Non-negative Matrix Factorization

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Partie I

Motivation and applications

Non-negative Matrix Factorization (NMF)

- Very popular tool for data analysis
- The original paper have got 7000+ citations
- One of the standard tools for data mining
- Has found place in many domains
  - Audio/Speech/Music processing
  - Computer vision
  - Recommender systems ...

Applications to audio

- Audio source separation
- Audio restoration
- Music transcription
- ...

Audio source separation :

Audio source separation :

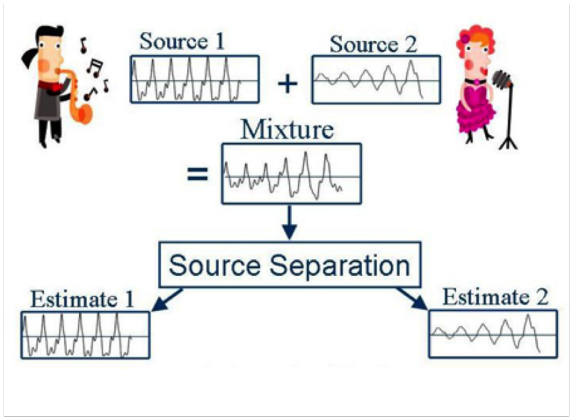


Image taken from http ://music.cs.northwestern.edu/

AUDIO SOURCE SEPARATION

Music remixing

emixing and content creation

Image taken from Bryan and Sun, 2013

AUDIO SOURCE SEPARATION

Spatial audio up-mixing

Spatial audio and upmixing

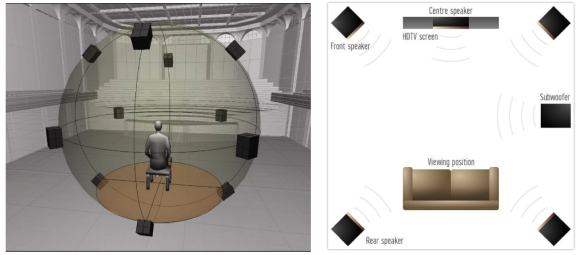


Image taken from Bryan and Sun, 2013

AUDIO SOURCE SEPARATION

Audio denoising

Audio restoration

Restore missing/corrupted parts of audio

Music transcription

Given the audio signal, find the musical score

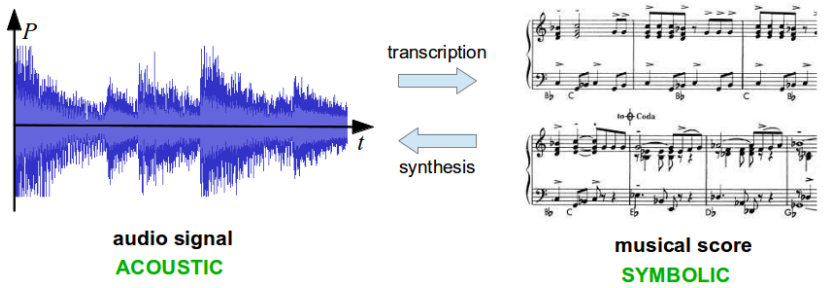


Image from http ://www-etud.iro.umontreal.ca/ boulanni/

Partie II

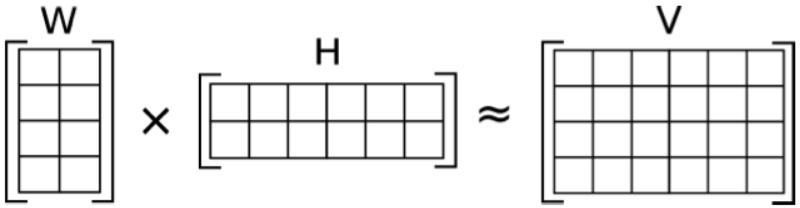
NMF

Non-Negative matrix factorization

- Aim : decompose a non-negative matrix V as :

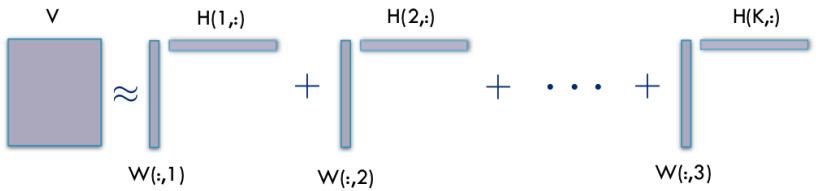
$$V \approx WH$$

- W and H are also non-negative



Non-Negative matrix factorization

Another point of view

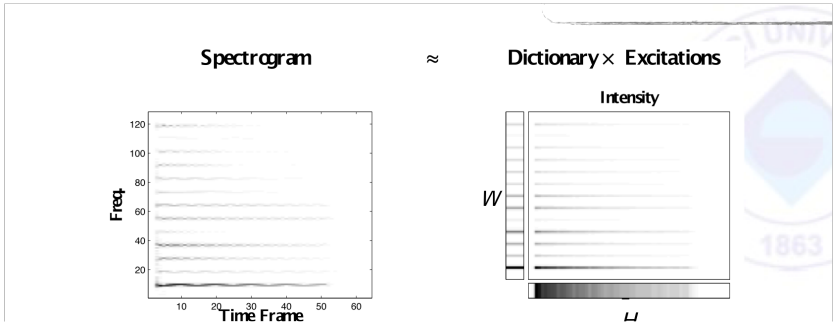


Non-Negative matrix factorization

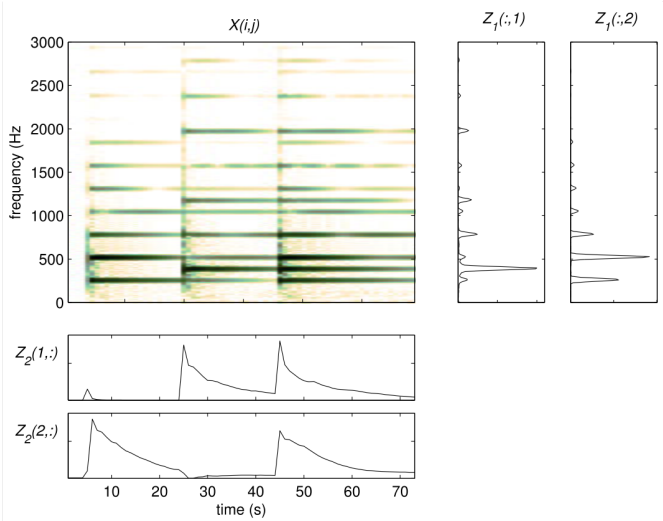
Matlab demo

NMF on audio

A simple audio example



NMF on audio



NMF on audio

Mary had a little lamb

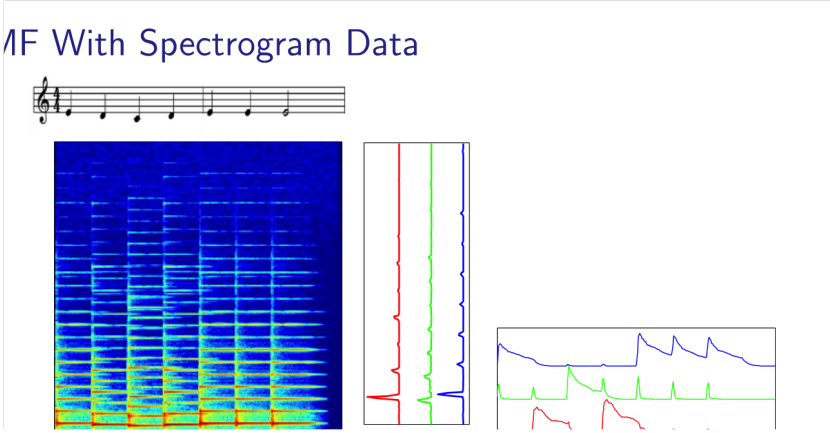


Image taken from Bryan and Sun, 2013

NMF on audio

Mary had a little lamb

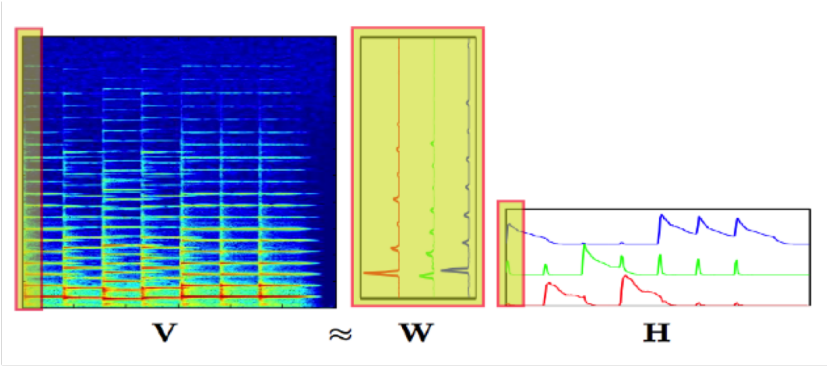
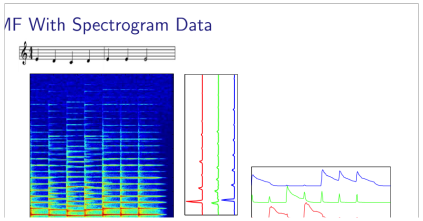


Image taken from Bryan and Sun, 2013

NMF on audio

Remember : 



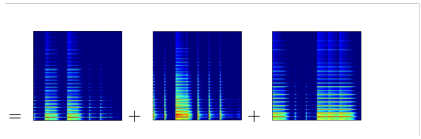
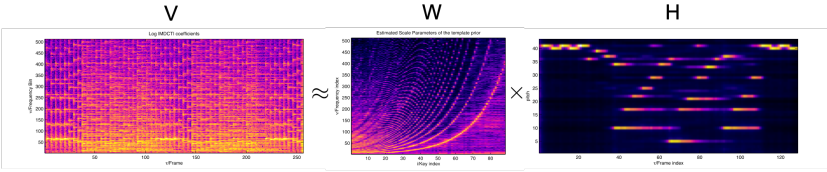


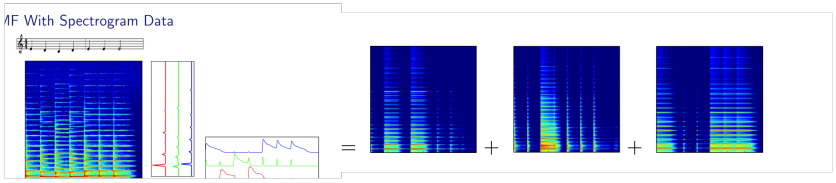
Image taken from Bryan and Sun, 2013

NMF on audio

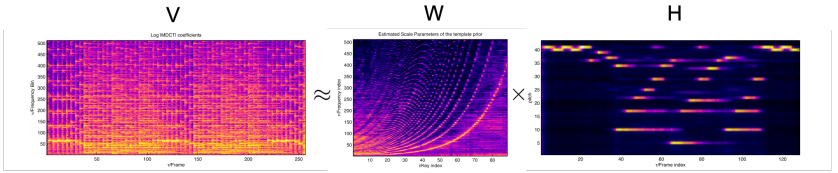
Polyphonic piano music



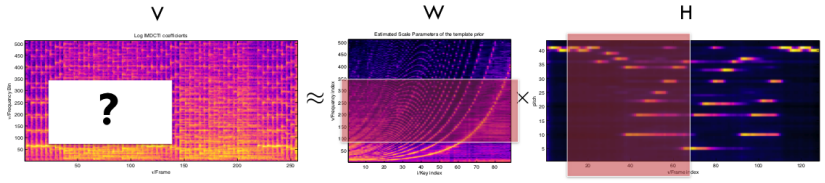
Source separation : by using W and H we can estimate the sources



Music transcription : if the columns of W correspond to different notes, H gives us an approximate transcription



Audio restoration : use W and H to restore corrupted parts



Partie III

How to estimate W and H?

Inference

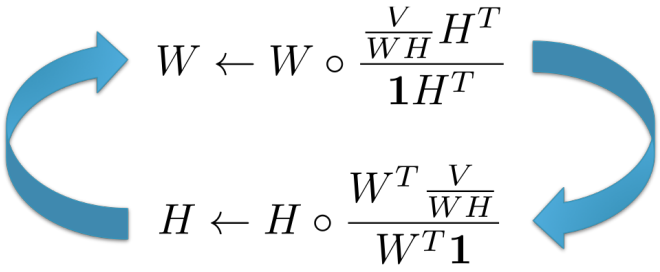
- We would like find W and H so that some "error" metric is minimized between V and WH

$$\min_{W,H} \quad d(V||WH)$$

- $d(V||WH)$ measured the error between V and WH
  - Euclidean $\rightarrow \|V - WH\|_2^2$
  - Kullback-Leibler $\rightarrow V \log(\frac{V}{WH}) - V + WH$ (we will use KL)
  - Itakura-Saito $\rightarrow$ also very popular in audio domain

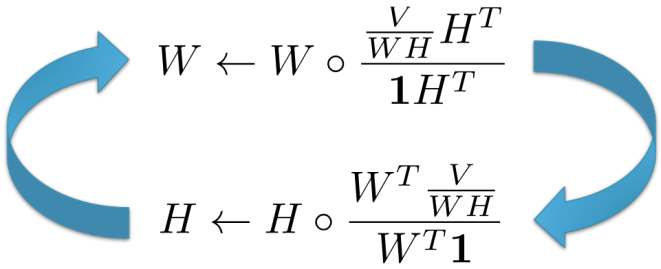
Multiplicative update rules

- Easy to code two lines in MATLAB
- Iteratively apply "multiplicative" updates to the factors (W and H)



Multiplicative update rules

Derivation on the board



Multiplicative update rules

Matlab demo

NMF on audio

- Recipe for audio applications
  1. Compute the complex spectrogram X
  2. Compute the magnitude spectrogram $V = |X|$
  3. Run the multiplicative update rules on V

Multiplicative update rules

Matlab demo

Partie IV

How to reconstruct the sources by using W and H?

Reconstructing the source

- For source separation applications, are we done after estimating W and H ?

NO!

- Wiener filtering, lossless reconstruction
- Reconstructing the complex spectrogram of source i :

$$X_i = X \otimes \frac{W(:,i)H(i,:)}{WH}$$

Multiplicative update rules

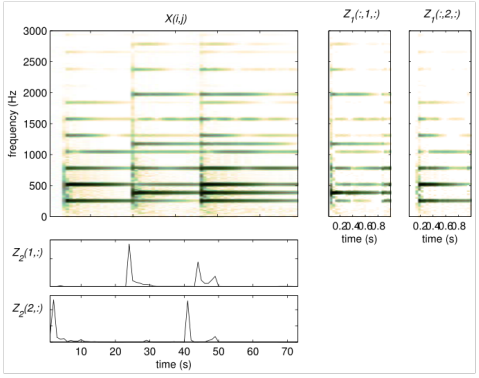
Matlab demo

Partie V

More on NMF

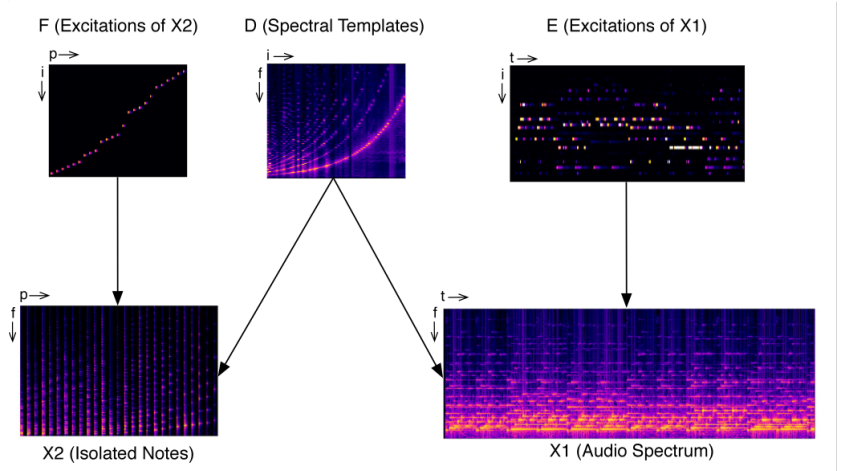
More on NMF

- Supervised vs Unsupervised
- Temporal information $\longrightarrow$ Convolutive models, non-negative dynamical systems



More on NMF

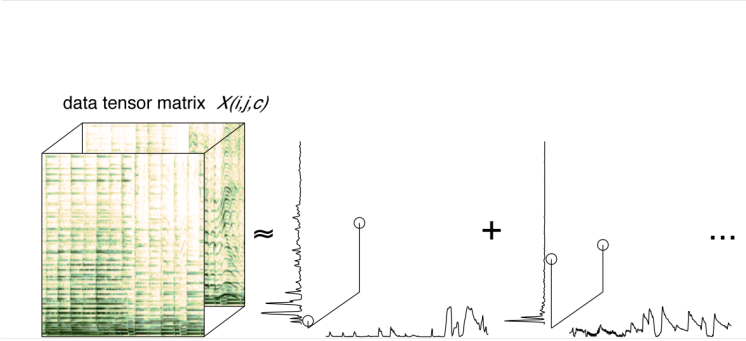
Side information $\longrightarrow$ Coupled factorization models



More on NMF

- Single channel vs Multi-channel
- Data is no more a matrix ; a tensor

- Tensor factorization models



Partie VI

Summary

Summary

- Applications of NMF (source separation, restoration, transcription, etc.)
- Model definition
- MUR for estimating the factors
- Wiener filtering for reconstructing the source signals