Ice Cream Maker's Problem

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Single Machine Model

Definitions

- $\mathcal{N}$: Set of N ice cream flavors
- $\mathbf{A}$: Matrix of allowed production orders where A_{ij} is 1 if flavor $j \in \mathcal{N}$ can be made immediately after flavor $i \in \mathcal{N}$ without cleaning, and 0 otherwise
- T: Number of time steps or allowable batches
- q_i: Required number of batches of flavor $i \in \mathcal{N}$

Decision Variables

- x_{it}: Equals 1 if flavor $i \in \mathcal{N}$ is made in period t, 0 otherwise
- y_t: Equals 1 if the machine is cleaned before period t, 0 otherwise

Formulation

$$
\begin{aligned}
\underset{x,y}{\text{minimize}} \quad & \sum_{t=1}^{T} y_t & & \\
\text{subject to} \quad & \sum_{i=1}^{N} x_{it} \leq 1 & & \forall t = 1, ..., T \\
& \sum_{t=1}^{T} x_{it} \geq q_i & & \forall i \in \mathcal{N} \\
& \sum_{j=1}^{N} A_{ij} x_{jt} + y_t \geq x_{i,t-1} & & \forall i \in \mathcal{N}, t = 2, ..., T \\
& x_{it} \in \{0, 1\} & & \forall i \in \mathcal{N}, t = 1, ..., T \\
& y_t \in \{0, 1\} & & \forall t = 1, ..., T.
\end{aligned}
$$

Multi-Machine Model

Definitions

- $\mathcal{N}$: Set of N ice cream flavors
- $\mathcal{M}$: Set of M available ice cream machines
- $\mathbf{A}$: Matrix of allowed production orders where A_{ij} is 1 if flavor $j \in \mathcal{N}$ can be made immediately after flavor $i \in \mathcal{N}$ without cleaning, and 0 otherwise
- T: Number of time steps or allowable batches
- q_i: Required number of batches of flavor $i \in \mathcal{N}$

Decision Variables

- x_{imt}: Equals 1 if flavor $i \in \mathcal{N}$ is made in machine $m \in \mathcal{M}$ in period t, 0 otherwise

- y_{mt}: Equals 1 if machine m is cleaned before period t, 0 otherwise

Formulation

$$
\begin{aligned}
\underset{x,y}{\text{minimize}} \quad & \sum_{t=1}^{T}\sum_{m=1}^{M} y_{mt} \\
\text{subject to} \quad & \sum_{i=1}^{N} x_{imt} \leq 1 && \forall m \in \mathcal{M}, t = 1, ..., T \\
& \sum_{t=1}^{T}\sum_{m=1}^{M} x_{imt} \geq q_i && \forall i \in \mathcal{N} \\
& \sum_{j=1}^{N} A_{ij} x_{jmt} + y_{mt} \geq x_{im,t-1} && \forall i \in \mathcal{N}, \forall m \in \mathcal{M}, t = 2, ..., T \\
& x_{imt} \in \{0, 1\} && \forall i \in \mathcal{N}, m \in \mathcal{M}, t = 1, ..., T \\
& y_{mt} \in \{0, 1\} && \forall m \in \mathcal{M}, t = 1, ..., T.
\end{aligned}
$$