The Unruh Effect and Hawking Radiation Explained by Thrust

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> The Unruh effect tells us that what we call particles is really just a matter of perspective.
>
> <div align="right">Lee Smolin</div>

Abstract

First we use classical mechanics and the holographic principle to derive the Unruh effect. Next, we consider the field theoretic picture where Unruh radiation was initially uncovered. In both cases, we demonstrate how thrust serves as a more accurate and direct explanation of the Unruh effect and Hawking radiation.

1 The Holographic Scenario

We first recall the setup outlined in [1]. Consider a holographic screen $\mathcal{S}$ with entropy S. Let there be a test particle which is a single wavelength Δx away from $\mathcal{S}$. Following Bekenstein [2], we regard the particle to be simultaneously near and on the screen.

Let the screen have energy $E_{\mathcal{S}}$ and mass $M_{\mathcal{S}}$. Suppose that the holographic screen is being propelled by a stream of photons as in Figure 1. We want to see how the behavior of the photons affects the dynamics and information carried by the screen.

1.1 Emergence Overloaded

We briefly outline some notions of "emergence". Items 1 and 2 we inherit from [1]. We supply item 3, where matter/energy emerges from the screen.

1. **(statistical mechanics)** The usual thermodynamic emergence of macrostate variables (such as temperature) from microstate variables (such as energies of individual particles). [1] includes the emergence of Newton's laws and entropic gravity.

2. **(space-time)** The emergence of "new" space-time beyond holographic screens, treating screens like stretched horizons, far away from the actual event horizons. The existence of a foliation of screens where test particles move from screen to screen.

3. **(coming out)** We actually discuss the time-reverse in this item. Known matter/energy in front of the screen passes to unknown matter/energy on the screen. The matter/energy in front of the screen has known information. When passing into the screen, the information is thermalized becoming unknown.
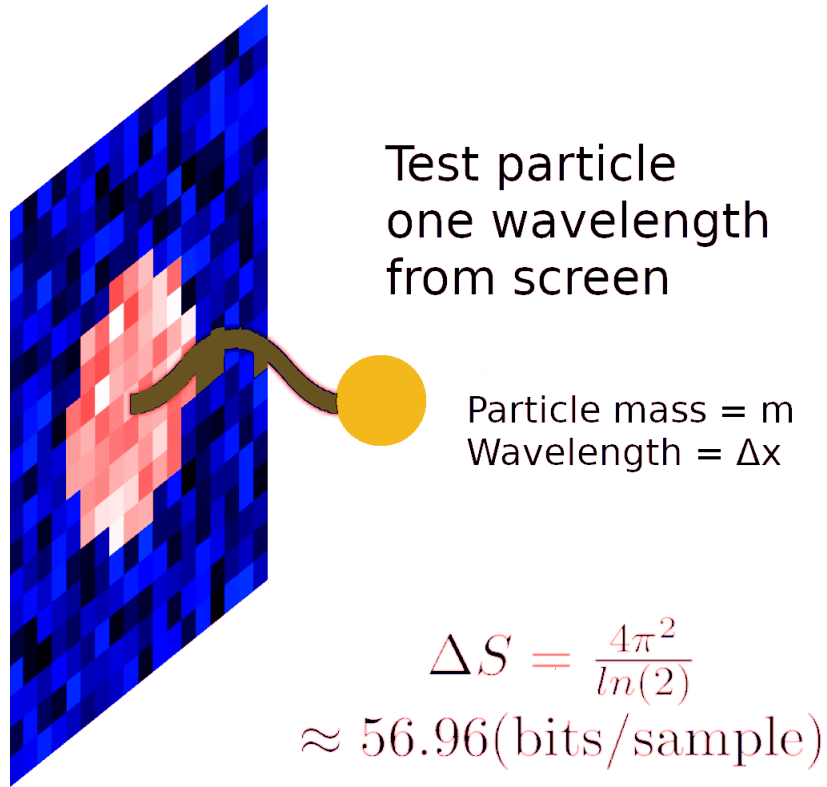
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Figure 1: This figure was adapted from [1]. A test particle with mass m and Compton wavelength Δx is being absorbed into the holographic screen. In this note, we replace the massive test particle with a single photon.

The information theoretic notions of emergence will be discussed in more detail in terms of Landauer's language in the next section.

2 Landauer Limit

We want to keep track of the information corresponding to degrees of freedom on the screen. It is interesting to compare this situation with the limiting case of Landauer [3]. The holographic situation is analogous, we let the screen be the separation between the system and the heat bath (between the known and the unknown). Forgetting (or learning) information is the same thing as letting the information "join the screen" ("emerge from the screen" resp.).

Let ΔS be the change in entropy before and after the particle joins the screen. The choice of ΔS is far from obvious. Consider the case where a single photon of energy E_p is joining the screen. The change of information, per photon, will be a constant. It is not given by $k\ln(2)$, from a bit, but by another number

$$\Delta S = 4\pi^2 k.$$

There is nothing mysterious here; we pick this constant so that we will match the constant in Unruh radiation.

3 The Unruh Effect from Classical Thrust

We derive the Unruh temperature equation from the information change of an accelerating screen due to the thrust of photons. For a single photon in the stream, consider the Planck relation for the energy and frequency of the screen

$$E_{\mathcal{S}} = h f_{\mathcal{S}} = \frac{h}{\Delta t}$$

where Δt is the period of $\mathcal{S}$. Let a be the acceleration due to the photon which we also write as $a = \Delta v / \Delta t$. Let p be the momentum of the photon, E_p its energy, and we equate the momenta $p_{\mathcal{S}} = p_p = p$ for an elastic collision with the screen. Then

$$a M_{\mathcal{S}} \Delta t = M_{\mathcal{S}} \Delta v = p = E_p / c$$

and setting a temperature T, with respect to E_p, we get

$$\frac{4\pi^2 k}{h} E_{\mathcal{S}} = \frac{\Delta S}{\Delta t} = \frac{E_p}{T \Delta t} = \frac{c a M_{\mathcal{S}}}{T} = \frac{a E_{\mathcal{S}}}{cT}$$

and canceling $E_{\mathcal{S}}$ we derive the Unruh temperature [4]

$$T = \frac{h}{4\pi^2 k c} a$$

4 The Unruh Effect from Field Theoretic Thrust

We consider a field theoretic version of thrust, driving sources. From this, we will derive the Unruh vacuum expectation. Let $\hbar = c = 1$.

Much of this section including notation and conventions are drawn directly from the overview [5], from the original source [4], and from [6]. As in these references, we only consider the scalar free field which is enough for the Unruh effect to become manifest.

4.1 Rindler and Minkowski Modes Review

Consider a $(1,1)$-dimensional spacetime[1] (t, z) with coordinates $u = -t + z$ and $v = -t - z$. Any function of purely u, $f(u)$, is made up of Minkowski modes that are constant on v. That is they are given by a 1-dimensional Fourier expansion

$$f(u) = \frac{1}{2\pi} \int e^{i p_u u} \hat{f}(p_u) dp_u.$$

supported on $p_v = 0$. The same is true for v, $g(v)$, p_v, and $\hat{g}(v)$. The full 2-dimensional transform of combinations of f and g is a linear combination of $\hat{f}(p_u)\delta(p_v)$ and $\hat{g}(p_v)\delta(p_u)$. These are supported on the massless shell — "the momentum light cone".

Let W be the $z > |t|$ Rindler wedge with coordinates

$$t = \frac{1}{a} e^{a\xi} \sinh(a\eta)$$

$$z = \frac{1}{a} e^{a\xi} \cosh(a\eta)$$

where $a > 0$ is an acceleration parameter (see Figure 2). We call u and v "event horizon" coordinates since they are defined along the event-horizons of an accelerating observer in W.

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[1]The full $(1,3)$-dimensional case does not add anything to the argument, so we stick to the dimensions where the linear acceleration boosts are taking place (t and z).
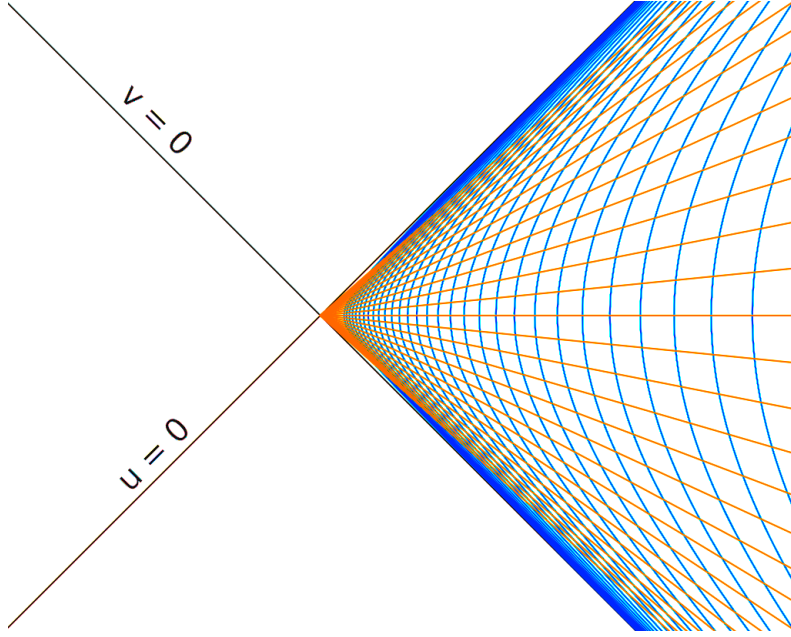
Figure 2: Rindler wedge W on the right.

For wave number k and positive frequency ω_k consider the functions

$$h_k^{(u)} = \frac{e^{\frac{\pi \omega_k}{2a}} (au)^{\frac{i\omega_k}{a}}}{\sqrt{2 \sinh\left(\frac{\pi \omega_k}{a}\right)}}$$

$$h_k^{(v)} = \frac{e^{\frac{\pi \omega_k}{2a}} (av)^{\frac{i\omega_k}{a}}}{\sqrt{2 \sinh\left(\frac{\pi \omega_k}{a}\right)}}$$

The scalar field in Rindler curved space has modes

$$r_k = \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k \eta - k\xi)}$$

which can be written as combinations of the "event horizon" modes

$$e^{\frac{\pi \omega_k}{2a}} h_k^{(u)} - e^{-\frac{\pi \omega_k}{2a}} h_k^{(v)} = \sqrt{2 \sinh\left(\frac{\pi \omega_k}{a}\right)} r_k$$

Unlike the Rindler mode, the event horizon modes extend analytically from W to all of spacetime.

When we switch between h_k and the usual Minkowski bases we find that the Minkowski vacuum has excitations visible in the Rindler frame. More specifically, there is a positive particle number vacuum expectation of

$$\frac{1}{e^{\frac{2\pi\omega_k}{a}} - 1} \tag{1}$$

which is usually interpreted as (Unruh) radiation [4].

4.2   Fourier Transform of the Sources

We will take several Fourier transforms to elucidate the various modes and set up for some integrals. Let ϕ be a scalar free field in the flat $(1,1)$-dimensional Minkowski spacetime. We will consider $h_k^{(u)}$ and $h_k^{(v)}$ as driving W-event horizon sources

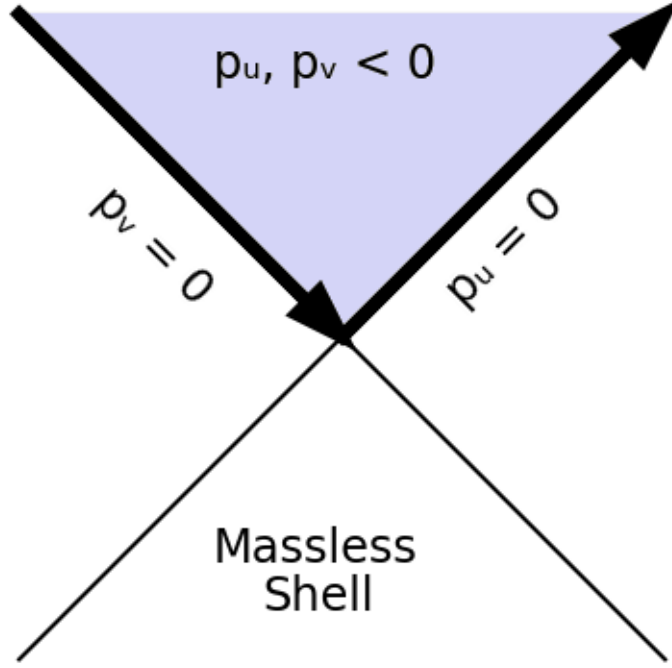$$\rho = \alpha h_k^{(u)} + \beta h_k^{(v)}. \tag{2}$$

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Figure 3: Massless shell is when $p_u = p_v = 0$

with positive real convex combination $(\alpha + \beta) = 1$. The drivers $h_k^{(u)}$ and $h_k^{(v)}$ are functions $f(u)$ of the past W-horizon and $g(v)$ of the future W-horizon respectively. Both generate excitations, which we identify with absorption and emission thrusts respectively.

The source can originate from a coupling term, $\rho\phi$, added to the Lagrangian.

$$\mathcal{L}_{driven} = \mathcal{L}_{original} + \rho\phi$$

This leads to an inhomogeneous Klein-Gordon equation

$$(\Box + m^2)\phi = \rho$$

as presented in [6][2]

We want to integrate ρ on shell in momentum space, which for a massless source is the positive energy part of the massless shell. The two positive energy "horizons" border $p_u <= 0$ and $p_v <= 0$, see Figure 3. Proceeding to take the Fourier transform of the function $f(u)$[3], we drop p_u to just p for the time being to increase legibility. We will continue to assume that ω_k and a are positive. Define the kernel

$$A = e^{-ipu}(au)^{\frac{i\omega_k}{a}}\, du$$

and then we have

$$\hat{f}(\_u) = \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} \int_{-\infty}^{\infty} A \tag{3}$$

where $L = \int_{-\infty}^{0} A$ and $R = \int_{0}^{\infty} A$ are the left and right sides of the total integral $L + R$.

We rewrite the integrals using a complex changes of variables, $s = ipu$, and contour integrals.

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[2]In [6] it is assumed that the source is only active for a finite amount of time. We let ρ be active for all time. The argument in [6] seem to be adaptable to ρ.
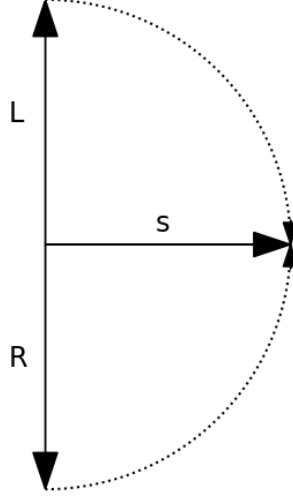
[3]WLOG since $g(v)$ is of the same form.

Figure 4: Using contours with large radius we convert the L integral that goes to $i\infty$, and the R integral that goes to $-i\infty$, to integrals with s going to real ∞.

The L integral for real $p < 0$ is

$$
\begin{aligned}
L(p) &= -\int_0^{i\infty} \left(\frac{ias}{-p}\right)^{\frac{i\omega_k}{a}} \left(\frac{i}{-p}\right) ds \\
&= \frac{-i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \int_0^\infty (is)^{\frac{i\omega_k}{a}} e^{-s} ds \\
&= \frac{-i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} \\
&= \frac{1}{2}\Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} B(p)
\end{aligned}
$$

where

$$
B(p) = \frac{-2i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1}
$$

is as shown. This is using a large radius contour which rotates the endpoint 90 degrees clockwise.

The same calculation for R is done using a counter-clockwise contour this time.

$$
R(p) = \frac{1}{2}\Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} B(p)
$$

We get back to $h_k^{(u)}$ and apply the normalization from equation (3)

$$
\hat{h_k}^{(u)}(p_u) = \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}(L(p_u) + R(p_u))
$$

So

$$
\begin{aligned}
\hat{h_k}^{(u)}(p_u) &= \frac{\Gamma\left(\frac{i\omega_k}{a}+1\right)}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} B(p_u) \\[2em]
\hat{h_k}^{(v)}(p_v) &= \frac{\Gamma\left(\frac{i\omega_k}{a}+1\right)}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} B(p_u)
\end{aligned}
\tag{4}
$$

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4.3 Interpretation

The driving source ρ, with mixed absorption and emission thrusts $\alpha + \beta = 1$, contribute excitations to the scalar field ϕ. Equations (2) and (4) let us write down the expected change of energy

$$
\begin{aligned}
\mathbb{E}[\Delta E] &= \frac{1}{4\pi} \int |\rho(p)|^2 dp \\
&= \frac{\alpha}{4\pi} \int \left| \hat{h_k}^{(u)}(p_u) \right|^2 dp_u + \frac{\beta}{4\pi} \int \left| \hat{h_k}^{(v)}(p_v) \right|^2 dp_v \\
&= \frac{\left| \Gamma\left(\frac{i\omega_k}{a} + 1\right) \right|^2}{2\sinh\left(\frac{\pi\omega_k}{a}\right)} \frac{1}{4\pi} \int |B(p)|^2 dp \\
&= \frac{\left| \Gamma\left(\frac{i\omega_k}{a} + 1\right) \right|^2}{2\pi a \sinh\left(\frac{\pi\omega_k}{a}\right)} \int a/|p|^2 dp \\
&= I(\omega_k) P
\end{aligned}
\tag{5}
$$

where the integrals are on the positive energy massless shell with contributions from p_u on the left piece and p_v on the right piece. We factored out $P = \int a/|p|^2$ with a remaining p independent positive real coefficient $I(\omega_k)$.

Without being more careful we end up with inferred problems — The integrals do not converge at zero, where P explodes. But this infinity cancels when we compare the spectral radiances $I(\omega_k)$ to each other.

The magnitude of our Gamma function has known asymptotics [7, Eq. 5.11.9]

$$
\left| \Gamma\left(\frac{i\omega_k}{a} + 1\right) \right|^2 \sim \left(\frac{2\pi\omega_k}{a}\right) e^{-\frac{\pi\omega_k}{a}}
$$

Plugging this into equation (5) we find the average energy of the mode, the (1,1) dimensional Planck distribution function, and thus recover the Unruh's radiation spectrum[4] from a thrust driven field.

$$
\frac{1}{P}\mathbb{E}[\Delta E] = I(\omega_k) \sim \frac{\omega_k}{e^{\frac{2\pi\omega_k}{a}} - 1}
$$

5 Hawking-Bekenstein Radiation and the Equivalence Principle

The equivalence principle allows us to transfer linear acceleration conclusions to accelerations due to gravity. Unruh's temperature maps to a temperature for acceleration due to gravity – for instance near the event horizon of a black hole[5]. This corresponds to a reference frame which continues to thrust away from a black hole, maintaining a given height. Thrust that explains Unruh radiation also explains Hawking-Bekenstein radiation.

6 Acceleration and Observation

We have seen how acceleration is paired with an accelerating thrust. Central to this idea is the implicit separation of the accelerated system from the rest of the universe. We have studied the fanciful holographic screens from entropic gravity, Landauer's principle, and

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[4]Compare to equation (1) and references [4] and [5].

[5]This leads to the celebrated black hole radiation. The equivalence principle is used to equate this with Unruh radiation see [4]

the Rindler horizons. In all of these cases, there is a separation between the accelerated system that we are a part of $\mathcal{A}$, and the remaining heat bath $\mathcal{H}$.

Suppose we add the particles responsible for the thrust, $\mathcal{T}$, back into our system. The resulting larger system, $\mathcal{B} = \mathcal{A} + \mathcal{T}$, would no longer be experiencing a net acceleration since all of the forces balance out. This is the essence of Newton's third law. So the very existence of a nonzero acceleration requires the separation of the system $\mathcal{A}$ from its thrust $\mathcal{T}$.
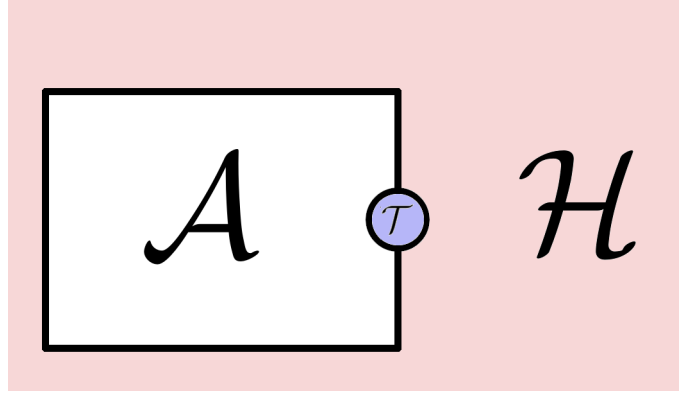

Figure 5: System $\mathcal{A}$ with thrust $\mathcal{T}$ in surrounding heat bath $\mathcal{H}$. Define $\mathcal{B} = \mathcal{A} + \mathcal{T}$.

The physical difference between $\mathcal{A}$ and $\mathcal{B}$ is the thrust $\mathcal{T}$. In accordance with Landauer, the following transitions are neatly summarized:

1. A direct application of Landauer's principle,
$$\mathcal{B} \to \mathcal{A} \text{ corresponds to \textbf{``Forgetting''} } \mathcal{T}.$$

2. A reverse application of Landauer's principle,
$$\mathcal{A} \to \mathcal{B} \text{ corresponds to \textbf{``Learning''} } \mathcal{T}.$$

The 2nd transition has $\mathcal{T}$ as a "measurable" in $\mathcal{A}$. This could be a full on apperception by a conscious observer, an interaction with a particle, or just a measurement potential. The key here is an increase in total information in $\mathcal{A}$. This is related exactly to an increase in total mass/energy by Landauer's principle. The 1st transition is not measurable in $\mathcal{A}$, but is measurable in $\mathcal{H}$

7   Conclusion and Prediction

If we do not account for the thrust required to accelerate a detector, then we recover it instead as a thermal unknown in the vacuum, Unruh radiation. But, it would seem that we can explain Unruh radiation directly using thrust in the illustrative ways outlined in this paper. The prediction of this note is that Unruh radiation and Hawking-Bekenstein radiation, separate from thrust, should not appear.

8   Acknowledgments

Thanks to Daniel Justice for useful discussions.

References

[1] E. Verlinde, "On the origin of gravity and the laws of newton," *Journal of High Energy Physics*, vol. 2011, no. 4, pp. 1–27, 2011.

[2] J. D. Bekenstein, "Black holes and entropy," *Phys. Rev. D*, vol. 7, pp. 2333–2346, Apr 1973.

[3] R. Landauer, "Irreversibility and heat generation in the computing process," *IBM Journal of Research and Development*, vol. 5, no. 3, pp. 183–191, 1961.

[4] W. G. Unruh, "Notes on black-hole evaporation," *Phys. Rev. D*, vol. 14, pp. 870–892, Aug 1976.

[5] E. Frodden and N. Valdés, "Unruh effect: Introductory notes to quantum effects for accelerated observers," *International Journal of Modern Physics A*, vol. 33, p. 1830026, sep 2018.

[6] E. Zurich, "Chapter 3 quantum field theory," 2012.

[7] "*NIST Digital Library of Mathematical Functions*." http://dlmf.nist.gov/, Release 1.1.8 of 2022-12-15. F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, B. V. Saunders, H. S. Cohl, and M. A. McClain, eds.

[8] "The derivation of the planck formula." Accessed: 2023-02-25.