Table .1 summarizes the individual symbols used in the Copula-fGARCH and count modeling framework.

Table .1: Summary of Notations Used in Copula-fGARCH and Count Modeling Framework

Symbol	Description
X_{ij}	Observed count for day i and interval j
Y_{ij}	Count increment: $X_{i,j+1} - X_{ij}$
Y_{ij}^2	Squared increment (raw volatility measure)
$Y_i^2(t)$	Smoothed functional volatility trajectory for day i
$\mu(t)$	Mean functional volatility curve
$\phi_k(t)$	k-th FPCA eigenfunction
Z_{ik}	FPCA score for day i and component k
p	Number of retained FPCA components
$F_{Z_k}(z)$	Marginal cumulative distribution function of Z_k
U_{ik}	Transformed uniform variable: $U_{ik} = F_{Z_k}(Z_{ik})$
$C(u_1, \ldots, u_p)$	Copula function modeling dependence among FPCA scores
σ_t^2	Latent volatility at time t (GARCH process)
$\hat{\sigma}_t^2$	Estimated latent volatility proxy used in count modeling and monitoring
$\lambda, \mu, \theta, \nu$	Parameters of count distributions (Poisson, NB, CMP)
$f(\cdot)$	Link function mapping latent volatility to conditional mean/rate
Combined effect	Average effect from Gaussian, Clayton, and empirical copulas

Table .2 summarizes the individual symbols used in the control chart procedure.

Table .3 summarizes all individual symbols used. Figure .1 is a compact flowchart of our proposed method. Figure .2 shows intra-day latent volatility for BTC buy transactions (11-day average perspective). Figure .3 shows intra-day latent volatility for BTC sell transactions (11-day average perspective.

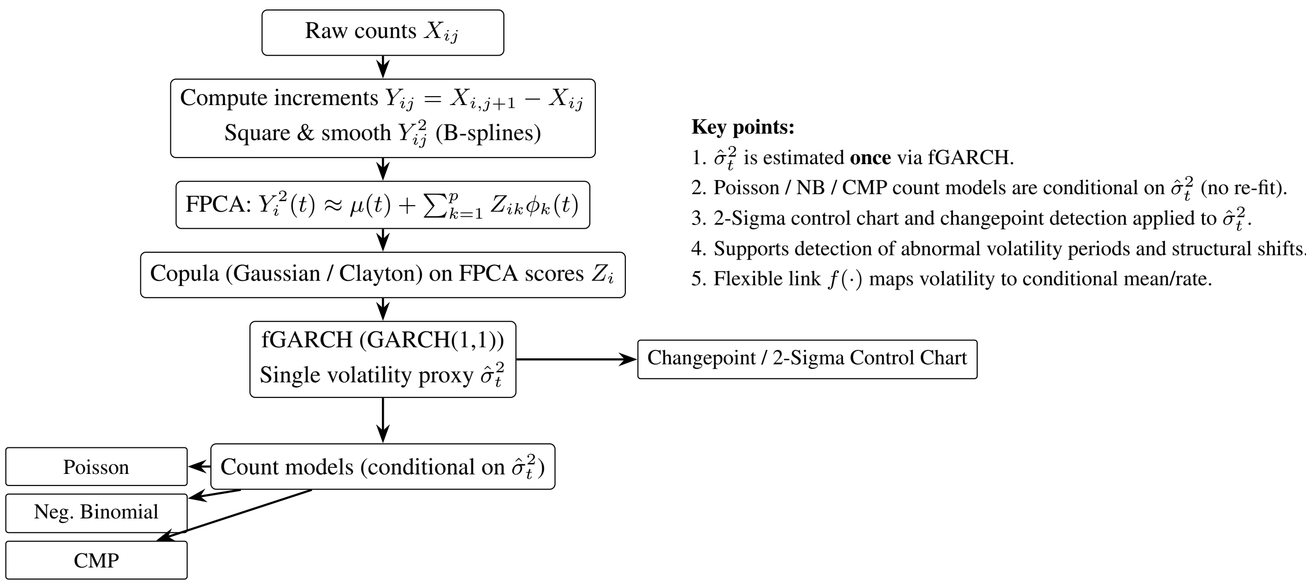
Table .2: Notation for Control Chart of Volatility Time Series

Symbol	Description
$\hat{\sigma}_t^2$	Estimated latent volatility at time t from Copula-fGARCH
T	Total number of time points in the volatility series
μ_{vol}	Mean of the volatility series $\hat{\sigma}_t^2$
σ_{vol}	Standard deviation of $\hat{\sigma}_t^2$
U	Upper control limit, $U = \mu_{\text{vol}} + 2\sigma_{\text{vol}}$
L	Lower control limit, $L = \mu_{\text{vol}} - 2\sigma_{\text{vol}}$
t	Index of the time point in the volatility series, $t = 1, \ldots, T$
i_k	Index of the k-th out-of-control point
m	Total number of out-of-control points
Δ_k	Interval between consecutive out-of-control points: $\Delta_k = i_{k+1} - i_k$
ARL	Average Run Length: mean interval between out-of-control signals
ARL_{SD}	Standard deviation of run lengths between out-of-control signals

Table .3: Notation for Changepoint Detection in Volatility Series

Symbol	Description
$\hat{\sigma}_t^2$	Estimated latent volatility proxy at time t
T	Total number of time points in the series
τ_k	Index of the k-th changepoint, $k = 1, \ldots, K$
K	Total number of changepoints detected
$\mathcal{C}(\cdot)$	Segment-specific cost function (e.g., based on mean/variance)
β	Penalty for adding a changepoint, controls model complexity
τ^*	Greedy estimate of changepoint in BinSeg algorithm
L_i	Cost associated with segment i: $L_i = \mathcal{C}(\hat{\sigma}^2_{t_{i-1}+1:t_i})$
t_i	Segment boundary index for segment i

Figure .1: Compact flowchart from raw counts X_{ij} to a single volatility proxy $\hat{\sigma}_t^2$, which is then used for alternative count distributions and monitored via control chart and changepoint detection.



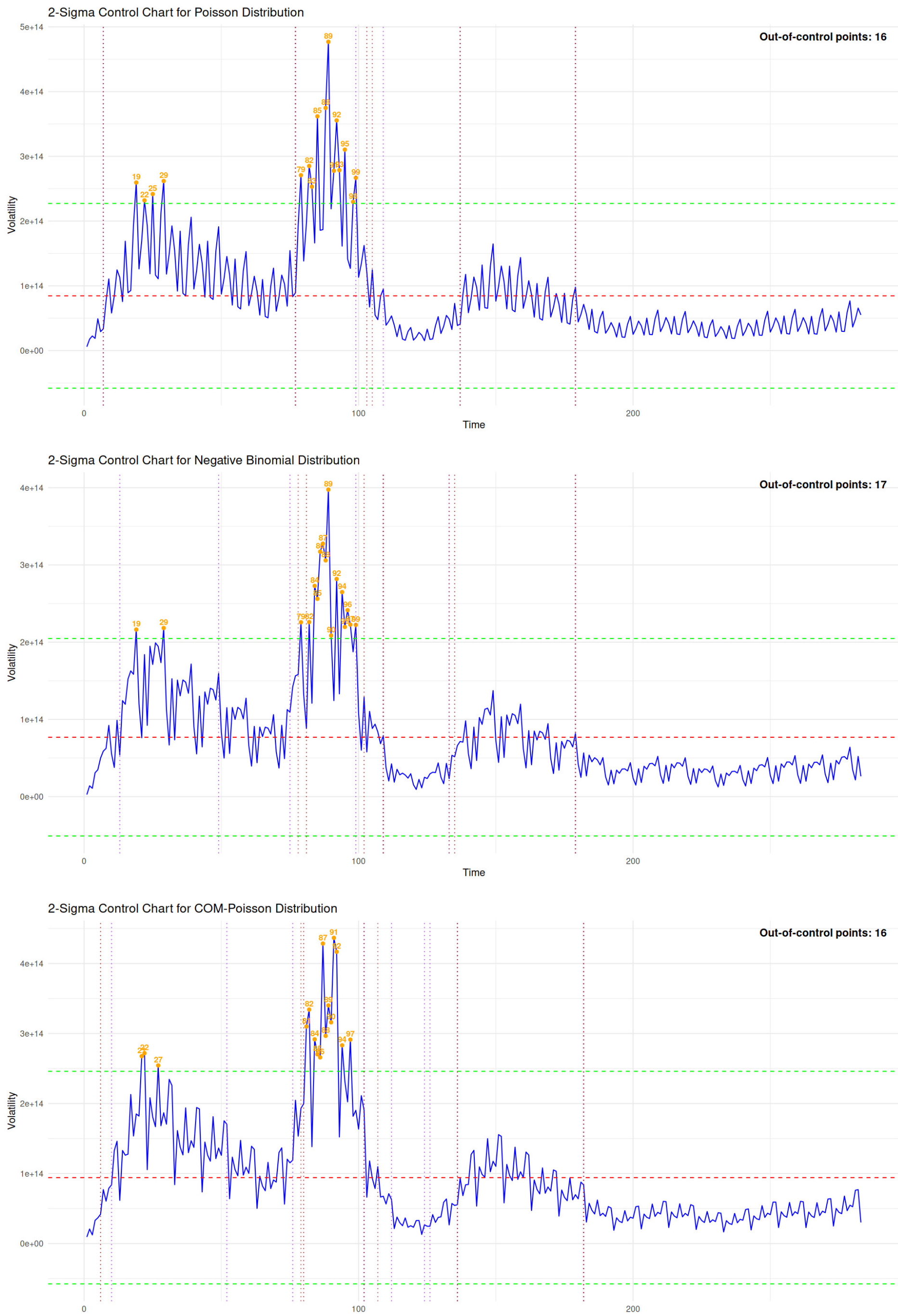


Figure .2: Intra-day latent volatility for BTC buy transactions (11-day average perspective).

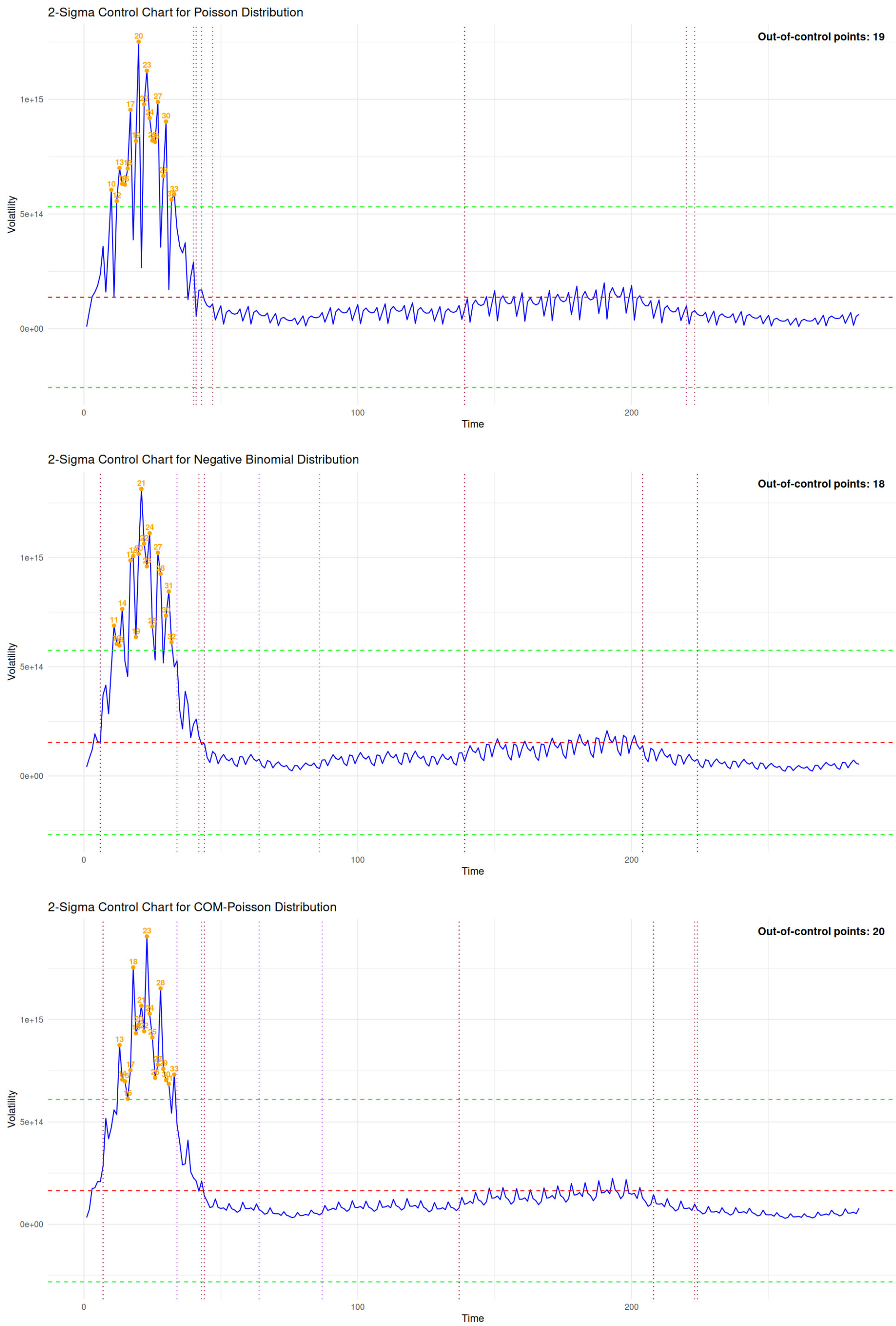


Figure .3: Intra-day latent volatility for BTC sell transactions (11-day average perspective).