Stat 422 – Homework 6

1. A random sample $X_1, X_2, \ldots, X_n$ is drawn from a population with pdf

$$f(x|\theta) = \frac{1}{2}(1+\theta x)\mathrm{I}_{(-1,1)}(x)$$

for $-1 < \theta < 1$. Find a consistent estimator of θ and show that it is consistent.

2. MGB Problem #29. Consider two estimators of $\tau(\theta) = P_\theta(X > 0)$ given by

$$T_1 = \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}_{(0,\infty)}(X_i) \quad \text{and} \quad T_2 = \Phi(\widehat{\Theta})$$

where $\widehat{\Theta}$ is MLE of θ.

   (a) Show that both estimators are weakly consistent.

   (b) Are both estimators asymptotically efficient? Justify your answers.

3. MGB # 32, MLE for θ is

$$\widehat{\Theta} = \frac{n}{\sum_{i=1}^{n} \log(1+X_i)}.$$

Show that this estimator is consistent and asymptotically efficient.

4. (G) MGB # 43, consider two estimators of θ^2 given by

$$T_1 = \frac{\sum X_i^2}{n} \quad \text{and} \quad T_2 = \frac{\sum X_i^2}{n+2}.$$

   (a) Show that both estimators are consistent.

   (b) Find the asymptotic distribution of T_1.

5. MGB, Chapter VIII: 1[b], 4