Integral Equations - Spring 2014 - Final Test

1. Is the sequence $y_n = x^n$ compact

   (a) in $C[0,1]$?

   (b) in $h[0,1]$?

2. Find characteristic values and eigenfunctions:

$$y(x) = \lambda \int_{-1}^{1} \left(xs + x^2 s^2\right) y(s) ds.$$

3. Construct the Neumann series for the Volterra equation of the second kind

$$y(x) = \lambda \int_{0}^{x} s\, y(s)\, ds + 1$$

   and find the solution.

4. Construct the resolvent kernel for the equation in the previous Problem and use it to find the solution.

5. Analyze the equation

$$y(x) = \lambda \int_{-1}^{1} (1 + xs) y(s)\, ds + \sin \pi x$$

   and solve it for any λ.

6. Construct the resolvent kernel for the equation in the previous Problem and use it to find the solution.