Multi-Observer Quantum Mechanics from Classical Bayesian Inference

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> I would not call [entanglement] **one** but rather **the** characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.
>
> Erwin Schrödinger

Abstract

We present several quintessential quantum ideas and shed them in a classical light. The ideas presented all have analogues in classical information theory. For instance, entanglement is presented as a generalization of classical correlation. We argue how quantum information theory should be understood as a generalization of classical information theory in a nonstandard way (without density matrices). In fact, classical information theory is embedded in quantum information theory as zero phase wavefunctions. We employ this embedding to motivate a new multi-observer version of quantum mechanics. Finally, we outline an experiment to test the existence of our multi-observer theory.

1 Quantum Weirdness List

If you ask a physicist on the street what kinds of things are uniquely quantum mechanical they might pick an item from this list

1. entanglement
2. wave function collapse
3. the measurement problem
4. tensor product of ensembles

It might be argued that entanglement is a uniquely quantum phenomenon with no classical instance. The wave function collapsing is, according to Penrose [1], an unsettlingly nonlinear mystery. The measurement problem is aptly named. And unlike the 8-dimensional 8-bit classical ensemble, 8 qbits require 256 dimensions from a tensor product to describe.

2 Some Thoughts

With 8 coins in hand, we conduct some thought experiments.

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2.1 Configurations

Let us try to describe the situation once we throw the 8 coins. Let

$$x_i \in \{T, F\}$$

be the outcome for the throws $i = 1, \cdots, 8$. We have 8 "dimensions" worth of information to describe the 8 coins. Lets move on and generalize.

2.2 Classical Information Theory

We follow a knowledge based or epistemic treatment.

2.2.1 Tensor Product for an Ensemble

We want to describe a general knowledge statement about the coins. This will actually involve more than 8 dimensions. We let

$$p_0, \cdots, p_{255} \in \mathbb{R}^+$$

with $\sum p_i = 1$. Each p_i is the probability of the coins being in a configuration state given by the binary expansion of i. For instance p_{71} is the probability of seeing $FTFFFTTT$. Notice that we have not moved to quantum information theory, but we are already motivated to use 256 dimensions instead of 8. This can be thought of as a tensor product. So cross "tensor product of ensembles" off the weirdness list.

2.2.2 Bayesian Projections as Classical Wave Function Collapse

The measurement of classical information is decidedly not weird, so this section should be familiar to all who observe. We don't always get to measure the exact configuration. For instance, we might only get to know that the first coin is T and that the 2nd and 3rd coins are the same. This corresponds to an exemplar subset of $\{0, \cdots, 255\}$, which we will call

$$S = \{s \in \{0, \cdots, 255\} | b_1(s) = T, b_2(s) = b_3(s)\},$$

where $b_i(s)$ is the value x_i. Someone could make a measurement that tells them that the state is in S and nothing else. This would be a specific case of a projection measurement. The wavefunction becomes zero exactly on p_i where $i \notin S$ and just renormalizes p_i on the remaining $i \in S$.

2.2.3 Bayesian Inference as General Classical Wave Function Collapse

A more general type measurement is a probabilistic measurement. Someone could learn that there is a 95% chance that the state is in S. In full generality we will call such a probabilistic observation $\mathcal{O}$. We can figure out how to update our knowledge statement from p_i, to $\hat{p}_i$, using a relative[1] version of Bayes's rule

$$\frac{\hat{p}_i}{\hat{p}_j} = \underbrace{\frac{P(i|\mathcal{O})}{P(j|\mathcal{O})}}_{\text{Posterior}} = \underbrace{\frac{P(\mathcal{O}|i)}{P(\mathcal{O}|j)}}_{\text{Bayes Factor}} \underbrace{\frac{p_i}{p_j}}_{\text{Prior}}$$

Pulling out the Bayes factor we find that we just multiply by the likelihood and renormalize

$$\hat{p}_k = P(\mathcal{O}|k)\ p_k.$$

A special case are projections $P(\mathcal{O}|k) \in \{0, 1\}$, like the S projection case in section 2.2.2. We call these Bayesian projections.

[1]In the ratio, the contribution of $P(\mathcal{O})$ is canceled out and accounted for during normalization. $P(\mathcal{O})$ only holds significance prior to its observation and it doesn't require consideration during the update process.

2.2.4 Classical Correlation as Classical Entanglement

Finally, we can also consider distributions such as the two coin distribution p_0, p_1, p_2, p_3

$$p_i = \begin{cases} \frac{1}{2} \text{ if } i \in \{0, 3\} \\ 0 \text{ else} \end{cases}$$

We have FF or TT with equal probabilities. If I give the first coin to Alice and the second coin to Bob then we have a classical correlation. If Bob finds that the first coin is T then we know that Alice will also find T. The reduction is purely epistemic, it is the separate knowledge of Alice and Bob that is changing. This should all seem very reasonable from every day experience; classical correlation is not weird.

2.3 Quantum Information Theory

2.3.1 Classical to Quantum Embedding

General quantum information[2] about 8-qbits can be expressed as a wave function.

$$q_0, \cdots, q_{255} \in \mathbb{C}$$

with $\sum |q_i|^2 = 1$. The Born rule is ostensibly a map $q_i \to |q_i|^2 = p_i$ to classical probability. Here we can instrument all of the classical information theoretic constructs by restricting the phase[3] of q_i to be zero. We can map backward $p_i \to \sqrt{p_i} = q_i$; which commutes with the Born rule (subject to normalization):

$$\text{(Quantum)} \qquad \mathbb{C}^n \rightleftarrows (\mathbb{R}_{\geq 0})^n \qquad \text{(Classical)}$$

So quantum information theory needs to generalize the classical. In particular, quantum measurement and entanglement restrict to classical measurement and classical correlation for zero phase.

2.3.2 Bayesian Projection

We illustrate zero phase Bayesian projection with an example.

Within the embedding we define a projection operator π which projects onto the space generated by the subset S, from the previous subsection

$$\pi = \sum_{i \in S} |i\rangle \langle i| \,.$$

Then let A be any operator with an eigenspace, with eigenvalue λ, equal to the image of π. An observation of λ would correspond to an application of π, which is a Bayesian projection in the classical picture.

2.3.3 Bayesian Inference

Consider, in the fashion of [2], a more general measurement with matrix $M_{\mathcal{O}}$ which is diagonal with entries

$$M_{\mathcal{O}}(i, i) = \sqrt{P(\mathcal{O}|i)}$$

This implements classical Bayesian inference using zero phase wavefunctions.

[2] We are purposely leaving out density matrices and POVMs and just dealing with pure states.

[3] We consider negative values as 180 degrees out of phase, so zero phase means non-negative real.

2.3.4 Entropy

Von Neumann entropy is left out of this note since it is not a generalization of classical entropy in the manner presented here. This is because the zero phase classical wavefunctions are pure states which all have zero Von Neumann entropy.

3 Overview of the Generalization

The quantum mechanical concepts mentioned in Section 1 are a direct generalization of their classical information theoretic analogs. This is not to say that they are not weird, but to say that they need to be generalizations of the classical concepts. Wave function collapsing should generalize Bayesian inference and classical measurement. Entanglement should be a generalization of classical correlation. This is enough to motivate new ways of doing quantum mechanics, which we will encounter in the next sections.

3.1 Multi-Observer Quantum Mechanics

We focus on quantum wave function collapse as a generalization of classical Bayesian inference. We can implement the classical Bayesian inference within zero phase quantum mechanics as outlined above. So the classical Bayesian theory has a direct tie in; that observation and measurement occur in tandem with a change in knowledge[4].

In the classical theory, knowledge is local to the observer[5], multiple observers each have their own knowledge. For instance, Wigner's friend and Wigner each have their own classical knowledge. It would seem then that Wigner's friend and Wigner must have different zero phase wavefunctions as well, if they are to be generalizing classical knowledge. This is a prediction on multi-observer quantum physics, that every observer has a local wavefunction.

3.2 Prediction and Experiment

We predict that multi-observer quantum mechanics will be required for a proper generalization of classical knowledge. We propose an experiment where we inject an "observer" into a quantum eraser[6]. The injected observer will have to be a small apparatus that is

- able to record a measurement ψ of the eraser particle path.
- able to forget the measurement ψ.
- able to demonstrate that a measurement was made with a record ρ.

Let ϕ be the lab technician's wavefunction. In ϕ the apparatus is entangled with the subject particles in the eraser experiment. We need to find out if there is another wavefunction in play that is not just part of ϕ. The key here is that the apparatus is able to demonstrate that it "knew" something, or in other words that another wavefunction ψ existed, using ρ.

The apparatus can not keep ψ for proper erasure, but must "forget" it. Observed erasure, via an interference pattern, proves that ψ is not a part of ϕ. Finally, there should be a record ρ in the apparatus that it did at one point in time record a measurement ψ. Receipt of the report ρ and eraser interference pattern together show that ψ and ϕ were necessarily part of a two-observer system.

[4]This change in knowledge is tangible, it always occurs as a transfer of matter/energy from the environment to the observer [3].

[5]Note that current quantum theory is a theory of one observer, usually the experimenter in a lab. An immediate subject is quantum key distribution(QKD), which requires at least three observers, Alice, Bob, and Eve; the security of QKD depends on multi-observer quantum ontology.

[6]Here we probe the boundaries of what constitutes an observer and a measurement. We claim that all versions of observer and measurement will be detectable in this experiment whenever it can be done.

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References

[1] R. Penrose, "The road to reality: a complete guide to the laws of the universe," 2007.

[2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition.* Cambridge University Press, 2011.

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