Big Data Analytics - Spring 2015 - Project 2
Real Time Signal Processing

In this project you will implement and demonstrate optimal signal processing with a sliding window. Some parts of this project are related to Homework 4. Here are suggested phases of the project.

1 Specifying problem settings:

(a) Specify main ingredients of your simulated measurement system a, f, σ^2:

i. Point spread function (influence function) a,

ii. Covariance function f for the signal x:

$$f = \mathrm{Cov}(x) = b * b^*.$$

For that you just need to specify some “smoothing” function b first.

iii. Variance σ^2 of the independent components of the additive random noise ν.

$$y = a * x + \nu, \quad \nu \sim (0, \sigma^2 \delta).$$

(b) Choose some support interval Δ for the point spread function of the processing algorithm:

i. For symmetric a it is natural to choose a symmetric interval $\Delta = [-d, d]$, where $d \geq 0$ is a parameter;

ii. For asymmetric a (which is usually the case in time series processing) it is better to choose $\Delta = [-T, \tau]$, where $T \geq 0$ and $\tau \geq 0$ (prediction delay) are parameters.

2 Computing optimal influence function r:

(a) Compute $p = a * f * a^* + s$ and $p = f * a^*$.

(b) Construct matrix P.

(c) Using p and q find r.

(d) Show graphs for a, f, r, and $r * a$.

(e) Do that for a range of parameters d (or T, τ) and show how estimation precision

$$H = \mathsf{E}(\widehat{x}_i - x_i)^2 = f_0 - \langle q, P^{-1} q \rangle_\Delta$$

depends on parameters. To illustrate a function of two variables you can show it, e.g., as a surface or as a pseudocolor image. It might be interesting to indicate contour lines (curve along which the function has a constant value).

(f) Using this graph determine a "good" choice for your parameter d (or parameters T, τ).

3 Simulation of Measurement and Processing:

(a) Randomly generate some "unknown" profile $x \sim (0, f)$. To do that you will need to generate random signal μ with i.i.d. components and "smooth" it with b by computing a convolution $b * \mu$ (to be more specific, take the "intenal" part of convolution).

(b) Simulate a measurement

$$y = a * x + \nu, \quad \nu \sim (0, \sigma^2 \delta).$$

Here again take the "intenal" part of convolution.

(c) Produce an estimate of x

$$\widehat{x} = r * y.$$

Again take the "intenal" part of convolution.

4 Illustrate the result of estimation. Show:

(a) The original signal x (a curve with components x_i),

(b) Observation y (a curve with components y_i),

(c) Estimate of x: $\widehat{x}$ (a curve with components $\widehat{x}_i$),

(d) Standard deviations for the estimates $\widehat{x}_i$

$$\sqrt{\mathsf{E}(\widehat{x}_i - x_i)^2} = \sqrt{H}$$

can be illustrated by showing the corresponding "corridor" around $\widehat{x}_i$).

Please do not hesitate to ask questions. For more details please see Lecture 13 (pages 5-14) and the sample code from `http://www.math.umt.edu/golubtsov/BD_2014_S/Prj2_Demo`.