**Simulation results for evaluating the performance of the protection routings on Qn and LTQn**

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**Graph automorphism**

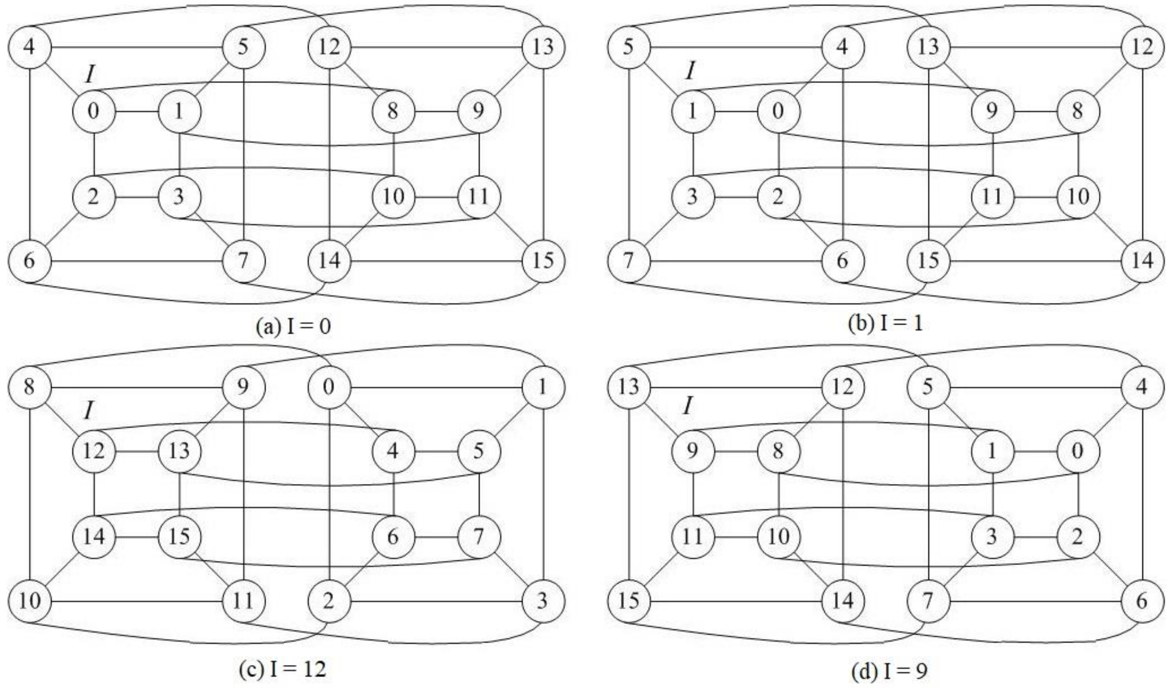
From Wikipedia,

In the mathematical field of graph theory, an automorphism of a graph is a form of symmetry in which the graph is mapped onto itself while preserving the edgevertex connectivity. Formally, an automorphism of a graph G = (V, E) is a permutation of the vertex set V, such that the pair of vertices (u, v) form an edge if and only if the pair (f(u), f(v)) also form an edge. That is, it is a graph isomorphism from G to itself.

**The full automorphism groups of Q_n**

Since Q_n is vertex-symmetric and there exists an automorphism y = f(x, I) = x xor I, where I is the identity node.

When I = 0, a permutation ( 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15) in Q4
When I = 1, a permutation ( 1, 0, 3, 2, 5, 4, 7, 6, 9, 8,11,10,13,12,15,14) in Q4
When I = 3, a permutation ( 2, 3, 0, 1, 6, 7, 4, 5,10,11, 8, 9,14,15,12,13) in Q4
When I = 4, a permutation ( 3, 2, 1, 0, 7, 6, 5, 4,11,10, 9, 8,15,14,13,12) in Q4
When I = 5, a permutation ( 4, 5, 6, 7, 0, 1, 2, 3,12,13,14,15, 8, 9,10,11) in Q4
When I = 6, a permutation ( 5, 4, 7, 6, 1, 0, 3, 2,13,12,15,14, 9, 8,11,10) in Q4
When I = 7, a permutation ( 6, 7, 4, 5, 2, 3, 0, 1,14,15,12,13,10,11, 8, 9) in Q4
When I = 8, a permutation ( 7, 6, 5, 4, 3, 2, 1, 0,15,14,13,12,11,10, 9, 8) in Q4
When I = 9, a permutation ( 8, 9,10,11,12,13,14,15, 0, 1, 2, 3, 4, 5, 6, 7) in Q4
When I =10, a permutation ( 9, 8,11,10,13,12,15,14, 1, 0, 3, 2, 5, 4, 7, 6) in Q4
When I =11, a permutation (10,11, 8, 9,14,15,12,13, 2, 3, 0, 1, 6, 7, 4, 5) in Q4
When I =12, a permutation (11,10, 9, 8,15,14,13,12, 3, 2, 1, 0, 7, 6, 5, 4) in Q4
When I =13, a permutation (12,13,14,15, 8, 9,10,11, 4, 5, 6, 7, 0, 1, 2, 3) in Q4
When I =14, a permutation (13,12,15,14, 9, 8,11,10, 5, 4, 7, 6, 1, 0, 3, 2) in Q4
When I =15, a permutation (14,15,12,13,10,11, 8, 9, 6, 7, 4, 5, 2, 3, 0, 1) in Q4
When I =16, a permutation (15,14,13,12,11,10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0) in Q4
For example, Figure 1 (a), (b), (c), (d) show Q4 with indentity node I = 0, 1, 12, 9, respectively.




# Figure 1.

**The full automorphism groups of LTQ_n**

The full automorphism group of LTQn with n >= 4 has exactly two orbits, and the odd and even vertices belong to the same group.

While identity node I is even, there exists an automorphism y = f(x, I) = x xor I.

When I = 0, a permutation ( 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15) in LTQ4

When I = 2, a permutation ( 2, 3, 0, 1, 6, 7, 4, 5,10,11, 8, 9,14,15,12,13) in LTQ4

When I = 4, a permutation ( 4, 5, 6, 7, 0, 1, 2, 3,12,13,14,15, 8, 9,10,11) in LTQ4

When I = 6, a permutation ( 6, 7, 4, 5, 2, 3, 0, 1,14,15,12,13,10,11, 8, 9) in LTQ4

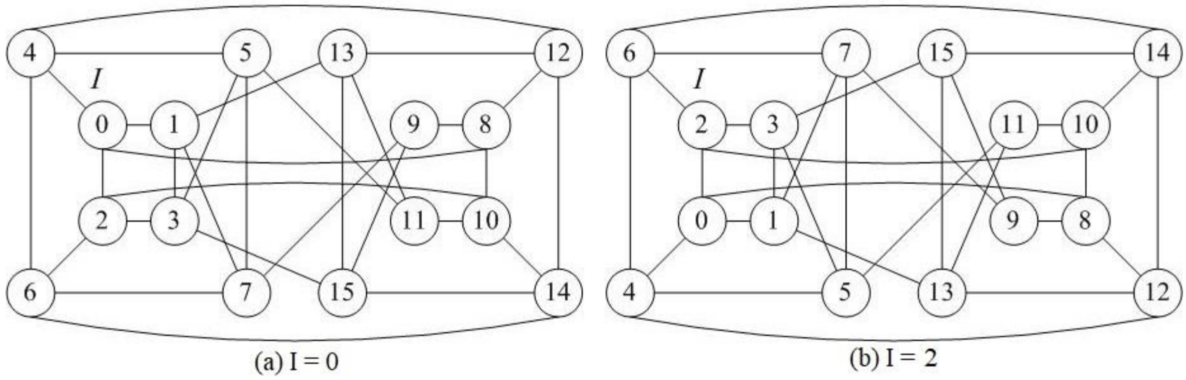
When I = 8, a permutation ( 8, 9,10,11,12,13,14,15, 0, 1, 2, 3, 4, 5, 6, 7) in LTQ4

When I =10, a permutation (10,11, 8, 9,14,15,12,13, 2, 3, 0, 1, 6, 7, 4, 5) in LTQ4

When I =12, a permutation (12,13,14,15, 8, 9,10,11, 4, 5, 6, 7, 0, 1, 2, 3) in LTQ4

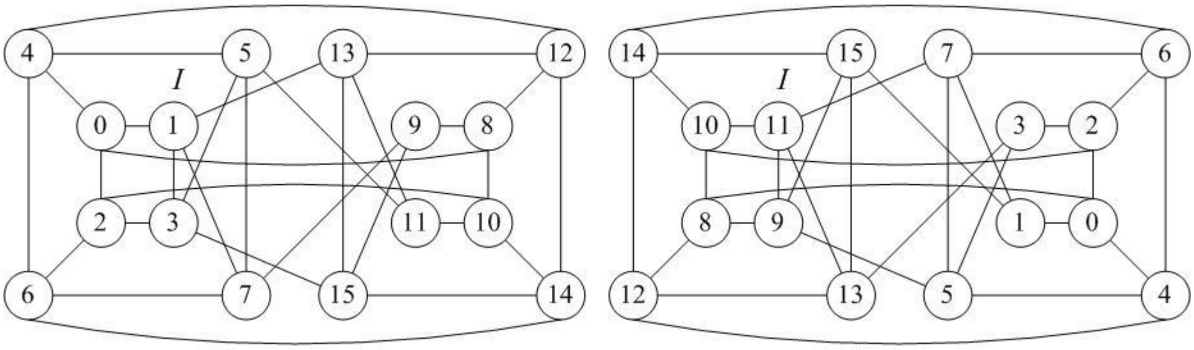
When I =14, a permutation (14,15,12,13,10,11, 8, 9, 6, 7, 4, 5, 2, 3, 0, 1) in LTQ4

For example, Figure 2 (a), (b) show LTQ4 with indentity node I = 0, 2, respectively.




# Figure 2.

While identity node I is odd, there exists an automorphism y = f(x, I) = x xor I.
When I = 1, a permutation ( 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15, 0) in LTQ4
When I = 3, a permutation ( 3, 0, 1, 6, 7, 4, 5,10,11, 8, 9,14,15,12,13, 2) in LTQ4
When I = 5, a permutation ( 5, 6, 7, 0, 1, 2, 3,12,13,14,15, 8, 9,10,11, 4) in LTQ4
When I = 7, a permutation ( 7, 4, 5, 2, 3, 0, 1,14,15,12,13,10,11, 8, 9, 6) in LTQ4
When I = 9, a permutation ( 9,10,11,12,13,14,15, 0, 1, 2, 3, 4, 5, 6, 7, 8) in LTQ4
When I = 11, a permutation (11, 8, 9,14,15,12,13, 2, 3, 0, 1, 6, 7, 4, 5,10) in LTQ4
When I = 13, a permutation (13,14,15, 8, 9,10,11, 4, 5, 6, 7, 0, 1, 2, 3,12) in LTQ4
When I = 15, a permutation (15,12,13,10,11, 8, 9, 6, 7, 4, 5, 2, 3, 0, 1,14) in LTQ4
For example, Figure 3 (a), (b) show LTQ4 with indentity node I = 1, 11, respectively.



(a) I = 1 (b) I = 11

# Figure 3.

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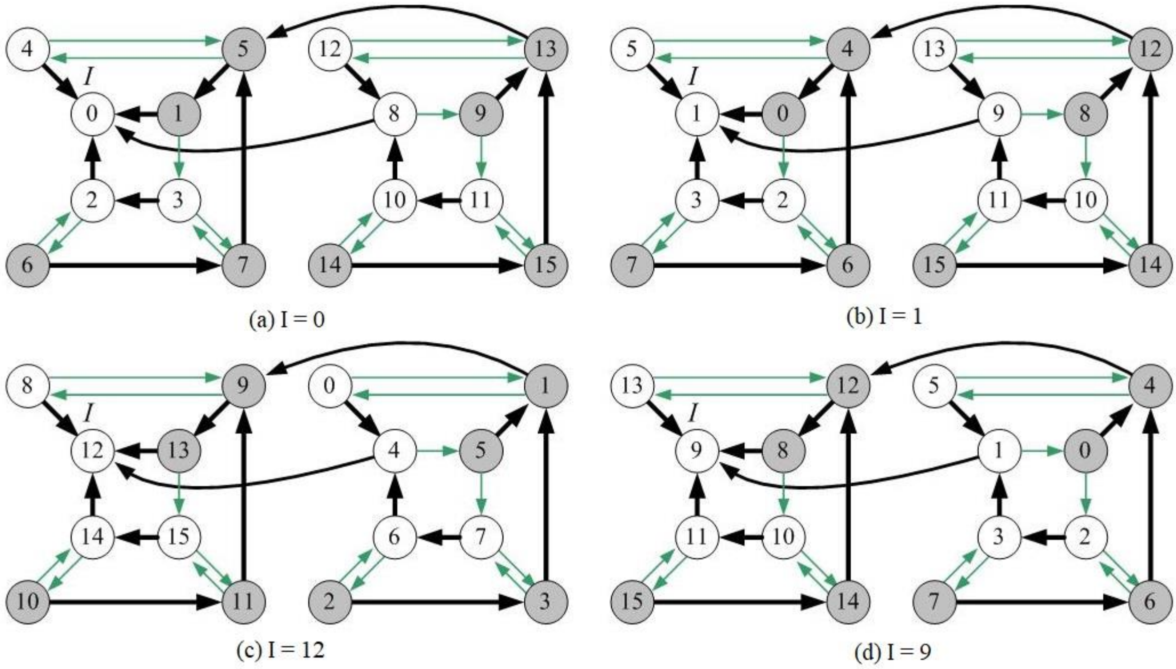
**Simulation on hypercube Qn**

Descriptions:

1. In Q_n, 3 <= n <= 9
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure

Example 1, instances (13, 0, 1), see Figure 4 (a),
(i) no failure, 13 -> 5 -> 1 -> 0
(ii) a single node failure, 13 -> 5 -> 1 fails, go to SNH 4 -> 0
Example 2, instances (13, 1, 9), see Figure 4 (b),
(i) no failure, 13 -> 9 -> 1
(ii) a single node failure, 13 -> 9 fails, go to SNH 12 -> 4 -> 0 -> 1
Example 3, instances (2, 12, 1), see Figure 4 (c),
(i) no failure, 2 -> 3 -> 1 -> 9 -> 13 -> 12
(ii) a single node failure, 2 -> 3 -> 1 fails, go to SNH 7 -> 6 -> 4 -> 12

Example 4, instances (4, 9, 12), see Figure 4 (d),
(i) no failure, 4 -> 12 -> 8 -> 9
(ii) a single node failure, 4 -> 12 fails, go to SNH 5 -> 1 -> 9

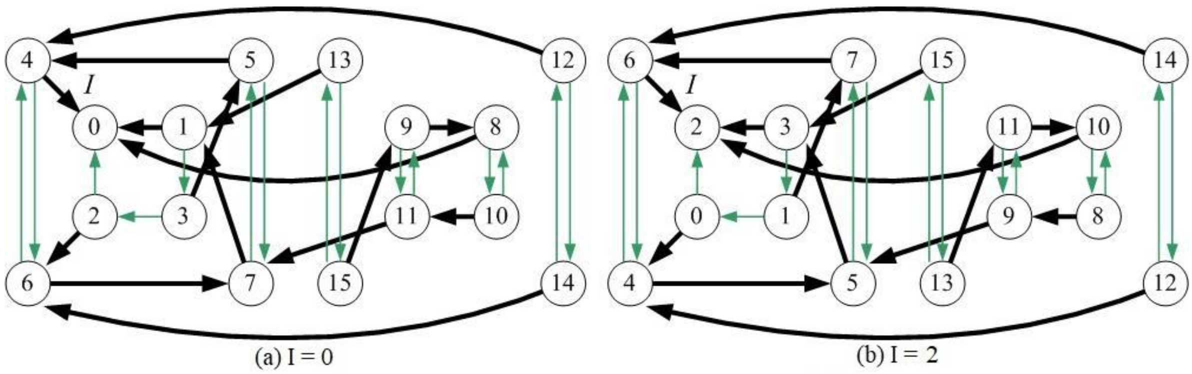



# Figure 4.

**Simulation on locally twisted cube LTQn**

Descriptions:
1. In LTQ_n, 3 <= n <= 9
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure

Example 1, instances (13, 0, 1), see Figure 5 (a),
(i) no failure, 13 -> 1 -> 0
(ii) a single node failure, 13 -> 1 fails, go to SNH 15 -> 9 -> 8 -> 0
Example 2, instances (8, 2, 3), see Figure 5 (b),
(i) no failure, 8 -> 9 -> 5 -> 3 -> 2
(ii) a single node failure, 8 -> 9 -> 5 -> 3 fails, go to SNH 7 -> 6 -> 2




## Figure 5.

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Simulation 1.

Descriptions:
1. In Q_3( 8 nodes, 12 links)
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure
4. Calculate three statistical quantities: the average path length (Avg.), the standard deviation of path length (Std), and the maximum path length and the number of its occurrences (Max.)
5. Calculate the number of times the fault falls into (s, d)

Simulation results by dual-PRTs: (i) Avg. = 2.000351 , Std = 0.925494 , Max. = 4 ( 142789 times); (ii) Avg. = 2.000187 , Std = 0.925361 , Max. = 4 ( 142707 times), the fault falls into (s, d)=166885 times (16.69%)

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Simulation 2.

Descriptions:
1. In Q_4( 16 nodes, 32 links)
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure
4. Calculate three statistical quantities: the average path length (Avg.), the standard deviation of path length (Std), and the maximum path length and the number of its occurrences (Max.)
5. Calculate the number of times the fault falls into (s, d)

Simulation results by dual-PRTs: (i) Avg. = 2.532640 , Std = 1.155438 , Max. = 5 ( 66970 times); (ii) Avg. = 2.580110 , Std = 1.191283 , Max. = 7 ( 4661 times), the fault falls into (s, d)=109839 times (10.98%)
Simulation results by 2 CISTs: (i) Avg. = 3.232948 , Std = 1.549985 , Max. = 7 ( 33326 times); (ii) Avg. = 3.393706 , Std = 1.851529 , Max. = 11 ( 2697 times), the fault falls into (s, d)=159376 times (15.94%)

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Simulation 3.

Descriptions:
1. In Q_5( 32 nodes, 80 links)
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure
4. Calculate three statistical quantities: the average path length (Avg.), the standard deviation of path length (Std), and the maximum path length and the number of its occurrences (Max.)
5. Calculate the number of times the fault falls into (s, d)

Simulation results by dual-PRTs: (i) Avg. = 3.031487 , Std = 1.233263 , Max. = 6 ( 32248 times); (ii) Avg. = 3.081363 , Std = 1.295897 , Max. = 8 ( 2178 times), the fault falls into (s, d)=67663 times (6.77%)
Simulation results by 2 CISTs: (i) Avg. = 4.271551 , Std = 1.776452 , Max. = 8 ( 32352 times); (ii) Avg. = 4.444931 , Std = 2.058815 , Max. = 12 ( 2058 times), the fault falls into (s, d)=109167 times (10.92%)

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Simulation 4.

Descriptions:
1. In Q_6( 64 nodes, 192 links)
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure
4. Calculate three statistical quantities: the average path length (Avg.), the standard deviation of path length (St$ length and the number of its occurrences (Max.)
5. Calculate the number of times the fault falls into (s, d)

Simulation results by dual-PRTs: (i) Avg. = 3.523229 , Std = 1.391281 , Max. = 7

( 15859 Times) (ii) Avg. = 3.561869 , Std = 1.400612 , Max. = 9 ( 766 Times), the fault falls into (s, d)= 40797 times (4.08%)
Simulation results by 2 CISTs: (i) Avg. = 5.277875 , Std = 1.941123 , Max. = 10 ( 8024 Times) (ii) Avg. = 5.417123 , Std = 2.156235 , Max. = 14 ( 503 Times), the fault falls into (s, d)= 69080 times (6.91% )

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Simulation 5.

Descriptions:
1. In Q_7( 128 nodes, 448 links)
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure
4. Calculate three statistical quantities: the average path length (Avg.), the standard deviation of path length (St$ length and the number of its occurrences (Max.)
5. Calculate the number of times the fault falls into (s, d)

Simulation results by dual-PRTs: (i) Avg. = 4.016004 , Std = 1.438708 , Max. = 8 ( 7940 Times) (ii) Avg. = 4.042182 , Std = 1.466437 , Max. = 10 ( 248 Times), the fault falls into (s, d)= 23983 times (2.40% )
Simulation results by 2 CISTs: (i) Avg. = 6.270518 , Std = 2.077178 , Max. = 12 ( 2038 Times) (ii) Avg. = 6.369486 , Std = 2.205245 , Max. = 16 ( 81 Times), the fault falls into (s, d)= 41668 times (4.17% )

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Simulation 6.

Descriptions:
1. In Q_8( 256 nodes, 1024 links)
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure
4. Calculate three statistical quantities: the average path length (Avg.), the standard deviation of path length (St$ length and the number of its occurrences (Max.)
5. Calculate the number of times the fault falls into (s, d)

Simulation results by dual-PRTs: (i) Avg. = 4.510443 , Std = 1.572223 , Max. = 9 ( 3820 Times) (ii) Avg. = 4.526833 , Std = 1.589101 , Max. = 11 ( 72 Times), the fault falls into (s, d)= 13722 times (1.37% )

Simulation results by 2 CISTs: (i) Avg. = 7.263877 , Std = 2.233631 , Max. = 14 ( 478 Times) (ii) Avg. = 7.327195 , Std = 2.294374 , Max. = 18 ( 10 Times), the fault falls into (s, d)= 24573 times (2.46% )

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Simulation 7.

Descriptions:
1. In Q_9( 512 nodes, 2304 links)
2. Randomly generate 1,000,000 instances of node lists (source s, destination d, failure node f) and s != d != f
3. For each instances, calculate the path length of (s, d, f) under
   (i) no failure
   (ii) a single node failure
4. Calculate three statistical quantities: the average path length (Avg.), the standard deviation of path length (St$ length and the number of its occurrences (Max.)
5. Calculate the number of times the fault falls into (s, d)

Simulation results by dual-PRTs: (i) Avg. = 5.008286 , Std = 1.608203 , Max. = 10 ( 2017 Times) (ii) Avg. = 5.018454 , Std = 1.617714 , Max. = 12 ( 29 Times), the fault falls into (s, d)= 7718 times (0.77% )
Simulation results by 2 CISTs: (i) Avg. = 8.257116 , Std = 2.360037 , Max. = 16 ( 127 Times) (ii) Avg. = 8.296446 , Std = 2.369716 , Max. = 20 ( 2 Times), the fault falls into (s, d)= 14191 times (1.42% )

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Table 1. Simulation results: 3 length quantities on Qn when routing is in Scenario 1 (no fault).

| | two CISTs | | | dual-PRTs | | |
|---|---|---|---|---|---|---|
| | Avg. | Std | Max. | Avg. | Std | Max. |
| Q3 | | | | 2.000351 | 0.925494 | 4 (142789 times) |
| Q4 | 3.232948 | 1.549985 | 7 (33326 times) | 2.53264 | 1.155438 | 5 (66970 times) |
| Q5 | 4.271551 | 1.776452 | 8 (32352 times) | 3.031487 | 1.233263 | 5 (32248 times) |
| Q6 | 5.277875 | 1.941123 | 10 (8024 times) | 3.523229 | 1.391281 | 7 (15859 times) |
| Q7 | 6.270518 | 2.077178 | 12 (2038 times) | 4.016004 | 1.438708 | 8 (7940 times) |
| Q8 | 7.263877 | 2.233631 | 14 (478 times) | 4.510443 | 1.572223 | 9 (3820 times) |
| Q9 | 8.257116 | 2.360037 | 16 (127 times) | 5.008286 | 1.608203 | 10 (2017 times) |

2. Simulation results: 3 length quantities on Qn when routing is in Scenario 2 (a single node fault).

| | two CISTs | | | dual-PRTs | | |
|---|---|---|---|---|---|---|
| | Avg. | Std | Max. | Avg. | Std | Max. |
| Q3 | | | | 2.000187 | 0.925361 | 4 (142707 times) |
| Q4 | 3.393706 | 1.851529 | 11 (2697 times) | 2.58011 | 1.191283 | 7 (4661 times) |

| | | | | | | |
|---|---|---|---|---|---|---|
| Q5 | 4.444931 | 2.058815 | 12 (2058 times) | 3.081363 | 1.295897 | 8 (2178 times) |
| Q6 | 5.417123 | 2.156235 | 14 (503 times) | 3.561869 | 1.400612 | 9 (766 times) |
| Q7 | 6.369486 | 2.205245 | 16 (81 times) | 4.042182 | 1.466437 | 10 (248 times) |
| Q8 | 7.327195 | 2.294374 | 18 (10 times) | 4.526833 | 1.589101 | 11 (72 times) |
| Q9 | 8.296446 | 2.369716 | 20 (2 times) | 5.018454 | 1.617714 | 12 (29 times) |

Table 3. Simulation results: 3 length quantities on LTQn when routing is in Scenario 1 (no fault).

| | two CISTs | | | dual-PRTs | | |
|---|---|---|---|---|---|---|
| | Avg. | Std | Max. | Avg. | Std | Max. |
| LTQ3 | | | | 2.285303 | 0.755827 | 4 (142789 times) |
| LTQ4 | 2.931516 | 1.19693 | 6 (24848 times) | 2.600489 | 0.949539 | 5 (66813 times) |
| LTQ5 | 3.869399 | 1.526712 | 8 (7999 times) | 3.289709 | 1.396974 | 7 (32254 times) |
| LTQ6 | 4.825682 | 1.734439 | 10 (1970 times) | 3.87026 | 1.581364 | 8 (15864 times) |
| LTQ7 | 5.793951 | 1.915318 | 12 (478 times) | 4.411415 | 1.637678 | 9 (7920 times) |
| LTQ8 | 6.774469 | 2.066624 | 14 (122 times) | 4.925867 | 1.757221 | 10 (3920 times) |
| LTQ9 | 7.765519 | 2.203155 | 16 (31 times) | 5.432122 | 1.829297 | 11 (1879 times) |

Table 4. Simulation results: 3 length quantities on LTQn when routing is in Scenario 2 (a single node fault)

| | two CISTs | | | dual-PRTs | | |
|---|---|---|---|---|---|---|
| | Avg. | Std | Max. | Avg. | Std | Max. |
| LTQ3 | | | | 2.236596 | 0.91194 | 5 (23620 times) |
| LTQ4 | 3.026907 | 1.379224 | 9 (582 times) | 2.636714 | 1.010564 | 6 (4782 times) |
| LTQ5 | 3.98309 | 1.670772 | 11 (133 times) | 3.325073 | 1.443063 | 10 (538 times) |
| LTQ6 | 4.913578 | 1.84209 | 13 (12 times) | 3.902035 | 1.615225 | 11 (392 times) |
| LTQ7 | 5.852506 | 1.978876 | 15 (1 times) | 4.434017 | 1.711811 | 12 (165 times) |
| LTQ8 | 6.810781 | 2.101302 | 16 (9 times) | 4.941104 | 1.774578 | 13 (51 times) |
| LTQ9 | 7.787632 | 2.221917 | 18 (1 times) | 5.441689 | 1.83916 | 14 (14 times) |