Master Project ASL

Frederike Dmbgen

May 3, 2016

Chapter 1

Rectification

1.1 Rectification test

The following test was performed to verify the correctness of the pixel association process. The difference of intensity at corresponding pixels in the original images I_j and rectified images I_j^{rect} are given by:

$$\Delta I_{i,j} = I_j(u_{i,j}) - I_j^{rect}(w_{i,j}) \qquad \text{for } i = 1 \ldots n, \quad j = 1,2 \quad , \tag{1.1}$$

where $u_{i,j}$, $w_{i,j}$ denote the pixel location of the object point X_i in the original and rectified image of camera j respectively.

If the rectified pixels are computed in accordance with the rectification process of the images given by image_proc of ROS, this difference should be approximately zero or

$$I_j(u_{j,i}) = I_j^{rect}(w_{i,j}) + \epsilon_{i,j} \qquad \text{for } i = 1 \ldots n, \quad j = 1,2 \quad , \tag{1.2}$$

with the error $\epsilon_{i,j}$ arising solely interpolation.

The pixels of the rectified image are computed using the projection matrices P_j obtained by the CameraInfo message [3] with

$$w_{i,j} = P_j \begin{pmatrix} X_{i,j}^{rect} \\ 1 \end{pmatrix} = \begin{bmatrix} K' | t_j \end{bmatrix} \begin{pmatrix} X_{i,j}^{rect} \\ 1 \end{pmatrix} \qquad \text{for } j = 1,2 \quad , \tag{1.3}$$

where $X_{i,j}^{rect}$ are computed such that $w_{i,j} = \tilde{u}_{i,j}$.

The points $\tilde{u}_{i,j}$ denote the pixel locations of X_i in the undistorted image of camera j, or

$$\tilde{u}_{i,j} = K'(T_j(X_i)) \qquad \text{, where} \tag{1.4}$$

$$T_j(X_i) = \pi(C_{C_jM}(X_i - {}_Mr_{MC_j})) \qquad \text{and} \tag{1.5}$$

$$\pi(\left(x, y, z\right)^T) = \left(x/z, y/z, z\right)^T \quad . \tag{1.6}$$

The object points $X_{i,j}^{rect}$ are therefore given by

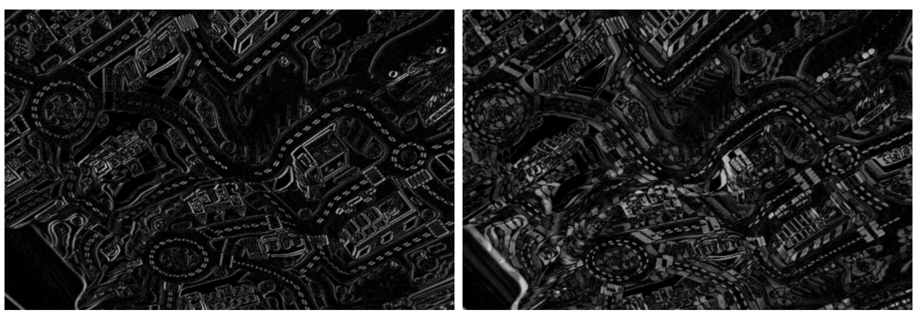
$$X_{i,j}^{rect} = K'^{-1}(\tilde{u}_{i,j} - t_j) \quad . \tag{1.7}$$

Remember that $u_{i,j}$ are given by

$$u_{i,j} = K_j(D_j(T_j(X_i))) \qquad \text{for } i = 1 \dots n, \quad j = 1, 2 \quad , \tag{1.8}$$

with K_j, D_j and T_j the camera matrix, distortion coefficient and projection transforms respectively.

A look at Figures 1.1a and 1.1b shows that the resulting error between the rectified image and the original image is indeed unformly small and with no patterns observable in both left and right images. (Unfortunately not the case...) The above equations for pixel correspondences are thereby validated for the rectification process given by image_proc of ROS.



(a) Left image (mean: 22.67) (b) Right image (mean: 30.2813)

Figure 1.1: Rectification intensity errors with $n_x = 600$ and $n_y = 400$.

1.2 Transformation test

Similar considerations can be applied for validating the interpretation of the camera parameters given in the camera_info message.

The following assumptions are to be tested.

$$C_{SC_j} = R_j \qquad \text{for j=1, 2} \implies C_{C_2C_1} = R_2^T R_1 \qquad \text{and} \tag{1.9}$$

$$ {}_{C_1}r_{C_1C_2} = C_{C_1S}({}_S r_{SC_2} - {}_S r_{SC_1}) = R_1^T \begin{pmatrix} -P_2(0,3)/P_2(0,0) & 0 & 0 \end{pmatrix}^T \quad . \tag{1.10}$$

The methodology applied is the following. If the above assumptions hold, then one can obtain the pixel coordinates in the second rectified image given the coordinates in the first image using $X_{i,1}^{rect}$ such that $w_{i,1} = \tilde{u}_{i,1}$, with $\tilde{u}_{i,1}$ defined as above, and

$$X_{i,2}^{rect} = {}_{C_2} r_{C_2X} = C_{C_2C_1}({}_{C_1} r_{C_1X_i} + {}_{C_1} r_{C_1C_2}) \tag{1.11}$$
$$= C_{C_2C_1}(X_{i,1}^{rect} + {}_{C_1} r_{C_1C_2}) \quad . \tag{1.12}$$

The residuals of the original images I_1 and I_2 and of the rectified images I_1^{rect} and I_2^{rect} are given by

$$\Delta r_i = I_1(u_{1,i}) - I_2(w_{2,i}) \tag{1.13}$$
$$\Delta r_i^{rect} = I_1^{rect}(w_{1,i}) - I_2^{rect}(w_{2,i}) \qquad \text{for } i = 1 \dots n \quad . \tag{1.14}$$

If the rectified pixels are computed in accordance with the rectification process of the images, we have

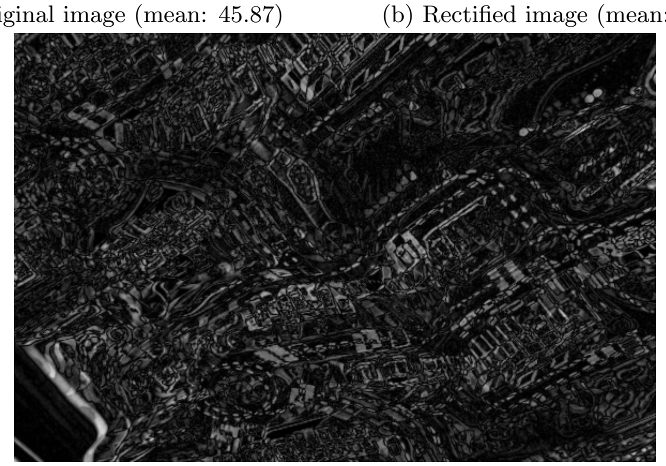
$$\Delta r_i = \Delta r_i^{rect} + \epsilon_i \qquad \text{for } i = 1 \dots n \quad , \tag{1.15}$$

with ϵ_i the error arising solely from interpolation errors between the left and the right image.

The photometric error in the original and rectified image are is shown in Figures 1.2a and 1.2b. Since at this stage, no reliable depth information is available, low photmetric errors cannot be expected, which explains the relatively high average value. However, from Equation 1.15 we know that the photometric errors should be approximately the same in both cases. A visualization of the difference between the two in Figure 1.2c shows that this is indeed the case. Therefore, trnsform between the left and right image can be computed from the camera_info message as given in 1.12.

(a) Original image (mean: 45.87)

(b) Rectified image (mean: 45.42)



(c) Difference (mean: 31.6758)

Figure 1.2: Rectification residual errors with $n_x = 600$ and $n_y = 400$.

Bibliography

[1] J. Nocedal and S. J. Wright, *Numerical Optimization*, Springer-Verlag New York, Inc., second ed., 1999.

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[3] ——, *sensor_msgs CameraInfo*. http://docs.ros.org/api/sensor_msgs/html/msg/CameraInfo.html, 2016.

[4] B. Siciliano and O. Khatib, *Springer, Handbook of Robotics*, Springer-Verlag New York, Inc., first ed., 2007.