

Stat 422 – Homework 5

1. A random sample X_1, X_2, \dots, X_n is drawn from a population with pdf

$$f(x|\theta) = \frac{1}{2}(1 + \theta x)I_{(-1,1)}(x)$$

for $-1 < \theta < 1$. Find a consistent estimator of θ and show that it is consistent.

2. MGB Problem #29. Consider two estimators of θ^2 given by

$$T_1 = \frac{1}{n} \sum_{i=1}^n I_{(0,\infty)}(X_i) \quad \text{and} \quad T_2 = \Phi(\hat{\Theta})$$

where $\hat{\Theta}$ is MLE of θ .

(a) Show that both estimators are weakly consistent.

(b) Are both estimators asymptotically efficient? Justify your answers.

3. MGB # 32, MLE for $1/\theta$ is

$$\hat{\Theta} = \frac{n}{\sum_{i=1}^n \log(1 + X_i)}.$$

Show that this estimator is consistent and asymptotically efficient.

4. (G) MGB # 43, consider two estimators of θ^2 given by

$$T_1 = \frac{\sum X_i^2}{n} \quad \text{and} \quad T_2 = \frac{\sum X_i^2}{n+2}.$$

(a) Show that both estimators are consistent.

(b) Find the asymptotic distribution of T_1 .

5. MGB, Chapter VIII: 1[b], 4