What is so Weird About Quantum Mechanics?

Kevin Player*

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> I would not call [entanglement] **one** but rather **the** characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.
>
> Erwin Schrödinger

Abstract

We present several quintessential quantum ideas and shed them in a classical light. The ideas presented all have analogues in classical information theory. For instance, entanglement is presented as a generalization of classical correlation. We argue how quantum information theory should be understood as a generalization of classical information theory. In fact, classical information theory is embedded in quantum information as positive zero phase wavefunctions.

1 The Weirdo List

If you ask a physicist on the street what kinds of things are uniquely quantum mechanical they might just pick an item from this list

1. entanglement
2. wave function collapse
3. the measurement problem
4. tensor product of ensembles

Entanglement is surely a uniquely quantum item with no classical analog. The wave function collapsing is according to Penrose [1], an unsettlingly nonlinear mystery. The measurement problem is aptly named. And unlike the classical ensembles, 8 qbits require 256 dimensions from a tensor product, not just 8.

2 Some Thoughts

With 8 coins in hand, we start some thought experiments.

*kjplaye@gmail.com

2.1 Configurations

Let us try to describe the situation once we throw the 8 coins. Let

$$x_i \in \{T, F\}$$

be the outcome for the throws $i = 1, \cdots, 8$. We have 8 "dimensions" worth of information to describe the 8 coins. Lets move on and generalize.

2.2 Classical Information Theory

2.2.1 Tensor Product for an Ensamble

We want to describe a general knowledge statement about the coins. This will actually involve more than 8 dimensions. We let

$$p_0, \cdots, p_{255} \in \mathbb{R}^+$$

with $\sum p_i = 1$. Each p_i is the probability of the coins being in a binary state given by i. For instance p_{71} is the probability of seeing $FTFFFTTT$. Notice that we have not moved to quantum information theory, but we are already motivated to use 256 dimensions instead of 8. This can be thought of as a tensor product. So cross "tensor product of ensembles" off the weirdness list.

2.2.2 Bayesian Projections as Classical Wave Function Collapse

The measurement of classical information is decidedly not weird. We don't in fact have to measure the exact configuration. For instance, we might just get to know that the first coin is T and that the 2nd and 3rd coins are the same. This cooresponds to a subset of $\{0, \cdots, 255\}$ which we will call S.

$$S = \{s \in \{0, \cdots, 255\} | b_1(s) = T, b_2(s) = b_3(s)\}$$

where $b_i(s)$ is the value x_i. Someone could measure this by being told that the state was in S.

2.2.3 Bayesian Inference as General Classical Wave Function Collapse

A more general type measurement is a probabilistic measurement. Someone could be told that there is a 95% chance that the state is in S. In full generality we will call such a probablistic observation $\mathcal{O}$. We can figure out how to update our knowledge statement, p_i, using a relative[1] version of Bayes's rule

$$\frac{\hat{p_i}}{\hat{p_j}} = \underbrace{\frac{P(i|\mathcal{O})}{P(j|\mathcal{O})}}_{\text{Posterior}} = \underbrace{\frac{P(\mathcal{O}|i)}{P(\mathcal{O}|j)}}_{\text{Bayes Factor}} \underbrace{\frac{p_j}{p_j}}_{\text{Prior}}$$

Pulling out the Bayes factor we find that we just multiply by the likelihood and renormalize.

$$\hat{p_i} = P(\mathcal{O}|i)\ p_i$$

A special case are projections $P(\mathcal{O}|i) \in \{0, 1\}$, like the 100% S case above. We call these Bayesian projections.

[1] Here the $P(\mathcal{O})$ cancels out and is wrapped up in the normalization. This omition reflects its non-physicality.

2.2.4 Classical Correlation as Classical Entanglement

Finally, we can also consider distributions such as the two coin distribution

$$p_i = \begin{cases} \frac{1}{2} \text{ if } i \in \{0,3\} \\ 0 \text{ else} \end{cases}$$

Looking at just the first two coins, we have FF or TT with equal probabilities. If I give the first coin to Alice and the second coin to Bob then we have a classical correlation. If Bob finds that the first coin is T then we know that Alice will also find T. This should all seem very reasonable; classical correlation is not weird.

2.3 Quantum Information Theory

General quantum information[2] about 8-qbits can be expressed as a wave function.

$$q_0, \cdots, q_{255} \in \mathbb{C}$$

with $\sum |q_i|^2 = 1$. The Born rule is ostensibly a map $q_i \to |q_i|^2 = p_i$ to classical probability. Here we can instrument all of the classical information theoretic constructs by restricting the phase and positivity of q_i. For instance, we can map backward $p_i \to \sqrt{p_i} = q_i$. So quantum information theory needs to generalize the classical. In particular, quantum measurement and entanglement restrict to classical measurement and classical correlation for zero phase and positivity. [3]

An example of this is to define a projection operator π which projects onto the space generated by the subset S, from the previous subsection.

$$\pi = \sum_{i \in S} |i\rangle \langle i|$$

Then to let A be any operator with an eigenspace, with eigenvalue λ, equal to the image of π. Then an observation of λ would correspond to an application of π, which is Bayesian projection in the classical picture.

Finally, consider, in the fashion of [2], a more general measurement with matrix $M_{\mathcal{O}}$ which is diagonal with entries.

$$M_{\mathcal{O}}(i,i) = \sqrt{P(\mathcal{O}|i)}$$

This implements classical Bayesian inference in the quantum realm.

3 Conclusion

It is the author's belief that the quantum mechanical concepts mentioned in section 1 are a direct generalization of their classical information theoretic analogs. This is not to say that they are not weird, but to say that they need to be generalizations of the classical concepts. Wave function collapsing should generalize Bayesian inference[4] and classical measurement. Entanglement should be a generalization of classical correlation. These ideas have direct classical motivations, only after acknowledging this can we appreciate the weirdness that remains.

[2]We are purposely leaving out density matrices and POVMs.

[3]Von Neumann entropy is left out of this note since it is not a generalization of classical entropy in the manner presented in this note. This is because the zero phase classical wavefunctions are pure states which all have zero Von Neumann entropy.

[4]QBism may be invoked here.

4 Acknowledgments

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References

[1] R. Penrose, "The road to reality: a complete guide to the laws of the universe," 2007.

[2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition.* Cambridge University Press, 2011.