Real Time Signal Processing with Symmetric and Asymmetric Support Intervals

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Outline

Importance of Real Time Signal Processing

What is real time signal processing?

- Applications
  - Speech recognition
  - Audio signal processing
  - Video compression
  - Weather forecasting
  - Economic forecasting
  - Medical imagining (e.g., CAT, MRI)
  - And more...

Outline

What is the problem?

Goal: We wish to reconstruct some generated signal $\hat{x}$ that has been distorted by some error and convolution processes.

Solution: Take the convolution inverse of $\hat{x}$ to reconstruct the signal.

Outline

Problem Simulation

Specify the main ingredients of simulated measurement system:

- Finitely supported point spread function (influence function),
  - Symmetric case: $a_i = \frac{1}{10}$ for $|i| < 15$.
  - Asymmetric case: $a_i = \frac{2}{10} e^{-i/40}$ for $0 \leq i \leq 40$.
- The covariance function ϕ for the signal x is given by

  $$\phi = \mathrm{Cov}\,(x) = b * b^*$$

  where $b_i = \frac{21}{100}(1 - |i|)$ for $|i| \leq 7$.
- Measurement noise is modeled with a zero mean Gaussian ν with a specified $\sigma^2 = \frac{1}{100}$.
- Finally, the data is given by

  $$y = a * x + \nu$$

Reconstruction Operator R

- Following Lecture 13, we seek a reconstruction operator R that is given by convolution with r supported on a specified interval Δ, so that $\widehat{x} = r * x$.
- Further, it was shown that

$$H(r) = E(\widehat{x} - x)^2 = \langle P(r - P^{-1}q), r - P^{-1}q \rangle_\Delta + f_0 - \langle q, P^{-1}q \rangle_\Delta$$

  where P is the operator associated with convolution by $p = a * \phi * a^* + \sigma^2 \delta$ and $q = a * \phi$.
- So, for a given Δ, the reconstruction kernel is uniquely determined by $r = P^{-1}q$, and

$$\mathrm{Var}\,\widehat{x} = H_{min} = f_0 - \langle q, r \rangle_\Delta.$$

Outline

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References

[1] Golubtsov, P. (2015). Theoretical Big Data Analytics course notes.