

Big Data Analytics - Spring 2015 - Project 2b

Real Time Signal Processing

In this project you will implement and demonstrate optimal signal processing with a sliding window. Some parts of this project are related to Homework 4. Here are suggested phases of the project.

1 Specifying problem settings:

- (a) Specify main ingredients of your simulated measurement system a, f, σ^2 :
 - i. Possibly asymmetric (which is usually the case when a signal evolves in time) point spread function (influence function) a ,
 - ii. Covariance function f for the signal x :

$$f = \text{Cov}(x) = b * b^*.$$

For that you just need to specify some “smoothing” function b first.

- iii. Variance σ^2 of the independent components of the additive random noise ν .

$$y = a * x + \nu, \quad \nu \sim (0, \sigma^2 \delta).$$

- (b) Choose some (possibly asymmetric) support interval $\Delta = [-T, \tau]$ for the point spread function of the processing algorithm. Here $T \geq 0$ and $\tau \geq 0$ (prediction delay) are parameters.

2 Computing optimal influence function r :

- (a) Compute $p = a * f * a^* + s$ and $q = f * a^*$.
- (b) Construct matrix P .
- (c) Using p and q find r .
- (d) Show graphs for a, f, r , and $r * a$.
- (e) Do that for a range of parameters T and τ and show how estimation precision

$$H(T, \tau) = \mathbf{E}(\hat{x}_i - x_i)^2 = f_0 - \langle q, P^{-1}q \rangle_{\Delta}$$

depends on parameters. To illustrate a function of two variables you can show it, e.g., as a surface or as a pseudocolor image. It might be interesting to indicate contour lines (curve along which the function has a constant value).

(f) Using this graph determine a “good” choice for parameters T, τ .

3 Simulation of Measurement and Processing:

(a) Randomly generate some “unknown” profile $x \sim (0, f)$. To do that you will need to generate random signal μ with i.i.d. components and “smooth” it with b by computing a convolution $b * \mu$ (to be more specific, take the “intenal” part of convolution).

(b) Simulate a measurement

$$y = a * x + \nu, \quad \nu \sim (0, \sigma^2 \delta).$$

Here again take the “intenal” part of convolution.

(c) Produce an estimate of x

$$\hat{x} = r * y.$$

Again take the “intenal” part of convolution.

4 Illustrate the result of estimation. Show:

(a) The original signal x (a curve with components x_i),

(b) Observation y (a curve with components y_i),

(c) Estimate of x : \hat{x} (a curve with components \hat{x}_i),

(d) Standard deviations for the estimates \hat{x}_i

$$\sqrt{\mathbf{E}(\hat{x}_i - x_i)^2} = \sqrt{H}$$

can be illustrated by showing the corresponding “corridor” around \hat{x}_i).

Please do not hesitate to ask questions. For more details please see Lecture 13 (pages 5-14) and the sample code from

http://www.math.umd.edu/golubtsov/BD_2014_S/Prj2_Demo.