

# Master Project ASL

Frederike Dmbgen

April 27, 2016

# Chapter 1

## Rectification

The following test was performed to verify the correctness of the pixel association process. The difference of intensity at corresponding pixels in the original images  $I_j$  and rectified images  $I_j^{rect}$  are given by:

$$\Delta I_{1,i} = I_1(u_{1,i}) - I_1^{rect}(w_{1,i}) \quad (1.1)$$

$$\Delta I_{2,i} = I_2(u_{2,i}) - I_2^{rect}(w_{2,i}) \quad \text{for } i = 1 \dots n, \quad (1.2)$$

where  $u_{1,i}$ ,  $u_{2,i}$  denote the pixels in the original image and  $w_{1,i}$ ,  $w_{2,i}$  denote the pixels in the rectified image.

If the rectified pixels are computed in accordance with the rectification process of the images given by `image_proc` of ROS, we would expect both differences to be approximately zero, since:

$$I_1(u_{1,i}) = I_1^{rect}(w_{1,i}) + \epsilon_{1,i} \quad (1.3)$$

$$I_2(u_{2,i}) = I_2^{rect}(w_{2,i}) + \epsilon_{2,i} \quad (1.4)$$

$$\text{for } i = 1 \dots n, \quad (1.5)$$

with  $\epsilon_{i,j}$  the error arising solely from interpolation of the rectified images.

The pixels of the rectified image need to obey

$$w_{i,j} = P_j \begin{pmatrix} X_{i,j}^{rect} \\ 1 \end{pmatrix} \quad \text{for } j = 1, 2, \quad (1.6)$$

where  $X_{i,j}^{rect}$  are computed such that  $w_{i,j} = \tilde{u}_{i,j}$ , with

$$\tilde{u}_{i,j} = K'(T_j(X_i)), \text{ with} \quad (1.7)$$

$$T_j(X_i) = \pi(C_{C_j M}(X_i - M r_{MC_j})) \text{ and} \quad (1.8)$$

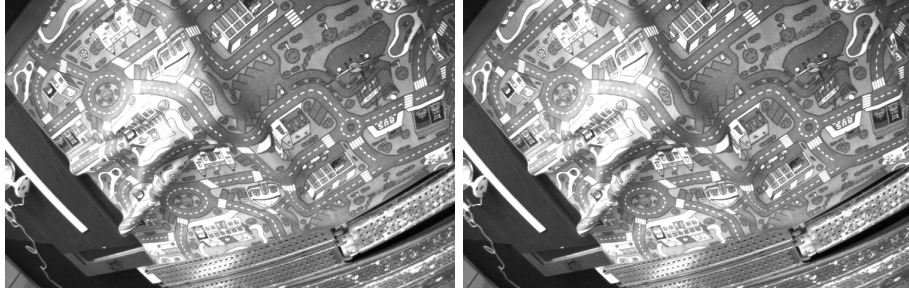
$$\pi((x, y, z)^T) = (x/z, y/z, z)^T. \quad (1.9)$$

Remember that  $u_{i,j}$  are given by

$$u_{i,j} = K_j(D_j(T(X_i))) \quad (1.10)$$

which concludes the above considerations.

All tools for computing the pixel correspondences being defined, a look at Figures 1.1a and 1.1b shows that the resulting error between the rectified image and the original image is indeed uniformly small and with no patterns observable in both left and right images. The above equations for pixel correspondences are thereby validated for the rectification process given by image\_proc of ROS.



(a) Intensity error of left image with  $n_x = ?$  and  $n_y = ?$ . (b) Intensity error of right image with  $n_x = ?$  and  $n_y = ?$ .

Figure 1.1: Rectification intensity errors of left and right image.

Similar considerations can be applied for validating the interpretation of the camera parameters given in the camera\_info message.

The following assumptions are to be tested.

$$C_{SC_j} = R_j \text{ for } j=1, 2 \implies C_{C_2C_1} = R_2^T R_1 \text{ and} \quad (1.11)$$

$$C_1 r_{C_1C_2} = \begin{pmatrix} -P_j(0,3)/P_j(0,0) & 0 & 0 \end{pmatrix}^T. \quad (1.12)$$

The methodology applied is the following. If the above assumptions hold, then one can obtain the pixel coordinates in the second rectified image given the coordinates in the first image using  $X_{i,1}^{rect}$  such that  $w_{i,1} = \tilde{u}_{i,1}$ , with  $\tilde{u}_{i,1}$  defined as above, and

$$X_{i,2}^{rect} = C_{C_2C_1}(C_1 r_{C_1C_2} X_i + C_1 r_{C_1C_2}) \quad (1.13)$$

$$= C_{C_2C_1}(X_{i,1}^{rect} + C_1 r_{C_1C_2}). \quad (1.14)$$

The residuals of the original images  $I_1$  and  $I_2$  and of the rectified images  $I_1^{rect}$  and  $I_2^{rect}$  are given by

$$\Delta r_i = I_1(u_{1,i}) - I_2(w_{2,i}) \quad (1.15)$$

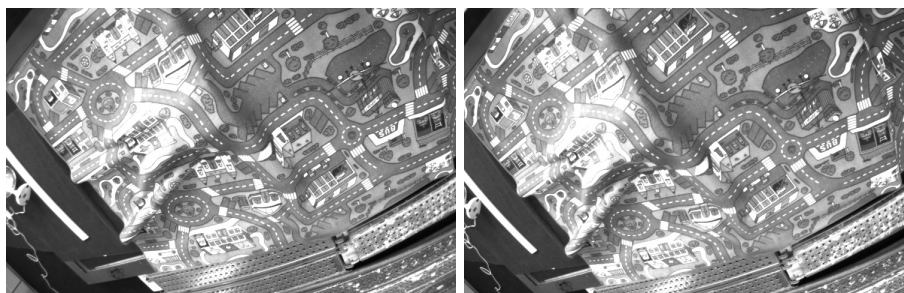
$$\Delta r_i^{rect} = I_1^{rect}(w_{1,i}) - I_2^{rect}(w_{2,i}), \quad (1.16)$$

$$\text{for } i = 1 \dots n, \quad (1.17)$$

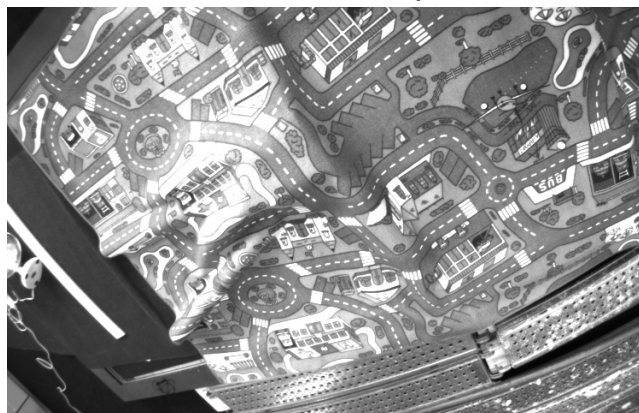
If the rectified pixels are computed in accordance with the rectification process of the images, we have

$$\Delta r_i = \Delta r_i^{rect} + \epsilon_i \text{ for } i = 1 \dots n, \quad (1.18)$$

with  $\epsilon_i$  the error arising solely from interpolation errors between the left and the right image. The photometric error in the original and rectified image are shown in Figures 1.2a and 1.2b. Since at this stage, no reliable depth theinformation is available, low photmetric errors cannot be expected, which explains the relatively high average value. However, from Equation 1.18 we know that the photometric errors should be approximately the same in both cases. A visualization of the difference between the two in Figure 1.2c shows that this is indeed the case.



(a) Residuals of original image with  $n_x = ?$  and  $n_y = ?$ . (b) Residuals of original image with  $n_x = ?$  and  $n_y = ?$ .



(c) Difference of residual errors.

Figure 1.2: Rectification residual errors of original and rectified images.