

1. Is the sequence $y_n = x^n$ compact

(a) in $C[0, 1]$?

Solution:

This sequence is not compact in this space. For $\{y_n\}$ we have

$$\|y_n - y_{2n}\|_{C[0,1]} = \sup_{x \in [0,1]} |x^n - x^{2n}| \geq \left| \left(\frac{1}{2}\right)^n - x^{2n} \right|$$

□

(b) in $h[0, 1]$?

2. Find characteristic values and eigenfunctions:

$$y(x) = \lambda \int_{-1}^1 (xs + x^2 s^2) y(s) ds$$

3. Construct the Neumann series for the Volterra equation of the second kind

$$y(x) = \lambda \int_0^x s y(s) ds + 1$$

and find the solution.

4. Construct the resolvent kernel for the equation in the previous problem and use it to find the solution.

5. Analyze the equation

$$y(x) = \lambda \int_{-1}^1 (1 + xs) y(s) ds + \sin \pi x$$

and solve it for any λ .

6. Construct the resolvent kernel for the equation in the previous problem and use it to find the solution