1. [12 points]Let R and S be relations on the set A, where A={1,2,3,4,5}, R={(1,2),~~(2,2)~~,(2,4) ,(4,3),(3,3),~~(3,5)~~}, S={(2,2),(3,5),(5,3)}.

   a) Find the symmetric closure of $R\circ S$

   对称闭包

计算问题， 矩阵的布尔积运算

   b) Find the following relations, and determine whether it is reflexive, symmetric, asymmetric, antisymmetric, and/or transitive?

   (1)R-S 性质判定， 记不清楚，

	1	2	3	4	5
1	0	1	0	0	0
2	0	0	0	1	0
3	0	0	1	0	0
4	0	0	1	0	0
5	0	0	0	0	0

   (2) S-R

   (3) $(R\cap S)^{-1}$

Inverse，作业中定义， 就是把序偶倒过来。{(2,2),(5,3)}

(4) R^2

2. [8 points]Use Warshall's algorithm to find the transitive closure of R , where the zero–one matrix of R is

$$M=\begin{bmatrix}1&1&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&1&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{bmatrix}$$

自反 对称 传递.

3. [3 points] Find the smallest equivalence relation on the set {1, 2, 3, 4, 5,6} containing the relation {(1, 2), (2, 3),(4,5)}.

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① $r(R)=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,2),(2,3),(4,5)\}$

② $s(r(R))=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,2),(2,1),(2,3),\cancel{(4,5)}(3,2),(4,5),(5,4)\}$

③ $tsr(R)=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,2),(2,1),(2,3),(3,2),(4,5),(5,4),(1,3),(3,1)\}$

①②③每个一分，若直接写出正确答案直接给3分，但若无过程直接写结果且结果错误则不得分.

4. [7 points] Let R be the relation on the set of ordered pairs of positive integers such that ((a, b), (c, d)) ∈ R if and only if a + d = b + c.

a) Show that R is an equivalence relation.

b) What are the equivalence classes of R?

a) ① 自反：(2分) $\forall$ ~~(a,b),(a,b)~~ $a,b\in Z^+$

则有 $a+b=b+a$，即 $((a,b),(a,b))\in R$.

② 对称：(2分) $\forall\ ((a,b),(c,d))\in R$.

则 $a+d=b+c$，即 $c+b=d+a$

$((c,d),(a,b))\in R$

③ 传递：(2分) $\forall\ ((a,b),(c,d))\in R$，$((c,d),(e,f))\in R$

则 $\begin{cases} a+d=b+c \\ c+f=d+e \end{cases}$ 得：$a+f=b+e$

即 $((a,b),(e,f))\in R$.

综上，R是等价关系.

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b) (1分) 故 R的等价类为 $[(a,b)]=\{(x,y)\mid x-y=a-b,\ x\in Z^+,\ y\in Z^+\}$

5. [7 points] Set A= {1, 2, 3, 4, 6, 9, 12, 24}.R is the divisibility on the set A.

a) Draw the Hasse diagram for the poset <A,R>.

b) Subset B={ 3, 4, 6}. Find the minimal elements, least element and least upper bound of B.

6. [3 points] Posets <L,R> consisting of the following sets L, Where R is defined as: let $n_1, n_2 \epsilon L$, $n_1 R n_2$ if and only if n_1 is a factor of n_2. Which posets are lattices?

a) L={1,2,3,4,6,12}

b) L={1,2,3,4,6,8,12,14}

c) L={1,2,3,4,5,6,7,8,9,10,11,12}

7. [3 points] Complete the operational table so that * is a commutative binary operation.

*	a	b	c
a		c	a
b			
c		b	c

8. [7 points] Let $(S_1,*_1)$, $(S_2,*_2)$, and $(S_3,*_3)$ be semigroups and f: $S_1 \rightarrow S_2$ and g: $S_2 \rightarrow S_3$ be homomorphisms. Prove that g°f is a homomorphism from S_1 to S_3.

$f(a_1) = a_2$ $f(b_1) = b_2$ $f(a \times b) = f(a) \times' f(b)$.

$f(a_1 *_1 a_2) = f(\ ) * = a_2 *_2 b_2 \in S_2$

9. [10 points] S = Z with ordinary addition and R defined by a R b if and only if a ≡ b(mod 3).

a) Prove the relation R on the semigroup (S,+) is a congruence relation.

b) Write the operation table of the quotient semigroup S/R.

对 自 传

aRb
cRd

10. [10 points] Let R* =R-{0},

a) Show the (R*, ×) be a group. × is ordinary multiplication.

b) Determine whether the function $f: R^* \rightarrow R^*$ defined by $f(x) = x^2$, for $x \in R^*$, is a homomorphism. and give your reason.

非空
封闭
结合
幺
逆

11. [10 points] Write the multiplication table for the group $Z_2 \times Z_3$. And find all the normal subgroups.

gH = Hg

Arithmetic Modulo *m*

Definitions: Let $\mathbf{Z}_m$ be the set of nonnegative integers less than *m*: {0,1, …., *m*−1}
- The operation $+_m$ is defined as $a +_m b = (a + b)$ **mod** *m*. This is *addition modulo m*.
- The operation $\cdot_m$ is defined as $a \cdot_m b = (a * b)$ **mod** *m*. This is *multiplication modulo m*.
- Using these operations is said to be doing *arithmetic modulo m*.

Example: Find $7 +_{11} 9$ and $7 \cdot_{11} 9$.
Solution: Using the definitions above:
- $7 +_{11} 9 = (7 + 9) \bmod 11 = 16 \bmod 11 = 5$
- $7 \cdot_{11} 9 = (7 \cdot 9) \bmod 11 = 63 \bmod 11 = 8$

p243,上学期给了定义， 本学期只是使用。

Table 9.10 Multiplication Table of $Z_2 \times Z_2$

	$(\bar{0},\bar{0})$	$(\bar{1},\bar{0})$	$(\bar{0},\bar{1})$	$(\bar{1},\bar{1})$
$(\bar{0},\bar{0})$	$(\bar{0},\bar{0})$	$(\bar{1},\bar{0})$	$(\bar{0},\bar{1})$	$(\bar{1},\bar{1})$
$(\bar{1},\bar{0})$	$(\bar{1},\bar{0})$	$(\bar{0},\bar{0})$	$(\bar{1},\bar{1})$	$(\bar{0},\bar{1})$
$(\bar{0},\bar{1})$	$(\bar{0},\bar{1})$	$(\bar{1},\bar{1})$	$(\bar{0},\bar{0})$	$(\bar{1},\bar{0})$
$(\bar{1},\bar{1})$	$(\bar{1},\bar{1})$	$(\bar{0},\bar{1})$	$(\bar{1},\bar{0})$	$(\bar{0},\bar{0})$

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$Z_2 \times Z_3$={0，1}×{0，1，2}

	(0，0)	(0，1)	(0，2)	(1，0)	(1，1)	(1，2)
(0，0)	(0，0)	(0，1)	(0，2)	(1，0)	(1，1)	(1，2)
(0，1)	(0，1)	(0，2)	(0，0)	(1，1)	(1，2)	(1，0)
(0，2)	(0，2)	(0，0)	(0，1)	(1，2)	(1，0)	(1，1)
(1，0)	(1，0)	(1，1)	(1，2)	(0，0)	(0，1)	(0，2)
(1，1)	(1，1)	(1，2)	(1，0)	(0，1)	(0，2)	(0，0)
(1，2)	(1，2)	(1，0)	(1，1)	(0，2)	(0，0)	(0，1)

1 最小的{（0，0）}

2 最大的{

(0，0)	(0，1)	(0，2)	(1，0)	(1，1)	(1，2)

}

3 三阶群

(0，0)	(0，1)	(0，2)

4 二阶群

{（0，0)，(1，0)}

12. [8 points] Consider the (2,5) encoding function e: $B^2 \rightarrow B^5$ defined by

$\mathrm{e}(00) = 00000 \quad \mathrm{e}(01) = 01100 \quad \mathrm{e}(10) = 10101 \quad \mathrm{e}(11) = 11001$

a) Show that the (2,5) encoding function e is a group code. [illegible]

b) Find the minimum distance of the (2, 5) encoding function e.

c) Determine the number of errors that e will **detect** and its associated decoding function will **correct**.

[illegible] [illegible]