1次近似（1）

$$f(v_1, v_2, I_S, K) = \frac{I_S}{2}\left(1 + \sqrt{\frac{2K}{I_S}}(v_1 - v_2)\sqrt{1 - \frac{K}{2I_S}(v_1 - v_2)^2}\right)$$

$$\cong \frac{I_S}{2}\left(1 + \sqrt{\frac{2K}{I_S}}(v_1 - v_2)\right)$$

$$I_{dB} = f(v_{in}, -v_{in}, I_C, K_B) \cong \frac{I_C}{2}\left(1 + 2\sqrt{\frac{2K_B}{I_C}}v_{in}\right)$$

$$I_{dB} = f(-v_{in}, v_{in}, I_C, K_B) \cong \frac{I_C}{2}\left(1 - 2\sqrt{\frac{2K_B}{I_C}}v_{in}\right)$$

$$I_{d1} = f(v_{CTRL}, -v_{CTRL}, I_{dB1}, K_A)$$

$$\cong \frac{I_{dB1}}{2}\left(1 + 2\sqrt{\frac{2K_A}{I_{dB1}}}v_{CTRL}\right)$$

$$\cong \frac{I_C}{4}\left(1 + 2\sqrt{\frac{2K_B}{I_C}}v_{in}\right)\left(1 + 2\sqrt{\frac{2K_A}{\frac{I_C}{2}}}\left(1 - \sqrt{\frac{2K_B}{I_C}}v_{in}\right)v_{CTRL}\right)$$

$$= \frac{I_C}{4}\left(1 + 2\sqrt{\frac{2K_B}{I_C}}v_{in}\right)\left(1 + 4\sqrt{\frac{K_A}{I_C}}v_{CTRL} - 4\sqrt{2}\frac{\sqrt{K_A K_B}}{I_C}v_{in}v_{CTRL}\right)$$

$$\cong \frac{I_C}{4}\left(1 + 2\sqrt{\frac{2K_B}{I_C}}v_{in} + 4\sqrt{\frac{K_A}{I_C}}v_{CTRL} + 4\sqrt{2}\frac{\sqrt{K_A K_B}}{I_C}v_{in}v_{CTRL}\right)$$

1次近似（2）

$$I_{\mathrm{d1}} = f(v_{\mathrm{CTRL}}, -v_{\mathrm{CTRL}}, I_{\mathrm{dB1}}, K_{\mathrm{A}})$$
$$\cong \frac{I_{\mathrm{C}}}{4}\left(1 + 2\sqrt{\frac{2K_{\mathrm{B}}}{I_{\mathrm{C}}}}v_{\mathrm{in}} + 4\sqrt{\frac{K_{\mathrm{A}}}{I_{\mathrm{C}}}}v_{\mathrm{CTRL}} + 4\sqrt{2}\frac{\sqrt{K_{\mathrm{A}}K_{\mathrm{B}}}}{I_{\mathrm{C}}}v_{\mathrm{in}}v_{\mathrm{CTRL}}\right)$$

$$I_{\mathrm{d2}} = f(-v_{\mathrm{CTRL}}, v_{\mathrm{CTRL}}, I_{\mathrm{dB1}}, K_{\mathrm{A}})$$
$$\cong \frac{I_{\mathrm{C}}}{4}\left(1 + 2\sqrt{\frac{2K_{\mathrm{B}}}{I_{\mathrm{C}}}}v_{\mathrm{in}} - 4\sqrt{\frac{K_{\mathrm{A}}}{I_{\mathrm{C}}}}v_{\mathrm{CTRL}} - 4\sqrt{2}\frac{\sqrt{K_{\mathrm{A}}K_{\mathrm{B}}}}{I_{\mathrm{C}}}v_{\mathrm{in}}v_{\mathrm{CTRL}}\right)$$

$$I_{\mathrm{d3}} = f(-v_{\mathrm{CTRL}}, v_{\mathrm{CTRL}}, I_{\mathrm{dB2}}, K_{\mathrm{A}})$$
$$\cong \frac{I_{\mathrm{C}}}{4}\left(1 - 2\sqrt{\frac{2K_{\mathrm{B}}}{I_{\mathrm{C}}}}v_{\mathrm{in}} - 4\sqrt{\frac{K_{\mathrm{A}}}{I_{\mathrm{C}}}}v_{\mathrm{CTRL}} + 4\sqrt{2}\frac{\sqrt{K_{\mathrm{A}}K_{\mathrm{B}}}}{I_{\mathrm{C}}}v_{\mathrm{in}}v_{\mathrm{CTRL}}\right)$$

$$I_{\mathrm{d4}} = f(v_{\mathrm{CTRL}}, -v_{\mathrm{CTRL}}, I_{\mathrm{dB2}}, K_{\mathrm{A}})$$
$$\cong \frac{I_{\mathrm{C}}}{4}\left(1 - 2\sqrt{\frac{2K_{\mathrm{B}}}{I_{\mathrm{C}}}}v_{\mathrm{in}} + 4\sqrt{\frac{K_{\mathrm{A}}}{I_{\mathrm{C}}}}v_{\mathrm{CTRL}} - 4\sqrt{2}\frac{\sqrt{K_{\mathrm{A}}K_{\mathrm{B}}}}{I_{\mathrm{C}}}v_{\mathrm{in}}v_{\mathrm{CTRL}}\right)$$

$$V_{\mathrm{out}} = R_{\mathrm{L}}(I_{\mathrm{d1}} - I_{\mathrm{d2}} + I_{\mathrm{d3}} - I_{\mathrm{d4}})$$
$$\cong 4\sqrt{2}\sqrt{K_{\mathrm{A}}K_{\mathrm{B}}}R_{\mathrm{L}}v_{\mathrm{in}}v_{\mathrm{CTRL}}$$

小信号等価回路を利用した結果と（当然ながら）一致

飽和領域でのプロット～準備～

$$\left(I_{\mathrm{C}} = 1000\ \mu\mathrm{A}, \qquad 2K_{\mathrm{A}} = K_{\mathrm{B}} = 500\ \mu\mathrm{S/V}, \qquad R_{\mathrm{L}} = 400\ \Omega, \qquad g(x) = x\sqrt{1-\left(\frac{x}{2}\right)^2}, \qquad h(x) = 1 + g(x) \right)$$

$$\begin{aligned}
f(v_1, v_2, I_{\mathrm{C}}, K_{\mathrm{B}}) &= \frac{I_{\mathrm{C}}}{2}\left(1 + \sqrt{\frac{2K_{\mathrm{B}}}{I_{\mathrm{C}}}}(v_1 - v_2)\sqrt{1 - \frac{K_{\mathrm{B}}}{2I_{\mathrm{C}}}(v_1 - v_2)^2}\right) \\
&= 500\ \mu\mathrm{A}\left(1 + (v_1 - v_2)\sqrt{1 - \left(\frac{v_1 - v_2}{2}\right)^2}\right) \\
&= 500\ \mu\mathrm{A} \cdot h(v_1 - v_2) \\
f(v_3, v_4, I_{\mathrm{dB1}}, K_{\mathrm{A}}) &= \frac{I_{\mathrm{dB1}}}{2}\left(1 + \sqrt{\frac{2K_{\mathrm{A}}}{I_{\mathrm{dB1}}}}(v_3 - v_4)\sqrt{1 - \frac{K_{\mathrm{A}}}{2I_{\mathrm{dB1}}}(v_3 - v_4)^2}\right) \\
&= 250\ \mu\mathrm{A} \cdot h(v_1 - v_2)\left(1 + \frac{v_3 - v_4}{\sqrt{h(v_1 - v_2)}}\sqrt{1 - \left(\frac{v_3 - v_4}{2\sqrt{h(v_1 - v_2)}}\right)^2}\right) \\
&= 250\ \mu\mathrm{A} \cdot h(v_1 - v_2) \cdot h\left(\frac{v_3 - v_4}{\sqrt{h(v_1 - v_2)}}\right)
\end{aligned}$$

$$\begin{aligned}
I_{\mathrm{dB1}} &= f(v_{\mathrm{in}}, -v_{\mathrm{in}}, I_{\mathrm{C}}, K_{\mathrm{B}}) = 500\ \mu\mathrm{A} \cdot g(2v_{\mathrm{in}}) \\
I_{\mathrm{dB2}} &= f(-v_{\mathrm{in}}, v_{\mathrm{in}}, I_{\mathrm{C}}, K_{\mathrm{B}}) = 500\ \mu\mathrm{A} \cdot g(-2v_{\mathrm{in}}) \\
I_{\mathrm{d1}} &= f(v_{\mathrm{CTRL}}, -v_{\mathrm{CTRL}}, I_{\mathrm{dB1}}, K_{\mathrm{A}}) \\
&= 250\ \mu\mathrm{A} \cdot h(2v_{\mathrm{in}})\left[1 + g\left(\frac{2v_{\mathrm{CTRL}}}{\sqrt{h(2v_{\mathrm{in}})}}\right)\right] \\
I_{\mathrm{d2}} &= f(-v_{\mathrm{CTRL}}, v_{\mathrm{CTRL}}, I_{\mathrm{dB1}}, K_{\mathrm{A}}) \\
&= 250\ \mu\mathrm{A} \cdot h(2v_{\mathrm{in}})\left[1 + g\left(-\frac{2v_{\mathrm{CTRL}}}{\sqrt{h(2v_{\mathrm{in}})}}\right)\right] \\
I_{\mathrm{d3}} &= f(-v_{\mathrm{CTRL}}, v_{\mathrm{CTRL}}, I_{\mathrm{dB2}}, K_{\mathrm{A}}) \\
&= 250\ \mu\mathrm{A} \cdot h(-2v_{\mathrm{in}})\left[1 + g\left(-\frac{2v_{\mathrm{CTRL}}}{\sqrt{h(-2v_{\mathrm{in}})}}\right)\right] \\
I_{\mathrm{d4}} &= f(v_{\mathrm{CTRL}}, -v_{\mathrm{CTRL}}, I_{\mathrm{dB2}}, K_{\mathrm{A}}) \\
&= 250\ \mu\mathrm{A} \cdot h(-2v_{\mathrm{in}})\left[1 + g\left(\frac{2v_{\mathrm{CTRL}}}{\sqrt{h(-2v_{\mathrm{in}})}}\right)\right]
\end{aligned}$$

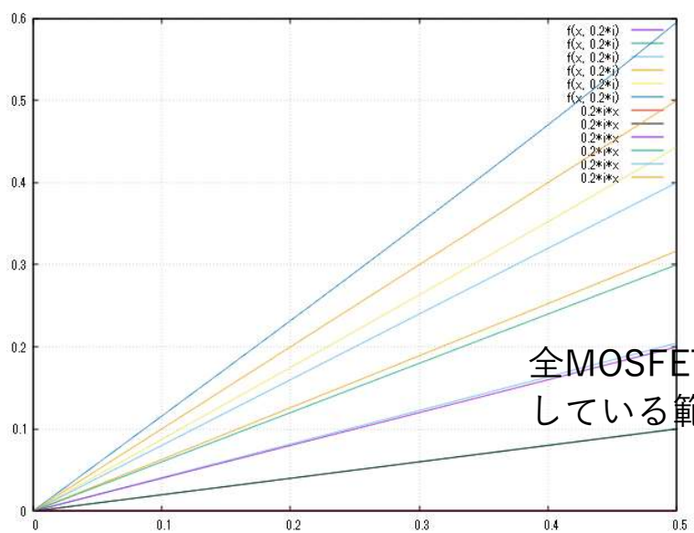
飽和領域でのプロット例～gnuplot～

$$\left(I_{\mathrm{C}} = 1000\ \mu\mathrm{A}, \quad 2K_{\mathrm{A}} = K_{\mathrm{B}} = 500\ \mu\mathrm{S/V}, \quad R_{\mathrm{L}} = 400\ \Omega, \quad g(x) = x\sqrt{1-\left(\frac{x}{2}\right)^2}, \quad h(x) = 1 + g(x) \right)$$

$g(x)$ が奇関数であることに注意して，

$$\begin{aligned}
(I_{\mathrm{d1}} - I_{\mathrm{d2}}) + (I_{\mathrm{d3}} - I_{\mathrm{d4}}) &= 250\ \mu\mathrm{A} \cdot h(2v_{\mathrm{in}}) \cdot 2g\left(\frac{2v_{\mathrm{CTRL}}}{\sqrt{h(2v_{\mathrm{in}})}}\right) + 250\ \mu\mathrm{A} \cdot h(-2v_{\mathrm{in}}) \cdot (-2)g\left(\frac{2v_{\mathrm{CTRL}}}{\sqrt{h(-2v_{\mathrm{in}})}}\right) \\
&= 500\ \mu\mathrm{A} \cdot [1 + g(2v_{\mathrm{in}})] \cdot g\left(\frac{2v_{\mathrm{CTRL}}}{\sqrt{1 + g(2v_{\mathrm{in}})}}\right) - 500\ \mu\mathrm{A} \cdot [1 - g(2v_{\mathrm{in}})] \cdot g\left(\frac{2v_{\mathrm{CTRL}}}{\sqrt{1 - g(2v_{\mathrm{in}})}}\right) \\
V_{\mathrm{out}} &= R_{\mathrm{L}}(I_{\mathrm{d1}} - I_{\mathrm{d2}} + I_{\mathrm{d3}} - I_{\mathrm{d4}}) \\
&= 0.2\ \mathrm{V} \cdot \left[\left(1 + g(2v_{\mathrm{in}})\right) \cdot g\left(\frac{2v_{\mathrm{CTRL}}}{\sqrt{1 + g(2v_{\mathrm{in}})}}\right) - \left(1 - g(2v_{\mathrm{in}})\right) \cdot g\left(\frac{2v_{\mathrm{CTRL}}}{\sqrt{1 - g(2v_{\mathrm{in}})}}\right) \right] \\
V_{\mathrm{out}} &\cong 4\sqrt{2}\sqrt{K_{\mathrm{A}}K_{\mathrm{B}}}R_{\mathrm{L}}v_{\mathrm{in}}v_{\mathrm{CTRL}} = 0.8\ [\mathrm{V}^{-2}] \cdot v_{\mathrm{in}}v_{\mathrm{CTRL}}
\end{aligned}$$

```
g(x)=x*sqrt(1-(x/2.0)**2)
f(x,y)=(1+g(x))*g(y/sqrt(1+g(x)))-(1-g(x))*g(y/sqrt(1-g(x)))
set xrange [0:0.5]
plot for [i=0:5] f(x, 0.2*i)
replot for [i=0:5] 0.2*i*x
```



全MOSFETが飽和領域で動作している範囲では高い直線性

プロット例～gnuplot～

$(I_{\mathrm{C}} = 1000\ \mu\mathrm{A}, \quad 2K_{\mathrm{A}} = K_{\mathrm{B}} = 500\ \mu\mathrm{S/V}, \quad R_{\mathrm{L}} = 400\ \Omega,)$

f(v1, v2, IS, K) = (abs(v1-v2) < sqrt(IS/K) ? (IS/2)*(1 + sqrt(2*K/IS)*(v1-v2)*sqrt(1 - K/(2*IS)*(v1-v2)**2)) : (IS/2)*(1 + sgn(v1-v2)))

Vout(x, y)=RL*(f(y, -y, f(x, -x, IC, K), K/2) - f(-y, y, f(x, -x, IC, K), K/2) + f(-y, y, f(-x, x, IC, K), K/2) - f(y, -y, f(-x, x, IC, K), K/2))

K = 0.0005

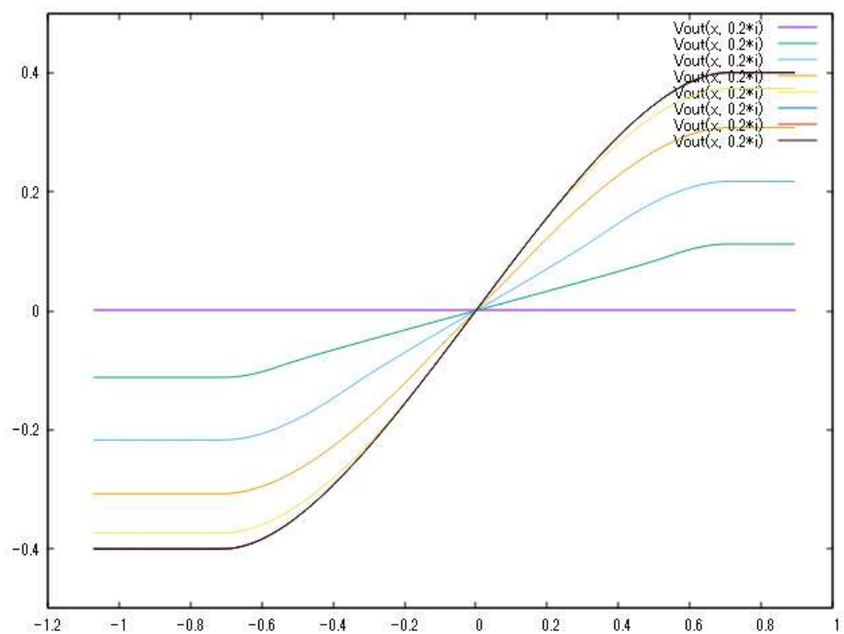
IC = 0.001

RL= 400

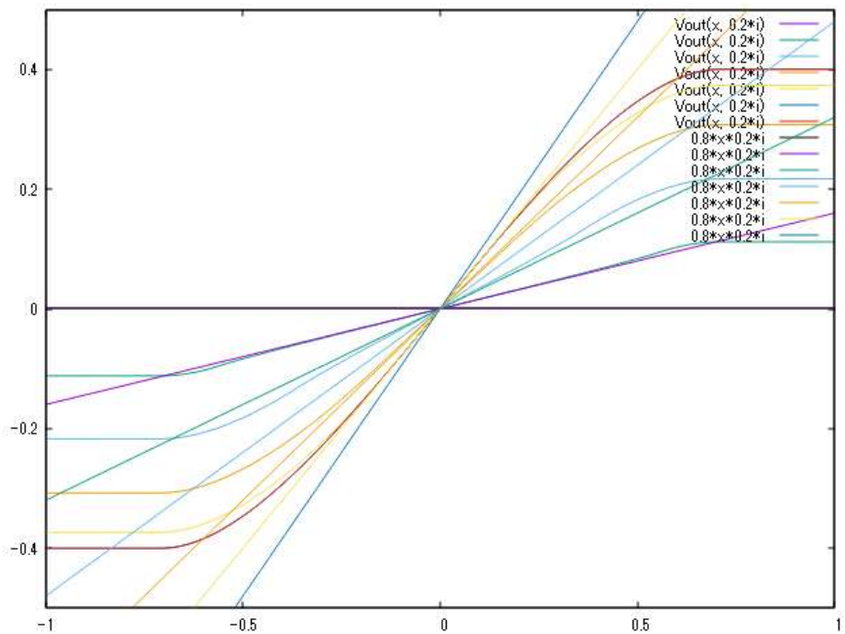
set xrange [-1:1]

plot for [i=0:6] Vout(x, 0.2*i)

replot for [i=0:6] 0.8*x*0.2*i



$v_1 = -v_2 = v_{\mathrm{in}}\ [\mathrm{V}]$



$v_1 = -v_2 = v_{\mathrm{in}}\ [\mathrm{V}]$

プロット例～gnuplot～

$(I_{\mathrm{C}} = 1000\ \mu\mathrm{A}, \quad 2K_{\mathrm{A}} = K_{\mathrm{B}} = 500\ \mu\mathrm{S/V}, \quad R_{\mathrm{L}} = 400\ \Omega,)$

f(v1, v2, IS, K) = (abs(v1-v2) < sqrt(IS/K) ? (IS/2)*(1 + sqrt(2*K/IS)*(v1-v2)*sqrt(1 - K/(2*IS)*(v1-v2)**2)) : (IS/2)*(1 + sgn(v1-v2)))

Vout(x, y)=RL*(f(y, -y, f(x, -x, IC, K), K/2) - f(-y, y, f(x, -x, IC, K), K/2) + f(-y, y, f(-x, x, IC, K), K/2) - f(y, -y, f(-x, x, IC, K), K/2))

K = 0.0005

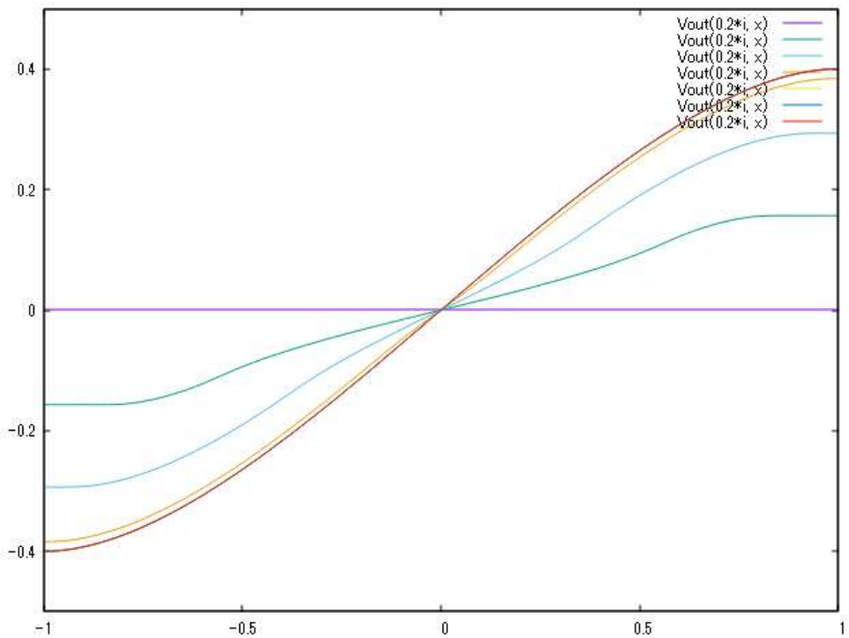
IC = 0.001

RL= 400

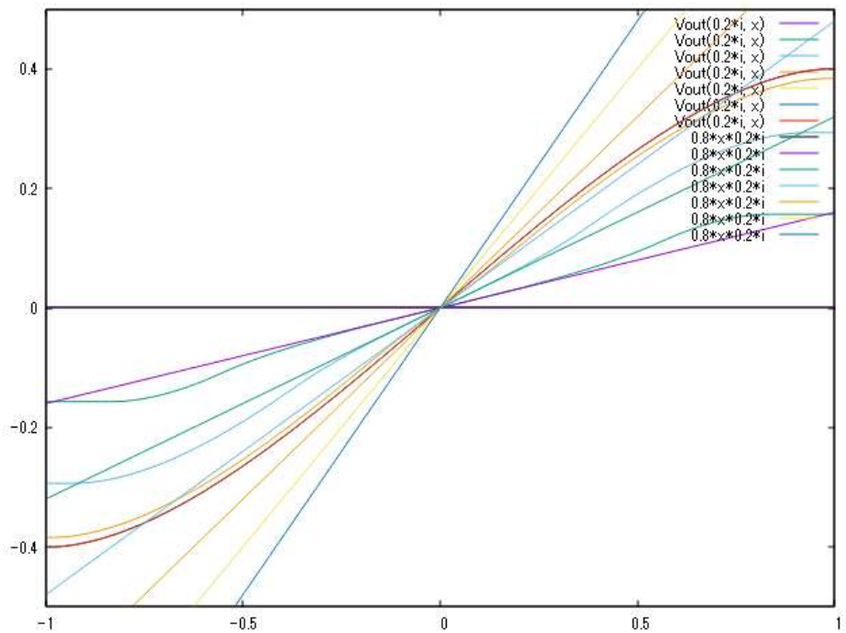
set xrange [-1:1]

plot for [i=0:6] Vout(0.2*i, x)

replot for [i=0:6] 0.8*x*0.2*i



$v_3 = -v_4 = v_{\mathrm{CTRL}}[\mathrm{V}]$



$v_3 = -v_4 = v_{\mathrm{CTRL}}[\mathrm{V}]$