Localization of the Unruh Effect

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> The Unruh effect tells us that what we call particles is really just a matter of perspective.
>
> Lee Smolin

Abstract

We explore a method to interpolate the Unruh effect between a fully thermal response to a completely localized driving source. In a uniformly accelerating frame, modes extend over long duration and experience red- and blue-shifts due to their extended support across the Rindler wedge, leading to a mixed-frequency response that appears thermal. To refine this picture, we develop a partially localized perspective using modular automorphisms to map modes between nested Rindler wedges. We then interpolate further to fully localized wave packets using parabolic cylinder functions. This framework enables a reinterpretation of Unruh radiation in terms of thrust, a localized, frequency-peaked driving effect that excites the field without producing a thermal response. This offers a complementary, non-thermal perspective on acceleration-observed radiation.

1 Introduction

We begin in Section 2 by introducing notation and reviewing the Unruh effect, including Rindler coordinates and explicit Bogoliubov transformations. In Section 3, we introduce a source in a standard way that exactly reproduces the particle creation associated with the Bogoliubov transformation for Rindler modes extended to Minkowski space. Section 4 explores partial localization by considering Rindler subwedges related by a spacelike translation, corresponding to a modular automorphism of the associated von Neumann algebra. We then use parabolic cylinder functions to interpolate between eternal Rindler modes and fully localized wave packets. Finally, in Section 5, we interpret and discuss the implications of these results.

2 Unruh Effect Review and Notation

We draw notation and standard results from Frodden and Valdés [1].

Let $\hbar = c = 1$. We consider a uniformly accelerating observer in 1+1 dimensional Minkowski spacetime with metric signature $\eta = (-1, +1)$. The extension to 1+3 dimensions does not affect the key physics of the Unruh effect, so we restrict to the (t,x) plane where the boost is occuring.

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Consider the free scalar massless Lagrangian

$$\mathcal{L}_{free} = -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{1}$$

We consider positive frequency modes with dispersion relation $\omega_k = |k| > 0$ as solutions to the resulting Klein-Gordon equation

$$\Box\phi = -\frac{\partial^2\phi}{\partial t^2} + \frac{\partial^2\phi}{\partial x^2} = 0, \tag{2}$$

where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$. We expand ϕ in terms of ladder operators $a_k, a_k^\dagger$

$$\phi(x,t) = \int \mathrm{d}k\, a_k\varphi_k(x,t) + \text{h.c.} \tag{3}$$

where

$$\varphi(x,t) = \frac{1}{\sqrt{4\pi\omega_k}} e^{i(kx-\omega_k t)}. \tag{4}$$

are pure Minkowski positive frequency waves normalized with respect to the Klein-Gordon inner product over a Cauchy surface Σ (usually $t = 0$)

$$\langle f, g\rangle_{KG} = i\int_\Sigma \mathrm{d}x(f^*\partial_t g - \partial_t f^* g). \tag{5}$$

2.1 Rindler Coordinates

To describe the physics from the point of view of a uniformly accelerating observer, we introduce Rindler coordinates covering a right wedge

$$W_c = \{(x,t) : x - c > |t|\} \tag{6}$$

with apex at $(t,x) = (0,c)$. We begin with the wedge at the origon, $W = W_0$, which coorosponds to region I pictured[1] in Figure 1; with coordinates

$$t = \frac{1}{a}e^{a\xi}\sinh(a\eta) \tag{7}$$

$$x = \frac{1}{a}e^{a\xi}\cosh(a\eta) \tag{8}$$

and a is a constant with dimensions of acceleration that sets the proper acceleration of the reference trajectory at fixed ξ. The parameter a is introduced explicitly to make the dependence of the Unruh temperature, $T = \frac{a}{2\pi}$, manifest in subsequent expressions. The coordinates (η, ξ) describe the proper time and position in the frame of a uniformly accelerating observer, with worldlines of constant ξ corresponding to hyperbolic trajectories in Minkowski spacetime.

The massless Klein-Gordon equation in Rindler coordinates is

$$\Box\phi = e^{-2a\xi}(-\partial_\eta^2 + \partial_\xi^2)\phi = 0 \tag{9}$$

The wave equation retains the same structure as the Minkowski case, up to the overall conformal factor $e^{-2a\xi}$. Since this factor does not affect the null structure of the equation, the mode solutions retain the same plane wave form but in the Rindler coordinates

$$r_k(\eta,\xi) = \frac{1}{\sqrt{4\pi\omega_k}}e^{-i(\omega_k\eta - k\xi)} + \text{h.c.} \tag{10}$$

for each wave number k and positive frequency $\omega_k = |k| > 0$. These "Rindler modes" are in terms of η and ξ and are thus confined to the Rindler wedge W. Since Rindler coordinates only cover region I (the right wedge), these modes are not defined globally in Minkowski space.

[1]All spacetime diagrams have time moving up and space to the right.

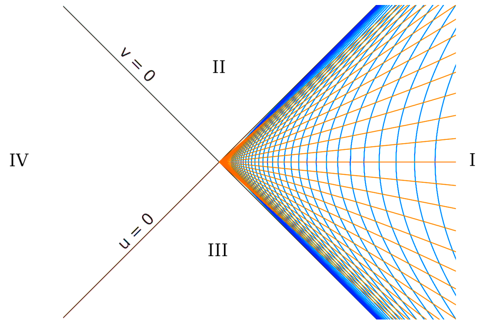

Figure 1: Rindler wedge I on the right.

2.2 Unruh Modes

For $\omega_k = k > 0$ define

$$\begin{aligned} \alpha_k &= \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\frac{\pi\omega_k}{a}}} = \sqrt{\frac{1}{1-e^{-2\pi\omega_k/a}}} \\ \beta_k &= \frac{e^{\frac{-\pi\omega_k}{2a}}}{\sqrt{2\sinh\frac{\pi\omega_k}{a}}} = \sqrt{\frac{1}{e^{2\pi\omega_k/a}-1}} \quad \text{(thermal form)} \end{aligned} \tag{11}$$

These coefficients satisfy $\alpha_k^2 - \beta_k^2 = 1$, reflecting the normalization condition for Bogoliubov transformations.

We analytically continue[2] r_k and r_{-k} to the (t, x) plane using the standard $i\epsilon$ prescription to define a specific branch of the log:

$$\begin{aligned} r_{+k} &= \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k\eta - k\xi)} = \frac{1}{\sqrt{4\pi\omega_k}} (a(-t+x+i\epsilon))^{\frac{i\omega_k}{a}} \\ r_{-k} &= \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k\eta + k\xi)} = \frac{1}{\sqrt{4\pi\omega_k}} (a(t+x-i\epsilon))^{\frac{-i\omega_k}{a}} \end{aligned} \tag{12}$$

Though r_k and r_{-k} are positive frequency in Rindler time η, they flip to negative frequency in Minkowski time t when analytically continued across the log branch to the left wedge; see the middle column in Figure 2. We mention again the standard Minkowski positive-frequency plane wave modes:

$$\begin{aligned} \varphi_{+k} &= \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k t - kx)} \\ \varphi_{-k} &= \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k t + kx)} \end{aligned} \tag{13}$$

We next construct the Unruh modes

$$\begin{aligned} \mu_k^R &= \alpha_k(r_{+k} - r_{-k}^*) \\ &= \frac{1}{\sqrt{4\pi\omega_k}\sqrt{2\sinh\frac{\pi\omega_k}{a}}} \left(e^{\frac{\pi\omega_k}{2a}} (a(-t+x+i\epsilon))^{\frac{i\omega_k}{a}} + e^{\frac{-\pi\omega_k}{2a}} (a(t+x-i\epsilon))^{\frac{i\omega_k}{a}} \right) \\ \mu_k^L &= \beta_k(r_{+k}^* - r_{-k}) \\ &= \frac{1}{\sqrt{4\pi\omega_k}\sqrt{2\sinh\frac{\pi\omega_k}{a}}} \left(e^{\frac{-\pi\omega_k}{2a}} (a(-t+x+i\epsilon))^{\frac{-i\omega_k}{a}} + e^{\frac{\pi\omega_k}{2a}} (a(t+x-i\epsilon))^{\frac{-i\omega_k}{a}} \right) \end{aligned} \tag{14}$$

The functions μ_k^R and μ_k^L are analytic in the lower-half complex t-plane and decay at infinity, so they qualify as positive-frequency Minkowski modes. They form an alternative orthonormal basis of solutions to the Klein-Gordon equation, distinct from the plane waves $\varphi_{\pm k}$. Most importantly, the Unruh modes diagonalize the Minkowski vacuum in terms of Rindler particle states and thus provide the natural framework for describing the Unruh effect and the thermal response perceived by uniformly accelerated observers.

[2]From the definitions and properties of sinh and cosh, it follows that $a(\mp t + x) = e^{a(\pm\eta+\xi)}$

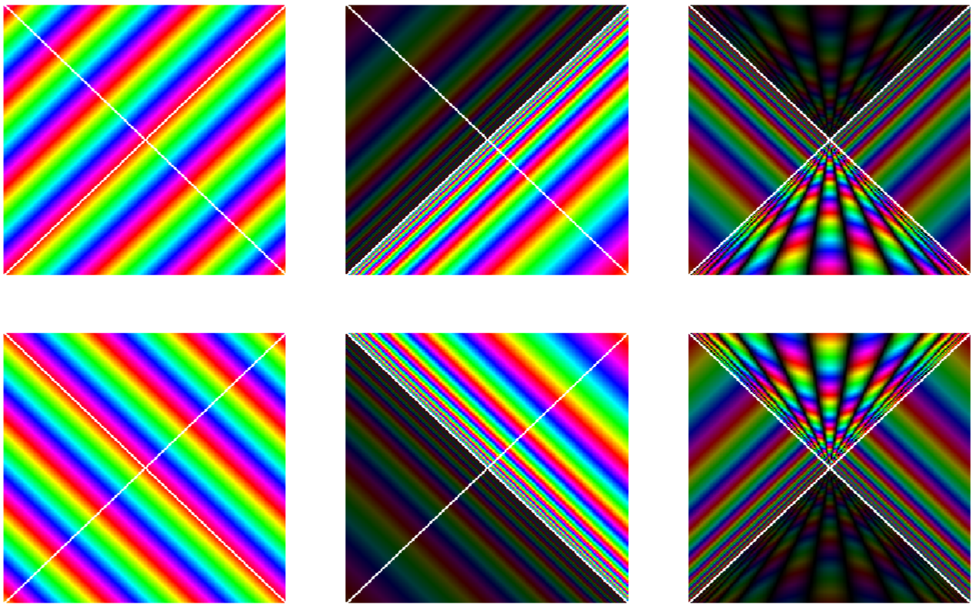

Figure 2: Space time diagrams for $\begin{bmatrix} \varphi_k & r_k & \mu_k^R \\ \varphi_{-k} & r_{-k} & \mu_k^L \end{bmatrix}$. Color represents phase; brightness shows magnitude. The consistent "rainbow" phase structure in φ_k and μ_k reflects their pure positive frequency content in Minkowski time, unlike r_k, which flips across log branch. The left moving Rindler modes r_k(top) correspond to emission and the right moving mode r_{-k} (bottom) to absorption.

2.3 Bogoliubov Transforms

Let the superscripts (0), (c), (M) represent the W_0, W_c, and Minkowski frames of reference respectively. ($0c$) represents a map from the Von Neumann algebras of W_c to W_0 and (cM) represents a map from the algebras of M to W_c. We have a Bogoliubov transformation matrix from M to W_0

$$\left[\begin{array}{c} a_k^{(0)} \\ a_{-k}^{(0)} \\ \hline a_k^{(0)\dagger} \\ a_{-k}^{(0)\dagger} \end{array}\right] = \left[\begin{array}{cc|cc} \alpha_k & 0 & 0 & \beta_k \\ 0 & -\alpha_k & -\beta_k & 0 \\ \hline 0 & \beta_k & \alpha_k & 0 \\ -\beta_k & 0 & 0 & -\alpha_k \end{array}\right]_{k,q} \left[\begin{array}{c} c_q^R \\ c_q^L \\ \hline c_q^{R\dagger} \\ c_q^{L\dagger} \end{array}\right] \tag{15}$$

for a change of basis from $a_q^{(M)}$ to c_q^R and c_q^L

$$\phi = \int \mathrm{d}q\ \mu_q^R c_q^R + \mu_q^L c_q^L + \text{h.c.} \tag{16}$$

So that we can compute the usual Unruh radiation equation with Planck spectrum (compare β_k with equation (11)) to obtain

$$a_k^{(0)} = \alpha_k c_q^R + \beta_k c_q^{L\dagger} \tag{17}$$

We next compute the more general mixed Bogoliubov transformations.

$$\begin{aligned} a_k^{(c)} &= \int \mathrm{d}q\ \alpha_{kq}^{(cM)} a_q^M + \beta_{kq}^{(cM)} a_q^{(M)\dagger} \\ &= \int \mathrm{d}q\ \alpha_{kq}^{(c0)} a_q^{(0)} + \beta_{kq}^{(c0)} a_q^{(0)\dagger} \end{aligned} \tag{18}$$

We make use of a gamma function for (cM) and a beta function for ($c0$). These occur naturally in the KG dot products as integrals over an exponential phase from φ_k, x-powers from $r_k^{(0)}$, and $(x - c)$ powers from $r_k^{(c)}$.

To go after (cM) we start with a useful formula obtained by taming Fourier oscillations and doing a contour integral

$$\int_{-\infty}^{\infty} e^{ikx} x^b \mathrm{d}x \ = -\frac{2i}{k^{b+1}} e^{\frac{-\pi b}{2}} \Gamma(b+1) \tag{19}$$

to obtain the Bogoliubov transform

$$\begin{aligned} \alpha_{kq}^{(cM)} &= \left\langle \varphi_q, r_k^{(c)} \right\rangle &= \tfrac{1}{a\pi} \sqrt{\tfrac{\omega_k}{\omega_q}} \left(\tfrac{a}{q}\right)^{\frac{i\omega_k}{a}} e^{\frac{\pi\omega_k}{a}} \Gamma\left(\tfrac{i\omega_k}{a}\right) \\ \beta_{kq}^{(cM)} &= \left\langle \varphi_q^*, r_k^{(c)} \right\rangle &= \tfrac{1}{a\pi} \sqrt{\tfrac{\omega_k}{\omega_q}} \left(\tfrac{a}{q}\right)^{\frac{-i\omega_k}{a}} e^{\frac{-\pi\omega_k}{a}} \Gamma\left(\tfrac{-i\omega_k}{a}\right) \end{aligned} \tag{20}$$

We next consider products of shifted powers to go after $(c0)$. We make use of a beta function for $(c0)$ which occurs natrually in the KG dot products as integrals over powers of x and $x-c$, from $r_k^{(0)}$ and $r_k^{(c)}$ respecitvely

$$\int_c^{\infty} x^a (x-c)^b \mathrm{d}x \ = c^{a+b+1} B(b+1, -a-b-1) \tag{21}$$

and over sign choices $b, d \in \{-1, 1\}$

$$f_{k,b,d} = \left(a\left(b(t-i\epsilon)+x\right)\right)^{\frac{id\omega_k}{a}} \tag{22}$$

we have the dot product

$$\left\langle f_{k,b_k,d_k}, f_{q,d_q,d_q} \right\rangle = \frac{1}{2\pi} \sqrt{\frac{\omega_k}{\omega_q}} (ac)^{\frac{i(d_k\omega_k - d_q\omega_q)}{a}} \left((-d_k)\frac{b_k+b_q}{2}\right) B\left(\frac{id_k\omega_k}{a}, \frac{-i(d_k\omega_k - d_q\omega_q)}{a}\right) \tag{23}$$

from which we compute the Bogoliubov coefficients as

$$\begin{aligned} \alpha_{kq}^{(c0)} &= \left\langle r_q^{(0)}, r_k^{(c)} \right\rangle &= \tfrac{1}{2\pi a} \sqrt{\tfrac{\omega_k}{\omega_q}} (ac)^{\frac{-i(\omega_q-\omega_k)}{a}} B\left(\tfrac{i\omega_k}{a}, \tfrac{i(\omega_q-\omega_k)}{a}\right) \\ \beta_{kq}^{(c0)} &= \left\langle r_q^{(0)*}, r_k^{(c)} \right\rangle &= \tfrac{1}{2\pi a} \sqrt{\tfrac{\omega_k}{\omega_q}} (ac)^{\frac{-i(\omega_q+\omega_k)}{a}} B\left(\tfrac{-i\omega_k}{a}, \tfrac{i(\omega_q+\omega_k)}{a}\right) \end{aligned} \tag{24}$$

We can compare absolute magnitudes for M v.s. W_c and see that they don't depend on q or c

$$\begin{aligned} \left|\beta_{kq}^{(c_1M)}\right|^2 &= \left|\beta_{kq}^{(c_2M)}\right|^2 \\ \left|\beta_{kq}^{(c_1M)}\right|^2 &= \left|\beta_{kq}^{(c_2M)}\right|^2 \end{aligned} \tag{25}$$

and in fact

$$\left|\beta_{kq}^{(cM)}\right|^2 / \left|\alpha_{kq}^{(cM)}\right|^2 = e^{\frac{-2\pi\omega_k}{a}} \tag{26}$$

The c independence is expected since Unruh radiation is translation invariant. The q independence can be strengthened as the expected number of particles in mode k

$$\int \mathrm{d}q \ \left|\beta_{kq}^{(cM)}\right|^2 = \frac{e^{\frac{-2\pi\omega_k}{a}}}{2\sinh\frac{\pi\omega_k}{a}} \int \mathrm{d}q \ \frac{2}{a\pi|q|} \tag{27}$$

where we factor out the divergent part to recover the radiation equation again [3].

We next turn to W_c v.s. W_0 and also find c independence there

$$\begin{aligned} \left|\alpha_{kq}^{(c0_1)}\right| &= \left|\alpha_{kq}^{(c0_2)}\right| \\ \left|\beta_{kq}^{(c0_1)}\right| &= \left|\beta_{kq}^{(c0_2)}\right| \end{aligned} \tag{28}$$

[3]We frequently use here and elsewhere $|\Gamma(ib)|^2 = \frac{\pi}{b\sinh\pi b}$

$$
\begin{aligned}
\left|\beta_{kq}^{(c0)}\right|^2 / \left|\alpha_{kq}^{(c0)}\right|^2 &= \left|\Gamma\left(\frac{i(\omega_q+\omega_k)}{a}\right)\right|^2 / \left|\Gamma\left(\frac{i(\omega_q-\omega_k)}{a}\right)\right|^2 \\
&= \frac{(\omega_q-\omega_k)\sinh\pi(\omega_q-\omega_k)}{(\omega_q+\omega_k)\sinh\pi(\omega_q+\omega_k)}
\end{aligned}
\tag{29}
$$

which is somewhat more surprising since this implies that $\int \mathrm{d}q \left|\beta_{kq}^{(c_2c_1)}\right|^2$ is a c_1 and c_2 independent factor for every shifted wedge inclusion. In other words, the expected number of particles for a mode $r_k^{(c_2)}$ of W_{c_2} in W_{c_1}'s vacuum is independent of the choice of shift c_2 and c_1.

More explicitly we have a transform matrix of Λ_c from W_0 to W_c

$$
\begin{bmatrix} a_k^{(c)} \\ a_{-k}^{(c)} \\ \hline a_k^{(c)\dagger} \\ a_{-k}^{(c)\dagger} \end{bmatrix} = \underbrace{\left[\begin{array}{cc|cc} A_c & 0 & B_c & 0 \\ 0 & -A_c & 0 & -B_c \\ \hline \overline{B_c} & 0 & \overline{A_c} & 0 \\ 0 & -\overline{B_c} & 0 & -\overline{A_c} \end{array}\right]_{k,q}}_{\Lambda_c} \begin{bmatrix} a_q^{(0)} \\ a_{-q}^{(0)} \\ \hline a_q^{(0)\dagger} \\ a_{-q}^{(0)\dagger} \end{bmatrix}
\tag{30}
$$

where $A_c = \alpha_{kq}^{(c0)} = P_c A_1 P_c^{-1}$ and $B_c = \beta_{kq}^{(c0)} = P_c B_1 P_c$ for a diagonal phase factor matrix

$$
P_c = P_{c,rs} = \delta(r-s) c^{\frac{i\omega_r}{a}} = e^{\frac{iH}{a}\log c}
\tag{31}
$$

We can write Λ_c out compactly out as

$$
\Lambda_c = Q_c \Lambda_1 Q_c^{-1}
\tag{32}
$$

where

$$
Q_c = \begin{bmatrix} P_c, & 0 & 0 & 0 \\ 0 & P_c & 0 & 0 \\ 0 & 0 & P_c^{-1} & 0 \\ 0 & 0 & 0 & P_c^{-1} \end{bmatrix}
\tag{33}
$$

Note that $\lim_{c\to 0} \Lambda_c = 1$ since the limit of $\lim_{c\to 0} \alpha_{kq}^{(c0)} = 1$ and $\lim_{c\to 0} \beta_{kq}^{(c0)} = 0$, which corresponds nicely to $\lim_{c\to 0} W_c = W_0$. The composition of Bogoliubov transforms, $\Lambda_{nc} = \Lambda_c^n$, yields

$$
\begin{aligned}
Q_{nc}\Lambda_1 Q_{nc}^{-1} &= \Lambda_{nc} \\
&= (Q_c\Lambda_c Q_c)\left(Q_c^{-1}\Lambda_c Q_c\right)\cdots(Q_c\Lambda_c Q_c) \\
&= Q_c \Lambda_c^n Q_c^{-1}
\end{aligned}
\tag{34}
$$

so that

$$
\begin{aligned}
\Lambda_c^n &= Q_c^{-1} Q_{nc}\Lambda_1 Q_{nc}^{-1} Qc \\
&= Q_n \Lambda_1 Q_n^{-1}
\end{aligned}
\tag{35}
$$

and more generally we have a one parameter group given by

$$
\left\{\Lambda_0^x = Q_x \Lambda_0 Q_x^{-1} : x \in \mathbb{R}\right\}.
\tag{36}
$$

These Bogoliubov transformations between shifted wedges define a one-parameter group, reflecting an underlying symmetry structure. This naturally connects to modular flow as studied in algebraic QFT, where such transformations correspond to automorphisms generated by the modular operator. This is an explicit realization of modular flow, due to the von Neumann algebra modular automorphism associated with the translation $W_0 \to W_c$, studied in detail in Tomita-Takesaki theory [2]. There we find thermal KMS states between open set inclusions in a much more general setting.

Consider a sequence

$$
W_{c_n} \subseteq \cdots \subseteq W_{c_i} \subseteq \cdots \subseteq W_{c_j} \subseteq W_{c_2} \subseteq W_{c_1}
\tag{37}
$$

Then each $W_{c_i} \subseteq W_{c_j}$ involves particle production with a fixed squared magnitude for mode k. We calculate this expected number of W_{c_i} particles for mode k in W_{c_j}'s vacuum

$$\langle 0_{W_{c_j}} | \, a_k^{(c_i)\dagger} a_k^{(c_i)} \, | 0_{W_{c_j}} \rangle = \frac{1}{2\pi^2 k \sinh \frac{\pi k}{a}} \int_{x=0}^{\infty} \frac{x \sinh x}{(x + \frac{\pi k}{a}) \sinh(x + \frac{\pi k}{a})} \tag{38}$$

which diverges. The integrand goes to $e^{-\frac{\pi k}{a}}$ as x gets large, so we can see that the expected number of particles in ratio goes to

$$\frac{1}{m(e^{2m}+1)} = \frac{1}{k(e^{\frac{2\pi\omega_k}{a}} - 1)}. \tag{39}$$

3 Driving Sources

We now ask a fundamental question: **"What exactly is accelerating the observer?"** Until this point, we've treated acceleration as a coordinate choice, without invoking any underlying physical mechanism. We have also not specified the observer's precise location within the Rindler wedge, nor the spatial origin of the observed excitations. These ambiguities reflect the effective coarse-graining over the observer's details, a feature that contributes to the thermal character of the Unruh effect.

Figure 3 illustrates the situation for a sharply peaked frequency wave packet made of Rindler modes. The modes r_k are left-moving, propagating toward the future horizon and are interpreted as **emission**. The r_{-k} modes are right-moving, originating from the past horizon and are interpreted as **absorption**. The Rindler modes are constructed as superpositions of Minkowski modes φ_q, effectively smeared over a range of frequencies. This is visually evident in Figure 2, where the local frequencies increase (blue-shift) near the horizons. This is made explicitly by the Bogoliubov coefficients $\alpha_{kq}^{(cM)}$ and $\beta_{kq}^{(cM)}$ which encode the Fourier decomposition of the Rindler modes through their Klein-Gordon inner products with the Minkowski modes φ_q and φ_q^*, respectively. This frequency delocalization, tied to the observer's acceleration horizon, underlies the apparent thermal character of the radiation.

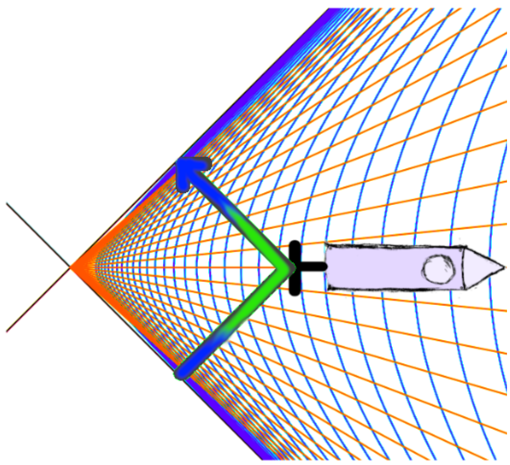

Figure 3: A Rindler mode's frequency is smeared out in Minkowski space, blue-shifted near the horizon. We diagram a particle as if it were striking a mirror at the rear of a rocket, where its reflection emerges as a combination of emission and absorption processes in the Rindler frame.

To address this, we introduce a driving source, a physical mechanism responsible for the field's excitation and, indirectly, for the observer's acceleration. This reframes the interpretation: the radiation is not spontaneous but instead emerges as a coherent response to the source. The apparent thermality, then, is tied to our ignorance of the source's detailed structure.

We aim to encode the effect of a creation operator by introducing a source term $J(x)$ into the Lagrangian at some distant time in the past, which directly excites the field in a specific mode. Since $J(x)$ couples linearly, it prepares a coherent state that excites the chosen mode in a controlled, phase-coherent manner. To replicate the action of a creation operator, the source must be engineered such that its overlap with the mode functions $u_k(x)$ matches the operator's action on the field.

The field can be expanded as in equation (16), and the β_k-term in equation (17) is responsible for the thermal particle content of the Minkowski vacuum as seen by Rindler observers. Without loss of generality[4], we encode the effect of a creation operator $c_k^{L\dagger}$ using

$$c_k^{L\dagger} = \langle \phi, \mu_k^{L*} \rangle_{KG} = \int \mathrm{d}x \ \mu_k^{L*}(x)\phi(x) \tag{40}$$

and the orthogonality of the mode functions in the Klein-Gordon inner product. In the generating functional formalism, setting $J_k^L(x) = -\beta_k u_k^{L*}$ the functional derivative $\frac{\delta}{\delta J_k^L} Z[J_k^L]|_{J_k^L=0}$ inserts ϕ into time-ordered correlators. Smearing this field insertion against $\mu_k^{L*}(x)$ thus projects onto $c_k^{L\dagger}$ and we have[5]

$$\begin{aligned} a_k^{(0)J} &= \alpha_k c_q^R + \beta_k c_q^{L\dagger} - \beta_k c_q^{L\dagger} \\ &= \alpha_k c_q^R \end{aligned} \tag{41}$$

In Rindler coordinates, this source term prepares a modified field state in which the Rindler mode occupation differs from the thermal distribution of the Minkowski vacuum. Rather than simply adding energy, the source introduces a coherent excitation that cancels the mode structure induced by the Bogoliubov β-terms, effectively replacing their contribution. This lets us construct a state where the Rindler response is vacuum-like for mode k

$$\langle J_k^L | \, b_k^\dagger b_k \, | J_k^L \rangle = \langle 0_M | \, b_k^{J_k^L \dagger} b_k^{J_k^L} \, | 0_M \rangle = 0 \tag{42}$$

4 Localization

4.1 Modular Automorphism

Consider the two nested Rindler wedges W_0 and W_c, with $W_c \subseteq W_0$ as shown in Figure 4. Let r_q denote a positive-frequency Rindler mode associated with W_0, analytically continued to the entire Minkowski space. The grayscale region illustrates the full support of r_q, while the rainbow segment shows the restriction of this mode to the smaller wedge W_c.

By considering the restriction of r_q to W_c, we have partially localized the observer and the excitation. The restriction effectively cuts off the high-frequency content of r_q near the future horizon[6]. The resulting mode still spans the full spatial extent of W_c, but it is now insulated from the highly oscillatory behavior near the horizons of W_0. The localization is not complete however, the observer can still be anywhere within the wedge W_c, and there is still some thermal character of r_k to contend with and lower frequency oscillations of the modes as well.

[4]We only consider a fixed frequency and the L-mode, and extend the argument linearly to any positive frequency Minkowski modes.

[5]The remaining α_k factor reflects the mismatch between the squeezed Unruh vacuum and the coherent state prepared by the source.

[6]Similarly, r_{-q} experiences suppression near the past horizon.

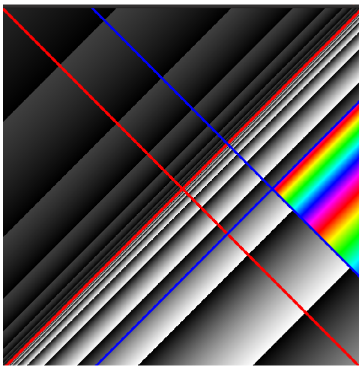

Figure 4: A Wedge W_c (blue) inside of the wedge W_0 (red). Rindler mode r_q of W_0 (grayscale) restricted to W_c (rainbow).

To further study the situation, consider the modulus squared dot product $\left|\left\langle r_q^{(0)}, r_k^{(c)}\right\rangle\right|^2$, also known as $\left|\alpha_{kq}^{(c0)}\right|$, from equation (24). We fix q and find that

$$\left|\left\langle r_q^{(0)}, r_k^{(c)}\right\rangle\right|^2 = \frac{\sinh\frac{\pi\omega_q}{a}}{4\pi a(\omega_q-\omega_k)\sinh\frac{\omega_q-\omega_k}{a}\sinh\frac{\pi\omega_k}{a}} \tag{43}$$

See Figure 5, where we now find a peaked response at $\omega_k = \omega_q$. We still find the thermal term from before, $\sinh\frac{\pi\omega_k}{a}$, contributing a spread near $\omega_k = 0$ and the thermal spread of $(\omega_q-\omega_k)\sinh\frac{\omega_q-\omega_k}{a}$ near the peak at $\omega_k = \omega_q$; but the large scale peaked response itself is due to the localization.

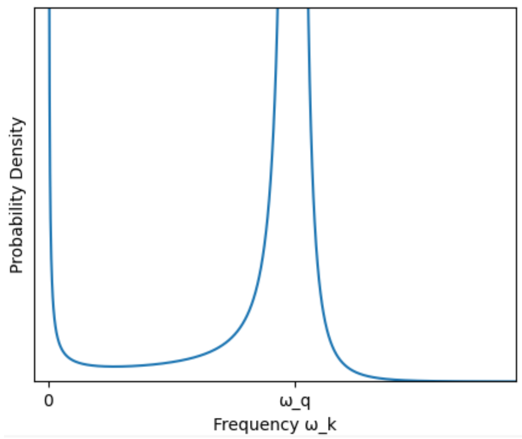

Figure 5: The Rindler modes r_k of W_c have peaked spectral response to r_q at $\omega_k = \omega_q$.

4.2 Wave Packet Interpolation

5 Conclusion and Prediction

If the thrust required to accelerate a detector is not explicitly accounted for, it manifests instead as an apparent thermal feature of the vacuum—Unruh radiation. However, as demonstrated in this paper, Unruh radiation can be directly explained as a consequence of thrust. This perspective leads to the prediction that neither Unruh radiation nor Hawking-Bekenstein radiation should appear independently of the thrust that drives the system.

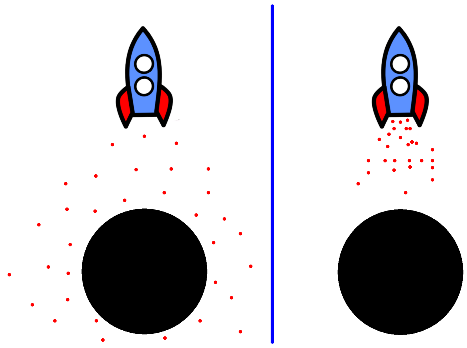

Figure 6: Hawking picture of black hole radiating on the left. Our picture of a rocket thrusting on the right.

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