

# Real Time Signal Processing with Symmetric and Asymmetric Support Intervals

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# Outline

Introduction and Motivation

Problem Outline

Problem Approach and Steps

Code Comments

Results and Conclusions

# Importance of Real Time Signal Processing

What is real time signal processing?

- ▶ Applications

- ▶ Speech recognition
- ▶ Audio signal processing
- ▶ Video compression
- ▶ Weather forecasting
- ▶ Economic forecasting
- ▶ Medical imaging (e.g., CAT, MRI)
- ▶ And more...

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# What is the problem?

**Goal:** We wish to reconstruct some generated signal  $\hat{x}$  that has been distorted by some error and convolution processes.

**Solution:** Take the convolution inverse of  $\hat{x}$  to reconstruct the signal.

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# Problem Simulation

Specify the main ingredients of simulated measurement system:

- ▶ Finitely supported point spread function (influence function),
  - ▶ Symmetric case:  $a_i = \frac{1}{10}$  for  $|i| < 15$ .
  - ▶ Asymmetric case:  $a_i = \frac{2}{10}e^{-i/40}$  for  $0 \leq i \leq 40$ .
- ▶ The covariance function  $\phi$  for the signal  $x$  is given by

$$\phi = \text{Cov}(x) = b * b^*$$

where  $b_i = \frac{21}{100}(1 - |i|)$  for  $|i| \leq 7$ .

- ▶ Measurement noise is modeled with a zero mean Gaussian  $\nu$  with a specified  $\sigma^2 = \frac{1}{100}$ .
- ▶ Finally, the data is given by

$$y = a * x + \nu$$

# Reconstruction Operator $R$

- ▶ Following Lecture 13, we seek a reconstruction operator  $R$  that is given by convolution with  $r$  supported on a specified interval  $\Delta$ , so that  $\hat{x} = r * x$ .
- ▶ Further, it was shown that

$$H(r) = E(\hat{x} - x)^2 = \langle P(r - P^{-1}q), r - P^{-1}q \rangle_{\Delta} + f_0 - \langle q, P^{-1}q \rangle_{\Delta}$$

where  $P$  is the operator associated with convolution by  $p = a * \phi * a^* + \sigma^2 \delta$  and  $q = a * \phi$ .

- ▶ So, for a given  $\Delta$ , the reconstruction kernel is uniquely determined by  $r = P^{-1}q$ , and

$$\text{Var } \hat{x} = H_{\min} = f_0 - \langle q, r \rangle_{\Delta}.$$



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# References

- [1] Golubtsov, P. (2015). Theoretical Big Data Analytics course notes.