Field Theoretic Perspective on Thrust of an Accelerating Frame

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> The Unruh effect tells us that what we call particles is really just a matter of perspective.
>
> Lee Smolin

Abstract

We analyze the quantum field theoretic framework where Unruh radiation was first described, interpreting acceleration as a driving source influencing the field. By conceptualizing this source as thrust, we provide a more intuitive mechanism for understanding radiation in accelerating frames. Using the equivalence principle, we extend this framework to Hawking radiation as well, demonstrating how a common source-response relationship manifests in accelerating frames.

1 Introduction

Buchholz and Verch [1] explain Unruh radiation as a thermal bath where an account is given

> ...the increase of temperature of accelerated thermometers is due to systematic quantum effects induced by the local coupling between the thermometer and the vacuum. This coupling inevitably creates excitations of the vacuum which transfer energy to the thermometer, gained by the acceleration, and thereby affect its readings.

Frodden and Valdés [2] further state that

> ...the interpretation of the Unruh effect as a thermal bath state for accelerated observers is controversial.

We present an argument that Unruh and Hawking radiation can be interpreted as a coupling to thrust particles instead of the vacuum. That the particles predicted for an accelerated observer are due to a driving force instead of thermal radiation, see Figures 5 and 6. In this note we start with thrust as a driving force and show how it can instead account for the transfer of energy to a detector. We pick the driving source to give the same distribution of energy as the radiation, but additionally show that more arbitrary driving sources can also be used.

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In Section 2, we present the preliminaries, following [2], with a review of Rindler coordinates, wave equations, and their solutions in 1+1-dimensional Minkowski and Rindler spacetimes. Section 3 introduces the concept of field-theoretic driving forces, drawing on [3], and extends the formalism by employing Fourier transforms to analyze sources as a combination of emission and absorption terms. In Section 4, we reinterpret Unruh radiation as thrust, providing a novel perspective on this phenomenon. Section 5 explores other types of driving sources, with Section 6 focusing specifically on the Ricci scalar as a driving source. Section 7 delves into the implications for interacting fields, and Section 8 concludes by examining Hawking radiation through the lens of the equivalence principle.

Much of the content, including notation and conventions, is directly adapted from the overview in [2], the foundational work in [4], and the detailed treatment in [3]. Consistent with these references, our focus is restricted to the free scalar field, which suffices to elucidate the Unruh effect.

2 Rindler and Minkowski Modes Review

Let $\hbar = c = 1$ and consider a uniform linear acceleration in $1+1$ dimensional spacetime. The full $1+3$ dimensional case does not add anything to the discussion, so without loss of generality we stick to the dimensions time t and space x where the boost is taking place. The massless Klein-Gordon equation

$$\Box\phi = \frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = 0, \tag{1}$$

has solutions which we describe using a basis $u = -t + x$ and $v = -t - x$, the null coordinates along the light cone. Any function of purely u, $f(u)$, is made up of modes that are constant on v. That is they are given by a 1-dimensional Fourier expansion

$$f(u) = \frac{1}{2\pi}\int e^{ip_u u}\hat{f}(p_u)dp_u. \tag{2}$$

supported on $p_v = 0$. The same is true for v, $g(v)$, p_v, and $\hat{g}(v)$ supported on $p_u = 0$. The full space of solutions to equation (1) are combinations of f and g, linear combinations of $\hat{f}(p_u)\delta(p_v)$ and $\hat{g}(p_v)\delta(p_u)$. These are supported on the massless shell $E = \pm p$. A basis of solutions are the "Minkowski modes"

$$\varphi_k(x,t) = \frac{1}{\sqrt{4\pi\omega_k}}e^{i(kx-\omega_k t)} \tag{3}$$

along with their complex conjugates, where $\omega^2 = k^2$, covering both the positive and negative frequency cases $\omega = \pm k$.

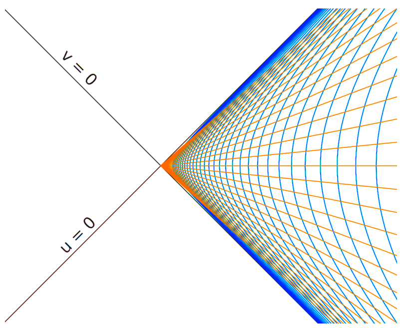

Figure 1: Rindler wedge W on the right.

The following notations and results for the rest of this section are taken from [2]. Let W be the $x > |t|$ Rindler "wedge" with coordinates

$$t = \frac{1}{a}e^{a\xi}\sinh(a\eta) \tag{4}$$

$$x = \frac{1}{a} e^{a\xi} \cosh(a\eta) \tag{5}$$

where $a > 0$ is an acceleration parameter (see Figure 1). These are hyperbolic polar coordinates with an exponential "radius". The massless Klein-Gordon equation in Rindler coordinates is

$$\Box\phi = e^{-2a\xi}(-\partial_\eta^2 + \partial_\xi^2)\phi = 0 \tag{6}$$

which has a basis of solutions

$$r_k(\eta, \xi) = \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k \eta - k\xi)} \tag{7}$$

(along with conjugates) for each wave number k and positive frequency $\omega_k = |k|$. These "Rindler modes" do not immediately extend to the entire (x, t)-plane but are confined to the Rindler wedge W.

Consider the analytic functions on the entire (x, t)-plane

$$f_k(u) = \frac{e^{\frac{\pi\omega_k}{2a}} (au)^{\frac{i\omega_k}{a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} \tag{8}$$

$$g_k(v) = \frac{e^{\frac{\pi\omega_k}{2a}} (av)^{\frac{i\omega_k}{a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} \tag{9}$$

These two functions are functions of u and v respectively, and so are solutions to equation (1). For $z = u$ or $z = v$ there is a choice of branch of the log

$$\begin{aligned} (-az)^{\frac{i\omega_k}{a}} &= e^{\frac{i\omega_k}{a}(\log(-1)+\log(az))} \\ &= e^{\frac{\pi\omega_k}{a}} (az)^{\frac{i\omega_k}{a}} \end{aligned} \tag{10}$$

where we choose $\log(-1) = -i\pi$ following [2]. Each r_k with can be extended to the entire plane as

$$r_k = \begin{cases} (au)^{\frac{i\omega_k}{a}} & \text{if } k > 0 \\ (av)^{\frac{i\omega_k}{a}} & \text{if } k < 0 \end{cases} \tag{11}$$

see Figure 2.

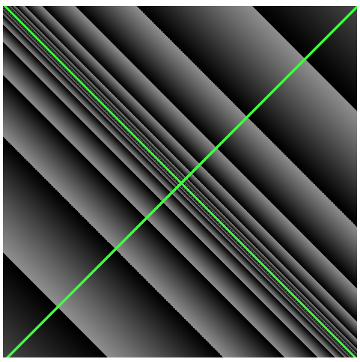

Figure 2: Phase of a Rindler mode r_k with $k < 0$ analyticlly extended to the whole x/t or u/v plane as $(av)^{\frac{i\omega_k}{a}}$.

Let $c_k^{(1)}$ and $c_k^{(2)}$ be annihilation and creation operators for the positive and negative frequency Minkowski modes respectively. Define

$$b_k = \frac{1}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} \left(e^{\frac{\pi\omega_k}{2a}} c_k^{(1)} + e^{-\frac{\pi\omega_k}{2a}} c_{-k}^{(2)\dagger} \right). \tag{12}$$

Then the number of particles that an accelerating observer will see in the Minkowski vacuum $|0_M\rangle$ is seen to be

$$\langle n\rangle = \langle 0_M|\, b_k^\dagger b_k \,|0_M\rangle = \frac{1}{e^{\frac{2\pi\omega_k}{a}} - 1} \tag{13}$$

which is usually interpreted as (Unruh) radiation, for instance:

> An accelerated observer measures her surrounding fields in excited thermal states. A simple thermometer carried by the observer marks a temperature directly proportional to the acceleration she is moving with. The more she accelerates, the higher the temperature.

We take the view that the thermometer is accelerated by thrust. This is a different interpretation of the apparent radiation. To make this explicit, we formalize thrust as field theoretic driving forces.

3 Fourier Transform of the Sources

We will take several Fourier transforms to study the various modes and set up for some integrals. Let ϕ be a free scalar field in the flat $1+1$ dimensional Minkowski spacetime. We will consider $f_k(u)$ and $g_k(v)$ as driving W-event horizon sources

$$\rho(u, v) = \alpha f_k(u) + \beta g_k(v). \tag{14}$$

with non-negative real convex combination $\alpha + \beta = 1$. The drivers $f_k(u)$ and $g_k(v)$ are functions $f_k(u)$ of the past W-horizon and $g_k(v)$ of the future W-horizon respectively. Both generate excitations, which we identify with absorption and emission thrusts respectively.

The sources can originate from a coupling term, $\rho\phi$, added to the free scalar Lagrangian

$$\mathcal{L}_{driven} = \mathcal{L}_{free} + \rho\phi \tag{15}$$

where

$$\mathcal{L}_{free} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi. \tag{16}$$

This leads to an inhomogeneous Klein-Gordon equation

$$\Box\phi = \rho \tag{17}$$

as presented in [3][1]

We want to integrate ρ on shell in momentum space, which for a massless source is the positive energy part of the massless shell. The two positive energy "horizons" border $p_u \le 0$ and $p_v \le 0$, see Figure 3. Proceeding to take the Fourier transform of the function $f(u)^2$, we drop p_u to just p for the time being to increase legibility. We will continue to assume that ω_k and a are positive. Define the kernel

$$A = e^{-ipu}(au)^{\frac{i\omega_k}{a}}\, du \tag{18}$$

and then we have

$$\hat{f}({}_u) = \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} \int_{-\infty}^{\infty} A \tag{19}$$

where $L = \int_{-\infty}^{0} A$ and $R = \int_{0}^{\infty} A$ are the left and right sides of the total integral $L + R$.

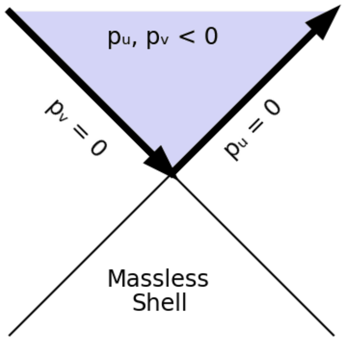

Figure 3: Massless shell is when $p_u = p_v = 0$

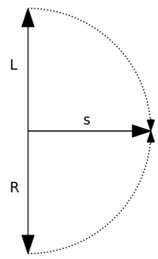

Figure 4: Using contours with large radius we convert the L integral that goes to $i\infty$, and the R integral that goes to $-i\infty$, to integrals with s going to real ∞.

We rewrite the integrals using a complex changes of variables, $s = ipu$, and contour integrals.

The L integral for real $p < 0$ is

$$\begin{aligned} L(p) &= -\int_0^{i\infty} \left(\frac{ias}{-p}\right)^{\frac{i\omega_k}{a}} \left(\frac{i}{-p}\right) ds \\ &= \frac{-i}{a} \left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \int_0^{\infty} (is)^{\frac{i\omega_k}{a}} e^{-s} ds \\ &= \frac{-i}{a} \left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} \\ &= \frac{1}{2}\Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} B(p) \end{aligned} \tag{20}$$

where

$$B(p) = \frac{-2i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \tag{21}$$

is as shown. This is using a large radius contour which rotates the endpoint 90 degrees clockwise.

The same calculation for R is done using a counter-clockwise contour this time.

$$R(p) = \frac{1}{2}\Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} B(p) \tag{22}$$

[1] In [3] it is assumed that the source is only active for a finite amount of time. We let ρ be active for all time. The argument in [3] seem to be adaptable to ρ.

[2] WLOG since $g(v)$ is of the same form.

We get back to $f_k(u)$ and apply the normalization from equation (19)

$$\widehat{f}_k(p_u) = \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}(L(p_u) + R(p_u)) \tag{23}$$

So

$$\begin{aligned} \widehat{f}_k(p_u) &= \frac{\Gamma\left(\frac{i\omega_k}{a}+1\right)}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}B(p_u) \\ \widehat{g}_k(p_v) &= \frac{\Gamma\left(\frac{i\omega_k}{a}+1\right)}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}B(p_u) \end{aligned} \tag{24}$$

4 Interpretation

The driving source ρ, with mixed absorption and emission thrusts $\alpha + \beta = 1$, contribute excitations to the scalar field ϕ. Equations (14) and (24) let us write down the expected change of energy

$$\begin{aligned} \mathbb{E}[\Delta E] &= \frac{1}{4\pi}\int |\rho(p)|^2 dp \\ &= \frac{\alpha}{4\pi}\int \left|\widehat{f}_k(p_u)\right|^2 dp_u + \frac{\beta}{4\pi}\int |\widehat{g}_k(p_v)|^2 dp_v \\ &= \frac{\left|\Gamma\left(\frac{i\omega_k}{a}+1\right)\right|^2}{2\sinh\left(\frac{\pi\omega_k}{a}\right)}\frac{1}{4\pi}\int |B(p)|^2 dp \\ &= \frac{\left|\Gamma\left(\frac{i\omega_k}{a}+1\right)\right|^2}{2\pi a\sinh\left(\frac{\pi\omega_k}{a}\right)}\int a/|p|^2 dp \\ &= I(\omega_k)P \end{aligned} \tag{25}$$

where the integrals are on the positive energy massless shell with contributions from p_u on the left piece and p_v on the right piece. We factored out $P = \int a/|p|^2$ with a remaining p independent positive real coefficient $I(\omega_k)$.

Without being more careful we end up with inferred problems — The integrals do not converge at zero, where P explodes. But this infinity cancels when we compare the spectral radiances to each other, $I(\omega_{k_1})/I(\omega_{k_2})$.

The magnitude of our Gamma function has known asymptotics [5, Eq. 5.11.9]

$$\left|\Gamma\left(\frac{i\omega_k}{a}+1\right)\right|^2 \sim \left(\frac{2\pi\omega_k}{a}\right)e^{-\frac{\pi\omega_k}{a}} \tag{26}$$

Plugging this into equation (25) we find the average energy of the mode, the 1 + 1 dimensional Planck distribution function, and thus recover the Unruh's radiation spectrum from a thrust driven field.

$$\frac{1}{P}\mathbb{E}[\Delta E] = I(\omega_k) \sim \frac{\omega_k}{e^{\frac{2\pi\omega_k}{a}} - 1} \tag{27}$$

Compare this to equation (13) and references [4] and [2].

5 Other Types of Driving Sources

TODO

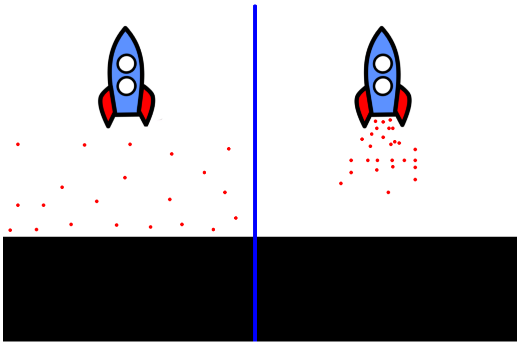

Figure 5: Unruh picture of an event horizon radiating in accelerating frame radiating on the left. Our picture of a rocket under inertial acceleration thrusting on the right.

6 Ricci Scalar as a Driving Source

TODO

7 Fields with Interaction

TODO

8 Conclusion and Prediction

If the thrust required to accelerate a detector is not explicitly accounted for, it manifests instead as an apparent thermal feature of the vacuum—Unruh radiation. However, as demonstrated in this paper, Unruh radiation can be directly explained as a consequence of thrust. This perspective leads to the prediction that neither Unruh radiation nor Hawking-Bekenstein radiation should appear independently of the thrust that drives the system.

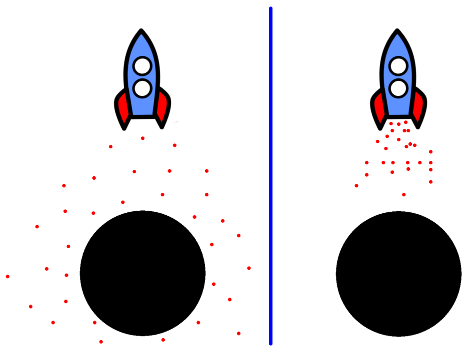

Figure 6: Hawking picture of black hole radiating on the left. Our picture of a rocket thrusting on the right.

9 Acknowledgments

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