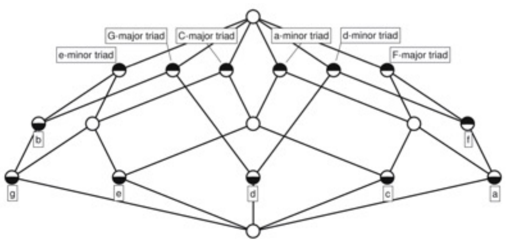
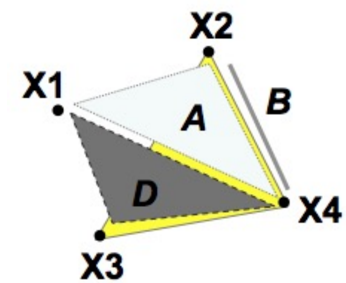
PARCOURS DU MASTER SCIENCES ET TECHNOLOGIES
UPMC • IRCAM • TELECOM PARISTECH

# Algebraic and geometrical models in computational musicology

**2024-2025**
**(part II)**





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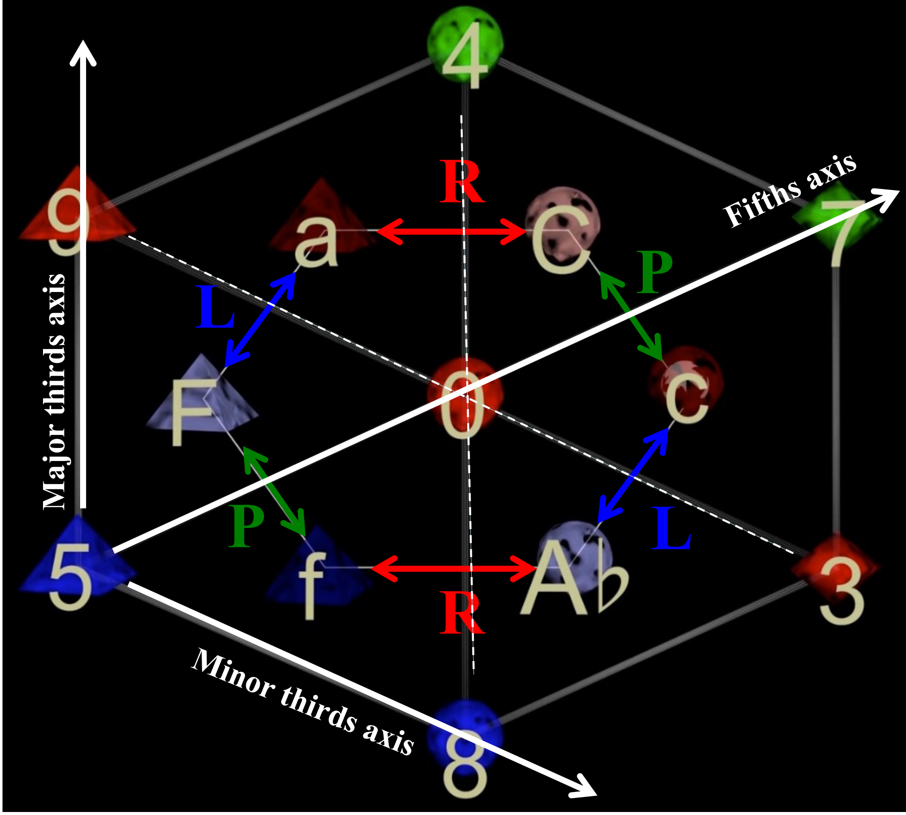
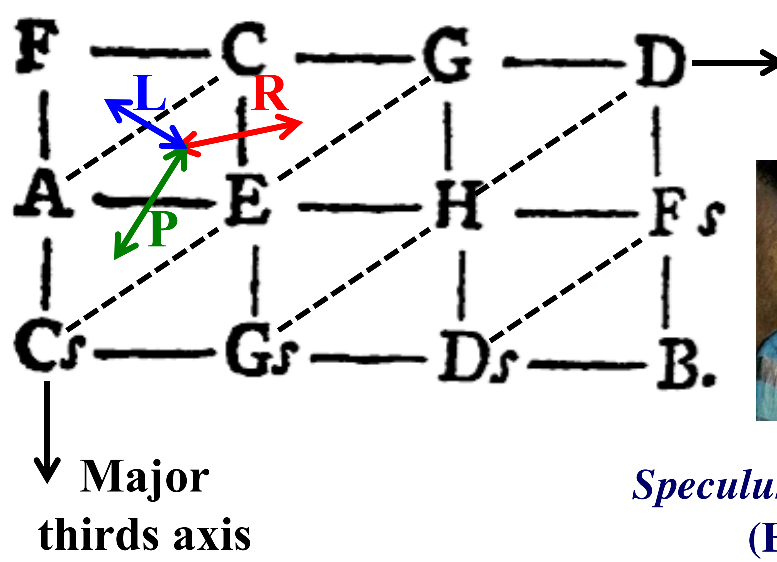
# The *Tonnetz*



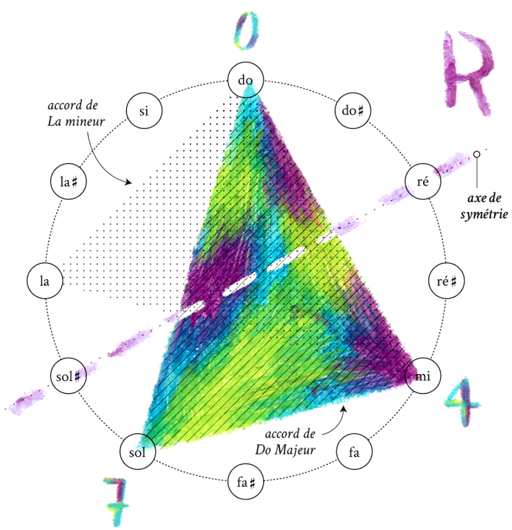


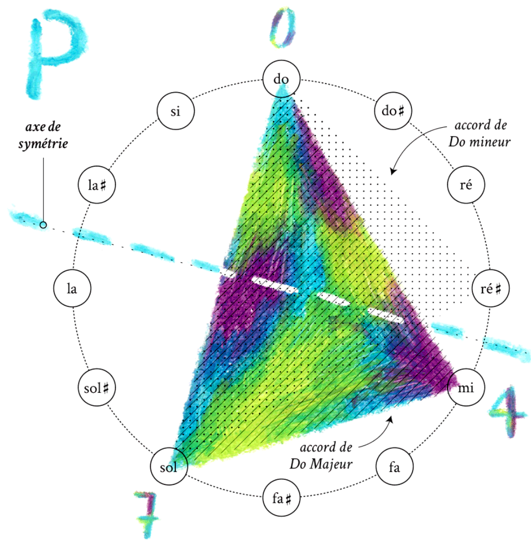
© Wiebke Drenckhan – Math'n Pop

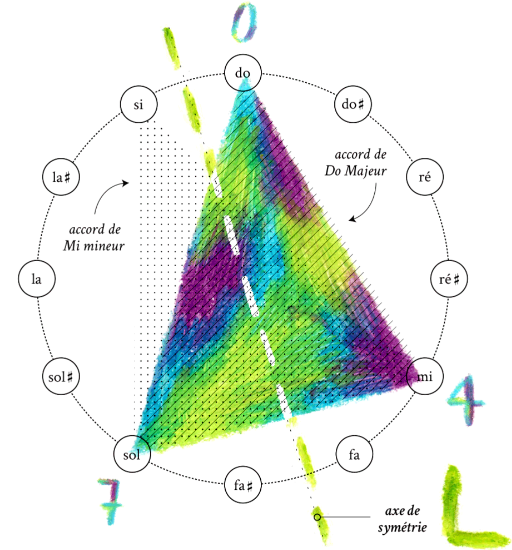



*Speculum Musicum*
(Euler, 1773)

# The three main major-minor symmetries



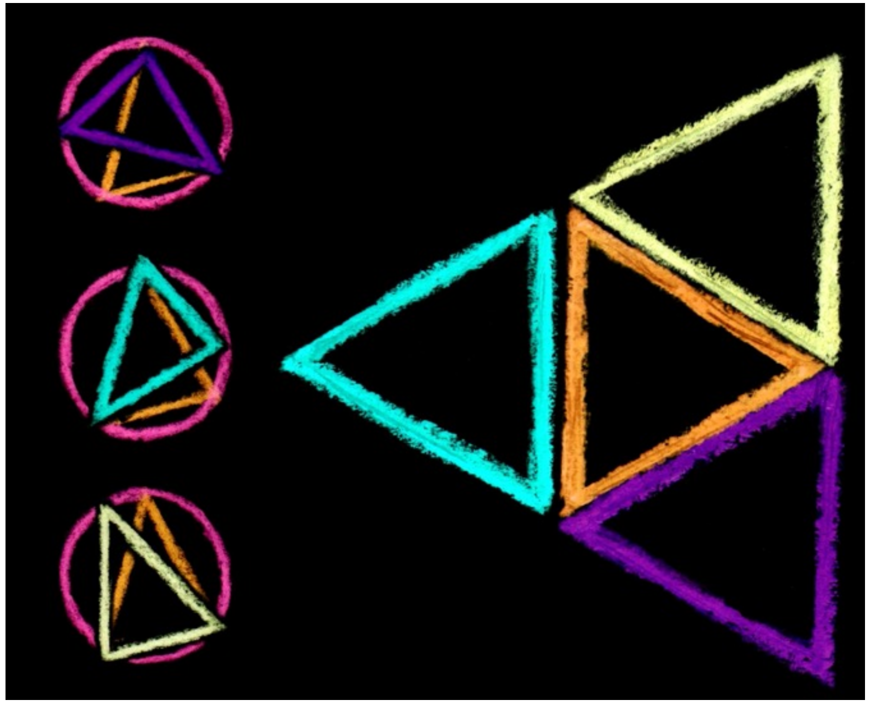

**R as RELATIVE** | **P as PARALLEL** | **L as LEADING-TONE (EXCHANGE)**

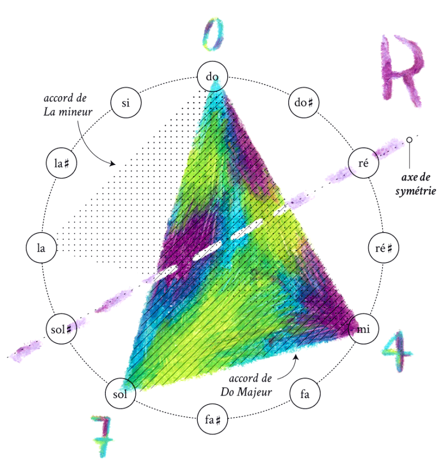
**C major** ↕ **A minor**

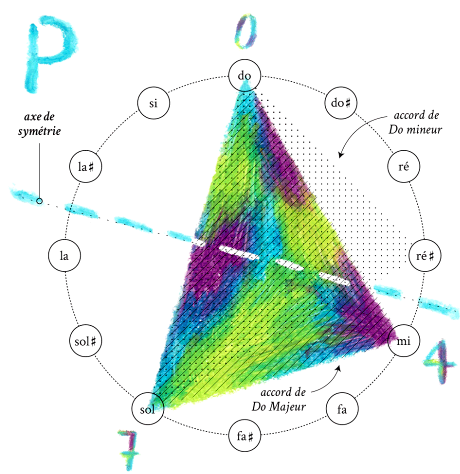
**C major** ↕ **C minor**

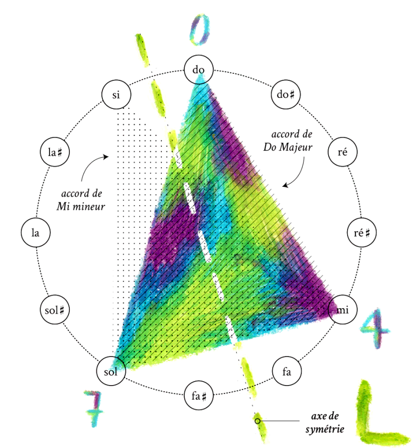
**C major** ↕ **E minor**

# The three main symmetries in the Tonnetz

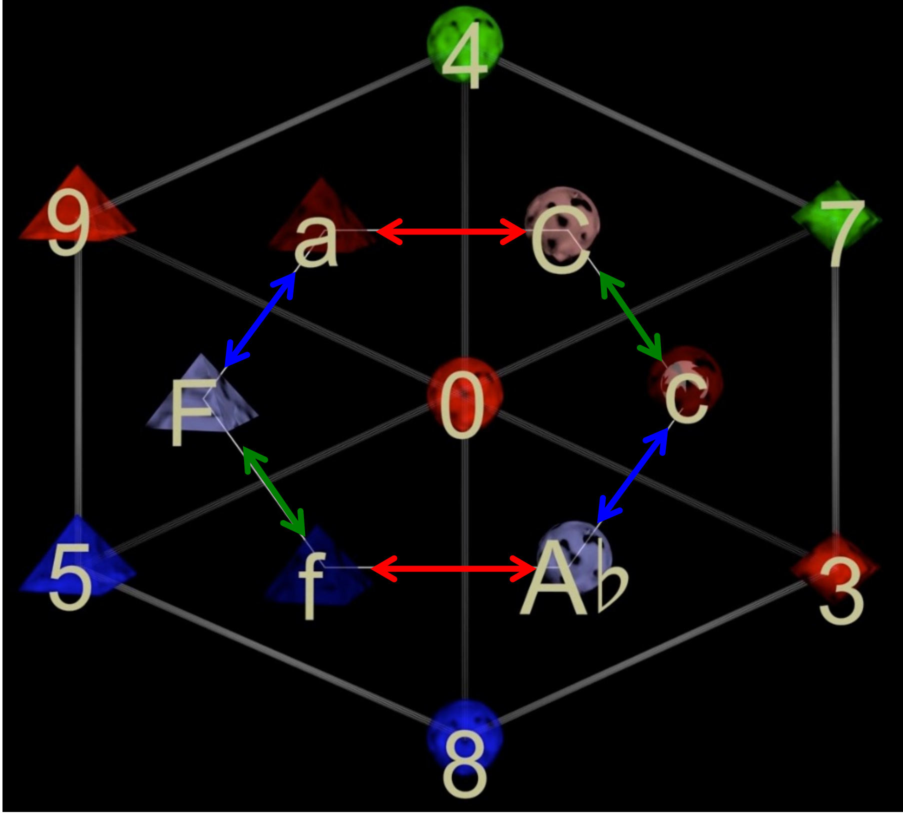


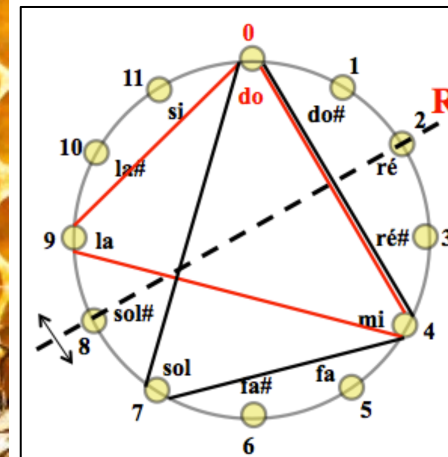


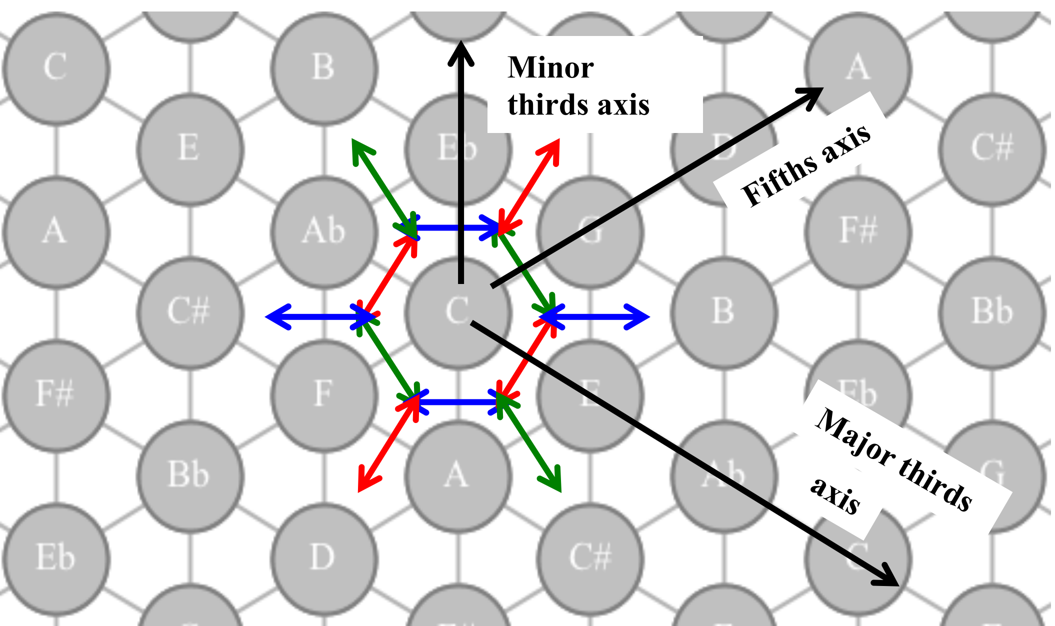



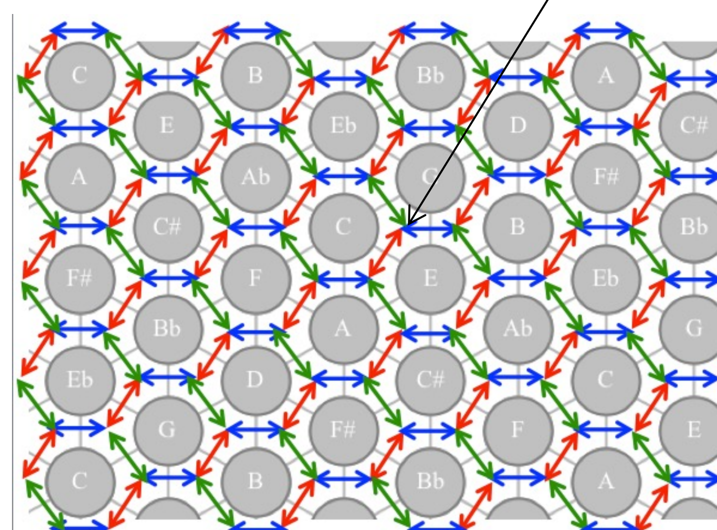



# The *Tonnetz*
## (or hexagonal tiling honeycomb)



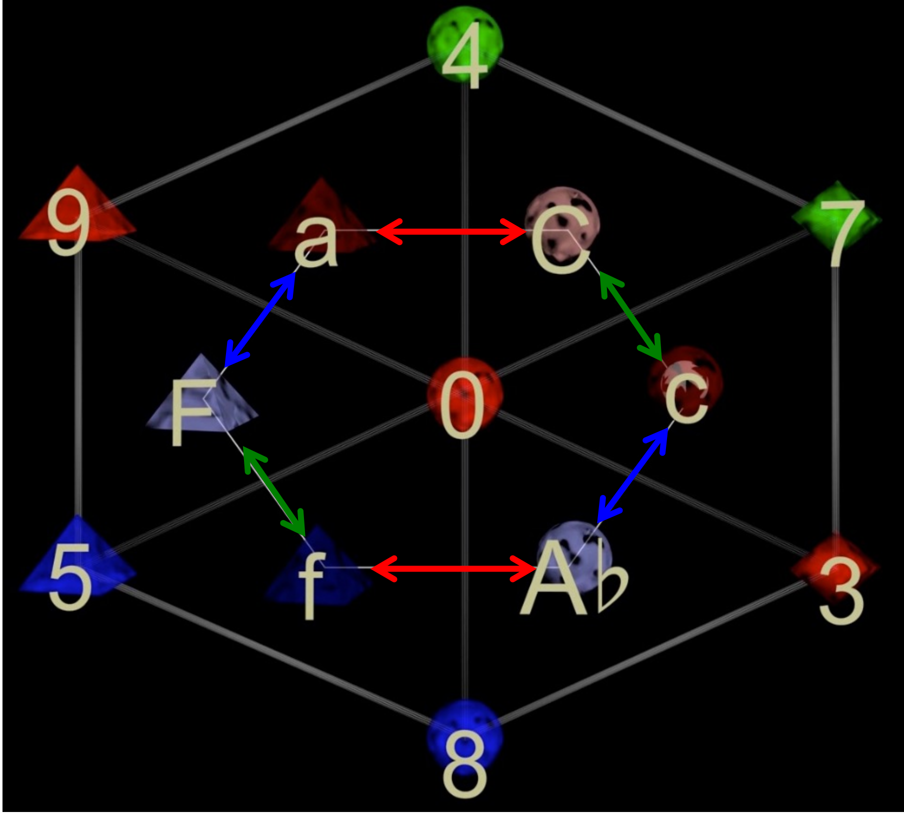



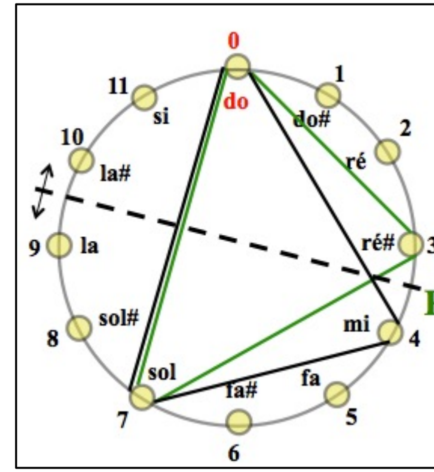



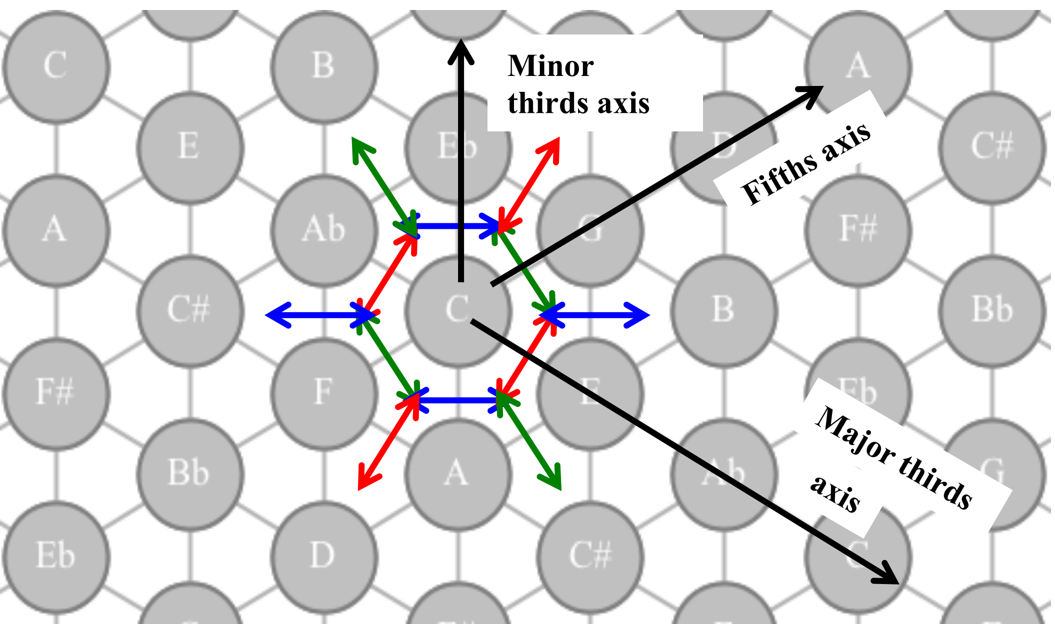



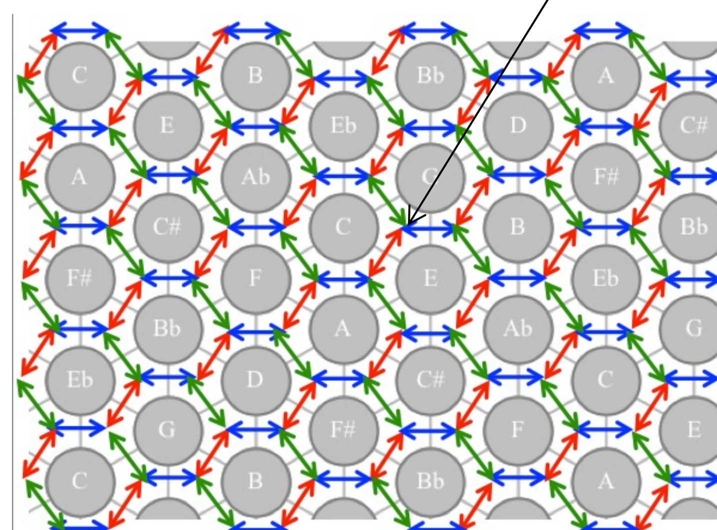
# The *Tonnetz*

## (or hexagonal tiling honeycomb)



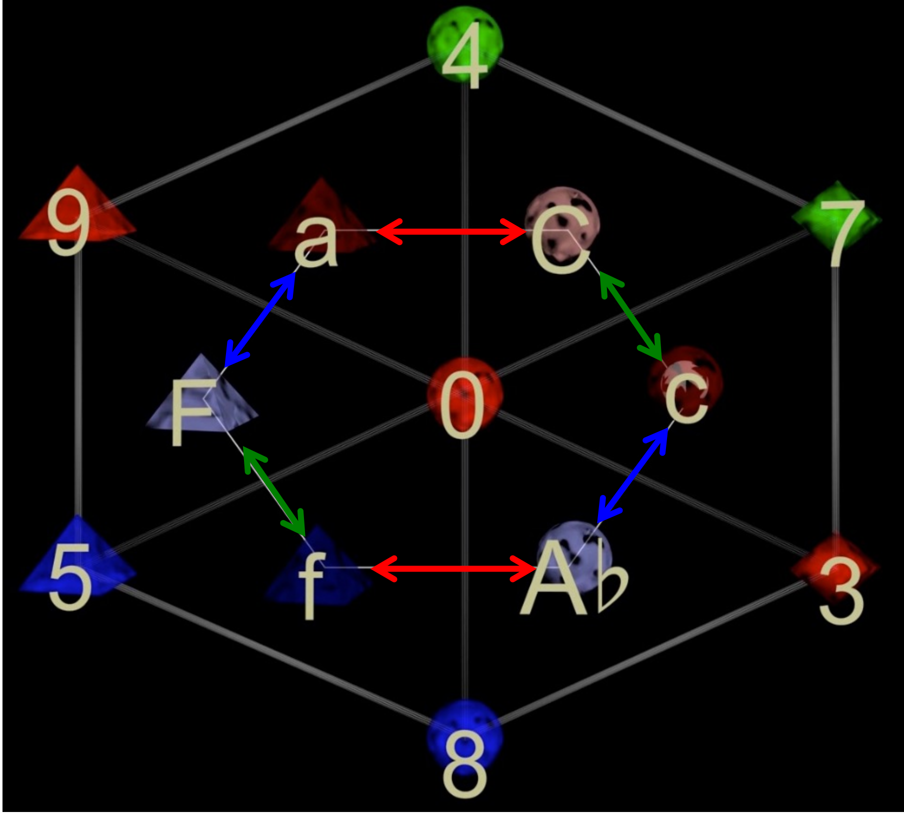



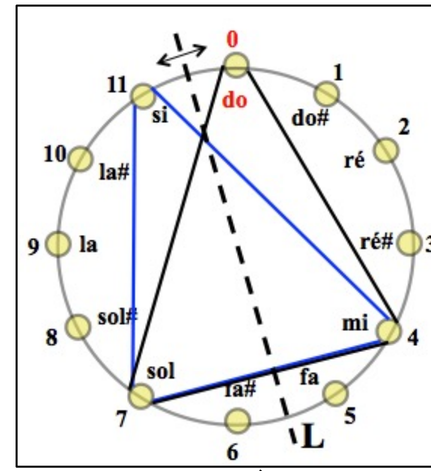



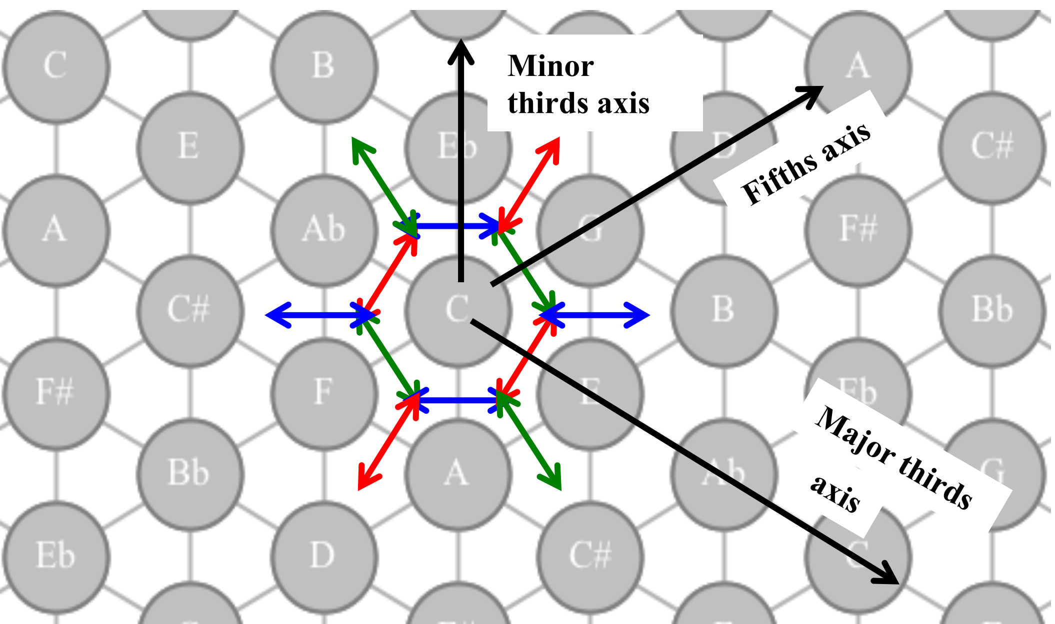



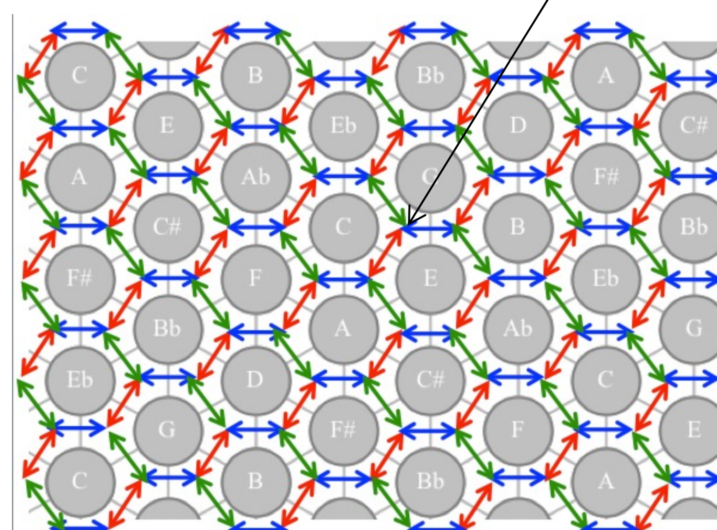
# The *Tonnetz*

## (or hexagonal tiling honeycomb)

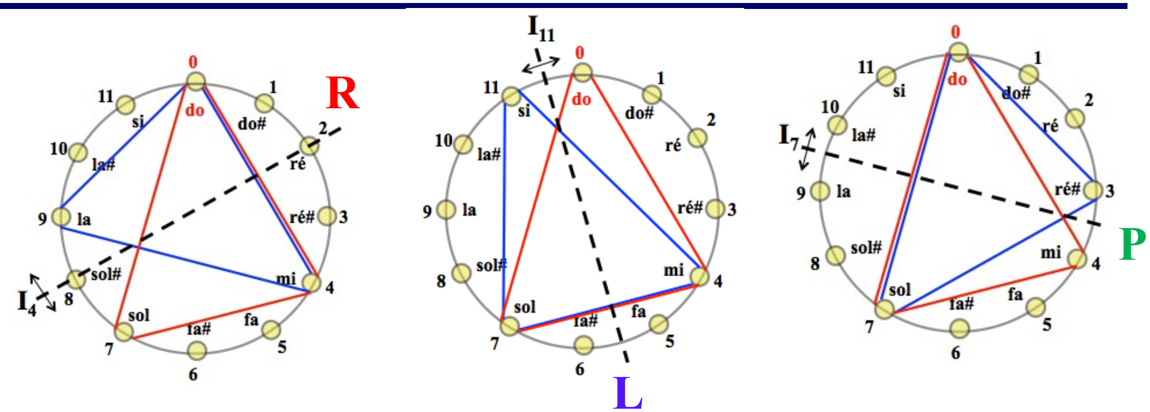


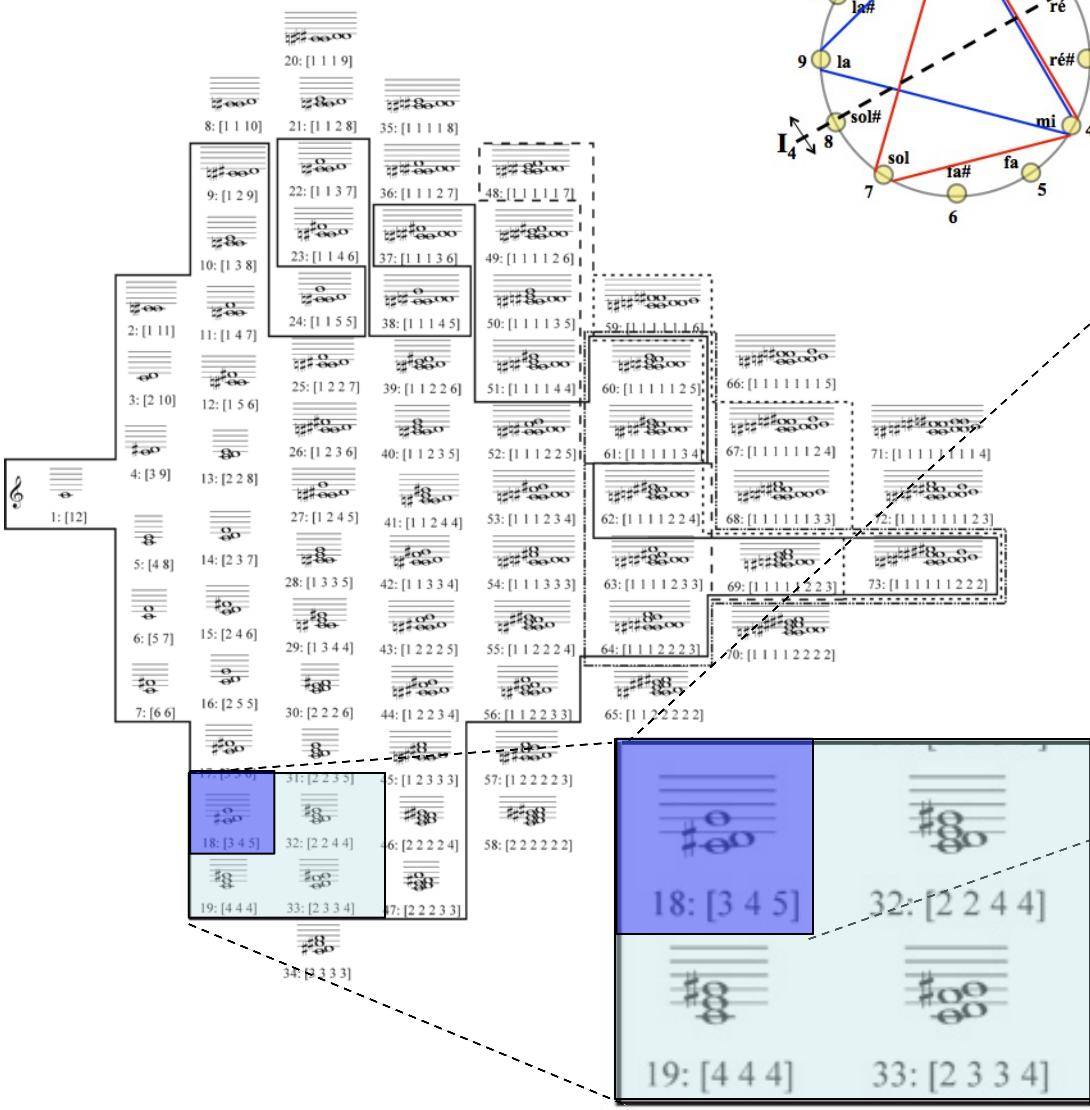



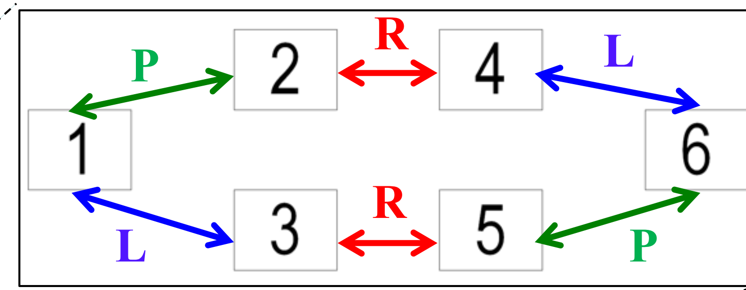



# Permutohedron and *Tonnetz*: a structural inclusion

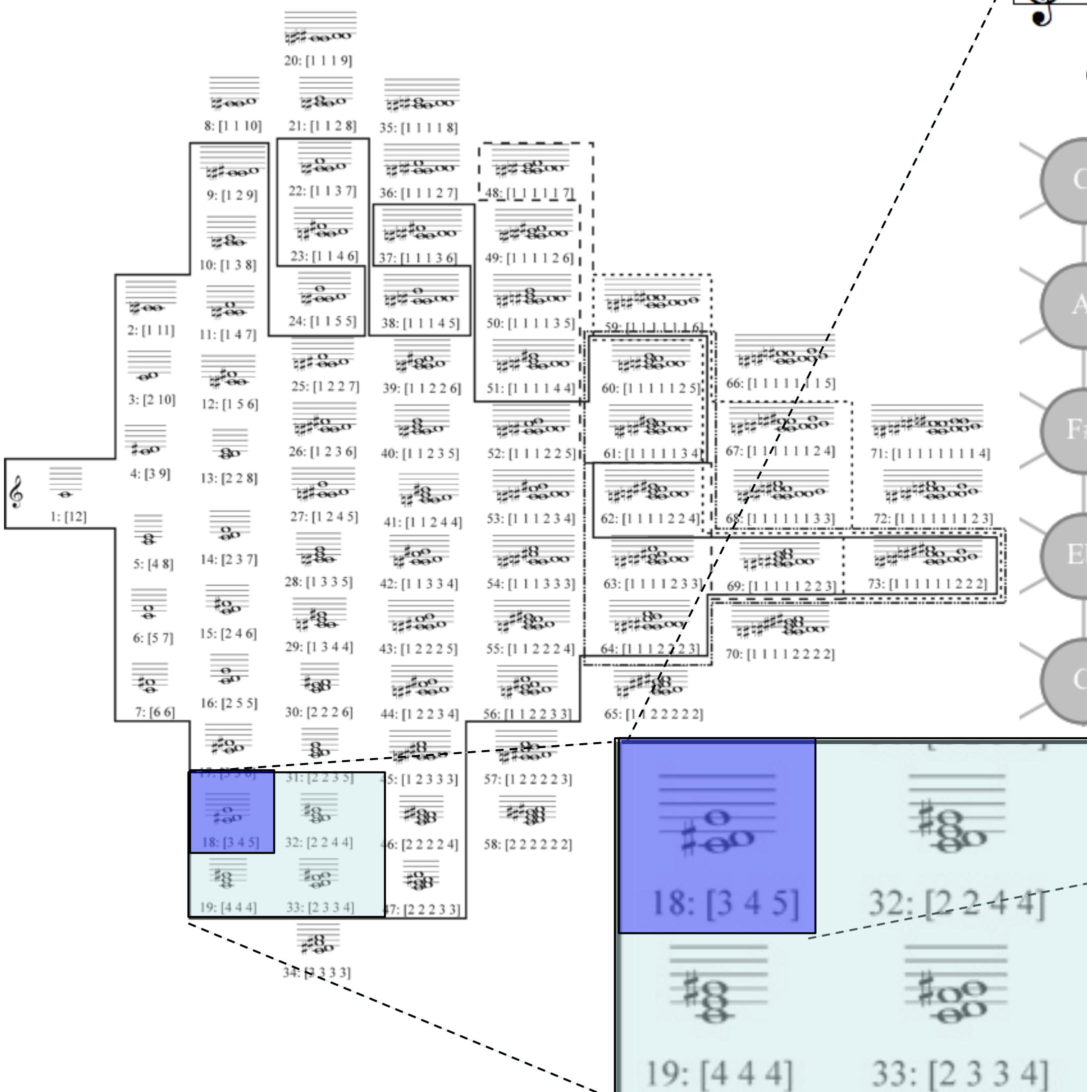


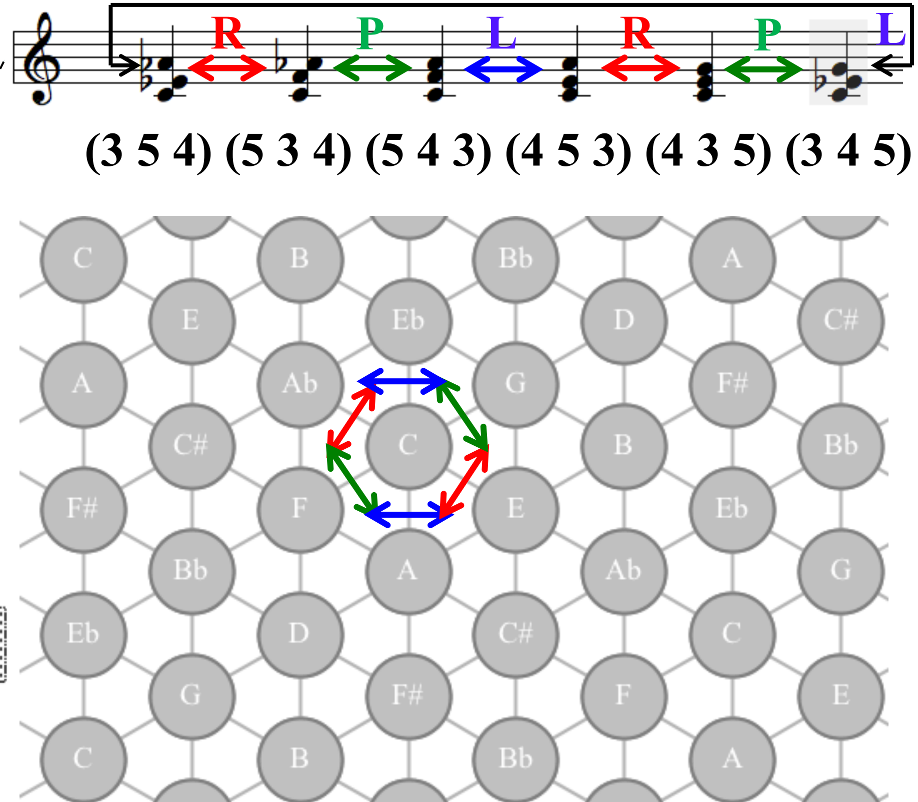




| | | |
|---|---|---|
| 1 | = | (3 4 5) |
| 2 | = | (4 3 5) |
| 3 | = | (3 5 4) |
| 4 | = | (4 5 3) |
| 5 | = | (5 3 4) |
| 6 | = | (5 4 3) |

# Permutohedron and *Tonnetz*: a structural inclusion



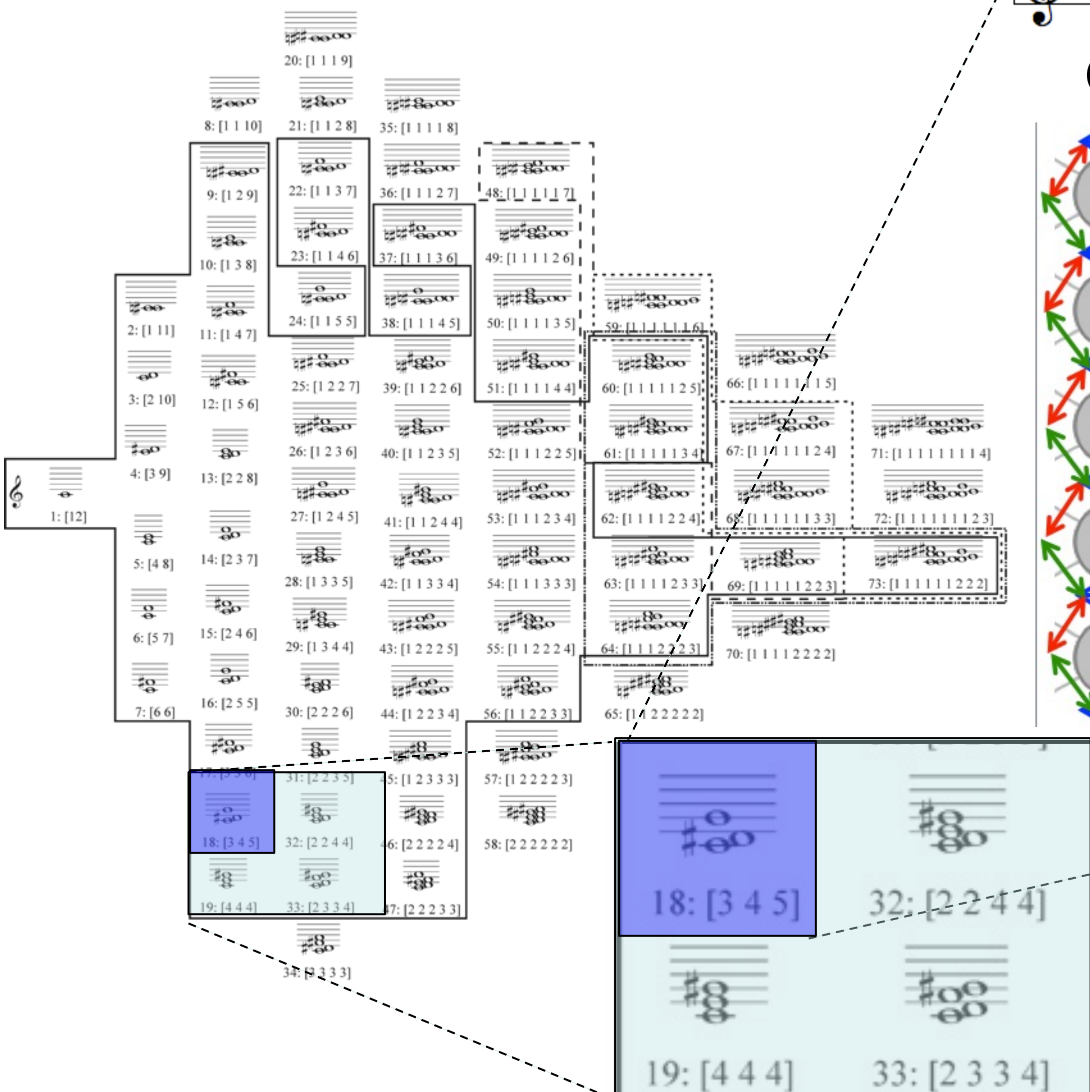



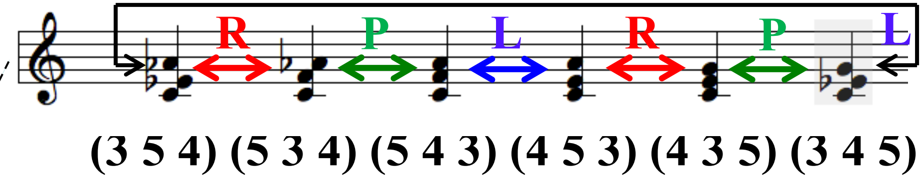

R: $C_{maj} \rightarrow A_{min}$

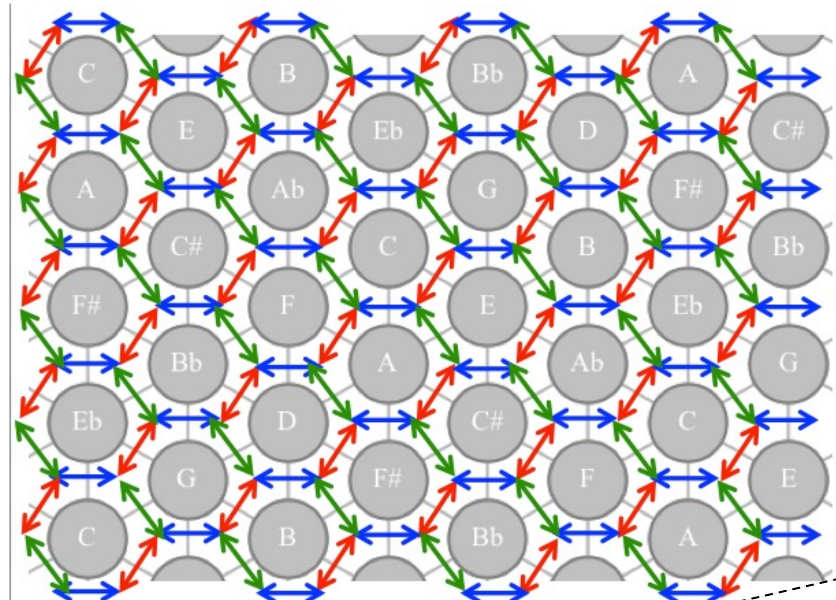
L: $C_{maj} \rightarrow E_{min}$

P: $C_{maj} \rightarrow C_{min}$

# Permutohedron and *Tonnetz*: a structural inclusion



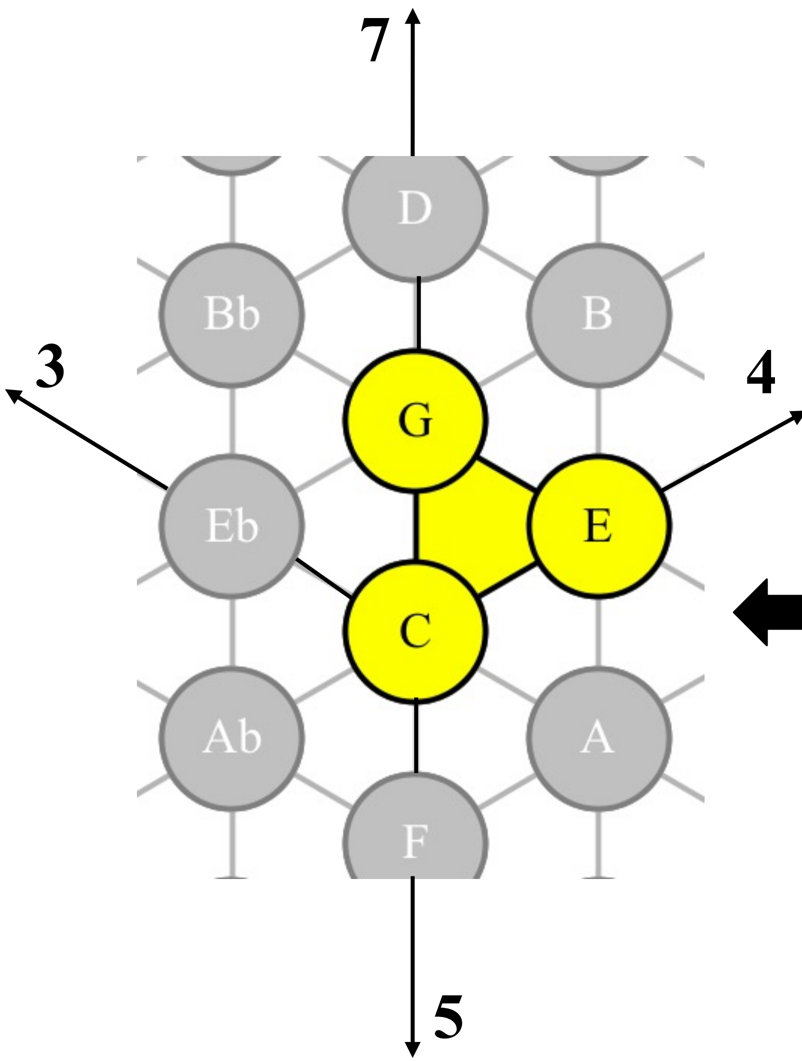



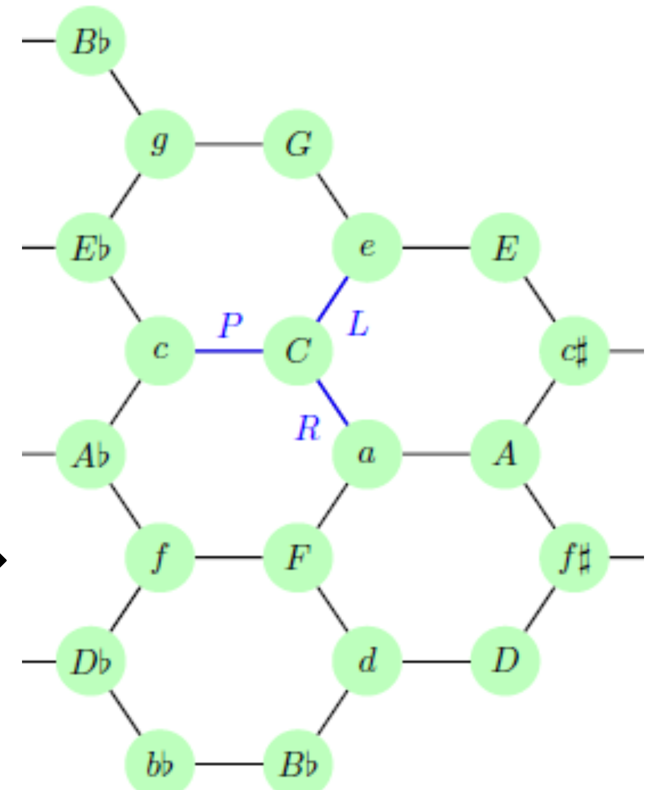

R: $C_{maj} \rightarrow A_{min}$

L: $C_{maj} \rightarrow E_{min}$

P: $C_{maj} \rightarrow C_{min}$

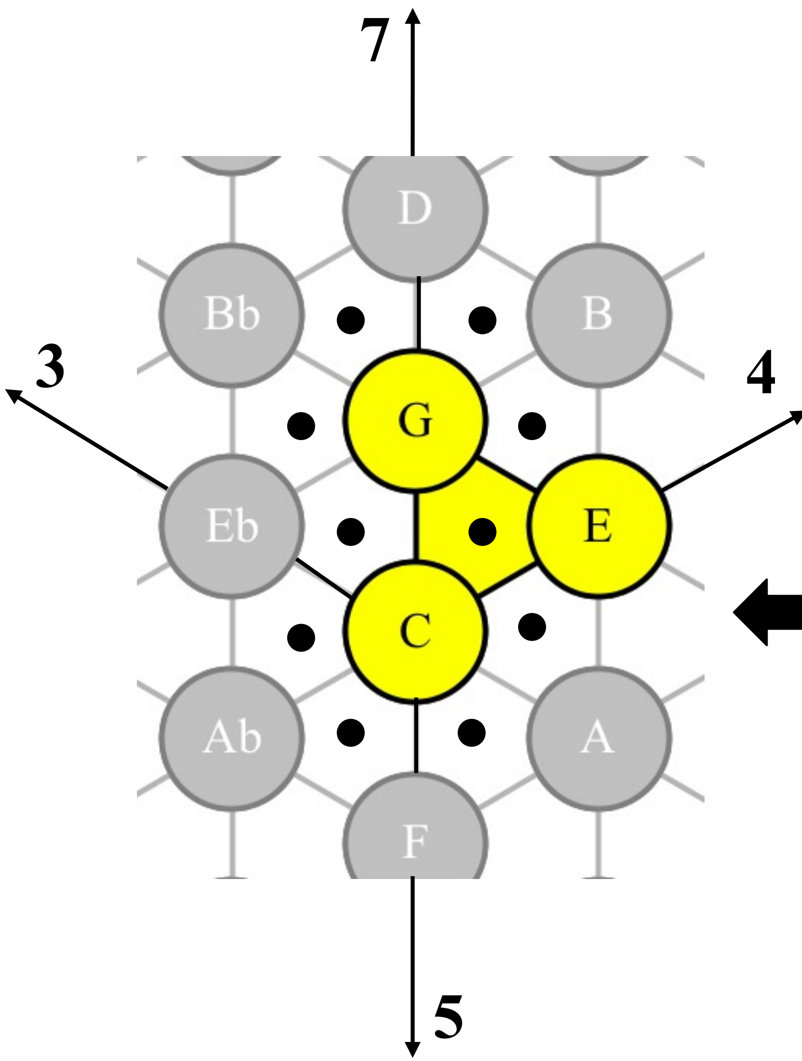
# From the Tonnetz to the dual one (and vice-versa)

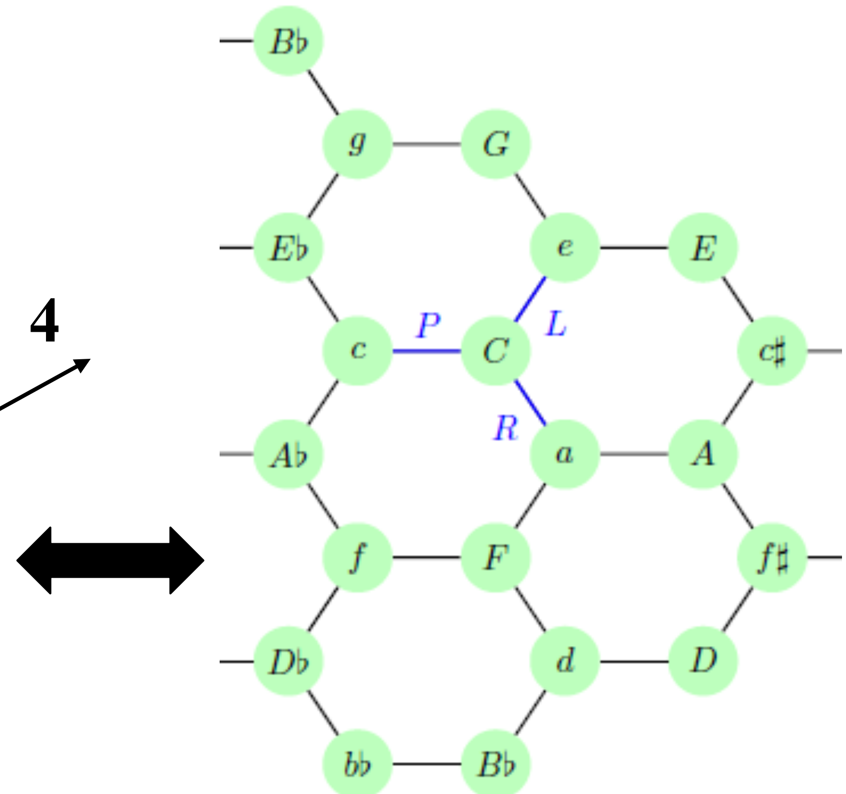



[Sonia Cannas, 2018]

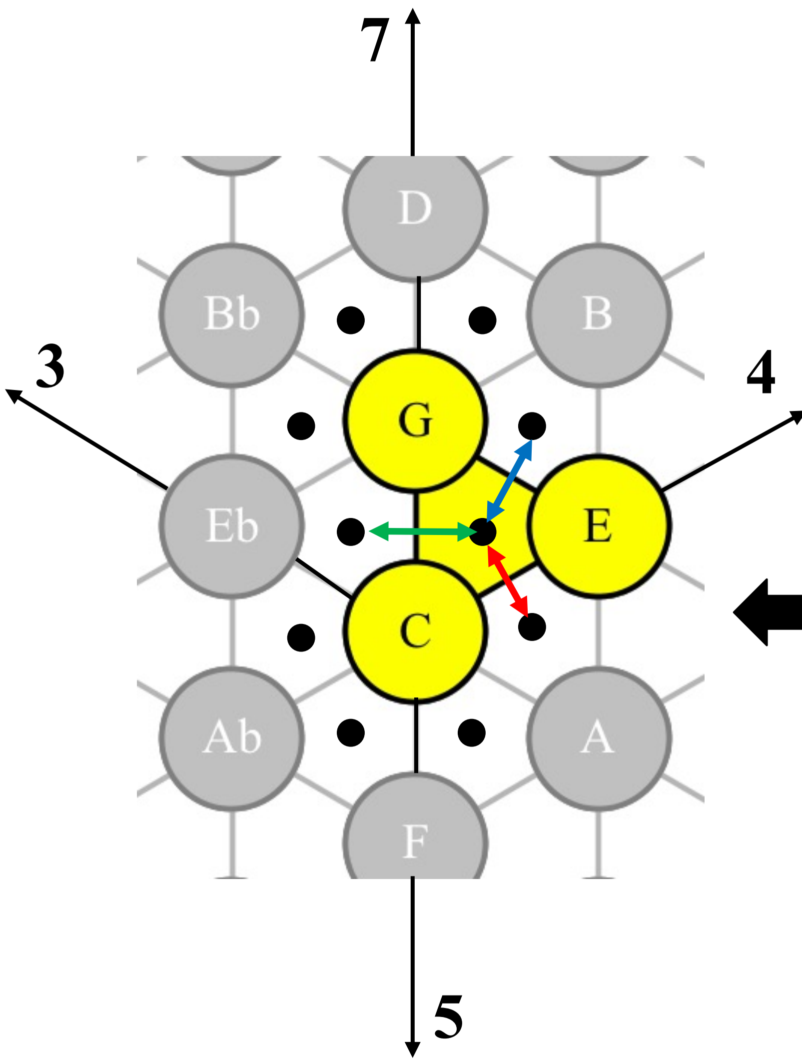
# From the Tonnetz to the dual one (and vice-versa)

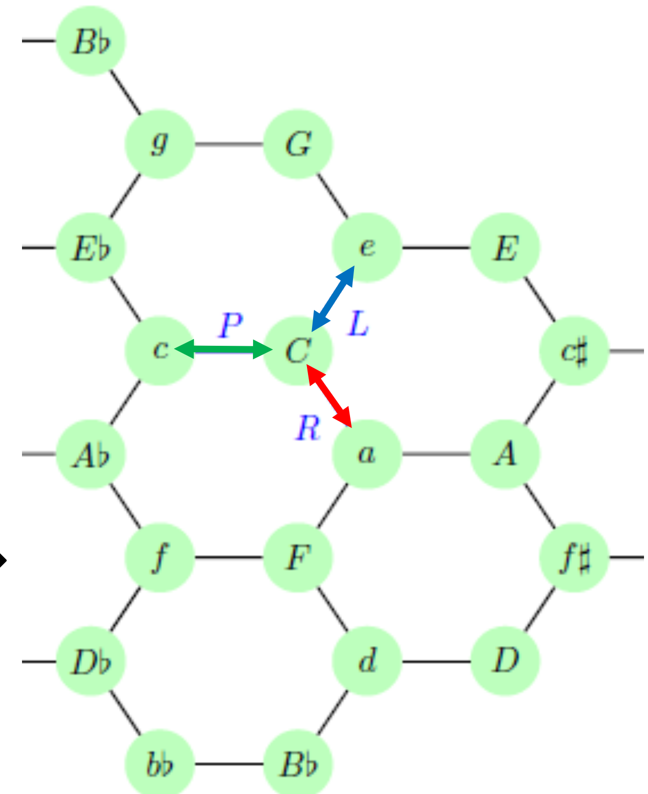



**[Sonia Cannas, 2018]**

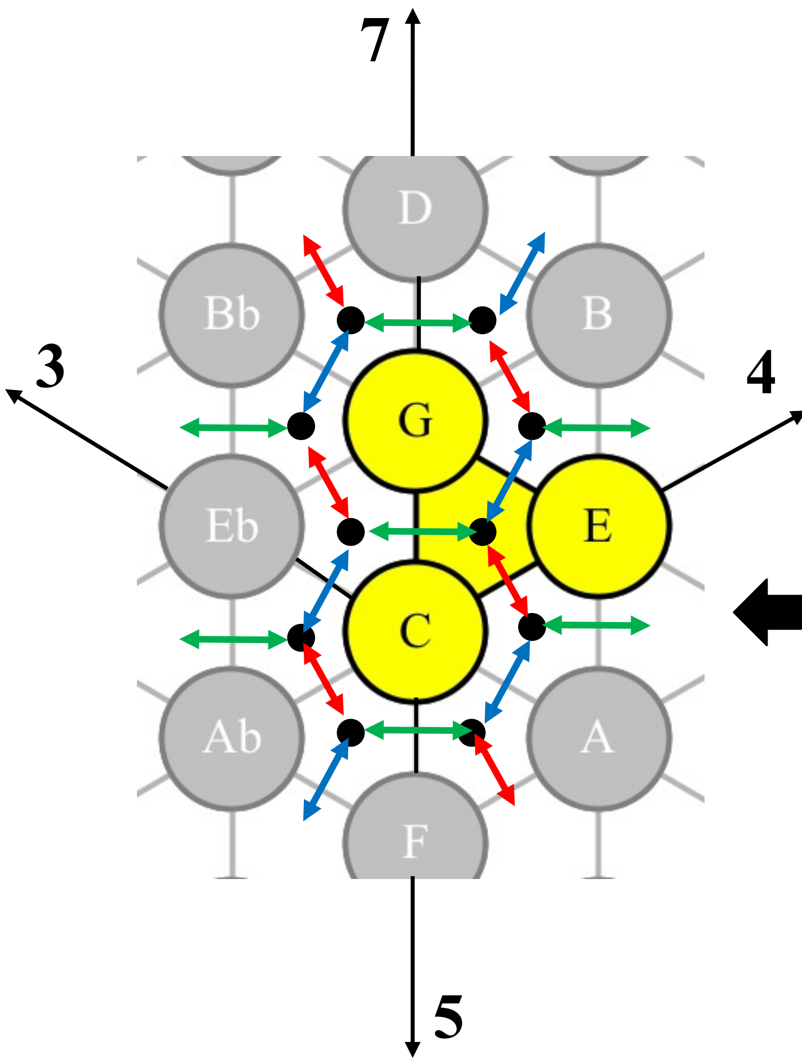
# From the Tonnetz to the dual one (and vice-versa)

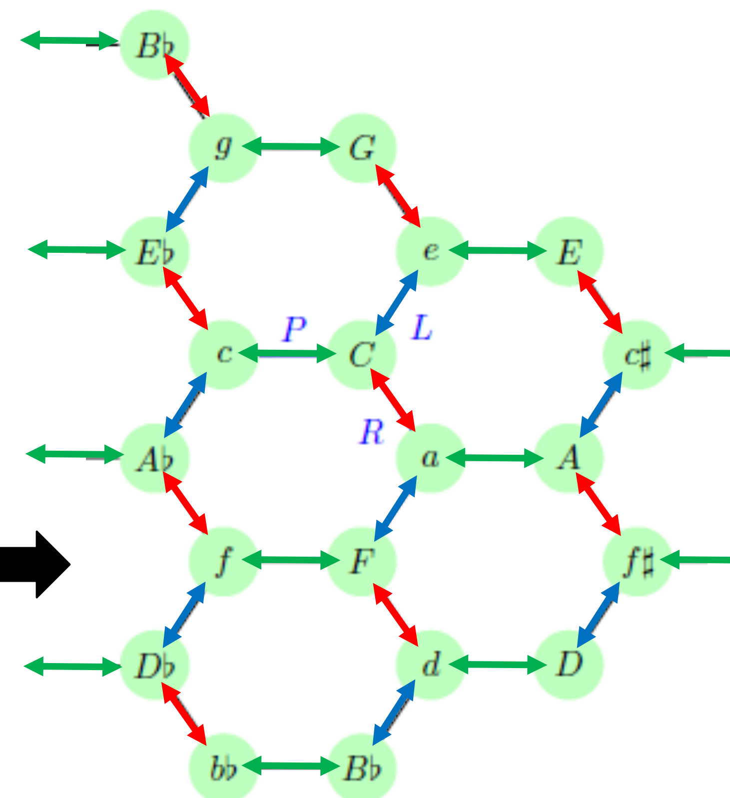



[Sonia Cannas, 2018]

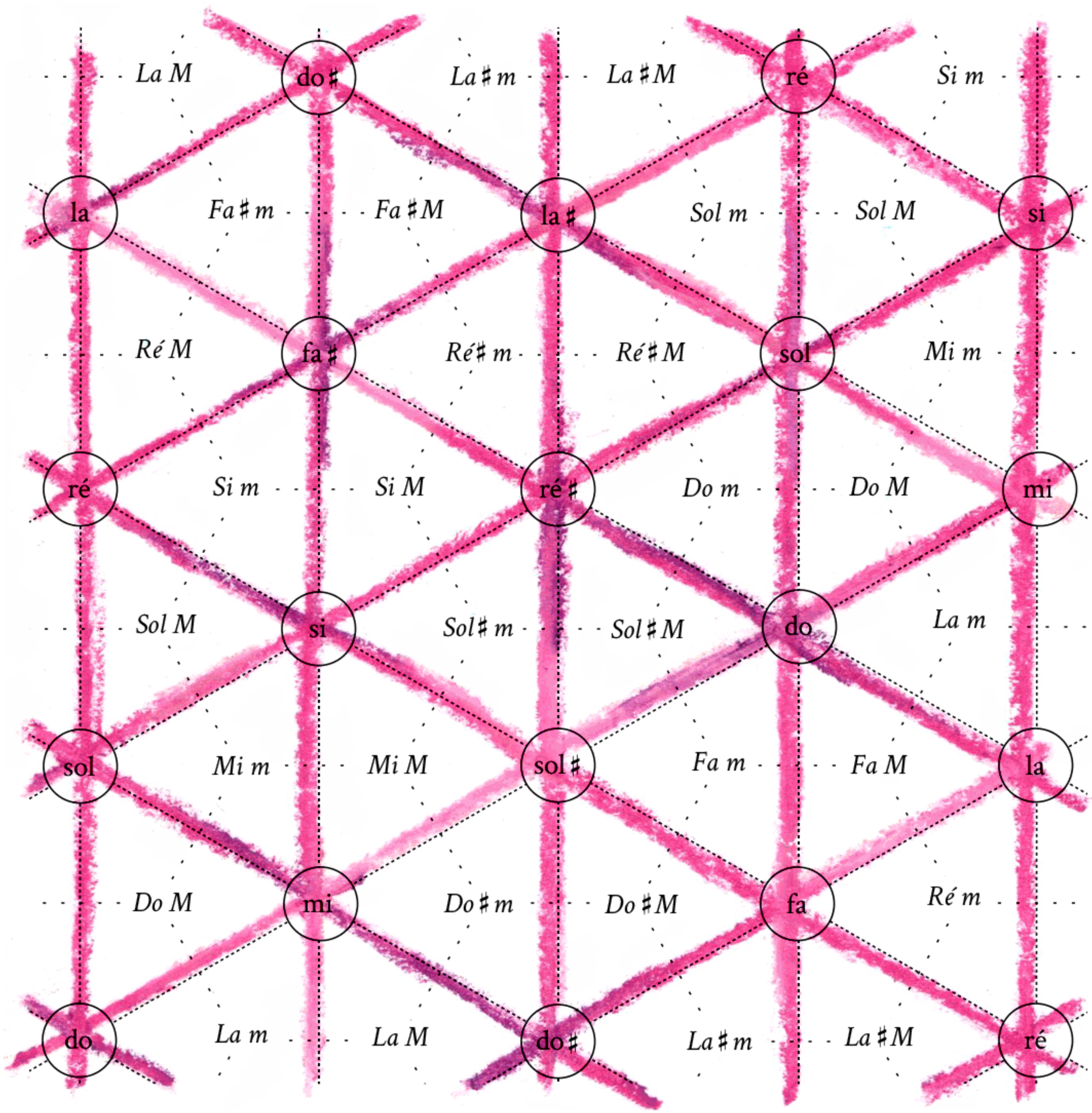
# From the Tonnetz to the dual one (and vice-versa)



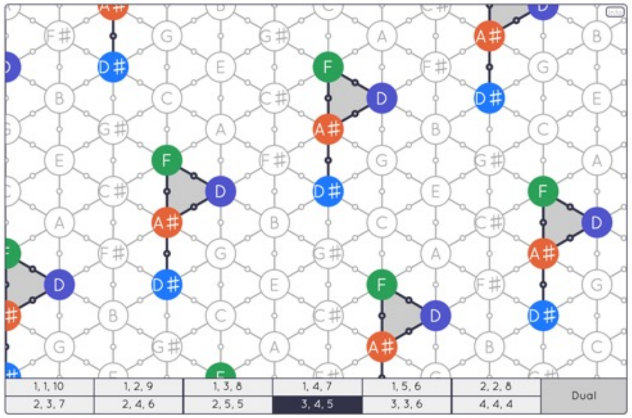




[Sonia Cannas, 2018]

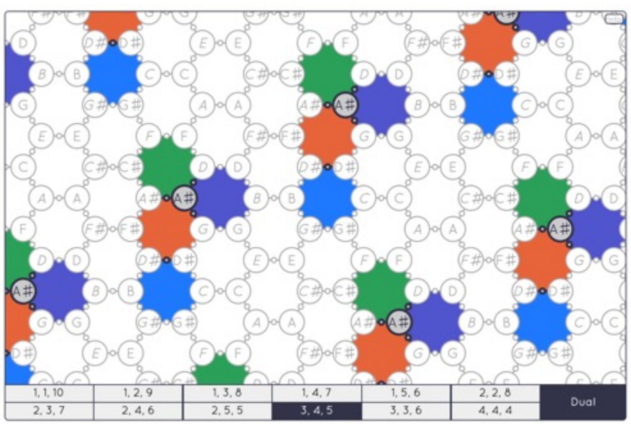
# From the Tonnetz to the dual one (and vice-versa)

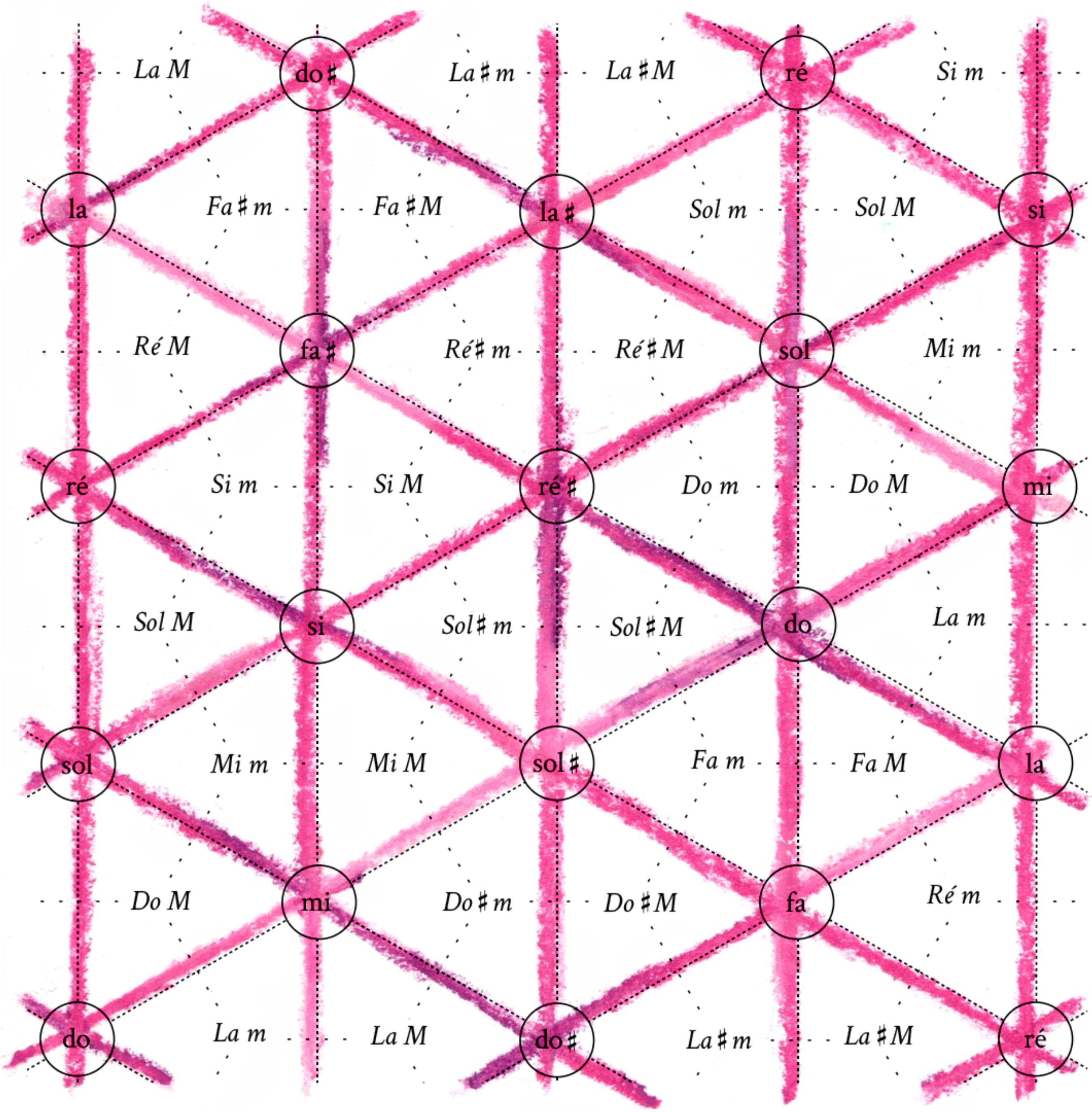



# From the *Tonnetz* to the dual one



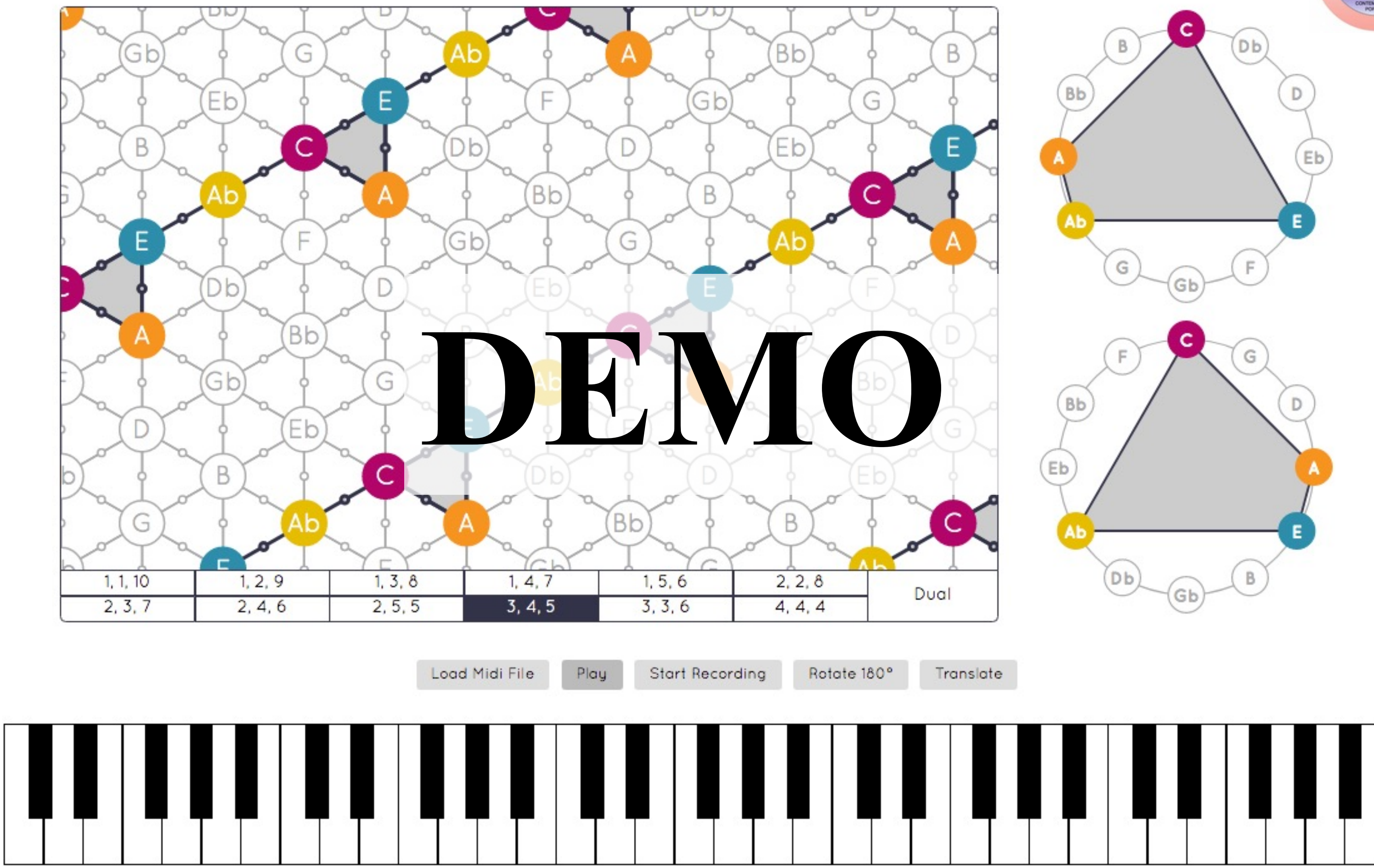
duality





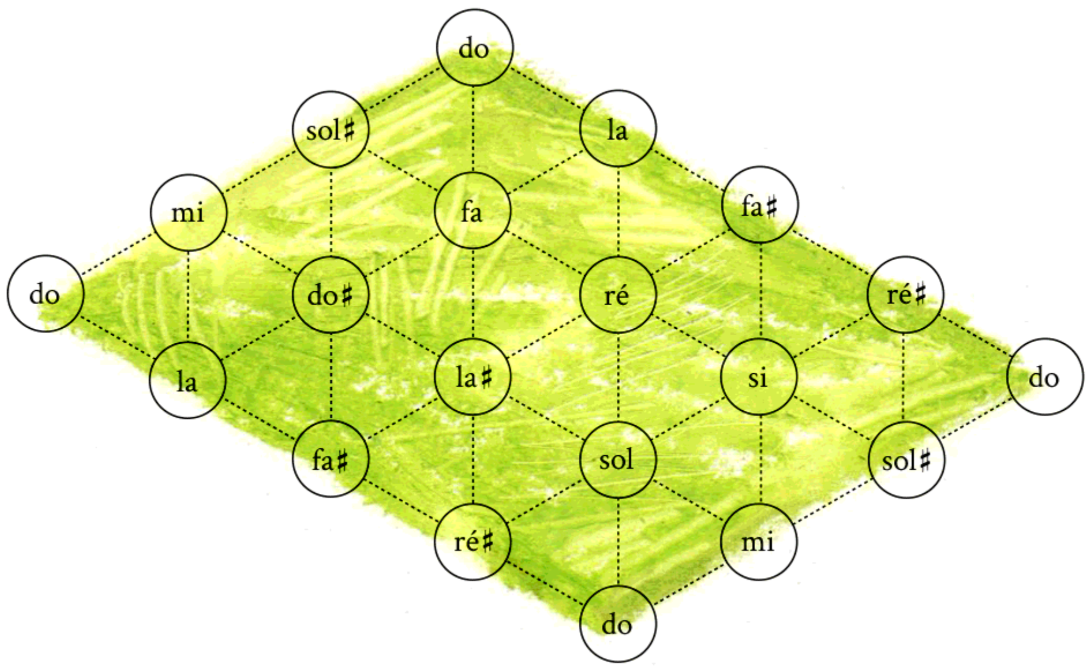
# THE TONNETZ

ONE KEY – MANY REPRESENTATIONS




➔ **https://thetonnetz.com/**

# The topological structure of the *Tonnetz*



# The topological structure of the *Tonnetz*

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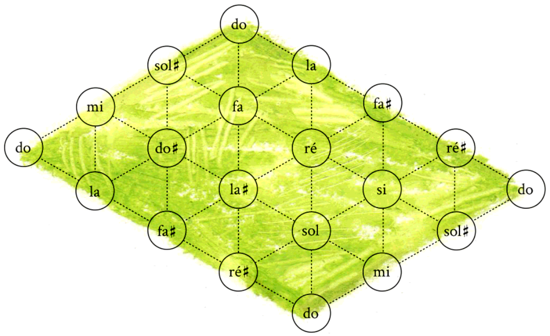
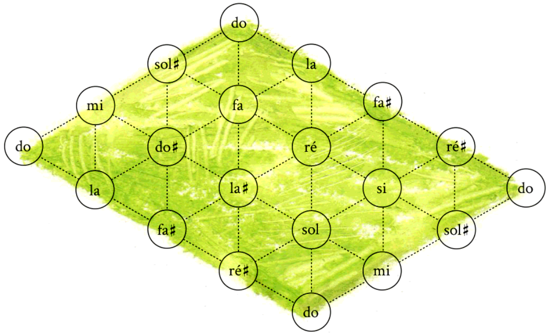
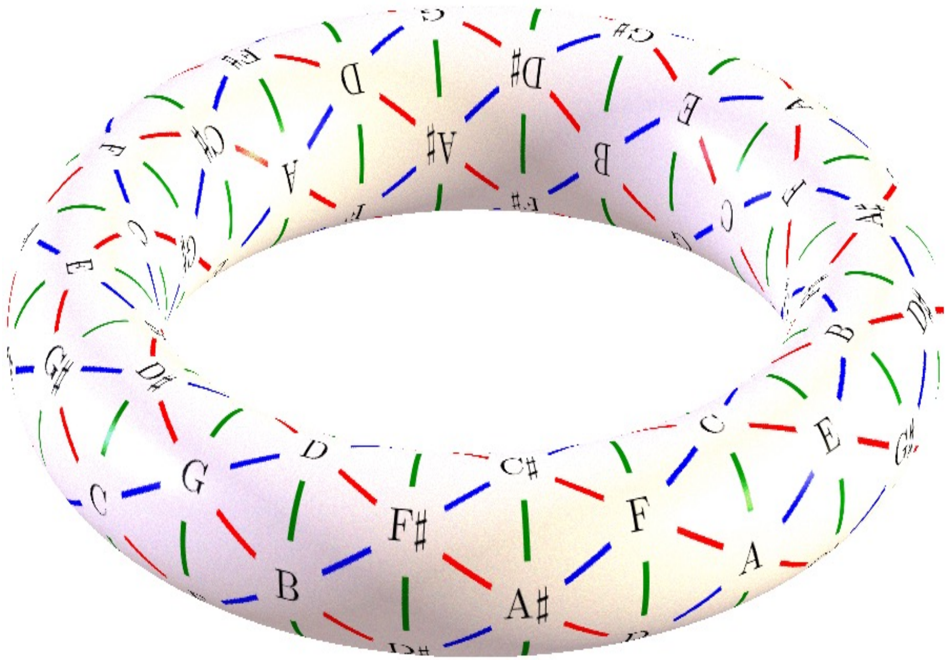



# The topological structure of the *Tonnetz*



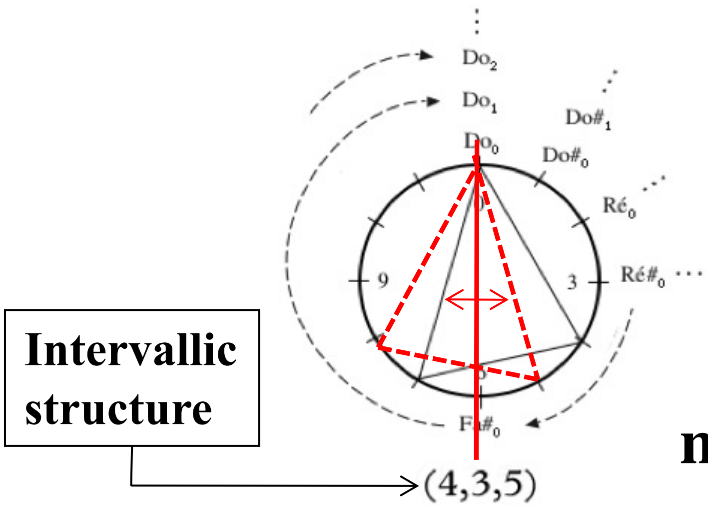
(Source: www.wikimedia.org/)

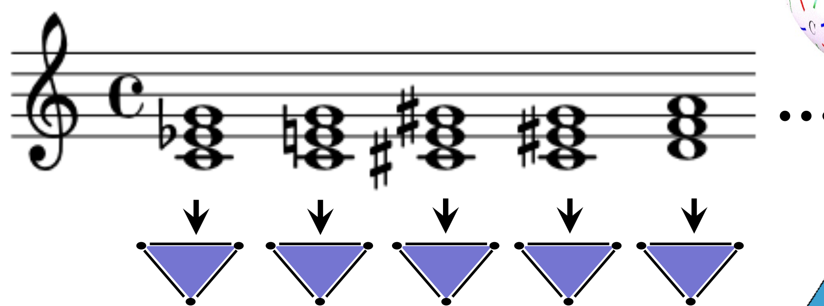
Louis Bigo

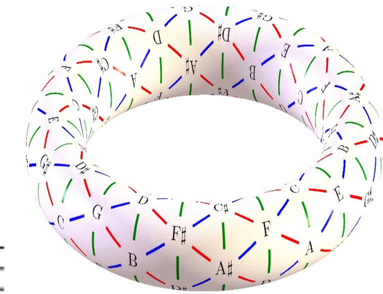
# The *Tonnetz* as a simplicial complex

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

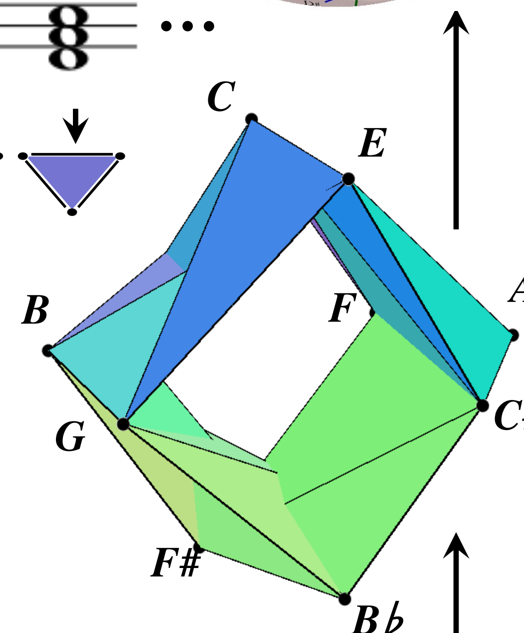
- Assembling chords related by some equivalence relation
  - Equivalence up to transposition/inversion:

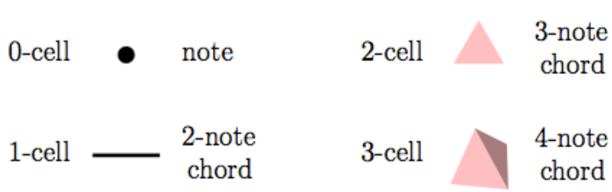


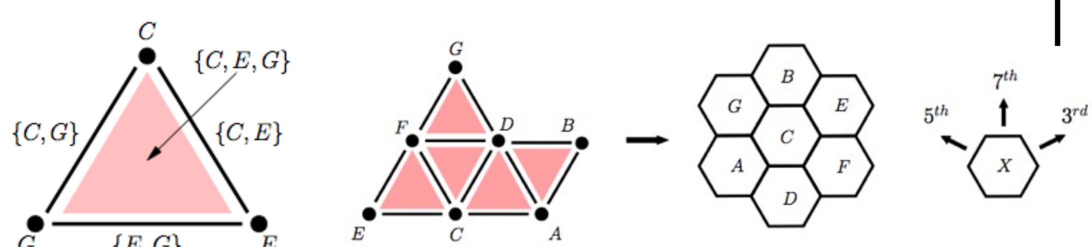





**major/minor triads**



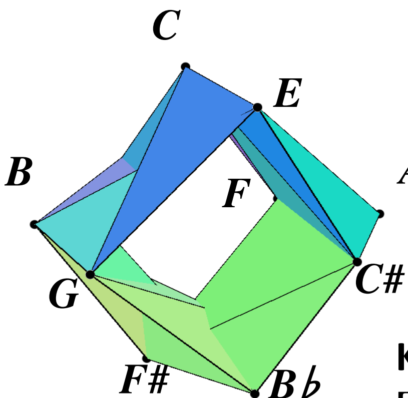



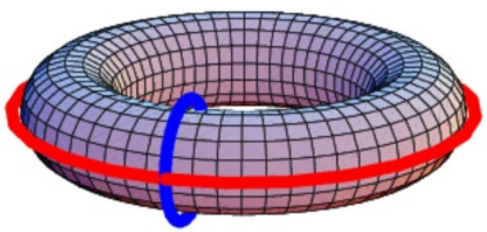
# Classifying Chord Complexes

L. Bigo, *Représentation symboliques musicales et calcul spatial*, PhD, Ircam / LACL, 2013

- Complexes enumeration in the chromatic system

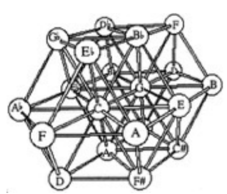



$K_{TI}[3,4,5]$
[Cohn – 1997]

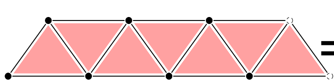
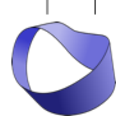


$\beta_0=1$
$\beta_1=2$
$\beta_2=1$

$K_{TI}[2,3,3,4]$
[Gollin - 1998]



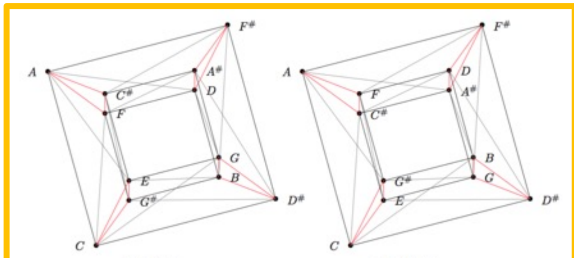
$K_{T}[2,2,3]$
[Mazzola – 2002]

 = 

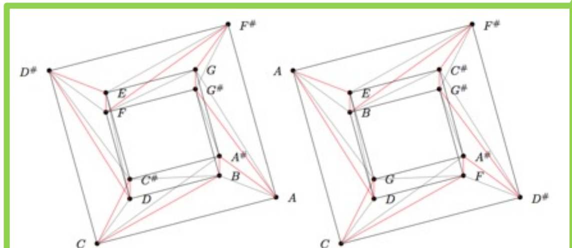
| $d$ | complexe | taille | $b_n$ | p-v | $\chi$ |
|---|---|---|---|---|---|
| - | $\mathcal{K}_\emptyset$ | 0 | 0 | | 0 |
| 0 | $\mathcal{K}_{TI}[0]$ | 0 | [0] | | 0 |
| 1 | $\mathcal{K}_{TI}[1,11]$ | 12 | [1, 1] | x | 0 |
| | $\mathcal{K}_{TI}[2,10]$ | 12 | [2, 2] | | 0 |
| | $\mathcal{K}_{TI}[3,9]$ | 12 | [3, 3] | | 0 |
| | $\mathcal{K}_{TI}[4,8]$ | 12 | [4, 4] | | 0 |
| | $\mathcal{K}_{TI}[5,7]$ | 12 | [1, 1] | x | 0 |
| | $\mathcal{K}_{TI}[6,6]$ | 6 | [6, 0] | | 6 |
| 2 | $\mathcal{K}_{TI}[1,1,10]$ | 12 | [1, 1, 0] | x | 0 |
| | $\mathcal{K}_{TI}[1,2,9]$ | 24 | [1, 2, 1] | x | 0 |
| | $\mathcal{K}_{TI}[1,3,8]$ | 24 | [1, 2, 1] | x | 0 |
| | $\mathcal{K}_{TI}[1,4,7]$ | 24 | [1, 2, 1] | x | 0 |
| | $\mathcal{K}_{TI}[1,5,6]$ | 24 | [1, 1, 6] | | 6 |
| | $\mathcal{K}_{TI}[2,2,8]$ | 12 | [2, 2, 0] | | 0 |
| | $\mathcal{K}_{TI}[2,3,7]$ | 24 | [1, 2, 1] | x | 0 |
| | $\mathcal{K}_{TI}[2,4,6]$ | 24 | [2, 2, 6] | | 6 |
| | $\mathcal{K}_{TI}[2,5,5]$ | 12 | [1, 1, 0] | x | 0 |
| | $\mathcal{K}_{TI}[3,3,6]$ | 12 | [3, 0, 3] | | 6 |
| | $\mathcal{K}_{TI}[3,4,5]$ | 24 | [1, 2, 1] | x | 0 |
| | $\mathcal{K}_{TI}[4,4,4]$ | 4 | [4, 0, 0] | | 4 |
| | $\mathcal{K}_{TI}[1,1,1,9]$ | 12 | [1, 1, 0, 0] | x | 0 |
| | $\mathcal{K}_{TI}[1,1,2,8]$ | 24 | [1, 1, 12, 0] | | 12 |
| | $\mathcal{K}_{TI}[1,1,3,7]$ | 24 | [1, 2, 13, 0] | | 12 |
| | $\mathcal{K}_{TI}[1,1,4,6]$ | 24 | [1, 1, 18, 0] | | 18 |
| | $\mathcal{K}_{TI}[1,1,5,5]$ | 12 | [1, 1, 6, 0] | | 6 |

# Classifying Tonnetze as Simplicial Chord Complexes

P. Lascabettes, *Homologie Persistante Appliquée à la Reconnaissance de Genres Musicaux*, M1, ENS Saclay, 2018

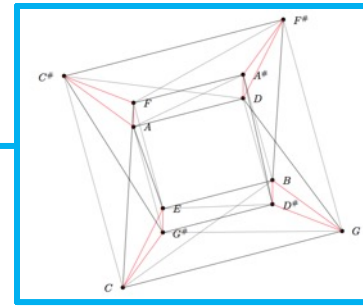



(a) T[3,4,5] (b) T[1,3,8]

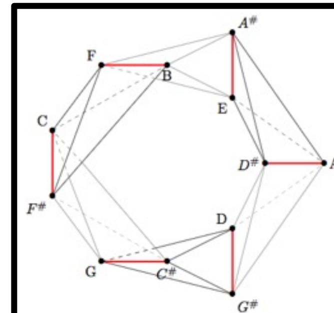



(a) T[1,2,9] (b) T[2,3,7]

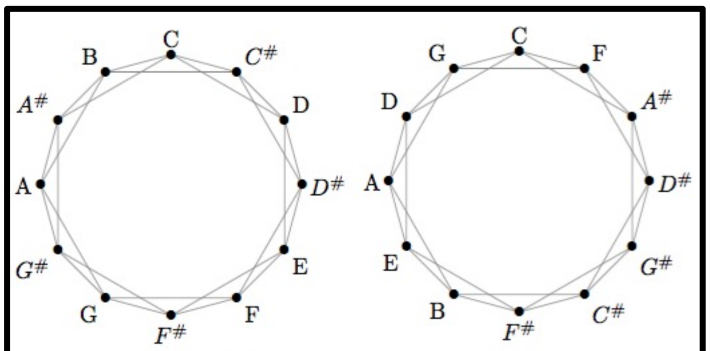
| Tonnetz | Nombres de Betti | | |
|---|---|---|---|
| | $\beta_0$ | $\beta_1$ | $\beta_2$ |
| $T[1,2,9]$ | 1 | 2 | 1 |
| $T[1,3,8]$ | 1 | 2 | 1 |
| $T[1,4,7]$ | 1 | 2 | 1 |
| $T[2,3,7]$ | 1 | 2 | 1 |
| $T[3,4,5]$ | 1 | 2 | 1 |
| $T[1,1,10]$ | 1 | 1 | 0 |
| $T[2,5,5]$ | 1 | 1 | 0 |
| $T[2,2,8]$ | 2 | 2 | 0 |
| $T[1,5,6]$ | 1 | 1 | 6 |
| $T[2,4,6]$ | 2 | 2 | 6 |
| $T[3,3,6]$ | 3 | 0 | 3 |
| $T[4,4,4]$ | 4 | 0 | 0 |



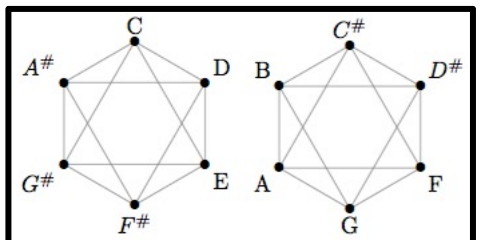

Paul Lascabettes



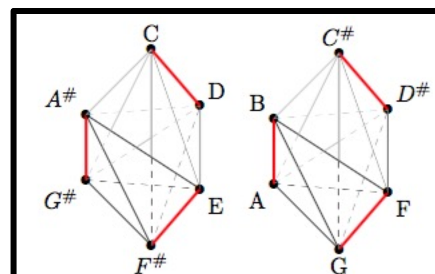

(a) $T[1,5,6]$



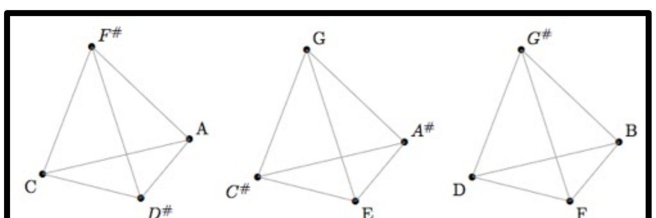

(a) $T[1,1,10]$ (b) $T[2,5,5]$



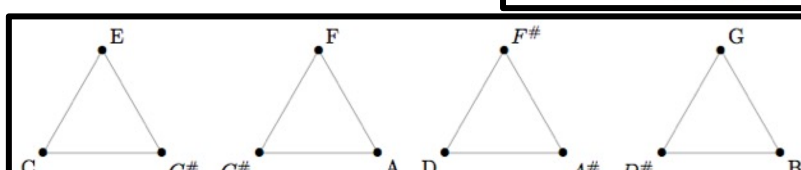

(c) $T[2,2,8]$




(b) $T[2,4,6]$

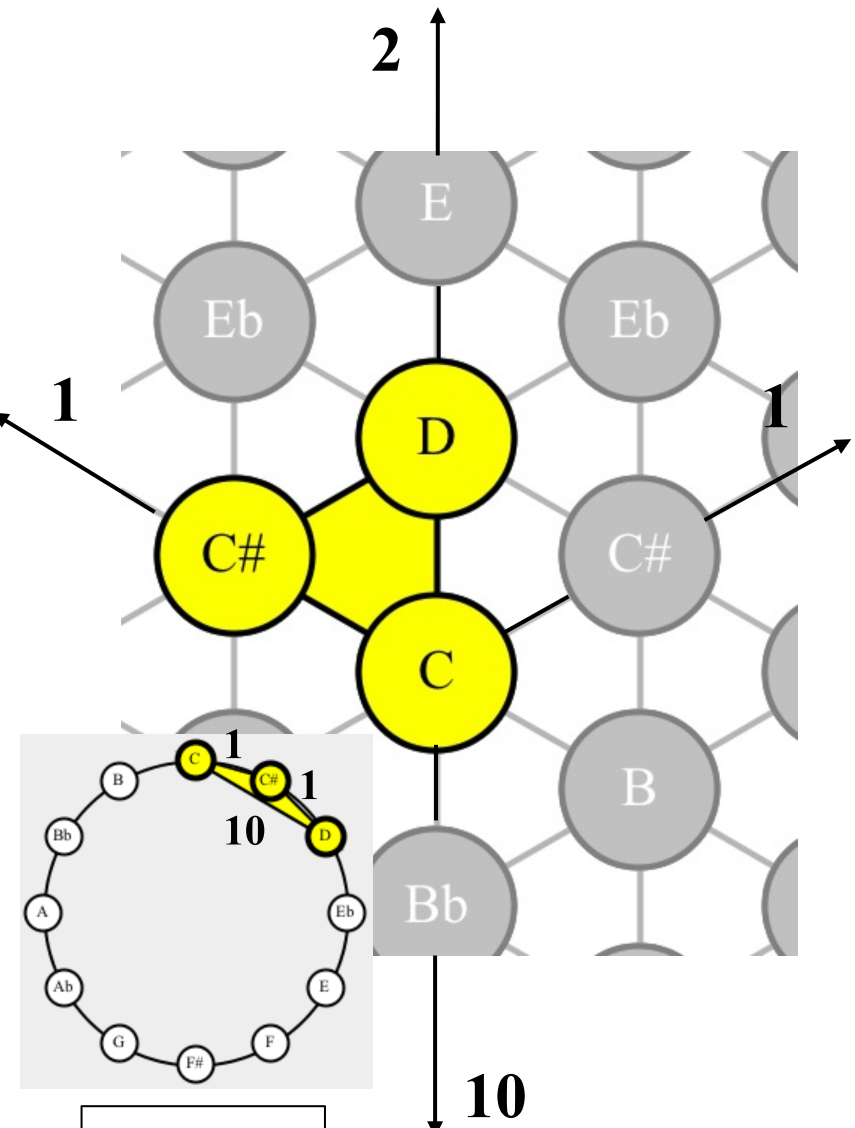



(a) $T[3,3,6]$

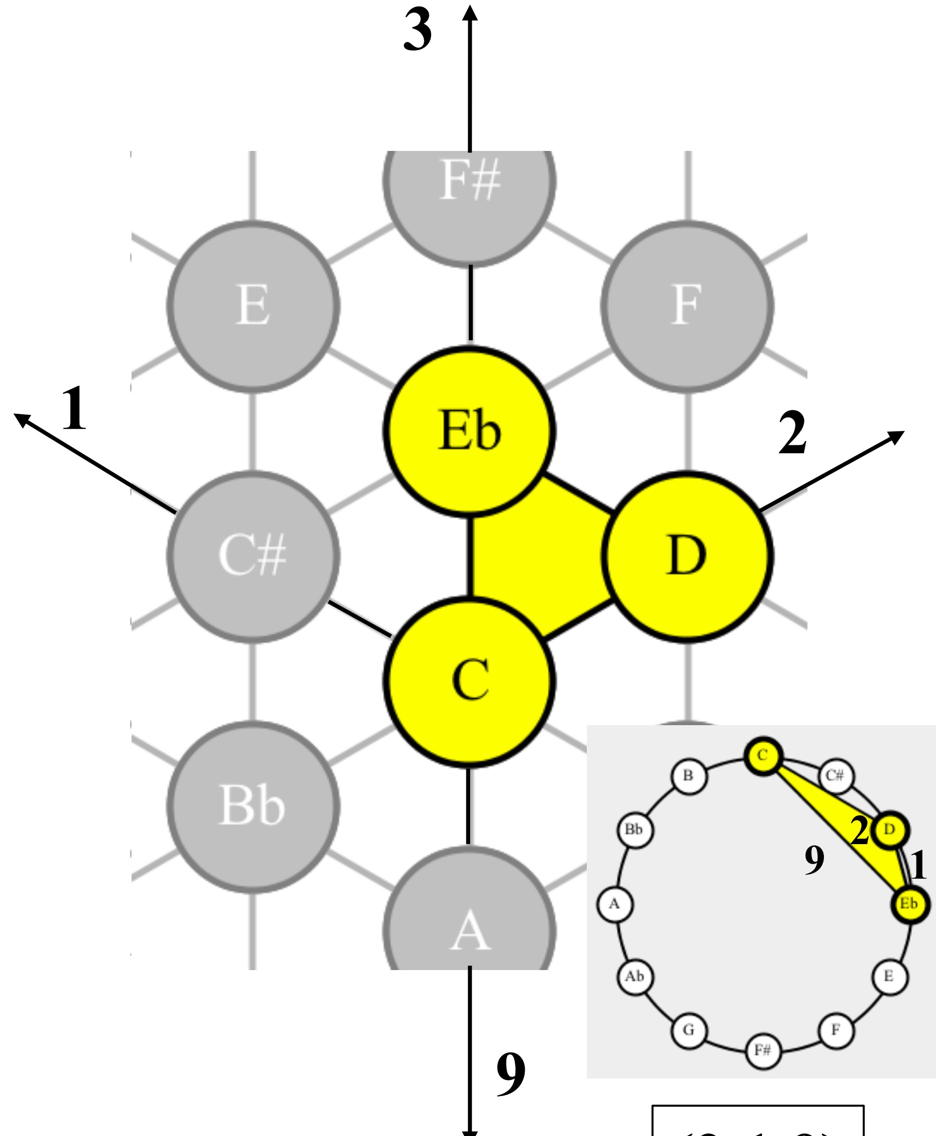



(b) T[4,4,4]

# The panoply of *Tonnetze* at the service of the analyst

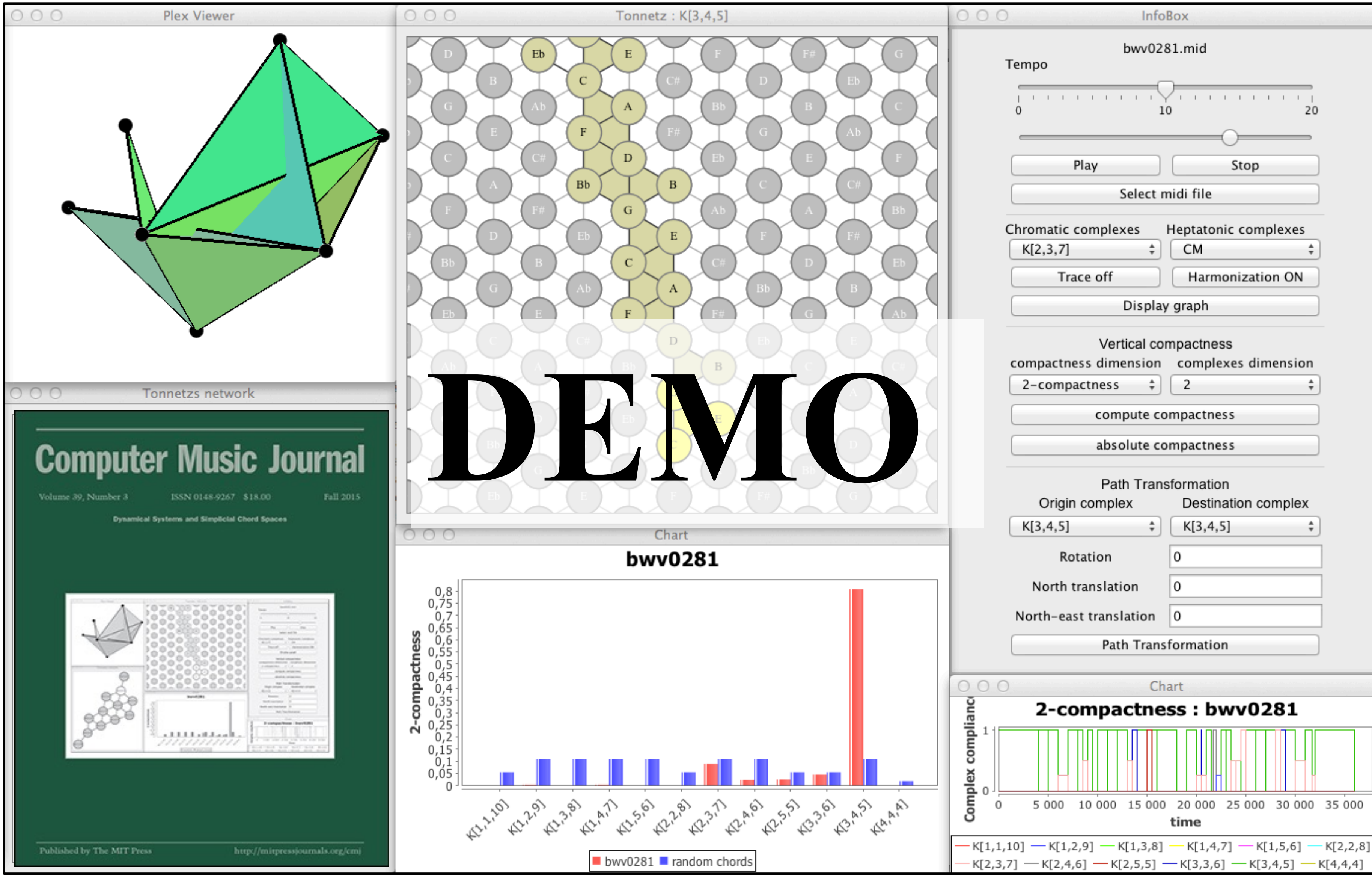



(1,1,10)



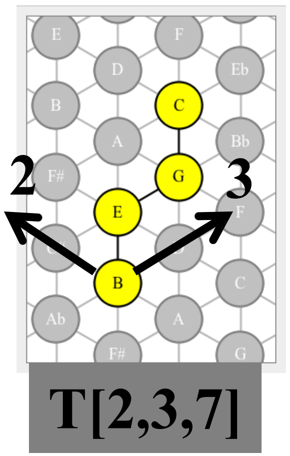

(2,1,9)

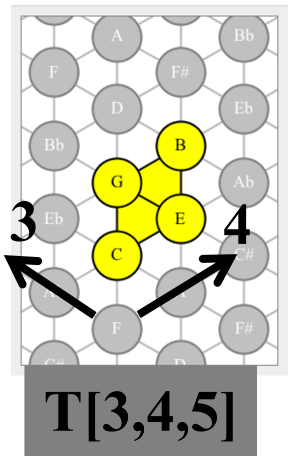
# Spatial music analysis via *Hexachord*

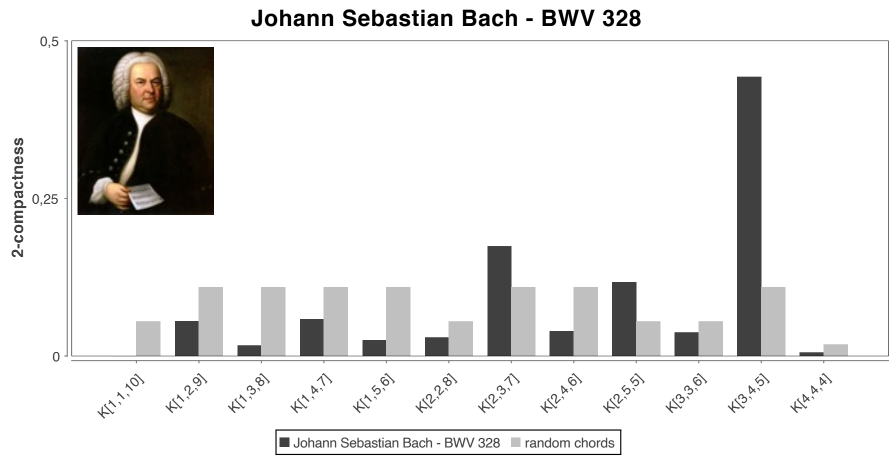


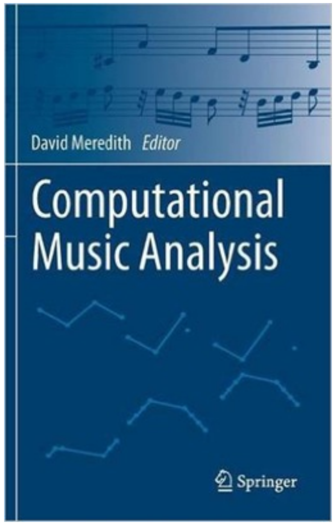

➔http://www.lacl.fr/~lbigo/hexachord

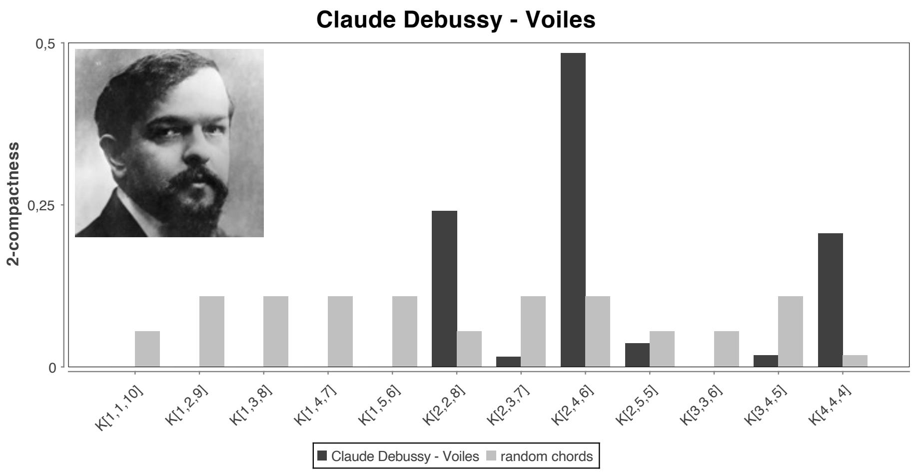
# The geometric character of musical logic

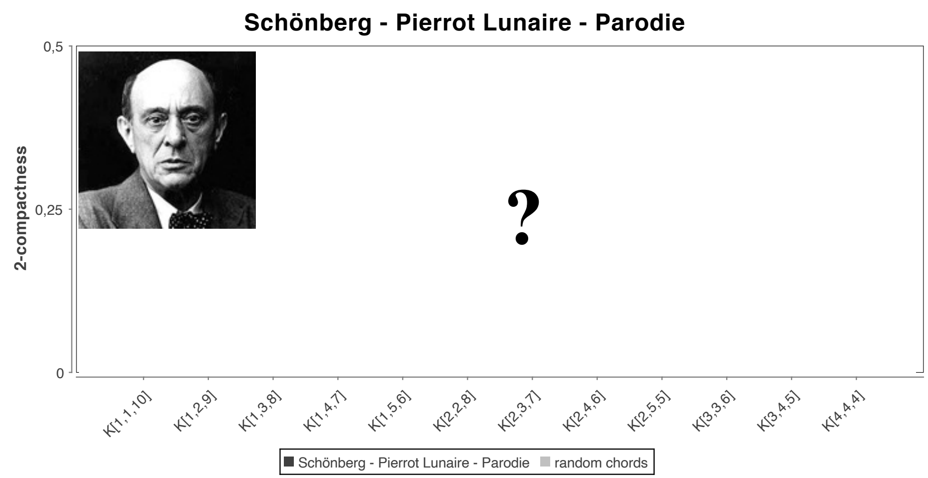


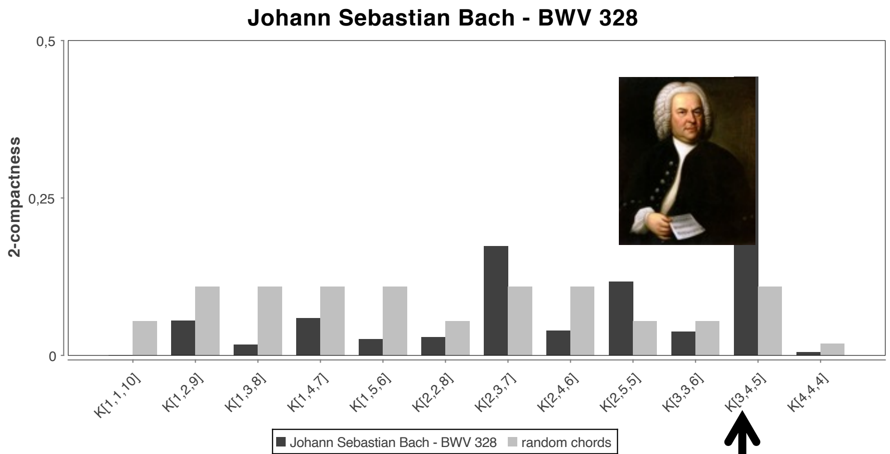



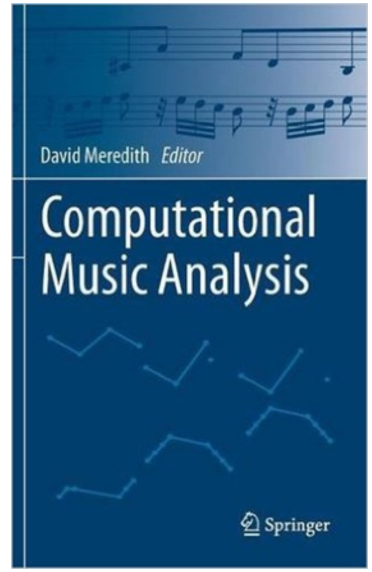



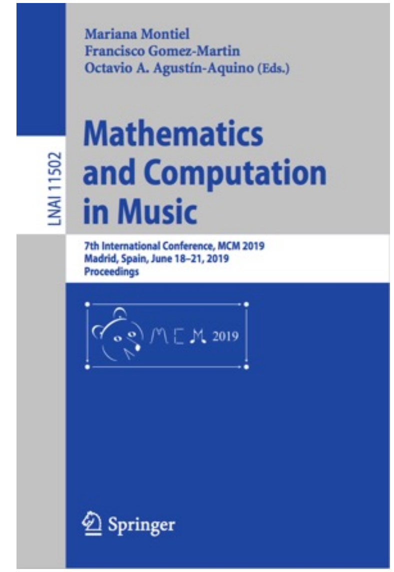

**Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015**

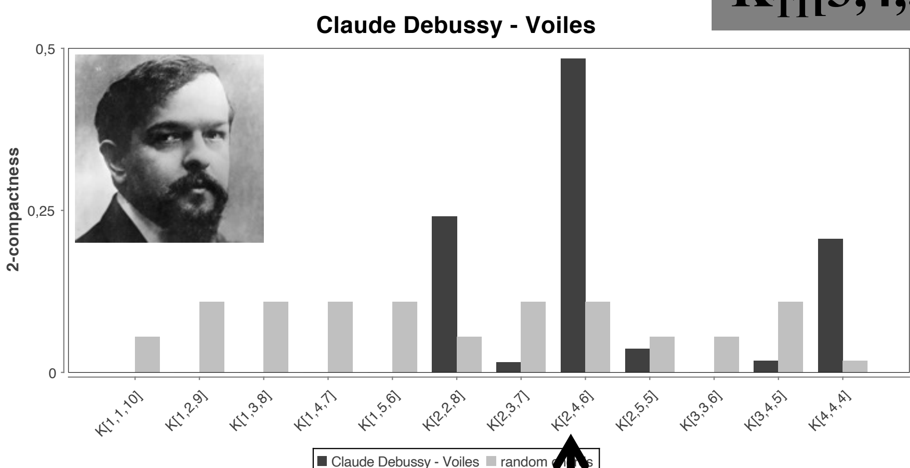
# The geometric character of musical logic



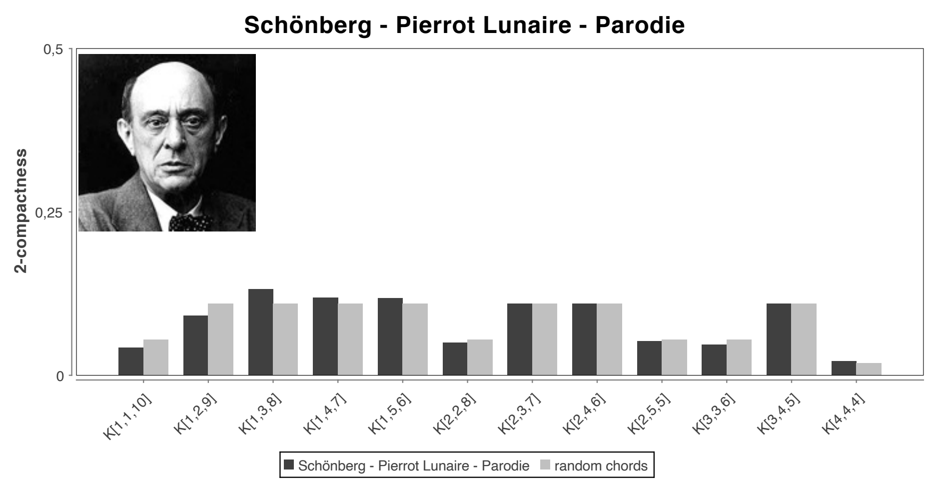

$K_{TI}[3,4,5]$



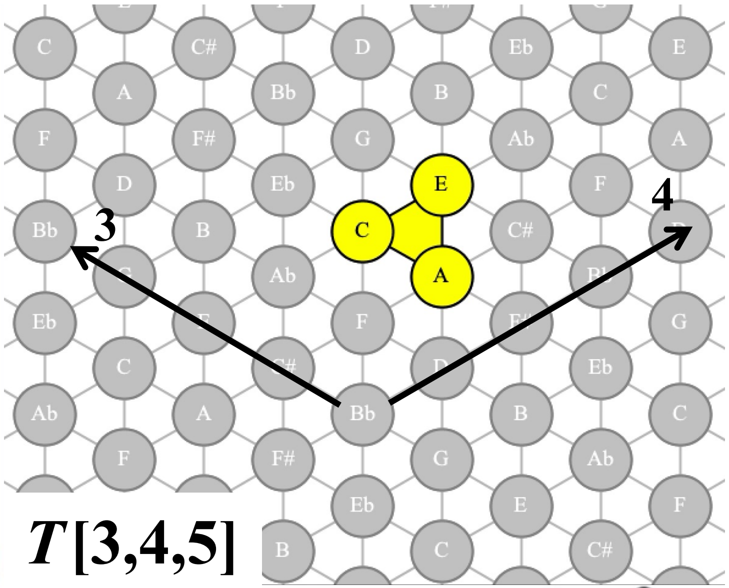




$K_{TI}[2,4,6]$

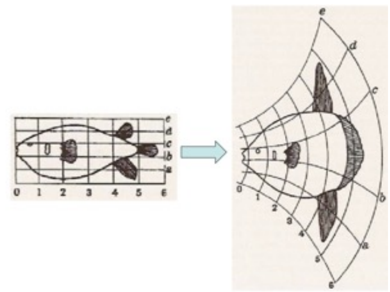


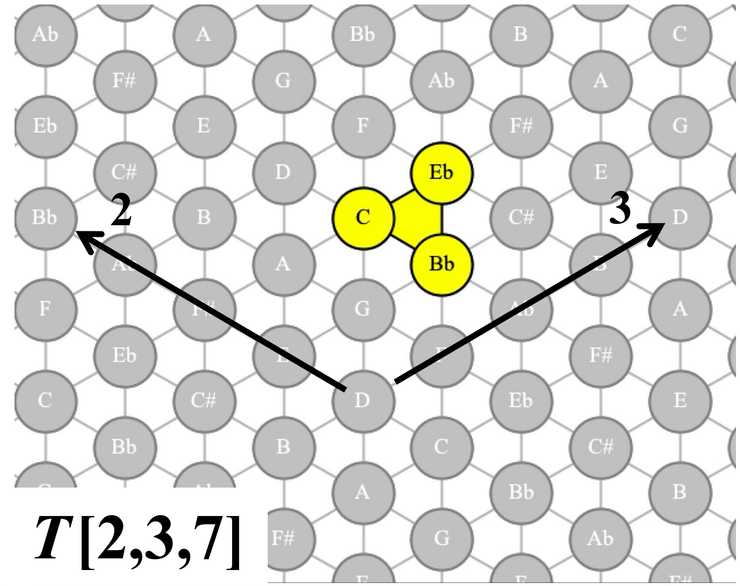

- **Bigo L., M. Andreatta, « Musical analysis with simplicial chord spaces », in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015**
- **Bigo L., M. Andreatta, « Filtration of Pitch-Class Sets Complexes », in M. Montiel et al. (eds.), Proceedings of MCM 2019, Madrid.**
- **Stage Master 2 Manos Karistineos (University Paris 7 / IRMA, 2018-2019 & Télécom-ParisTech / IRCAM 2019-2020)**

# Musical style and space trajectories

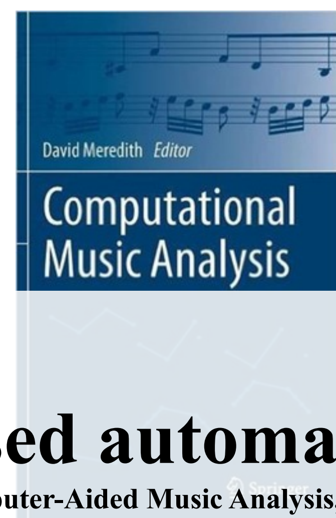



*T*[3,4,5]





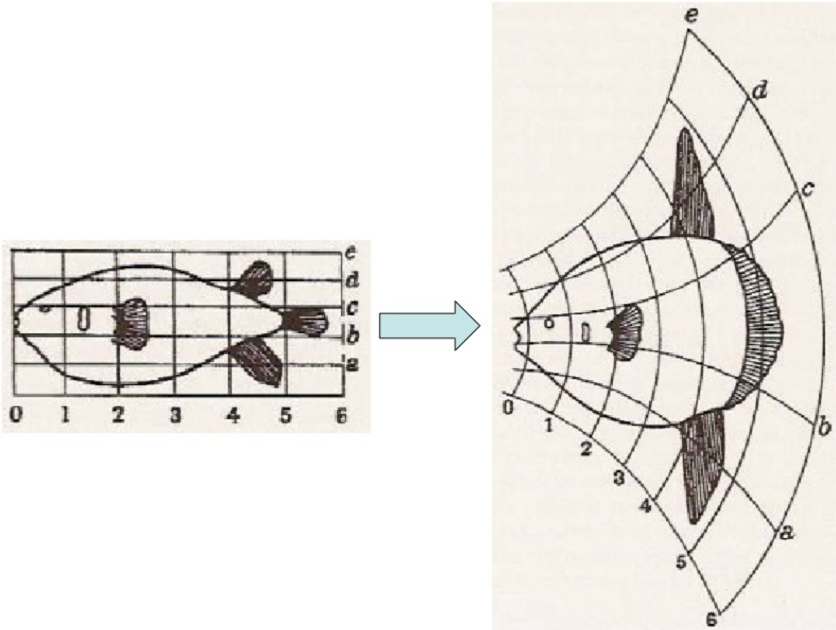

*T*[2,3,7]

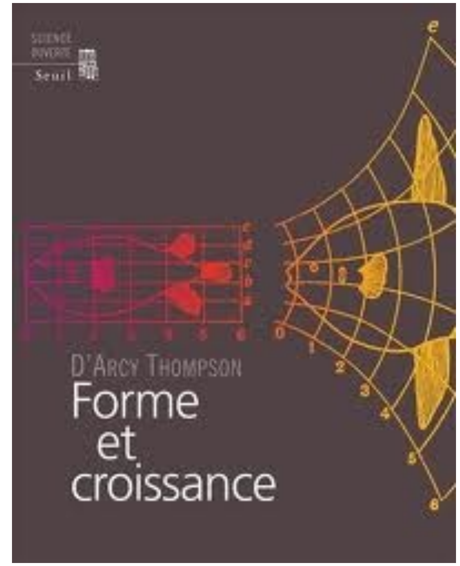



## Towards a geometry-based automatic musical style analysis

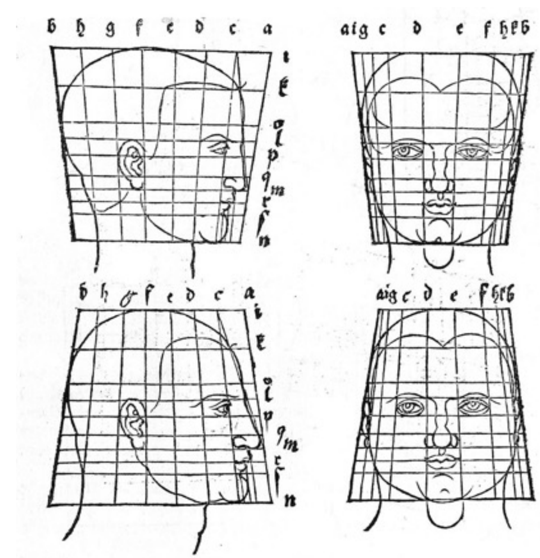
**Bigo L., M. Andreatta (2015), Topological Structures in Computer-Aided Music Analysis, in D. Meredith (ed.), *Computational Music Analysis*, Springer**

# The morphological *vs* the mathematical genealogy of the structuralism






"[The notion of **transformation**] comes from a work which played for me a very important role and which I have read during the war in the United States : *On Growth and Form*, in two volumes, by **D'Arcy Wentworth Thompson**, originally published in 1917. The author (...) proposes an interpretation of the visible transformations between the species (animals and vegetables) within a same gender. This was fascinating, in particular because I was quickly realizing that this perspective had a long tradition: behind Thompson, there was **Goethe's** botany and behind Goethe, **Albert Dürer** with his *Treatise of human proportions*" (Lévi-Strauss, conversation with Eribon, 1988).



# The study of the trajectories in the Tonnetz

La trajectoire spatiale dans le Tonnetz
comme outil de classification stylistique automatique
et de génération

Mémoire de stage

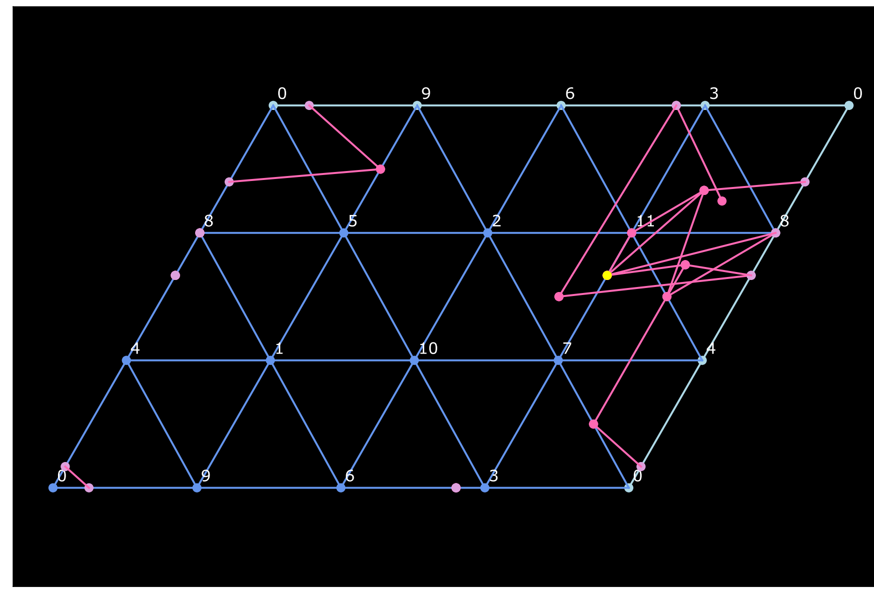
*Christophe WEIS*

*Encadrants :*
Isabelle BLOCH, Carlos AGON, Moreno ANDREATTA

*Structures d'accueil :*
LIP6 (Sorbonne Université), IRCAM (Équipe Représentations musicales)

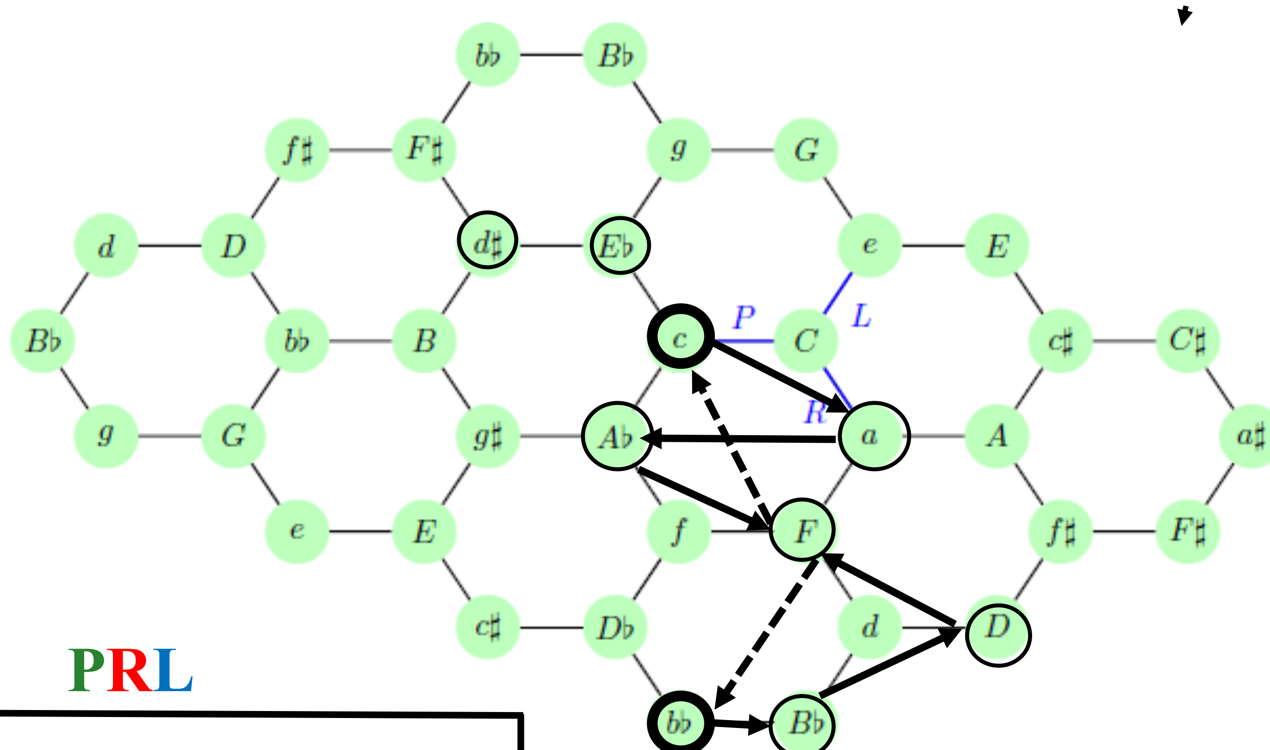
*1er mars - 31 août 2022*

WEIS
Christophe



# Harmonic progressions in the music by Arthur H


**Les Parures Secrètes**
**(album *Pour Madame X*, 2000)**




PRL

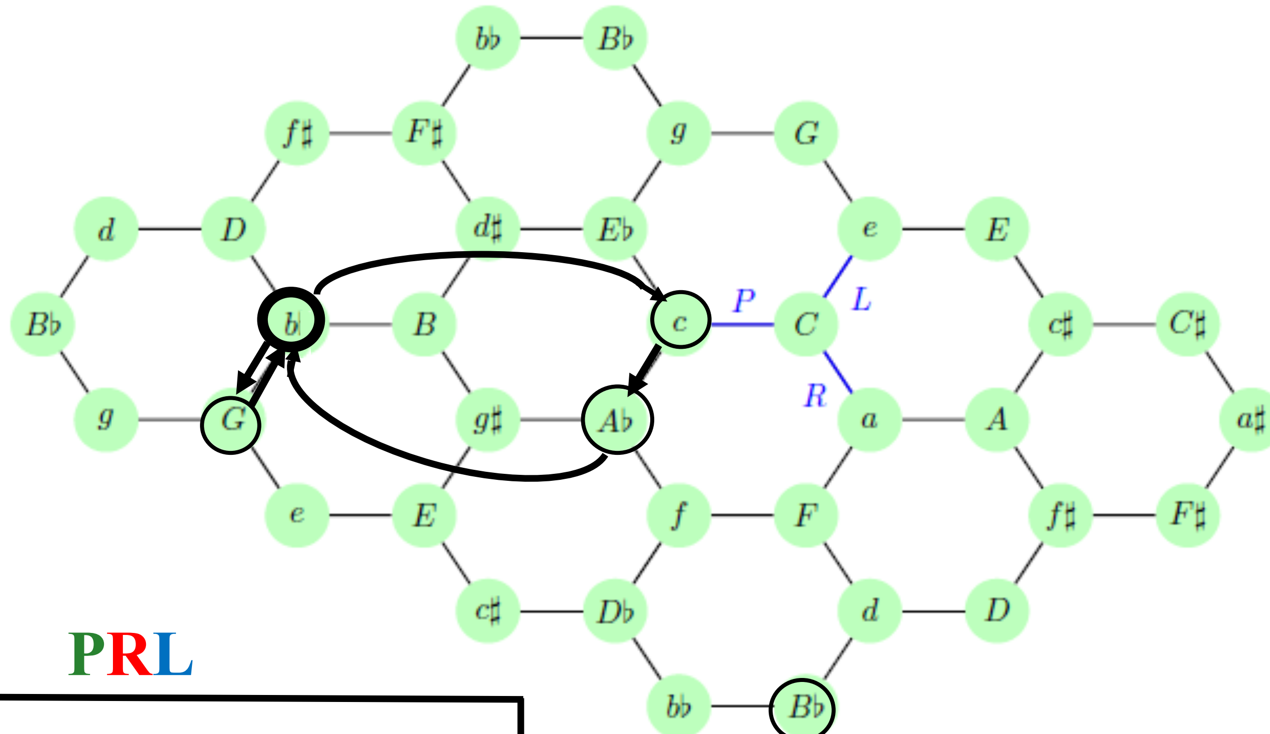
Cycle 1 : Cm $\xrightarrow{PR}$ Am $\xrightarrow{LPR}$ Ab $\xrightarrow{RP}$ F

Cycle 1 : Bbm $\xrightarrow{P}$ Bb $\xrightarrow{LP}$ D $\xrightarrow{PR}$ F

PLR=N

# Harmonic progressions in the music by Arthur H


**Le Baron noir**
**(album *Piano solo*, 2002)**

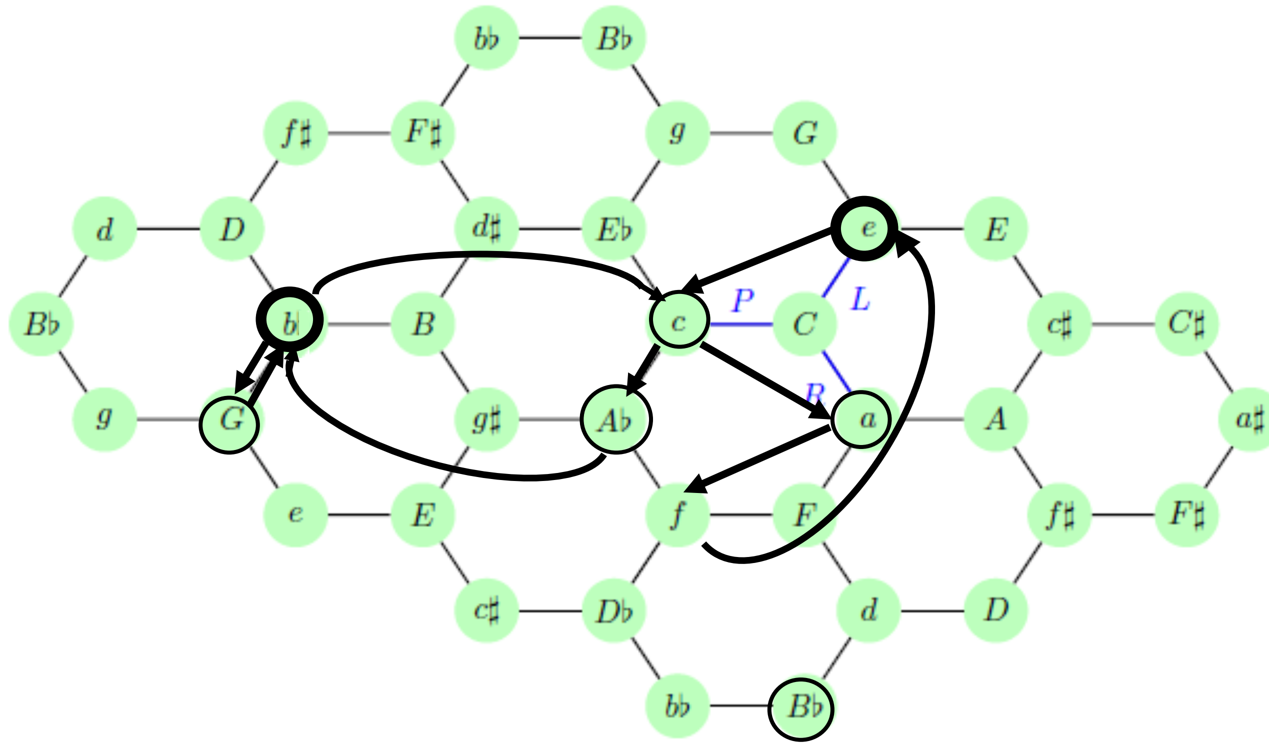



**PRL**

**Cycle 1 :** Bm $\xrightarrow{PLPR}$ Cm $\xrightarrow{LPR}$ Ab $\xrightarrow{PRP}$ Bm $\xrightarrow{L}$ G $\xrightarrow{L}$ Bm $\xrightarrow{L}$ G $\xrightarrow{L}$ Bm

# Harmonic progressions in the music by Arthur H


**Le Baron noir**
**(album *Piano solo*, 2002)**

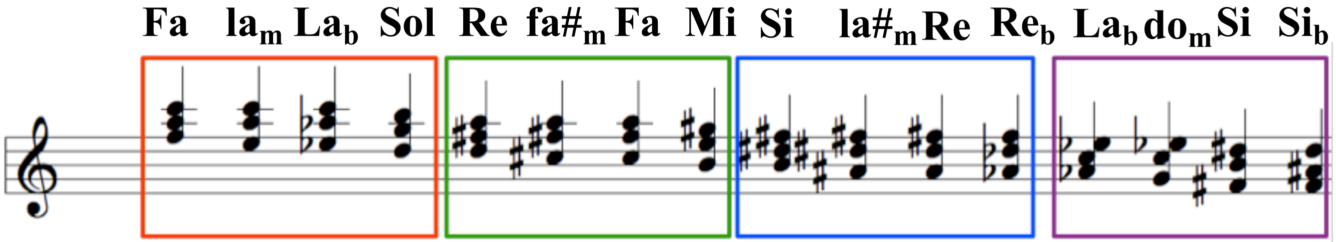


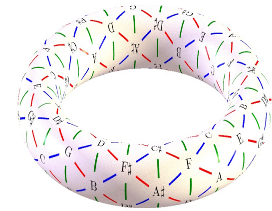

$$\mathrm{Em} \xrightarrow{LP} \mathrm{Cm} \xrightarrow{PL} \mathrm{Em} \xrightarrow{LP} \mathrm{Cm} \xrightarrow{PR} \mathrm{Am} \xrightarrow{LP} \mathrm{Fm}$$

PLRL=NL

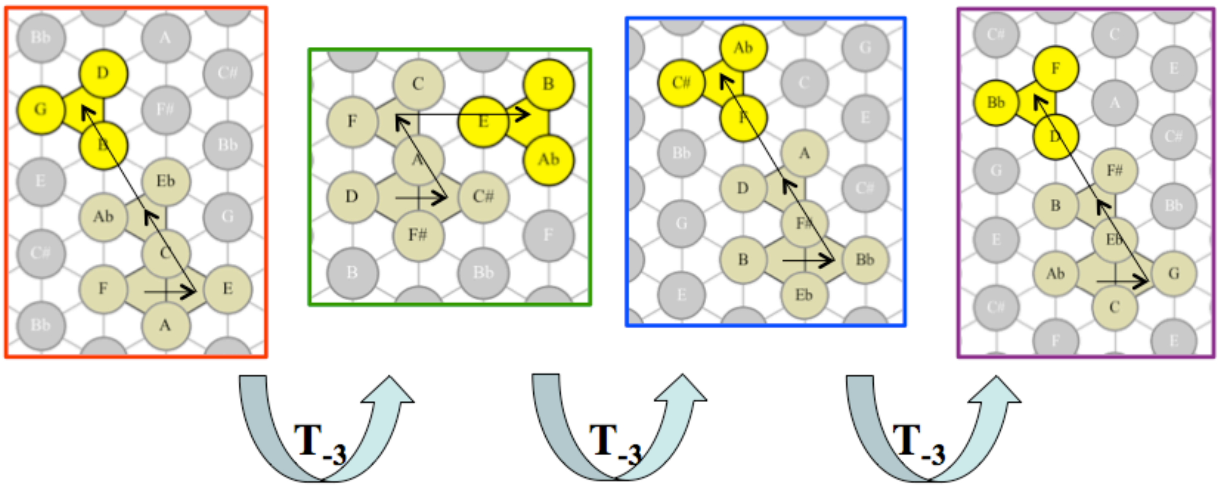

➔ **www.mathemusic.net**

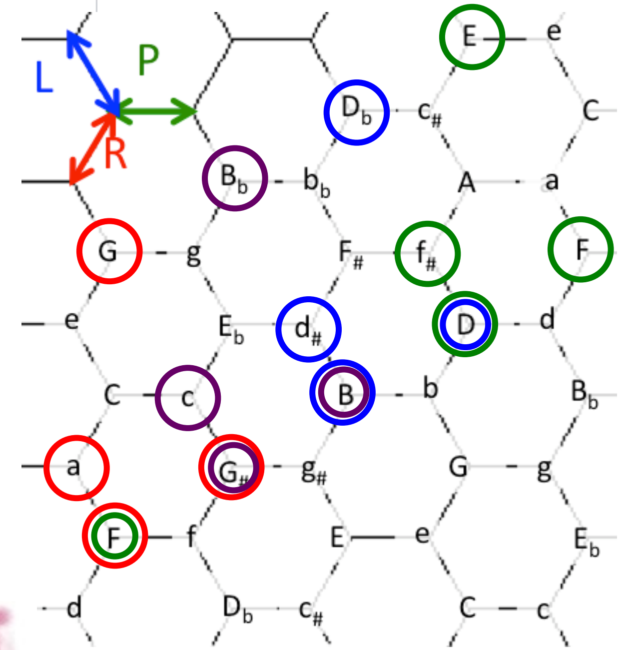
# Symmetries in Frank Zappa's music



➔ Source: Wikipedia



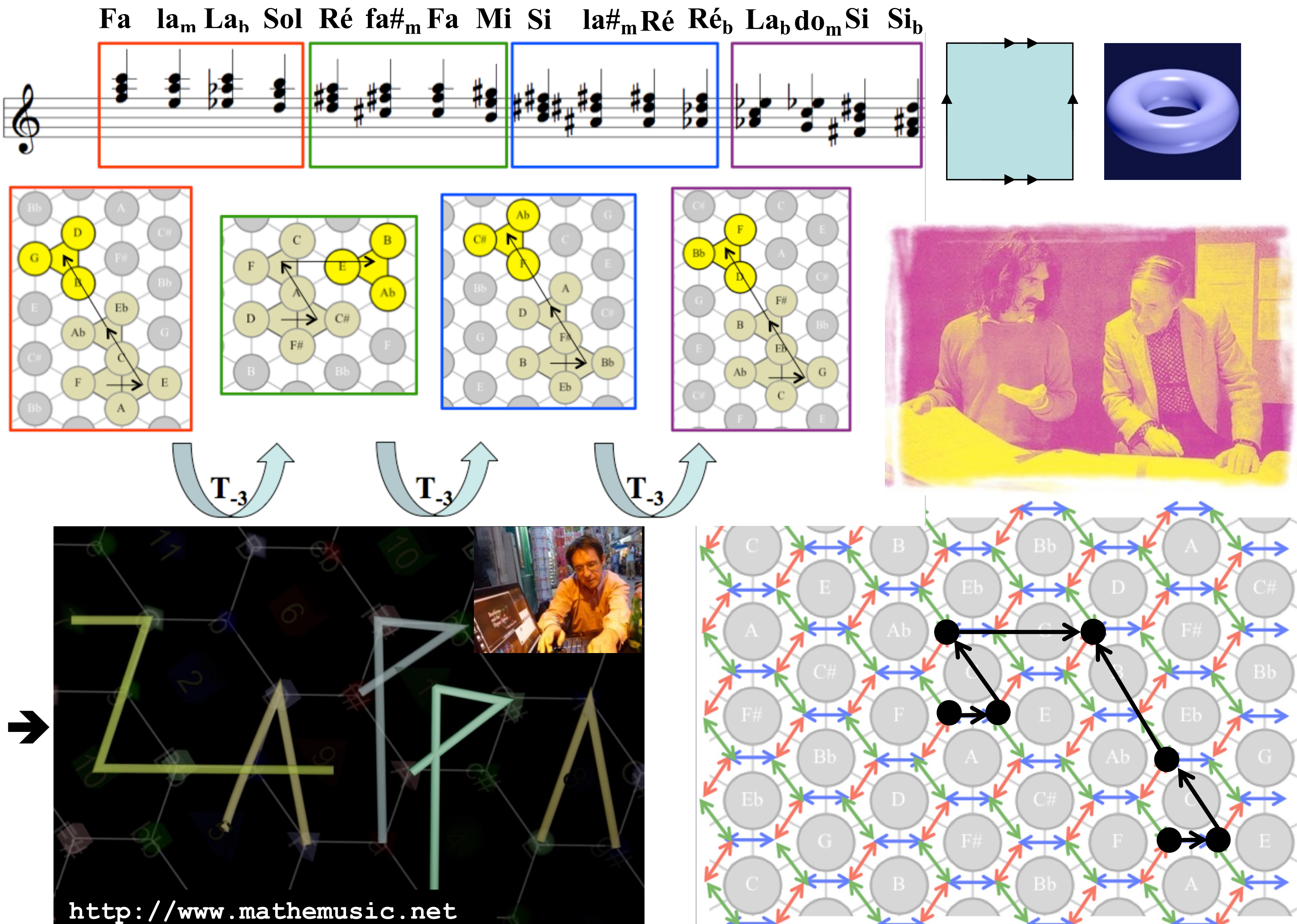




« Easy Meat » - 1981 (Frank Zappa)
min. 1'44" – 2'39"

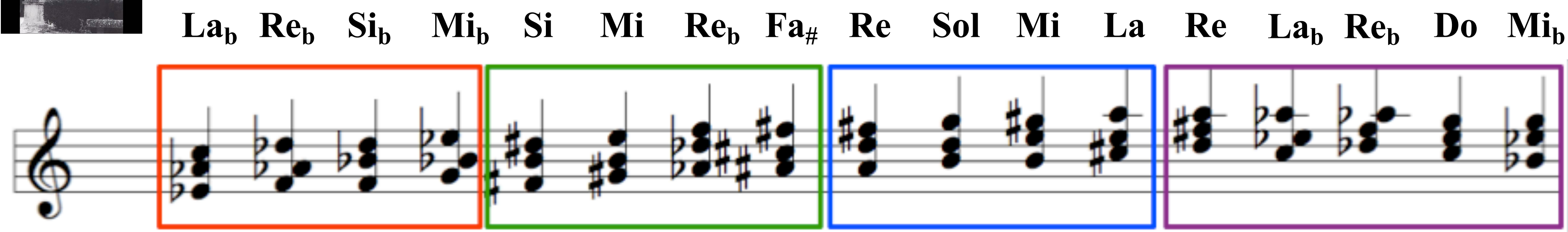
**S = LPR**

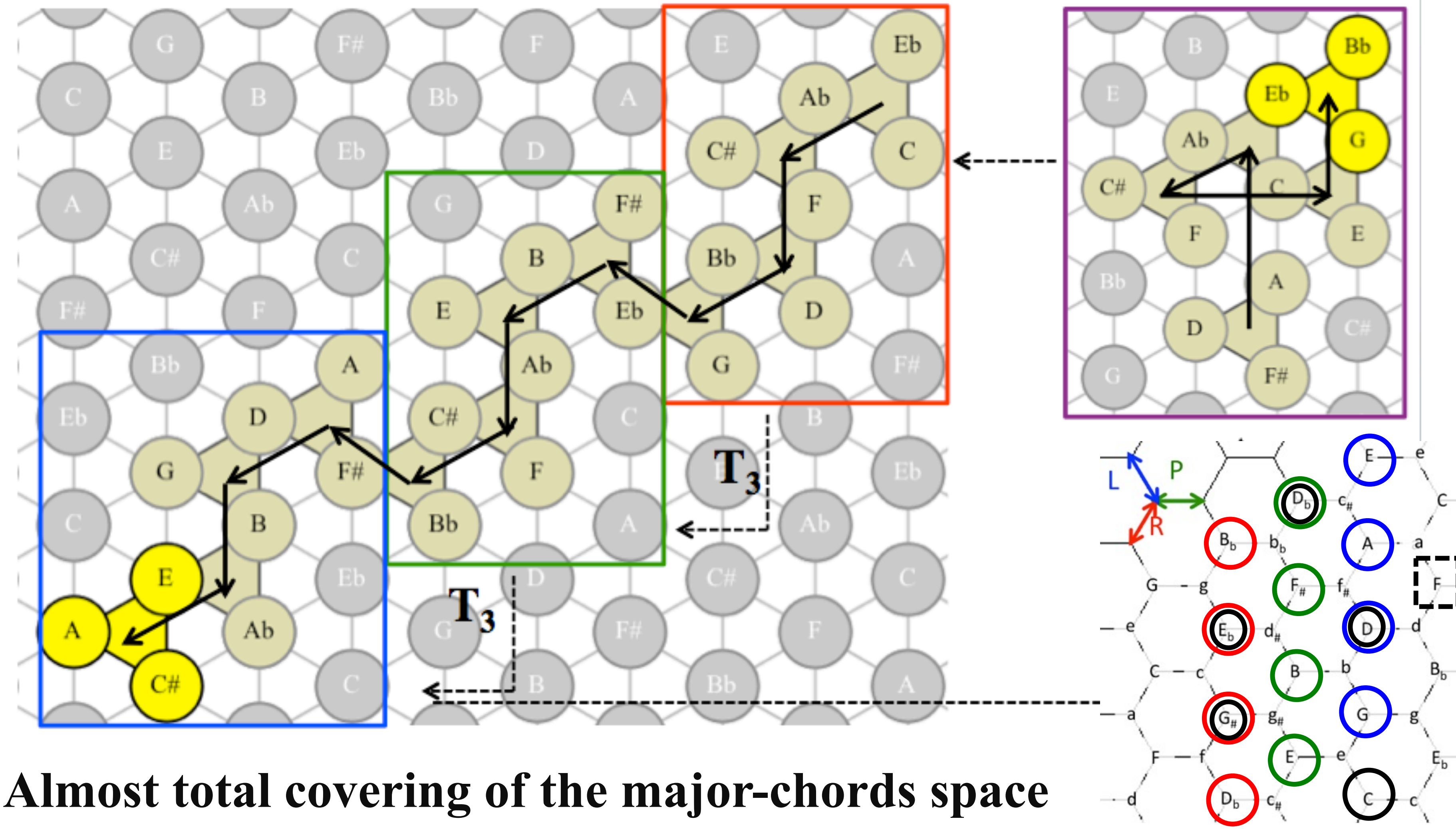
**Am ⟷ Ab**

# The trajectory of the harmonic progression

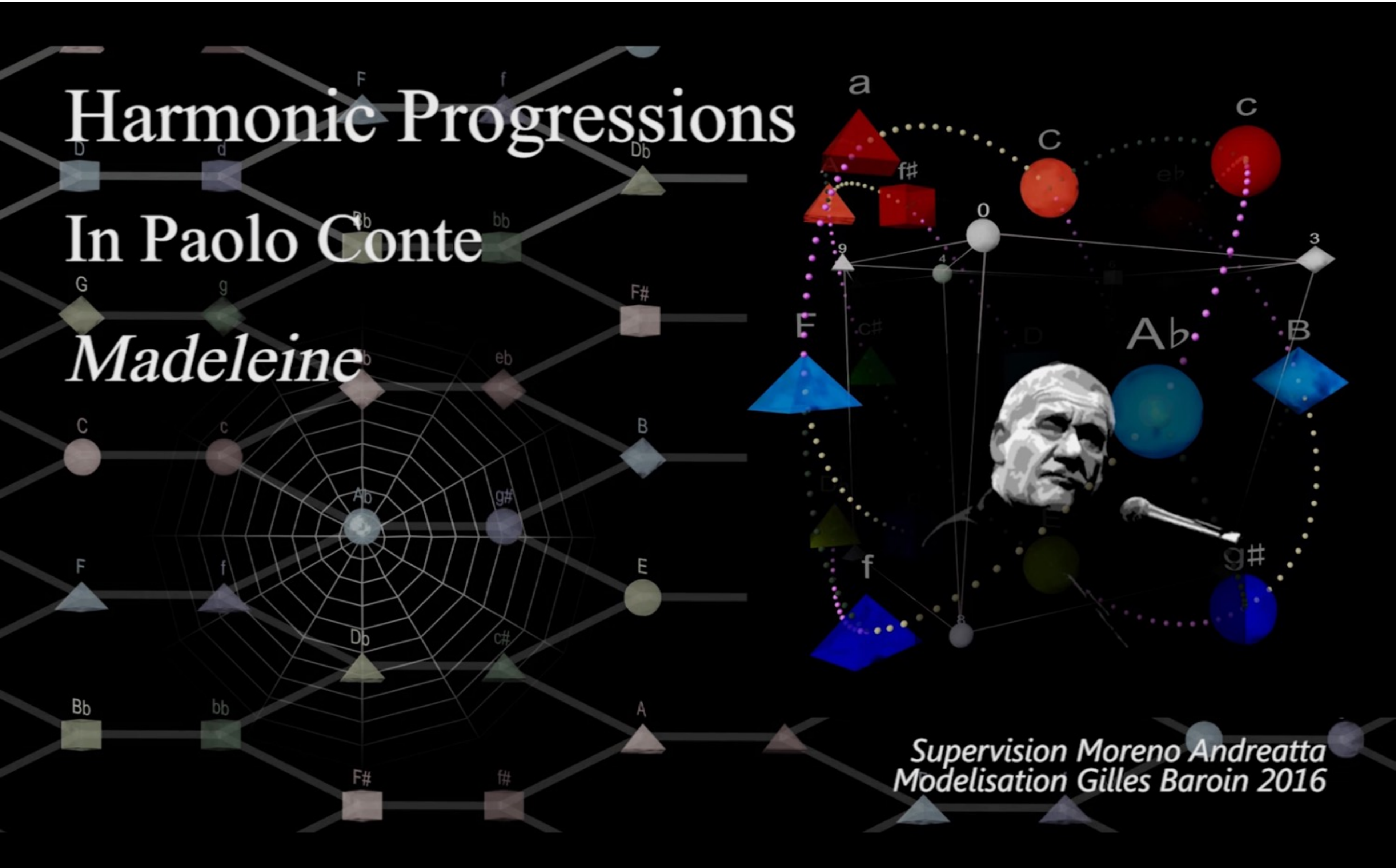


# Symmetries in Paolo Conte's *Madeleine*




**Almost total covering of the major-chords space**




➔ **www.mathemusic.net**

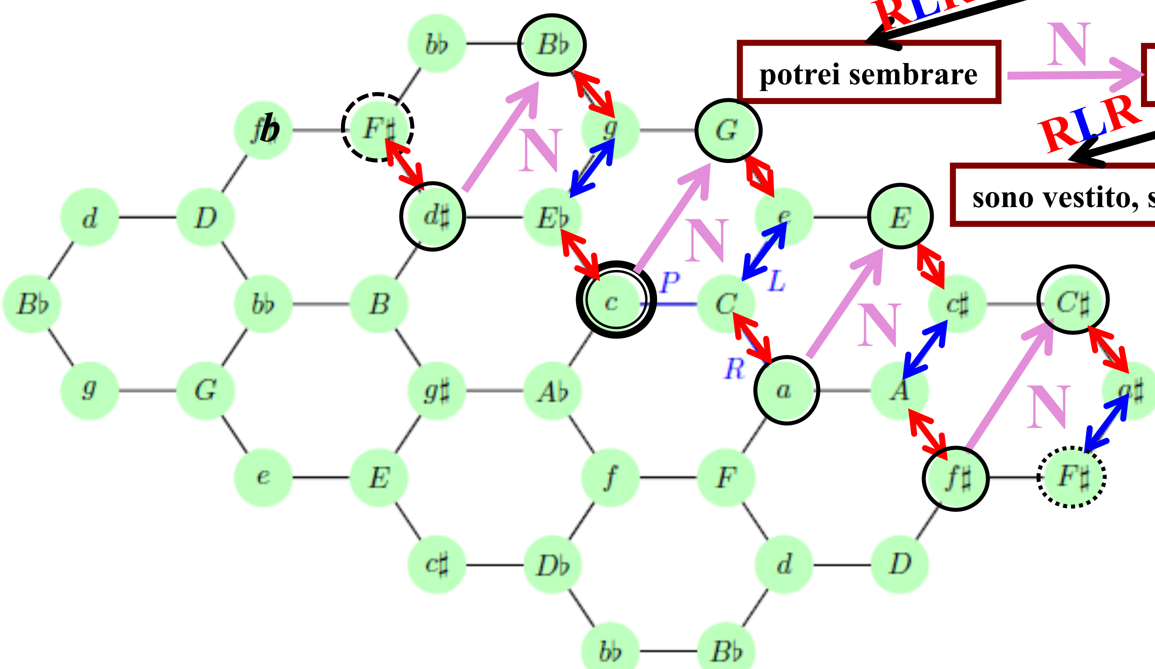
# The zig-zag of the *Nebenverwandt*

## IL REGNO DEL TANGO (Paolo Conte)

Non son neanche del $\xrightarrow{N}$ paese $\xrightarrow{RLR}$
ho una valigia di $\xrightarrow{N}$ carton $\xrightarrow{RLR}$
sono vestito, sì $\xrightarrow{N}$ in borghese, $\xrightarrow{RLR}$
ma dentro c'è $\xrightarrow{N}$ il bandoneon... $\xrightarrow{RLR}$
potrei sembrare $\xrightarrow{N}$ in borghese, $\xrightarrow{RLR}$
sono vestito, sì $\xrightarrow{N}$ un ragioniere, $\xrightarrow{RLR}$
anche un geometra $\xrightarrow{N}$ potrei, $\xrightarrow{RLR}$
ma un tango sento io $\xrightarrow{N}$ gridare $\xrightarrow{RLR}$
in fondo ai sentimenti $\xrightarrow{N}$ miei


N = PLR

Cm $\longleftrightarrow$ G

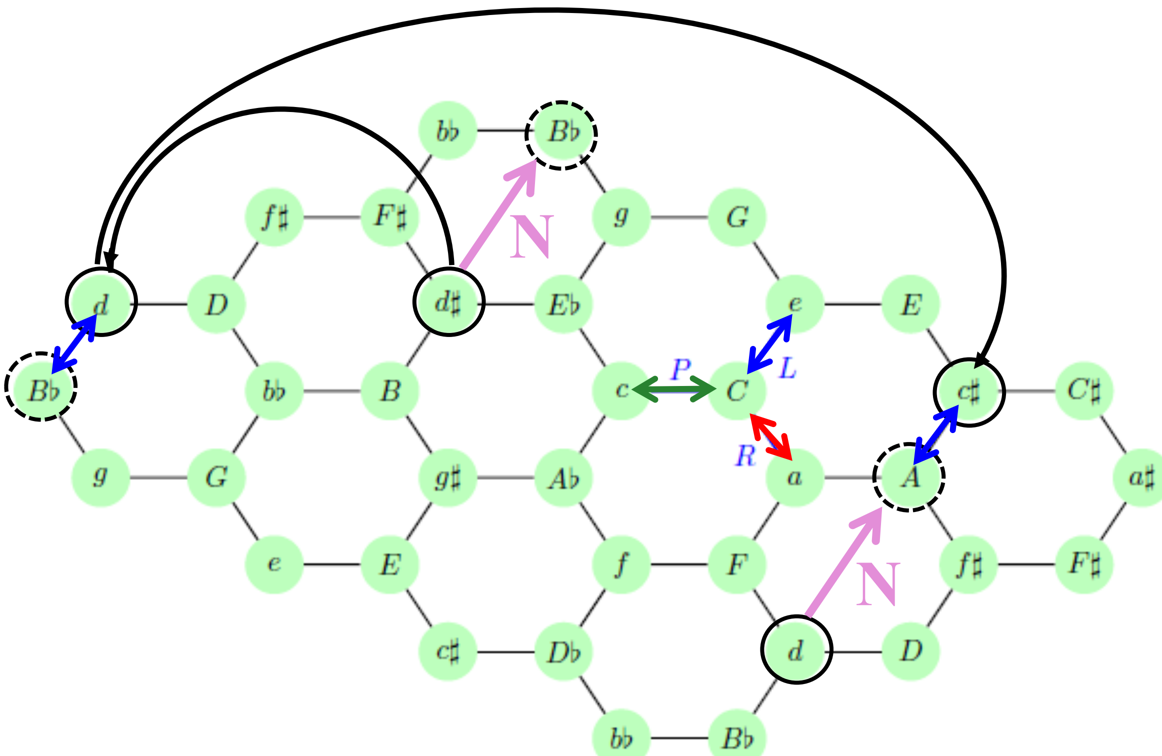



# ‘Shortcuts’ in the Tonnetz



**01:30**

## ANTEFATTO
## (L. Mello / M. Andreatta)




$d_{\#}$

non ci saranno stelle già sfinite
a raccontare stanche i nostri inizi
non ci saranno immagini sfuocate
dell’alba fatta dolce degli abbracci

**NL**

$B_b$

$d$

non ci saranno frasi come lame
e baci di un raccolto più prezioso
non sagome di vetro a cancellare
la schiuma del tuo volto che compare

**NL**

$A$

$c_{\#}$

nel mio respiro fragile d’argilla
non ci sarà la notte a distanziare
la brace dei tuoi angoli di labbra
la luce che nel tuo danzare brilla

# ‘Shortcuts’ in the Tonnetz




## FRANGIFLUTTI
## (L. Mello / M. Andreatta)

01:29

Nel rifrangere i nostri cuori,
– gente nata che esige cura –
sono stato tra i meno bravi
nella tua favola insicura.

NL

Ora è tardi lo so oramai
venti freddi come una fiaba
però tengo a cantare il canto
Eva sola che resta maga.

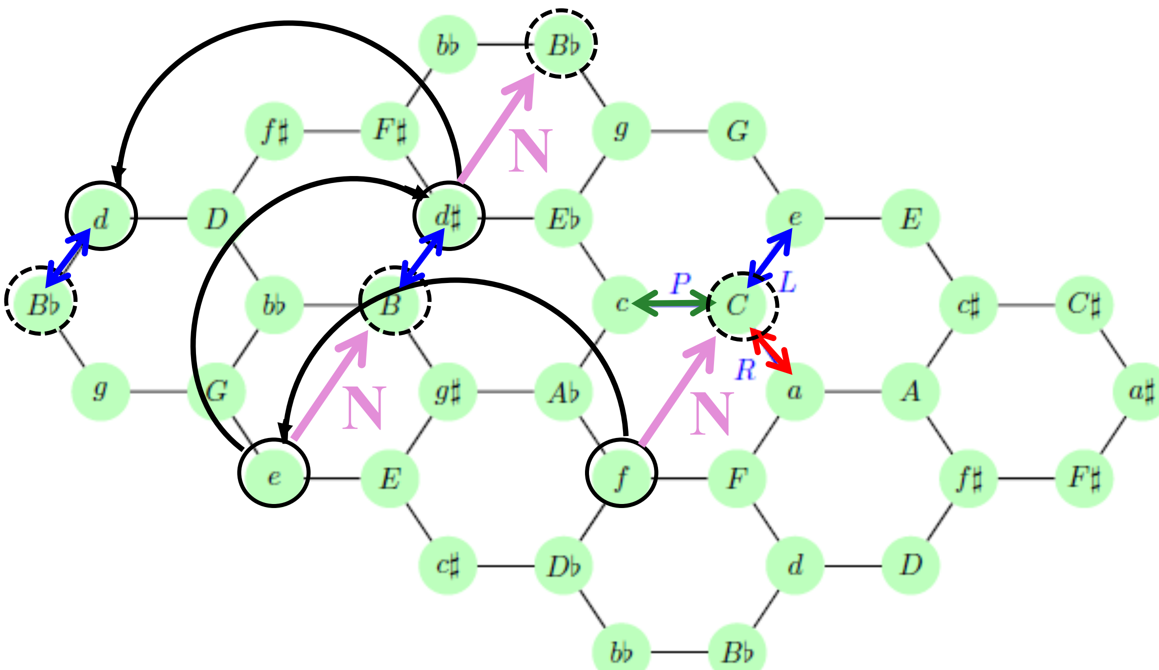
NL

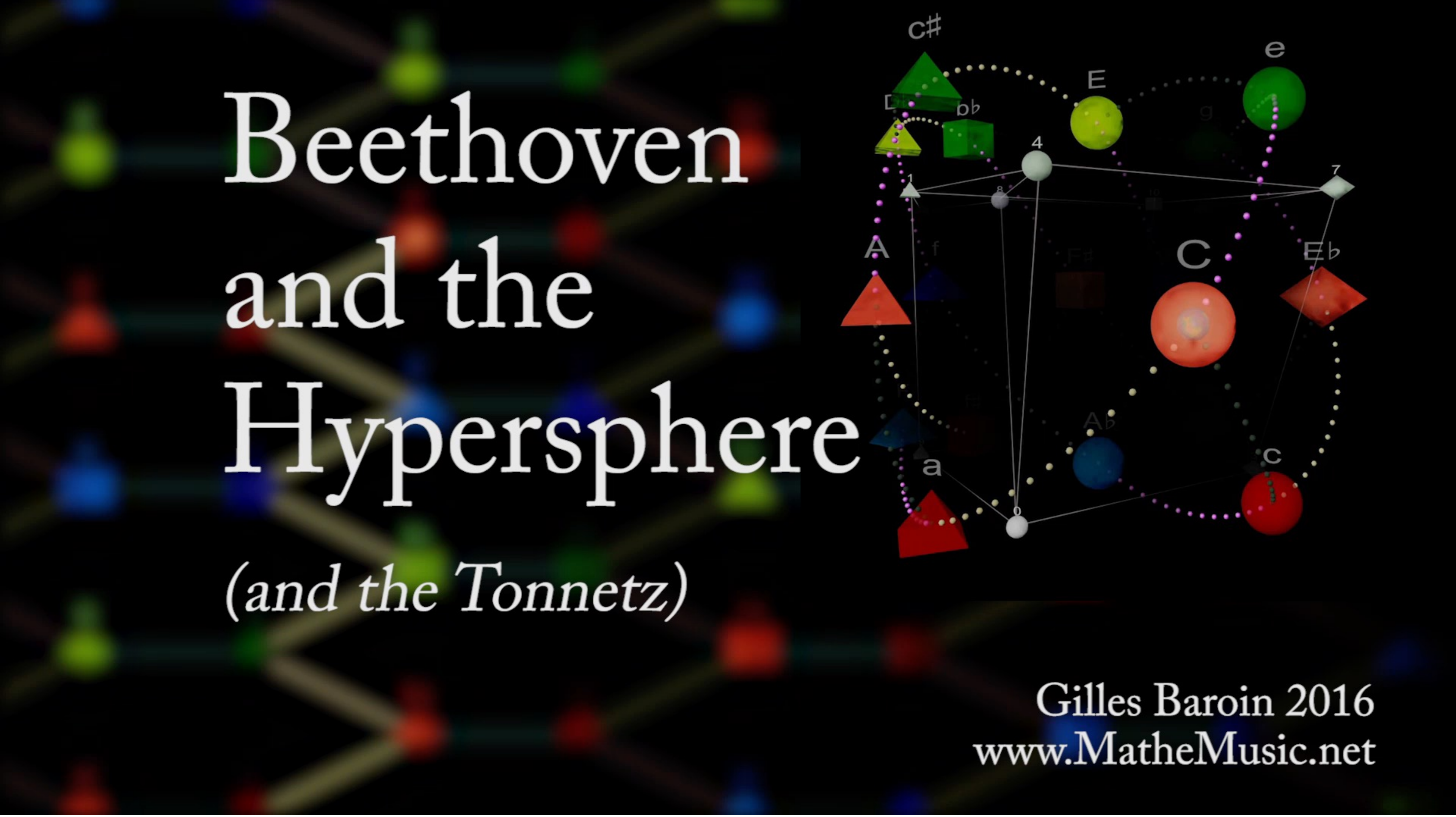
Eva cenno di ribellione
Eva fascio pronto all’addio
Eva donna che sa parlare
Al silenzio fatto d’oblio.

NL

Sai parole da incatenare
sai sorridere per piacere
sai scappare senza tornare
sai ferire e sai far l’amore.

*f* ↓ N *C* ↓ L *e* ↓ N *B* ↓ L *d#* ↓ N *Bb* ↓ L *d*






Gilles Baroin

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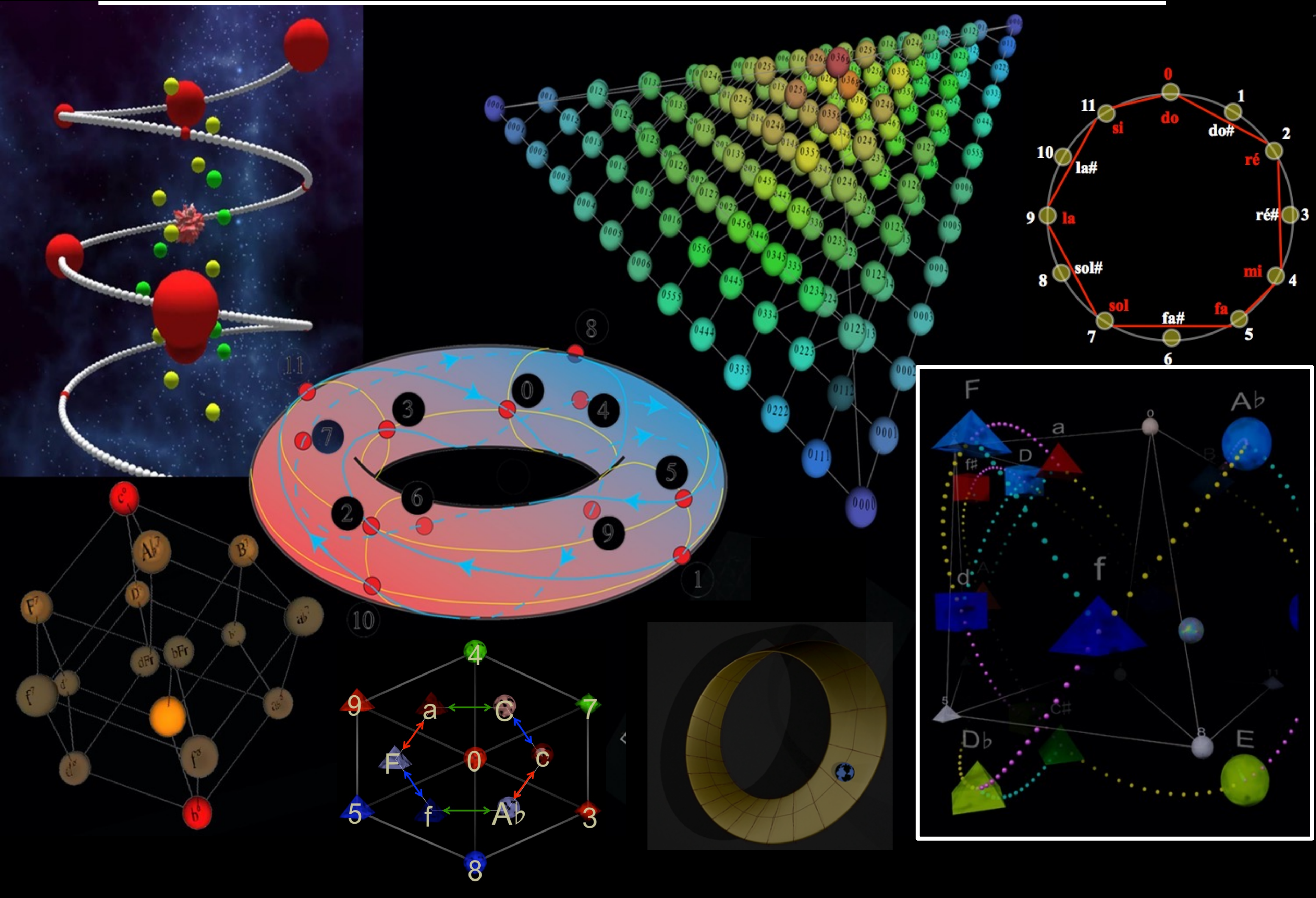
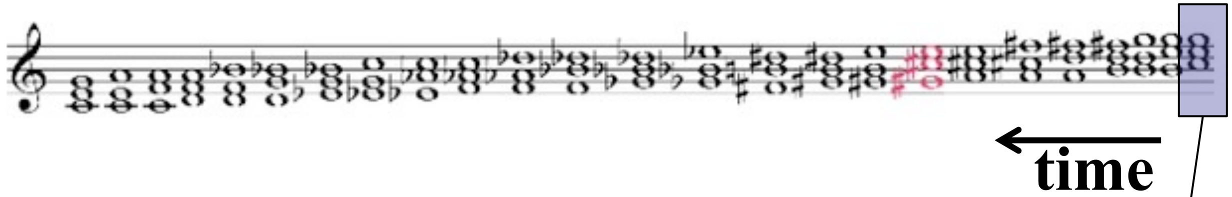
# The hypersphere of chords as an additional geometric space



# Reading Beethoven backwards




## Le Blé en Herbe

*(Polo/Moreno/Dieu)*

Plonger comme un enfant, cheveux au vent
Sous l'océan du blé en herbe
Marée d'épis couleur d'amande
Qui tendent à caresser le ciel

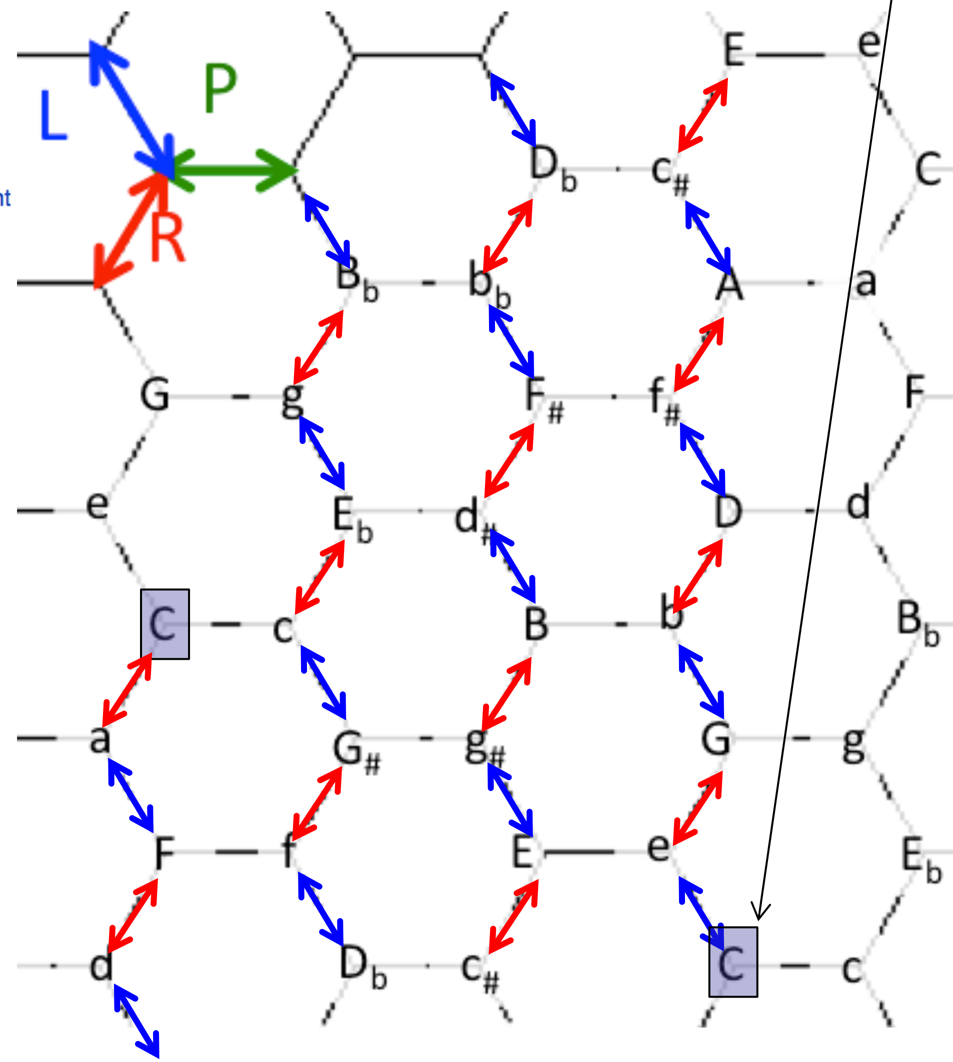
Algues tendres de mille plages
Frôlant le ventre des nuages
Cheveux de pluie, dos de poissons
Qui frissonnent à l'unisson

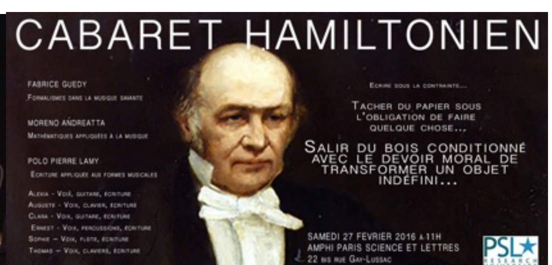
Suivre le bord des continents
Dans l'océan du blé en herbe
Pêcher le corail du pavot
Dans le sang des coquelicots

Croiser matin dans l'herbe folle
Deux tourterelles qui s'envolent
Suivre les jeux des hirondelles
Sur le paysage éternel
Nager comme un enfant, cheveux au vent
Sous l'océan
Du blé en herbe

Marée de fruits au goût amer
Acide et salée comme la mer

Vers l'ilôt d'un petit village
Vers un château d'eau sur la plage
Quand tout s'éteint avant l'orage
Quand se lève le vent du large
Sur le blé vert



# From poetry to song writing: hamiltonian compositional strategies

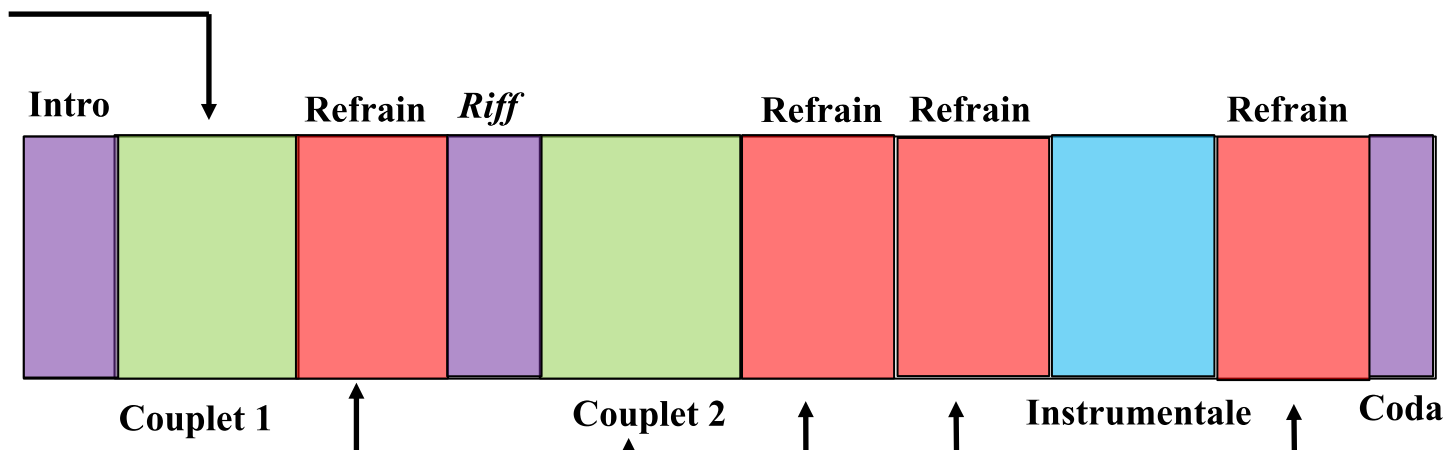

**A part** (Andrée Chedid, poème tiré du recueil *Rhymes* Collection Poésie/Gallimard (n. 527), Gallimard, 2018)

À part le temps
Et ses rouages
À part la terre
En éruptions
À part le ciel
Pétrisseur de nuages
À part l'ennemi
Qui génère l'ennemi

À part le désamour
Qui ronge l'illusion
À part la durée
Qui moisit nos visages

À part les fléaux
À part la tyrannie
À part l'ombre et le crime
Nos batailles nos outrages

Je te célèbre ô Vie
Entre cavités et songes
Intervalle convoité
Entre le vide et le rien

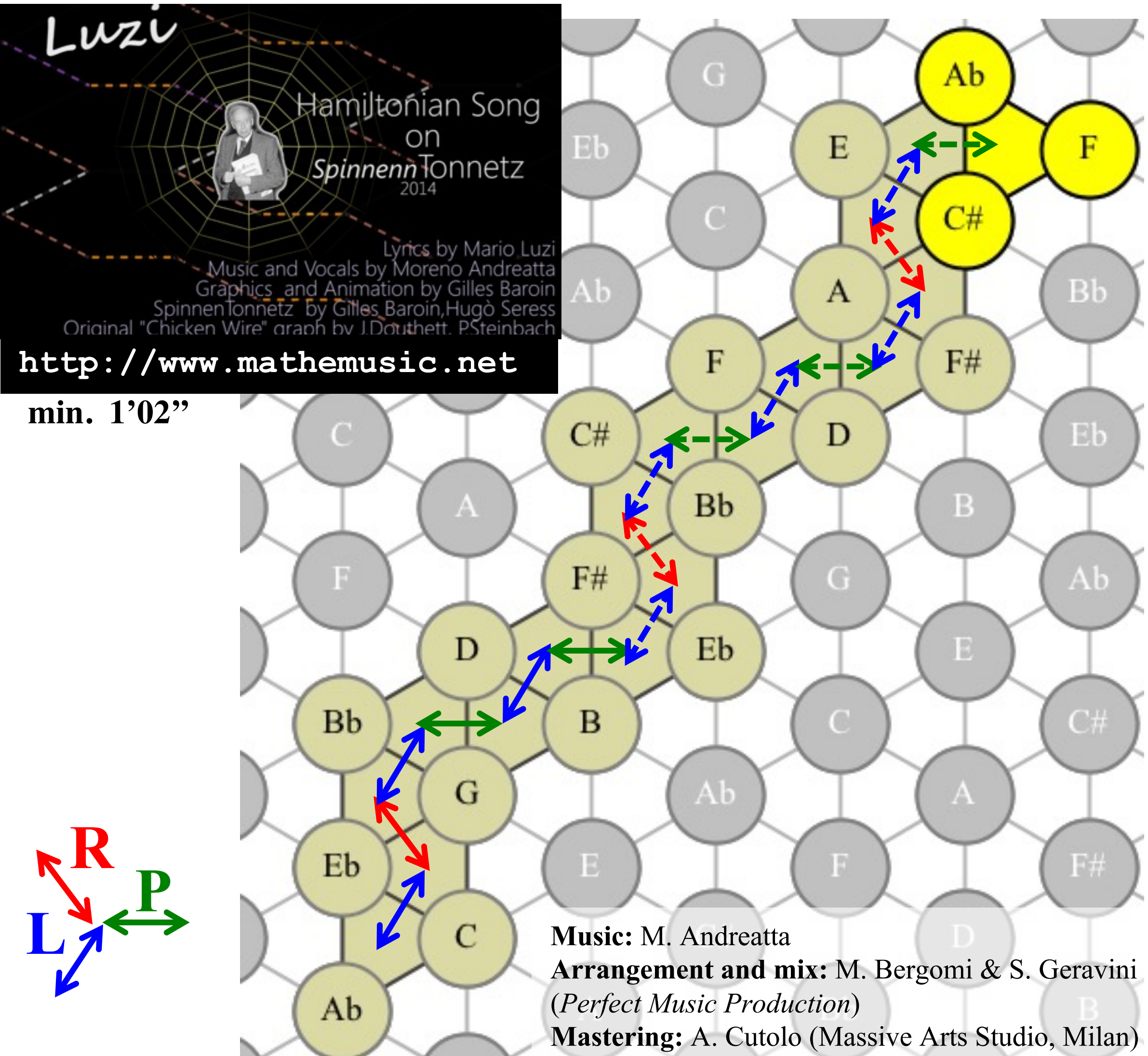


# Hamiltonian Cycles with inner periodicities

8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR

→

L P L P L R …
P L P L R L …
L P L R L P …
P L R L P L …
**L R L P L P …**
R L P L P L …




min. 1’02”



**Music:** M. Andreatta
**Arrangement and mix:** M. Bergomi & S. Geravini (*Perfect Music Production*)
**Mastering:** A. Cutolo (Massive Arts Studio, Milan)



**La sera non è più la tua canzone**
**(Mario Luzi, 1945, in *Poesie sparse*)**

**La sera non è più la tua canzone,**
**è questa roccia d’ombra traforata**
**dai lumi e dalle voci senza fine,**
**la quiete d’una cosa già pensata.**

**Ah questa luce viva e chiara viene**
**solo da te, sei tu così vicina**
**al vero d’una cosa conosciuta,**
**per nome hai una parola ch’è passata**
**nell’intimo del cuore e s’è perduta.**

**Caduto è più che un segno della vita,**
**riposi, dal viaggio sei tornata**
**dentro di te, sei scesa in questa pura**
**sostanza così tua, così romita**
**nel silenzio dell’essere, (compiuta).**

**L’aria tace ed il tempo dietro a te**
**si leva come un’arida montagna**
**dove vaga il tuo spirito e si perde,**
**un vento raro scivola e ristagna.**


➔ **www.mathemusic.net**

# *Aprile*, a Hamiltonian « decadent » song

$\text{Do} \leftarrow \text{do}_m \leftarrow \text{Sol\#} \leftarrow \text{fa}_m \leftarrow \text{Fa} \leftarrow \text{la}_m \leftarrow \text{La} \leftarrow \text{fa\#}_m \leftarrow \text{Fa\#} \leftarrow \text{sib}_m \leftarrow \text{Do\#} \leftarrow \text{do\#}_m$

La

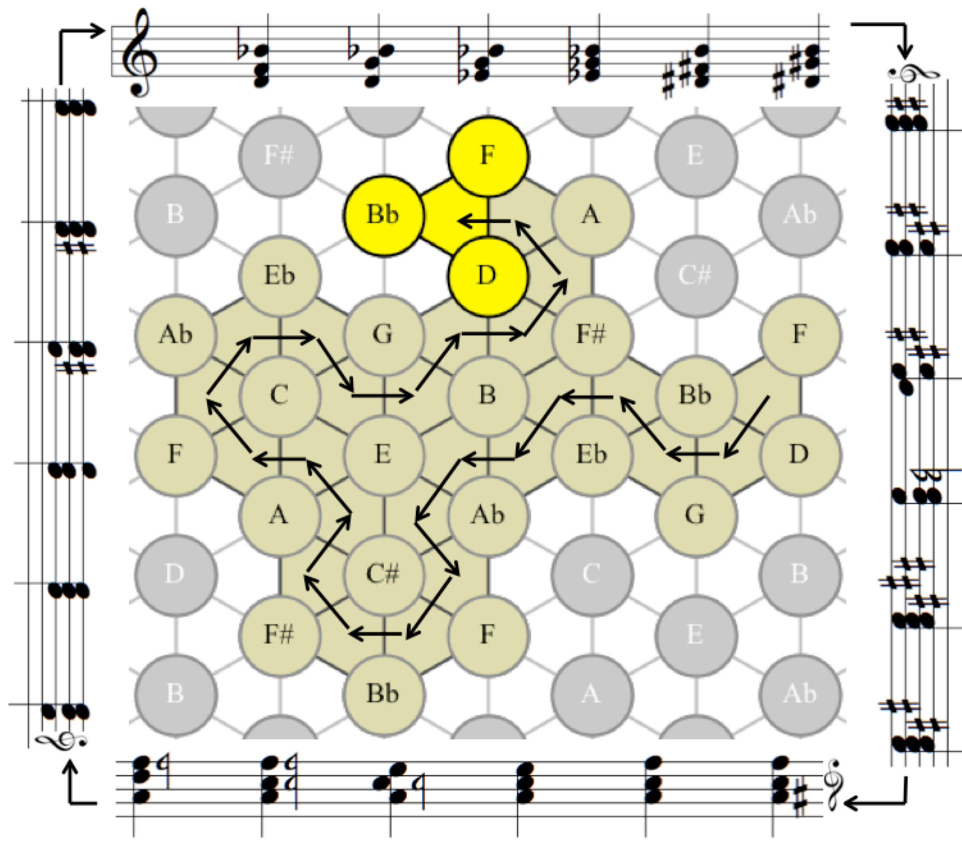
$\text{mi}_m \rightarrow \text{Sol} \rightarrow \text{si}_m \rightarrow \text{Ré} \rightarrow \text{ré}_m \rightarrow \boxed{\text{Sib}} \rightarrow \text{sol}_m \rightarrow \text{Mib} \rightarrow \text{mib}_m \rightarrow \text{Si} \rightarrow \text{sol\#}_m \rightarrow \text{Mi}$

*Socchiusa è la finestra, sul giardino.*
*Un'ora passa lenta, sonnolenta.*
*Ed ella, ch'era attenta, s'addormenta*
*A quella voce che già si lamenta,*
*Che si lamenta in fondo a quel giardino.*

*Non è che voce d'acque su la pietra:*
*E quante volte, quante volte udita!*
*Quell'amore e quell'ora in quella vita*
*S'affondan come ne l'onda infinita*
*Stretti insieme il cadavere e la pietra.*

*Ella stende l'angoscia sua nel sonno.*
*L'angoscia è forte, e il sonno è così lieve!*
*(Par la luce d'april quasi una neve*
*che sia tiepida.)*
*Ed ella certo deve soffrire,*
*Vagamente, anche nel sonno.*

G. D'Annunzio (1863-1938)




# The collection of 124 Hamiltonian Cycles

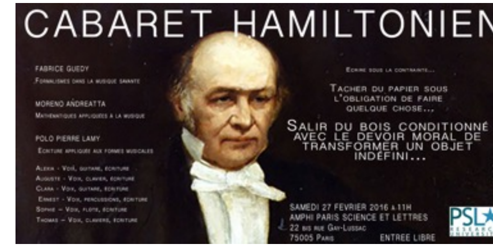
ACTIONS

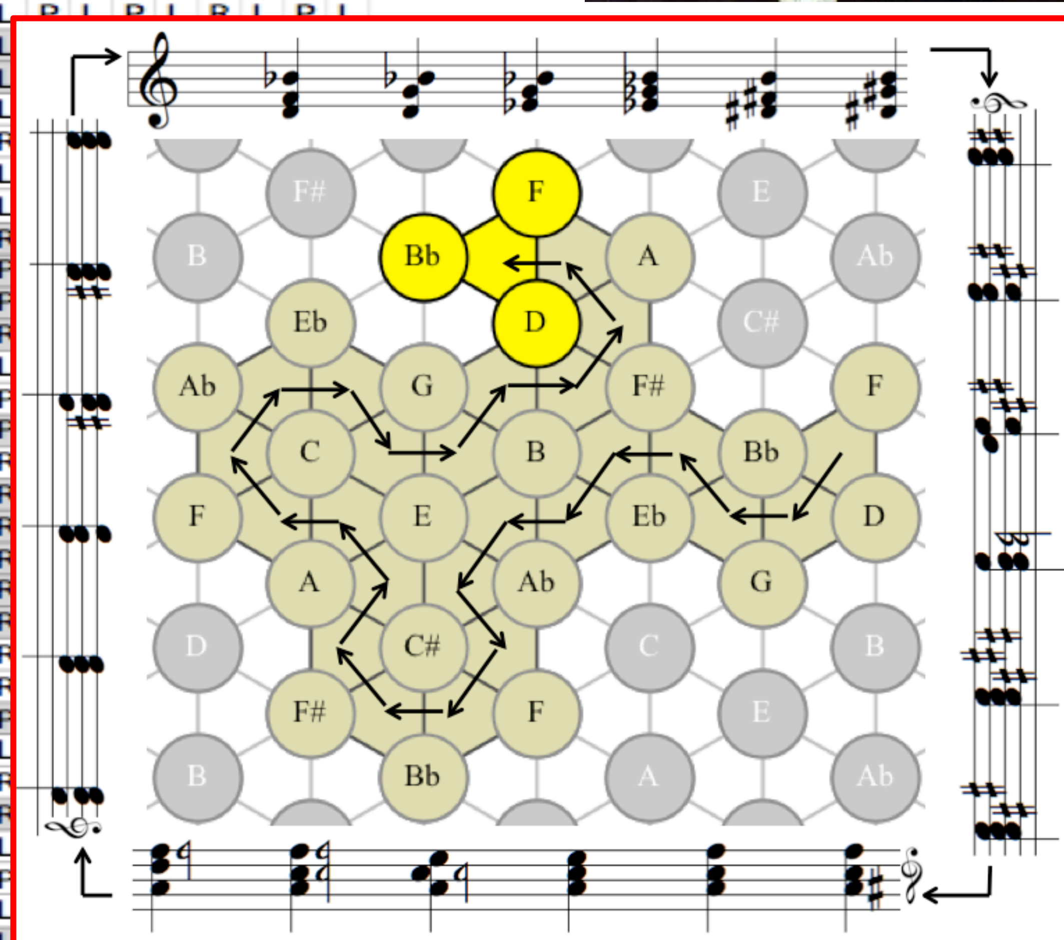
Math'n'pop

*Aprile* (d'après Gabriele D'Annunzio)

Tangente
L'aventure mathématique
pour mieux comprendre le monde

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| #41 | L | R | L | R | L | R | L | R | L | R | L | R | L | R | L | R | L | R | L | R | L | R | L | R |
| #62 | L | P | L | P | L | R | L | P | L | P | L | R | L | P | L | P | L | R | L | P | L | P | L | R |
| #13 | P | L | R | L | P | L | P | L | R | L | P | L | P | L | R | L | | | | | | | | |
| #4 | P | L | P | L | R | L | P | L | P | L | R | L | P | L | P | | | | | | | | | |
| #58 | L | P | L | P | L | R | P | L | P | L | P | R | P | L | P | | | | | | | | | |
| #19 | P | R | L | P | L | P | L | R | P | L | P | L | P | R | P | | | | | | | | | |
| #7 | P | L | P | R | L | P | L | P | L | R | P | L | P | L | P | | | | | | | | | |
| #27 | P | L | P | L | P | R | L | P | L | P | L | R | P | L | P | | | | | | | | | |
| #21 | P | R | P | L | P | L | P | R | L | P | L | P | L | R | P | | | | | | | | | |
| #8 | P | L | P | R | P | L | P | L | P | R | L | P | L | P | L | | | | | | | | | |
| #30 | P | L | P | L | P | R | P | L | P | L | P | R | L | P | L | | | | | | | | | |
| #22 | P | R | P | L | P | L | P | R | P | L | P | L | P | R | L | | | | | | | | | |
| #10 | P | L | P | R | P | L | P | L | P | R | P | L | P | L | P | | | | | | | | | |
| #31 | P | L | P | L | P | R | P | L | P | L | P | R | P | L | P | | | | | | | | | |
| #9 | P | L | R | P | L | P | L | P | R | P | L | P | L | P | R | | | | | | | | | |
| #6 | P | L | P | L | R | P | L | P | L | P | R | P | L | P | L | | | | | | | | | |
| #33 | P | R | P | R | P | R | L | R | P | R | P | R | P | R | L | | | | | | | | | |
| #44 | L | R | P | R | P | R | P | R | L | R | P | R | P | R | P | | | | | | | | | |
| #40 | P | R | L | R | P | R | P | R | P | R | L | R | P | R | P | | | | | | | | | |
| #38 | P | R | P | R | L | R | P | R | P | R | P | R | L | R | P | | | | | | | | | |
| #34 | P | R | P | R | L | R | L | R | P | R | P | R | L | R | L | | | | | | | | | |
| #42 | L | R | P | R | P | R | L | R | L | R | P | R | P | R | L | | | | | | | | | |
| #43 | L | R | L | R | P | R | P | R | L | R | L | R | P | R | P | | | | | | | | | |
| #39 | P | R | L | R | L | R | P | R | P | R | L | R | L | R | P | | | | | | | | | |
| #50 | L | R | L | P | L | R | L | P | R | L | R | L | P | L | R | | | | | | | | | |
| #37 | P | R | L | R | L | P | L | R | L | P | R | L | R | L | P | | | | | | | | | |
| #25 | P | L | R | P | R | L | R | L | P | L | R | L | P | R | L | | | | | | | | | |
| #16 | P | R | P | L | R | P | R | L | R | L | P | L | R | L | P | | | | | | | | | |
| #23 | P | L | P | R | P | L | R | P | R | L | R | L | P | L | R | | | | | | | | | |
| #1 | P | L | R | P | L | P | R | P | L | R | P | R | L | R | L | | | | | | | | | |
| #59 | L | P | L | R | P | L | P | R | P | L | R | P | R | L | R | | | | | | | | | |
| #48 | L | R | L | P | L | R | P | L | P | R | P | L | R | P | R | | | | | | | | | |
| #18 | P | R | L | R | L | P | L | R | P | L | P | R | P | L | R | P | R | L | R | L | P | L | R | L |
| #60 | L | P | R | L | R | L | P | L | R | P | L | P | R | P | L | R | P | R | L | R | L | P | L | R |




G. Albini & S. Antonini, University of Pavia, 2008

**ACTIONS** Math'n'pop

**Aprile (d'après Gabriele D'Annunzio)**

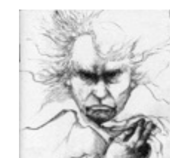
Tangente
L'aventure mathématique
pour mieux comprendre le monde

**M. Andreatta, « Math'n pop : symétries et cycles hamiltoniens en chanson », *Tangente***

# The catalogue of 28 hamiltonian cycles (with inner symmetry)

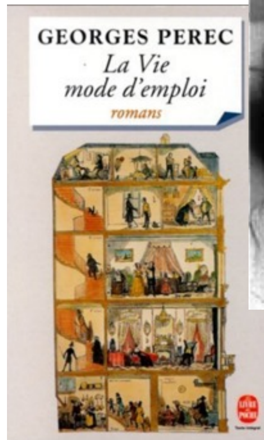
1. C-Cm-Ab-Abm-E-C#m-A-Am-F-Fm-C#-Bbm-F#-F#m-D-Dm-Bb-Gm-Eb-Ebm-B-Bm-G-Em--PLPLRL
2. C-Cm-Ab-Fm-C#-C#m-A-Am-F-Dm-Bb-Bbm-F#-F#m-D-Bm-G-Gm-Eb-Ebm-B-Abm-E-Em--PLRLPL
3. C-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Bm-D-Dm-F-Am--PRPRPRLR
4. C-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Em-G-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Bm-D-F#m-A-Am--PRPRLRPR
5. C-Cm-Eb-Ebm-F#-Bbm-C#-Fm-Ab-Abm-B-Bm-D-F#m-A-C#m-E-Em-G-Gm-Bb-Dm-F-Am--PRPRLRLR
6. C-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Em-G-Bm-D-Dm-F-Fm-Ab-Abm-B-Ebm-F#-F#m-A-Am--PRLRPRPR
7. C-Cm-Eb-Gm-Bb-Bbm-C#-Fm-Ab-Abm-B-Ebm-F#-F#m-A-C#m-E-Em-G-Bm-D-Dm-F-Am--PRLR
8. C-Cm-Eb-Gm-Bb-Dm-F-Fm-Ab-Abm-B-Ebm-F#-Bbm-C#-C#m-E-Em-G-Bm-D-F#m-A-Am--PRLRLRPR
9. C-Em-E-Abm-Ab-Cm-Eb-Gm-G-Bm-B-Ebm-F#-Bbm-Bb-Dm-D-F#m-A-C#m-C#-Fm-F-Am--LPLPLR
10. C-Em-E-Abm-B-Ebm-Eb-Gm-G-Bm-D-F#m-F#-Bbm-Bb-Dm-F-Am-A-C#m-C#-Fm-Ab-Cm--LPLRLP
11. C-Em-G-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-Am--LRPRPRPR
12. C-Em-G-Gm-Bb-Bbm-C#-Fm-Ab-Cm-Eb-Ebm-F#-F#m-A-C#m-E-Abm-B-Bm-D-Dm-F-Am--LRPRPRLR
13. C-Em-G-Gm-Bb-Dm-F-Fm-Ab-Cm-Eb-Ebm-F#-Bbm-C#-C#m-E-Abm-B-Bm-D-F#m-A-Am--LRPR
14. C-Em-G-Bm-B-Ebm-Eb-Gm-Bb-Dm-D-F#m-F#-Bbm-C#-Fm-F-Am-A-C#m-E-Abm-Ab-Cm--LRLPLP
15. C-Em-G-Bm-D-Dm-F-Fm-Ab-Cm-Eb-Gm-Bb-Bbm-C#-C#m-E-Abm-B-Ebm-F#-F#m-A-Am--LRLRPRPR
16. C-Em-G-Bm-D-F#m-A-C#m-E-Abm-B-Ebm-F#-Bbm-C#-Fm-Ab-Cm-Eb-Gm-Bb-Dm-F-Am--LR
17. C-Am-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-G-Em--RPRPRPRL
18. C-Am-A-F#m-F#-Ebm-B-Abm-Ab-Fm-F-Dm-D-Bm-G-Em-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm--RPRPRLRP
19. C-Am-A-F#m-F#-Ebm-B-Abm-E-C#m-C#-Bbm-Bb-Gm-Eb-Cm-Ab-Fm-F-Dm-D-Bm-G-Em--RPRPRLRL
20. C-Am-A-F#m-D-Bm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-G-Em-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm--RPRLRPRP
21. C-Am-A-F#m-D-Bm-B-Abm-E-C#m-C#-Bbm-F#-Ebm-Eb-Cm-Ab-Fm-F-Dm-Bb-Gm-G-Em--RPRL
22. C-Am-A-F#m-D-Bm-G-Em-E-C#m-C#-Bbm-F#-Ebm-B-Abm-Ab-Fm-F-Dm-Bb-Gm-Eb-Cm--RPRLRLRP
23. C-Am-F-Fm-C#-C#m-A-F#m-D-Dm-Bb-Bbm-F#-Ebm-B-Bm-G-Gm-Eb-Cm-Ab-Abm-E-Em--RLPLPL
24. C-Am-F-Dm-D-Bm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em-E-C#m-A-F#m-F#-Ebm-Eb-Cm--RLRPRPRP
25. C-Am-F-Dm-D-Bm-B-Abm-E-C#m-A-F#m-F#-Ebm-Eb-Cm-Ab-Fm-C#-Bbm-Bb-Gm-G-Em--RLRPRPRL
26. C-Am-F-Dm-D-Bm-G-Em-E-C#m-A-F#m-F#-Ebm-B-Abm-Ab-Fm-C#-Bbm-Bb-Gm-Eb-Cm--RLRP
27. C-Am-F-Dm-Bb-Gm-G-Em-E-C#m-A-F#m-D-Bm-B-Abm-Ab-Fm-C#-Bbm-F#-Ebm-Eb-Cm--RLRLRPRP
28. C-Am-F-Dm-Bb-Gm-Eb-Cm-Ab-Fm-C#-Bbm-F#-Ebm-B-Abm-E-C#m-A-F#m-D-Bm-G-Em--RL

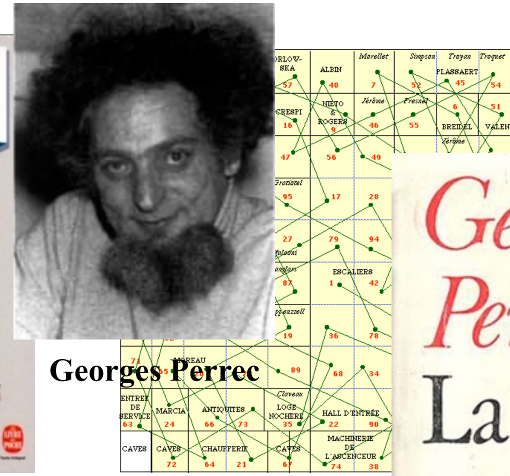




# The use of constraints in arts

*Cent mille milliards de poèmes*, 1961

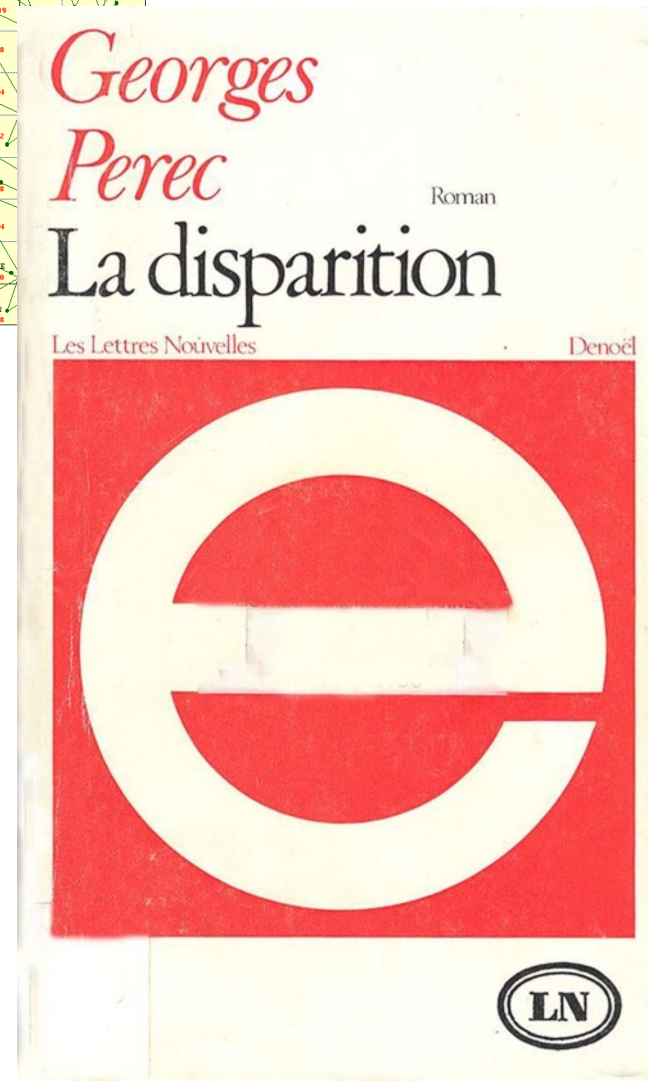


**Georges Perrec**

*La vie mode d'emploi*,

**OuLiPo (Ouvroir de Littérature Potentielle)**



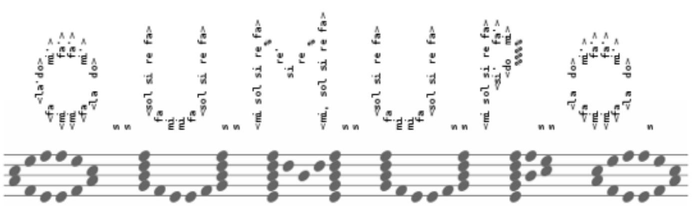


**Raymond Queneau**

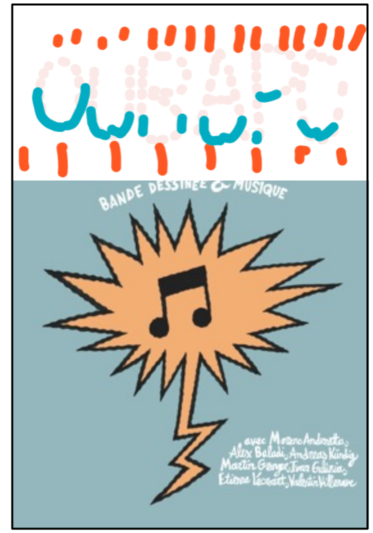
**Italo Calvino**

*Il castello dei destini incrociati,* 1969

# From the OuLiPo to the OuMuPo (ouvroir de musique potentielle)

http://oumupo.org/

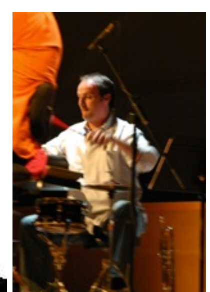




M. Andreatta et al., « Music, mathematics and language: chronicles from the Oumupo sandbox », in Kapoula, Z., Volle, E., Renoult, J., Andreatta, M. (Eds.), *Exploring Transdisciplinarity in Art and Sciences*, Springer, 2018

Valentin Villenave

Mike Solomon

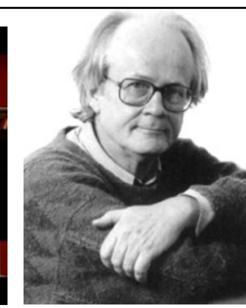


Jean-François Piette

Martin Granger

Joseph Boisseau

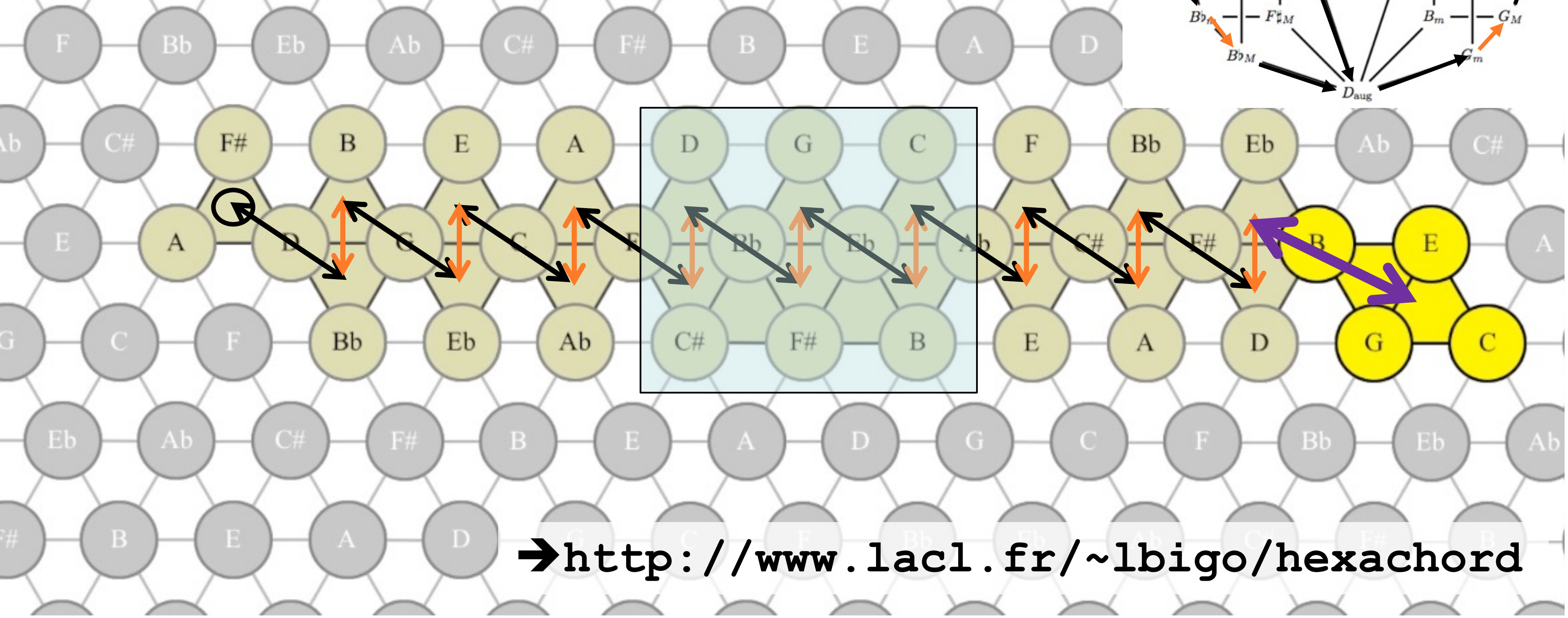
Moreno Andreatta

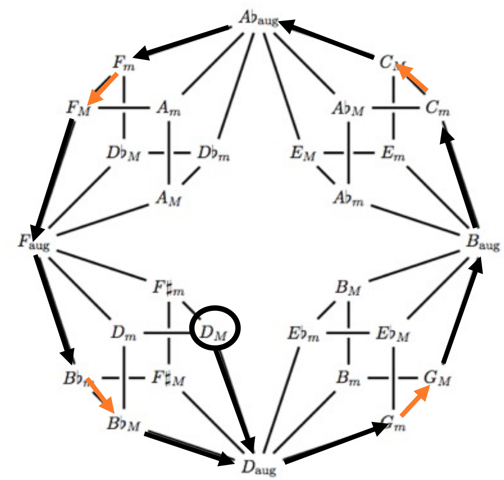


Tom Johnson

# Tonnetz *versus* Cube Dance Analysis for Muse's *Take a bow*


"Take a bow" (*Black Holes and Revelations*, 2006)



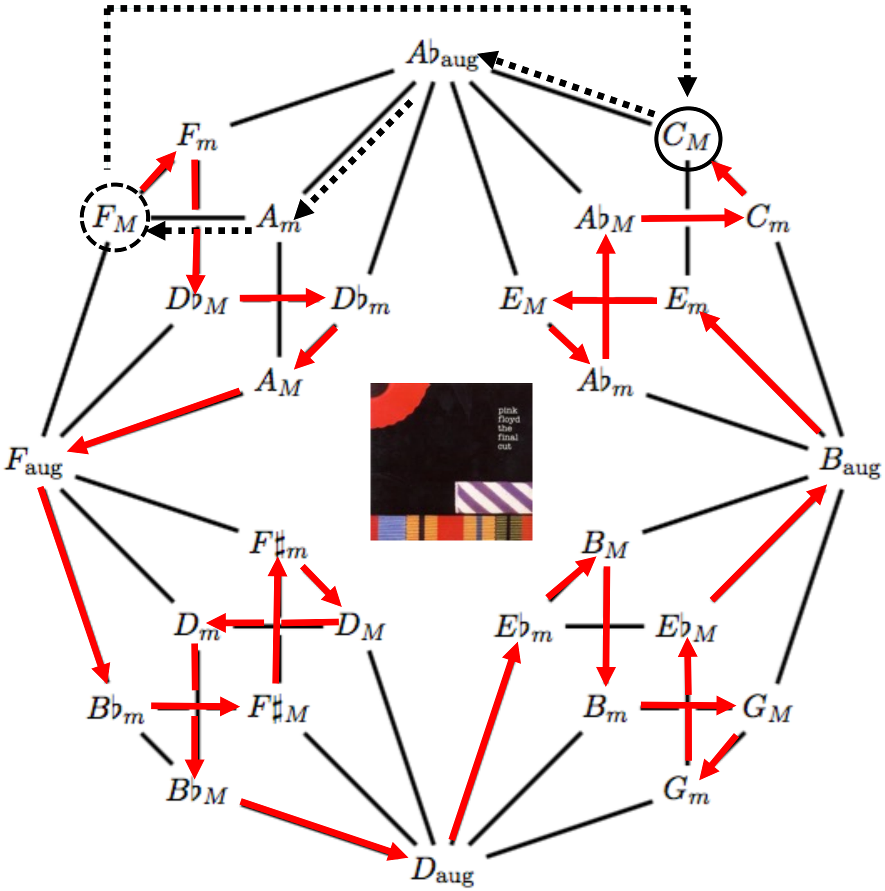




➔http://www.lacl.fr/~lbigo/hexachord

**Muse - Take A Bow (Tonnetz harmonic analysis)** YouTube

**30,364 views Jan 20, 2016** Harmonic analysis of the song Take A Bow composed by Matthew Bellamy performed by Muse.

# The Gunner's Hamiltonian Dream (a *OuMuPo* experience around Pink-Floyd)




**The Gunner's dream (R. Waters, 1983 / M. Andreatta, 2018)**

C C+
Floating down through the clouds
Am F
Memories come rushing up to meet me now.
Fm
In the space between the heavens
C# C#m
and in the corner of some foreign field
A F+ Bbm
I had a dream.
F# F#m D Dm
I had a dream.
Bb
Good-bye Max.
D+
Good-bye Ma.
Ebm B
After the service when you're walking slowly to the car
Bm G
And the silver in her hair shines in the cold November air

Gm
You hear the tolling bell
Eb
And touch the silk in your lapel
G+ Em E G#m
And as the tear drops rise to meet the comfort of the band
G# Cm
You take her frail hand
C
And hold on to the dream.

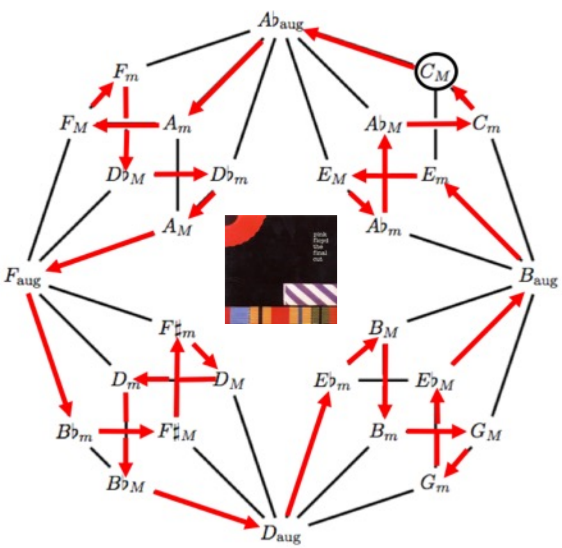
## The three Hamiltonian Cycles ($C_M$ = C, $C_m$ = Cm, $C_{aug}$ = C+)

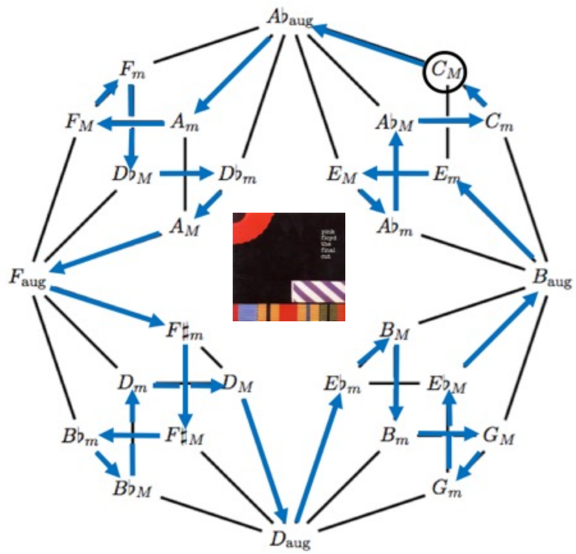
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

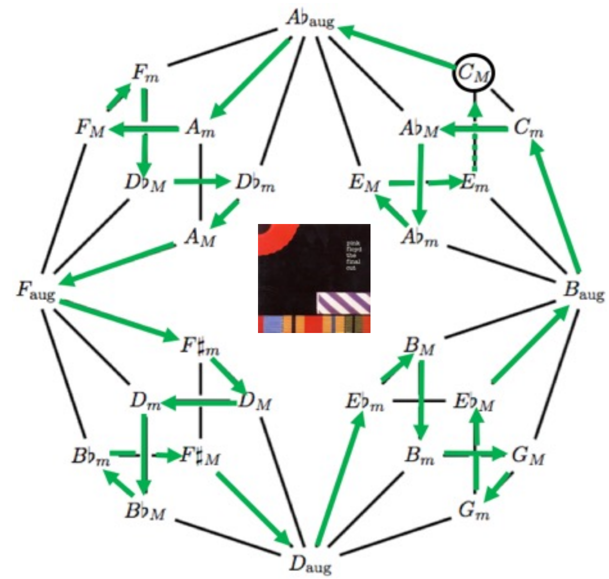
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

# The Gunner's Hamiltonian Dream (a *OuMuPo* experience around Pink-Floyd)




## The three Hamiltonian Cycles ($C_M$ = C, $C_m$ = Cm, $C_{aug}$ = C+)

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->Bbm-->F#-->F#m-->D-->Dm-->Bb-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->F#-->Bbm-->Bb-->Dm-->D-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Em-->E-->G#m-->G#-->Cm-->C

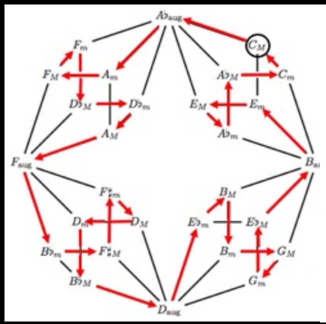
C-->C+-->Am-->F-->Fm-->C#-->C#m-->A-->F+-->F#m-->D-->Dm-->Bb-->Bbm-->F#-->D+-->Ebm-->B-->Bm-->
-->G-->Gm-->Eb-->G+-->Cm-->G#-->G#m-->E-->Em-->C

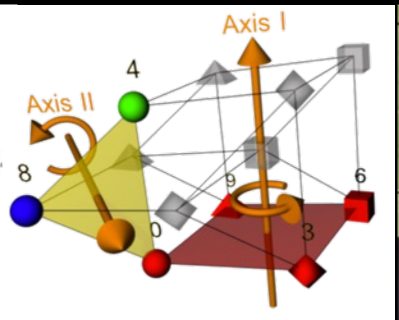
# HamilFloyd

4D & 2D Visualizations
Hamiltonian Cycles
M.Andreatta, G.Baroin 2020

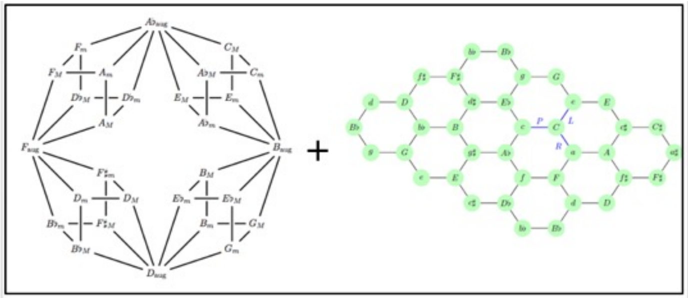
**Available at www.morenoandreatta.com**

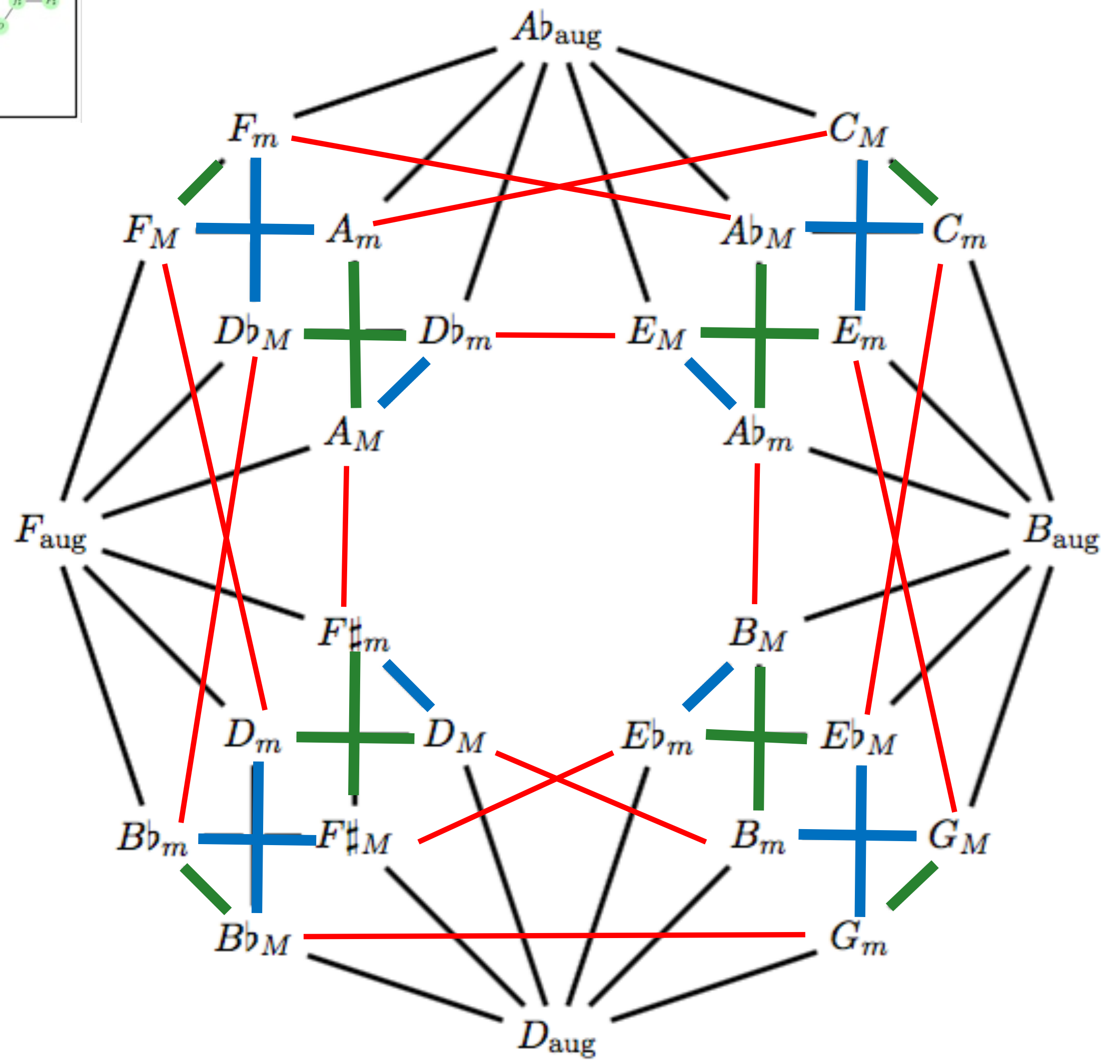
Composition, Performance: Moreno Andreatta
Hypersphere, Graphics, Animations: Gilles Baroin
Original "Cube Dance" graph: J.Douthett, P.Steinbach




# Embedding the Cube Dance into the Tonnetz






➔ **http://repmus.ircam.fr/moreno/chanson**

# A computational model of the Cube Dance (and its extensions)

Parsimonious graphs on triads for Douthett's and Steinbach's $P_{m,n}$ relations

➔ **https://alexpof.github.io/interactive_mathmusic/Pmn_graphs/pmn_graphs.html**

Select chords to display:

- ○ Major / Minor chords
- ◉ Major / Minor / Augmented chords
- ○ Major / Minor / Augmented / Sus4 chords

Select $P_{m,n}$ relations to display:

- ☑ $P_{1,0}$
- ☐ $P_{0,1}$
- ☐ $P_{1,1}$
- ☐ $P_{2,0}$
- ☐ $P_{0,2}$
- ☐ $P_{2,1}$
- ☐ $P_{1,2}$

Play Erase

Add chords to the progression by shift-clicking on the nodes.

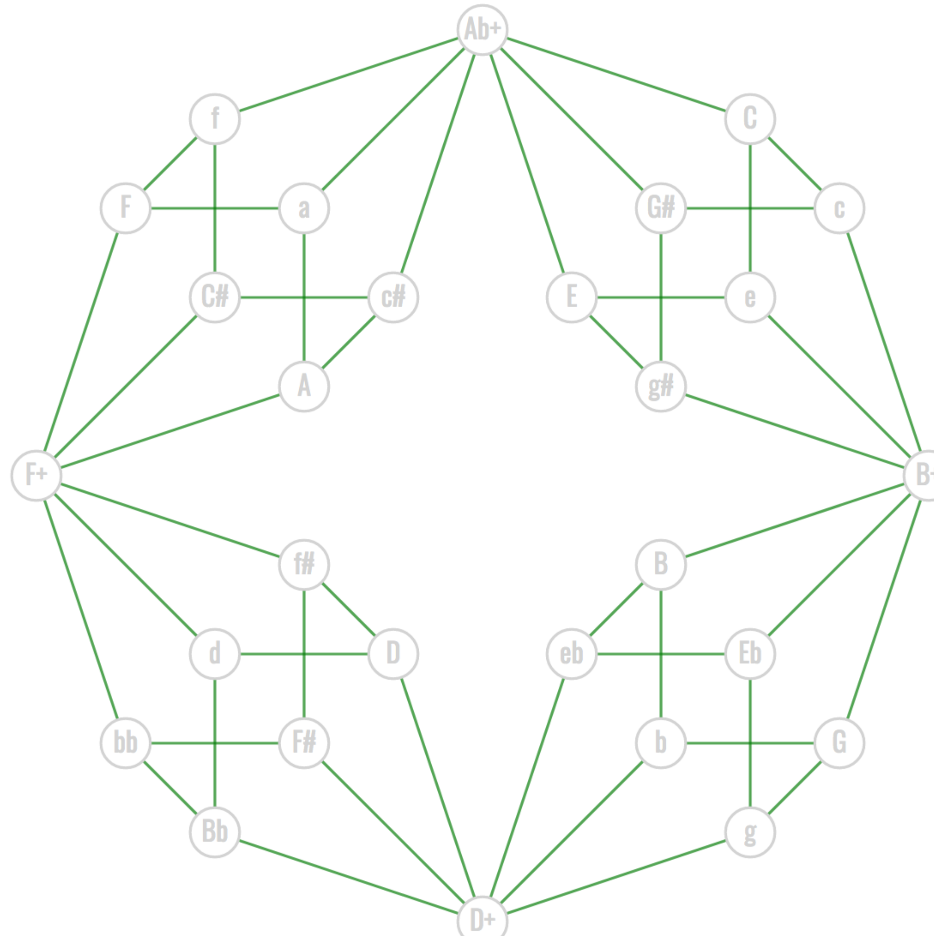
Two triads are said to be $P_{m,n}$-related if *m* pitch classes move by a semitone, while *n* pitch classes move by a whole tone, the rest of the pitch classes being identical.

Based on the original paper of Douthett and Steinbach : Douthett, Jack, and Peter Steinbach. 1998. "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition." Journal of Music Theory 42 (2): 241–263.

Visualization and code by Alexandre Popoff.
Best viewed with Chrome or Firefox. Compatibility with Internet Explorer and Microsoft Edge is not guaranteed.

**Jack Douthett**

**Alexandre Popoff**

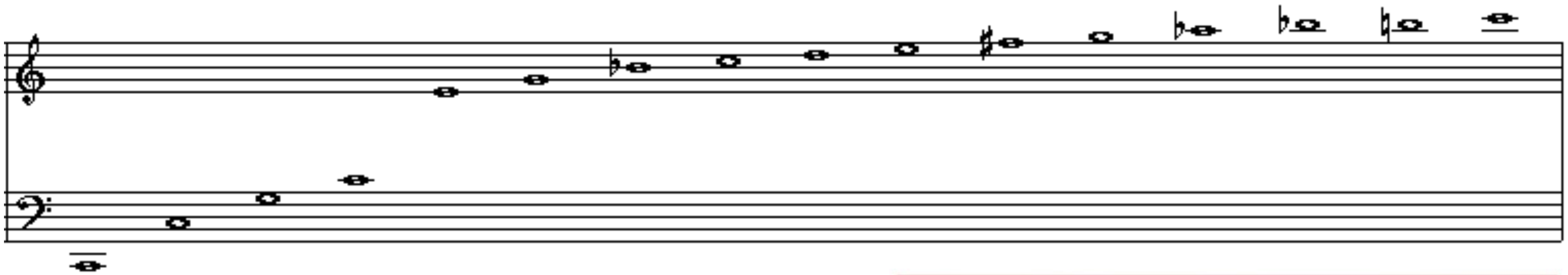


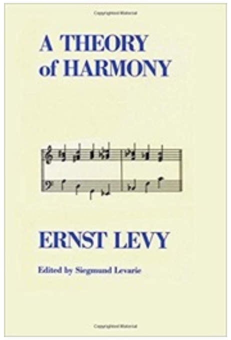

- The $\mathcal{P}_{1,0}$ binary relation connects two chords if they differ by the movement of only one pitch class by one semitone.

[1] Douthett, J., Steinbach, P., Journal of Music Theory, 42(2), 1998, pp. 241–263. // Cohn, R. 'Audacious Euphony: Chromaticism and the Triad's Second Nature', Oxford University Press, 2012

# The Negative/Symmetric Harmony (Jacob Collier/Steve Colemann)

## The series of harmonics






"The raw material of theories are facts. The raw materials of musical theory is music. Music is not, as some contemporary acousticians would like us to believe, 'something that happens in the air.' It is something that, first and last, happens in the soul."

Ernst Lévy (1895-1981)

Steve Coleman

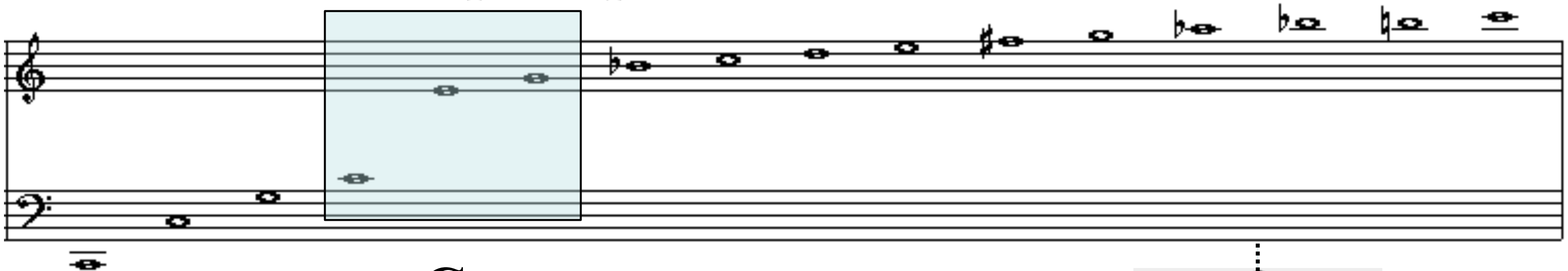
## The (imaginary) series of descending harmonics

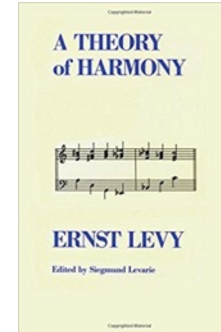
C. Corea, J. Collier & H. Hancock

- Ernst Lévy, *A Theory of Harmony*, Albany, New Yortk, 1985
- Jacob Collier, « Negative Harmony » (vidéos online)
- Steve Coleman, « Symmetrical Movement Concept » (online)

# The Negative/Symmetric Harmony (Jacob Collier/Steve Colemann)

## La série des harmoniques supérieurs

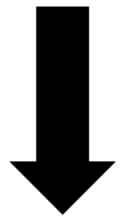


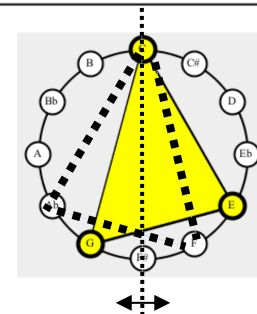



**C**

↓

**Fm**






Ernst Lévy
(1895-1981)

Steve Coleman

## La série (imaginaire) des harmoniques inférieurs

C. Corea, J. Collier & H. Hancock

- Ernst Lévy, *A Theory of Harmony*, Albany, New Yortk, 1985
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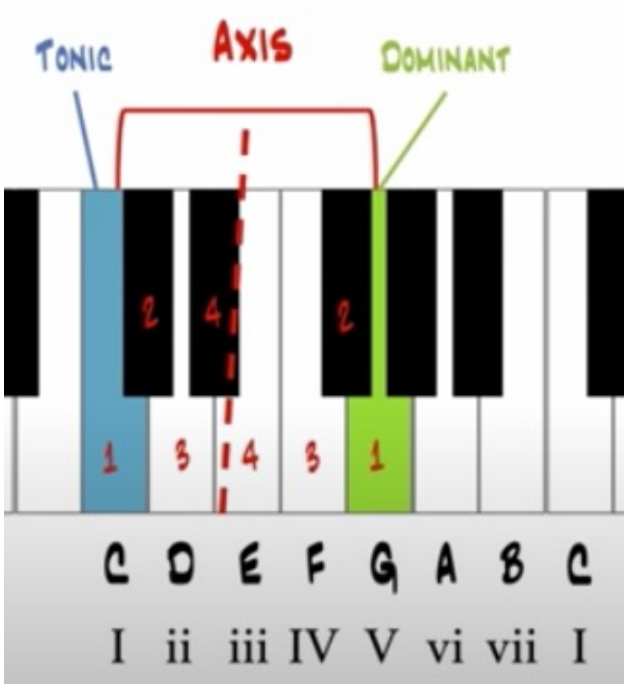
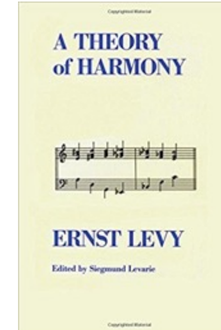
# The Negative/Symmetric Harmony (Jacob Collier/Steve Colemann)

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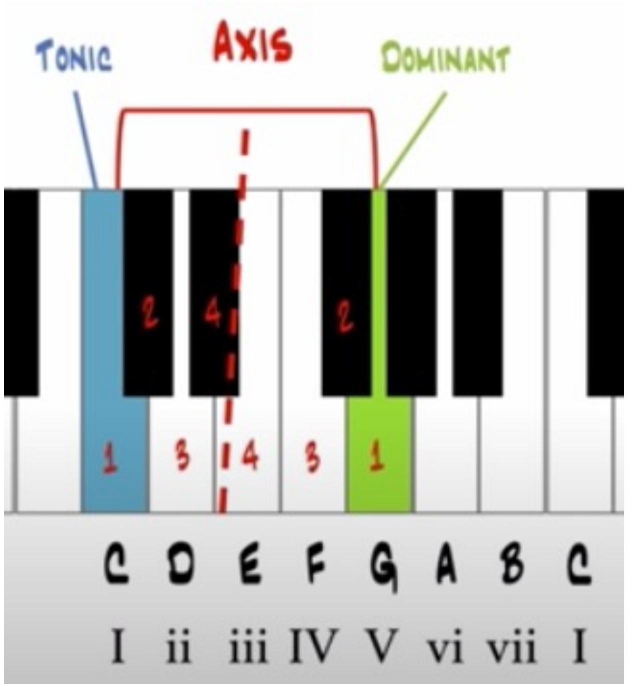
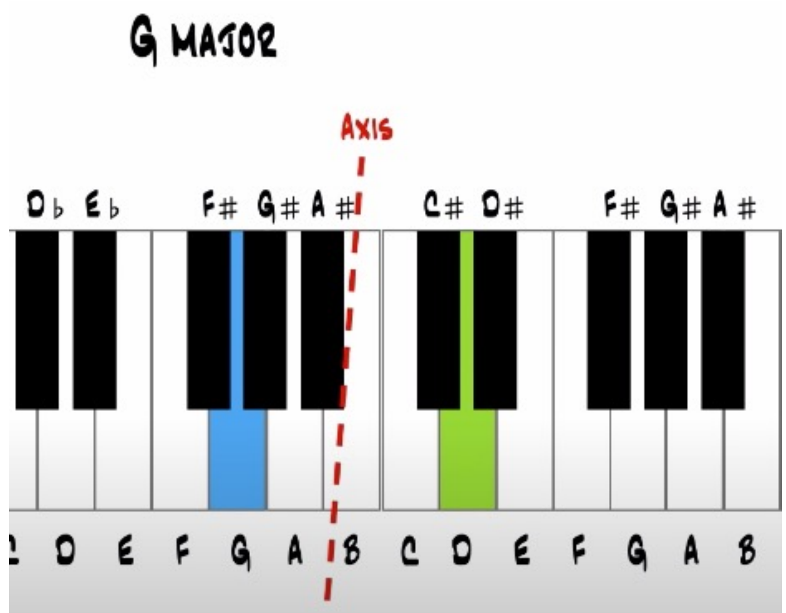
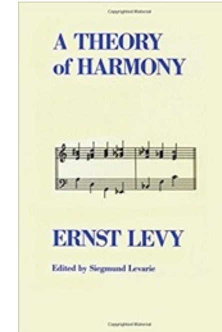



L'axe de symétrie dépend de la tonalité !

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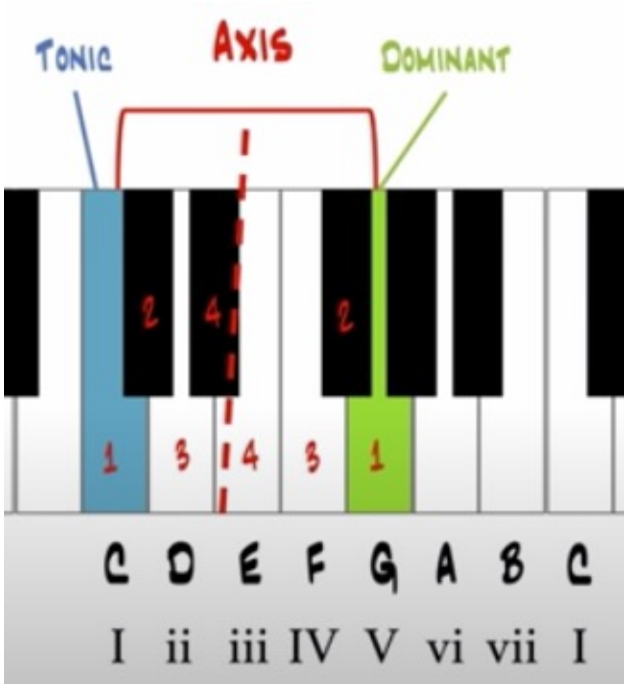


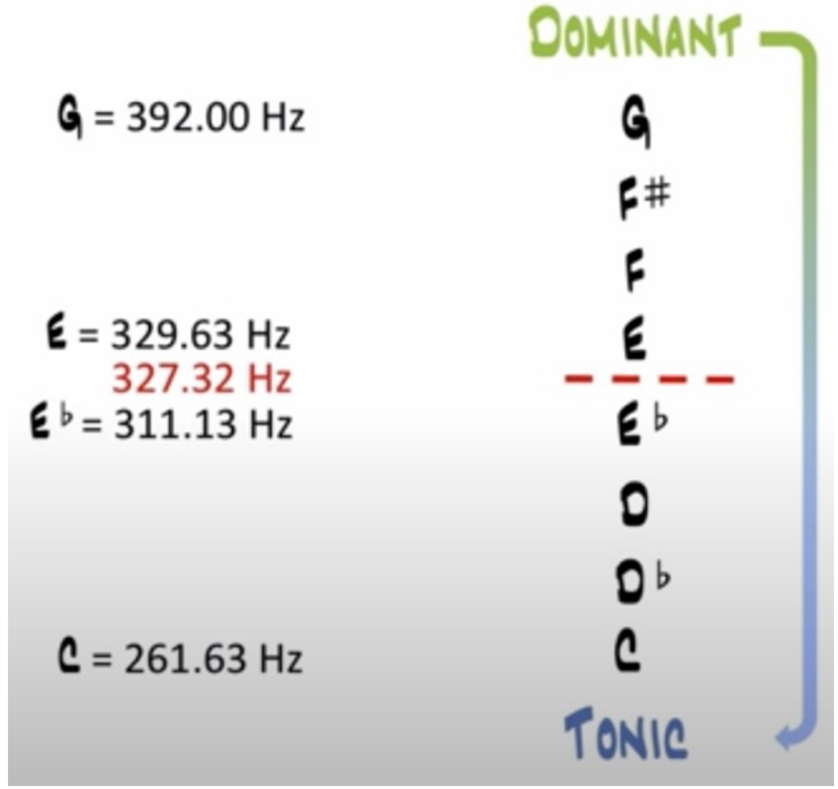


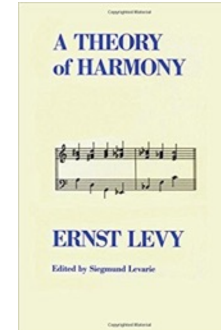


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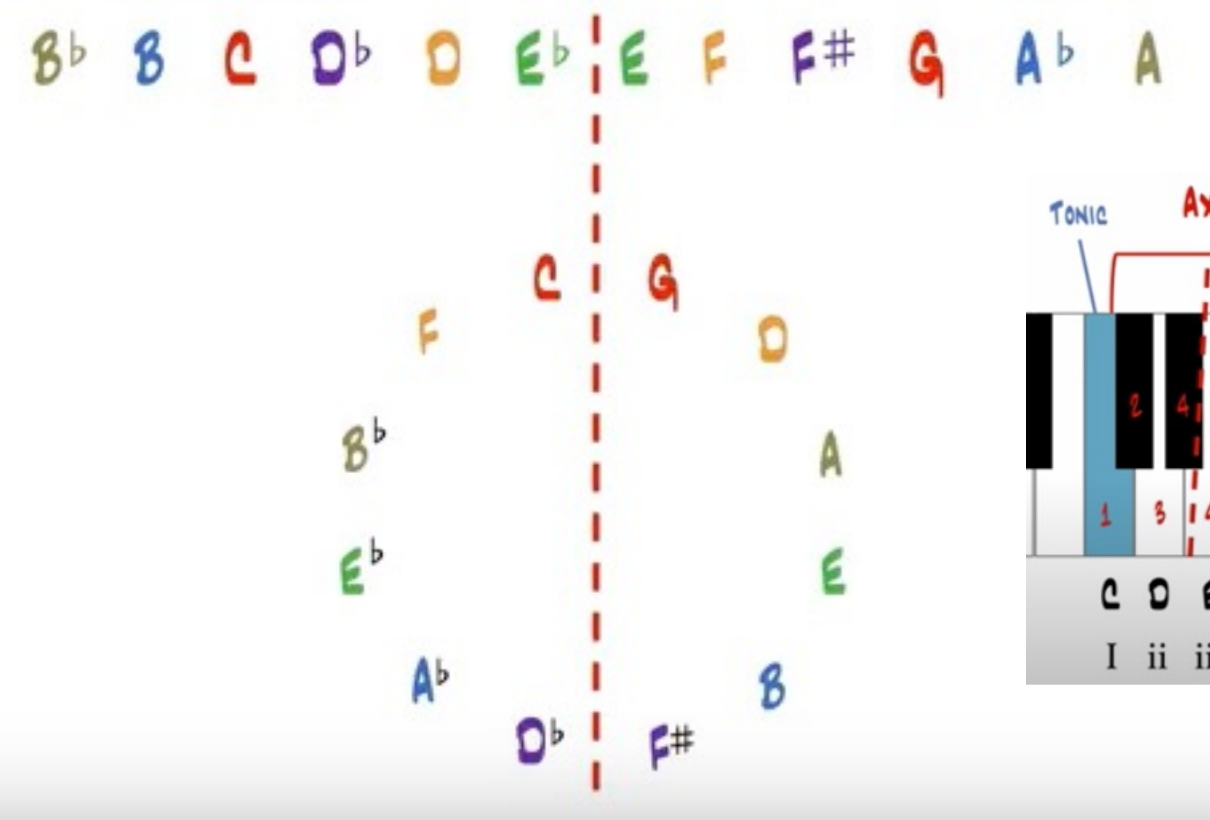


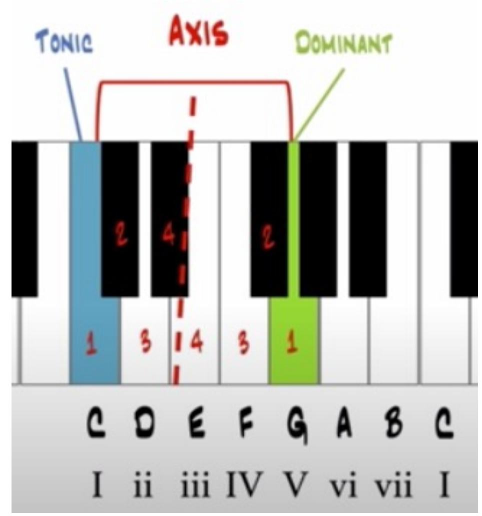


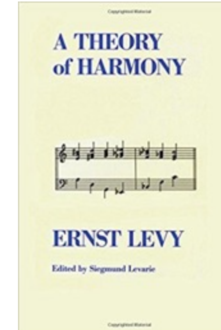


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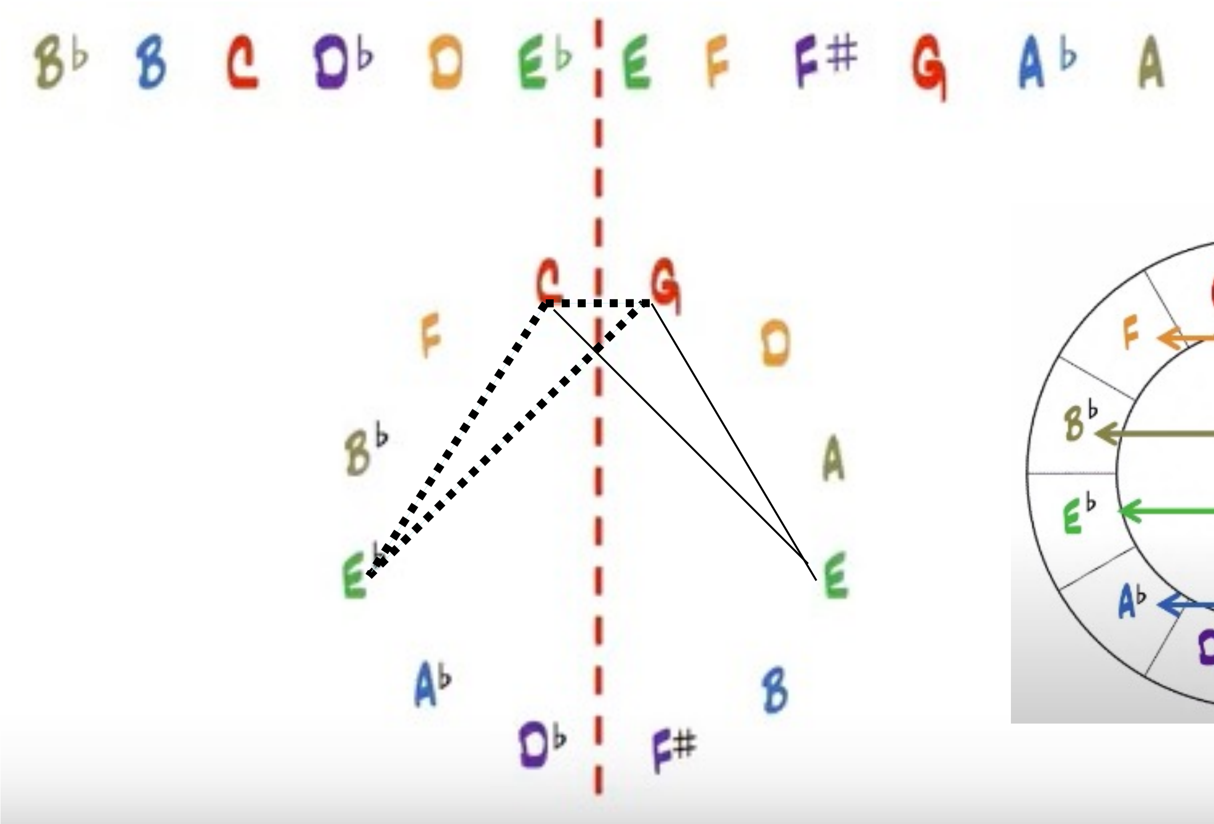


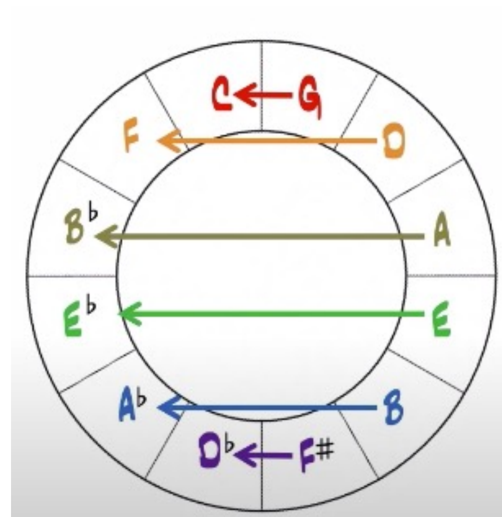


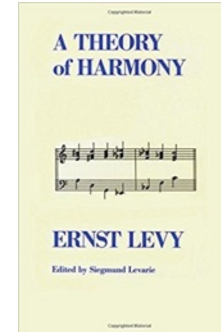


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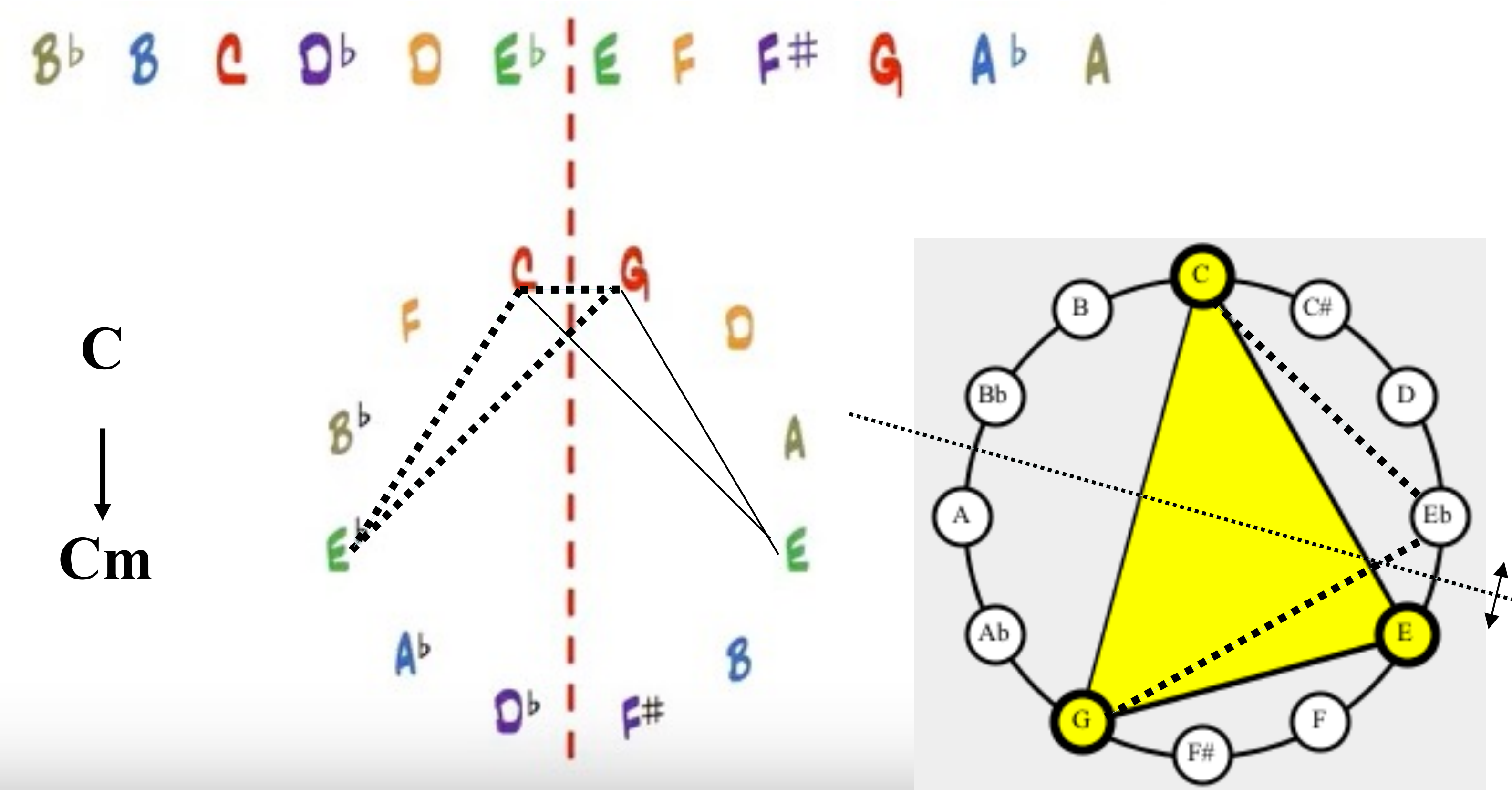
C. Corea, J. Collier & H. Hancock

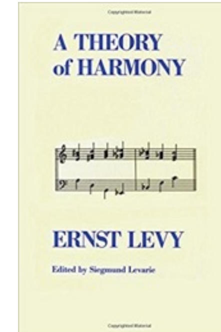
# The Negative/Symmetric Harmony (Jacob Collier/Steve Colemann)

C

↓

Cm

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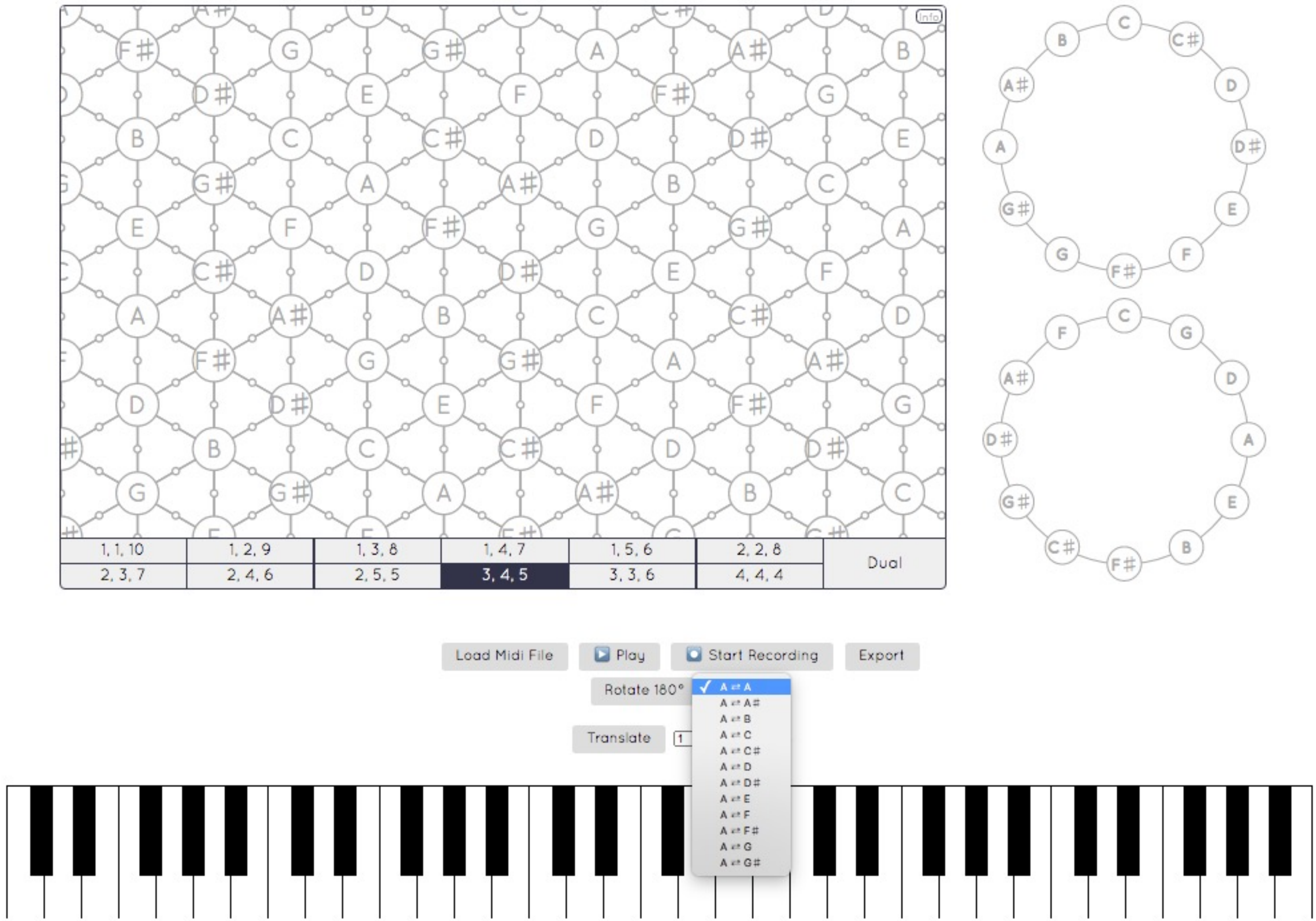
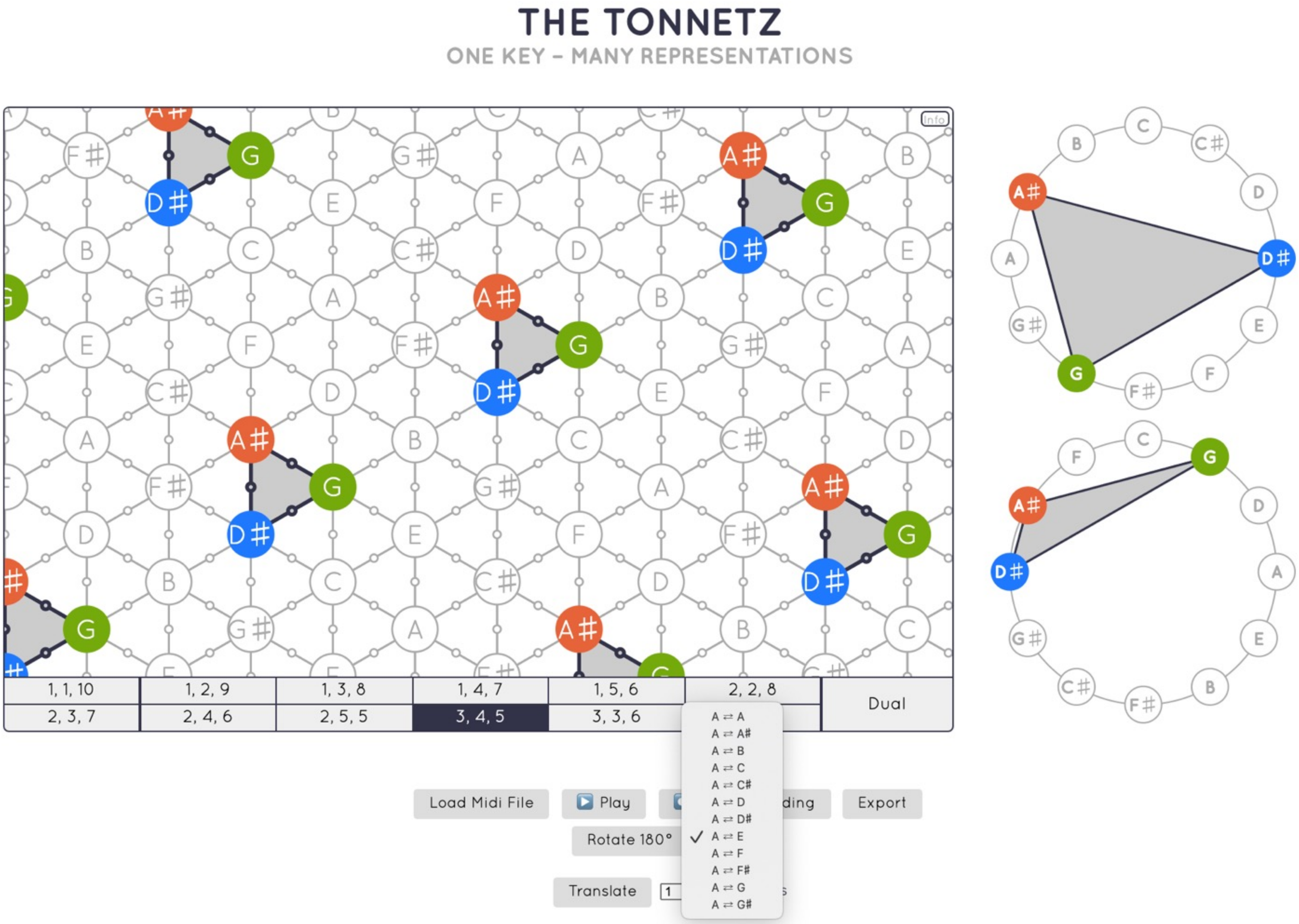
C. Corea, J. Collier & H. Hancock

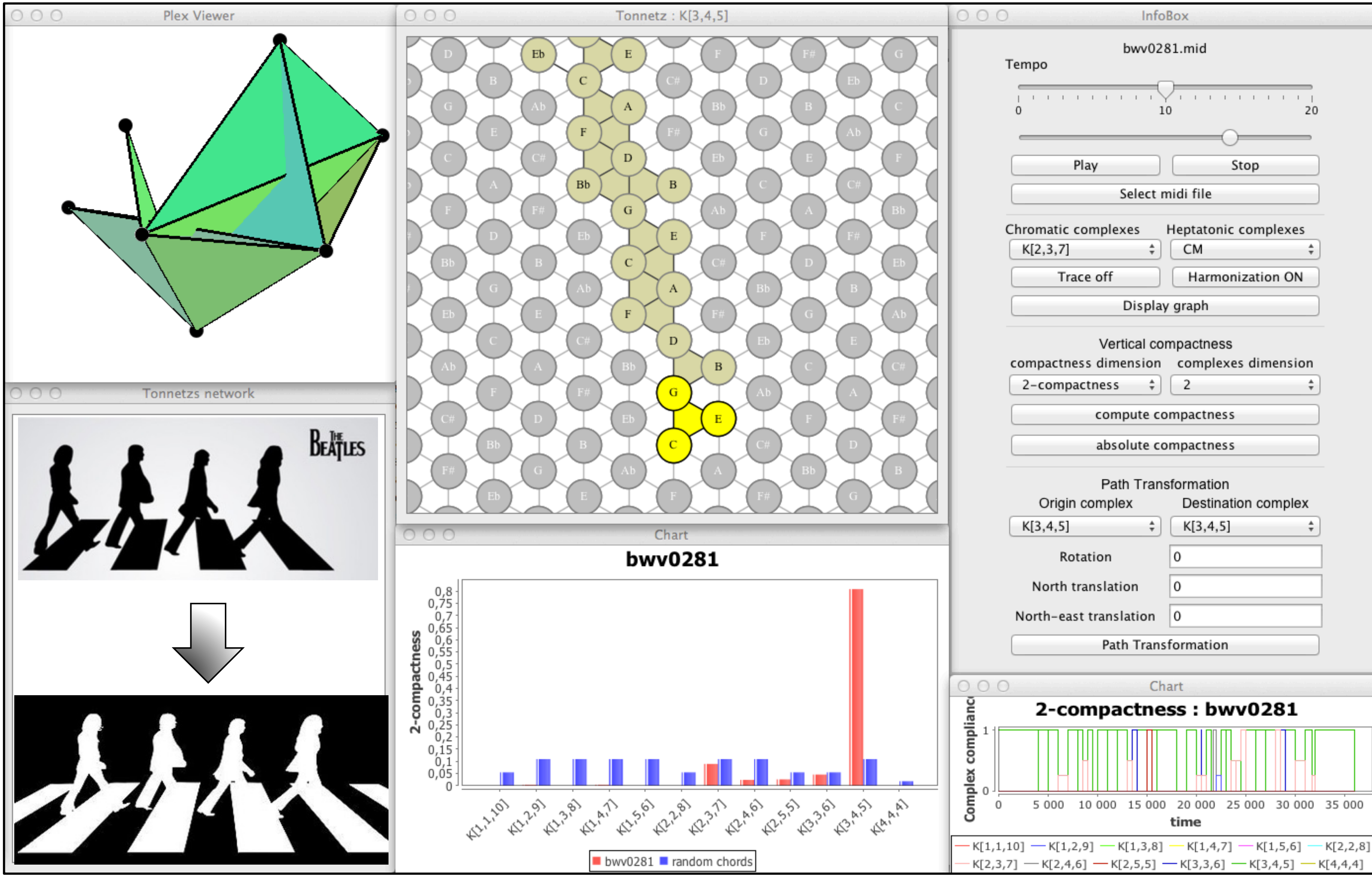
# Negative Harmony in the TONNETZ environnement



# Example: Negative Harmony of *Your Song* (E. John)



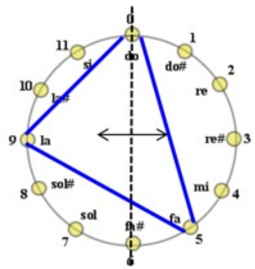
# Back to the Beatles exemple



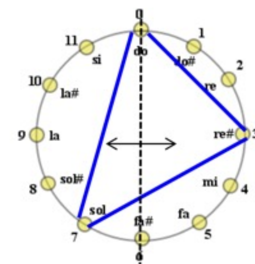

➔http://www.lacl.fr/~lbigo/hexachord

# Back to the Beatles exemple

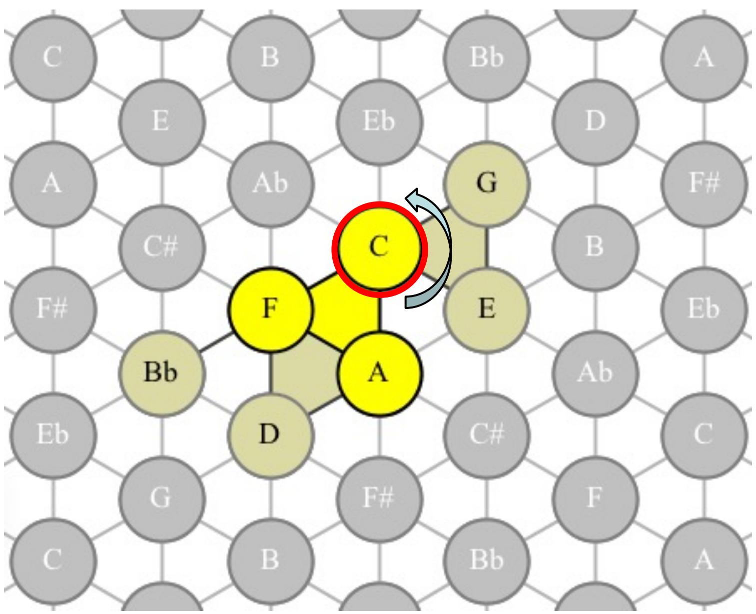
*F* major chord



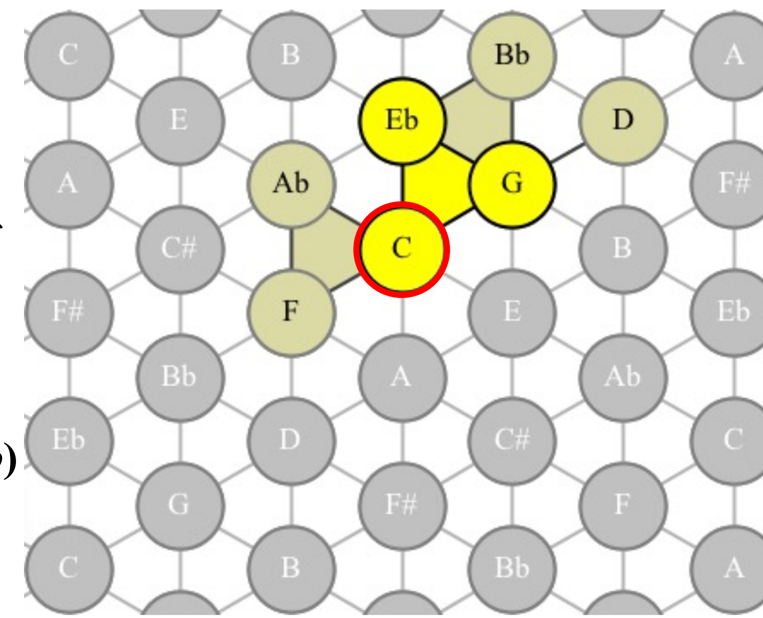

inversion




*C* minor chord

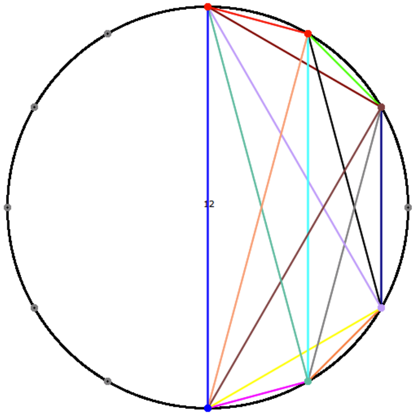



**Rotation**
**(autour du *do*)**



# Additional slides

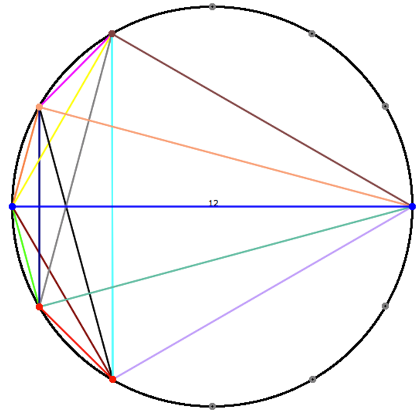
# The shortest proof of Babbitt's Theorem?

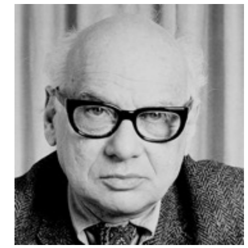


$\approx$

$\longleftrightarrow$

**Hexachord Theorem**





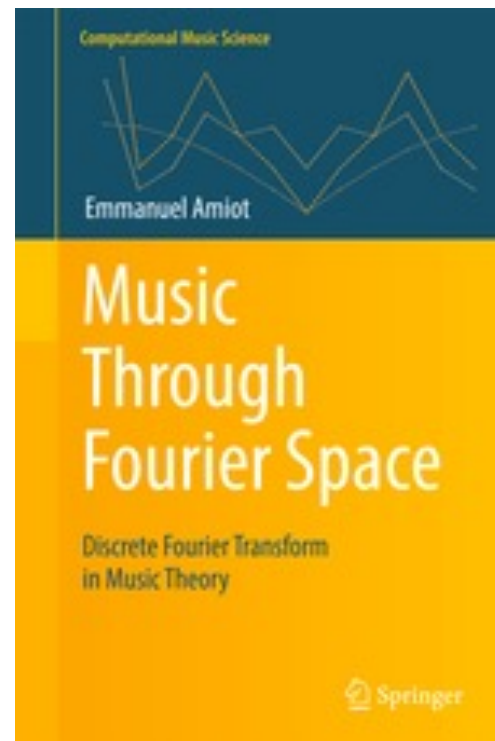
**M. Babbitt**

$$IC_A = [4, 3, 2, 3, 2, 1\,] = [4, 3, 2, 3, 2, 1\,] = IC_{A'}$$

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\forall \mathrm{k}\ \mathcal{F}(\mathrm{IC}_{\mathbb{Z}_c \setminus A})(\mathrm{k}) = \mathcal{F}(\mathrm{IC}_A)(\mathrm{k})$$




**E. Amiot : « Une preuve élégante du théorème de Babbitt par transformée de Fourier discrète », Quadrature, 61, 2006.**

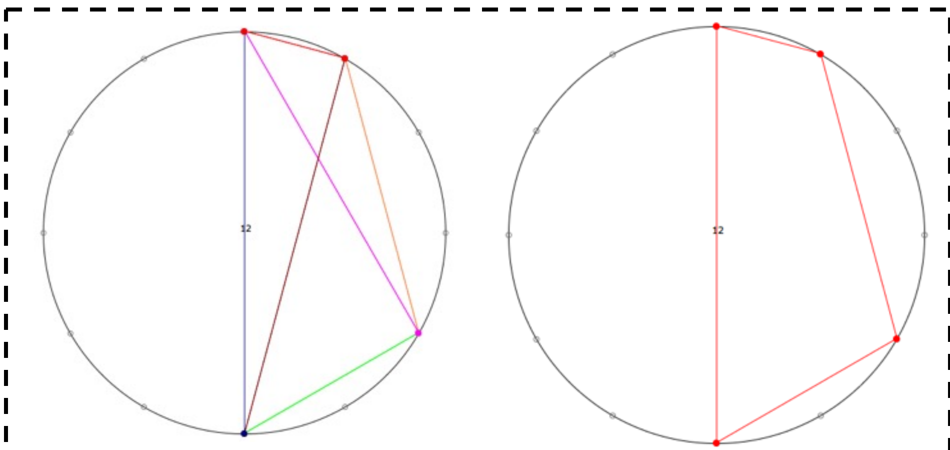
# Z-relation, homometry and phase retrieval problem

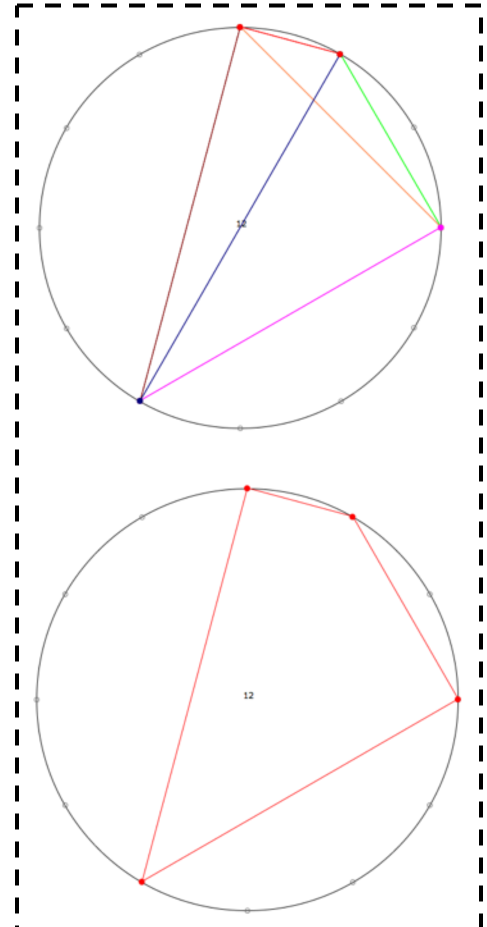
- Two sets are Z-related if they have the same module of the DFT

$$IC_A(k) = (1_A \star 1_{-A})(k)$$

$$\mathcal{F}_A : t \mapsto \sum_{k \in A} e^{-2i\pi kt/c}$$

$$\mathcal{F}(IC_A) = \mathcal{F}_A \times \mathcal{F}_{-A} = |\mathcal{F}_A|^2$$



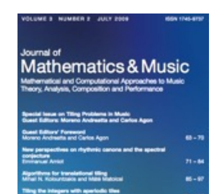


**Z-relation**

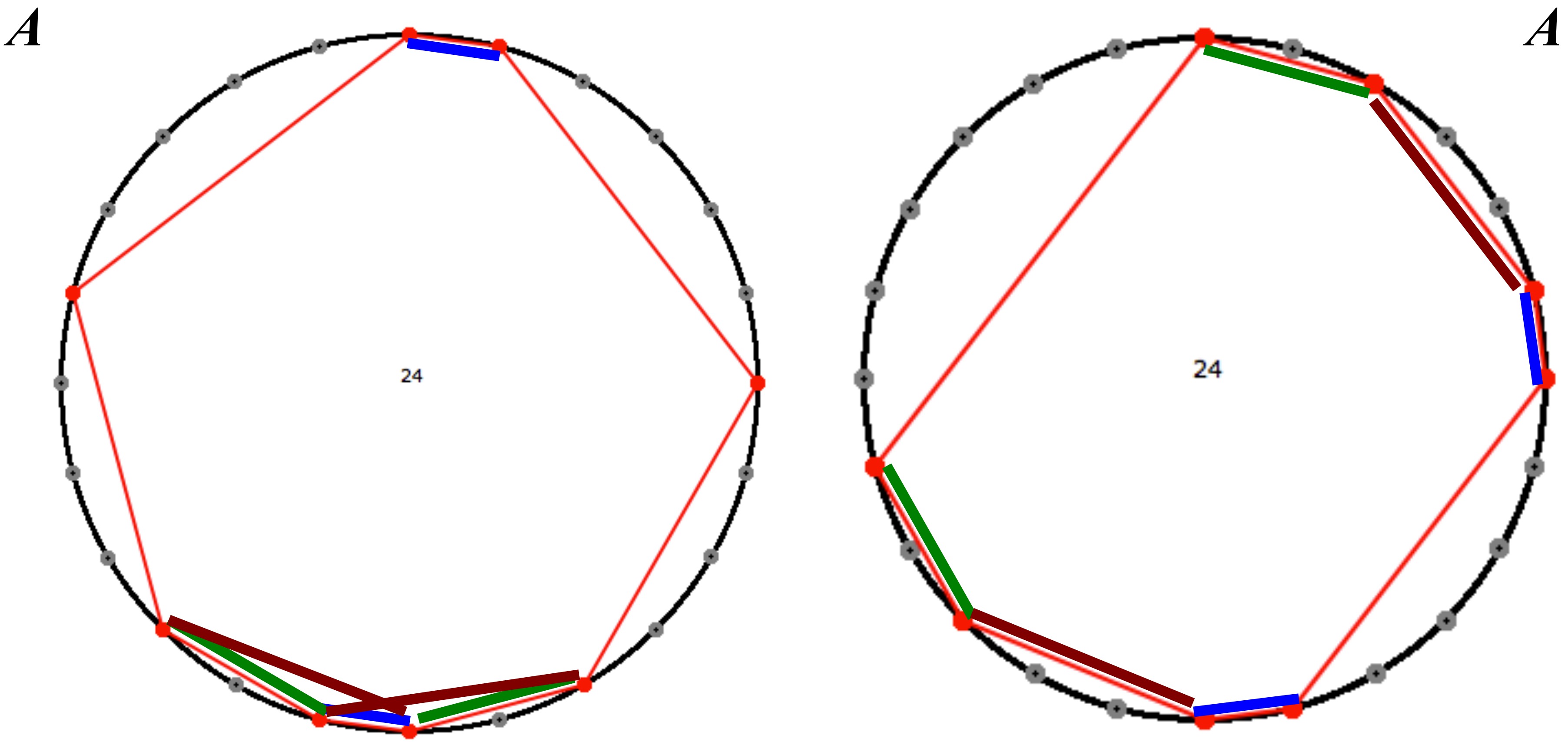
$\longleftrightarrow$

**homometry**

- **Mandereau J., D. Ghisi, E. Amiot, M. Andreatta, C. Agon, (2011), « Z-relation and homometry in musical distributions »,** ***Journal of Mathematics and Music*****, vol. 5, n° 2, p. 83-98.**
- **Mandereau J, D. Ghisi, E. Amiot, M. Andreatta, C. Agon (2011), « Discrete phase retrieval in musical structures »,** ***Journal of Mathematics and Music*****, vol. 5, n° 2, p. 99-116.**



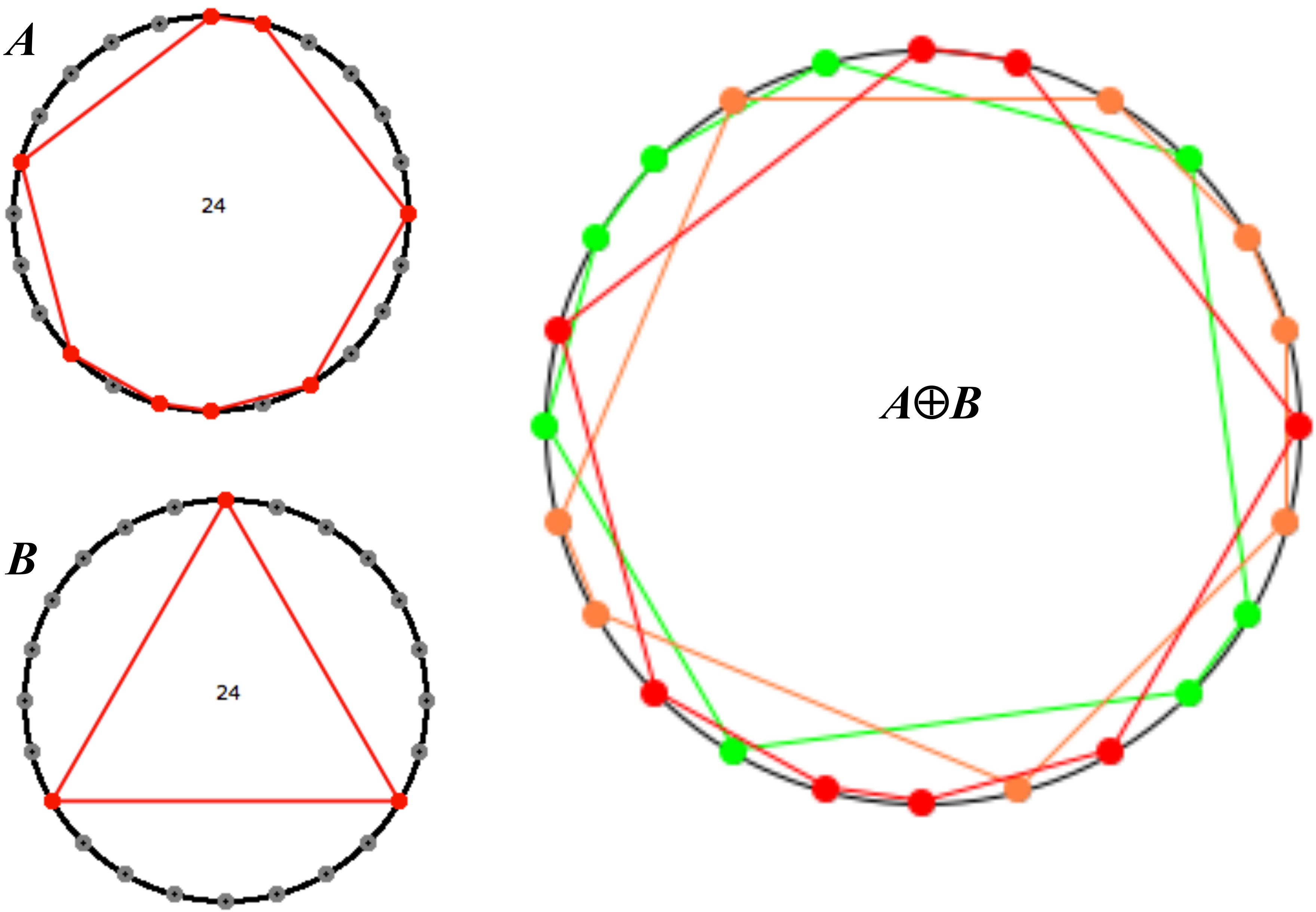
# Z-relation (music) and homometry (cristallography)




$$IC_A(k) = (1_A \star 1_{-A})(k)$$

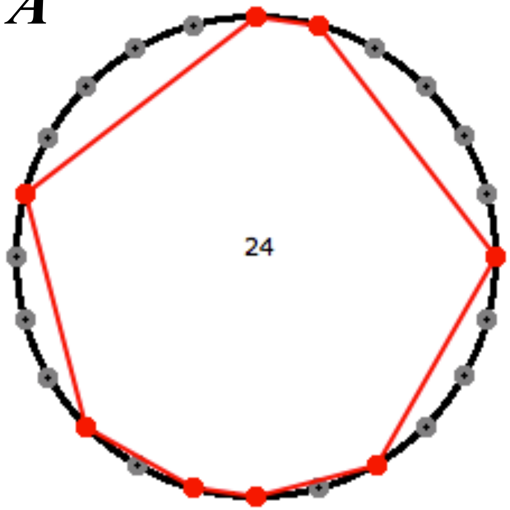
$$A \sim A' \quad \overset{Z\text{-relation}}{\underset{Homometry}{\Longleftrightarrow}} \quad IC_A(k) = IC_{A'}(k) \quad \Longleftrightarrow \quad |F_A|^2 = |F_{A'}|^2$$

# Tiling Rhythmic Canons and Homometry

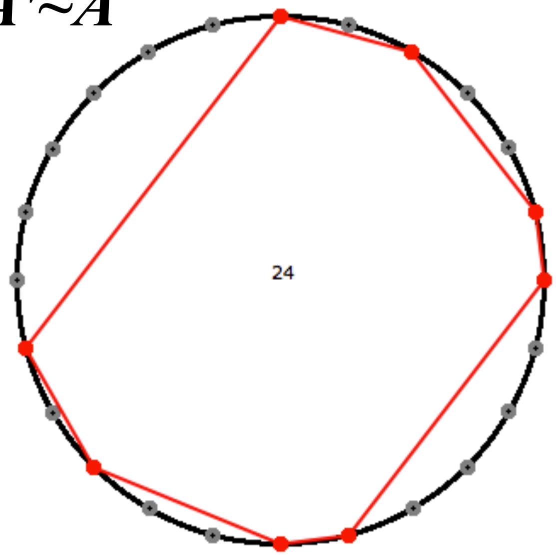


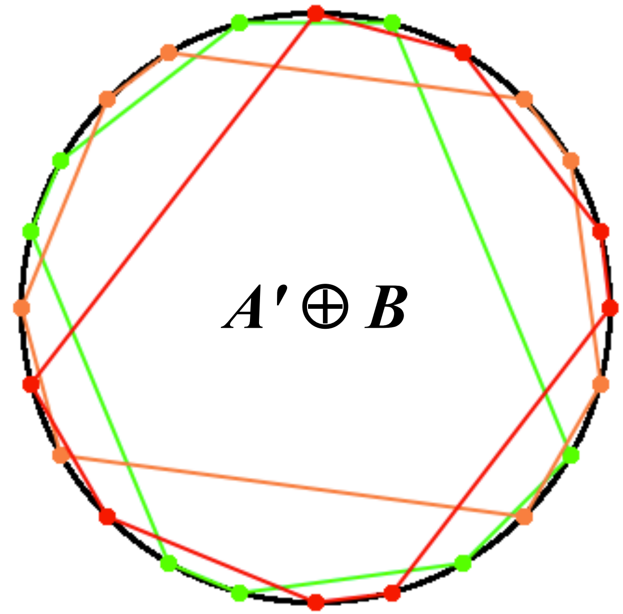
# Tiling Rhythmic Canons and Homometry

*A*

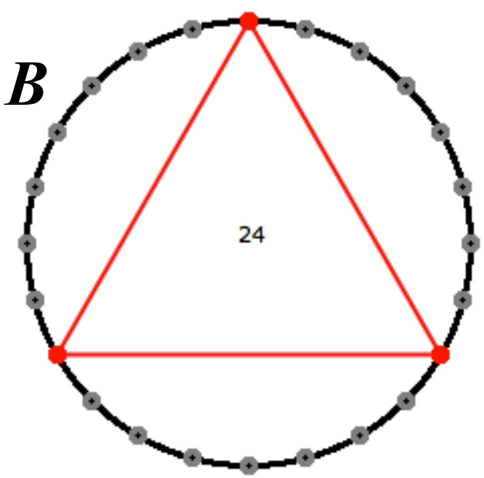



*A′~A*




*B*

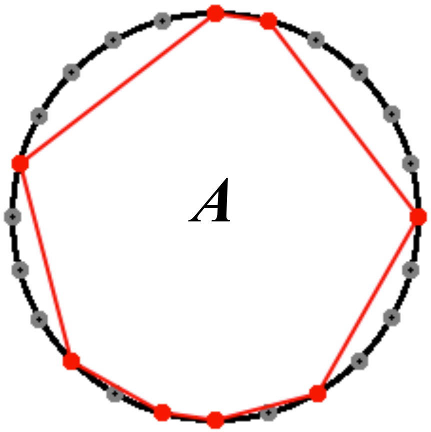


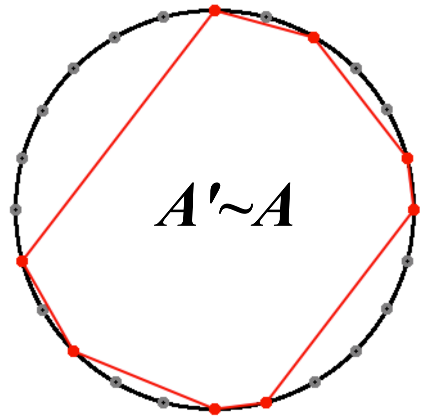

*A musical offering*:

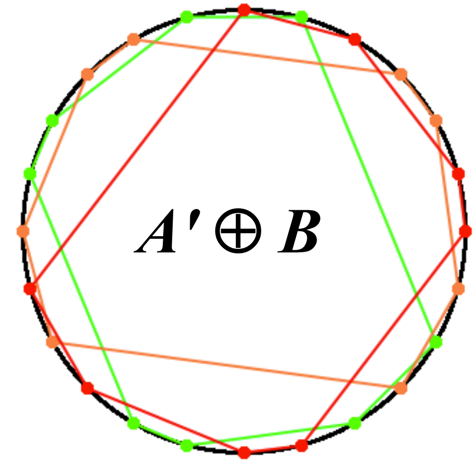
- ***Theorem***:
  If A tiles with B and A' has the same IC, then A' tiles with B, too.

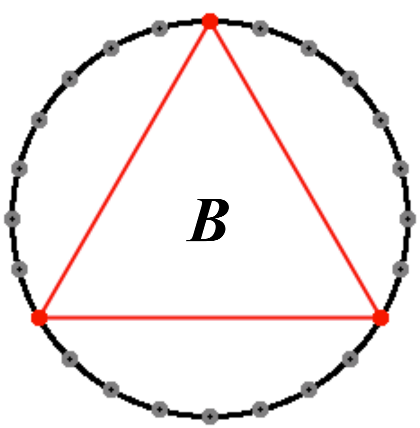
E. Amiot, « New Perspectives on Rhythmic Canons and the Spectral Conjecture », *JMM*, 2009.

# Tiling Rhythmic Canons and Homometry



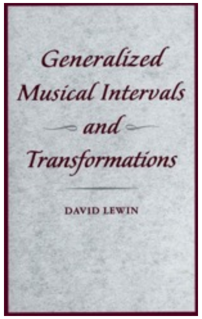

## Tiling

Let $Z_A = \{ t \in \mathbb{Z}_c,\ F_A(t) = 0 \}$

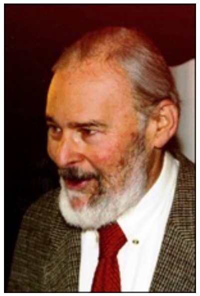
A tiles $\mathbb{Z}_c$ when equivalently:

- There exists B, $A \oplus B = \mathbb{Z}_c$
- $1_A \star 1_B = 1$
- $F_A \times F_B\,(t) = 1 + e^{-2i\pi t/c} + \ldots e^{-2i\pi t(c-1)/c}$ (0 unless t=0)
- $Z_A \cup Z_B = \{1, 2 \ldots c-1\}$ AND Card A × Card B = c
- $IC_A \star IC_B = IC(\mathbb{Z}_c) = c$ and Card A × Card B = c

E. Amiot, « New Perspectives on Rhythmic Canons and the Spectral Conjecture », *JMM*, 2009.




# Système d'Intervalles Généralisés - Système Généralisé d'Intervalles
## David Lewin' s *Generalized Interval System* [GMIT, 1987]



D. Lewin

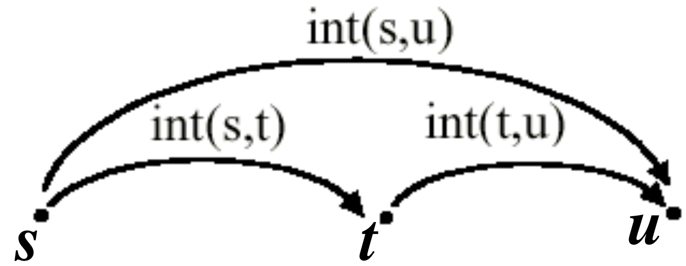
**GIS = (*S*, *G*, int)**

*S* = set

(*G*,•) = group of intervals

int = intervallic function

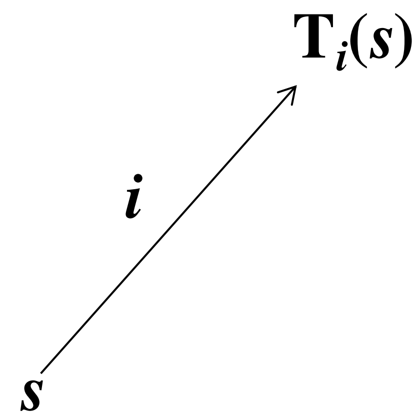
$$S \times S \xrightarrow{\text{int}} G$$




**Simply transitive action**

1. For all objects *s*, *t*, *u* in *S* :
   int(*s*, *t*)•int(*t*, *u*) = int(*s*, *u*)

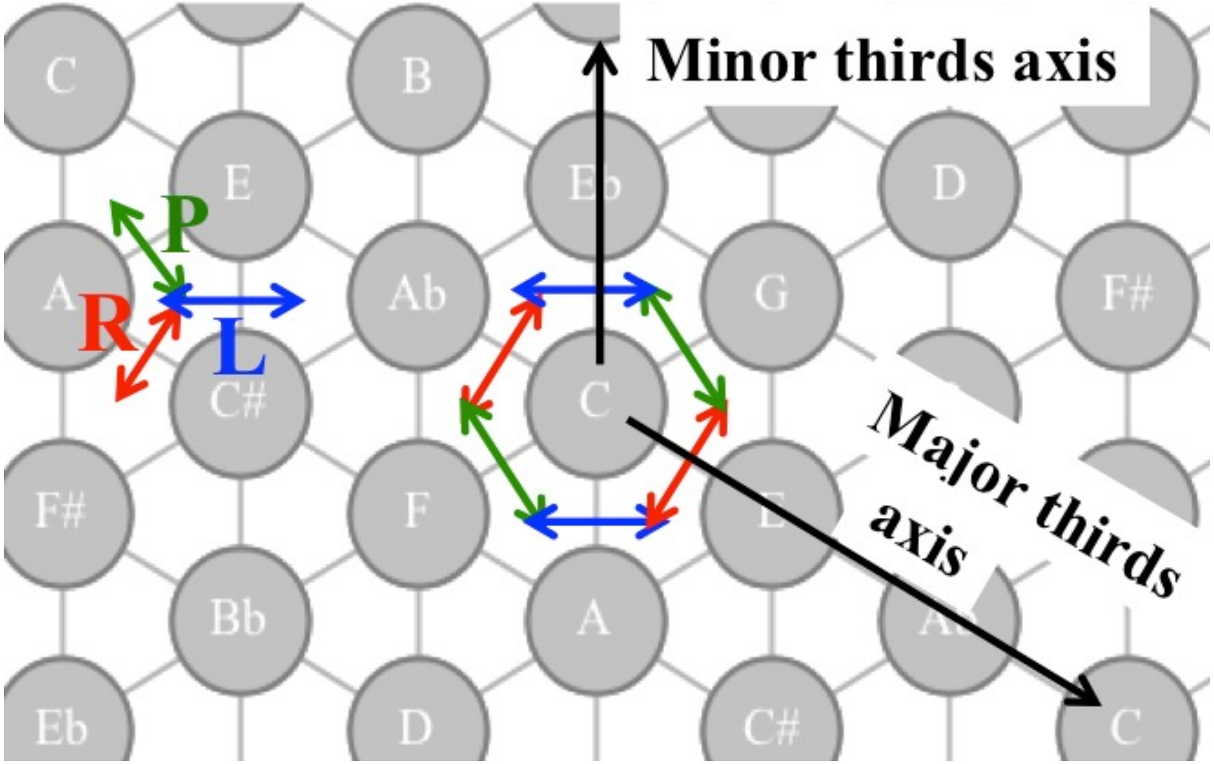
2. For all object *s* in *S* and for all interval *i* in *G* there exists a unique object *t* in *S* such that int(*s*, *t*) = *i*

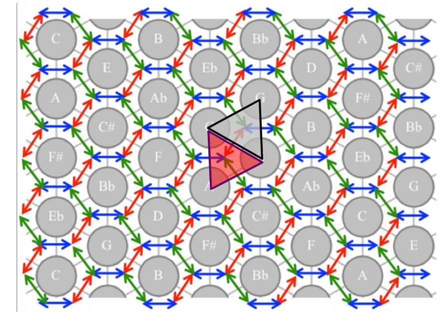


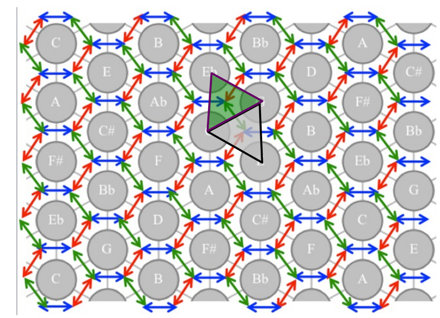

Let $\tau = \{T_i \,;\, i \in G\}$ be the group of transpositions

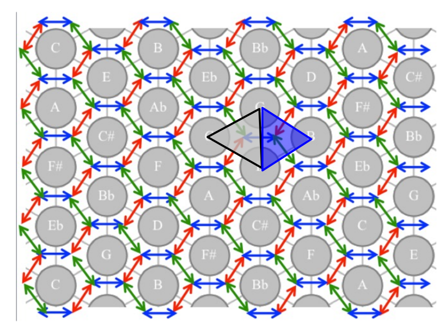
GIS = (*S*, *G*, int) $\Leftrightarrow$ $\tau \times S \rightarrow S$ such that $(T_i, s) \rightarrow T_i(s)$ where $\text{int}(s, T_i(s)) = i$

# The *Tonnetz* as "Generalized Interval System"



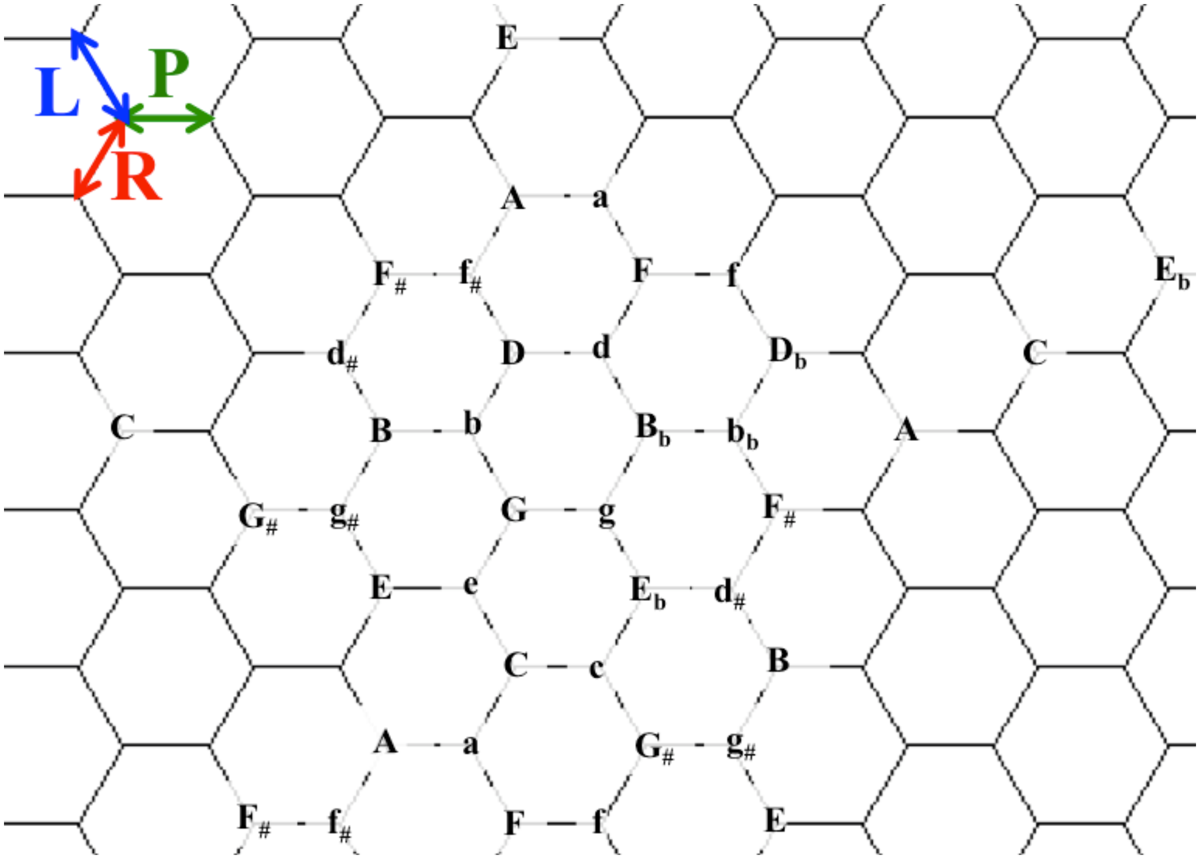


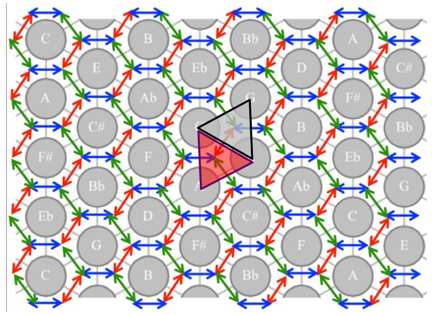


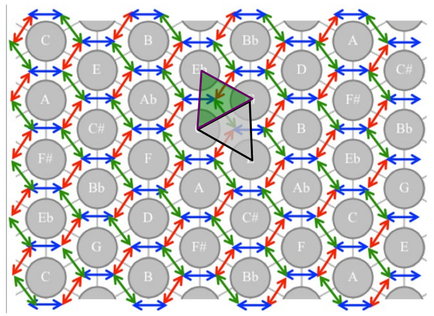
$$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$$

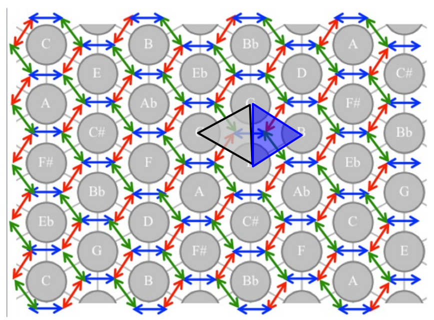
**ρ acts in a simply transitive way on the set *S* of the 24 consonant triads**

# The *Tonnetz* as "Generalized Interval System"



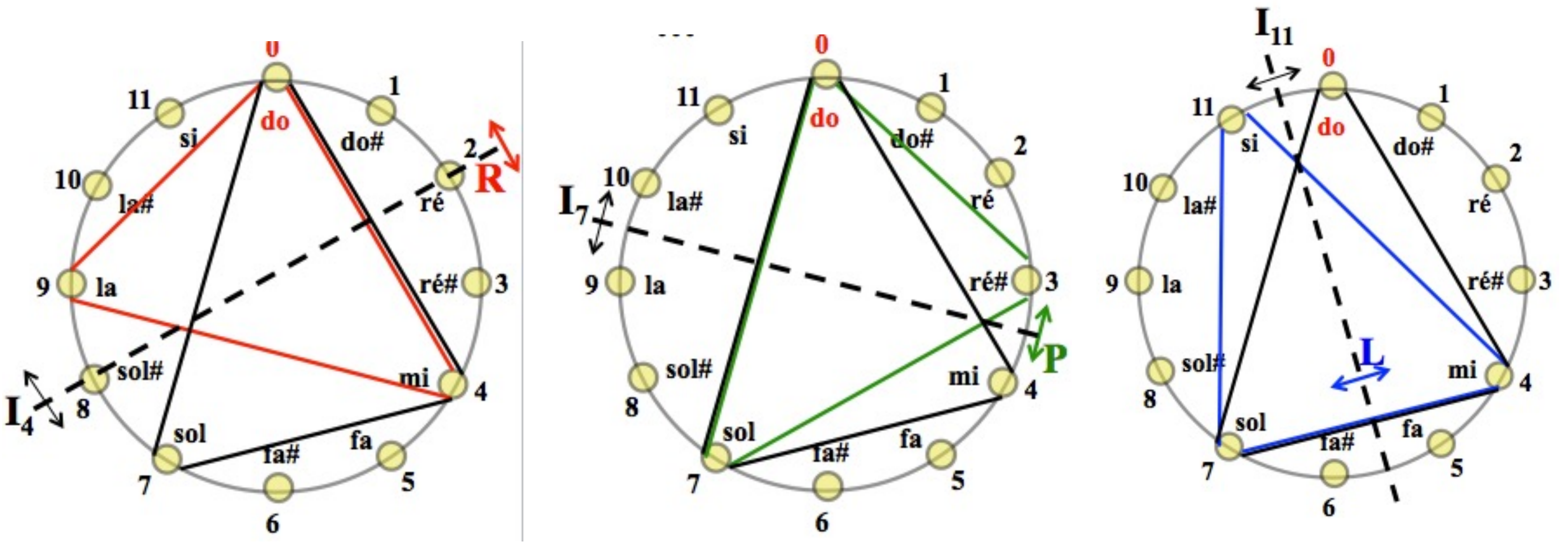




$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \; ; \; LRL = L(LR)^{-1} \rangle$

**ρ acts in a simply transitive way on the set *S* of the 24 consonant triads**

# A different GIS structure on the same set *S*



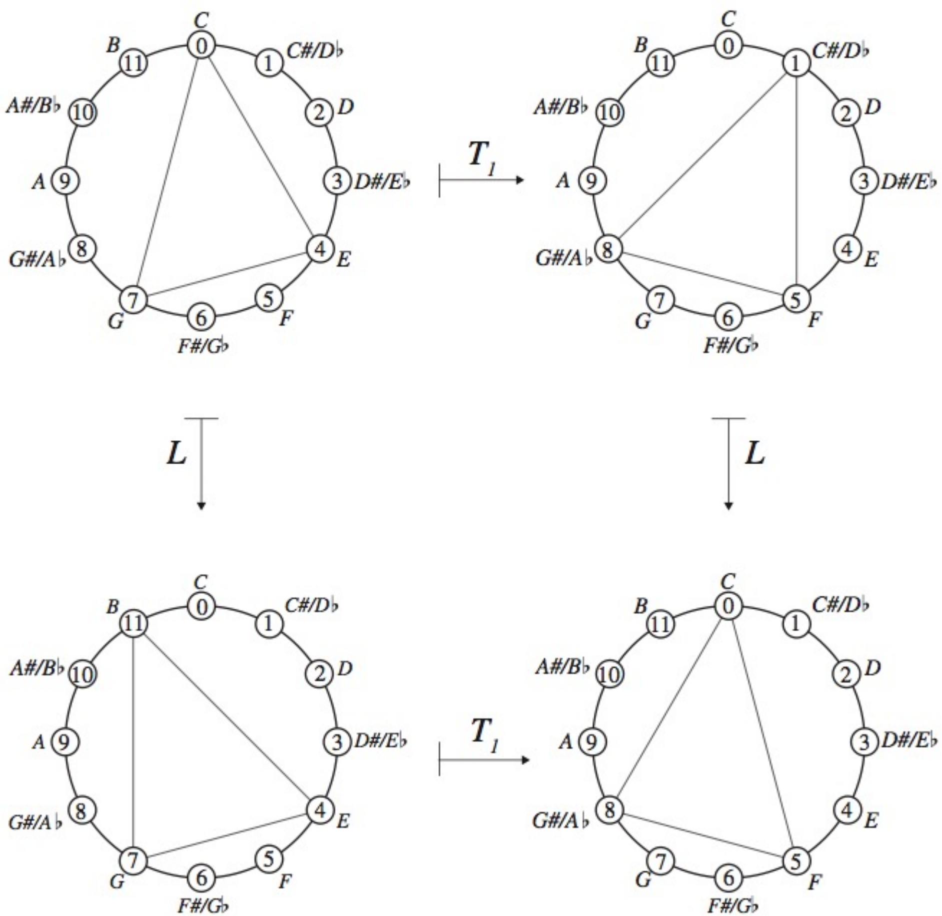

$I_4 : x \to 4 - x$ $\quad$ $I_7 : x \to 7 - x$ $\quad$ $I_{11} : x \to 11 - x$

$$D_{12} = \langle I, T \mid I^2 = T^{12} = 1 \; ; \; ITI = I(IT)^{-1} \rangle$$

**$D_{12}$ acts in a simply transitive way on the set *S* of the 24 consonant triads**

# Two “dual” actions on the set of consonant triads

$\rho = \langle L, R \mid L^2 = (LR)^{12} = 1 \ ; \ LRL = L(LR)^{-1} \rangle$ $\leftrightarrow$ $D_{12} = \langle I, T \mid I^2 = T^{12} = 1 \ ; \ ITI = I(IT)^{-1} \rangle$




$$\begin{tikzcd} \circ \arrow[r, "f"] \arrow[d, "g"'] & \circ \arrow[d, "g"] \\ \circ \arrow[r, "f"] & \circ \end{tikzcd}$$

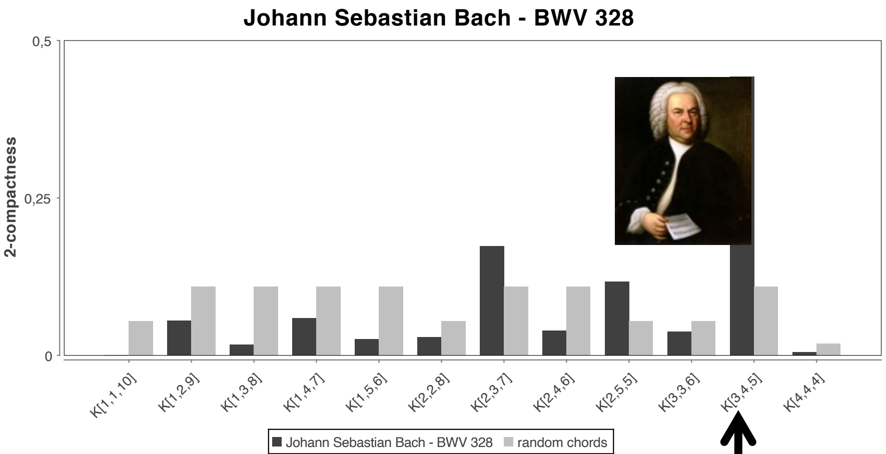
**Every diagram commutes**

$\forall f \in D_{12}$

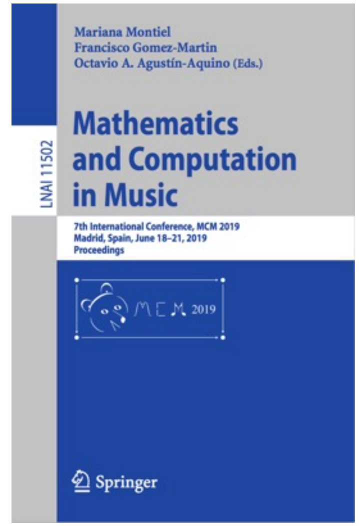
$\forall g \in \rho$

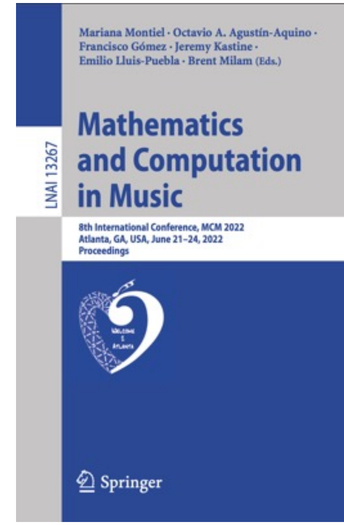
➔ **Alissa Crans, Tom Fiore and Ramon Satyendra, « Musical Actions of Dihedral Groups »,** ***The American Mathematical Monthly*****, Vol. 116 (2009), No. 6: 479 - 495**

# The geometric and topological character of musical style




$K_{TI}[3,4,5]$




## Persistent Homology

### Filtration and Persistence

- **Filtered simplicial complex** (figure 1): $\emptyset = K^{-1} \subset K^0 \subset \ldots \subset K^N = K$.
- **Persistent Homology**: computing simplicial homology $H_*(K^i)$ over $\mathbb{F}_2$ for each time $i$.
- **Barcodes** (figure 2): graph where the horizontal axis shows the progress in the filtration and a bar that starts at time $s$ and ends at time $t$ is a generator of $H_*(K^s)$ that is still one for $H_*(K^{t-1})$, but not at time $t$.

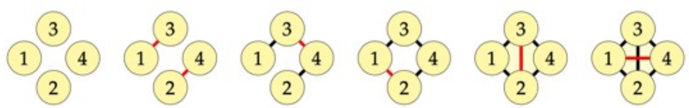
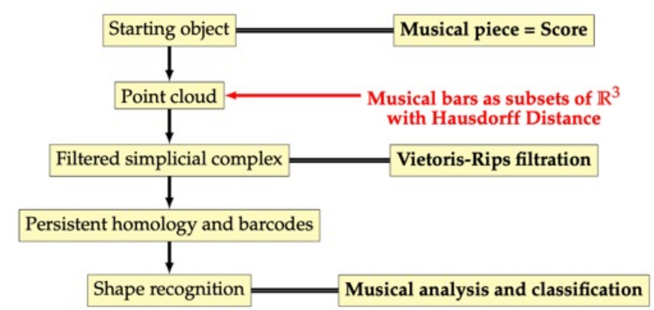


**Figure 1** – A filtered complex with 6 times of filtration.

**Figure 2** – Barcodes for filtration of figure 1 in degree 0 (left) and degree 1 (right).

### Context

**Topological Data Analysis:**

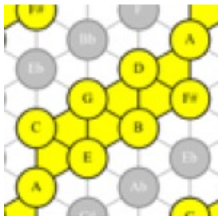



**How should we associate a filtered complex with a given musical piece?**


- **Bigo L., M. Andreatta, "Musical analysis with simplicial chord spaces", in D. Meredith (ed.), *Computational Music Analysis*, Springer, 2015**
- **Bigo L., M. Andreatta, "Filtration of Pitch-Class Sets Complexes", in M. Montiel et al. (eds.), Proceedings of MCM 2019, Madrid.**
- **Callet V., "Persistent Homology on Musical Bars", in M. Montiel et al. (eds), Proceedings of MCM 2022, Atlanta**

# Towards an anisotropic *Tonnetz*

Mattia Bergomi



Ionian mode

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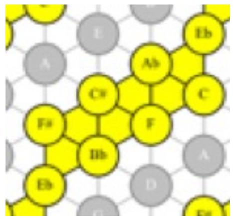
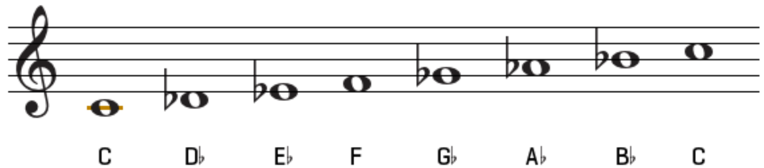
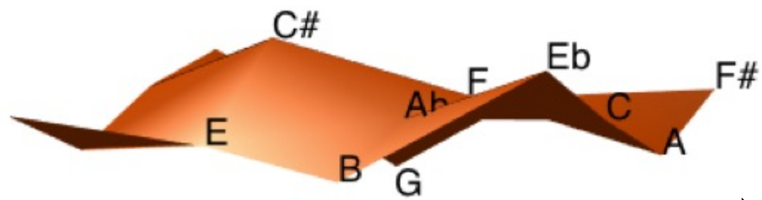
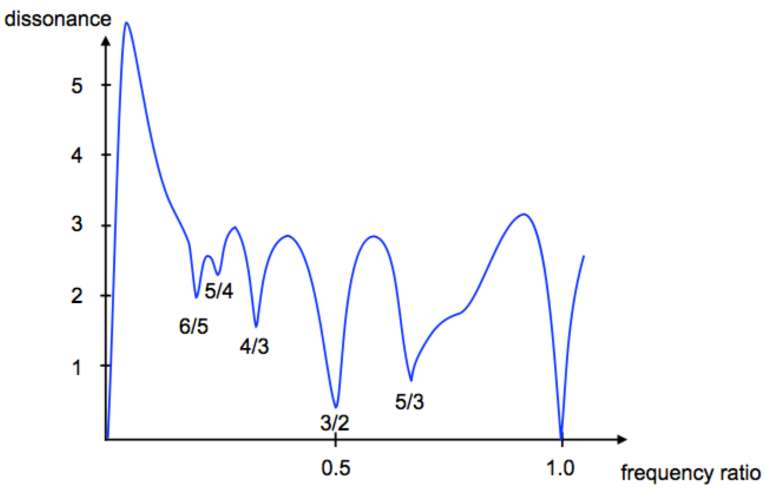
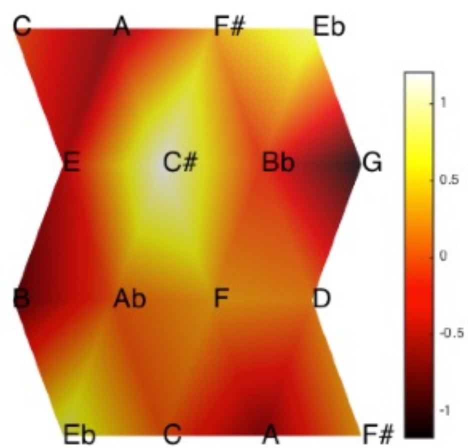


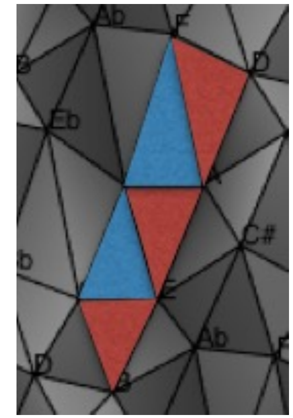
Locrian mode



Ionian mode

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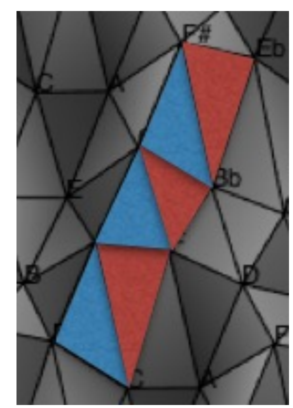

Locrian mode

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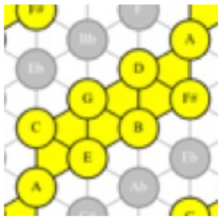
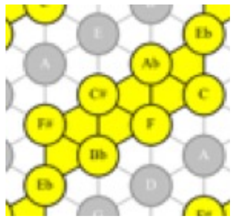


**Ionian mode**



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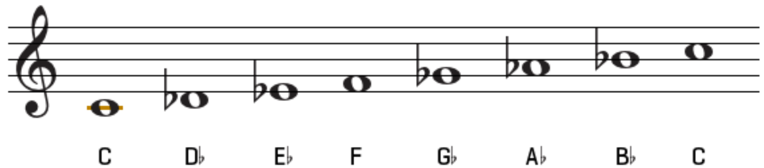


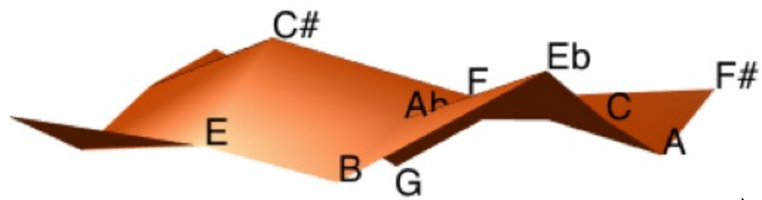


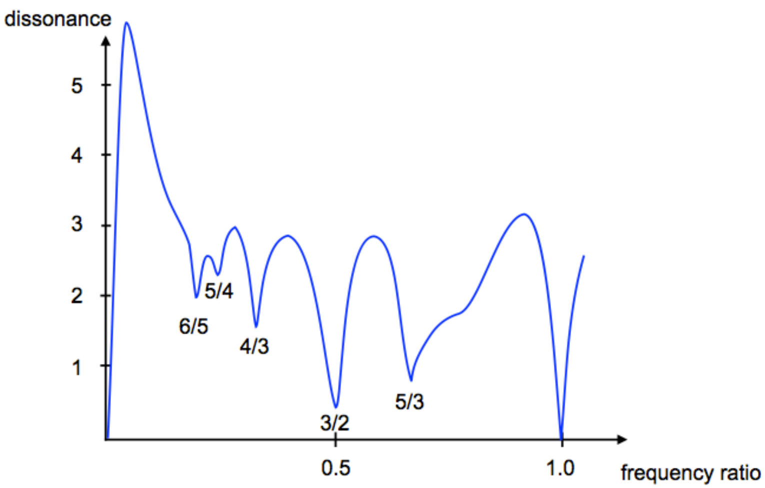


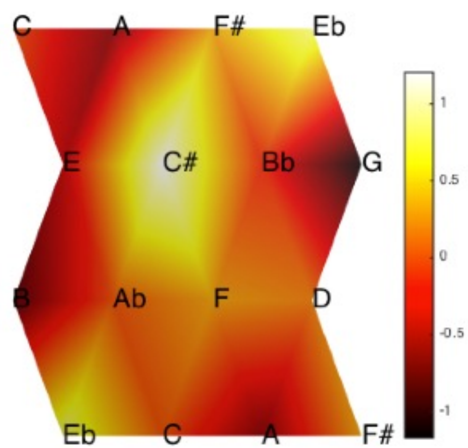


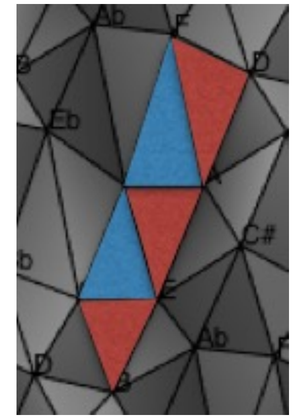

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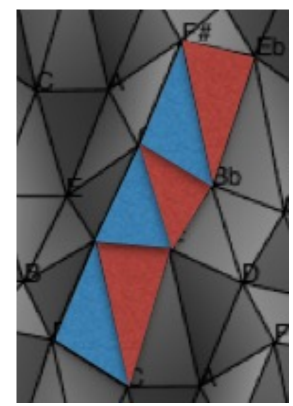


**Locrian mode**

- **Mattia Bergomi, *Dynamical and topological tools for (modern) music analysis,* Sorbonne/LIM Milan, 2015.**

# Persistent homology and music

Mattia Bergomi

## Homological persistence in time series: an application to music classification


Mattia G. Bergomi [a*] and Adriano Baratè [b]

[a] *Veos Digital, Milan, Italy;* [b] *Music and Computer Science Laboratory, University of Milan, Milan, Italy*



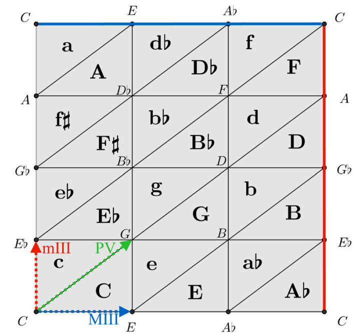
(*Received 22 May 2019; accepted 2 June 2020*)



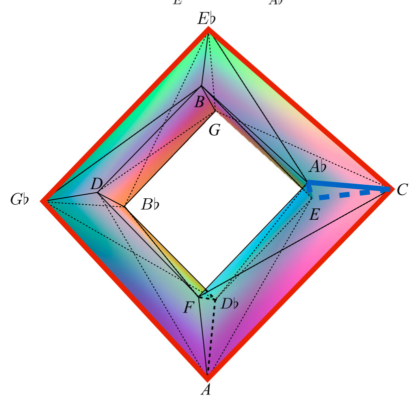
Meaningful low-dimensional representations of dynamical processes are essential to better understand the mechanisms underlying complex systems, from music composition to learning in both biological and artificial intelligence. We suggest to describe time-varying systems by considering the evolution of their geometrical and topological properties in time, by using a method based on persistent homology. In the static case, persistent homology allows one to provide a representation of a manifold paired with a continuous function as a collection of multisets of points and lines called persistence diagrams. The idea is to fingerprint the change of a variable-geometry space as a time series of persistence diagrams, and afterwards compare such time series by using dynamic time warping. As an application, we express some music features and their time dependency by updating the values of a function defined on a polyhedral surface, called the *Tonnetz*. Thereafter, we use this time-based representation to automatically classify three collections of compositions according to their style, and discuss the optimal time-granularity for the analysis of different musical genres.



**Keywords:** *Tonnetz*; topology; time-series analysis; persistent homology; dynamic time warping; classification; style





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- **Mattia Bergomi, "Homological persistence in time series: an application to music classification", Journal of Mathematics and Music, Vol. 14, Nr. 2, pp. 204-221, 2020 (Special Issue on Geometry and Topology in Music; Guest Editors: M. Andreatta, E. Amiot, and J. Yust).**

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**Mattia Bergomi**

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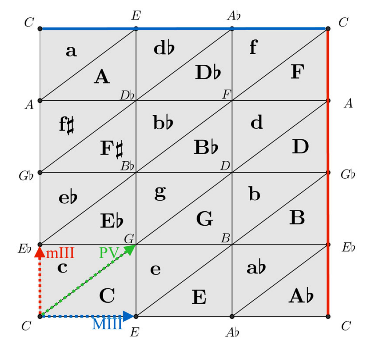


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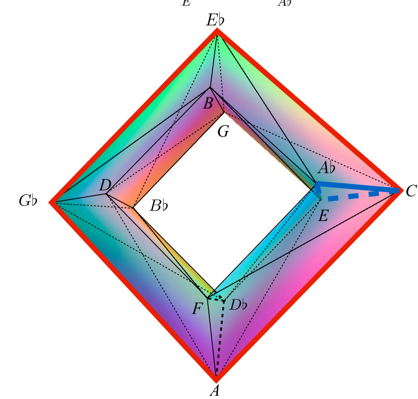


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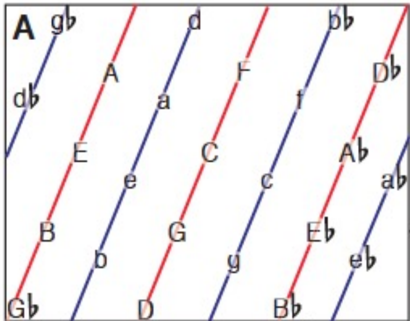
➔ **Victoria Callet**, *Modélisation topologique de structures et processus musicaux*, ongoing PhD, Université de Strasbourg

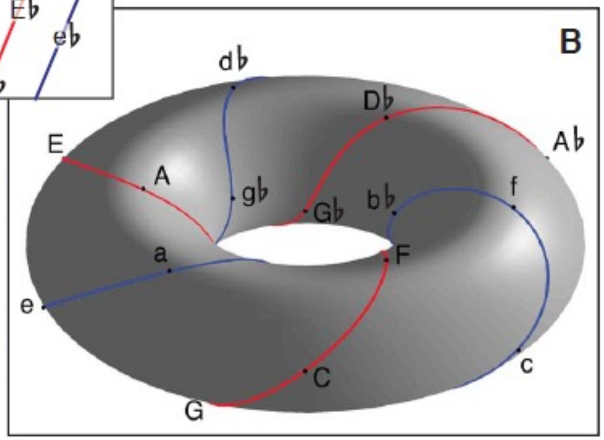
# Neurosciences and *Mathemusical* Learning



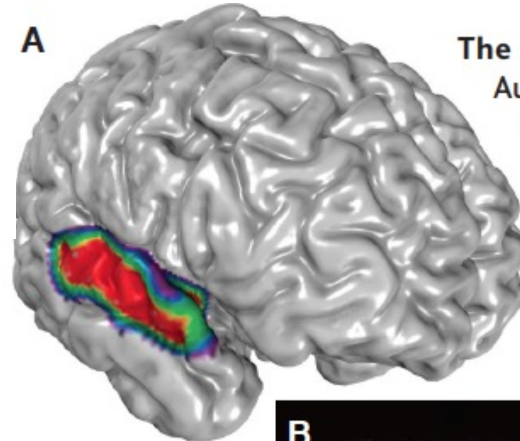

PERSPECTIVES: NEUROSCIENCE

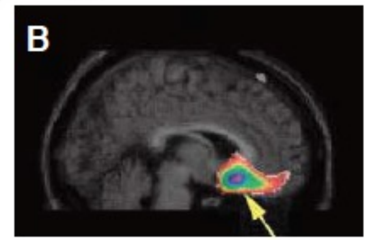
## Mental Models and Musical Minds

Robert J. Zatorre and Carol L. Krumhansl




**Mental key maps.** (**A**) Unfolded version of the key map, with opposite edges to be considered matched. There is one circle of fifths for major keys (red) and one for minor keys (blue), each wrapping the torus three times. In this way, every major key is flanked by its relative minor on one side (for example, C major and a minor) and its parallel minor on the other (for example, C major and c minor). (**B**) Musical keys as points on the surface of a torus.



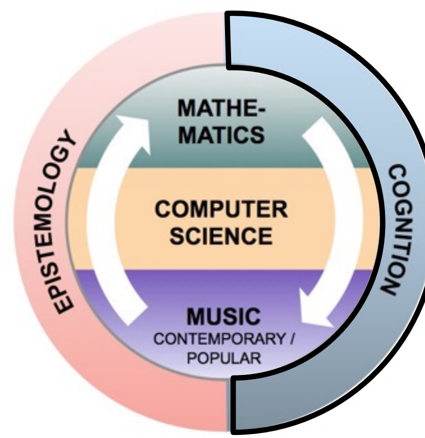

**The sensation of music.** (**A**) Auditory cortical areas in the superior temporal gyrus that respond to musical stimuli. Regions that are most strongly activated are shown in red. (**B**) Metabolic activity in the ventromedial region of the frontal lobe increases as a tonal stimulus becomes more consonant.


E. Bisesi

J. L. Besada

C. Guichaoua

M. Andreatta

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