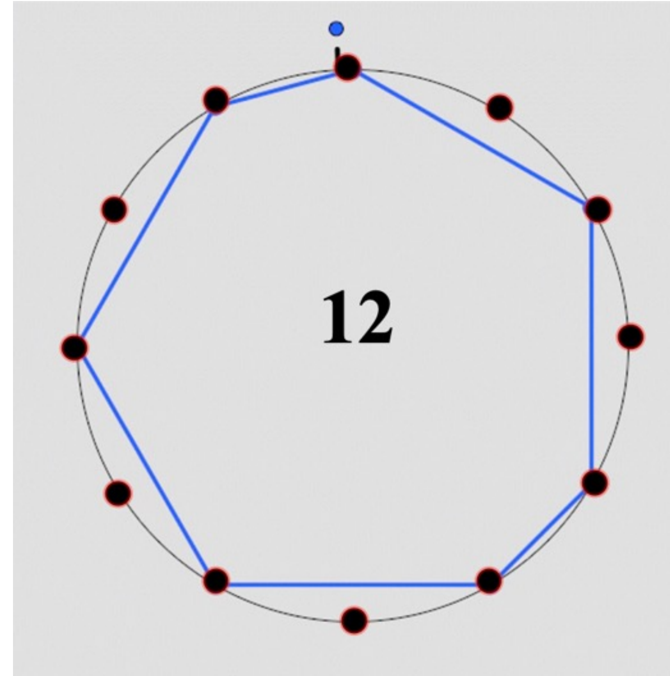

(L. Lalay, 2023)
Nombre d'unissons
Numéro de la rotation

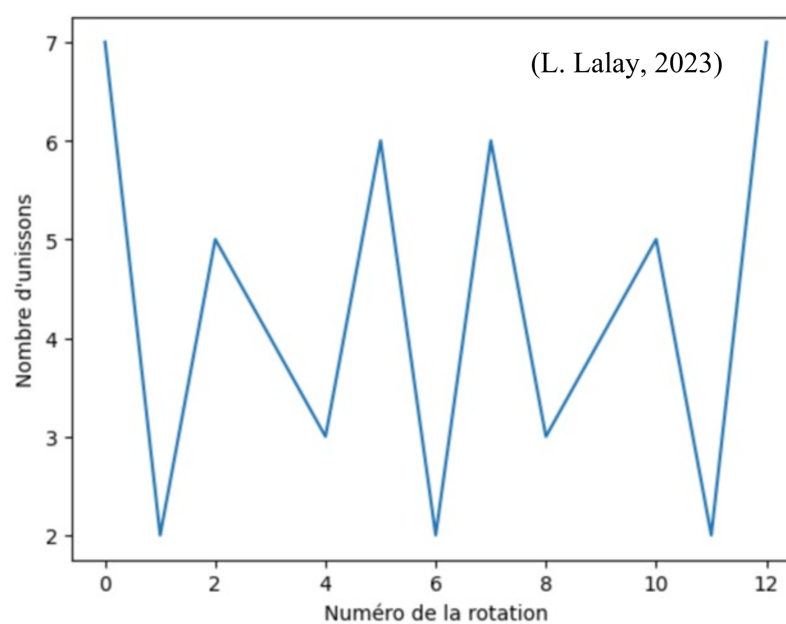

12

Clapping Music sur le rythme Euclidien (E,7,12)

(Pierre Chouteau, 2023)

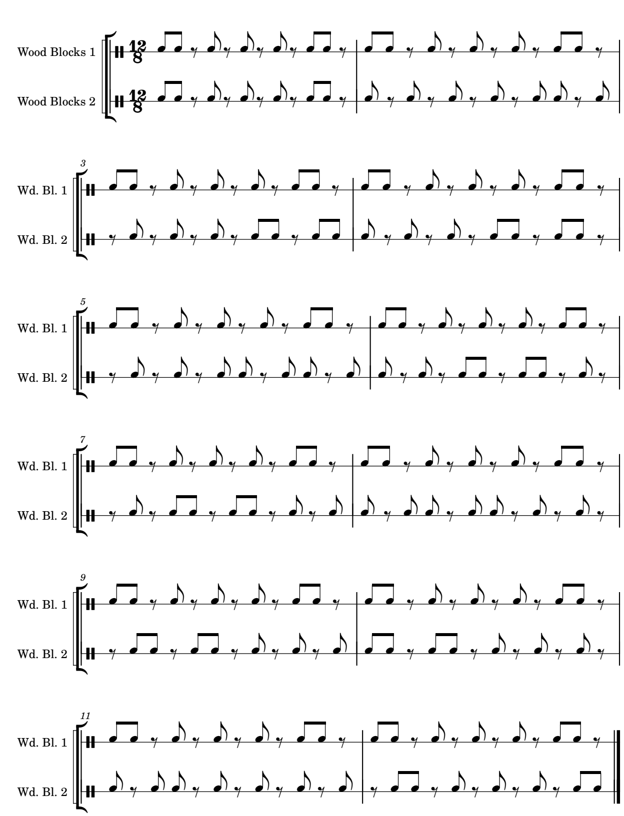


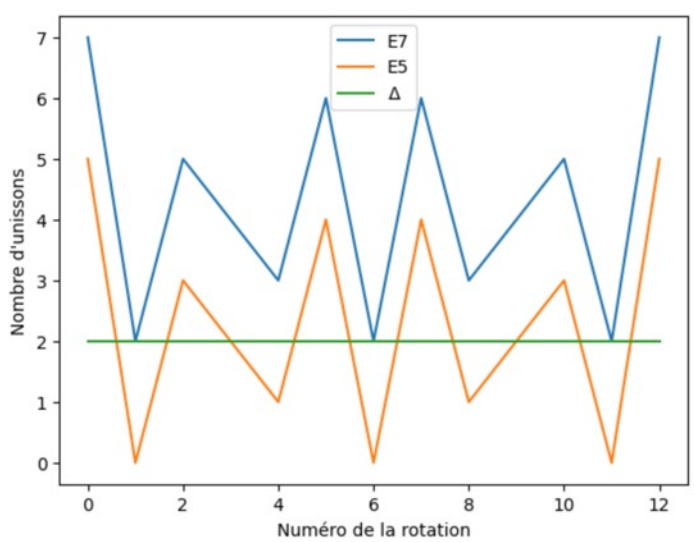




Clapping music à 5 temps

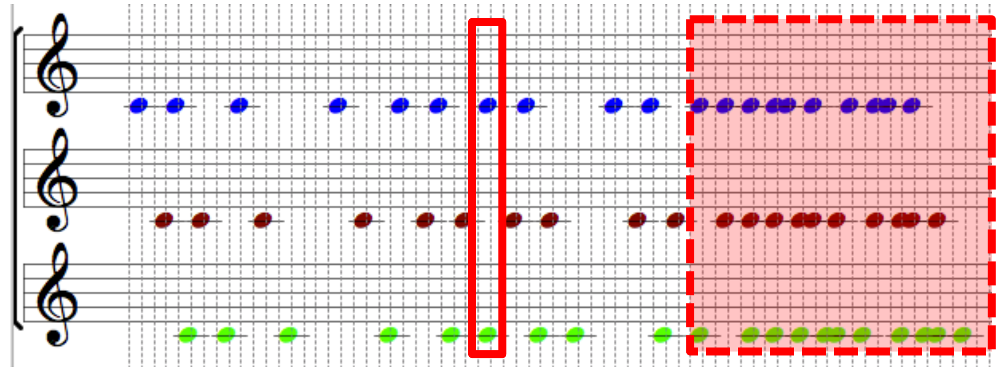
Clapping music à 7 temps



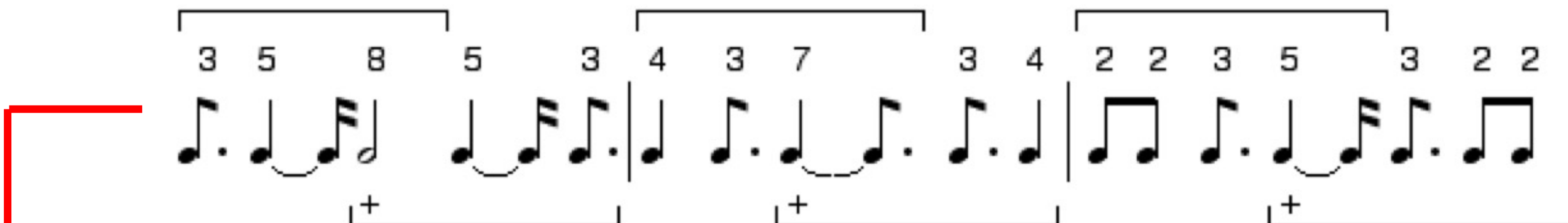


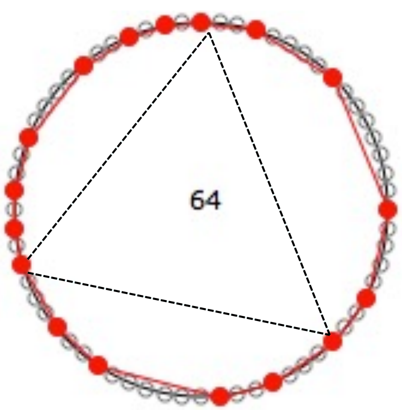
Periodic rhythmic sequences and tiling canons

Harawi (1945)



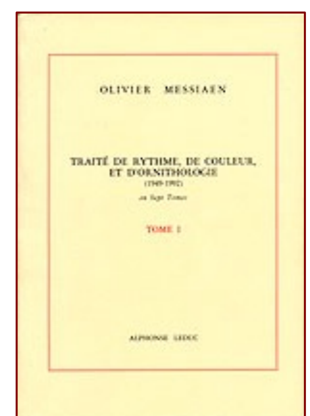
Harawi: rhythmic reduction

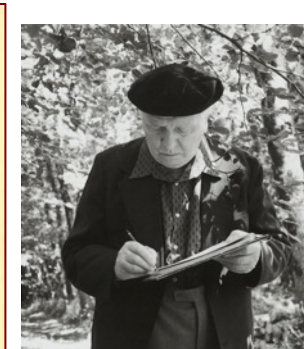




« …il résulte de tout cela que les différentes sonorités se mélangent ou s'opposent de manières très diverses, jamais au même moment ni au même endroit […]. C'est du désordre organisé »

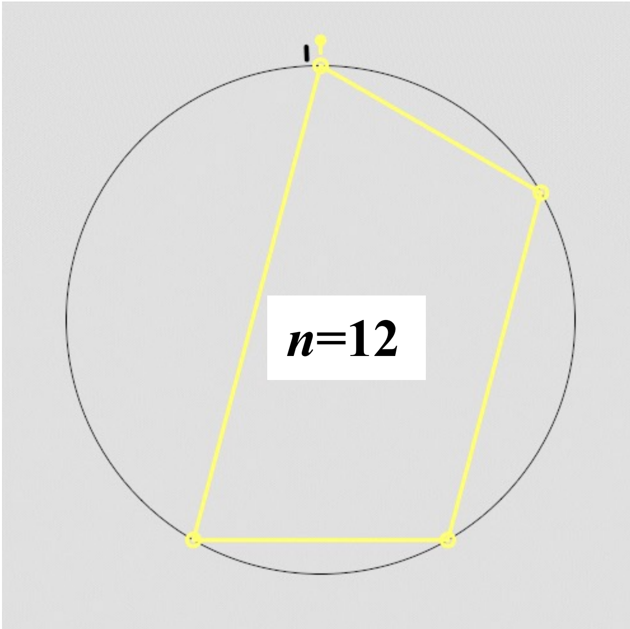
O. Messiaen: *Traité de Rythme, de Couleur et d'Ornithologie,* tome 2, Alphonse Leduc, 1992.

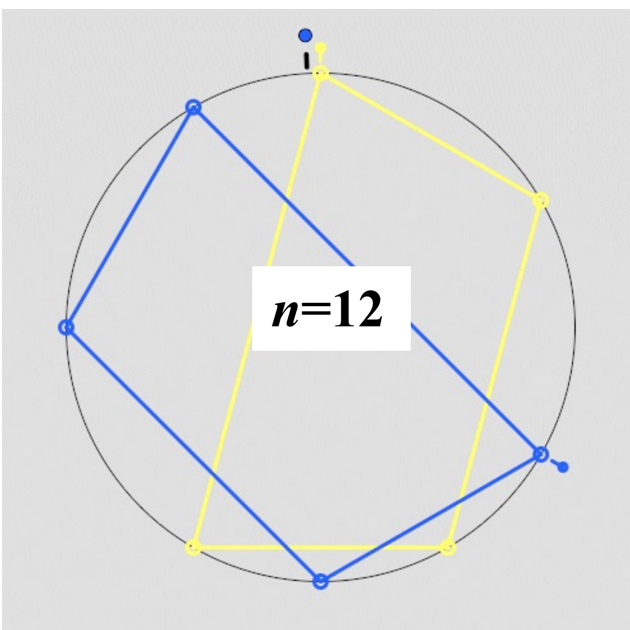


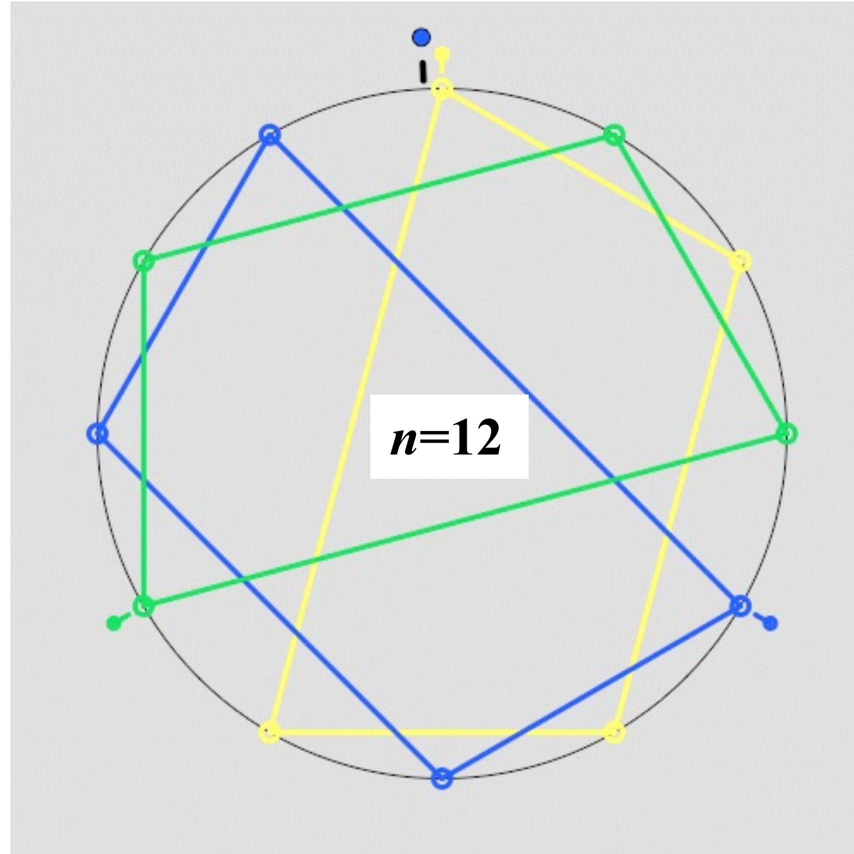


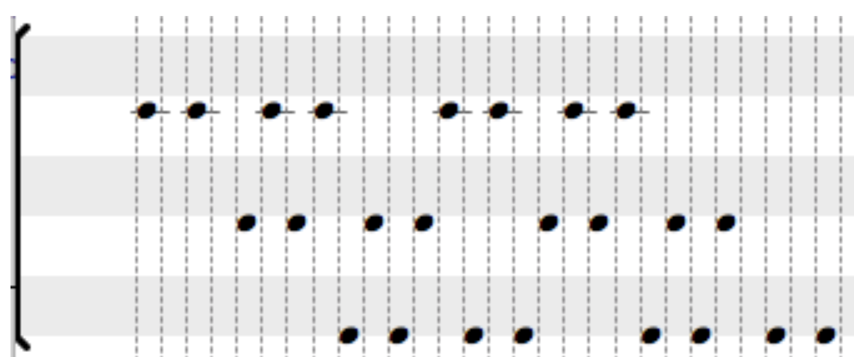
Olivier Messiaen

Tiling the time axis with translates of one tile

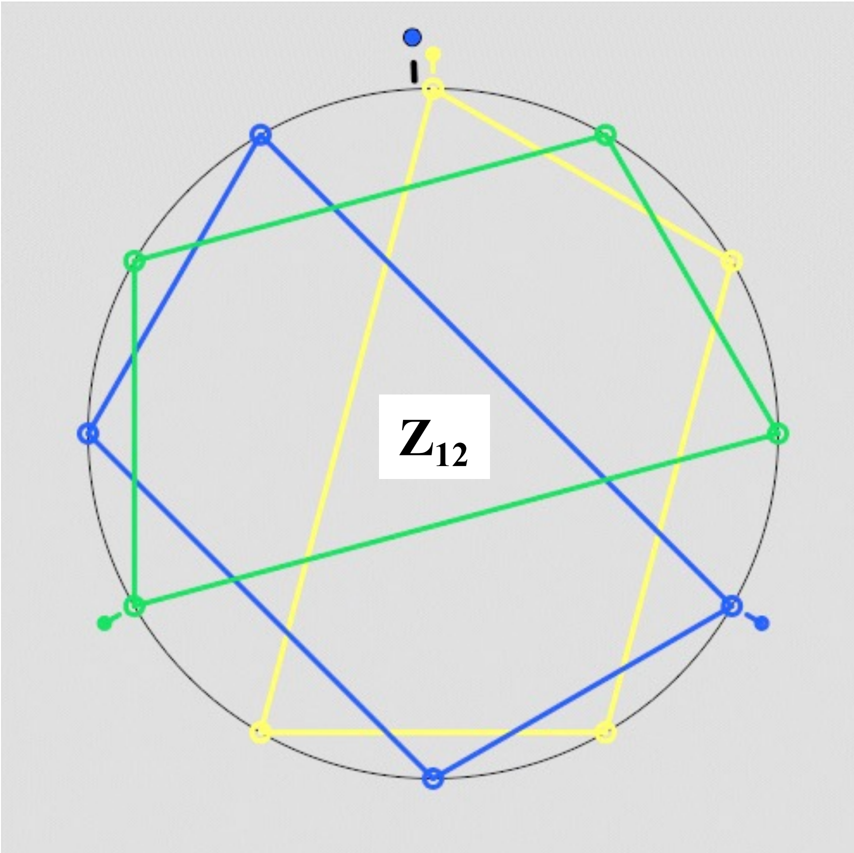


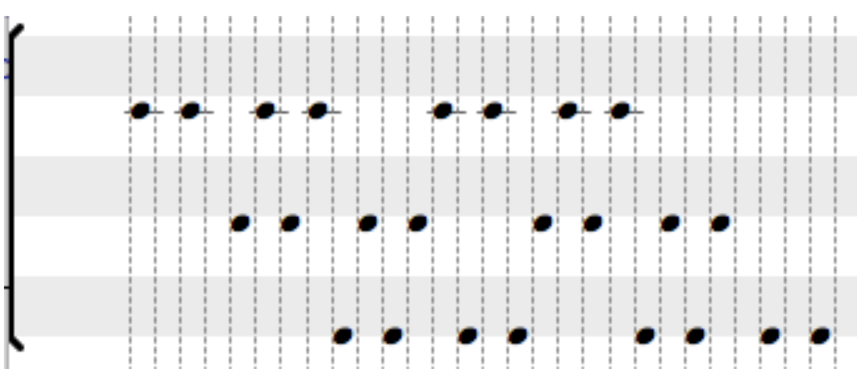






Formalizing the tiling process as set-theoretical operations





$A_1 = \{0, 2, 5, 7\}$

$\downarrow T_4$ $\quad$ $T_k : x \rightarrow x + k \bmod 12$

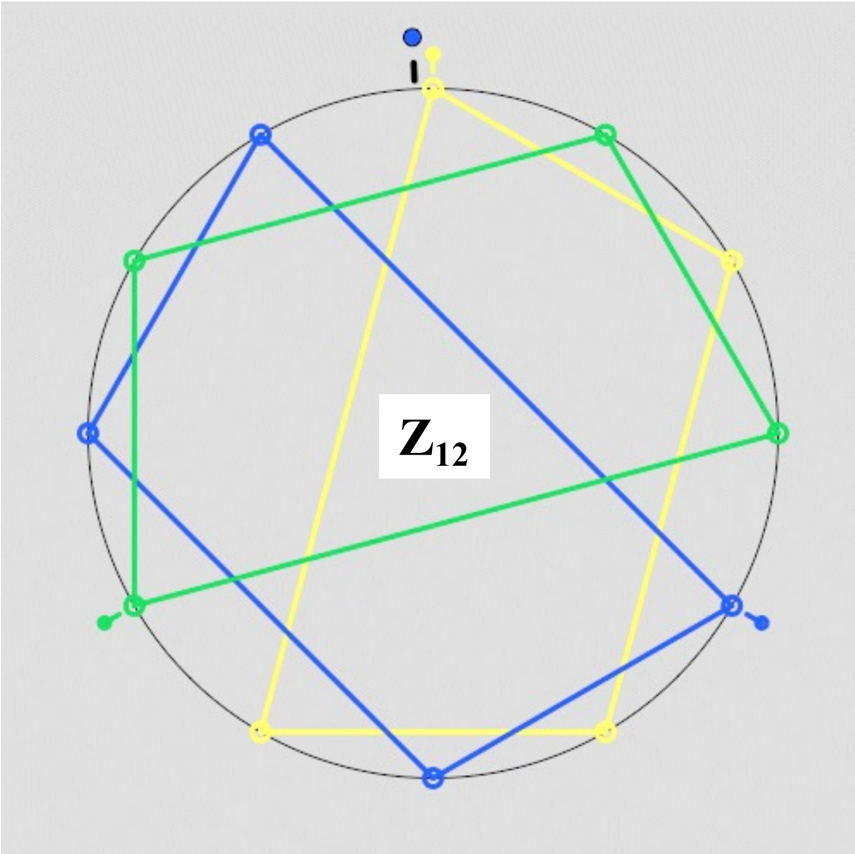
$A_2 = \{4, 6, 9, 11\}$

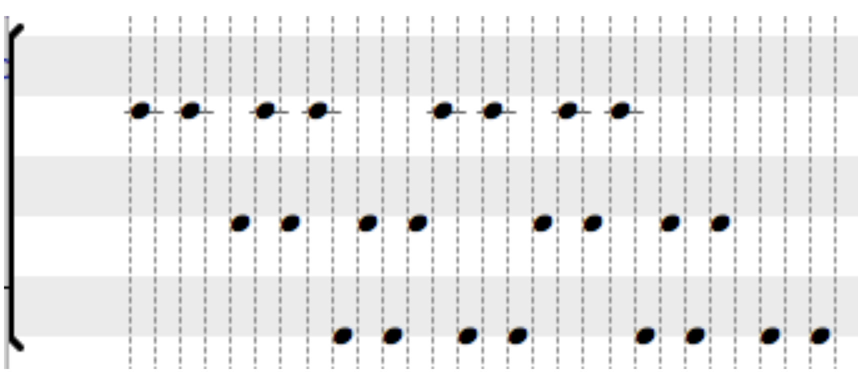
$\downarrow T_4$

$A_3 = \{8, 10, 1, 3\}$

$Z_{12} = A_1 \cup A_2 \cup A_3$

Formalizing the tiling process as a direct sum of subsets





$A_1 = \{0, 2, 5, 7\}$

$\downarrow T_4$

$A_2 = \{4, 6, 9, 11\}$

$\downarrow T_4$

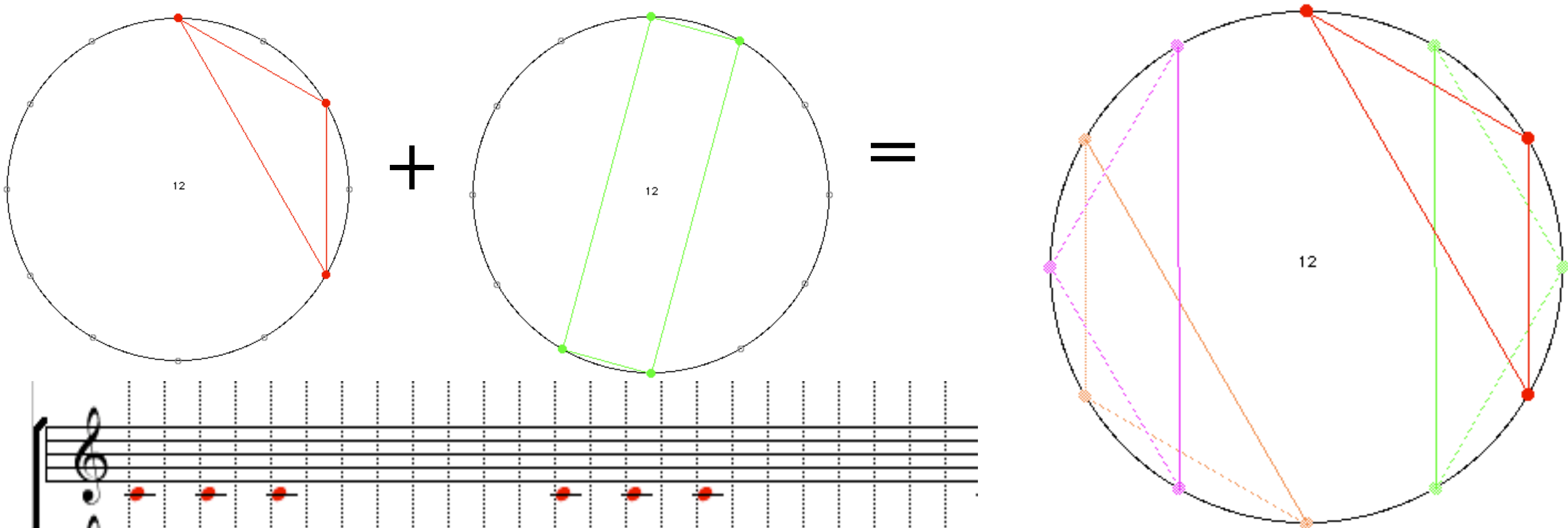
$A_3 = \{8, 10, 1, 3\}$

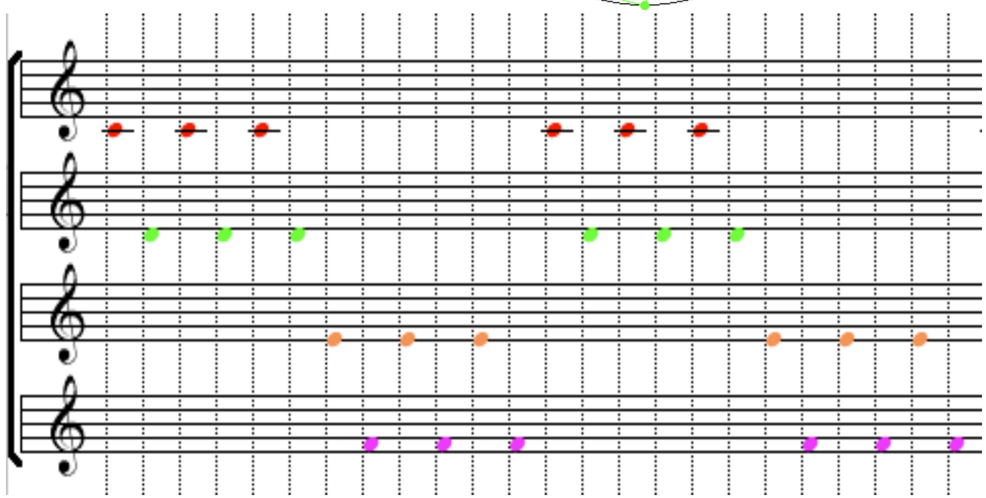
$Z_{12} = A_1 \cup A_2 \cup A_3$

$Z_{12} = A \oplus B$

$A = \{0,2,5,7\}$
$B = \{0,4,8\}$

Rhythmic tiling canons with no regular entries



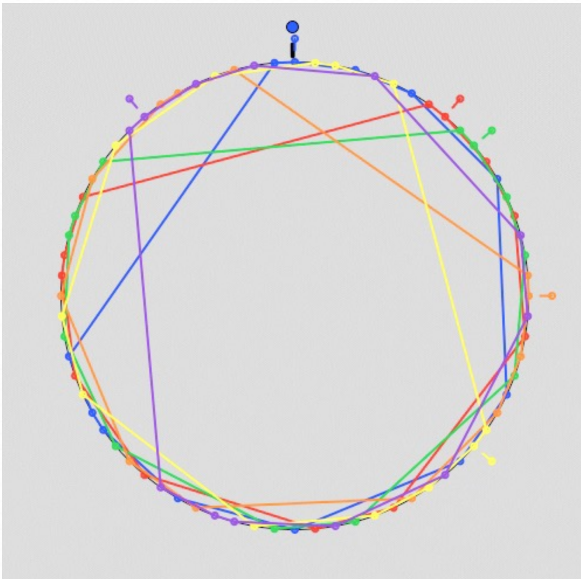


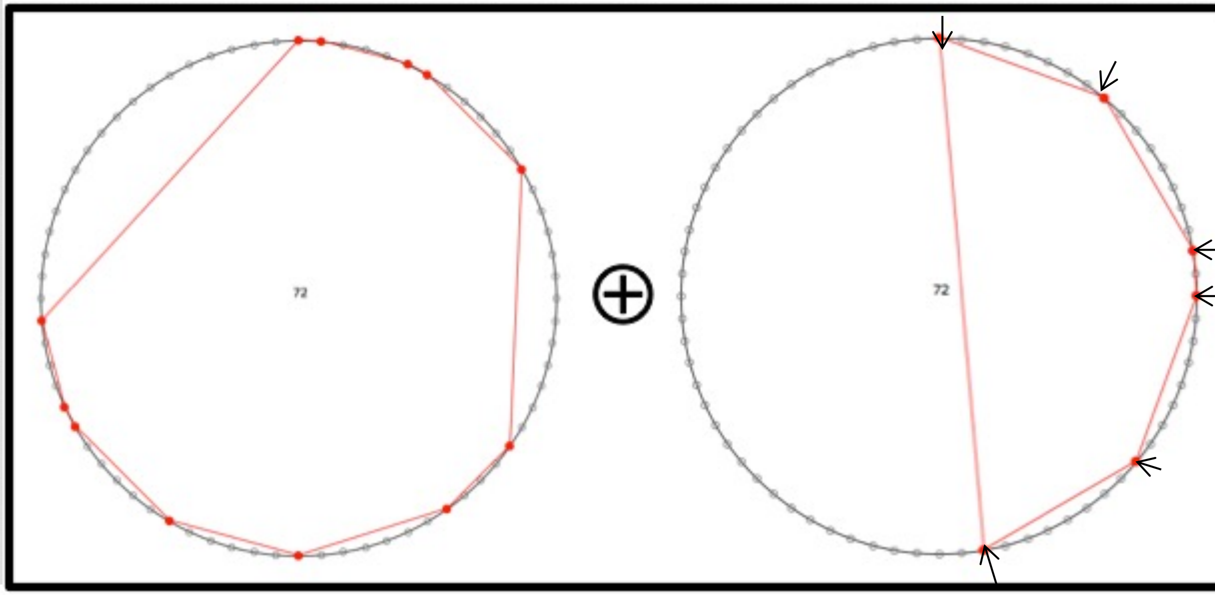
transpositional combination

$$\{0,2,4\} \oplus \{0,1,6,7\} = \mathbb{Z}_{12} = (2\ 2\ 8) \bullet (1\ 5\ 1\ 5)$$

One of the two factors is a Messiaen's mode of limited transposition

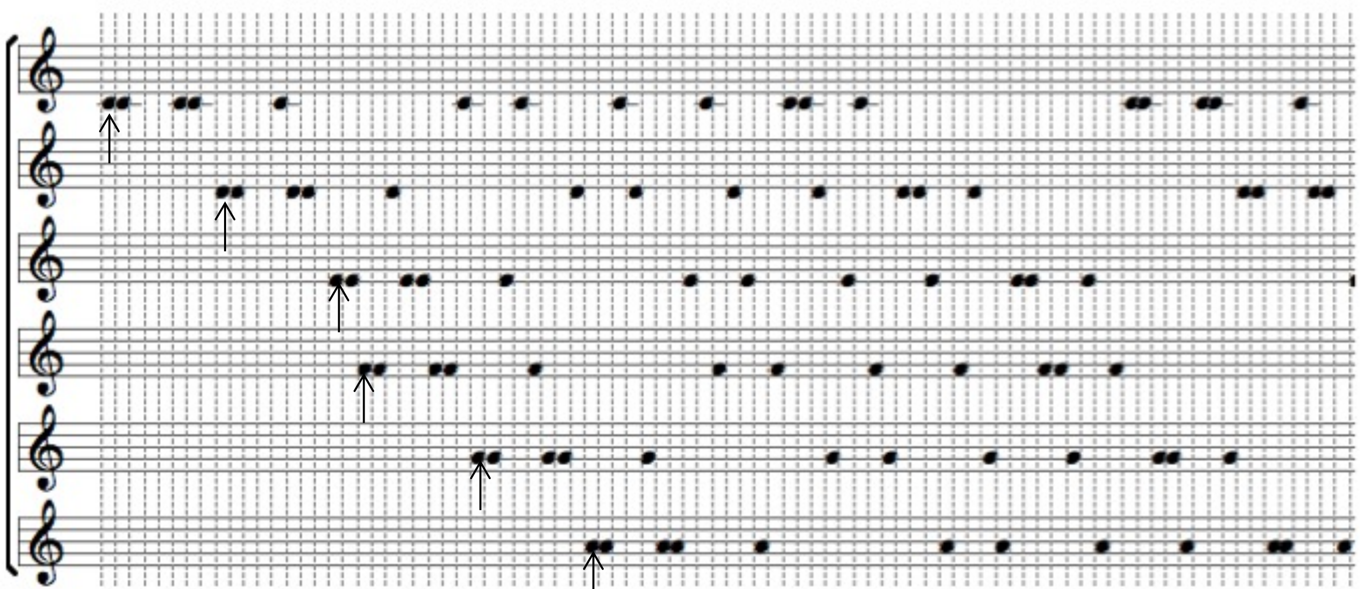
Aperiodic Rhythmic Tiling Canons (Vuza Canons)



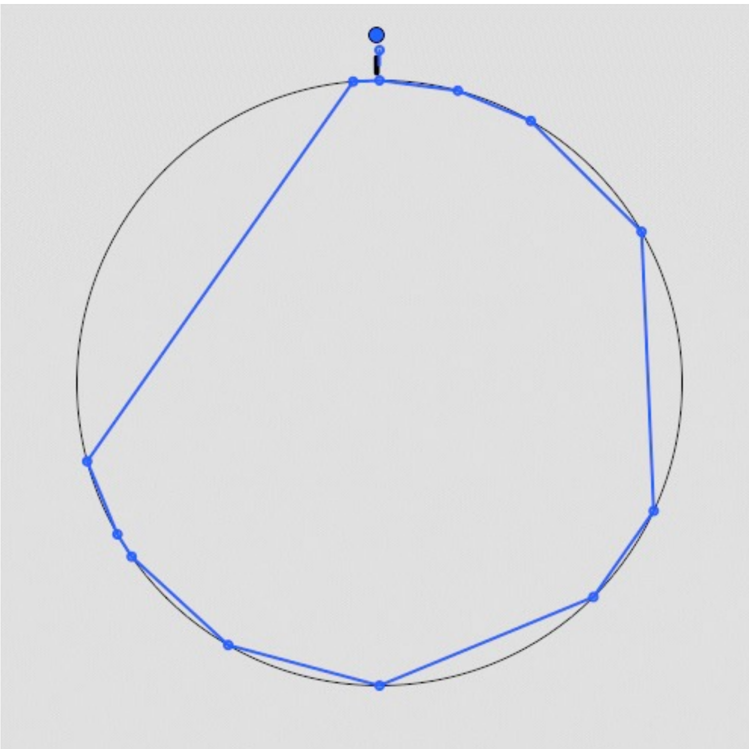


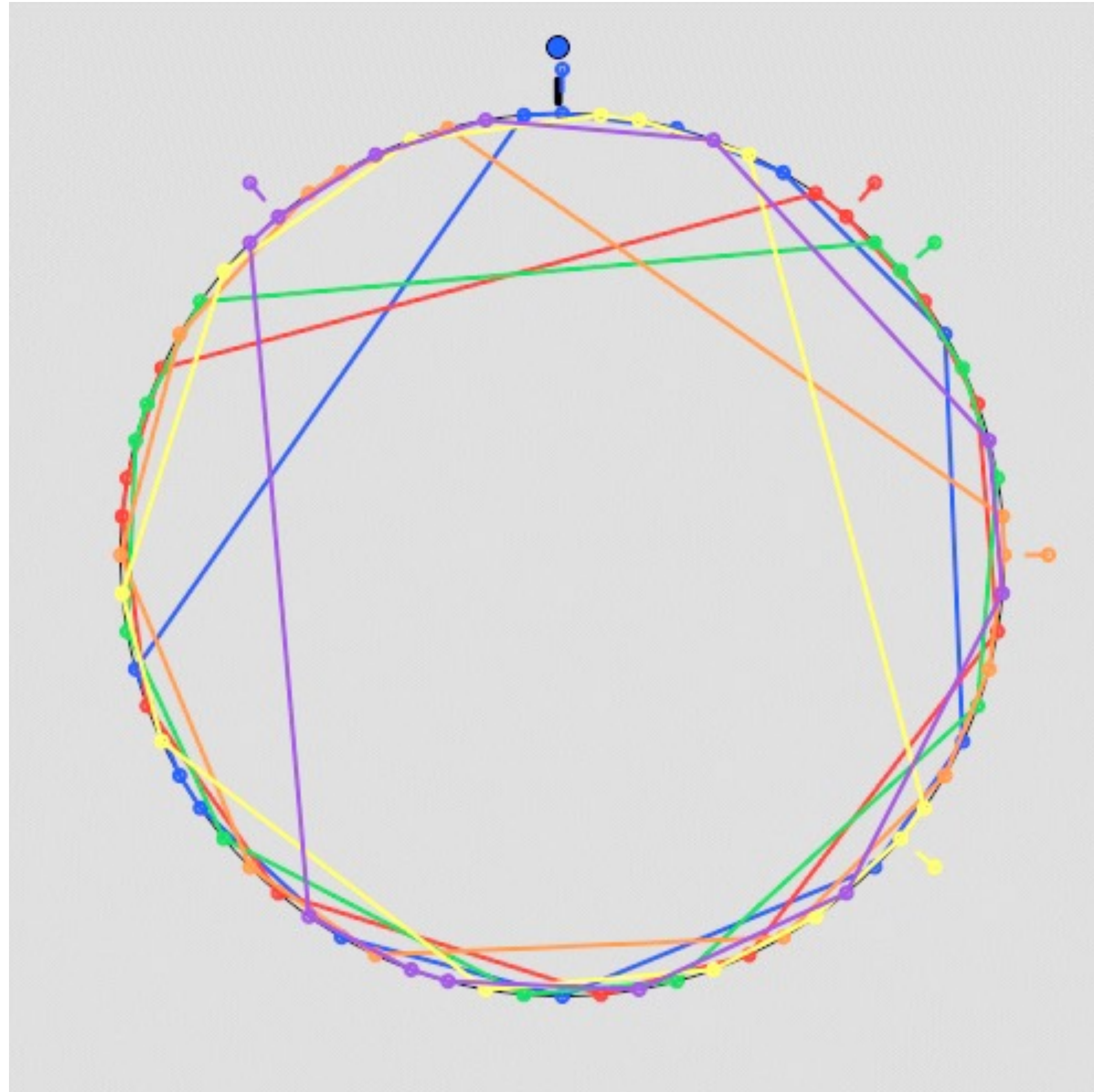

Dan Vuza


Anatol Vieru

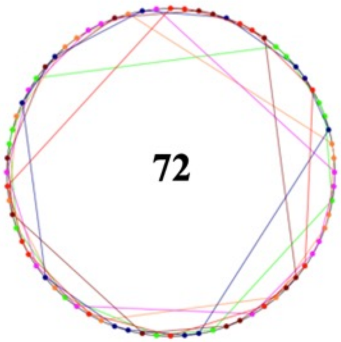


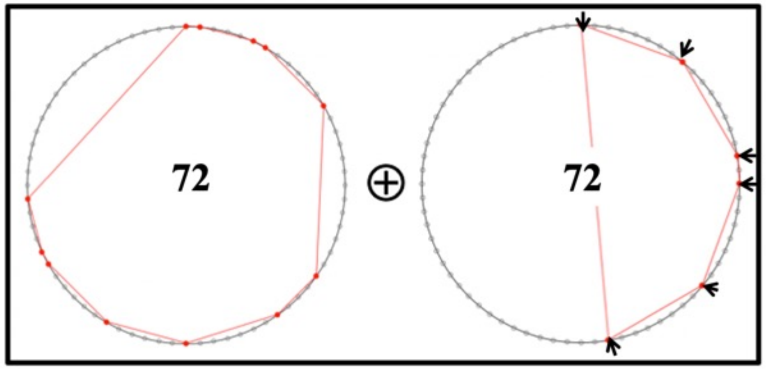
The melody of a Vuza Canon

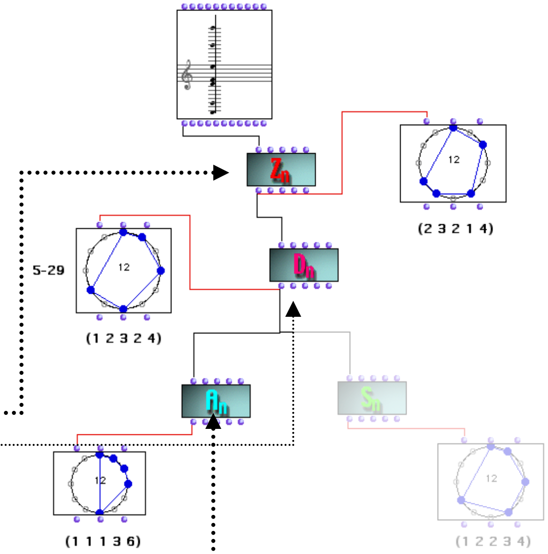




Towards the complete classification of Vuza Canons







Z_n

R

(1 3 3 6 11 4 9 6 5 1 3 20)
(20 3 1 5 6 9 4 11 6 3 3 1)
(1 4 1 19 4 1 6 6 7 4 13 6)
(6 13 4 7 6 6 1 4 19 1 4 1)
(1 5 15 4 5 6 6 3 4 17 3 3)
(3 3 17 4 3 6 6 5 4 15 5 1)

S

(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

D_n

R

(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)
(1 5 15 4 5 6 6 3 4 17 3 3)

S

(8 8 2 8 8 38)
(16 2 14 2 16 22)
(14 8 10 8 14 18)

A_n

R

(1 3 3 6 11 4 9 6 5 1 3 20)
(1 4 1 19 4 1 6 6 7 4 13 6)

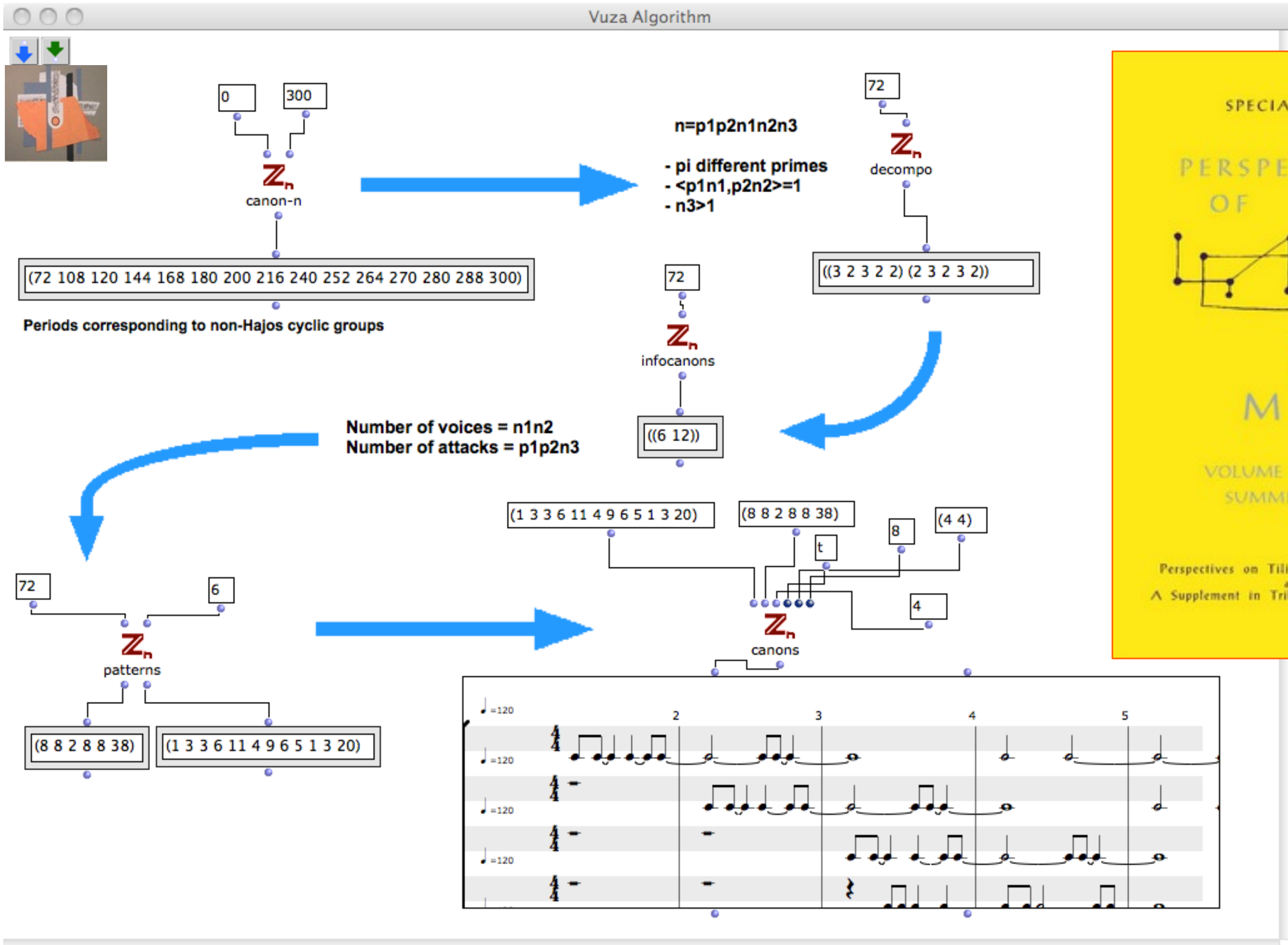
S

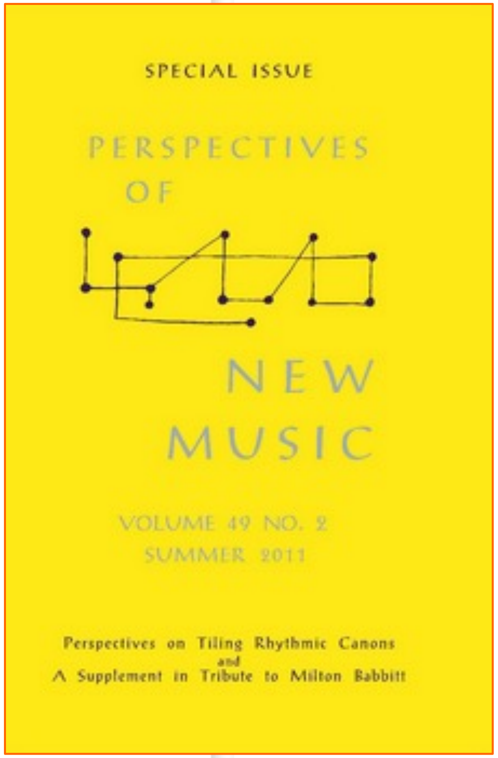
(8 8 2 8 8 38)

There are only two « types » of Vuza Canons of period 72 (up to an affine transformation)

(Agon & Andreatta 1998), Amiot, Jedrzejewski, Fripertinger, Lanzarotto & Pernazza 2022), …)

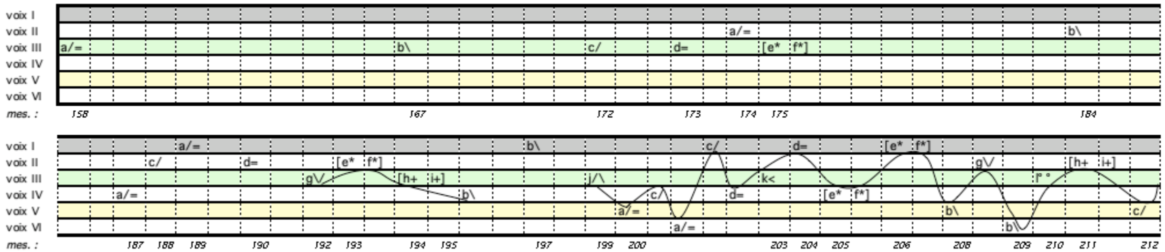
Exploring Vuza Canons in OpenMusic

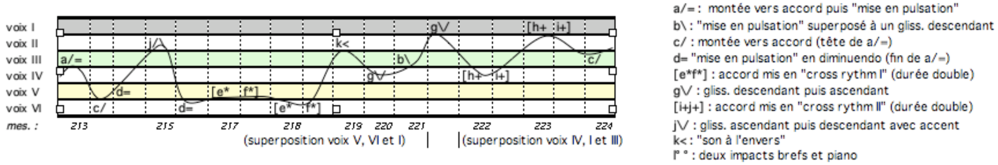




- Agon C. et M. Andreatta (2011), “Modelling and Implementing Tiling Rhythmic Canons in OpenMusic Visual Programming Language”, *Perspectives of New Music*, Special Issue, vol. 1-2, n° 49, p. 66-91.

Some compositional applications of the Vuza Canons model





Coïncidences (1999)

Coïncidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)

F. Lévy

M. Lanza

La bataille de caresme et de charnage **(2012)**
(pour violoncelle et accompagnement)

A piece based on Monk **(2007)**
(« Well You Need'nt »)

G. Bloch

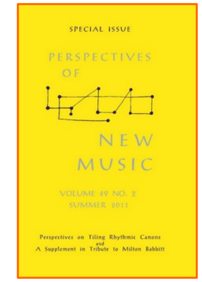
La notte poco prima della foresta **(2009)**
(opéra de chambre
pour acteur, mezzo-soprano, baryton, ensemble et électronique)

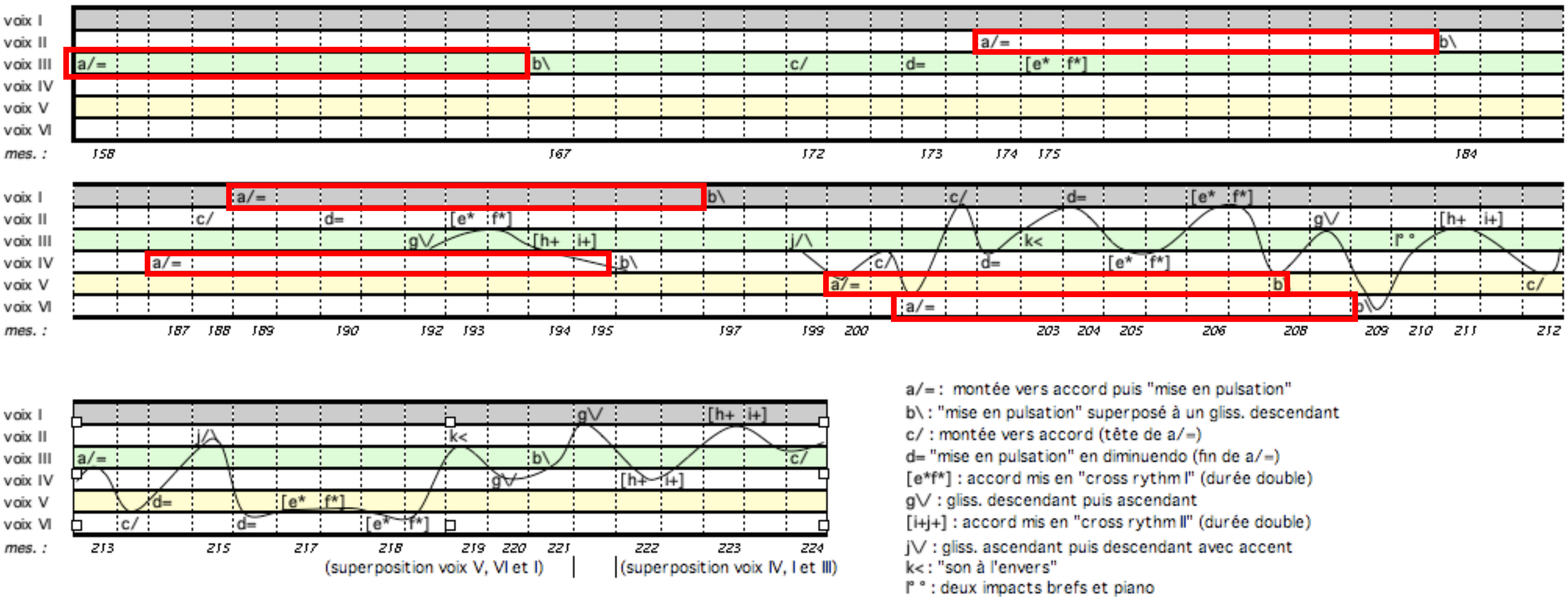
D. Ghisi

Fabien Lévy

Morphological Tiling Canons

- *Coïncidences* (pour 33 musiciens, 1999-2007)





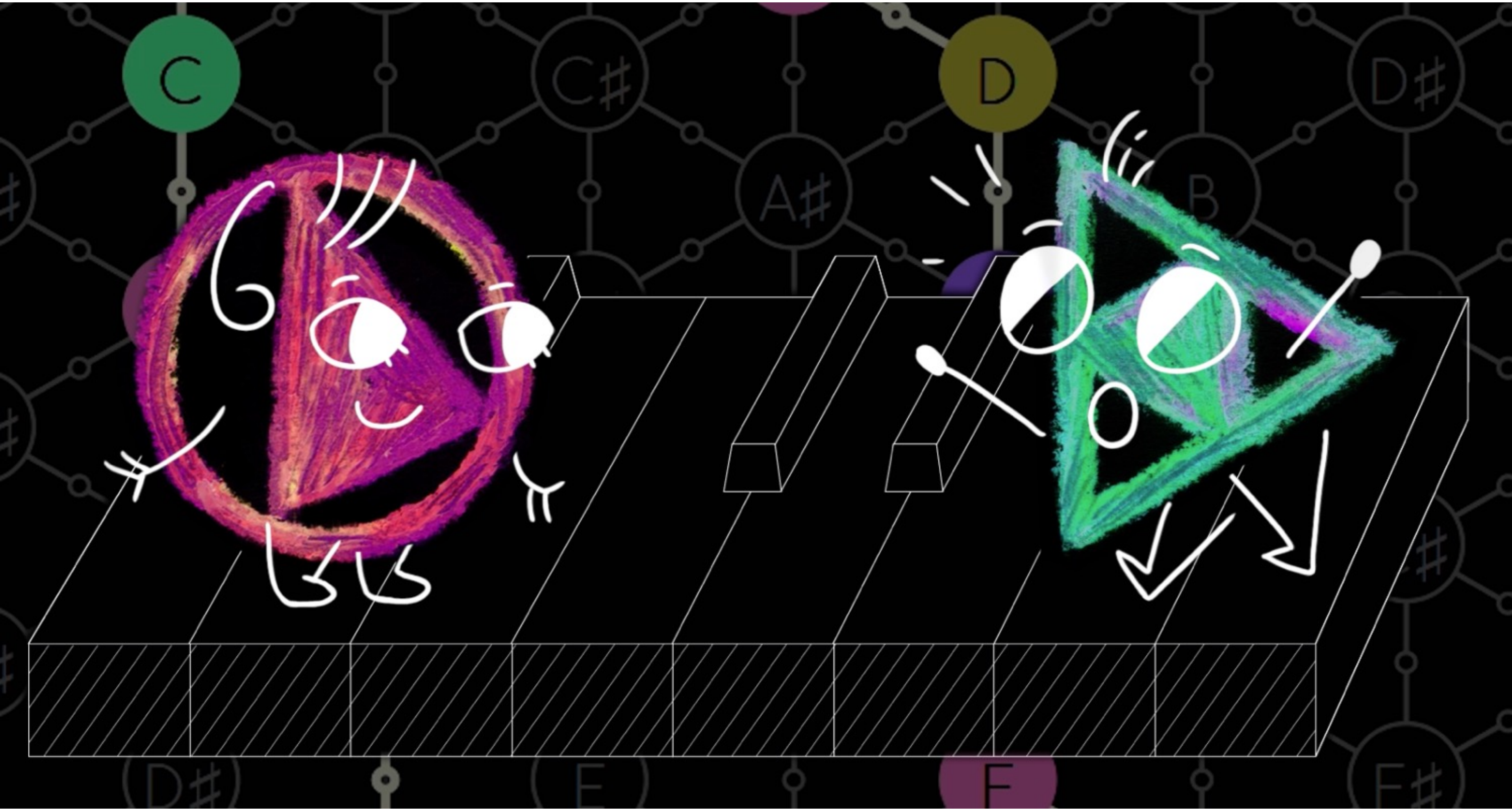
Coïncidences - Fabien Levy : déroulement du canon (mes. 158 à 226)
(chaque impact fait 3 temps)

Tokyo Symphony Orchestra, Dir.: Kazuyoshi Akiyama, 05/09/2007, Suntory Hall, Tokyo, Japon

F. Lévy, « Three uses of Vuza Canons », *Perspectives of New Music*, 49(2), 2011, p. 23-31.

Additional slides

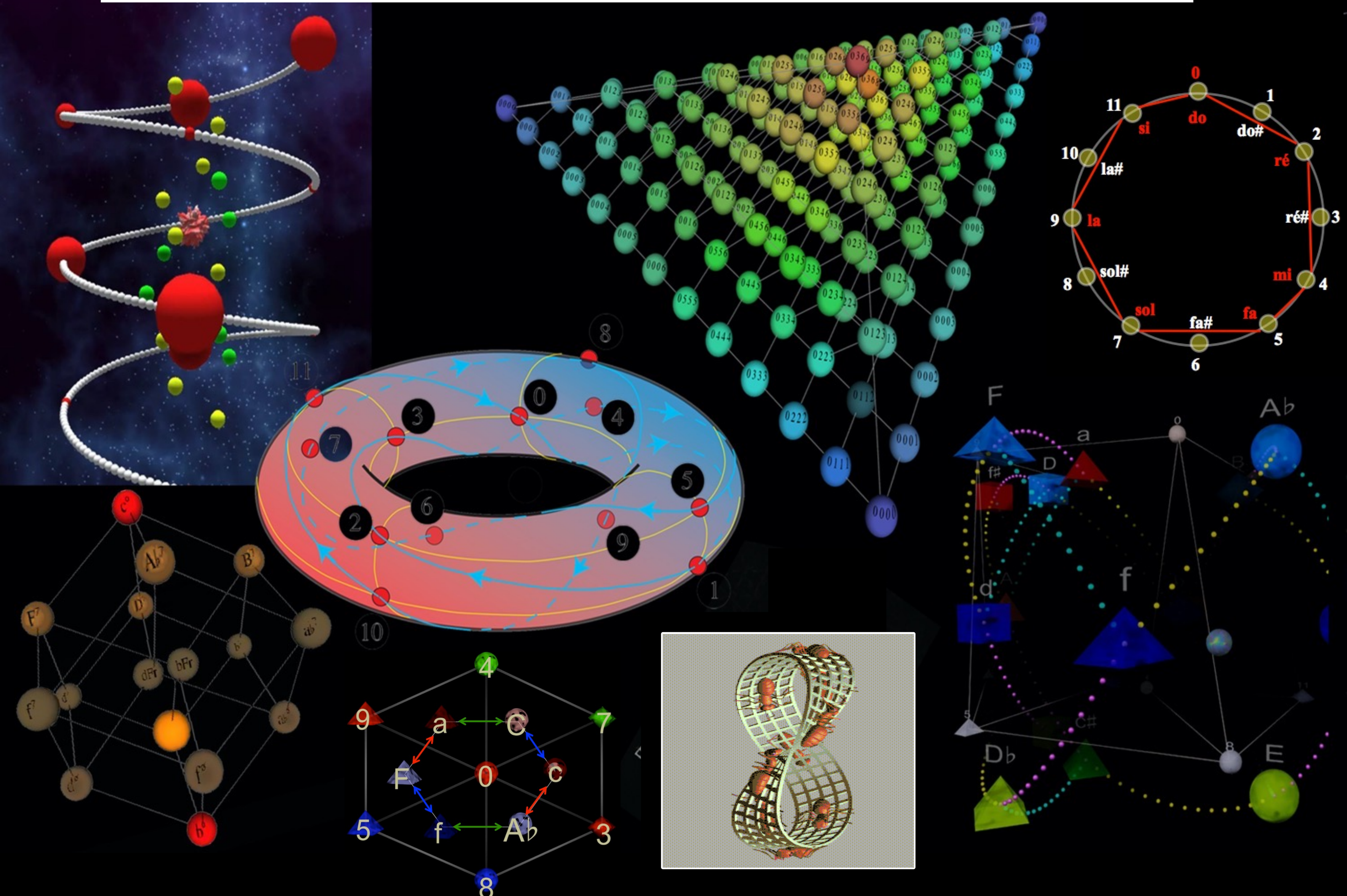
« Musique et mathématiques »: a pedagogical film



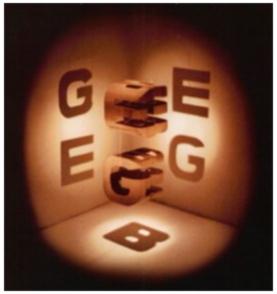
➔ www.morenoandreatta.com

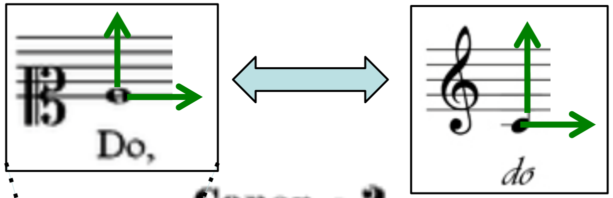
CNRS AuDiMATH AUTOUR DE LA DIFFUSION DES MATHÉMATIQUES

The galaxy of geometrical models at the service of music

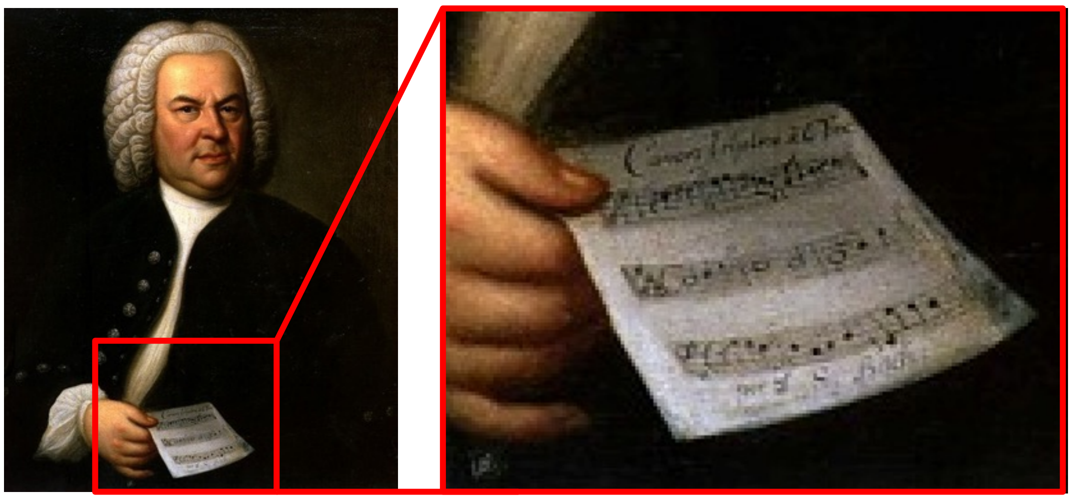


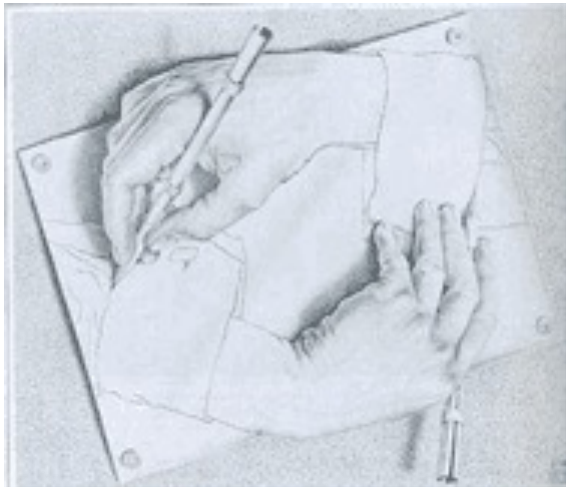
Bach’s enigmatic canons and geometry



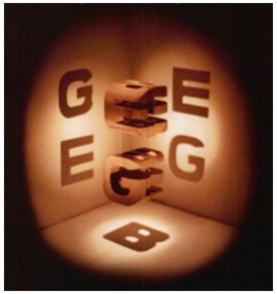


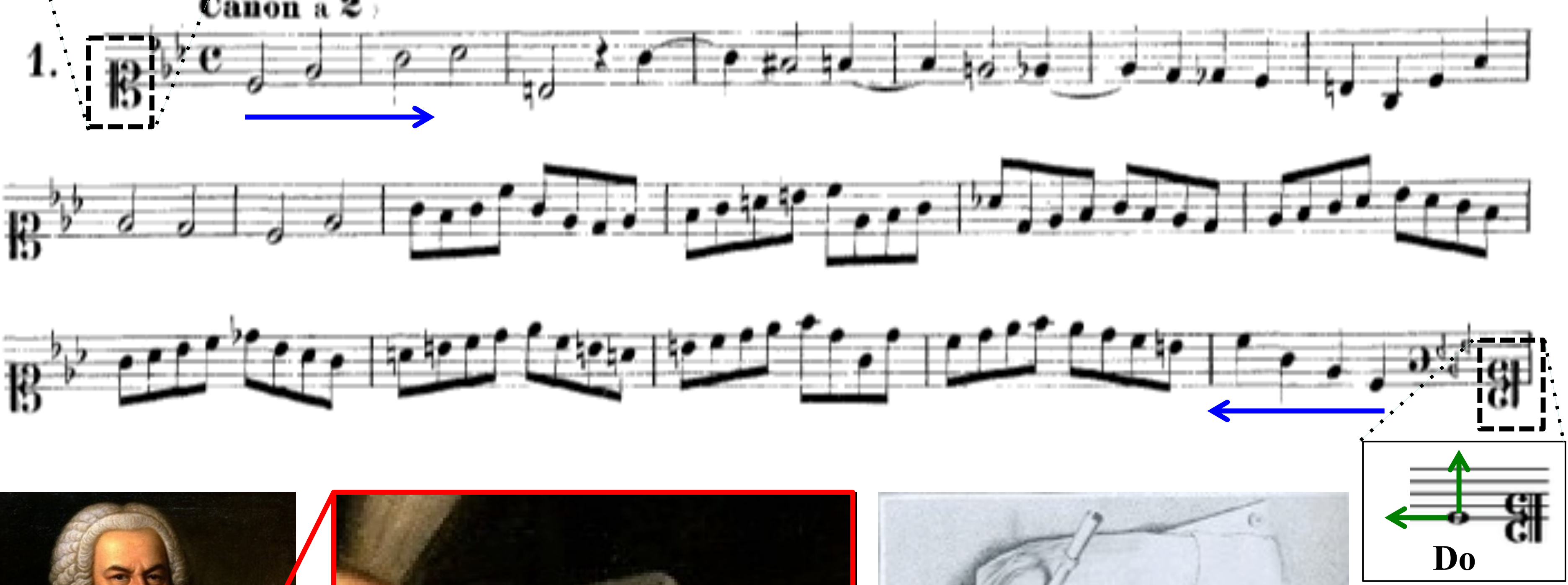
Canones diversi
super thema regium

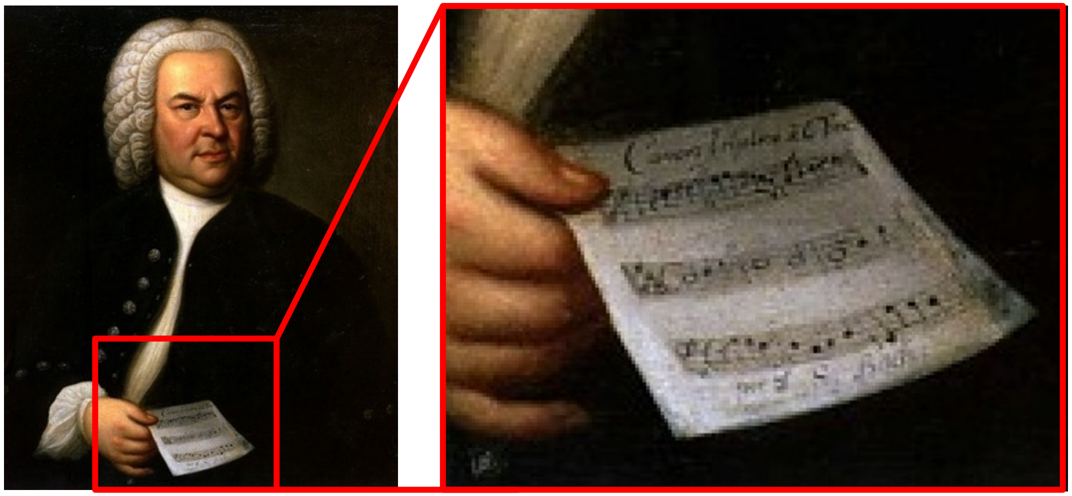


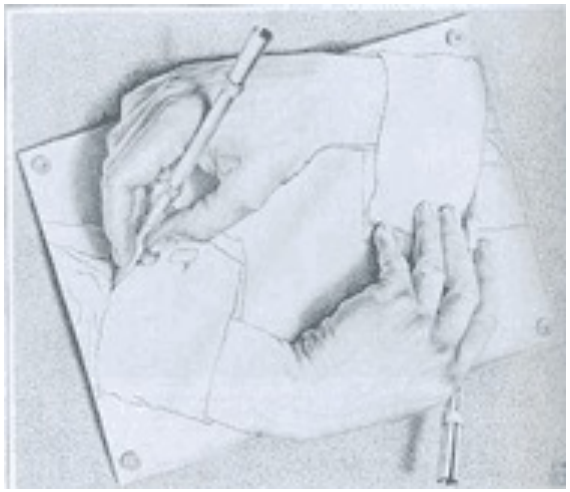


Bach’s enigmatic canons and geometry

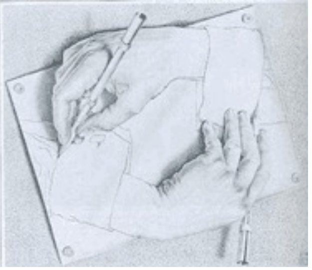


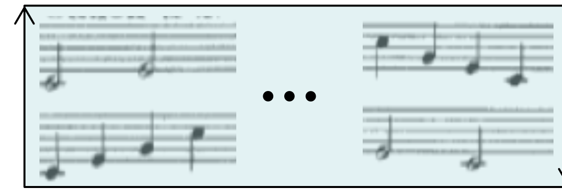






My end is my beginning (but twisted!)





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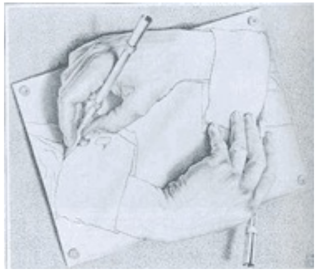
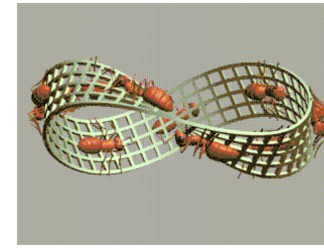
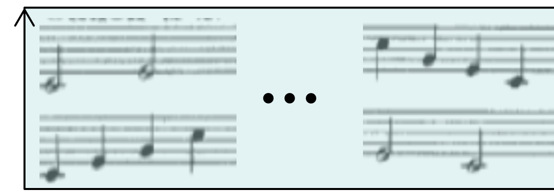
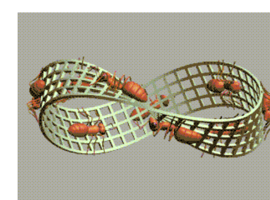






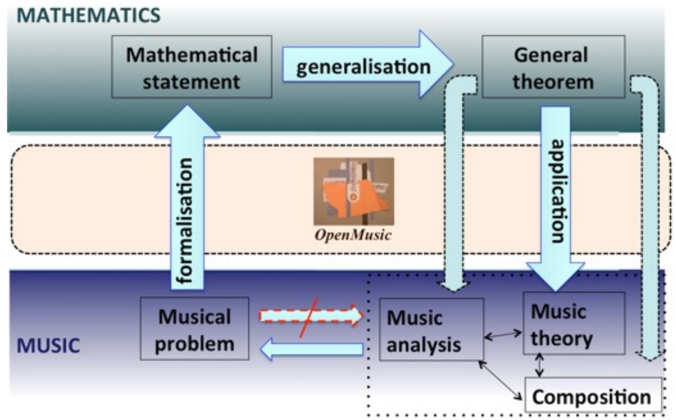
http://www.josleys.com/Canon/Canon.html

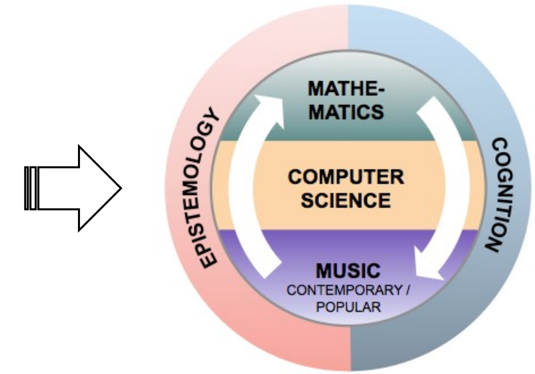
[min. 1’14”]

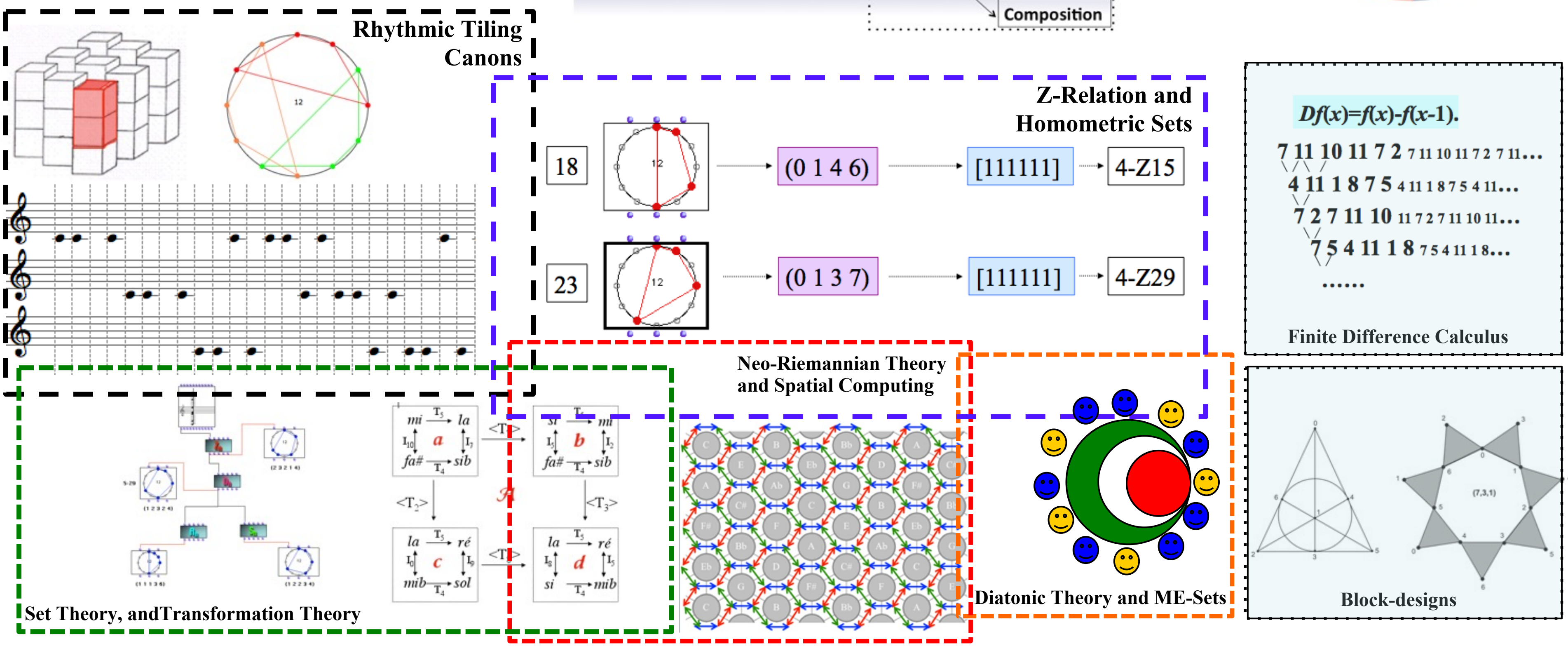


Some musically-driven mathematical problems

- Tiling Rhythmic Canons
- *Z* relation and homometry
- Transformational Theory
- Music Analysis, SC and FCA
- Diatonic Theory and ME-Sets
- Periodic sequences and FDC
- Block-designs in composition







➔ M. Andreatta : *Mathematica est exercitium musicae*, Habilitation Thesis, IRMA University of Strasbourg, 2010

Some musically-driven mathematical problems

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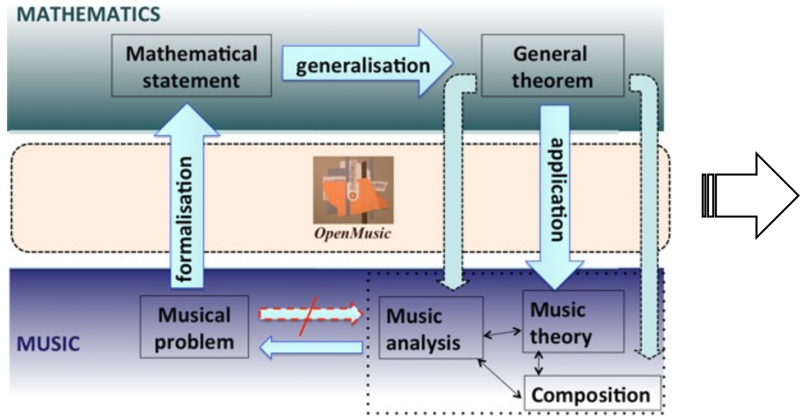
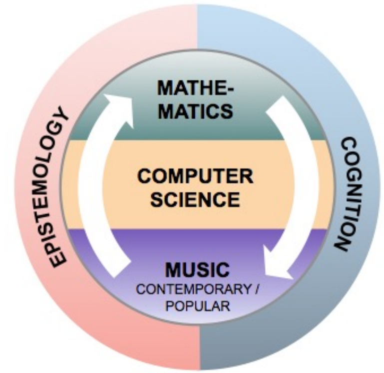
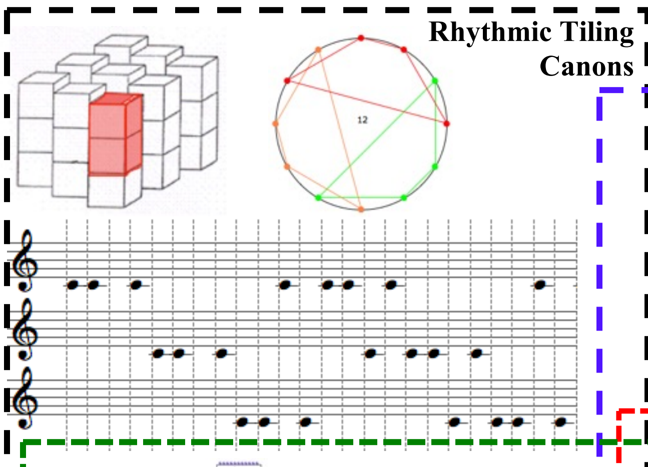
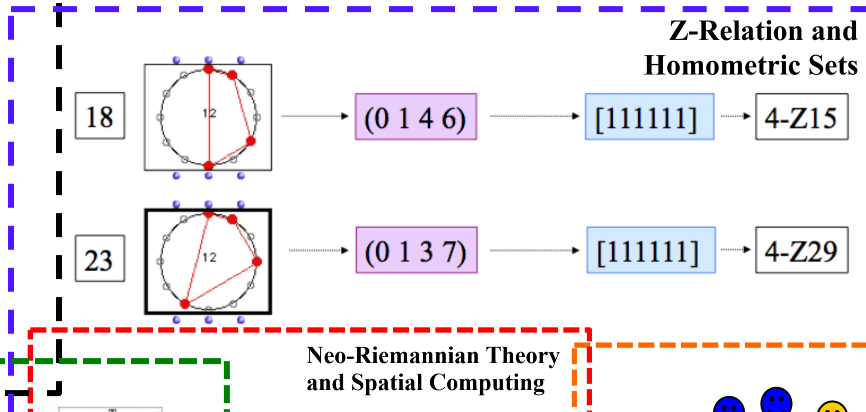
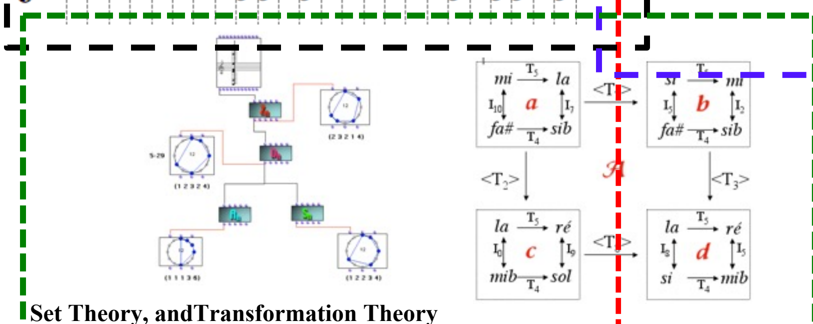
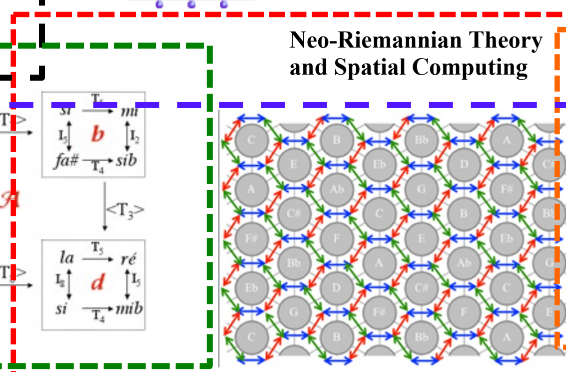


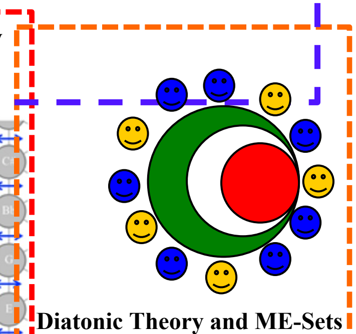


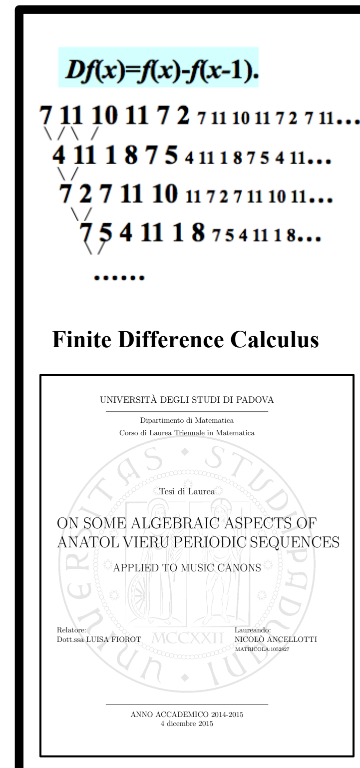










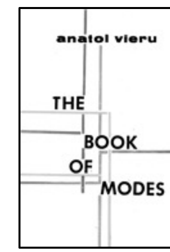


Periodic sequences and finite difference calculus

$$Df(x)=f(x)-f(x-1)$$

f	=	7 11 10 11 7 2 7 11 10 11 7 2 7 11…
Df	=	4 11 1 8 7 5 4 11 1 8 7 5 4 11…
$D^2 f$	=	7 2 7 11 10 11 7 2 7 11 10 11…
$D^3 f$	=	7 5 4 11 1 8 7 5 4 11 1 8…
$D^k f$	=	……

Anatol Vieru



V	0	3	8	7	11	0	11	10	6	9	0	9	1	2	9	8	4	3	6
VIII	0	0	0	0	3	3	7	2	0	0	0	6	3	3	3	4	8	0	0
IV	3	3	4	4	1	11	11	8	3	3	9	4	1	7	11	8	11	3	9
IX	0	0	0	0	0	3	6	[1]	3	3	3	3	9	0	3	6	[10]	6	6
IV	0	10	3	9	10	0	9	7	0	6	7	9	6	4	9	3	4	6	3

Anatol Vieru: *Zone d'oubli* pour alto (1973)

Reducible and reproducible sequences

$$f = 11\ 6\ 7\ 2\ 3\ 10\ 11\ 6 \ldots$$
$$Df = 7\ 1\ 7\ 1\ 7\ 1\ 7\ 1 \ldots$$
$$D^2f = 6\ 6\ 6\ 6\ 6 \ldots$$
$$D^4f = 0\ 0\ 0$$

Reducible sequences:

$\exists\ k \geq 1$ **such that**

$D^k f = 0$

$$f = 7\ 11\ 10\ 11\ 7\ 2\ 7\ 11 \ldots$$
$$Df = 4\ 11\ 1\ 8\ 7\ 5\ 4\ 11\ 1 \ldots$$
$$D^2f = 7\ 2\ 7\ 11\ 10\ 11\ 7\ 2\ 7 \ldots$$
$$D^3f = 7\ 5\ 4\ 11\ 1\ 8\ 7\ 5\ 4\ 11\ 1\ 8 \ldots$$
$$D^4f = 10\ 11\ 7\ 2\ 7\ 11\ 10\ 11 \ldots$$
$$D^5f = 1\ 8\ 7\ 5\ 4\ 11\ 1\ 8 \ldots$$
$$D^6f = 7\ 11\ 10\ 11\ 7\ 2\ 7\ 11 \ldots$$

Reproducible sequences:

$\exists\ k \geq 1$ **such that**

$D^k f = f$

A decomposition property of any periodic sequence

$Df(x)=f(x)-f(x-1)$.

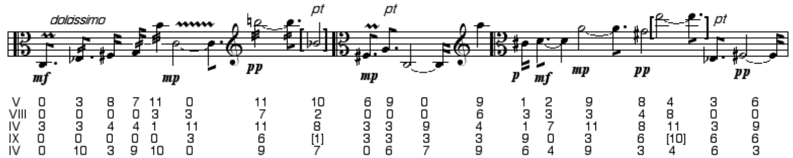
7 11 10 11 7 2 7 11 10 11 7 2 7 11…

4 11 1 8 7 5 4 11 1 8 7 5 4 11…

7 2 7 11 10 11 7 2 7 11 10 11…

7 5 4 11 1 8 7 5 4 11 1 8…

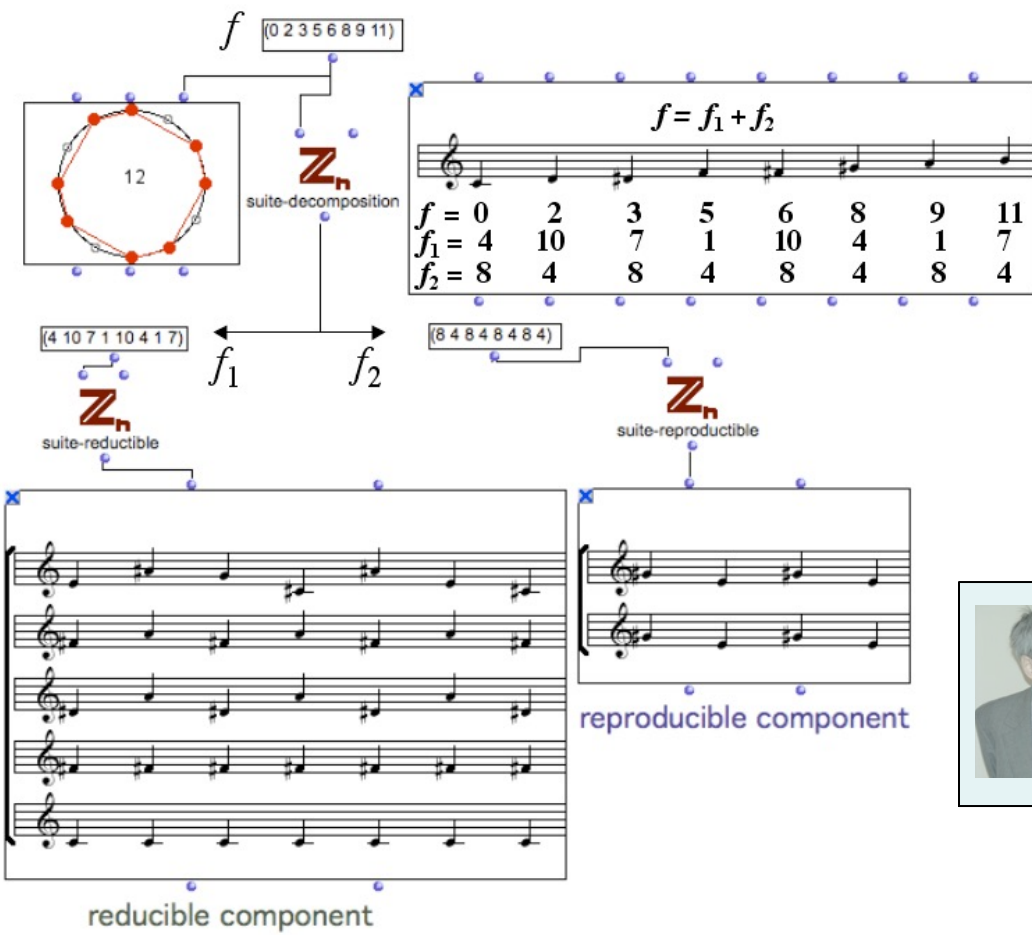
……



Anatol Vieru: *Zone d'oubli* for viola (1973)

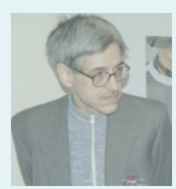
Reducible sequences:
$\exists\ k \geq 1$ such that $D^k f = 0$

Reproducible sequences:
$\exists\ k \geq 1$ such that $D^k f = f$



- **Decomposition theorem**

(Vuza & Andreatta, *Tatra M.*, 2001)
Every periodic sequence f can be decomposed in a unique way as a sum $f_1 + f_2$ of a reducible sequence f_1 and a reproducible sequence f_2



D. Vuza, M. Andreatta (2001), « On some properties of periodic sequences in Anatol Vieru's modal theory », Tatra Mountains Mathematical Publications, Vol. 23, p. 1-15

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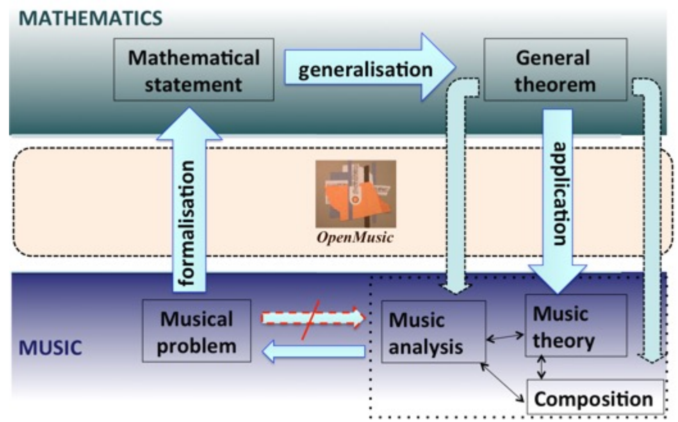
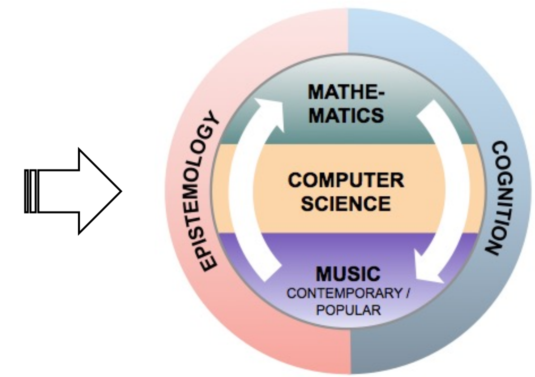
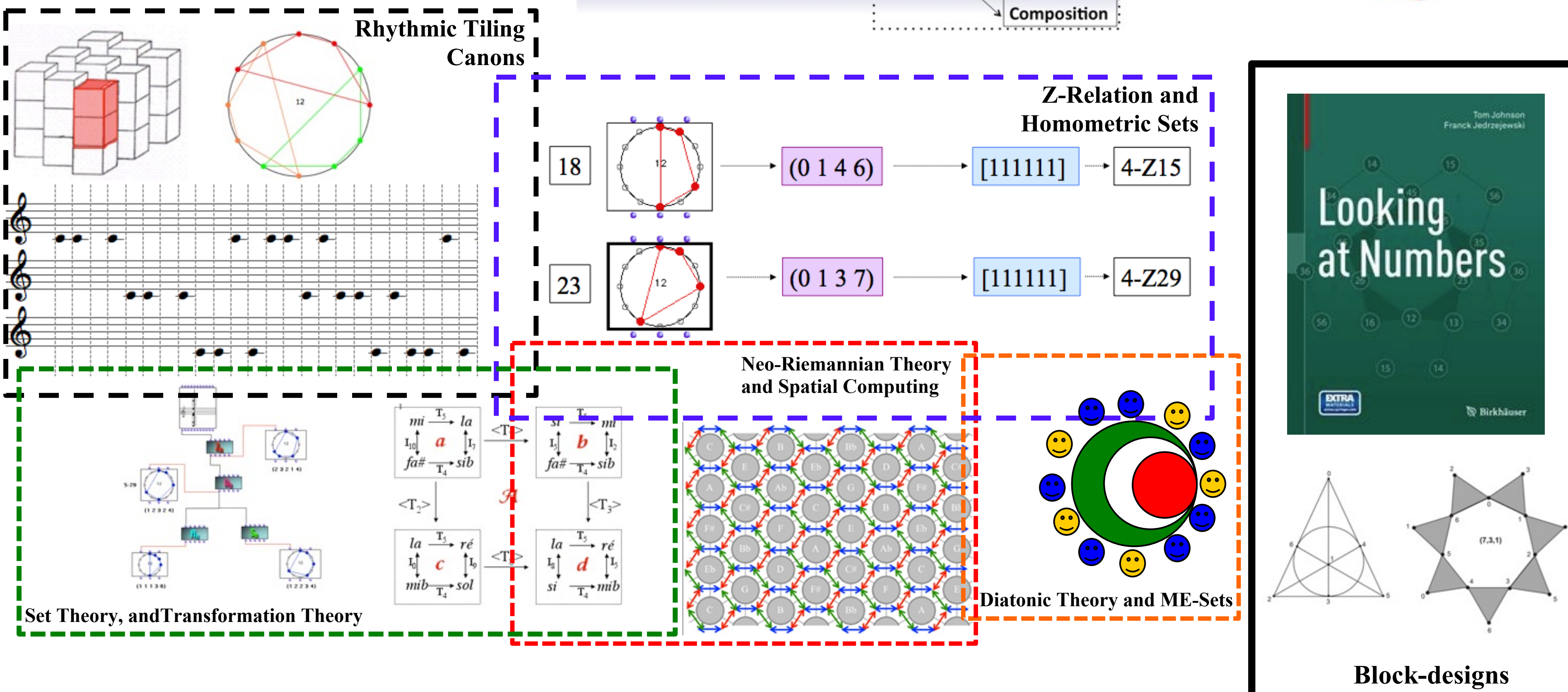
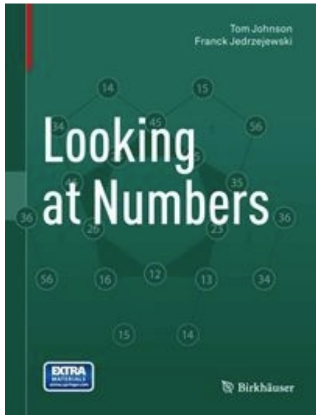


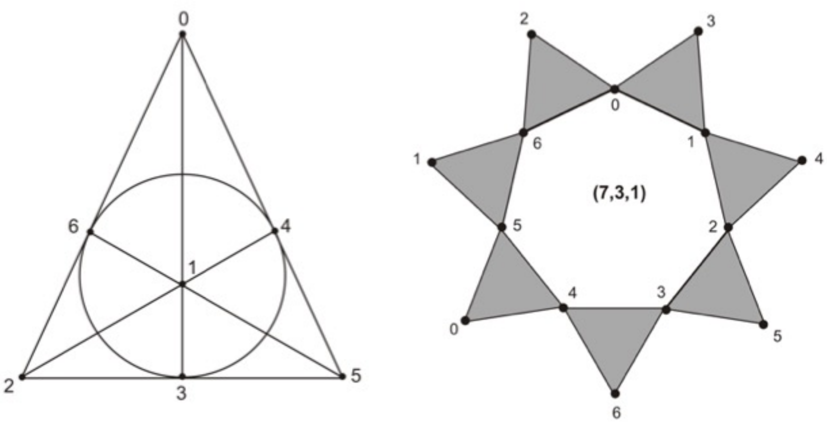




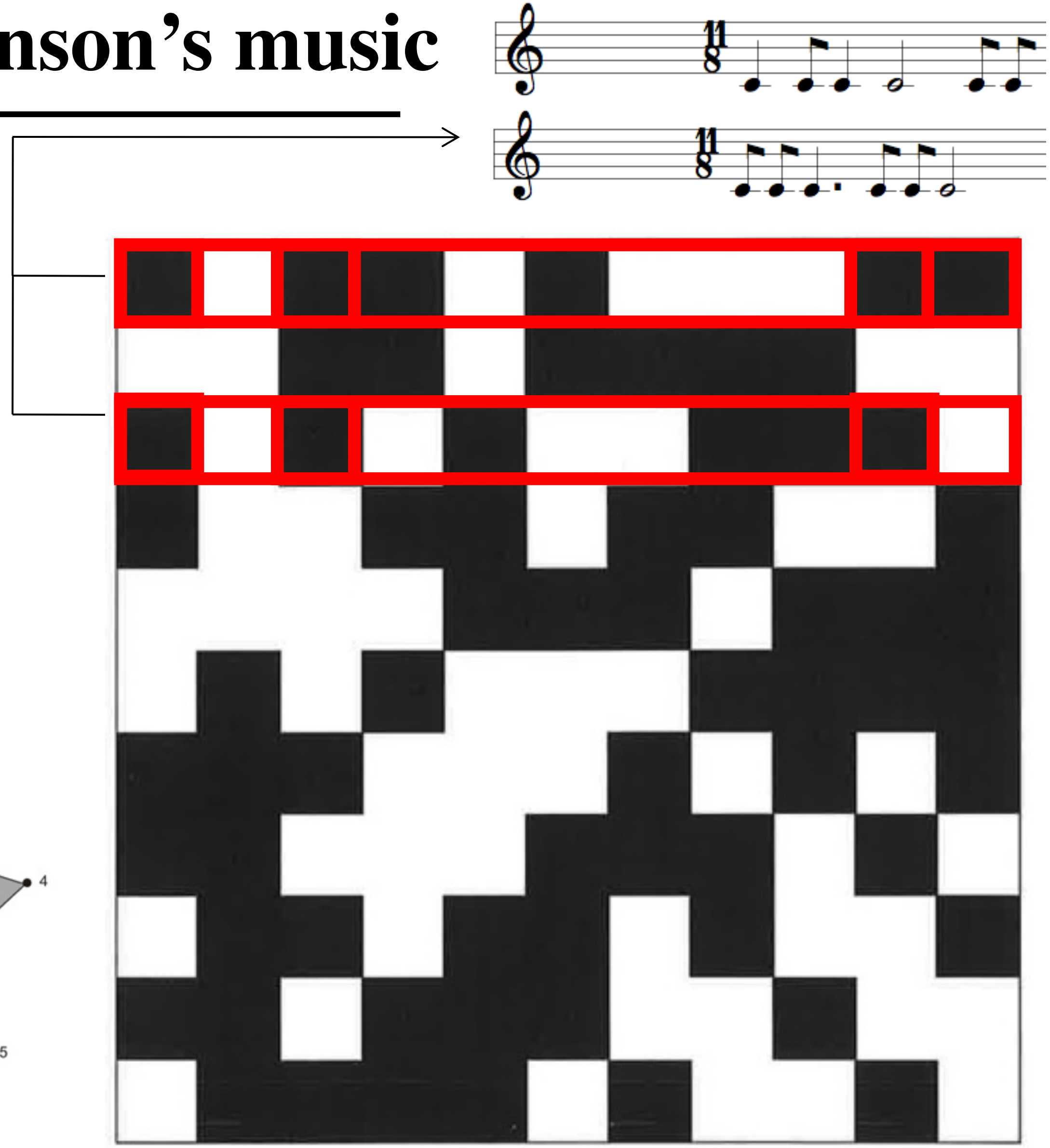
Block-Designs in Johnson's music

Tom Johnson





Fano plane = (7,3,1)



Block-design (11,6,3)

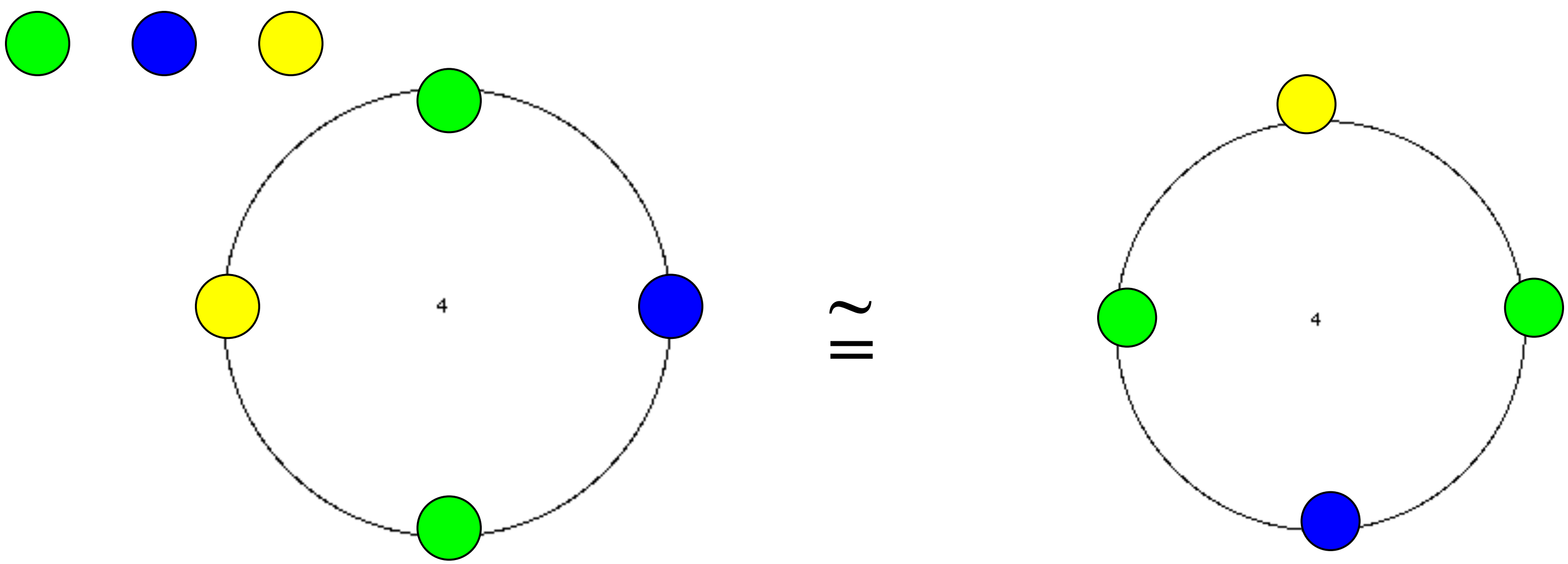
➔ Jedrzejewski, F., M. Andreatta, T. Johnson (2009), « Musical experiences with Block Designs », *Proceedings of the Conference MCM 2009*, vol. 38, New Haven, p. 154-165

Enumeration of chord classes (modulo a group action)

Burnside Lemma

$$n = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

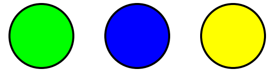
$$X^g = \{x \in X : gx = x\}$$



?

How many possible configurations could you find?

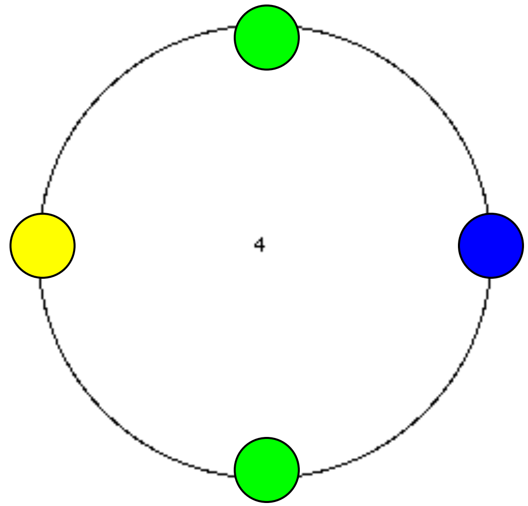
Enumeration of chord classes (modulo a group action)

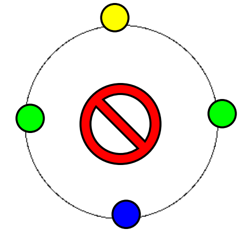


Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G}|X^g|.$$

$$X^g = \{x \in X : gx = x\}$$





Action of **Z/4Z**

T_0 = identity

T_1 = rotation by 90°

T_2 = rotation by 180°

T_3 = rotation by 270°

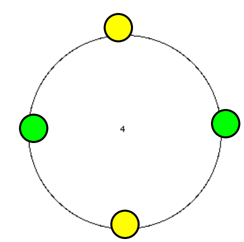
Possible configurations = 3^4 = 81

T_0 fixes all configurations => $|X^{T_0}|$=81

T_1 fixes all monochromatic configurations => $|X^{T_1}|$=3

T_3 idem

T_2 fixes all «double-diameter » configurations => $|X^{T_1}| = 3^2$ =9



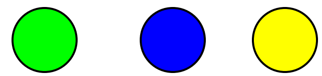
⇒ n = 1/4 (81+3+3+9) = 24

Enumeration of chord classes (modulo a group action)

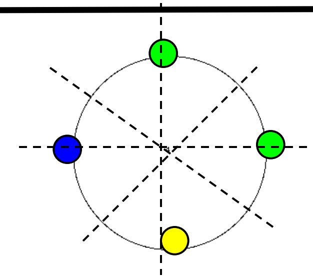
Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



Action de $\mathbf{Z_4}$



Transformation	*Action*	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	$0\to0,\ 1\to1,\ 2\to2,\ 3\to3$	(0)(1)(2)(3)	4	$3^4 = 81$	1^4	$t_1^{\,4}$
T_1	$0\to1\to2\to3\to0$	(0 1 2 3)	1	$3^1 = 3$	4^1	$t_4^{\,1}$
T_2	$0\to2\to0,\ 1\to3\to1$	(0 2)(1 3)	2	$3^2 = 9$	2^2	$t_2^{\,2}$
T_3	$0\to3\to2\to1\to0$	(0 3 2 1)	1	$3^1 = 3$	4^1	$t_4^{\,1}$

Julian Hook, « Why are there 29 Tetrachords? A Tutorial on Combinatorics and Enumeration in Music Theory », MTO, 13(4), 2007

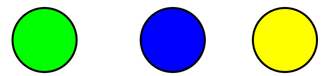
$$n = 1/4\ (81+3+3+9) = 24$$

Enumeration of chord classes (modulo a group action)

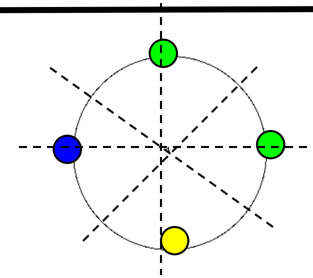
Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x \in X : gx = x\}$$



Action of $\mathbf{D}_4$



Transformation	*Action*	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	$0\to0,\ 1\to1,\ 2\to2,\ 3\to3$	(0)(1)(2)(3)	4	$3^4 = 81$	1^4	$t_1^{\,4}$
T_1	$0\to1\to2\to3\to0$	(0 1 2 3)	1	$3^1 = 3$	4^1	$t_4^{\,1}$
T_2	$0\to2\to0,\ 1\to3\to1$	(0 2)(1 3)	2	$3^2 = 9$	2^2	$t_2^{\,2}$
T_3	$0\to3\to2\to1\to0$	(0 3 2 1)	1	$3^1 = 3$	4^1	$t_4^{\,1}$
I	$0\to0,\ 1\to3\to1,\ 2\to2$	(0)(1 3)(2)	3	$3^3 = 27$	$1^2 2^1$	$t_1^{\,2}t_2^{\,1}$
T_1I	$0\to1\to0,\ 2\to3\to2$	(0 1)(2 3)	2	$3^2 = 9$	2^2	$t_2^{\,2}$
T_2I	$0\to2\to0,\ 1\to1,\ 3\to3$	(0 2)(1)(3)	3	$3^3 = 27$	$1^2 2^1$	$t_1^{\,2}t_2^{\,1}$
T_3I	$0\to3\to0,\ 1\to2\to1$	(0 3)(1 2)	2	$3^2 = 9$	2^2	$t_2^{\,2}$

Julian Hook, « Why are there 29 Tetrachords? A Tutorial on Combinatorics and Enumeration in Music Theory », MTO, 13(4), 2007

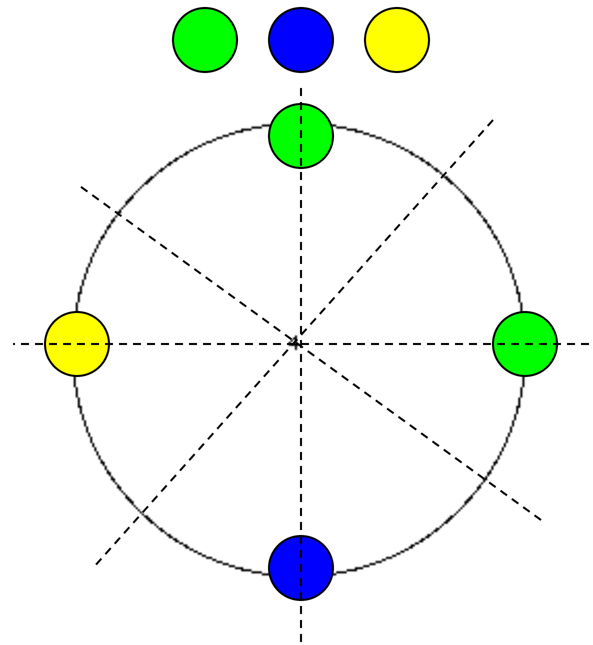
$$n = 1/8\ (81+3+3+9+27+9+27+9) = 168/8=21$$

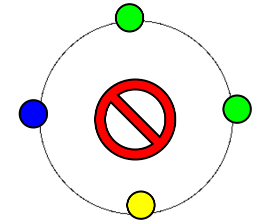
Enumeration of chord classes (modulo a group action)

Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$$X^g = \{x\in X : gx=x\}$$





Action of $\mathbf{D_4}$

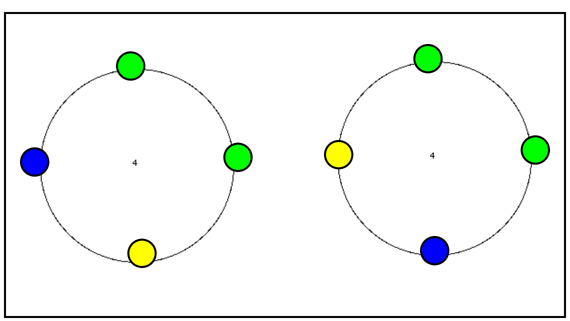
$T_0 = \text{id}$ $\quad T_0 I = \text{inversion}$

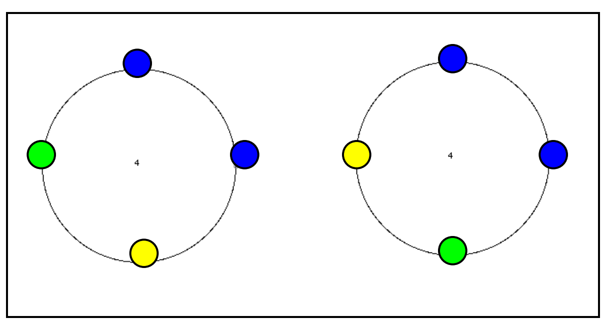
$T_1 = \text{rot } 90^\circ$ $\quad T_1 I = \text{inv.}$

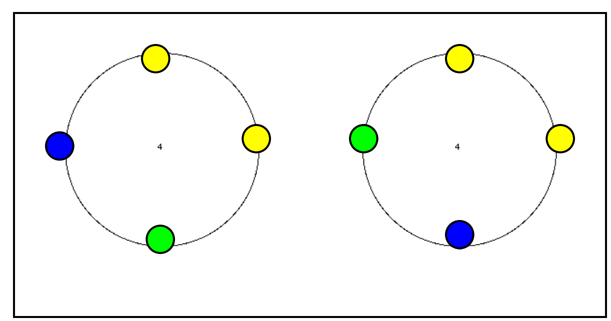
$T_2 = \text{rot } 180^\circ$ $\quad T_2 I = \text{inv.}$

$T_3 = \text{rot } 270^\circ$ $\quad T_3 I = \text{inv.}$

21=24-3







Enumeration of transposition chord classes

Transformation	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1

Burnside Lemma

$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$X^g = \{x \in X : gx = x\}$

Action of $\mathbf{Z}_{12}$

(Hook, MTO)

\# chords = 1/12[4096+2+4+8+16+2+64+2+16+8+4+2]=4224/12=352

Enumeration of pitch-class sets

Transformation	*Cycle representation*	*No. of cycles*	*Fixed configs.*	*Cycle type*	*Cycle index*
T_0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(A)(B)	12	$2^{12} = 4096$	1^{12}	t_1^{12}
T_1	(0 1 2 3 4 5 6 7 8 9 A B)	1	$2^1 = 2$	12^1	t_{12}^1
T_2	(0 2 4 6 8 A)(1 3 5 7 9 B)	2	$2^2 = 4$	6^2	t_6^2
T_3	(0 3 6 9)(1 4 7 A)(2 5 8 B)	3	$2^3 = 8$	4^3	t_4^3
T_4	(0 4 8)(1 5 9)(2 6 A)(3 7 B)	4	$2^4 = 16$	3^4	t_3^4
T_5	(0 5 A 3 8 1 6 B 4 9 2 7)	1	$2^1 = 2$	12^1	t_{12}^1
T_6	(0 6)(1 7)(2 8)(3 9)(4 A)(5 B)	6	$2^6 = 64$	2^6	t_2^6
T_7	(0 7 2 9 4 B 6 1 8 3 A 5)	1	$2^1 = 2$	12^1	t_{12}^1
T_8	(0 8 4)(1 9 5)(2 A 6)(3 B 7)	4	$2^4 = 16$	3^4	t_3^4
T_9	(0 9 6 3)(1 A 7 4)(2 B 8 5)	3	$2^3 = 8$	4^3	t_4^3
T_{10}	(0 A 8 6 4 2)(1 B 9 7 5 3)	2	$2^2 = 4$	6^2	t_6^2
T_{11}	(0 B A 9 8 7 6 5 4 3 2 1)	1	$2^1 = 2$	12^1	t_{12}^1
I	(0)(1 B)(2 A)(3 9)(4 8)(5 7)(6)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_1I	(0 1)(2 B)(3 A)(4 9)(5 8)(6 7)	6	$2^6 = 64$	2^6	t_2^6
T_2I	(0 2)(1)(3 B)(4 A)(5 9)(6 8)(7)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_3I	(0 3)(1 2)(4 B)(5 A)(6 9)(7 8)	6	$2^6 = 64$	2^6	t_2^6
T_4I	(0 4)(1 3)(2)(5 B)(6 A)(7 9)(8)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_5I	(0 5)(1 4)(2 3)(6 B)(7 A)(8 9)	6	$2^6 = 64$	2^6	t_2^6
T_6I	(0 6)(1 5)(2 4)(3)(7 B)(8 A)(9)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_7I	(0 7)(1 6)(2 5)(3 4)(8 B)(9 A)	6	$2^6 = 64$	2^6	t_2^6
T_8I	(0 8)(1 7)(2 6)(3 5)(4)(9 B)(A)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
T_9I	(0 9)(1 8)(2 7)(3 6)(4 5)(A B)	6	$2^6 = 64$	2^6	t_2^6
$T_{10}I$	(0 A)(1 9)(2 8)(3 7)(4 6)(5)(B)	7	$2^7 = 128$	$1^2 2^5$	$t_1^2 t_2^5$
$T_{11}I$	(0 B)(1 A)(2 9)(3 8)(4 7)(5 6)	6	$2^6 = 64$	2^6	t_2^6

Burnside Lemma

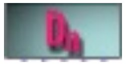
$$n = \frac{1}{|G|}\sum_{g\in G} |X^g|.$$

$X^g = \{x \in X : gx = x\}$

Action of $\mathbf{D_{12}}$

(Hook, MTO)

chords = 1/12[4096+2+4+8+16+2+64+2+16+8+4+2]=4224/12=352



chords = 1/24[4224+1152] = 224