Multi-Observer Quantum Mechanics from Classical Bayesian Inference

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> Solipsism may be logically consistent with present Quantum Mechanics, Monism in the sense of Materialism is not.
>
> Eugene Wigner

Abstract

We present several quintessential quantum ideas and shed them in a classical light. We argue how quantum information theory can be understood as a generalization of classical information theory in a nonstandard way (without density matrices). In fact, we show how classical information theory can be embedded in quantum information theory using zero phase wavefunctions. We employ this embedding to motivate a new multi-observer extension of quantum mechanics. Finally, we outline an experiment to test the existence of our multi-observer theory.

1 Some Thought Experiments

Lets start with a 3-bit example, where each bit is realized by coins which are heads (H) or tails (T). With these 3 coins in hand, we conduct some thought experiments.

1.1 Classical Configurations

Let us try to describe the situation once we throw the 3 coins. Let

$$x_i \in \{H, T\}$$

be the outcome for the throws $i = 1, 2, 3$ and let $x = (x_1, x_2, x_3)$ denote the resulting configuration sequence. Nominally, we have 3 "dimensions" worth of information to describe the 3 coins. Lets move on and generalize using classical statistics.

1.2 Classical Information Theory

In the following subsections we outline an epistemic treatment of classical information where we try to represent our classical knowledge of the 3 coin ensemble. We want to describe a classical probabilistic knowledge statement[1] about the coins. We will construct

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[1]Whenever we say "statement" in this paper, we mean knowledge statement, or Bayesian belief statement.

a general such statement using basic probabilistic statements and convex superpositions from classical statistics.

For any configuration x we have the knowledge statement b_x, that says we know x with probability 1. We call this a basic statement. There are 8 such configurations, so there are 8 basic statements.

A classical convex superposition can be made for any probabilistic statements s_1 and s_2 and $\alpha \in [0,1]$

$$\text{super}_\alpha(s_1, s_2) = \alpha s_1 + (1-\alpha)s_2. \tag{1}$$

This formal combination means to take statement s_1 with probability α and statement s_2 with probability $(1-\alpha)$ – This is a new statement. Repeatedly applying superpositions starting with the basic statements and then also to the resulting sub-statements allows us to consider knowledge statements as general probability distributions on the ensemble of coins. This is our starting point for the next subsection.

1.2.1 Distributions on the Ensemble

In the previous subsection we found that the general knowledge statements about 3 coins are covered by an 8 dimensional space, X_3, spanned by the 8 basic statements b_x. We identify the 8 configuration sequences x with the integers $i = 0, \cdots, 7$ by identifying i with its binary expansion and using $0 \to H$ and $1 \to T$. For instance $5 \to THT$. Then for a general knowledge statement, the basic statements $b_0, \cdots, b_7$ have coefficients p_i with

$$p_0, \cdots, p_7 \in \mathbb{R}^+$$

normalized with $\sum p_i = 1$. Each p_i is the probability of the coins being in a configuration state given by the binary expansion of i. For instance p_5 is the probability of seeing THT. Note that now (1) is not just formally true but actually true inside of X_3.

We also pause to note that each coin corresponds to a 2 dimensional space $X = \langle H, T\rangle$ spanned by formal vectors H and T. The 8 dimensional space $X^{\otimes 3}$ has a basis with formal vectors like $T \otimes H \otimes T$ or THT. Thus, the space of classical knowledge statements of the ensemble, X_3, is naturally identified with the tensor power $X^{\otimes 3}$. In general the classical knowledge statements about n bit ensembles naturally live in $X^{\otimes n}$. This generalizes to wavefunctions for n qubits in quantum mechanics.

1.2.2 Bayesian Projections as Classical Wave Function Collapse

We don't always get to measure the exact configuration. For instance, we might only get to know that the first coin is T and that the 2nd and 3rd coins are the same. This corresponds to an subset of $\{0, \cdots, 7\}$, which we will call

$$\begin{aligned} S &= \{s \in \{0, \cdots, 7\} | c_1(s) = T, c_2(s) = c_3(s)\} \\ &= \{4, 7\} \qquad \text{(THH, TTT)} \end{aligned}$$

where $c_i(s)$ is the value x_i. We will use S as an example in the remaining text.

Someone could make a measurement that tells them that the state is in S and nothing else. This would be a specific case of a projection measurement. The statement becomes zero exactly on p_i where $i \notin S$ and just renormalizes p_i on the remaining $i \in S$. It is exactly

$$(p_0, \cdots, p_7) = \left(0, 0, 0, 0, \frac{1}{2}, 0, 0, \frac{1}{2}\right) \tag{2}$$

1.2.3 Bayesian Inference as General Classical Wave Function Collapse

A more general type of measurement is a probabilistic measurement. Someone could learn that there is a 95% chance that the state is in S. In full generality we will call such a

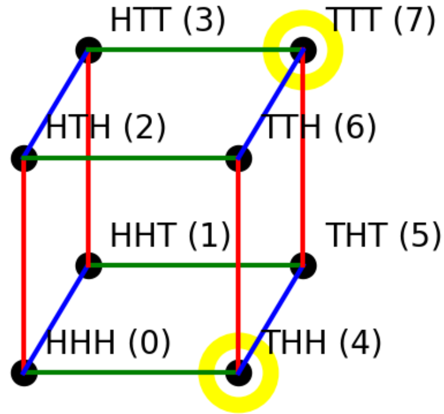

Figure 1: Coin configurations with S circled in yellow.

probabilistic observation $\mathcal{O}$. We can figure out how to update our knowledge statement from p_i, to $\hat{p}_i$, using a relative[2] version of Bayes's rule

$$\frac{\hat{p}_i}{\hat{p}_j} = \underbrace{\frac{P(i|\mathcal{O})}{P(j|\mathcal{O})}}_{\text{Posterior}} = \underbrace{\frac{P(\mathcal{O}|i)}{P(\mathcal{O}|j)}}_{\text{Bayes Factor}} \underbrace{\frac{p_i}{p_j}}_{\text{Prior}}$$

Pulling out the Bayes factor we find that we just multiply by the likelihood and re-normalize

$$\hat{p_k} = P(\mathcal{O}|k)\ p_k.$$

A special case are projections $P(\mathcal{O}|k) \in \{0, v\}$, for some fixed value v, like the S projection case in section 1.2.2. We call these Bayesian projections.

1.2.4 Classical Correlation as Classical Entanglement

Finally, we can also consider again the distribution (2). We know the first coin is T, which we promptly throw away.

We have HH or TT with equal probabilities for the second and third coins. If I give the second coin to Alice and the third coin to Bob then we have a classical correlation. If Bob finds that the third coin is T then we know that Alice will also find that the second coin is T, and similarly for F. The reduction is purely epistemic, it is the separate knowledge of Alice and Bob that is changing. Classical correlation should seem very familiar from every day experience.

1.3 Quantum Information Theory

1.3.1 Classical to Quantum Embedding

General quantum information[3] about 8-qubits can be expressed as a wave function.

$$q_0, \cdots, q_7 \in \mathbb{C}$$

with $\sum |q_i|^2 = 1$. The Born rule is ostensibly a map $q_i \to |q_i|^2 = p_i$ to classical probability. Here we can instrument all of the classical information theoretic constructs by restricting the phase[4] of q_i to be zero. We can map backward $p_i \to \sqrt{p_i} = q_i$; which commutes with the Born rule (subject to normalization):

[2] In the ratio, the contribution of $P(\mathcal{O})$ is canceled out and accounted for during normalization. $P(\mathcal{O})$ only holds significance prior to its observation and it doesn't require consideration during the update process.

[3] We are purposely leaving out density matrices and POVMs and just dealing with pure states.

[4] We consider negative values as 180 degrees out of phase, so zero phase means non-negative real.

$$\text{(Quantum)} \qquad \mathbb{C}^n \rightleftarrows (\mathbb{R}_{\geq 0})^n \qquad \text{(Classical)}$$

So quantum information theory needs to generalize the classical. In particular, quantum measurement and entanglement restrict to classical measurement and classical correlation for zero phase.

1.3.2 Bayesian Projection

We illustrate zero phase Bayesian projection with an example.

Within the embedding we define a projection operator π which projects onto the space generated by the subset S, from the previous subsection

$$\pi = \sum_{i \in S} |i\rangle \langle i| .$$

Then let A be any operator with an eigenspace, with eigenvalue λ, equal to the image of π. An observation of λ would correspond to an application of π, which is a Bayesian projection in the classical picture.

1.3.3 Bayesian Inference

Consider, in the fashion of [1], a more general measurement with matrix $M_{\mathcal{O}}$ which is diagonal with entries

$$M_{\mathcal{O}}(i,i) = \sqrt{P(\mathcal{O}|i)}$$

This implements classical Bayesian inference using zero phase wavefunctions.

1.3.4 Entropy

Von Neumann entropy is left out of this note since it is not a generalization of classical entropy in the manner presented here. This is because the zero phase classical wavefunctions are pure states which all have zero Von Neumann entropy.

2 Overview of the Generalization

The quantum mechanical concepts wave function collapse, entanglements and ensembles are direct generalizations of the classical information theoretic concepts of Bayesian inference, correlation and statistical ensembles respectively. This is not to say that they are not strange, but to say that they need to be generalizations of the classical concepts. Wave function collapsing should generalize Bayesian inference and classical measurement. Entanglement should be a generalization of classical correlation. This is enough to motivate new ways of doing quantum mechanics, which we will encounter in the next sections.

2.1 Multi-Observer Quantum Mechanics

We focus on quantum wave function collapse as a generalization of classical Bayesian inference. We can implement the classical Bayesian inference within zero phase quantum mechanics as outlined above. So the classical Bayesian theory has a direct tie in; that observation and measurement occur in tandem with a change in knowledge[5].

[5] This change in knowledge is tangible, it always occurs as a transfer of matter/energy from the environment to the observer [2].

In the classical theory, knowledge is local to the observer[6], multiple observers each have their own knowledge. For instance, Wigner's friend and Wigner each have their own classical knowledge. It would seem then that Wigner's friend and Wigner must have different zero phase wavefunctions as well, if they are to be generalizing classical knowledge. This is our prediction for multi-observer quantum physics, that every observer has a local wavefunction.

2.2 Prediction and Experiment

We predict that multi-observer quantum mechanics will be required for a proper generalization of classical knowledge. We propose an experiment where we inject an "observer" into a quantum eraser[7]. The injected observer will have to be a small apparatus that is

- able to record a measurement ψ of the eraser particle path.
- able to forget the measurement ψ.
- able to demonstrate that a measurement was made with a record ρ.

Let ϕ be the lab technician's wavefunction. In ϕ the apparatus is entangled with the subject particles in the eraser experiment. We need to find out if there is another wavefunction in play that is not just part of ϕ. The key here is that the apparatus is able to demonstrate that it "knew" something, or in other words that another wavefunction ψ existed, using ρ.

The apparatus can not keep ψ for proper erasure, but must "forget" it. Observed erasure, via an interference pattern, proves that ψ is not a part of ϕ. Finally, there should be a record ρ in the apparatus that it did at one point in time record a measurement ψ. Receipt of the report ρ and eraser interference pattern together show that ψ and ϕ were necessarily part of a two-observer system.

3 Acknowledgments

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References

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition.* Cambridge University Press, 2011.

[2] K. Player, "The unruh effect as holographic thrust," 2023.

[6] Note that current quantum theory is a theory of one observer, usually the experimenter in a lab. An immediate subject is quantum key distribution(QKD), which requires at least three observers, Alice, Bob, and Eve; the security of QKD depends on multi-observer quantum ontology.

[7] Here we probe the boundaries of what constitutes an observer and a measurement. We claim that all versions of observer and measurement will be detectable in this experiment whenever it can be done.