Travaux Dirigés d'Electromagnétisme

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Chapter 1

Tutorial 1 : Vector analysis - Gauss theorem

1.1 Vector analysis

Demonstrate the following results :

1. Divergence of the product between a scalar function and a vector field :

$$\mathrm{div}(f\ \vec{a}) = \mathrm{grad}\, f \cdot \vec{a} + f\ \mathrm{div}\vec{a}$$

2. Divergence of a curl product :

$$\mathrm{div}(\vec{a} \wedge \vec{b}) = \vec{b} \cdot \mathrm{rot}\vec{a} - \vec{a} \cdot \mathrm{rot}\vec{b}$$

3. Divergence of a curl :

$$\mathrm{div}(\mathrm{rot}\vec{a}) = 0$$

4. Curl of a gradient :

$$\mathrm{rot}(\mathrm{grad}f) = \vec{0}$$

5. Curl of the product between a scalar fucntion and a vector field :

$$\mathrm{rot}(f\vec{a}) = \mathrm{grad}f \wedge \vec{a} + f\ \mathrm{rot}\ \vec{a}$$

6. Curl of a Curl :

$$\mathrm{rot}\,\mathrm{rot}\,\vec{a} = \mathrm{grad}(\mathrm{div}\ \vec{a}) - \triangle\vec{a}$$

1.2 Gradient of a scalar function

Let us consider two points M and M' located in a cartesian coordinate system with its origin O such that $\mathbf{r} = \mathbf{OM}$ et $\mathbf{r}' = \mathbf{OM}'$. We set $R = \parallel \mathbf{R} \parallel = \parallel \mathbf{MM}' \parallel = \parallel \mathbf{r}' - \mathbf{r} \parallel$.

1. Establish the relationships existing between the gradient of R in M and the gradient of R in M'.

1.3 Evaluation of line integrals :

Let us consider a cartesian coordinate system $(0, \mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z)$, we consider a vector field which for every point M gives the following vector :

$$\mathbf{F}(x, y, z) = 5y\mathbf{a}_x + x^2\mathbf{a}_z \tag{1.1}$$

1. Calculate the line integral of $\mathbf{F}$ between $(0, 0, 0)$ and $(1, 2, 3)$ along the following path

$$\begin{cases} 2x = y \\ 9x = z^2 \end{cases} \tag{1.2}$$

1.4 Electrostatic field and Potential created by a distribution of charges

We consider a uniform volumic distribution of charges located in a sphere of radius a and centered at the origin of a coordinate system. The volumic charge density is denoted by ρ.

1. Considering an observation point M located inside or outside the sphere give the expression of the total quantity of charges Q included in the sphere centred in 0 and passing by M.
2. Apply the Gauss theorem in order to determine the electrostatic field $\mathbf{E}$.
3. Deduce the expression of the electrostatic potential V.
4. Plot the variations of the elctrostatic field and potential with respect to distance between the observation point and the center of the sphere.
5. Answer the same questions considering in that case, that the charges are located on an empty spherical shell of radius a and the surface charge density is denoted by σ.
6. Show that the normal component of the electrostatic field is discontinuous when the observation point is on the sphere.
7. Is it normal ?
8. Find again the value of the electrostatic potential by doing a direct integration of :

$$V(M) = \frac{1}{4\pi\varepsilon_0} \iint_{\mathcal{S}} \frac{\sigma(M_i)}{\|\mathbf{M_i M}\|} \, ds$$

Chapter 2

Tutorial 2 : Calculaton of electrostatic fields - Capacitance

2.1 Field created by two charges

We consider a cartesian coordinate system. Two point charges are respectively located at $(-2,0,0)$ and $(1,0,0)$, $q_1 = 12\pi\varepsilon_0$ while $q_2 = -4\pi\varepsilon_0$. Evaluate the electrostatic field and potential at the point $M(0,0,1)$.

2.2 Field created by a thin loop

We consider a circular horizontal loop of radius r, placed in the xOy plane and centered at the origin of the cartesian coordinate system. The loop carries a uniform charge density per unit length λ.

1. Evaluate the expression of the electrostatic field for a point located along the z axis.

2. Evaluate the expression of the electrostatic potential for the same point.

2.3 Field created by a disk

A disk of radius R is uniformly charged to Q and placed in the xOy plane with its centre at the origin.

1. Find the electric field along the z axis.

2.4 Electric dipole

We consider a sytem of two points charges $-q$ and $+q$ respectively located at $(-\frac{l}{2},0)$ and $(\frac{l}{2},0)$. This system is placed in the vacuum. We are considering all the phenomenas in the $x0y$ plane.

1. Find the electrostatic potential created by this system in a point $M(r,\phi)$

2. We assume that $r >> l$. Derive from the previous expression the approximate value of the electric potential.

3. In which power law of r this potential varies.
   We will call dipolar moment the vector $\mathbf{p} = lq\mathbf{e}_x$.

4. Deduce the electric field generated by this electric dipole. (Use the gradient in cylindrical coordinate system)

5. Sketch the electric field lines generated by this dipole.

2.5 Image Theory

We consider a pointcharge q located at $(0, 0, a)$ above an infinite ground plane located at $z = 0$.

1. Determine the differential equation and the boundary conditions satisfied by the electric potential.
2. We want to determine the expression of the potential in the half space $z \geq 0$. Find the equavilent distribution of charges which yields the same eqation and boundary conditions for V in the half space $z \geq 0$.
3. Deduce the value of the potential
4. Sketch the values of the electric field lines.

Chapter 3

Tutorial 3 : Induction field

3.1 Induction field created by a straight wire

We assume that a current with a given intensity I is circulating in a straight wire between the points x_1 and x_2 as depicted in the figure.

1. Evaluate the induction field created by this wire at point P.
2. Deduce the expression of the induction field in the case of an infinite wire.

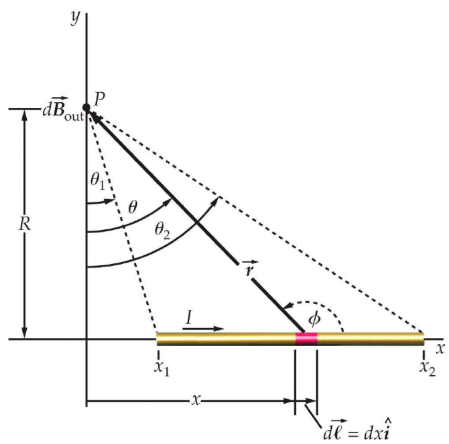

Figure 3.1: sketch of the proposed problem

3.2 Induction field created by a wire loop

A current with a given intensity I is circulating in a circular wire loop of radius a located in the (xOy) plane and centered at the origin of the coordinate system.

1. Calculate the induction field at a given point $M(0, 0, z)$.

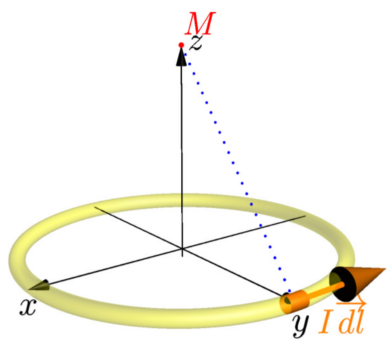

Figure 3.2: Configuration of the loop

3.3 Induction field generated by a solenoid

We consider the figure 3.3. A solenoid of finite lenght L is made of N circular wire loops with radius R.

1. Give the expression of the induction field for a given point M along the z axis.
2. Deduce the value $\mathbf{B}$ for an infinite solenoid.

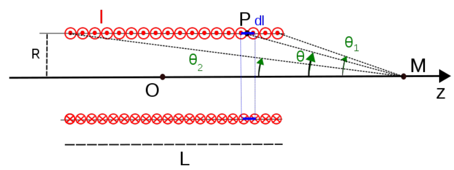

Figure 3.3: Solenoid of finite length

3.4 Toroidal Coil

A toroidal coil consists of a circular ring around which a long wire is wrapped. Each turn of wire can be considered as a plane closed loop. The cross section of the coil is rectangular.

1. Proove that the induction field of the toroidal coil is circumferential at all points, inside and outside the coil.

2. Give the value of the field for a point inside or outside the tore.

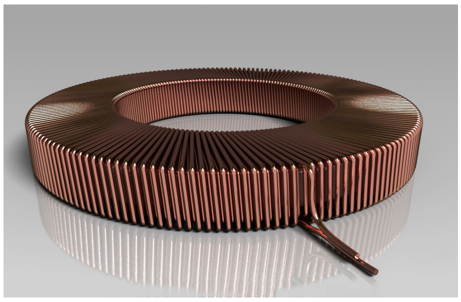

Figure 3.4: A toroidal coil

3.5 Magnetic dipole

A current with a given intensity I is circulating in a circular wire loop of radius a located in the (xOy) plane and centered at the origin of the coordinate system.

1. Give the general expression for the vector potential $\mathbf{A}$.

2. By taking into account the symetries of the problem give the unique non null component of the vector potential.

3. We want calculate the induction field $\mathbf{B}$ generated by this loop when the observation point is far from the loop *i.e* $r >> r'$. By doing a Taylor expansion of $\frac{1}{\|\mathbf{r}-\mathbf{r}'\|}$ at first order with respect to $\frac{r'}{r}$. Give the value of the vector potential in this framework.

4. We call $\mathbf{m} = I\pi a^2\mathbf{n}$ the magnetic dipole moment, deduce from $\mathbf{A}$ the value of $\mathbf{B}$.

5. Reveal the analogies with the electric field generated by an electric dipole

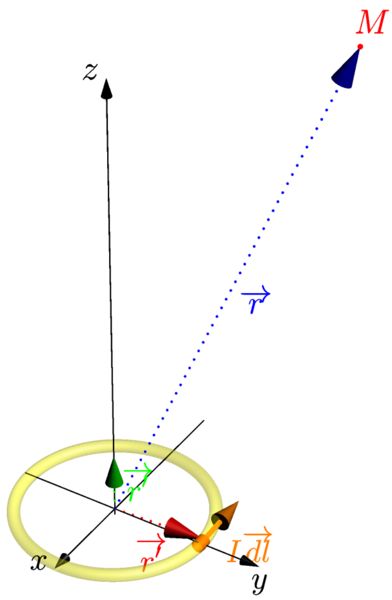

Figure 3.5: Configuration of the loop

Chapter 4

Tutorial 4 : Electromotive force

4.1 Faraday disc

A metal disk of radius a rotates with angular velocity ω through a uniform induction field $\mathbf{B}$. A circuit is made by connecting one end of a resistor to the axis and the other end to a sliding contact, which touches the outer edge of the disk.

1. Find the current in the resistor

4.2 Potential at the terminals of a square loop

We consider a square loop of conducting material rotating at the angular frequency ω in the (xOy) plane. This loop is placed in a uniform induction field $\mathbf{B} = B_0\mathbf{e}_x$

1. Find the value of the electromitve force at the terminals of the square loop

4.3 Self induction of a solenoid

We consider a solenoid of radius a and length L with N turns of conducting wires. The solenoid is assumed to be infinite and its axis is along $\mathbf{e}_z$.
This solenoid is placed in a time varying induction field :

$$\mathbf{B} = \mathbf{B}_0 \cos(\omega t)\mathbf{e}_z$$

The terminals of the solenoid are closed on a resistor with value R. The resistance of the conducting wires is neglected.

1. Calculate the induced current in the solenoid.
2. This current generates an self induction field named B_1. Evaluate its expression

In turn this field is going to produce an induced current . . . If we want to solve this problem in a rigorous way, we have to consider the total induction as the sum of two fields :

$$\mathbf{B}(t) = \mathbf{B}_0(t) + \mathbf{B}_2(t)$$

where $\mathbf{B}_2(t)$ describes all the induction process.

1. Establish the link between $B_2(t)$ and $i(t)$ the current flowing through the solenoid.
2. Establish the link between the electromitve force at the terminals of the solenoid and $B(t)$
3. Find the ordinary differential equation verified by $B_2(t)$.
4. Integrate this equation and find the solution of $B_2(t)$.
5. We set
$$\tau = \frac{LR}{\mu_0 N^2 \pi a^2}$$
6. Find the limit of $B_2(t)$ when $\tau\omega >> 1$

4.4 Airplane over the North Pole

An airplane flies over Antarctica at 1000kmh^{-1} , where the magnetic field of the earth mostly directed upward away from the ground.The geographical north is corresponding to the magnetic field south pole, the magnetic field is directed downward and its magnitude is $B_T = 50\mu T$. The windspan is $80m$.

1. Calculate the value of emf appearing between the end of both wings.

Chapter 5

Tutorial 5 : Heating by induction - Strucutre of wave propagation

5.1 Heating with eddy currents

Induction heating is similar to the Joule Heating Effect, but with one important modification. The currents that heat the material are induced by means of electromagnetic induction; it is a noncontact heating process.By applying a high-frequency alternating current to an induction coil, a time-varying magnetic field is generated. The material to be heated, known as the workpiece, is placed inside the magnetic field, without touching the coil. Note that not all materials can be heated by induction, only those with high electrical conductivity (such as copper, gold, and aluminum, to name a few). The alternating electromagnetic field induces eddy currents in the workpiece, resulting in resistive losses, which then heat the material up.We are going to study this phenomena.
We consider a cylindrical solenoid of radius a made of N turns of conducting wires and with a total length L. A current is flowing in the solenoid and it is of the following form :

$$I(t) = I_0 \cos(\omega t)$$

In the first part of this tutorial we will neglect all the self-induction phenomena.

1. Give the analytical expression of the induction field inside the solenoid assuming taht it is inifinite.

The space inside the solenoid is now filled with a conductive material of conductivity σ. The current density is following the ohmic law :

$$\mathbf{j} = \sigma \mathbf{E}$$

1. What kind of phenomena is appearing inside the conductor.
2. With the help of the symetries and invariances of the problem, determine the expression of the induced electric field inside the conductor.
3. We set that in a cylindrical coordinate system :

   $$\mathbf{E}(r,t) = E(r,t)\mathbf{e}_\theta$$

   and

   $$E(r,t) = E(r)\sin(\omega t)$$

   By applying the Maxwell-Faraday theorem (assuming that there is no induction phenomena) find the differential equation verified by $E(r)$
4. The following ODE :

   $$\frac{df}{dx} + \frac{1}{x} f(x) = A$$

   admits as a general solution $f(x) = \frac{K}{x} + \frac{1}{2}xA$ where K is a constant to be determined. What are the values of $\mathbf{E}(r,t)$ and $\mathbf{j}(r,t)$ along the axis of the system.
5. Find the solution of the previous ODE and deduce the expression of the current density.

6. The power density dissipated by Joule effect is equal to :

$$P = \mathbf{j} \cdot \mathbf{E}$$

Calculate its average for one period of the signal.

7. Evaluate the total power received by the material inside the coil.

8. What is the time required in order to melt an aluminium rod. We give the following parameters : $f = 50\text{Hz}$, $N = 10$, $L = 30\text{cm}$, $a = 3\text{cm}$, $I_0 = 3\text{kA}$. The conductivity of the aluminium is $\sigma = 3.7710^7\text{Sm}^{-1}$, its bulk density is 2700kgm^{-3} and its latent heat of fusion is 393kJkg^{-1}. At the begining of the process we are already at the melting point temperature.

We ara going to evaluate now te self-induction parameter in this experiment. If the frequecny is sufficiently low the Ampere law is still valid :

$$\text{rot}\mathbf{B} = \mu_0 \mathbf{j}$$

where $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$, since $\mathbf{B}_0$ is uniform :

$$\text{rot}\mathbf{B} = \text{rot}\mathbf{B}_1$$

and by virtue of symetries and invariances :

$$\mathbf{B}_1(r,t) = B_1(r) \sin \omega t \mathbf{e}_z$$

We assume that expression found for $\mathbf{j}$ is still valid.

1. Find the expression of $\mathbf{B}_1(r,t)$ assuming that $\mathbf{B}_1(r = a) = \mathbf{0}$
2. Find the condition on a such that the maximal value of $\mathbf{B}_1(r,t)$ is much lower than $\mathbf{B}_0(r,t)$.
3. Calulate its value for the given problem and conclude.

Curl of F in a cylindrical coordinate system

$$\text{rot}\mathbf{F} = \left(\frac{1}{\rho}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right)\mathbf{e}_\rho + \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho}\right)\mathbf{e}_\phi + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho F_\phi) - \frac{\partial F_\rho}{\partial \phi}\right)\mathbf{e}_z$$

5.2 General solutions of wave propagation

1. Show that the general soltuion of :

$$\frac{\partial^2 f}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$$

is

$$f(x,t) = F(x - ct) + G(x + ct)$$

2. Find in spherical coordinate system the solution of :

$$\Delta f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$$

assuming that f depends only of r

Scalar Laplacian in a spherical coordinate system

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

Chapter 6

Tutorial 6 : Interferences between plane waves - Polarisation

A time dependance in $\exp^{-i\omega t}$ is assumed in this tutorial.

6.1 Interferences between two plane waves

We consider two plane waves propagating in vaccum with two wave vectos $\mathbf{k}_1$ and $\mathbf{k}_2$. These wave vectors are lying in the yOz plane and are at an angle θ with respect to the z axis. The phase difference between both fields is equal to 2φ. Both fields have a constant amplitude E_1 and E_2.

$$\begin{aligned}
\mathcal{E}_1(\vec{r}) &= \mathbf{E}_1 \exp\left[i(\mathbf{k}_1 \cdot \mathbf{r})\right], \\
\mathcal{E}_2(\vec{r}) &= \mathbf{E}_2 \exp\left[i(\mathbf{k}_2 \cdot \mathbf{r} + 2\varphi)\right].
\end{aligned}$$

6.1.1 Case of parallel electric field

We assume that both electric fiels are along the x axis.

1. Write the expressions of the complex vectors $\mathcal{E}_1$ and $\mathcal{E}_2$.
2. Deduce from Maxwell equations the value of the associated $\mathcal{B}_1$ and $\mathcal{B}_2$.
3. Deduce the expression of the electric and magnetic fields resulting from the superpostion of bith fields.
4. Calulate the value of complex Poynting vector of this field in terms of I_1 and I_2 where I_1 and I_2 are respectively equal to :

$$\begin{aligned}
I_1 &= \frac{1}{2\eta} E_1^2 \\
I_2 &= \frac{1}{2\eta} E_2^2
\end{aligned}$$

5. Show that the intensity equal to $\|\mathcal{P}|$ can be written as :

$$I = \left[(I_1 - I_2)^2 \sin^2\theta + \left(I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(2\alpha) \right)^2 \cos^2\theta \right]^{\frac{1}{2}}$$

where $\alpha = ky\sin\theta - \varphi$

6. Rewrite this expression in the case where $\theta = 0$ and $E_1 = E_2$.
7. Sketch the plot of the intensity pattern in the xOy plane
8. Give the value of the period of the pattern

6.2 Polarisation state of a plane wave

We consider the following complex vector :

$$\mathcal{E} = [E_1(0.71 - j0.71)\vec{a}_y + jE_2(0.5\vec{a}_x - 0.866\vec{a}_z)]\exp^{\left(-j\left[(\frac{40\pi}{\sqrt{3}})x + (\frac{40\pi}{3})z\right]\right)}$$

1. What is the wavevector for this wave
2. Determine its frequency
3. Show that this elctric field can be seen as the sum of two fields with two separate polarisation states.
4. Give the value of the corresponding induction field.

Chapter 7

Tutorial 7 :Radiation

We assume that we are in the harmonic regime and with a time depende in $-i\omega t$. The expressions of the scalar and vector potential in vaccum are related to the distribtuon of current sources in Ω by the following relations :

$$\begin{aligned}\mathcal{A}(\mathbf{r}(&= \frac{\mu}{4\pi}\int_\Omega G(R)\mathbf{j}(\mathbf{r}')d\mathbf{r}' \\ \mathcal{V}(\mathbf{r}) &= \frac{1}{i\omega\epsilon_0\mu_0}\mathrm{div}\mathcal{A}(\mathbf{r})\end{aligned}$$

avec $G(\mathbf{r},\mathbf{r}') = \frac{1}{R}e^{ikR}$ G is called the Green function of the vaccum. η is the impedance of vaccum and is equal to : $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$

We set :

$$\begin{aligned}\mathbf{u}(\mathbf{r},\mathbf{r}') &= \frac{\mathbf{r}-\mathbf{r}'}{R} \\ \mathrm{grad}\left[\mathbf{j}(\mathbf{r}')\cdot\mathbf{u}(\mathbf{r},\mathbf{r}')\right] &= -\frac{1}{R}\left[\mathbf{j}(\mathbf{r}')\cdot\mathbf{u}(\mathbf{r},\mathbf{r}')\right]\mathbf{u}(\mathbf{r},\mathbf{r}') + \frac{1}{R}\mathbf{j}(\mathbf{r}') \\ \mathbf{j}_u(\mathbf{r},\mathbf{r}') &= \left[\mathbf{j}(\mathbf{r}')\cdot\mathbf{u}(\mathbf{r},\mathbf{r}')\right]\mathbf{u}(\mathbf{r},\mathbf{r}')\end{aligned}$$

Starting from the expressions linking $\mathcal{E}$ et $\mathcal{H}$ to the potential previously defined :

$$\begin{aligned}\mathcal{H}(\mathbf{r}) &= \frac{1}{\mu}\mathrm{rot}\mathcal{A}(\mathbf{r}) \\ \mathcal{E}(\mathbf{r}) &= -\mathrm{grad}\mathcal{V}(\mathbf{r}) + j\omega\mathcal{A}(\mathbf{r})\end{aligned}$$

We show that :

$$\begin{aligned}\mathcal{H}(\mathbf{r}) &= \frac{jk}{4\pi}\int_\Omega\left(1+\frac{j}{kR}\right)G(R)\mathbf{u}(\mathbf{r},\mathbf{r}')\wedge\mathbf{j}(\mathbf{r}')d\mathbf{r}' \\ \mathcal{E}(\mathbf{r}) &= \frac{jk\eta}{4\pi}\int_\Omega\left(\frac{3}{k^2R^2}-\frac{3j}{kR}-1\right)G(R)\,\mathbf{j}_u(\mathbf{r},\mathbf{r}')d\mathbf{r}' \\ &\quad -\frac{jk\eta}{4\pi}\int_\Omega\left(-1-\frac{j}{kR}+\frac{1}{k^2R^2}\right)G(R)\mathbf{j}(\mathbf{r}')d\mathbf{r}'\end{aligned}$$

1. Simplify $\mathcal{E}$ et $\mathcal{H}$ when $R >> \lambda$. This condition will be assumed hereafter.

$$\begin{aligned}\mathcal{H}(\mathbf{r}) &= \frac{jk}{4\pi}\int_\Omega G(R)\,\mathbf{u}(\mathbf{r},\vec{r'})\wedge\mathbf{j}(\mathbf{r}')d\mathbf{r}' \\ \mathcal{E}(\mathbf{r}) &= \frac{jk\eta}{4\pi}\int_\Omega G(R)\,\left(\mathbf{j}(\mathbf{r}')-\mathbf{j}_u(\mathbf{r},\mathbf{r}')\right)d\mathbf{r}'\end{aligned}$$

Assume also that $\mathbf{u}$ is independant of $\mathbf{r}'$ (the observation point $\mathbf{r}$ is located far compared to the dimension of the antenna.

2. Show that in this framework the radiated field has a local structure of plane wave, we can write :

$$\begin{aligned}\mathcal{H}(\mathbf{r}) &= \frac{jk}{4\pi}\frac{e^{jkr}}{r}\mathbf{u}_0(\vec{r}) \wedge \left(\int_\Omega e^{-jk\mathbf{u}_0(\mathbf{r})\cdot\mathbf{r}'}\mathbf{j}(\mathbf{r}')d\mathbf{r}'\right) \\ \mathcal{E}(\mathbf{r}) &= -\eta\mathbf{u}_0(\vec{r}) \wedge \mathcal{H}(\mathbf{r})\end{aligned}$$

Write $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$ when the currents are localized along a thin wire.

7.1 Radiation by simple structure

7.1.1 Hertzian dipole

1. We consider that $\mathbf{I} = Il\mathbf{e}_z\delta(\vec{r'})$, find the corresponding value of $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$.
2. Calculate the averaged power radiated by the dipole. Plot it with repect to θ

7.1.2 Dipole of finite length

We assume now that :

$$\mathbf{I} = I_0\cos(kz)\mathbf{e}_z \text{ for } -d \leq z \leq d$$

1. Evaluate $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$ for $d = \frac{\lambda}{4}$
2. Calculate also its average power and plot it with repect to θ.

7.1.3 Magnetic dipole

Assume that the current distribution is a circular loop of radius a lying in the (xOy) plane centered at the origin.

1. Evaluate $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$.

Chapter 8

Tutorial 8 : Reflection-Refraction

8.1 Reflection - Refraction at normal incidence

The half-space $z < 0$ is the vaccum while the region $z > 0$ is filled with a dielectric material $\varepsilon_r = 25$ et $\mu_r = 1$. A uniform plane wave is incoming from vaccum to the interface along the z axis. The electric field is directed along Ox and its modulus is equal to $\mathbf{E}_0$. The signal frequency is equal to 1GHz.

1. Provide the expression of the incident wavevector
2. Provide the expression of the complex electric field
3. Give the values of the reflection and transmission coefficients
4. Provide the analytic forms of both transmitted and reflected fields.

8.2 Reflection - Refraction - TM polarisation case

An incident plane wave is incoming from vaccum $x < 0$ to a plane interface $x = 0$. The half space $x > 0$ is filled with a non magnetic material and with a permittivity equal to $\varepsilon_r = 9.0$. The value of the incident magnetic field is equal to :

$$\mathcal{H}_i = H_0 \vec{a}_y \exp^{j\sqrt{2}\pi\left(\frac{\sqrt{3}}{2}z+\frac{1}{2}x\right)}$$

1. Determine the frequency of the signal and the components of its wavevector.
2. Provide the expression of the incident electric field.
3. Provide the expressions of the reflection and transmission coefficients. Give their numerical values.
4. Give, if it exists for this type of polarisation the value of the Brewster angle.

8.3 Reflection - Refraction - Mixed polarisation case

The region $z < 0$ is the vaccum while the half space $z > 0$ is filled with a material characterized by $\varepsilon_r = 25$ and $\mu_r = 1$. A plane wave is incoming from vaccum to the interface with an angle of incidence of 60°. The frequency of the signal is 6 GHz. The wavevector and the normal to the interface are defining the (xOz) plane.
The complex amplitude of the electric field is equal to :

$$\vec{E}_i = [E_1 0.71 \vec{a}_y + E_2(0.5\vec{a}_x - 0.866\vec{a}_z)]$$

Provide the analytical expression of the incident electric field $\mathcal{E}_i$
Provide the analytical expression for the reflected and the transmitted field.