instrument to another. Nylon strings typically require tensions of 50 to 80 N, whereas steel strings require tensions of 100 to 180 N. There appears to be some advantage in selecting string gauges in such a way that the tensions in all six strings will be nearly the same (Houtsma, 1975).

9.4 Modes of Vibration of Component Parts

A guitar top plate vibrates in many modes; those of low frequency bear considerable resemblance to the modes of a rectangular plate described in Chapter 3. The mode shape and frequencies change quite markedly when the braces are added, however, and, in addition, they are totally different if the plate is tested with its edge free, clamped, or simply supported (hinged). Relatively few studies of guitar top plate modes, especially with a free edge, have been reported.

Figure 9.6 shows the observed mode shapes and frequencies of a guitar plate without braces (with a free edge), and Fig. 9.7 shows the modes calculated for a plate with traditional fan bracing (also with a free edge), which are reported to be in good agreement with observed mode frequencies and shapes (Richardson and Roberts, 1985). Mode shapes and frequencies for the first five modes in a classical guitar sans back are shown in Fig. 9.8. These are also in reasonably good agreement with the modes calculated by Richardson and Roberts (1985) for a clamped edge, although the actual boundary condition probably is somewhere between clamped and hinged.

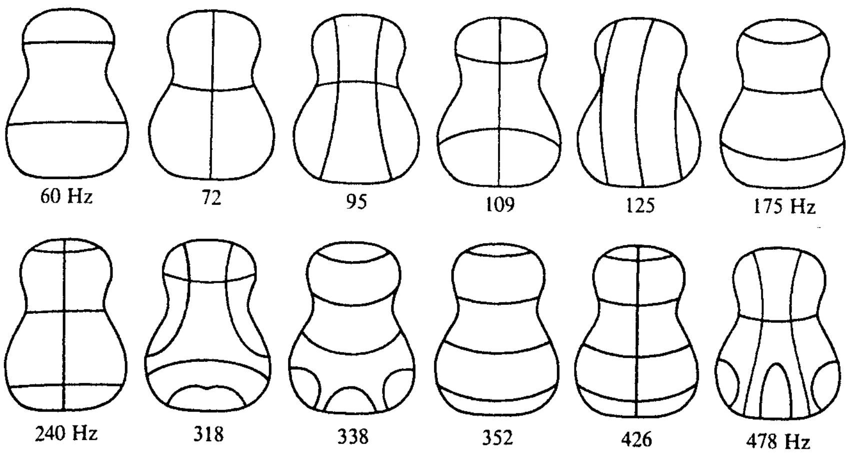

FIGURE 9.6. Vibration modes of a guitar plate blank (without braces) with a free edge (adapted from Rossing, 1982b).

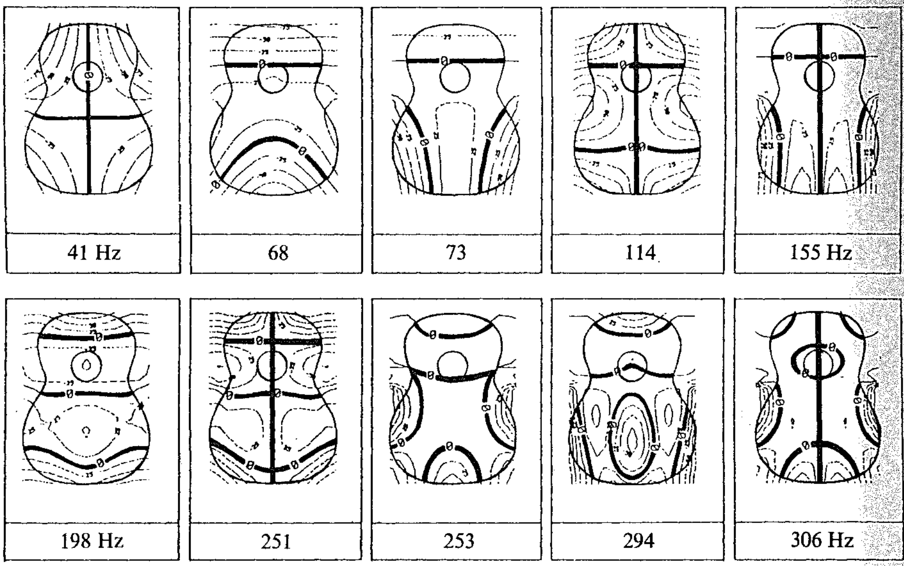

FIGURE 9.7. Vibration modes of a classical guitar top plate with traditional fan bracing (adapted from Richardson and Roberts, 1985).

Obviously, the observed modal shapes and frequencies of the top plate depend upon the exact boundary conditions and acoustic environment during testing. A very convenient and readily reproducible arrangement is to immobilize the back and ribs of the guitar (in sand, for example) and to close the soundhole; a number of guitars have been tested in this way by various investigators.

Figure 9.9(a) shows the modes of a folk guitar top measured with the back and ribs in sand and the soundhole closed by a lightweight sheet of balsa wood. The modes are quite similar to those of the classical guitar in Fig. 9.8, except that the (1, 0) mode now occurs at a higher frequency

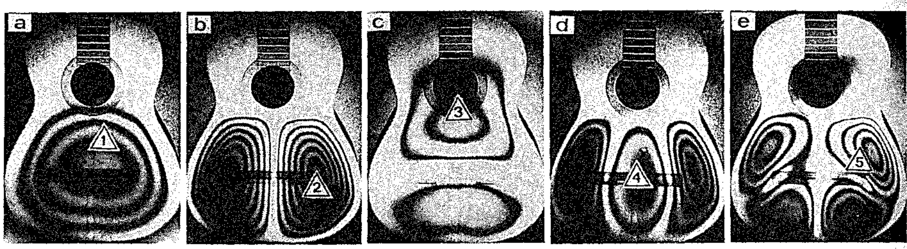

FIGURE 9.8. Vibration modes of a classical guitar top plate glued to fixed ribs but without the back (Jansson, 1971).

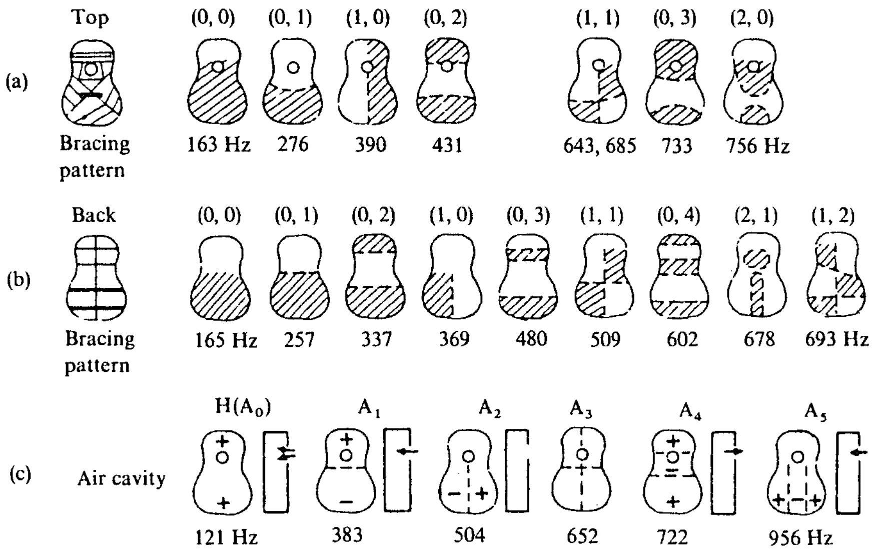

FIGURE 9.9. (a) Modes of a folk guitar top (Martin D-28) with the back and ribs in sand. (b) Modes of the back with the top and ribs in sand. (c) Modes of the air cavity with the guitar body in sand. Modal designations are given above the figures and modal frequencies below (Rossing et al., 1985).

than the (0, 1) mode, and the (2, 0) mode has moved up in frequency and changed its shape because of the crossed bracing.

Generally, the back plate of a guitar is rather simply braced with a center strip and three (most classical guitars) or four (folk guitars) cross braces, as shown in Fig. 9.9(b). Some vibrational modes of the back are shown in Fig. 9.9(b).

Also shown in Fig. 9.9 are the modes of the air cavity of a folk guitar. These were measured with the top, back, and ribs immobilized in sand but with the soundhole open. The lowest mode is the Helmholtz resonance, whose frequency is determined by the cavity volume and the soundhole diameter. There is also a small dependence upon the cavity shape and the soundhole placement, but these are usually not variables in guitar design. The term "Helmholtz resonance" is sometimes applied to the lowest resonance of the guitar (around 90–100 Hz), but this resonance involves considerable motion of the top and back plates and so is not a simple Helmholtz cavity resonance. Higher air modes resemble the standing waves in a rectangular box.

Frequencies of the principal modes of the top plate, back plate, and air cavity in two folk guitars and two classical guitars are given in Table 9.1. The main difference is in the relative frequencies of the (1, 0) and (0, 1) modes in the top plates. In fan-braced classical guitars, the (0, 1)

TABLE 9.1. Frequencies of the principal modes of the top plate, back plate, and air cavity in four guitars.*

Top plate	(0,0)	(0,1)	(1,0)	(0,2)	(1,1)	(0,3)	(2,0)	(1,2)
Folk								
Martin D-28	163	326	390	431	643	733	756	
Martin D-35	135	219	313	397	576	626	648	777
Classical								
Kohno 30	183	388	296	466	558		616	660
Conrad	163	261	228	382	474		497	
Back plate	(0,0)	(0,1)	(0,2)	(1,0)	(0,3)	(1,1)	(2,0)	(1,2)
Folk								
Martin D-28	165	257	337	369	480	509	678	693
Martin D-35	160	231	306	354	467	501	677	
Classical								
Kohno 30	204	285	368	417	537	566	646	856
Conrad	229	277	344	495	481	573	830	611
Air cavity	A_0 (Helmholtz)	A_1 (0,1)	A_2 (1,0)	A_3 (1,1)	A_4 (0,2)	A_5 (2,0)		
Folk								
Martin D-28	121	383	504	652	722	956		
Martin D-35	118	392	512	666	730	975		
Classical								
Kohno 30	118	396	560	674	780			
Conrad	127	391	558	711	772	1033		

*From Rossing et al. (1985).

mode occurs at a higher frequency than the $(1,0)$ mode, while in the cross-braced top plate of the folk guitars and in the back plates of both types, the reverse is generally true. In the Martin D-28, the fundamental modes of the top plate and back plate are tuned to almost the same frequency.

9.5 Coupling of the Top Plate to the Air Cavity: Two-Oscillator Model

If we ignore, for the moment, motion of the back plate and ribs, the guitar can be viewed as a two-mass vibrating system of the type discussed in Chapter 4, particularly Section 4.6. The two-mass model and its electrical equivalent circuit are shown in Fig. 9.10. The vibrating strings apply a force $F(t)$ to the top plate, whose mass and stiffness are represented

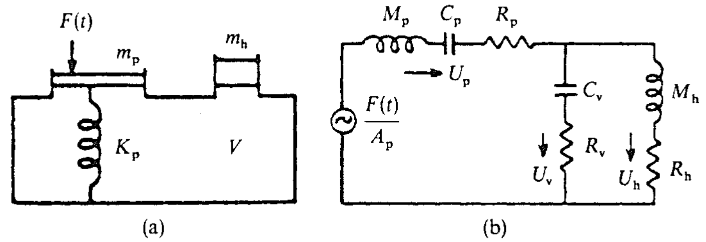

FIGURE 9.10. (a) Two-mass model representing the motion of a guitar with a rigid back plate and ribs. (b) Equivalent electrical circuit for the two-mass model. The equivalent currents are volume velocities (Rossing et al., 1985).

by m_p and K_p. A second piston of mass m_h represents the mass of air in the soundhole, and the volume V of enclosed air acts as the second spring.

The equivalent electrical circuit in Fig. 9.10(b), one of several possible choices for representing Fig. 9.10(a), is an acoustical impedance representation (Beranek, 1954). The equivalent voltage is the force applied to the top plate divided by the effective top plate area, and the equivalent currents are volume velocities (in m^3/s). The following symbols are used:

$M_\mathrm{p} = m_\mathrm{p}/A_\mathrm{p}^2$ is the inertance (mass/area^2) of the top plate (kg/m^4),
$M_\mathrm{h} = m_\mathrm{h}/A_\mathrm{h}^2$ is the inertance of air in the soundhole (kg/m^4),
$C_\mathrm{p} = A^2/K_\mathrm{p}$ is the compliance of the top plate (N/m^5),
$C_\mathrm{v} = V/\rho c^2$ is the compliance of the enclosed air (N/m^5),
U_p is the volume velocity of the top plate (m^3/s),
U_h is the volume velocity of air in the soundhole (m^3/s),
U_v is the volume velocity of air into the cavity (m^3/s),
R_p is the loss (mechanical and radiative) in the top plate,
R_h is the due to radiation from the soundhole, and
R_v is the loss in the enclosure.

The two-mass model predicts two resonances with an antiresonance between them. At the lower resonance, air flows out of the soundhole in phase with the inward moving top plate. In the equivalent circuit in Fig. 9.9(b), this corresponds to U_p and U_v being essentially in phase (they would be exactly in phase if $R_\mathrm{h} = R_\mathrm{v} = 0$). At the upper resonance, U_p and U_h are essentially opposite in phase; that is, air moves into the soundhole when the top plate moves inward. The antiresonance represents the Helmholtz resonance of the enclosure; U_v and U_h are equal and opposite, and thus U_p is a minimum. This behavior, which is dominant in a guitar at low frequency, is analogous to that of a loudspeaker in a bass reflex enclosure (Caldersmith, 1978).

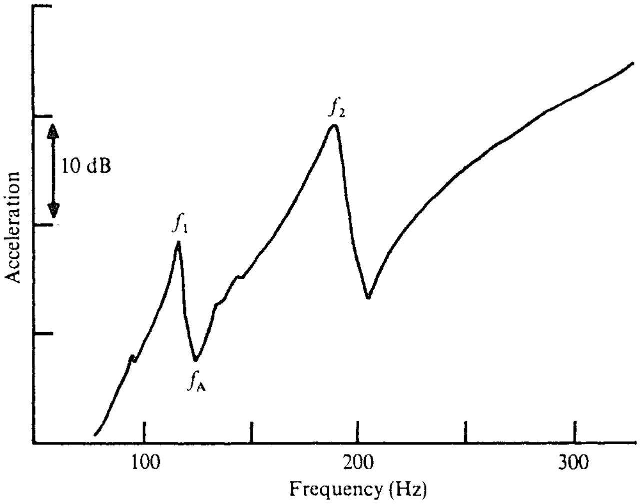

FIGURE 9.11. Low-frequency response curve for a Martin D-28 folk guitar with its back plate and ribs immobilized in sand. The bridge was driven on its treble side by a sinusoidal force of constant amplitude, and the acceleration was recorded at the driving point.

In Fig. 9.11, the response curve for a folk guitar, with its back and ribs immobilized, illustrates the two-mass model. The two resonances occur at f_1 and f_2 and the antiresonance at f_A. The two resonances f_1 and f_2 will span the lowest top plate mode f_p and the Helmholtz resonance f_A; that is, f_A and f_p will lie between f_1 and f_2. In fact, it can be shown that $f_1^2 + f_2^2 = f_A^2 + f_p^2$ (Ross and Rossing, 1979; Christensen and Vistisen, 1980). If $f_p > f_A$ (as it is in most guitars), f_A will lie closer to f_1 than to f_2 (Meyer, 1974).

9.6 Coupling to the Back Plate: Three-Oscillator Model

A three-oscillator model that includes the motion of the back is shown in Fig. 9.12 along with its electrical equivalent circuit. The ribs are immobilized, so that the coupling of the top plate to the back plate is via the enclosed air. Additional circuit elements are the mass M_b, compliance C_b, loss R_b, and volume velocity U_b of the back plate.

The response curve for the three-mass model has a third resonance and a second antiresonance, as shown in Fig. 9.13. In addition f_1 has been moved to a slightly lower frequency, and f_2 may be moved either upward (for $f_b < f_p$) or downward (for $f_b > f_p$), depending upon the resonance

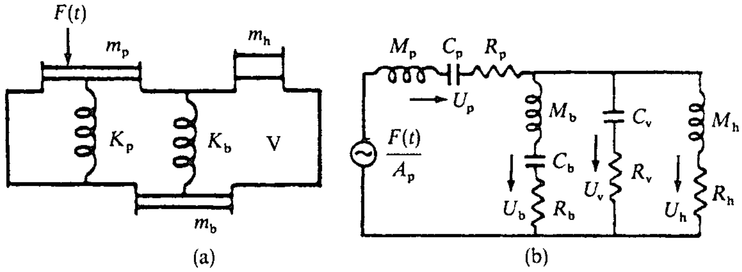

FIGURE 9.12. (a) Three-mass model of a guitar representing coupled motion of the top plate, back plate, and enclosed air. (b) Equivalent electrical circuit for three-mass model. Symbols are similar to those used in Fig. 9.10 (Rossing et al., 1985).

frequencies f_p and f_b of the top and back alone (Christensen, 1982). In most guitars, $f_\mathrm{b} > f_\mathrm{p}$, so both f_1 and f_2 are shifted downward by interaction with the flexible back (Meyer, 1974).

The three-mass model predicts that $f_1^2 + f_2^2 = f_\mathrm{A}^2 + f_\mathrm{B}^2$. This relationship has been verified by experimental measurements in several guitars with the ribs immobilized (Rossing, et al., 1985).

9.7 Resonances of a Guitar Body

The frequency response of a guitar is characterized by a series of resonances and antiresonances. In order to determine the vibrational configuration of the instrument at each of its major resonances, it is usually driven sinusoidally at one or more points, and its motion observed optically, acoustically, electrically, or mechanically. Optical sensing techniques include

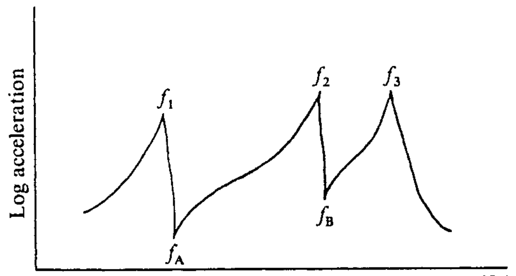

FIGURE 9.13. Frequency response curve predicted by the three-mass model. A third resonance and a second antiresonance have been added to the response curve of the two-mass model.

holographic interferometry (Stetson, 1981) and laser velocimetry (Boullosa, 1981). Acoustical detection techniques have included using an array of microphones (Strong et al., 1982) and scanning with a single microphone (Ross and Rossing, 1979). An electrical pickup relies on variation in capacitance as the instrument vibrates, and a mechanical pickup consists of an accelerometer or a velocity transducer of very small mass (such as a phonograph cartridge).

The response function depends upon the location of the driving point and sensing point and also on how the guitar is supported. A driving point on the bridge is usually selected, but for at least one prominent resonance there is a nodal line near the bridge, and thus it may be overlooked when the drive point or the sensing point lies on the bridge. It is difficult to overemphasize the importance of clearly describing the driving and sensing points and the method of support when reporting experimental results. Suspending the guitar by rubber bands has proved to be quite a satisfactory test configuration.

The configuration of a guitar at one of its resonances is often called a mode of vibration, but it is not necessarily a normal mode or eigenmode of the system. A resonance may result from exciting two or more normal modes. Only when the spacing of the normal modes is large compared with their natural widths does the vibration pattern at a resonance closely resemble that of a normal mode of vibration (Arnold and Weinreich, 1982).

When a guitar is driven at the bridge, the lowest resonance is usually a barlike bending mode. In the Martin D-28 folk guitar that we tested, it occurred at 55 Hz, and probably has little or no musical importance because it lies well below the lowest string frequency. Barlike bending modes at higher frequencies should not be overlooked, however.

Most guitars have three strong resonances in the 100–200 Hz range due to coupling between the (0,0) top and back modes and the A_0 (Helmholtz) air mode. In addition to the coupling via air motion discussed in Section 9.6, the top and back plates in a free guitar are coupled through the motion of the ribs. At the lowest of the three resonances, however, the top and back plates move in opposite directions (i.e., the guitar "breathes" in and out of the soundhole), and so the motion of the free guitar is very little different from the case in which the ribs are clamped. In the D-28, this breathing mode occurs at 102 Hz.

The other two resonances that result from (0,0)-type motion of the component parts usually occur a little above and below 200 Hz, depending upon the stiffness of the top and back plates. In the D-28, they occur at 193 and 204 Hz, as shown in Fig. 9.14. Note the motion of the air in the soundhole; in the upper two resonances, it moves in the same direction as the top plate, thus resulting in strong radiation of sound. At the middle resonance, the lower ribs and tail block move opposite to the main part of the top and back plates. Thus, clamping the ribs lowers this resonance considerably (from 193 to 169 Hz in the D-28; compare Fig. 9.13).

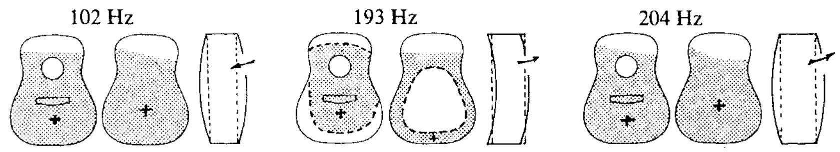

FIGURE 9.14. Vibrational motion of a freely supported Martin D-28 folk guitar at three strong resonances in the low-frequency range.

The $(1, 0)$ modes in the top plate, back plate, and air cavity generally combine to give at least one strong resonance around 300 Hz in a classical guitar but closer to 400 Hz in a cross-braced folk guitar. In the D-28, this coupling is quite strong at 377 Hz. Motion of the $(0, 1)$ type also leads to fairly strong resonances around 400 Hz in most guitars (Fig. 9.15).

Above 400 Hz, the coupling between top and back plate modes appears to be relatively weak, and so the observed resonances are due to resonances in one or the other of the two plates. A fairly prominent $(2, 0)$ top plate resonance is usually observed in classical guitars around 550 Hz, but this mode is much less prominent in folk guitars. Vibrational configurations of a classical guitar top plate at its principal resonance are shown in Fig. 9.16.

9.8 Response to String Forces

In Section 9.3, we considered the parallel and perpendicular forces a string exerts on the bridge when it is plucked. The parallel force at twice the fundamental string frequency was found to be small at ordinary playing amplitudes but increases quadratically, so it can become a factor in loud playing. This force exerts a torque whose magnitude depends upon the bridge height and which could be a factor if it occurred at a frequency near a resonance of the $(0, 1)$, $(0, 2)$, or similar mode having a node near the bridge.

There are, of course, an infinite number of planes in which the string can vibrate; we consider the directions parallel and perpendicular to the bridge.

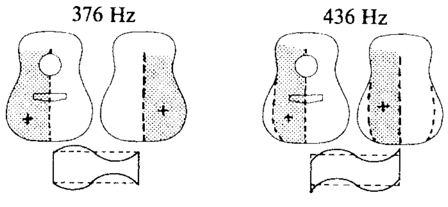

FIGURE 9.15. Vibrational configurations of a Martin D-28 guitar at two resonances resulting from "see-saw" motion of the (1,0) type.

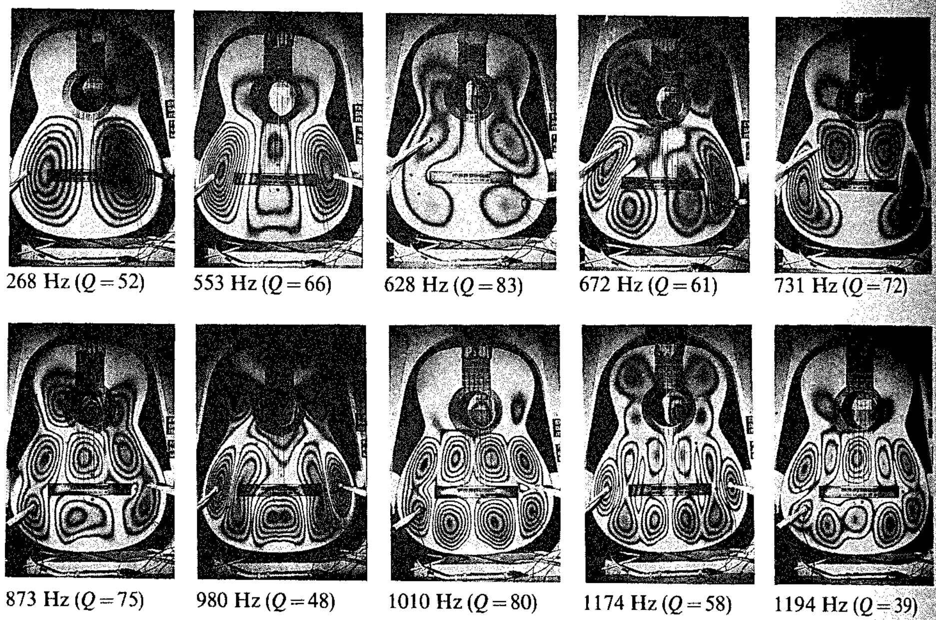

FIGURE 9.16. Time-averaged holographic interferograms of top-plate modes of a guitar (Guitar BR11). The resonant frequencies and Q values of each mode are shown below the interferograms (Richardson and Roberts, 1985).

The force parallel to the bridge encourages rocking motion, and thus it can easily excite resonances of the $(1,0)$ type. A perpendicular force anywhere on the bridge can excite the fundamental $(0,0)$ resonances, and if it is applied at the treble or bass sides, it can excite the $(1,0)$ resonances as well.

The effect of various string forces can be better understood by referring to the holographic interferograms in Fig. 9.17. These show the distortion of the top plate that results from static perpendicular and parallel bridge forces and also a torque caused by twisting one of the center strings.

Presumably, a player can greatly alter the tone of a guitar by adjusting the angle through which the string is plucked. Not only do forces parallel and perpendicular to the bridge excite different sets of resonances, but they result in tones that have different decay rates, as shown in Fig. 9.18. When the string is plucked perpendicular to the top plate, a strong but rapidly decaying tone is obtained. When the string is plucked parallel to the plate, on the other hand, a weaker but longer tone results. Thus, a guitar tone can be regarded as having a compound decay rate, as shown in Fig. 9.18(c). The spectra of the initial and final parts of the tone vary substantially, as do the decay rates.