The Unruh Effect as Holographic Thrust

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Abstract

We outline the holographic scenario from entropic gravity and additionally consider the dynamics of the holographic screen itself in an accelerating frame. We consider the case where the screen's acceleration is due to mass/energy joining with the screen, increasing the entropy of the screen. Finally, we consider the field theoretic picture where Unruh radiation was first predicted. In both cases, we explain the Unruh effect using thrust.

1 The Holographic Scenario

We first recall the setup outlined in [1]. Consider a holographic screen with entropy S. Let there be a test particle which is a single wavelength Δx away from S. Following Bekenstein [2], we can regard the particle to be simultaneously near and on the screen.

Let the screen have energy E_S and mass M_S. Suppose that the holographic screen is being propelled by a stream of photons in place of the test particle. We want to see how the dynamics of the photons affects the dynamics and information carried by the screen.

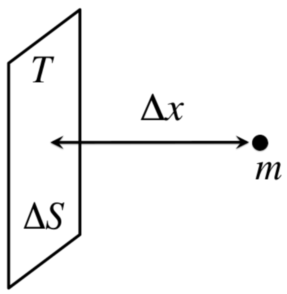

Figure 1: This figure was copied from [1]. A test particle with mass m and Compton wavelength Δx is being absorbed into the holographic screen. In this note, we replace the test particle with a single photon.

There are some complementary notions of "emergence" that we inherit from [1]. A third notion, item 3, that we supply here is the time-reversal of matter/energy emerging from the screen.

1. The usual thermodynamic emergence of macrostate variables (such as temperature) from microstate variables (energies of individual particles). This includes the emergence of Newton's laws and entropic gravity.

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2. The emergence of "new" space-time beyond holographic screens, treating screens like stretched horizons. The existence of a foliation of screens where test particles move from screen to screen.
3. Known matter/energy in front of the screen passes to unknown matter/energy on the screen. The matter/energy in front of the screen has known information. When passing into the screen, the information is thermalized, and becomes unknown.

The information theoretic notions of emergence will be discussed in more detail in terms of Landauer's language in the next section.

2 Landauer Limit

We want to keep track of the information corresponding to degrees of freedom on the screen. It is interesting to compare this situation with the limiting case of Landauer [3]. The situation is identical, the holographic screen in his case is the separation between the system and the heat bath (between the known and the unknown). Forgetting (or learning) information is the same thing as letting the information "join the screen" ("emerge from the screen" resp.).

We pause to note that there is nothing canonical about choosing a bit to express this information. For instance, Landauer considered logical gates and as an example, we briefly consider the AND gate on two unbiased bits of input. The output is a bit, but it is biased, and so it carries less than one bit of information. The AND gate actually forgets more than a bit[1]. All of this is to say that the choice of ΔS is far from obvious.

Consider the case where a single photon of energy E_p is joining the screen. The change of information, per photon, will be a constant. It is not given by $ln(2)k$, from a bit, but by another number

$$\Delta S = 4\pi^2 k.$$

There is nothing mysterious here; we pick this constant so that we will match the constant in Unruh radiation.

3 Information Theoretic Unruh Effect

We will derive the Unruh temperature equation from the information change of an accelerating screen due to the thrust of photons. For a single photon in the stream, consider the Planck relation for the energy and frequency of the screen

$$E_S = hf_S = \frac{h}{\Delta t}$$

where Δt is the period of S. Let a be the acceleration due to the photon which we also write as $a = \Delta v/\Delta t$. Let p be the momentum of the photon, E_p its energy, and we equate the momenta $p_S = p_p = p$. Then

$$aM_S\Delta t = M_S\Delta v = p = E_p/c$$

and setting a temperature T, with respect to E_p, we get

$$\frac{4\pi^2 k}{h}E_S = \frac{\Delta S}{\Delta t} = \frac{E_p}{T\Delta t} = \frac{caM_S}{T} = \frac{aE_S}{cT}$$

and canceling E_S we derive the Unruh temperature [4]

$$T = \frac{h}{4\pi^2 kc}a$$

[1] It actually forgets around 1.19 bits

4 Field Theoretic Considerations

Let $\hbar = \mathrm{c} = 1$.

4.1 Rindler and Minkowski Modes Review

Much of this section including notation and conventions are drawn directly from the overview [5], from the original source [4], and from [6].

Consider a $(1,1)$-dimensional spacetime[2] (t, z) with "event horizon" coordinates $u = -t + z$ and $v = -t - z$. Any function of purely u, $f(u)$, is made up of Minkowski modes that are constant on v. That is they are given by a 1-dimensional Fourier expansion

$$f(u) = \frac{1}{2\pi} \int e^{ip_u u} \hat{f}(p_u) dp_u.$$

The same is true for v, $g(v)$, p_v, and $\hat{g}(v)$. The full 2-dimensional transform of combinations of f and g is a linear combination of $\hat{f}(p_u)\delta(p_v)$ and $\hat{g}(p_v)\delta(p_u)$. These are supported on the massless shell — "the momentum light cone".

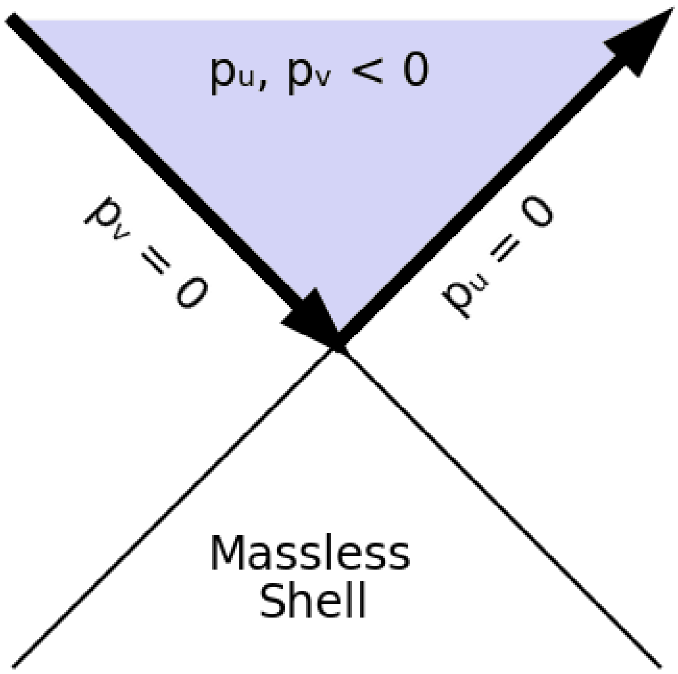

Figure 2

Let W be the $z > |t|$ Rindler wedge with coordinates

$$t = \frac{1}{a} e^{a\xi} \sinh(a\eta)$$

$$z = \frac{1}{a} e^{a\xi} \cosh(a\eta)$$

where $a > 0$ is an acceleration parameter (see Figure 3).

For wave number k and positive frequency ω_k consider the functions

$$h_k^{(u)} = \frac{e^{\frac{\pi\omega_k}{2a}} (au)^{\frac{i\omega_k}{a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}$$

[2]It is not useful to consider the full $(1,3)$-dimensional case, so we stick to the dimensions where the linear acceleration boosts are taking place – t and z.

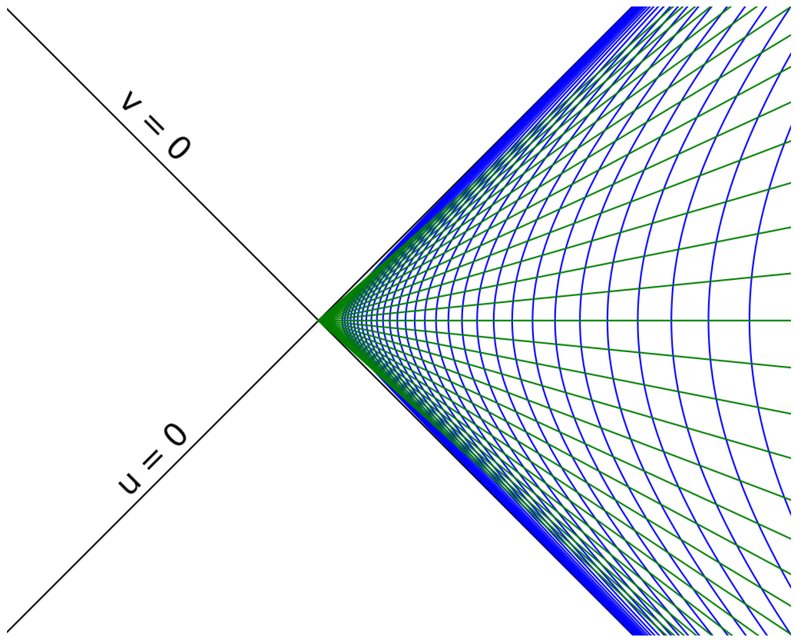

Figure 3: Rindler wedge W on the right.

$$h_k^{(v)} = \frac{e^{\frac{\pi\omega_k}{2a}}(av)^{\frac{i\omega_k}{a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}$$

The scalar field in Rindler curved space has modes

$$r_k = \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k\eta - k\xi)}$$

which can be written as combinations of the "event horizon" modes x

$$e^{\frac{\pi\omega_k}{2a}} h_k^{(u)} - e^{-\frac{\pi\omega_k}{2a}} h_k^{(v)} = \sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)} r_k$$

Unlike the Rindler mode, the event horizon modes extend analytically from W to all of spacetime.

When we switch between h_k and the usual Minkowski bases we find that the Minkowski vacuum has excitations coming from the Rindler modes. More specifically, there is a positive particle number vacuum expectation of

$$\frac{1}{e^{\frac{2\pi\omega_k}{a}} - 1} \tag{1}$$

which is interpreted as (Unruh) radiation [4].

4.2 Fourier Transform of the Sources

Let ϕ be a scalar field in the flat $(1,1)$-dimensional Minkowski spacetime[3]. We will consider $h_k^{(u)}$ and $h_k^{(u)}$ as driving W-event horizon sources

$$\rho = \alpha h_k^{(u)} + \beta h_k^{(v)}. \tag{2}$$

with positive real coefficients $(\alpha + \beta) = 1$. The drivers $h_k^{(u)}$ and $h_k^{(v)}$ are functions $f(u)$ of the past W-horizon and $g(v)$ of the future W-horizon respectively. Both generate excitations, which we identify with absorption and emission thrusts respectively.

[3]Note that we only consider the scalar case. Generalization to other fields is straightforward and does not add any intuition to the presentation.

The source can originate from a coupling term, $\rho\phi$, added to the Lagrangian.

$$\mathcal{L}_{driven} = \mathcal{L}_{original} + \rho\phi$$

This leads to an inhomogeneous Klein-Gordon equation

$$(\Box + m^2)\phi = \rho$$

as presented in [6]. It is assumed that the source will only be active for some finite amount of time. We ignore this issue and instead go after the general asymptotic form of our solution in Fourier space.

We want to integrate ρ on shell in momentum space, which for a massless source is the positive energy part of the massless shell. The two positive energy "horizons" border $p_u <= 0$ and $p_v <= 0$, see Figure 2. Proceeding to take the Fourier transform of the function $f(u)$[4], we drop p_u to just p for the time being to increase legibility. We will continue to assume that ω_k and a are positive. Define the kernel

$$A = e^{-ipu}(au)^{\frac{i\omega_k}{a}}du$$

and then we have

$$\hat{f}(\rho_U) = \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}\int_{-\infty}^{\infty} A \tag{3}$$

where $L = \int_{-\infty}^{0} A$ and $R = \int_{0}^{\infty} A$ are the left and right sides of the total integral $L + R$.

We rewrite the integrals using a complex changes of variables, $s = ipu$, and contour integrals.

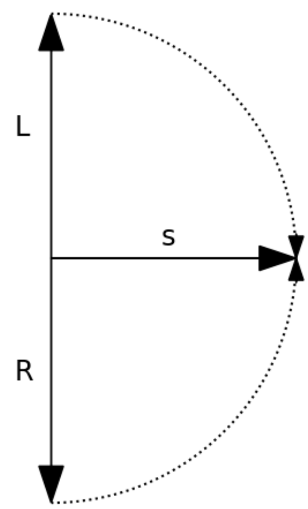

Figure 4: Using contours with large radius we convert the L integral that goes to $i\infty$, and the R integral that goes to $-i\infty$, to integrals with s going to real ∞.

[4] WLOG since $g(v)$ is of the same form.

The L integral for real $p < 0$ is

$$\begin{aligned} L(p) &= -\int_0^{i\infty} \left(\frac{ias}{-p}\right)^{\frac{i\omega_k}{a}} \left(\frac{i}{-p}\right) ds \\ &= \frac{-i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \int_0^{\infty} (is)^{\frac{i\omega_k}{a}} e^{-s} ds \\ &= \frac{-i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} \\ &= \frac{1}{2}\Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} B(p) \end{aligned}$$

where

$$B(p) = \frac{-2i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1}$$

is as shown. This is using a large radius contour which rotates the endpoint 90 degrees clockwise.

The same calculation for R is done using a counter-clockwise contour this time.

$$R(p) = \frac{1}{2}\Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} B(p)$$

We get back to $h_k^{(u)}$ and apply the normalization from (3)

$$\hat{h_k}^{(u)}(p_u) = \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}(L(p_u) + R(p_u))$$

So

$$\begin{aligned} \hat{h_k}^{(u)}(p_u) &= \frac{\Gamma\left(\frac{i\omega_k}{a}+1\right)}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} B(p_u) \\ \hat{h_k}^{(v)}(p_v) &= \frac{\Gamma\left(\frac{i\omega_k}{a}+1\right)}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} B(p_u) \end{aligned} \tag{4}$$

4.3 Interpretation

The driving source ρ, with mixed absorption and emission thrusts $\alpha + \beta = 1$, contribute excitations to the scalar field ϕ. Equations (2) and (4) let us write down the expected number of new particles

$$\begin{aligned} \mathbb{E}[\Delta N] &= \langle\rho|\rho\rangle \\ &= \frac{1}{4\pi}\int \frac{|\rho(p)|^2}{|p|} dp \\ &= \frac{\alpha}{4\pi}\int \frac{\left|\hat{h_k}^{(u)}(p_u)\right|^2}{|p_u|} dp_u + \frac{\beta}{4\pi}\int \frac{\left|\hat{h_k}^{(v)}(p_v)\right|^2}{|p_u|} dp_v \\ &= \frac{\left|\Gamma\left(\frac{i\omega_k}{a}+1\right)\right|^2}{2\sinh\left(\frac{\pi\omega_k}{a}\right)} \frac{1}{4\pi}\int |B(p)|^2 |p| dp \\ &= \frac{\left|\Gamma\left(\frac{i\omega_k}{a}+1\right)\right|^2}{2\pi a\sinh\left(\frac{\pi\omega_k}{a}\right)} \int a/|p|^3 dp \\ &= I(\omega_k)P \end{aligned} \tag{5}$$

where the integrals are on the positive energy massless shell with contributions from p_u on the left piece and p_v on the right piece. We factored out $P = \int a/|p|^3$ with a remaining p independent positive real coefficient $I(\omega_k)$.

Without being more careful about the finite time and shape of the source we end up with inferred problems — The integrals do not converge at zero, where P explodes. But this infinity cancels when we compare the spectral radiances $I(\omega_k)$ to each other.

The magnitude of our Gamma function has known asymptotics [7, Eq. 5.11.9]

$$\left|\Gamma\left(\frac{i\omega_k}{a}+1\right)\right|^2 \sim \left(\frac{2\pi\omega_k}{a}\right) e^{-\frac{\pi\omega_k}{a}}$$

Plugging this into equation (5) we find a $(1,1)$-dimensional version of Plank's law[5], and thus recover the Unruh radiation spectrum[6] from a thrust driven field.

$$\frac{1}{P}\mathbb{E}[\Delta N] = I(\omega_k) \sim \frac{\omega_k}{e^{\frac{2\pi\omega_k}{a}}-1}$$

5 Hawking-Bekenstein Radiation and the Equivalence Principle

The equivalence principle allows us to transfer linear acceleration conclusions to accelerations due to gravity. Unruh's temperature maps to a temperature for acceleration due to gravity – for instance near the event horizon of a black hole[7]. This corresponds to a reference frame which continues to thrust away from a black hole, maintaining a given height. Thrust that explains Unruh radiation also explains Hawking-Bekenstein radiation as well.

6 Conclusion and Prediction

If we do not account for the thrust required to accelerate a detector, then we recover it instead as a thermal unknown in the vacuum, Unruh radiation. But, it would seem that we can explain Unruh radiation directly using thrust in the illustrative ways outlined in this paper. The prediction of this note is that Unruh radiation, separate from thrust, should not appear.

References

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[4] W. G. Unruh, "Notes on black-hole evaporation," *Phys. Rev. D*, vol. 14, pp. 870–892, Aug 1976.

[5] There is a counting argument, in [8] for instance, that depends on the dimension $(1,n)$. In general, the formula (1) gets multiplied by a ω_k^n term.

[6] Compare to equation (1) and references [4] and [5].

[7] This leads to the celebrated black hole radiation. The equivalence principle is used to equate this with Unruh radiation see [4]

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