

1. Let a and b be positive real numbers with $a \geq b$. Define sequences $\{a_n\}$ and $\{b_n\}$ recursively as follows: $a_1 = a, b_1 = b, a_{n+1} = \frac{a_n + b_n}{2}, b_{n+1} = \sqrt{a_n b_n}$. Prove that for every natural number n , $a_n \geq a_{n+1} \geq b_{n+1} \geq b_n$. Show that the sequences $\{a_n\}$ and $\{b_n\}$ are convergent and have the same limit.