The Unruh Effect as Holographic Thrust

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Abstract

We outline the holographic scenario in [**?**] and additionally consider the dynamics of the holographic screen itself in an accelerating frame. We consider the case where the screen's acceleration is due to mass/energy joining with the screen, increasing the entropy of the screen. Finally, we consider the field theoretic picture. On both cases, we derive the Unruh effect[**?**] using thrust.

1 The Holographic Scenario

We first recall the setup outlined in [**?**]. Consider a holographic screen with entropy S. Let there be a test particle which is a single wavelength Δx away from S. Following Bekenstein, we can regard the particle to be simultanously near and on the screen.

Let the screen have energy E_S and mass M_S. Suppose that the holographic screen is being propelled by a stream of photons in place of the test particle. We want to see how the dynamics of the photons affects the dynamics and information carried by the screen.

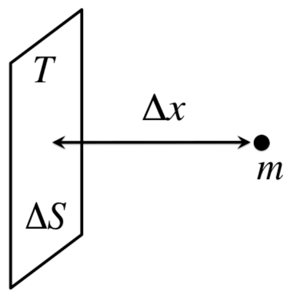

Figure 1: This figure was copied from [**?**]. A test particle with mass m and Compton wavelength Δx is being absorbed into the holographic screen. In this note, we replace the test particle with a single photon.

There are some complementary notions of "emergence" that we inherit from [**?**]. A third notion, item 3, that we supply here is the time-reversal of matter/energy emerging from the screen.

1. The usual thermodynamic emergence of macrostate variables (such as temperature) from microstate variables (energies of individual particles). This includes the emergence of Newton's laws and entropic gravity.

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2. The emergence of "new" space-time beyond holographic screens, treating screens like stretched horizons. The existence of a foliation of screens where test particles move from screen to screen.
3. Known matter/energy in front of the screen passes to unknown matter/energy on the screen. The matter/energy in front of the screen has known information. When passing into the screen, the information is thermalized, and becomes unknown.

The information theoretic notions of emergence will be discussed in more detail in terms of Landauer's language in the next section.

2 Landauer Limit

We want to keep track of the information corresponding to degrees of freedom on the screen. It is interesting to compare this situation with the limiting case of Landauer [**?**]. The situation is identical, the holographic screen in his case is the separation between the system and the heat bath (between the known and the unknown). Forgetting (or learning) information is the same thing as letting the information "join the screen".

We pause to note that there is nothing canonical about choosing a bit to express this information. For instance, Landauer considered logical gates and as an example, we briefly consider the AND gate on two unbiased bits of input. The output is a bit, but it is biased, and so it carries less than one bit of information. The AND gate actually forgets more than a bit[1]. All of this is to say that the choice of ΔS is far from obvious.

Consider the case where a single photon of energy E_p is joining the screen. The change of information, per photon, will be a constant. It is not given by $ln(2)k$, from a bit, but by another number

$$\Delta S = 4\pi^2 k.$$

There is nothing mysterious here, we pick this constant so that we will match the constant in Unruh radiation. All of the dimensional analysis will also be seen to match up and be motivated in the next section. We leave in universal constants mainly for context.

3 Information Theoretic Unruh Effect

We will derive the Unruh temperature equation from the information change of an accelerating screen using the thrust of photons. For a single photon, consider the Planck relation for the energy and frequency of the screen

$$E_S = hf_S = \frac{h}{\Delta t}$$

where Δt is the period of S. Let a be the acceleration due to the photon which we also write as $a = \Delta v/\Delta t$. Let p be the momentum of the photon, E_p its energy, and we equate the momenta $p_S = p_p = p$. Then

$$aM_S\Delta t = M_S\Delta v = p = E_p/c$$

and setting a temperature T, with respect to E_p, we get

$$\frac{4\pi^2 k}{h}E_S = \frac{\Delta S}{\Delta t} = \frac{E_p}{T\Delta t} = \frac{caM_S}{T} = \frac{aE_S}{cT}$$

and canceling E_S we derive the Unruh temperature [**?**]

$$T = \frac{h}{4\pi^2 kc}a$$

[1] It actually forgets around 1.19 bits

4 Hawking-Bekenstein Radiation

The equivalence principle allows us to derive a temperature for acceleration due to gravity near the event horizon of a black hole. This corresponds to a reference frame which continues to thrust away from a black hole maintaining a given height. Much like the Unruh radiation, we find this thrust accounts for Hawking-Bekenstien radiation.

5 Field Theoretic Considerations

5.1 Rindler and Minkowski Modes Review

Consider a two dimensional spacetime[2] (t, z) with "event horizon" coordinates $u = -t + z$ and $v = -t - z$. Any function of purely u, $f(u)$, is made up of Minkowski modes that are constant on v. That is they are given by a one dimensional Fourier expansion

$$f(u) = \frac{1}{2\pi} \int e^{ip_u u} \hat{f}(p_u) dp_u.$$

The same is true for v, $g(v)$, p_v, and $\hat{g}(v)$. The full two dimensional transform of combinations of f and g is a linear combination of $\hat{f}(p_u)\delta(p_v)$ and $\hat{g}(p_v)\delta(p_u)$. These are supported on the massless shell — the light cone.

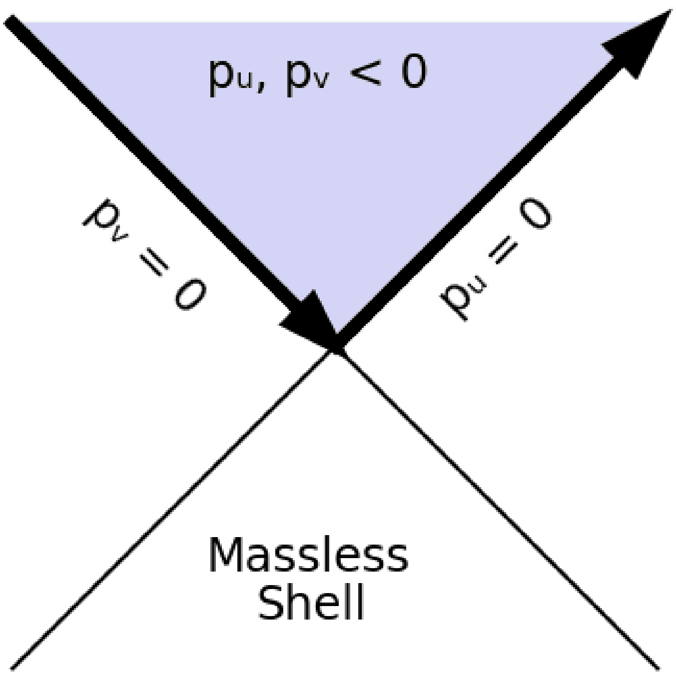

Figure 2

Let W be the $z > |t|$ Rindler wedge with coordinates

$$t = \frac{1}{a} e^{a\xi} \sinh(a\eta)$$

$$z = \frac{1}{a} e^{a\xi} \cosh(a\eta)$$

where $a > 0$ is an acceleration parameter (see Figure rindlerw).

[2]it is not informative to consider the full 4 dimensional case, so we stick to the dimensions where the linear acceleration boosts are taking place t and z.

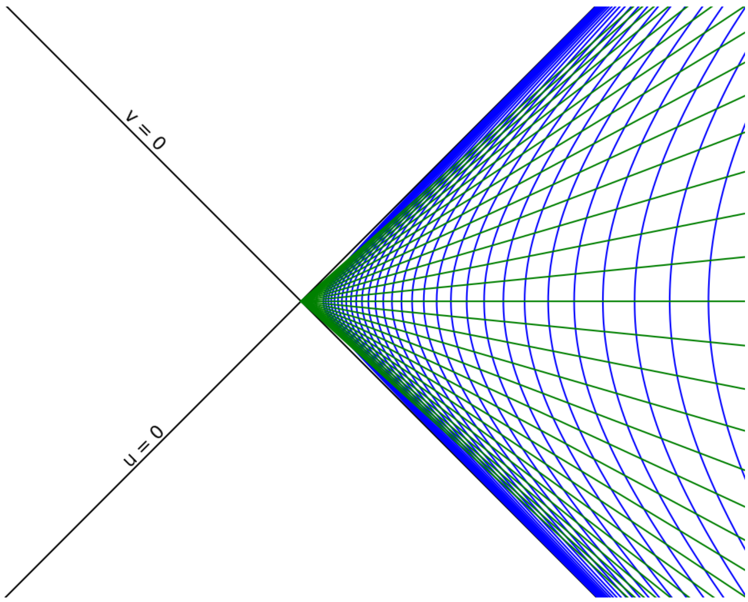

Figure 3: Rindler wedge W on the right.

For wave number k and positive frequency ω_k consider the functions

$$h_k^{(u)} = \frac{e^{\frac{\pi\omega_k}{2a}}(au)^{\frac{i\omega_k}{a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}$$

$$h_k^{(v)} = \frac{e^{\frac{\pi\omega_k}{2a}}(av)^{\frac{i\omega_k}{a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}$$

The scalar field in Rindler curved space has modes

$$r_k = \frac{1}{\sqrt{4\pi\omega_k}}e^{-i(\omega_k\eta - k\xi)}$$

which can be writen as combinations of the "event horizon" modes x

$$e^{\frac{\pi\omega_k}{2a}}h_k^{(u)} - e^{-\frac{\pi\omega_k}{2a}}h_k^{(v)} = \sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}r_k$$

Unlike the Rindler mode, the event horizon modes extend analytically from W to all of spacetime.

When we switch between h_k and the usual Minkowski bases we find that the Minkowski vacuum has excitations coming from the Rindler modes. More specifically, there is a positive particle number vacuum expectation of

$$\frac{1}{e^{\frac{2\pi\omega_k}{a}} - 1} \tag{1}$$

which is interpreted as (Unruh) radiation.

5.2 Fourier Transform of the Sources

Let ϕ be a scalar field in the flat 2 dimensional Minkowski spacetime[3]. We will consider $h_k^{(u)}$ and $h_k^{(u)}$ as driving W-event horizon sources

$$\rho = \alpha h_k^{(u)} + \beta h_k^{(v)}.$$

[3]Note that we only consider the scalar case since generalization to other fields is straightforward and does not add any intution to the presentation.

with positive real coeffienccts $(\alpha + \beta) = 1$. The drivers $h_k^{(u)}$ and $h_k^{(v)}$ are functions $f(u)$ of the past W-horizon and $g(v)$ of the future W-horizon respectively. Both generate excitations, which we identify with absorption and emission thrusts respectively.

The source can originate from a coupling term, $\rho\phi$, added to the Lagrangian.

$$\mathcal{L}_{driven} = \mathcal{L}_{original} + \rho\phi$$

This leads to an inhomogenous Klein-Gordon equation

$$(\Box + m^2)\phi = \rho$$

[?]. It is usually assumed that the source will only be active for some finite amount of time. We delay ignore this issue and instead go after the general asymptotic form of our solution in Fourier space.

We want to integrate ρ on shell in momentum space, which for a massless source is the positive time part of the light cone. The two positive time horizons border $p_u <= 0$ and $p_v <= 0$, see Figure 2. Proceeding to take the Fourier transform of the function $f(u)^4$, p_u drops to just p for the time being to increase legibility. We will continue to assume that w_k and a are positive. Let

$$A = e^{-ipu}(au)^{\frac{i\omega_k}{a}}du$$

and then we have

$$\hat{f}(\rho_U) = \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}\int_{-\infty}^{\infty} A \tag{2}$$

where $L = \int_{-\infty}^{0} A$ and $R = \int_{0}^{\infty} A$ are the left and right sides of the total integral $I = L+R$.

We rewrite the integrals using a complex changes of variables, $s = ipu$, and contour integrals.

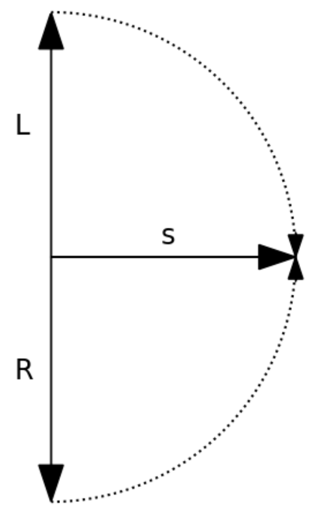

Figure 4: Using contours with large radius we convert the L integral that goes to $i\infty$, and the R integral that goes to $-i\infty$, to integrals with s going to real ∞.

[4] WLOG since $g(v)$ is of the same form.

The L integral for real $p < 0$ is

$$\begin{aligned}
L(p) &= -\int_0^{i\infty} \left(\frac{ias}{-p}\right)^{\frac{i\omega_k}{a}} \left(\frac{i}{-p}\right) ds \\
&= \frac{-i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \int_0^{\infty} (is)^{\frac{i\omega_k}{a}} e^{-s} ds \\
&= \frac{-i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1} \Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} \\
&= \frac{1}{2}\Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} B(p)
\end{aligned}$$

where

$$B(p) = \frac{-2i}{a}\left(\frac{a}{-p}\right)^{\frac{i\omega_k}{a}+1}$$

is as shown. This is using a large radius contour which rotates the endpoint 90 degrees clockwise.

The same calculation for R is done using a counter-clockwise countour this time.

$$R(p) = \frac{1}{2}\Gamma\left(\frac{i\omega_k}{a}+1\right) e^{-\frac{\pi\omega_k}{2a}} B(p)$$

We get back to $h_k^{(u)}$ and apply the normalization from (2)

$$\hat{h_k}^{(u)}(p_u) = \frac{e^{\frac{\pi\omega_k}{2a}}}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}}(L(p_u) + R(p_u))$$

So

$$\begin{aligned}
\hat{h_k}^{(u)}(p_u) &= \frac{\Gamma\left(\frac{i\omega_k}{a}+1\right)}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} B(p_u) \\
\hat{h_k}^{(u)}(p_v) &= \frac{\Gamma\left(\frac{i\omega_k}{a}+1\right)}{\sqrt{2\sinh\left(\frac{\pi\omega_k}{a}\right)}} B(p_u)
\end{aligned} \tag{3}$$

5.3 Interpretation

The driving source ρ contributes excitatons to the scalar field ϕ. The expected number of new particles is

$$\mathbb{E}[\Delta N] = \langle\rho|\rho\rangle = \int \frac{|\rho(p)|^2}{4\pi|p|} dp \tag{4}$$

where the integral is on the positive time light cone with contibutions from p_u on the left piece and p_v on the right piece.

Without being more careful about the finite time and shape of the source we end up with inferred problems. The integral does not converge at zero where $\int 1/p^3$ explodes. But we can still understand the high frequency asymptotic spectral radiance $I(\omega_k)$ by factoring out the common divergant p dependent part of (4), using (3), and taking a squared magnitude.

$$|I(\omega_k)| \sim \frac{\left|\Gamma\left(\frac{i\omega_k}{a}+1\right)\right|^2}{\sinh\left(\frac{\pi\omega_k}{a}\right)}$$

The squared magnitude of the Gamma function $\left|\Gamma\left(\frac{i\omega_k}{a}+1\right)\right|^2$ is dominated by $e^{-\frac{\pi\omega_k}{a}}$ for large ω_k, see [?]. So we recover the Unruh radiation spectrum (1)

$$\mathbb{E}[\Delta N] \sim I(\omega_k) \sim \frac{1}{e^{\frac{2\pi\omega_k}{a}}-1}$$

from a thrust driven field.

6 Prediction

If we do not account for the thrust required to accelerate a detector, then we recover it instead as a thermal unknown in the vacuum. It would seem that we can explain Unruh radiation directly using thrust. The prediction of this note is that Unruh radiation, separate from thrust, should not appear.